QCDF Amplitudes from Flavor Symmetries: Beyond the SU(3) Symmetric Case

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Based on arXiv:2604.19612

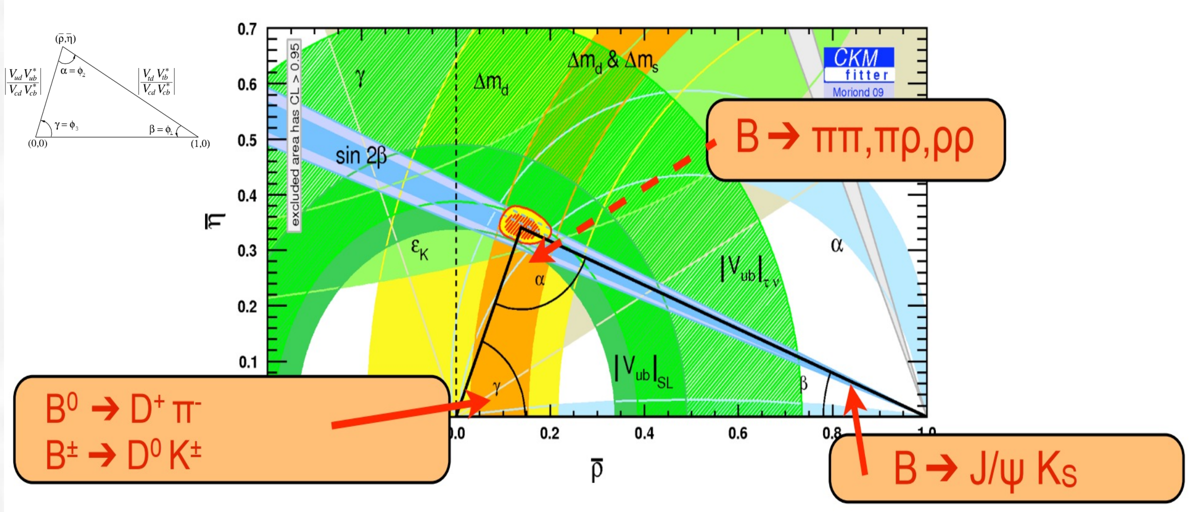
第八届全国重味物理与量子色动力学研讨会，2026/04/25，重庆

Outline

- ☐ Introduction & motivation
- ☐ Theoretical framework
- ☐ Numerical results and implications
- ☐ Summary

Why non-leptonic B-hadron decays

- ☐ **direct access to the CKM parameters, especially to the three angles of UT**

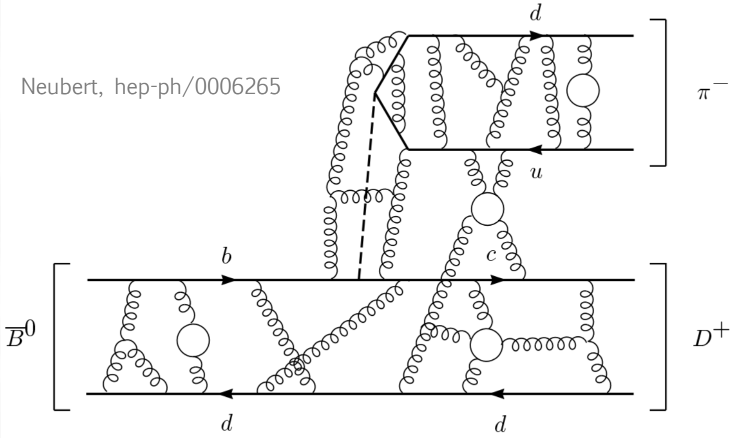


$B \to \pi\pi, \pi\rho, \rho\rho$

$B^0 \to D^+\pi^-$
$B^\pm \to D^0 K^\pm$

$B \to J/\psi\, K_S$

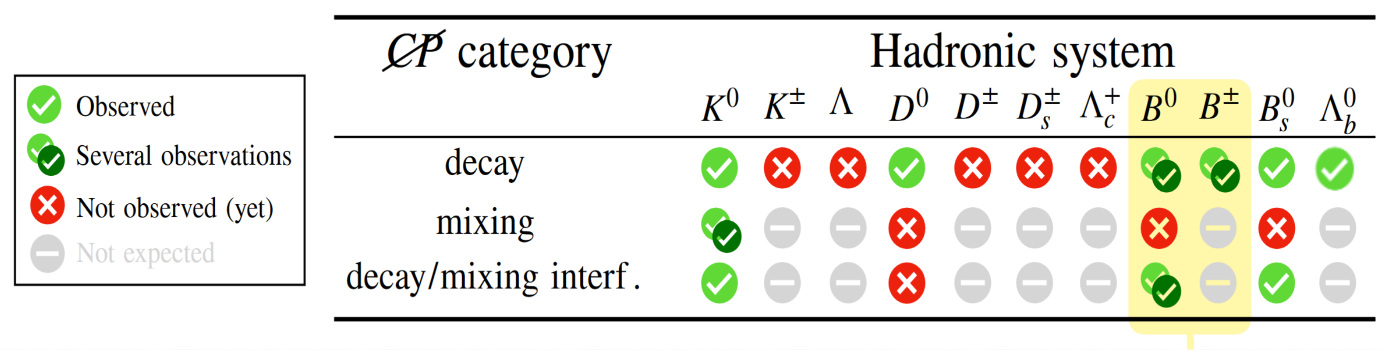
- ☐ **further insight into the strong-interaction effects involved in these weak decays**



Neubert, hep-ph/0006265

factorization? strong phase origin? ...

- ☐ **deepen our understanding of the quark mixing & CP-violation mechanism**



✓ Observed
✓✓ Several observations
✗ Not observed (yet)
⊖ Not expected

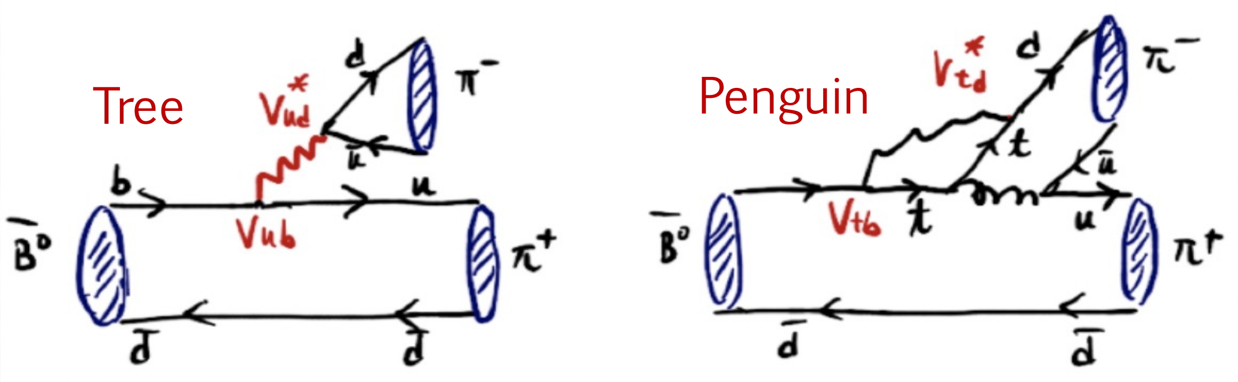
$\not{CP}$ category	Hadronic system										
	K^0	$K^\pm$	Λ	D^0	$D^\pm$	$D_s^\pm$	Λ_c^+	B^0	$B^\pm$	B_s^0	Λ_b^0
decay	✓	✗	✗	✓	✗	✗	✗	✓✓	✓✓	✓	✓
mixing	✓✓	⊖	⊖	✗	⊖	⊖	⊖	✗	⊖	✗	⊖
decay/mixing interf.	✓	⊖	⊖	✗	⊖	⊖	⊖	✓✓	⊖	✓	⊖

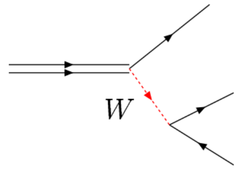
➡ ***non-leptonic B-hadron decays always play a key role in precisely testing SM!***

Why non-leptonic B-hadron decays

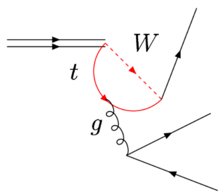
☐ **SM incomplete and BSM needed: possible new sources of flavor & CP violations?**

→ efficiently probed indirectly by high-precision flavor frontier

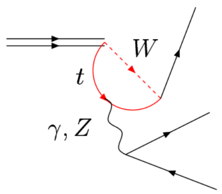




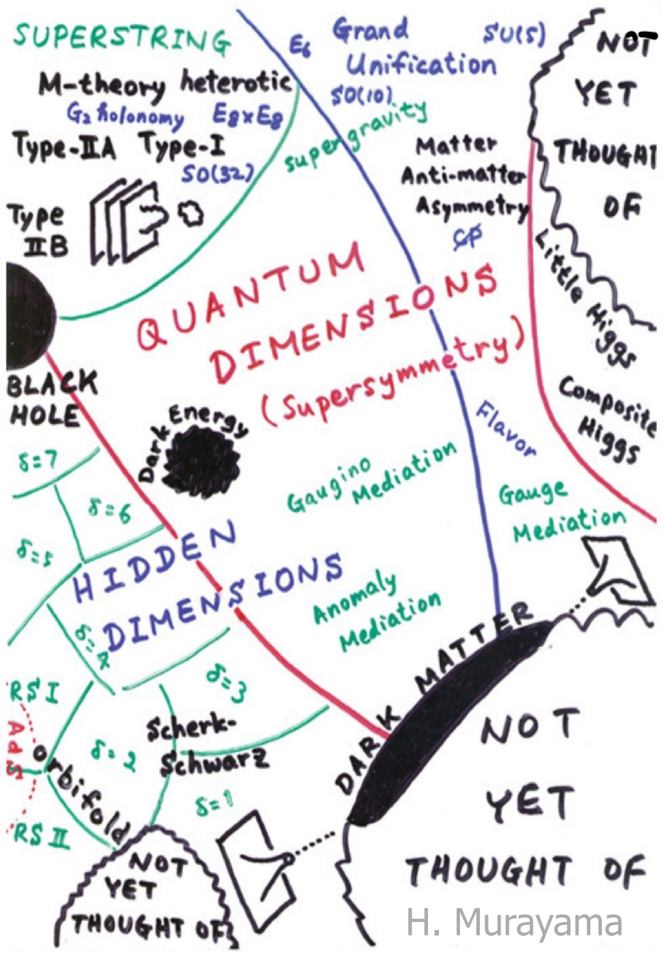
tree



QCD penguin



EW penguin



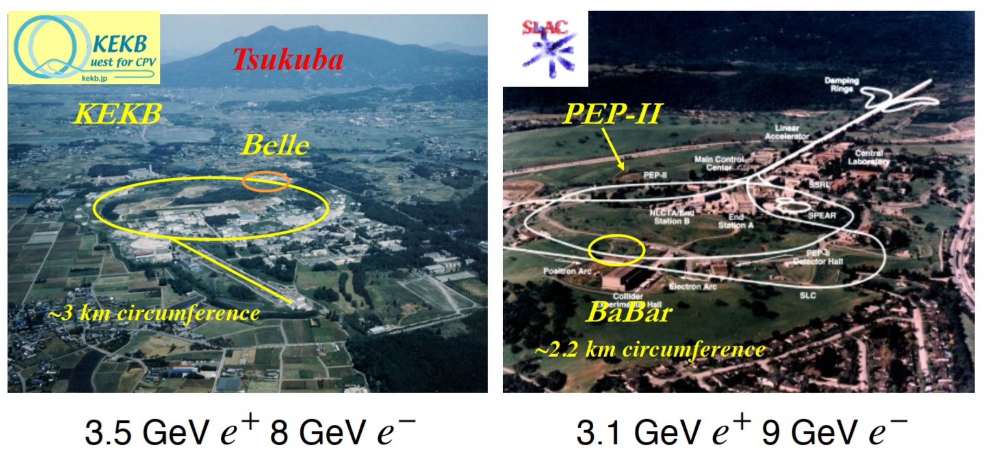
H. Murayama

b-quark massive enough to have many decay modes and many observables like branching ratios, CP asymmetries, polarizations, . . .

