Complete Next-to-Next-to-Leading-Order QCD Correction to $J/\psi \to 3\gamma$ Decay

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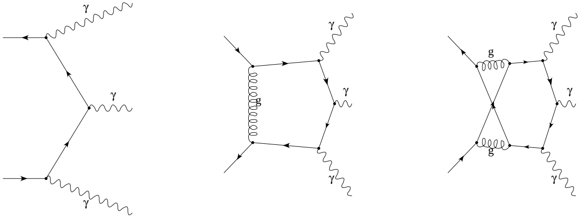
Introduction

- $J/\psi \to 3\gamma$: pristine probe for testing perturbative QCD within NRQCD factorization
- Experimental progress:
  - CLEO-c (2008): $\mathrm{Br}(J/\psi \to 3\gamma) = (1.2 \pm 0.3 \pm 0.2) \times 10^{-5}$
  - BESIII (2013): $\mathrm{Br}(J/\psi \to 3\gamma) = (11.3 \pm 1.8 \pm 2.0) \times 10^{-6}$
- Theoretical challenge (perturbative expansion in α_s and v^2):
  - LO NRQCD prediction exceeds experimental values
  - NLO QCD correction (1981, Caswell, Lepage, Sapirstein): large and negative
  - $\mathcal{O}(v^2)$ relativistic correction (1983, Keung, Muzinich): also large and negative
  - $e^+e^- \to 2J/\psi$, NLO (2008, Gong, Wang), NNLO(Sang, et al; Huang,et al), negtive,
  - $\Rightarrow$ "Sign crisis": non-physical negative rates
  - $J/\psi \to 3\gamma$, renormalo, Borel transform, Large -N_f, (1999, Bodwin, Chen) convergence?
  - $\mathcal{O}(\alpha_s v^2)$ joint correction (2012, Feng, Jia, Sang): large and positive, resolved the crisis
  - lattice QCD gave $Br(J/\psi \to 3\gamma) = (2.13 \pm 0.14 \pm 0.29) \times 10^{-5}$ (2019,Meng, Liu, Zhang)
- This work: First complete NNLO QCD correction to $J/\psi \to 3\gamma$
  - Propose amplitude-level NRQCD factorization as systematic prescription
  - Markedly improved agreement with BESIII data

Traditional Expansion at NNLO: the Problem



Typical Feynman diagrams for LO, NLO, NNLO

Traditional (amplitude-square level) perturbative expansion result:

$$\Gamma = \Gamma_{LO}\left(1 - 12.6\frac{\alpha_s}{\pi} - 8.23\langle v^2\rangle + 68.9\frac{\alpha_s}{\pi}\langle v^2\rangle - 28.7\frac{\alpha_s^2}{\pi^2}\right)$$

- The α_s^2 term (our NNLO result): large and negative again $\Rightarrow$ negative total rate
- Long-standing problem: negative decay/production rates in perturbative QCD for exclusive processes

Solution: Amplitude-level NRQCD Factorization

Key Proposal

For exclusive processes, factorization should be implemented at the amplitude level:

$$\Gamma_M = \int d\phi(J/\psi \to 3\gamma) \sum_{\text{pol}} \left| \sum_{l,i} \mathcal{A}_i^{(l)} \langle 0|\hat{O}_i|J/\psi\rangle \right|^2$$

where $M =$ LO, NLO, NNLO, . . . corresponds to summing l up to 0, 1, 2, . . .

- Expand at amplitude level, then square ⇒ guaranteed positive rates at every order
- Traditional approach: expand at amplitude-square level ⇒ can give negative rates

$$|M|^2 = |M_0{+}M_1{+}M_2|^2 = \underbrace{|M_0|^2 + 2\text{Re}(M_0 M_1^*) + \cdots}_{\text{traditional: fixed-order terms}} + \underbrace{|M_1|^2 + 2\text{Re}(M_1 M_2^*) + \cdots}_{\text{certain higher-order terms}}$$

- This prescription can be generalized to any exclusive process as a systematic prescription

Calculation Overview

- For $J/\psi \to 3\gamma$: no real corrections $\Rightarrow$ amplitude is finite and gauge invariant
- Initially attempted converting phase space integrals into 4-loop integrals with 3 cut propagators, but IBP/AMFlow encountered insurmountable difficulties $\Rightarrow$ abandoned
- Adopted conventional path: amplitude projection + phase space integration
  - Project amplitude onto complete set of 14 Lorentz structures: $\mathcal{A}^{(l)} = c_i^{(l)} \epsilon_i$
  - Inner product: $\langle \epsilon_j, \epsilon_i \rangle = \sum_{\text{pol}} \epsilon_i \epsilon_j^* \equiv G_{ij}$
  - $\det(G) \propto \varepsilon^2$ in $D = 4 - 2\varepsilon \Rightarrow$ first 12 structures suffice in $D = 4$
- Feynman diagrams: 6 (tree) + 48 (1-loop) + 894 (2-loop); 6+96 integral families
- IBP reduction: Kira; Master integrals: AMFlow (differential equation method)

$$k_l \cdot \frac{\partial I}{\partial k_i} = \frac{\partial I}{\partial (k_i \cdot k_j)} \frac{\partial (k_i \cdot k_j)}{\partial k_i} \cdot k_l, \quad (i, j, l = 1, 2, 3)$$

- Phase space: Gauss-Kronrod quadrature (G7-K15), $15 \times 15 = 225$ sampling points