→ ***non-leptonic B-hadron decays always play a key role in probing BSM!***

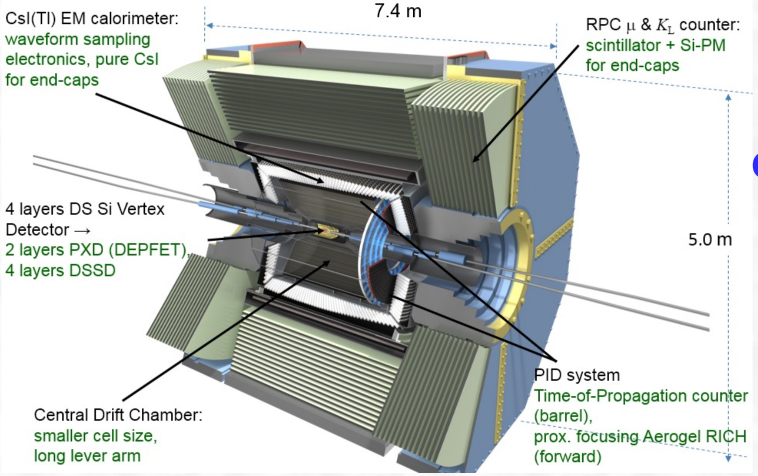
B physics experiments

☐ B-factories (e^+e^-): **Belle & BaBar**



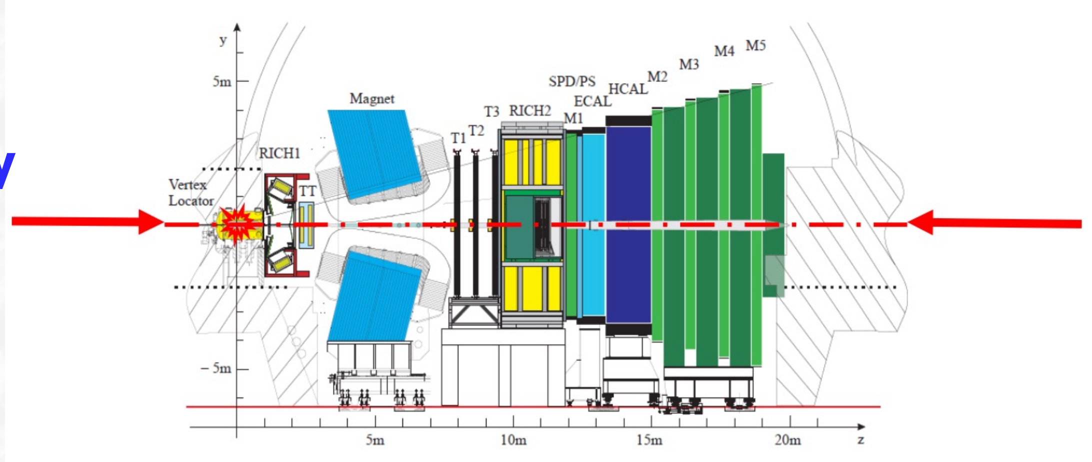
☐ Hadron colliders ($p\bar{p}$): **CDF & D0 @ Tevatron**

☐ Super B-factories (e^+e^-): **Belle-II @ KEK**



complementary to each other!

☐ Hadron colliders (pp): **LHCb @ LHC**

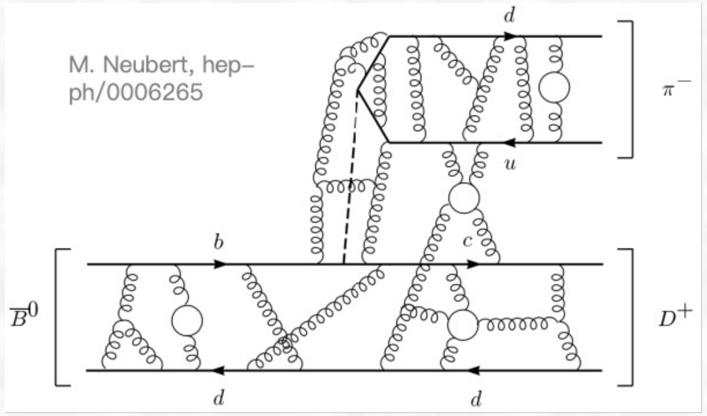


$\mathcal{H}_{\rm eff}$ for non-leptonic B-hadron decays

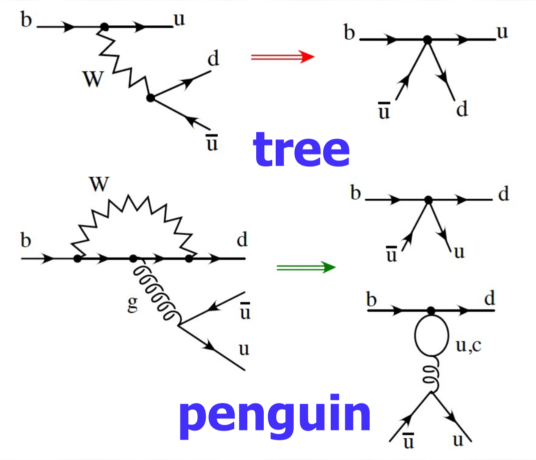
- ☐ **For non-leptonic B-hadron decays: typical multi-scale problem**

multi-scale problem with highly hierarchical scales!

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV


M. Neubert, hep–ph/0006265

➡ **OPE, RG-improved PT, & EFT formalism more suitable!**

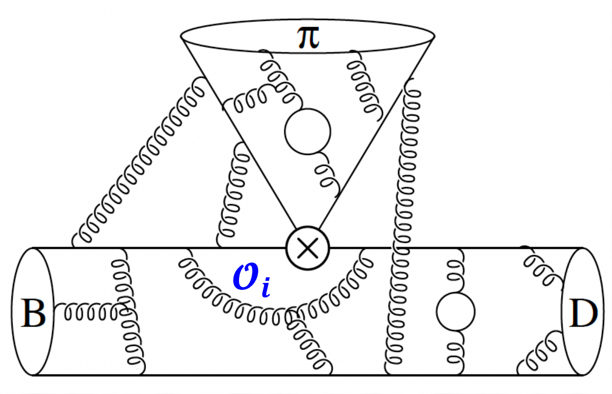


- ☐ **Starting point $\mathcal{H}_{\rm eff} = -\mathcal{L}_{\rm eff}$: obtained after integrating out heavy d.o.f. ($m_{W,Z,t} \gg m_b$)** [Buras, Buchalla, Lautenbacher '96; Chetyrkin, Misiak, Munz '98]

$$\boxed{\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb}V_{pD}^*\left(C_1\mathcal{O}_1 + C_2\mathcal{O}_2 + \sum_{i={\rm pen}} C_i\mathcal{O}_{i,{\rm pen}}\right)}$$

- ☐ **Wilson coefficients C_i: all physics above m_b; perturbatively calculable & NNLL program now complete!** [Gorbahn, Haisch '04; Misiak, Steinhauser '04]

Hadronic matrix elements



- ☐ **For a two-body non-leptonic decay $\bar{B} \to M_1 M_2$:**

$$\boxed{\mathcal{A}(\bar{B} \to M_1 M_2) = \sum_i [\lambda_{\text{CKM}} \times C_i \times \langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle]}$$

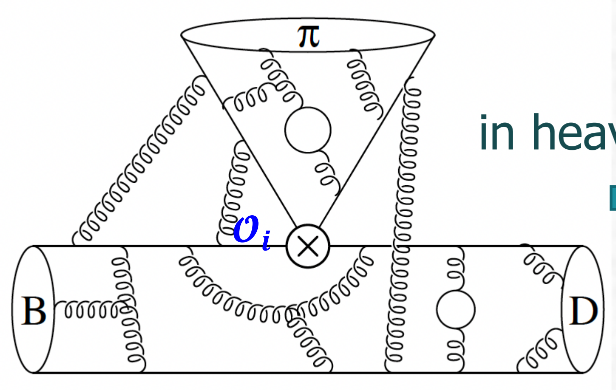
➡ physics below m_b; process–dependent; FSI introduces strong phases, and hence direct CPV!

- ☐ **Different methods proposed for $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$:** naïve fact., generalized fact., ……

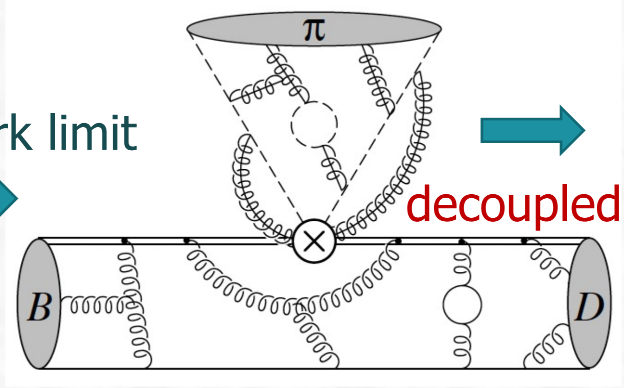
- Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, · · ·
[Keum, Li, Sanda, Lü, Yang ’00;
Beneke, Buchalla, Neubert, Sachrajda, ’00;
Bauer, Flemming, Pirjol, Stewart, ’01; Beneke, Chapovsky, Diehl, Feldmann, ’02]

- Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, · · ·
[Zeppenfeld, ’81;
London, Gronau, Rosner, He, Chiang, Cheng *et al.*]

- ☐ **QCDF/SCET:** systematic framework from QCD, valid to all orders in α_s, limited by $\frac{\Lambda_{\text{QCD}}}{m_b}$ corrections



in heavy-quark limit ➡



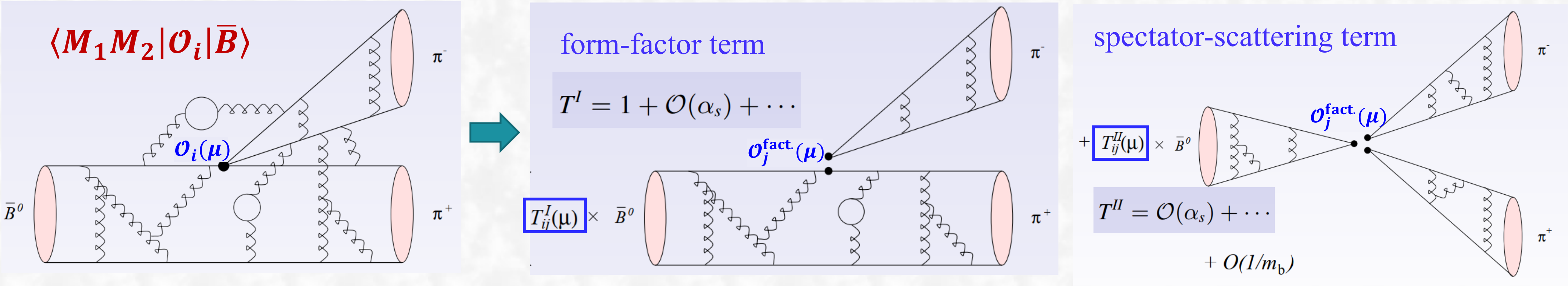
decoupled ➡

$$\langle D_q^{(*)+} L^- | \mathcal{Q}_i | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du \, \boxed{T_{ij}(u)} \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$$

$$\boxed{T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)}$$

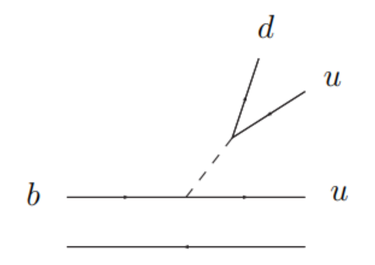
Hadronic matrix elements

- **QCDF formula for two-body charmless B decays:** [BBNS '99-'03]

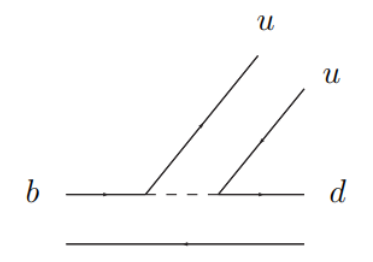


➡ $\langle M_1 M_2 | Q_i | \bar{B} \rangle$ factorized into $\langle M | j_\mu | \bar{B} \rangle$ (transition form factors), $\langle M | j_\mu | 0 \rangle$, $\langle 0 | j_\mu | \bar{B} \rangle$ (decay constants & LCDAs)

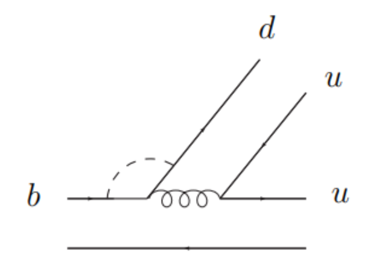
- **Status @ NNLO:** [Beneke, Huber and Xin-Qiang Li '10; Bell, Beneke, Huber and Xin-Qiang Li '15, '20; Huber, Susanne and Xin-Qiang Li '16]



colour-allowed tree α_1



colour-suppressed tree α_2

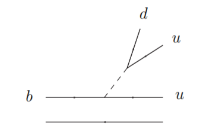


QCD penguins α_4

Final results for $\alpha_{1,2}$

☐ Numerical results including the NNLO corrections:

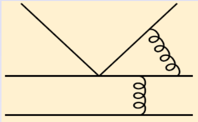
$$\alpha_1(\pi\pi) = 1.009 + [0.023 + 0.010\,i]_{\rm NLO} + [0.026 + 0.028\,i]_{\rm NNLO}$$



colour-allowed tree α_1

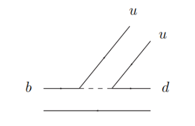
$$- \left[\frac{r_{\rm sp}}{0.445}\right] \left\{[0.014]_{\rm LOsp} + [0.034 + 0.027i\,]_{\rm NLOsp} + [0.008]_{\rm tw3}\right\}$$

$$= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i$$



Beneke, Jager '05
Kivel '06, Pilipp '07

$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077\,i]_{\rm NLO} - [0.031 + 0.050\,i]_{\rm NNLO}$$

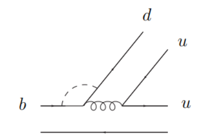


colour-suppressed tree α_2

$$+ \left[\frac{r_{\rm sp}}{0.445}\right] \left\{[0.114]_{\rm LOsp} + [0.049 + 0.051i\,]_{\rm NLOsp} + [0.067]_{\rm tw3}\right\}$$

$$= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i$$

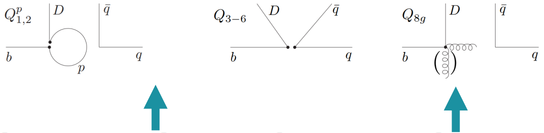
$$a_4^u(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{\rm V_1} + [0.49 - 1.32i]_{\rm P_1} - [0.32 + 0.71\,i]_{\rm P_2,\,Q_{1,2}} + [0.33 + 0.38i]_{\rm P_2,\,Q_{3-6,8}}$$



QCD penguins α_4

$$+ \left[\frac{r_{\rm sp}}{0.434}\right] \left\{[0.13]_{\rm LO} + [0.14 + 0.12i]_{\rm HV} - [0.01 - 0.05i]_{\rm HP} + [0.07]_{\rm tw3}\right\}$$

$$= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,,$$

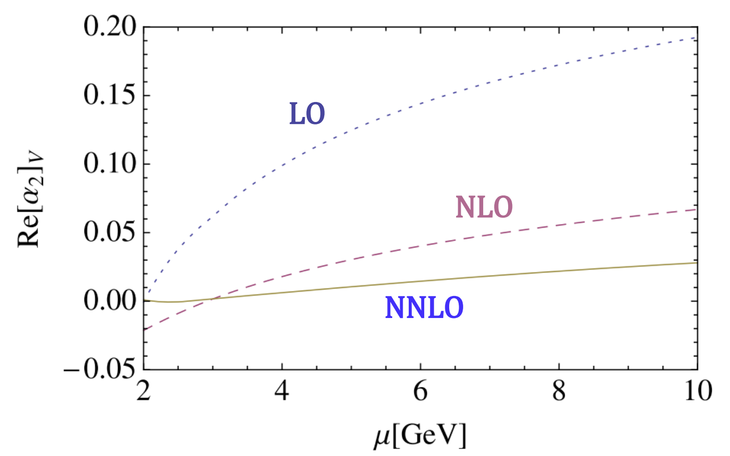


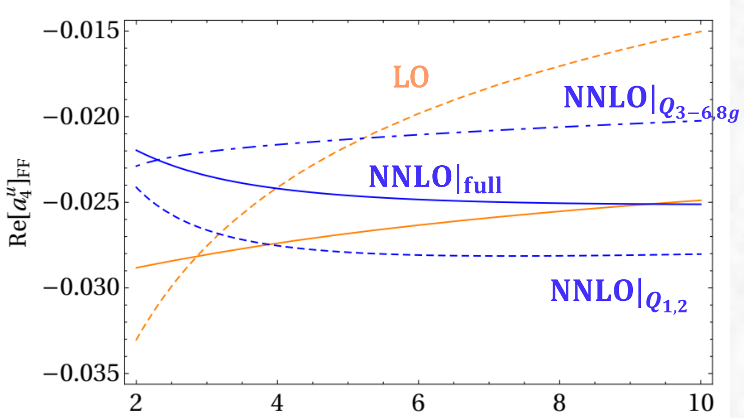
$$a_4^c(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{\rm V_1} + [0.05 - 0.62i]_{\rm P_1} - [0.77 + 0.50i]_{\rm P_2,\,Q_{1,2}} + [0.33 + 0.38i]_{\rm P_2,\,Q_{3-6,8}}$$

$$+ \left[\frac{r_{\rm sp}}{0.434}\right] \left\{[0.13]_{\rm LO} + [0.14 + 0.12i]_{\rm HV} + [0.01 + 0.03i]_{\rm HP} + [0.07]_{\rm tw3}\right\}$$

$$= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i\,.$$

☐ Scale-dependence much reduced



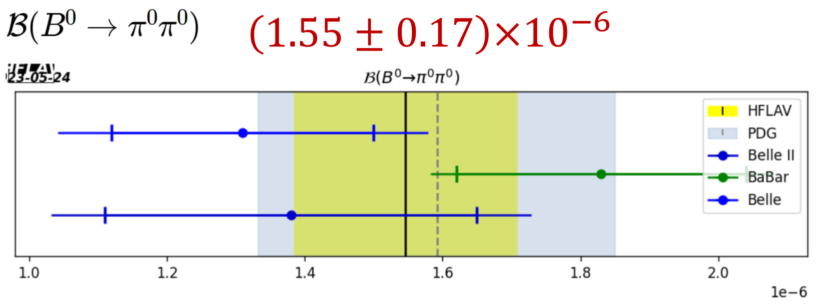


$\pi\pi$ and πK puzzles

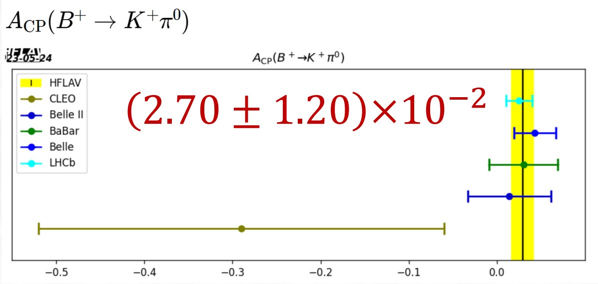
- ☐ **Long-standing puzzles in $\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$:** [HFLAV '24]

$\mathrm{Br}(B^0 \to \pi^0\pi^0) = (0.3 - 0.9)\times 10^{-6}$

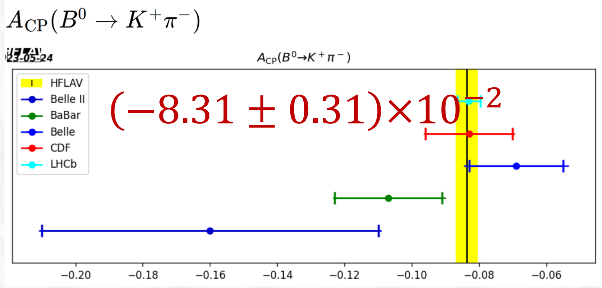
$\Delta A_{CP}(\pi K) = (11.0 \pm 1.2)\%$ **differs from 0 by ~9.2σ**



$(1.55 \pm 0.17)\times 10^{-6}$



$(2.70 \pm 1.20)\times 10^{-2}$

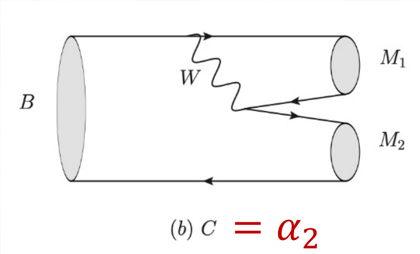


$(-8.31 \pm 0.31)\times 10^{-2}$

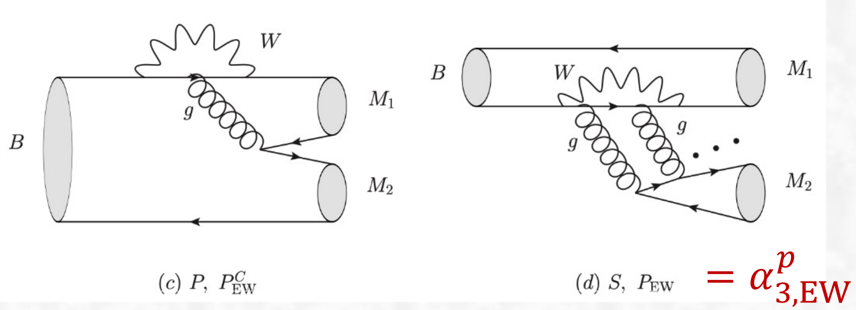
- ☐ **Decay amplitudes in QCDF:**

$$-\mathcal{A}_{\bar{B}^0\to\pi^0\pi^0} = A_{\pi\pi}\left[\delta_{pu}(\alpha_2-\beta_1) - \hat{\alpha}_4^p - 2\beta_4^p\right]$$

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar{K}}\left[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\right] + A_{\bar{K}\pi}\left[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha^c_{3,\mathrm{EW}}\right],$$
$$\mathcal{A}_{\bar{B}^0\to\pi^+K^-} = A_{\pi\bar{K}}\left[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\right],$$



(b) C $= \alpha_2$



(c) $P,\ P^C_{\mathrm{EW}}$

(d) $S,\ P_{\mathrm{EW}}$ $= \alpha^p_{3,\mathrm{EW}}$

some mechanism or sub-leading power corrections or even NP to enhance $C = \alpha_2$ or $P_{\mathrm{EW}} = \alpha^p_{3,\mathrm{EW}}$?