Complete NNLO Result

$$\Gamma_{LO} = \frac{16(\pi^2 - 9) q_c^6 \alpha^3 |R_{J/\psi}|^2}{3\pi M_{J/\psi}^2}, \quad \Gamma_{NLO} = \Gamma_{LO}\left[1 - 12.63(\alpha_s/\pi) + 45.28(\alpha_s/\pi)^2\right],$$

$$\Gamma_{NNLO} = \Gamma_{LO}\Big\{1 - 12.63(\alpha_s/\pi) + (\alpha_s/\pi)^2\left[5.369 n_l - 1.986\bar{q}^2 - 41.77 + (4.210 n_l - 65.26) l_R - 51.18 l_\Lambda\right]$$
$$+ (\alpha_s/\pi)^3\left[(467.4 - 30.16 n_l) l_R + 323.2 l_\Lambda - 42.27 n_l + 10.10\bar{q}^2 + 647.4\right] + (\alpha_s/\pi)^4\left[654.7 l_\Lambda^2\right.$$
$$+ (1670. - 107.7 n_l) l_R l_\Lambda + (1208. - 155.9 n_l + 5.028 n_l^2) l_R^2$$
$$+ (52.21\bar{q}^2 - 3.368 n_l \bar{q}^2 + 3346. - 434.3 n_l + 14.10 n_l^2) l_R + (50.82\bar{q}^2 + 2228. - 137.4 n_l) l_\Lambda$$
$$\left. + 4.018\bar{q}^4 + 63.97\bar{q}^2 - 3.448 n_l \bar{q}^2 + 2367. - 314.0 n_l + 10.65 n_l^2\right]\Big\}.$$

where $l_R = \ln\frac{\mu_R}{m_c}$, $l_\Lambda = \ln\frac{\mu_\Lambda}{m_c}$, $q_c = \frac{2}{3}$, $\bar{q}^2 = (q_u^2 + q_d^2 + q_s^2)/q_c^2$, $n_l = 3$.

- Positive decay widths at LO, NLO, and NNLO (amplitude expansion)
- $\Gamma_{NLO}/\Gamma_{LO} = 0.2771, \quad \Gamma_{NNLO}/\Gamma_{LO} = 0.1273$ $\Rightarrow$ better convergence

Renormalization Scale Dependence

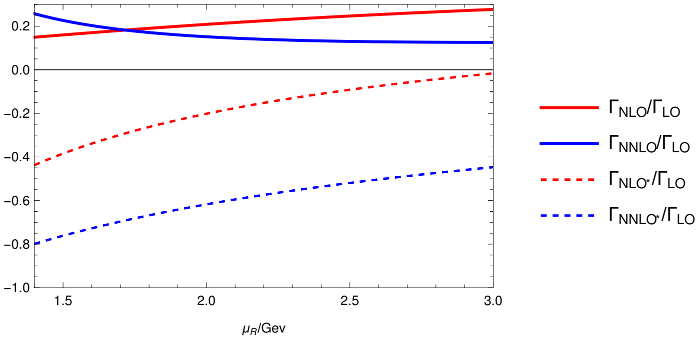


Figure: *

Renormalization scale dependence: amplitude expansion (solid) vs. traditional expansion (dashed)

- Amplitude expansion yields much better scale dependence than traditional expansion
- Traditional expansion gives negative results at both NLO and NNLO

Numerical Results and Comparison with Experiment

Theoretical predictions for $\Gamma(J/\psi \to 3\gamma)$, all values in eV

Contribution	Central Value	$\Delta\mu_R$	Δm_c	$\Delta\mu_\Lambda$	$\Delta\vert R(0)\vert^2$
Γ_{NNLO}	0.6659	$^{+0.5181}_{-0}$	$^{+0.1053}_{-0}$	$^{+0.2299}_{-0}$	$^{+0.3421}_{-0.0884}$
Γ_{Rel}	0.2970	$^{+2.474}_{-0}$	$^{+0.2502}_{-0}$	$^{-0.2346}_{+0}$	$^{+0.1653}_{-0.0394}$
Γ_{Full}	0.9629	$^{+2.992}_{-0}$	$^{+0.3555}_{-0}$	$^{-0.0047}_{+0}$	$^{+0.5074}_{-0.1278}$
BESIII (2013)	$1.046 \pm 0.167 \pm 0.185$				

- $\Gamma_{\text{Full}} = \Gamma_{NNLO} + \Gamma_{Rel}$ (Γ_{Rel}: known $\mathcal{O}(\alpha_s v^2)$ relativistic correction)
- Final prediction: $\Gamma(J/\psi \to 3\gamma) = 0.96^{+3.85}_{-0.13}$ eV, markedly improved agreement with BESIII
- Dominant uncertainty from renormalization scale μ_R variation

Summary

- First complete NNLO QCD correction to $J/\psi \to 3\gamma$ within NRQCD factorization, 48 years after NLO calculation
- The traditional (amplitude-square level) perturbative expansion again yields unphysical negative rates at NNLO — a long-standing problem in exclusive processes
- We propose amplitude-level NRQCD factorization as a systematic prescription:
  - Expanding at the amplitude level before squaring guarantees positive rates
  - Substantially improves perturbative convergence and scale dependence
  - Can be generalized to any exclusive process
- Final prediction $\Gamma(J/\psi \to 3\gamma) = 0.96^{+3.85}_{-0.13}$ eV, in markedly improved agreement with BESIII data $(1.046 \pm 0.167 \pm 0.185$ eV)
- Dominant uncertainty from μ_R variation, underscoring the need for higher-order calculations or resummation techniques

Thank you for your attention!

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