Other interesting puzzles

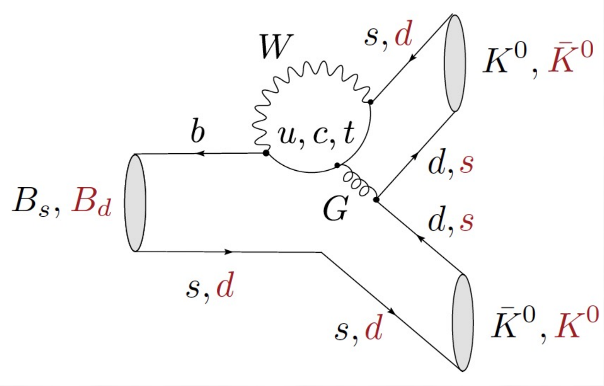
- ☐ **Ratios among $B_{(s)} \to KK$ decays:** [Grossman, Neubert, Nir, Shpilman, Viernik 2407.13506]

$$R^{ss}_{KK} \equiv \frac{\Gamma(B_s \to K^0\overline{K}^0)}{\Gamma(B_s \to K^+K^-)} = 0.66 \pm 0.13,$$

related by iso-spin ($u \leftrightarrow d$)

$$R^{sd}_{KK} \equiv \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Gamma(B_s \to K^0\overline{K}^0)}{\Gamma(B^0 \to K^0\overline{K}^0)} = 0.61 \pm 0.13,$$

related by U-spin ($d \leftrightarrow s$)



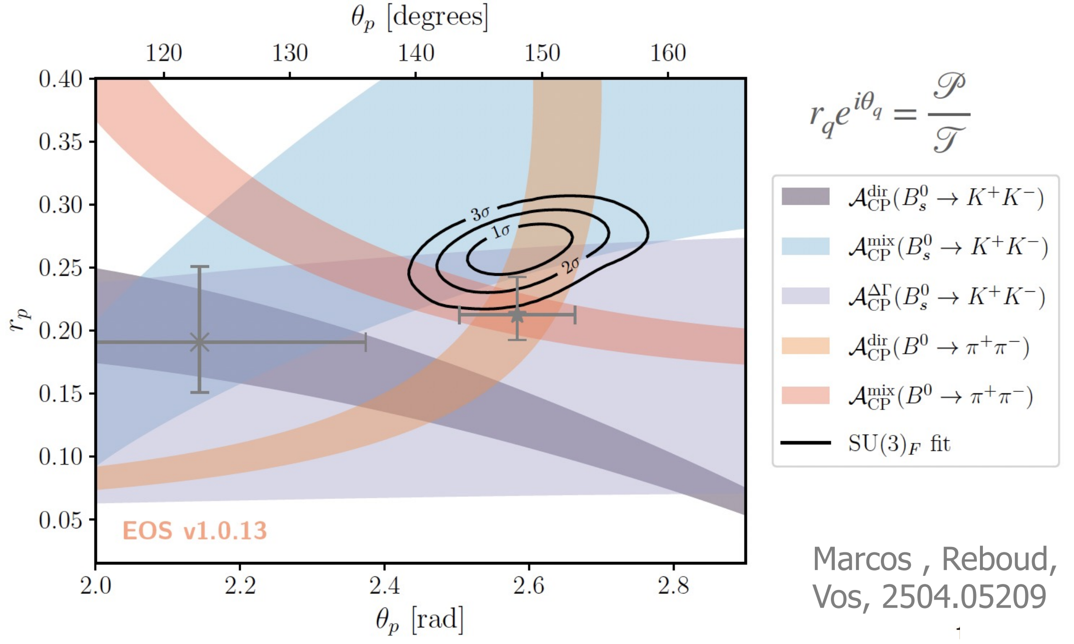
- ☐ **Decay amplitudes in QCDF:**

$$\mathcal{A}(\bar{B}_d \to K^0\bar{K}^0) = \frac{G_F}{\sqrt{2}}\lambda_p^d\Bigg[\left(\alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p\right)A_{\bar{K}K} + \left(b_3^p + b_4^p - \frac{1}{2}b_{3,\mathrm{EW}}^p - \frac{1}{2}b_{4,\mathrm{EW}}^p\right)B_{\bar{K}K} + \left(b_4^p - \frac{1}{2}b_{4,\mathrm{EW}}^p\right)B_{K\bar{K}}\Bigg],$$

$$\mathcal{A}(\bar{B}_s \to K^0\bar{K}^0) = \frac{G_F}{\sqrt{2}}\lambda_p^s\Bigg[\left(\alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p\right)A_{K\bar{K}} + \left(b_3^p + b_4^p - \frac{1}{2}b_{3,\mathrm{EW}}^p - \frac{1}{2}b_{4,\mathrm{EW}}^p\right)B_{K\bar{K}} + \left(b_4^p - \frac{1}{2}b_{4,\mathrm{EW}}^p\right)B_{\bar{K}K}\Bigg],$$

$$\mathcal{A}(\bar{B}_s \to K^+K^-) = \frac{G_F}{\sqrt{2}}\lambda_p^s\Bigg[\left(\delta_{pu}\alpha_1 + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p\right)A_{K\bar{K}} + \left(b_3^p + b_4^p - \frac{1}{2}b_{3,\mathrm{EW}}^p - \frac{1}{2}b_{4,\mathrm{EW}}^p\right)B_{K\bar{K}} + \left(\delta_{pu}b_1 + b_4^p + b_{4,\mathrm{EW}}^p\right)B_{\bar{K}K}\Bigg],$$

➡ **differ only in tree, EWP, & annihilation**



Marcos , Reboud, Vos, 2504.05209

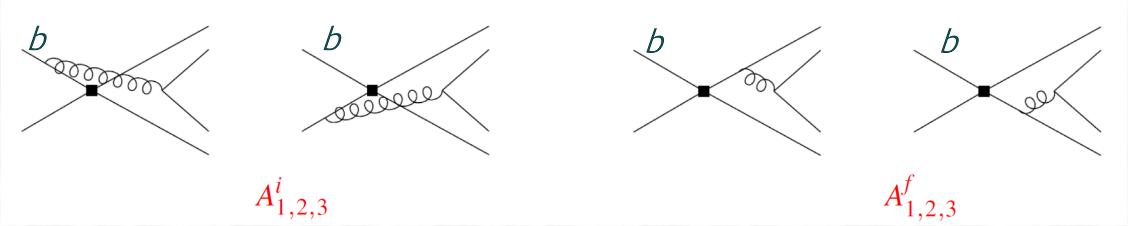
Pure annihilation B decays

- ☐ **Two typical pure annihilation decay modes:** $\bar{B}_s^0 \to \pi^+\pi^-$ vs $\bar{B}_d^0 \to K^+K^-$ related by U-spin

$$\mathcal{A}(\bar{B}_s \to \pi^+\pi^-) = B_{\pi\pi}\left[\delta_{pu} b_1 + 2b_4^p + \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$\mathcal{A}(\bar{B}_d \to K^+K^-) = A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p + b_{4,\mathrm{EW}}^p\right] + B_{K\bar{K}}\left[b_4^p - \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$= A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p\right] + B_{K\bar{K}}\left[b_4^p\right]$$



- ☐ **QCDF vs data for branching ratios:** large SU(3)-flavor symmetry breaking?

Mode	Theory	S1 (large γ)	S2 (large a_2)	S3 ($\varphi_A = -45^\circ$)	S4 ($\varphi_A = -55^\circ$)	Exp.
$\bar{B}_s^0 \to \pi^+\pi^-$	$0.024^{+0.003+0.025+0.000+0.163}_{-0.003-0.012-0.000-0.021}$	0.027	0.032	0.149	0.155	0.72 ± 0.11
$\bar{B}^0 \to K^-K^+$	$0.013^{+0.005+0.008+0.000+0.087}_{-0.005-0.005-0.000-0.011}$	0.007	0.014	0.079	0.070	0.080 ± 0.015

- ☐ **State-of-the-art $\mathrm{SU(3)_F}$ fit:** [D. London et al., 2311.18011; 2505.11492; 2510.13969]
  - ✓ for ΔS = 0 decays: excellent fit; for ΔS = 1 decays: good fit
  - ✓ for combined ΔS = 0 & ΔS = 1 decays: very poor fit
  - ✓ 1000% $\mathrm{SU(3)_F}$-breaking effect required ($f_K/f_\pi - 1 \sim 20\%$)

$\Delta S = 0$ fit:

$\lvert\tilde{T}\rvert$	$\lvert\tilde{C}\rvert$	$\lvert\tilde{P}_{uc}\rvert$	$\lvert\tilde{A}\rvert$	$\lvert P_{tc}\rvert$
4.0 ± 0.5	6.6 ± 0.7	3 ± 4	6 ± 5	0.8 ± 0.4

$\Delta S = 1$ fit:

$\lvert\tilde{T}'\rvert$	$\lvert\tilde{C}'\rvert$	$\lvert\tilde{P}'_{uc}\rvert$	$\lvert\tilde{A}'\rvert$	$\lvert P'_{tc}\rvert$
48 ± 14	41 ± 14	48 ± 15	81 ± 28	0.78 ± 0.16

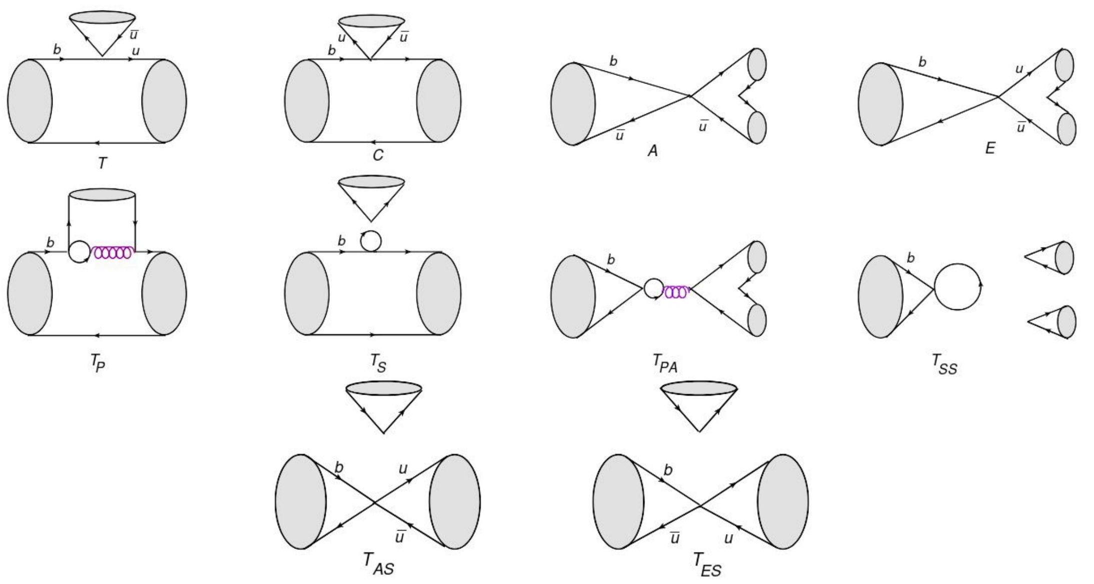
Decay amplitudes of $B \to PP$ based on $\mathrm{SU}(3)_{\mathrm{F}}$

- ☐ **Decay amplitudes in TDA:** [Zeppenfeld '81; He and Wang '18; Jia, Wang and Yu '21]

$$\boxed{\mathcal{A}^{\mathrm{TDA}} = \frac{iG_F}{\sqrt{2}}\left[\mathcal{T}^{\mathrm{TDA}} + \mathcal{P}^{\mathrm{TDA}}\right]}$$

$$\begin{aligned}
\mathcal{T}^{\mathrm{TDA}} = {} & T\, B_i (M_1)^i_j\, \bar{H}^j_k (M_2)^k_l + C\, B_i (M_1)^i_j\, \bar{H}^l_k (M_2)^k_l + A\, B_i\, \bar{H}^l_j (M_1)^j_k (M_2)^k_i \\
& + E\, B_i\, \bar{H}^{li}_j (M_1)^j_k (M_2)^k_l + T_{ES}\, B_i\, \bar{H}^{ij}_l (M_1)^l_j (M_2)^k_k + T_{AS}\, B_i\, \bar{H}^{ji}_l (M_1)^l_j (M_2)^k_k \\
& + T_S\, B_i (M_1)^i_j\, \bar{H}^{lj}_l (M_2)^k_k + T_{PA}\, B_i\, \bar{H}^{li}_l (M_1)^j_k (M_2)^k_j + T_P\, B_i (M_1)^i_j (M_2)^j_k\, \bar{H}^{lk}_l \\
& + T_{SS}\, B_i\, \bar{H}^{li}_l (M_1)^j_j (M_2)^k_k + (M_1 \leftrightarrow M_2)\,,
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}^{\mathrm{TDA}} = {} & P\, B_i (M_1)^i_j (M_2)^j_k\, \tilde{H}^k + P_T\, B_i (M_1)^i_j\, \tilde{H}^{jl}_k (M_2)^k_l + S\, B_i (M_1)^i_j\, \tilde{H}^j (M_2)^k_k \\
& + P_C\, B_i (M_1)^i_j\, \tilde{H}^{lj}_k (M_2)^k_l + P_{TA}\, B_i\, \tilde{H}^{il}_j (M_1)^j_k (M_2)^k_l + P_A\, B_i\, \tilde{H}^i (M_1)^j_k (M_2)^k_j \\
& + P_{TE}\, B_i\, \tilde{H}^{ji}_k (M_1)^k_l (M_2)^l_j + P_{AS}\, B_i\, \tilde{H}^{ji}_l (M_1)^l_j (M_2)^k_k + P_{SS}\, B_i\, \tilde{H}^i (M_1)^j_j (M_2)^k_k \\
& + P_{ES}\, B_i\, \tilde{H}^{ij}_l (M_1)^l_j (M_2)^k_k + (M_1 \leftrightarrow M_2)\,.
\end{aligned}$$



- ☐ **Also equivalent with IRA:** [He and Wang '18]

$$\begin{aligned}
\mathcal{A}^{\mathrm{IRA}}_{\mathrm{c-less}} = {} & a_{15}(\overline{B}_\gamma)_i (H(15))^{ij}_k (P_\alpha)^l_j (P_\beta)^k_l + b_{15}(\overline{B}_\gamma)_i (H(15))^{ij}_k (P_\alpha)^k_j (P_\beta)^l_l + c_{15}(\overline{B}_\gamma)_i (H(15))^{jl}_k (P_\alpha)^i_j (P_\beta)^k_l \\
& + a_6(\overline{B}_\gamma)_i (H(\overline{6}))^{ji}_k (P_\alpha)^l_j (P_\beta)^k_l + b_6(\overline{B}_\gamma)_i (H(\overline{6}))^{ji}_k (P_\alpha)^k_j (P_\beta)^l_l + c_6(\overline{B}_\gamma)_i (H(\overline{6}))^{lj}_k (P_\alpha)^i_j (P_\beta)^k_l \\
& + a_3(\overline{B}_\gamma)_i (H(3))^i (P_\alpha)^k_j (P_\beta)^j_k + b_3(\overline{B}_\gamma)_i (H(3))^i (P_\alpha)^k_k (P_\beta)^j_j + c_3(\overline{B}_\gamma)_i (H(3))^k (P_\alpha)^i_k (P_\beta)^j_j \\
& + d_3(\overline{B}_\gamma)_i (H(3))^k (P_\alpha)^i_j (P_\beta)^j_k + a'_3(\overline{B}_\gamma)_i (H(3'))^i (P_\alpha)^k_j (P_\beta)^j_k + b'_3(\overline{B}_\gamma)_i (H(3'))^i (P_\alpha)^k_k (P_\beta)^j_j \\
& + c'_3(\overline{B}_\gamma)_i (H(3'))^k (P_\alpha)^i_k (P_\beta)^j_j + d'_3(\overline{B}_\gamma)_i (H(3'))^k (P_\alpha)^i_j (P_\beta)^j_k + +a''_3(\overline{B}_\gamma)_i (H(3''))^i (P_\alpha)^k_j (P_\beta)^j_k \\
& + b''_3(\overline{B}_\gamma)_i (H(3''))^i (P_\alpha)^k_k (P_\beta)^j_j + c''_3(\overline{B}_\gamma)_i (H(3''))^k (P_\alpha)^i_k (P_\beta)^j_j + d''_3(\overline{B}_\gamma)_i (H(3''))^k (P_\alpha)^i_j (P_\beta)^j_k \\
& + a'''_3(\overline{B}_\gamma)_i (H(3'''))^i (P_\alpha)^k_j (P_\beta)^j_k + b'''_3(\overline{B}_\gamma)_i (H(3'''))^i (P_\alpha)^k_k (P_\beta)^j_j + c'''_3(\overline{B}_\gamma)_i (H(3'''))^k (P_\alpha)^i_k (P_\beta)^j_i \\
& + d'''_3(\overline{B}_\gamma)_i (H(3'''))^k (P_\alpha)^i_j (P_\beta)^j_k + \alpha \leftrightarrow \beta.
\end{aligned}$$

- ☐ **Key point:** no any theoretical assumptions on RMEs ⇒ completely rigorous on group-theoretical side, but need enough data to fit them!

Decay amplitudes of $B \to PP$ based on $\mathrm{SU(3)_F}$

☐ Decay amplitudes in QCDF: [BBNS '03; Huber and Tetlalmatzi '21]

$$\begin{aligned}
\mathcal{A}^{\rm QCDF} = & i\frac{G_F}{\sqrt{2}} \sum_{p=u,c} A_{M_1M_2} \Big\{ B M_1 \left(\alpha_1 \delta_{pu} \hat{U}_p + \alpha_4^p \hat{I} + \alpha_{4,\rm EW}^p \hat{Q}\right) M_2 \Lambda_p \\
& + B M_1 \Lambda_p \cdot {\rm Tr}\left[\left(\alpha_2 \delta_{pu} \hat{U}_p + \alpha_3^p \hat{I} + \alpha_{3,\rm EW}^p \hat{Q}\right) M_2\right] \\
& + B \left(\beta_2 \delta_{pu} \hat{U}_p + \beta_3^p \hat{I} + \beta_{3,\rm EW}^p \hat{Q}\right) M_1 M_2 \Lambda_p \\
& + B \Lambda_p \cdot {\rm Tr}\left[\left(\beta_1 \delta_{pu} \hat{U}_p + \beta_4^p \hat{I} + \beta_{4,\rm EW}^p \hat{Q}\right) M_1 M_2\right] \\
& + B \left(\beta_{S2} \delta_{pu} \hat{U}_p + \beta_{S3}^p \hat{I} + \beta_{S3,\rm EW}^p \hat{Q}\right) M_1 \Lambda_p \cdot {\rm Tr}\, M_2 \\
& + B \Lambda_p \cdot {\rm Tr}\left[\left(\beta_{S1} \delta_{pu} \hat{U}_p + \beta_{S4}^p \hat{I} + \beta_{S4,\rm EW}^p \hat{Q}\right) M_1\right] \cdot {\rm Tr}\, M_2 \Big\} + (M_1 \leftrightarrow M_2)
\end{aligned}$$

$$S = A_{M_1M_2}\left[\alpha_3^c + \beta_{S3}^c - \frac{\alpha_{3,\rm EW}^c}{2} - \frac{\beta_{S3,\rm EW}^c}{2}\right],$$

$$P = A_{M_1M_2}\left[\alpha_4^c + \beta_3^c - \frac{\alpha_{4,\rm EW}^c}{2} - \frac{\beta_{3,\rm EW}^c}{2}\right],$$

Penguin sector

$$P_A = A_{M_1M_2}\left[\beta_4^c - \frac{\beta_{4,\rm EW}^c}{2}\right], \qquad P_{SS} = A_{M_1M_2}\left[\beta_{S4}^c - \frac{\beta_{S4,\rm EW}^c}{2}\right],$$

$$P_C = \frac{3}{2} A_{M_1M_2} \alpha_{3,\rm EW}^c, \qquad P_T = \frac{3}{2} A_{M_1M_2} \alpha_{4,\rm EW}^c,$$

$$P_{TA} = \frac{3}{2} A_{M_1M_2} \beta_{3,\rm EW}^c, \qquad P_{TE} = \frac{3}{2} A_{M_1M_2} \beta_{4,\rm EW}^c,$$

$$P_{AS} = \frac{3}{2} A_{M_1M_2} \beta_{S4,\rm EW}^c, \qquad P_{ES} = \frac{3}{2} A_{M_1M_2} \beta_{S3,\rm EW}^c,$$

$$T = A_{M_1M_2}\left[\alpha_1 + \frac{3}{2}\alpha_{4,\rm EW}^u - \frac{3}{2}\alpha_{4,\rm EW}^c\right], \qquad C = A_{M_1M_2}\left[\alpha_2 + \frac{3}{2}\alpha_{3,\rm EW}^u - \frac{3}{2}\alpha_{3,\rm EW}^c\right]$$

$$E = A_{M_1M_2}\left[\beta_1 + \frac{3}{2}\beta_{4,\rm EW}^u - \frac{3}{2}\beta_{4,\rm EW}^c\right], \qquad A = A_{M_1M_2}\left[\beta_2 + \frac{3}{2}\beta_{3,\rm EW}^u - \frac{3}{2}\beta_{3,\rm EW}^c\right];$$

$$T_{AS} = A_{M_1M_2}\left[\beta_{S1} + \frac{3}{2}\beta_{S4,\rm EW}^u - \frac{3}{2}\beta_{S4,\rm EW}^c\right],$$

$$T_{ES} = A_{M_1M_2}\left[\beta_{S2} + \frac{3}{2}\beta_{S3,\rm EW}^u - \frac{3}{2}\beta_{S3,\rm EW}^c\right],$$

Tree sector

$$T_{PA} = A_{M_1M_2}\left[\beta_4^u - \beta_4^c - \left(\frac{\beta_{4,\rm EW}^u}{2} - \frac{\beta_{4,\rm EW}^c}{2}\right)\right],$$

$$T_{SS} = A_{M_1M_2}\left[\beta_{S4}^u - \beta_{S4}^c - \left(\frac{\beta_{S4,\rm EW}^u}{2} - \frac{\beta_{S4,\rm EW}^c}{2}\right)\right],$$

$$T_S = A_{M_1M_2}\left[\alpha_3^u - \alpha_3^c - \left(\frac{\alpha_{3,\rm EW}^u}{2} - \frac{\alpha_{3,\rm EW}^c}{2}\right) + (\beta_{S3}^u - \beta_{S3}^c) - \left(\frac{\beta_{S3,\rm EW}^u}{2} - \frac{\beta_{S3,\rm EW}^c}{2}\right)\right],$$

$$T_P = A_{M_1M_2}\left[\alpha_4^u - \alpha_4^c - \left(\frac{\alpha_{4,\rm EW}^u}{2} - \frac{\alpha_{4,\rm EW}^c}{2}\right) + (\beta_3^u - \beta_3^c) - \left(\frac{\beta_{3,\rm EW}^u}{2} - \frac{\beta_{3,\rm EW}^c}{2}\right)\right],$$

- All 17 ($\Delta S = 0$)+17 ($\Delta S = 1$) two-body charmless $B \to PP$ decays described by 20 TDA or QCDF parameters

Dynamical information from QCDF

☐ **Direct calculation in QCDF @ NLO and NNLO:** [BBNS '03; Bell, Beneke, Huber, Xin-Qiang Li '00; '20]

$$\mathcal{A}(B \to M_1 M_2) = i\frac{G_F}{\sqrt{2}}[\lambda_u(\alpha_i^u+\beta_i^u) + \lambda_c(\alpha_i^u+\beta_i^u)]$$

$$\begin{aligned}
\alpha_1(M_1M_2) &= a_1(M_1M_2),\\
\alpha_2(M_1M_2) &= a_2(M_1M_2),\\
\alpha_3^p(M_1M_2) &= a_3^p(M_1M_2) - a_5^p(M_1M_2)\\
\alpha_4^p(M_1M_2) &= a_4^p(M_1M_2) + r_\chi^{M_2} a_6^p(M_1M_2)\\
\alpha_{3,\mathrm{EW}}^p(M_1M_2) &= a_9^p(M_1M_2) - a_7^p(M_1M_2)\\
\alpha_{4,\mathrm{EW}}^p(M_1M_2) &= a_{10}^p(M_1M_2) + r_\chi^{M_2} a_8^p(M_1M_2)
\end{aligned}$$

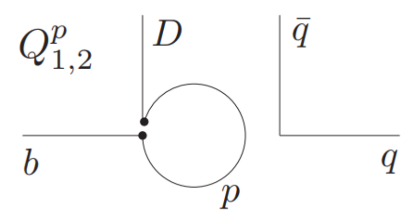
$$a_i^p(M_1M_2) = \left(C_i + \frac{C_{i\pm1}}{N_c}\right) N_i(M_2) + \frac{C_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_i(M_2) + \frac{4\pi^2}{N_c}H_i(M_1M_2)\right] + P_i^p(M_2)$$

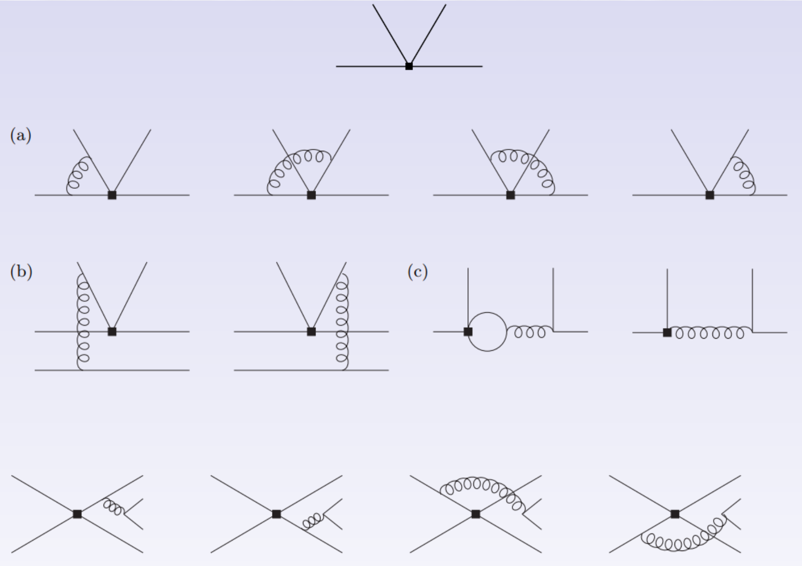
$$\alpha_3^u = \alpha_3^c = \alpha_3, \qquad \alpha_{3,EW}^u = \alpha_{3,EW}^c = \alpha_{3,EW},$$

$$\beta_i^u = \beta_i^c = \beta_i, \qquad b_i^u = b_i^c = b_i$$

$$|\alpha_{4,EW}^c - \alpha_{4,EW}^u| < \mathcal{O}(10^{-3}),$$

$$|a_4^c - a_4^u| = 0.013_{-0.007}^{+0.008}$$





reduce the independent TDA amplitudes from 20 to 17 or 16:

$$T_{PA} = 0, \qquad T_{SS} = 0, \qquad T_S = 0, \qquad |T_P|/A_{M_1M_2} < 0.02,$$

Factorizable SU(3) breaking effects

- ☐ **Independent TDA amplitudes: 17 or 16**

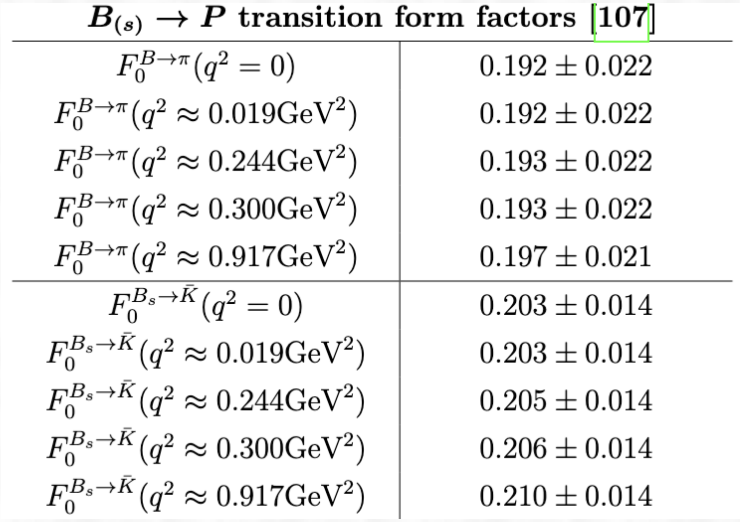
$$T = A_{M_1M_2}\,\alpha_1, \qquad C = A_{M_1M_2}\,\alpha_2, \qquad E = B_{M_1M_2}\,b_1,$$
$$A = B_{M_1M_2}\,b_2, \qquad T_{AS} = B_{M_1M_2}\,b_{S1}, \qquad T_{ES} = B_{M_1M_2}\,b_{S2},$$
$$T_{PA} = 0, \qquad T_{SS} = 0, \qquad T_S = 0, \qquad |T_P|/A_{M_1M_2} < 0.02,$$

$$T = A_{M_1M_2}\,\tilde{T}, \qquad C = A_{M_1M_2}\,\tilde{C}, \qquad E = B_{M_1M_2}\,\tilde{E},$$
$$A = B_{M_1M_2}\,\tilde{A}, \qquad T_{AS} = B_{M_1M_2}\,\tilde{T}_{AS}, \qquad T_{ES} = B_{M_1M_2}\,\tilde{T}_{ES}.$$
$$S = A_{M_1M_2}\,\tilde{S}, \qquad P = A_{M_1M_2}\,\tilde{P}, \qquad P_A = B_{M_1M_2}\,\tilde{P}_A,$$
$$P_{SS} = B_{M_1M_2}\,\tilde{P}_{SS}, \qquad P_C = A_{M_1M_2}\,\tilde{P}_C, \qquad P_T = A_{M_1M_2}\,\tilde{P}_T,$$
$$P_{TA} = B_{M_1M_2}\,\tilde{P}_{TA}, \qquad P_{TE} = B_{M_1M_2}\,\tilde{P}_{TE}, \qquad P_{AS} = B_{M_1M_2}\,\tilde{P}_{AS},$$
$$P_{ES} = B_{M_1M_2}\,\tilde{P}_{ES}.$$

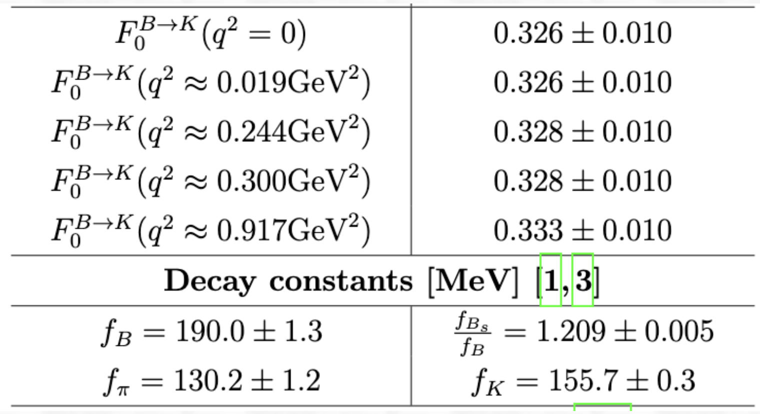
- ☐ **SU(3)-breaking effects from transition form factors, decay constants & phase space:**

$$A_{M_1M_2} = (m_B^2 - m_{M_1}^2)F_0^{B\to M_1}(m_{M_2}^2)f_{M_2}\,, \qquad B_{M_1M_2} = f_{B_q}f_{M_1}f_{M_2}\,,$$

➡ all sub-amplitudes symmetric, and the dominant SU(3)-breakings from $A_{M_1M_2}$ & $B_{M_1M_2}$ included



$B_{(s)} \to P$ transition form factors [107]	
$F_0^{B\to\pi}(q^2 = 0)$	0.192 ± 0.022
$F_0^{B\to\pi}(q^2 \approx 0.019\text{GeV}^2)$	0.192 ± 0.022
$F_0^{B\to\pi}(q^2 \approx 0.244\text{GeV}^2)$	0.193 ± 0.022
$F_0^{B\to\pi}(q^2 \approx 0.300\text{GeV}^2)$	0.193 ± 0.022
$F_0^{B\to\pi}(q^2 \approx 0.917\text{GeV}^2)$	0.197 ± 0.021
$F_0^{B_s\to\bar{K}}(q^2 = 0)$	0.203 ± 0.014
$F_0^{B_s\to\bar{K}}(q^2 \approx 0.019\text{GeV}^2)$	0.203 ± 0.014
$F_0^{B_s\to\bar{K}}(q^2 \approx 0.244\text{GeV}^2)$	0.205 ± 0.014
$F_0^{B_s\to\bar{K}}(q^2 \approx 0.300\text{GeV}^2)$	0.206 ± 0.014
$F_0^{B_s\to\bar{K}}(q^2 \approx 0.917\text{GeV}^2)$	0.210 ± 0.014



$F_0^{B\to K}(q^2 = 0)$	0.326 ± 0.010
$F_0^{B\to K}(q^2 \approx 0.019\text{GeV}^2)$	0.326 ± 0.010
$F_0^{B\to K}(q^2 \approx 0.244\text{GeV}^2)$	0.328 ± 0.010
$F_0^{B\to K}(q^2 \approx 0.300\text{GeV}^2)$	0.328 ± 0.010
$F_0^{B\to K}(q^2 \approx 0.917\text{GeV}^2)$	0.333 ± 0.010
Decay constants [MeV] [1,3]	
$f_B = 190.0 \pm 1.3$	$\frac{f_{B_s}}{f_B} = 1.209 \pm 0.005$
$f_\pi = 130.2 \pm 1.2$	$f_K = 155.7 \pm 0.3$

$\mathcal{O}(20-30\%)$ SU(3)-breaking effects predicted between $B \to \pi$ & $B \to K$ FFs in large recoil region

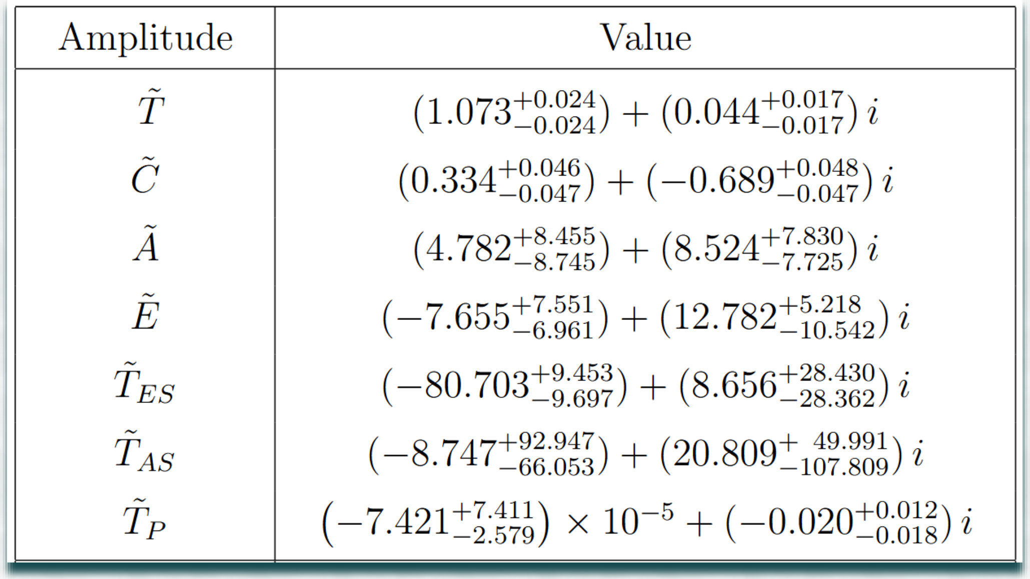
[B.-Y. Cui, Y.-K. Huang, Y.-L. Shen, C. Wang, and Y.-M. Wang, arXiv:2212.11624]

Global fit to data

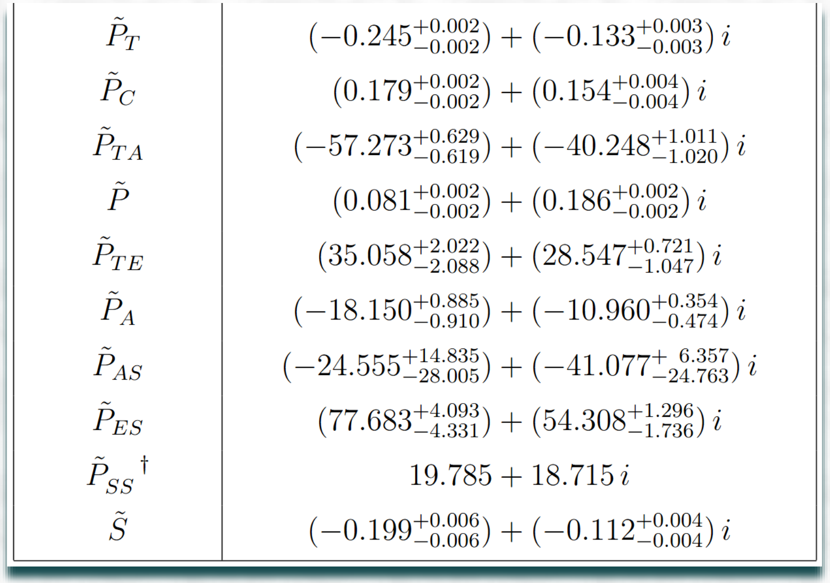
☐ **Enough exp. data to extract the TDA & QCDF amplitudes:**

23 branching ratios + 16 direct CP + 6 mixing-induced CP

☐ **Two indep. fitting methodologies:** Frequentist (CKMfitter) + Bayesian inference (Stan) framework



Amplitude	Value
$\tilde{T}$	$(1.073^{+0.024}_{-0.024}) + (0.044^{+0.017}_{-0.017})\,i$
$\tilde{C}$	$(0.334^{+0.046}_{-0.047}) + (-0.689^{+0.048}_{-0.047})\,i$
$\tilde{A}$	$(4.782^{+8.455}_{-8.745}) + (8.524^{+7.830}_{-7.725})\,i$
$\tilde{E}$	$(-7.655^{+7.551}_{-6.961}) + (12.782^{+5.218}_{-10.542})\,i$
$\tilde{T}_{ES}$	$(-80.703^{+9.453}_{-9.697}) + (8.656^{+28.430}_{-28.362})\,i$
$\tilde{T}_{AS}$	$(-8.747^{+92.947}_{-66.053}) + (20.809^{+\ 49.991}_{-107.809})\,i$
$\tilde{T}_{P}$	$(-7.421^{+7.411}_{-2.579}) \times 10^{-5} + (-0.020^{+0.012}_{-0.018})\,i$



$\tilde{P}_{T}$	$(-0.245^{+0.002}_{-0.002}) + (-0.133^{+0.003}_{-0.003})\,i$
$\tilde{P}_{C}$	$(0.179^{+0.002}_{-0.002}) + (0.154^{+0.004}_{-0.004})\,i$
$\tilde{P}_{TA}$	$(-57.273^{+0.629}_{-0.619}) + (-40.248^{+1.011}_{-1.020})\,i$
$\tilde{P}$	$(0.081^{+0.002}_{-0.002}) + (0.186^{+0.002}_{-0.002})\,i$
$\tilde{P}_{TE}$	$(35.058^{+2.022}_{-2.088}) + (28.547^{+0.721}_{-1.047})\,i$
$\tilde{P}_{A}$	$(-18.150^{+0.885}_{-0.910}) + (-10.960^{+0.354}_{-0.474})\,i$
$\tilde{P}_{AS}$	$(-24.555^{+14.835}_{-28.005}) + (-41.077^{+\ 6.357}_{-24.763})\,i$
$\tilde{P}_{ES}$	$(77.683^{+4.093}_{-4.331}) + (54.308^{+1.296}_{-1.736})\,i$
$\tilde{P}_{SS}{}^{\dagger}$	$19.785 + 18.715\,i$
$\tilde{S}$	$(-0.199^{+0.006}_{-0.006}) + (-0.112^{+0.004}_{-0.004})\,i$

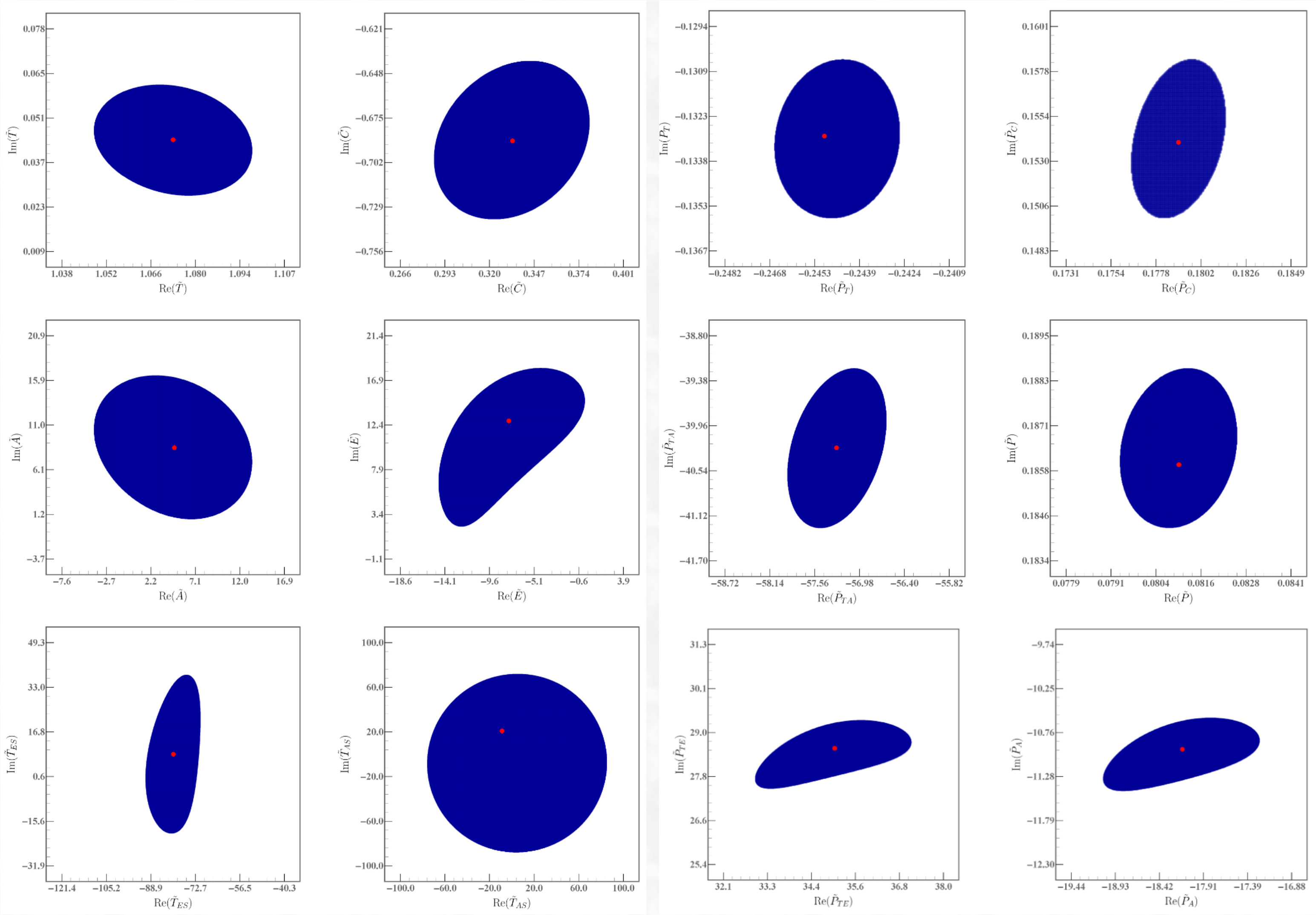
- central values resemble many dynamical features obtained in QCDF/SCET framework

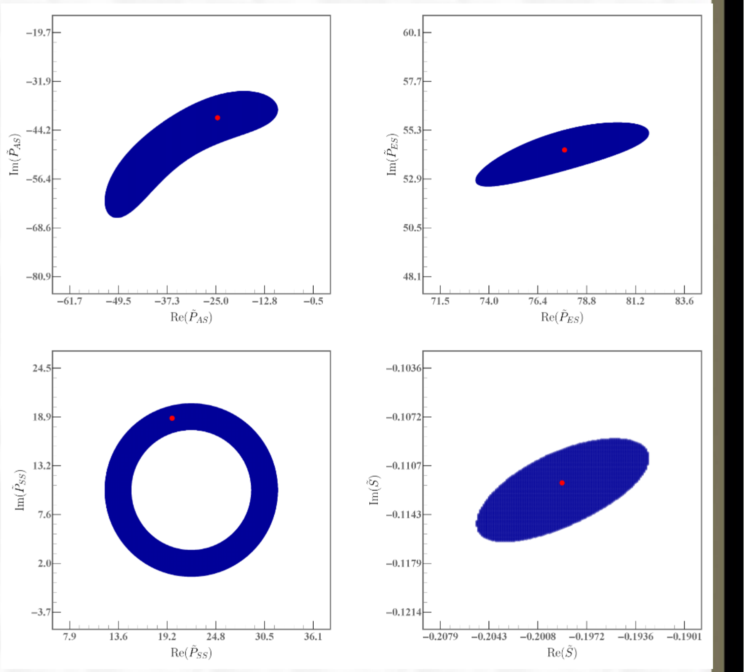
$$\alpha_1(\pi\pi) = 1.000^{+0.029}_{-0.069} + 0.011^{+0.023}_{-0.050}i, \qquad \alpha_2(\pi\pi) = 0.240^{+0.217}_{-0.125} - 0.077^{+0.115}_{-0.078}i$$

- no strong indications of numerically enhancements for annihilations beyond Λ_{QCD}/m_b scaling

Two-dimensional confidence regions

☐ 68% confidence regions of TDA amplitudes:





- Factorizable SU(3) breaking describes data perfectly, with a p-value of 0.072
- Electroweak penguin topologies larger than generally expected

Phenomenological implications

☐ **$B \to \pi\pi, \pi K$ and KK puzzles:**

$$R = \frac{\mathrm{Br}(\bar{B}^0 \to \pi^+K^-)}{\mathrm{Br}(B^- \to \pi^-\bar{K}^0)} \cdot \frac{\tau_{B^-}}{\tau_{B^0}} = 0.89 \pm 0.03 \ \text{vs} \ 0.90^{+0.24}_{-0.22}$$

$$R_c = \frac{2\mathrm{Br}(B^- \to \pi^0K^-)}{\mathrm{Br}(B^- \to \pi^-\bar{K}^0)} = 1.10 \pm 0.04 \ \text{vs} \ 1.11^{+0.28}_{-0.25}$$

$$R_n = \frac{1}{2}\frac{\mathrm{Br}(\bar{B}^0 \to \pi^+K^-)}{\mathrm{Br}(\bar{B}^0 \to \pi^0\bar{K}^0)} = 0.97 \pm 0.04 \ \text{vs} \ 0.98^{+0.31}_{-0.25}$$

$$R^{\pi\pi}_{+-} = \frac{2\mathrm{Br}(B^- \to \pi^-\pi^0)}{\mathrm{Br}(\bar{B}^0 \to \pi^+\pi^-)} \cdot \frac{\tau_{B^0}}{\tau_{B^-}} = 1.83 \pm 0.11 \ \text{vs} \ 1.86^{+0.76}_{-0.54}$$

$$R^{\pi\pi}_{00} = \frac{2\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)}{\mathrm{Br}(\bar{B}^0 \to \pi^+\pi^-)} = 0.58 \pm 0.07 \ \text{vs} \ 0.56^{+0.24}_{-0.17}$$

$$\Delta A_{CP}(\pi K) = A_{CP}(\pi^0K^-) - A_{CP}(\pi^+K^-)$$

$$= (11.0 \pm 1.2)\% \ \text{vs} \ (11.1 \pm 2.9)\%$$

☐ **Badly Broken of EWP to tree ratios:**

[Gronau and Rosner, hep-ph/9809384]

$$P_{EW} = P_T = -\frac{3}{2}\frac{C_9 + C_{10}}{C_1 + C_2}C,$$

$$P^C_{EW} = P_C = -\frac{3}{2}\frac{C_9 + C_{10}}{C_1 + C_2}T.$$

orders of magnitude larger than predicted by EWP-tree ratios

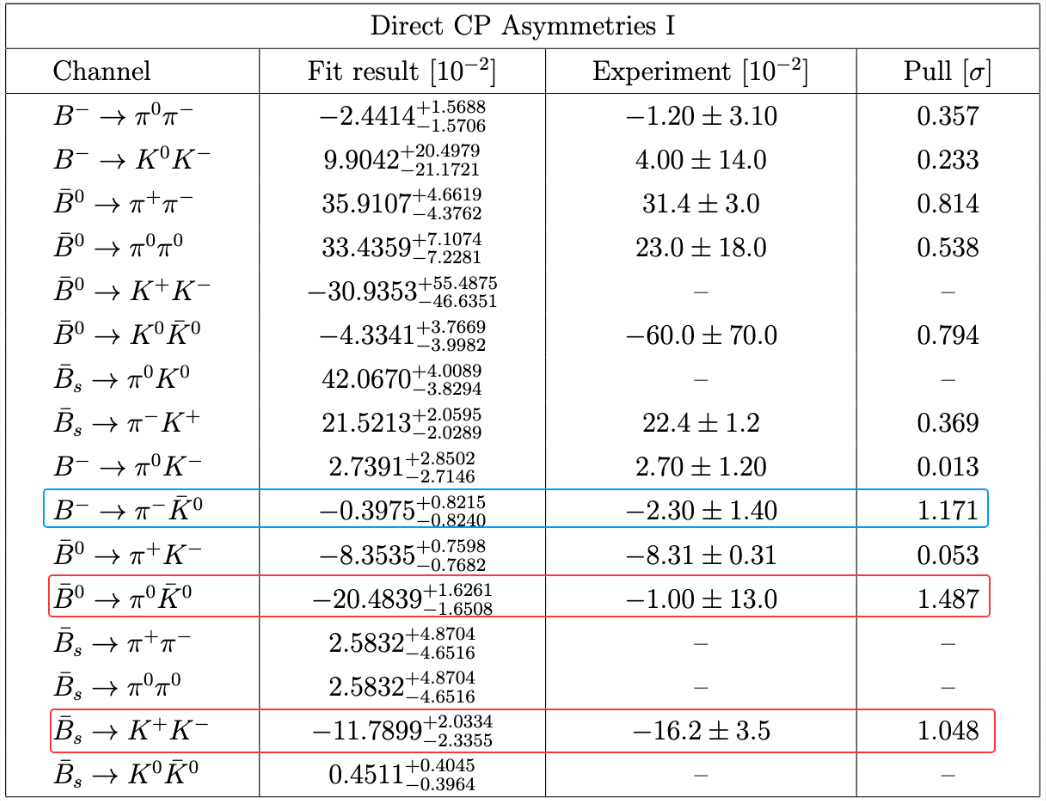
☐ **Sum rule for $B \to \pi K$ decays:** [Gronau, hep-ph/0508047]

$$\Delta_{\mathrm{SR}} = A_{CP}(B^- \to \pi^0K^-)\frac{2\mathrm{Br}(B^- \to \pi^0K^-)}{\mathrm{Br}(\bar{B}^0 \to \pi^+K^-)}\frac{\tau_{B^0}}{\tau_{B^-}} + A_{CP}(\bar{B}^0 \to \pi^0\bar{K}^0)\frac{2\mathrm{Br}(\bar{B}^0 \to \pi^0\bar{K}^0)}{\mathrm{Br}(\bar{B}^0 \to \pi^+K^-)}$$

$$-A_{CP}(\bar{B}^0 \to \pi^+K^-) - A_{CP}(B^- \to \pi^-\bar{K}^0)\frac{\mathrm{Br}(B^- \to \pi^-\bar{K}^0)}{\mathrm{Br}(\bar{B}^0 \to \pi^+K^-)}\frac{\tau_{B^0}}{\tau_{B^-}} = 0.03 \pm 0.14 \ \text{vs} \ -0.09 \pm 0.03$$

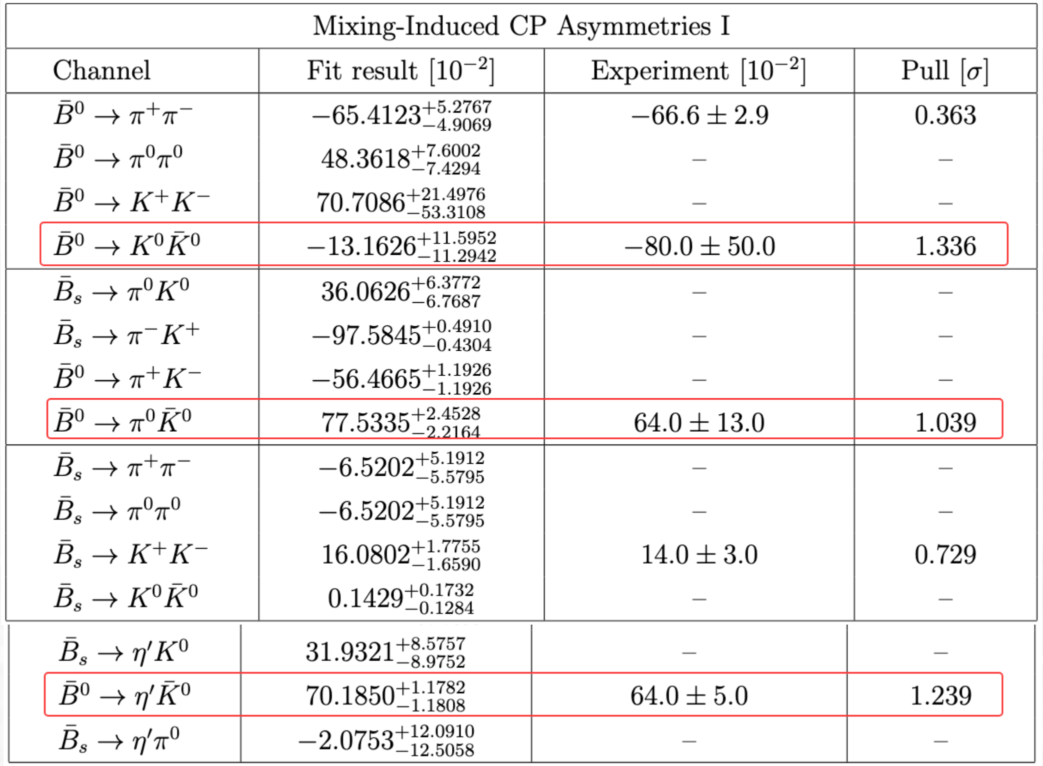
Phenomenological implications

- **small tensions in $B \to \pi K, KK, \eta' K$ direct & mixing-induced CPV, but not exceed 1.5σ**



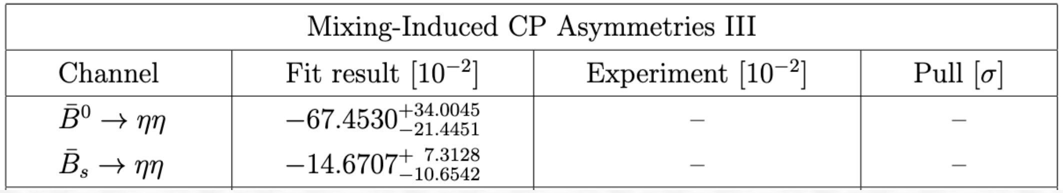
Direct CP Asymmetries I			
Channel	Fit result $[10^{-2}]$	Experiment $[10^{-2}]$	Pull $[\sigma]$
$B^- \to \pi^0\pi^-$	$-2.4414^{+1.5688}_{-1.5706}$	-1.20 ± 3.10	0.357
$B^- \to K^0K^-$	$9.9042^{+20.4979}_{-21.1721}$	4.00 ± 14.0	0.233
$\bar{B}^0 \to \pi^+\pi^-$	$35.9107^{+4.6619}_{-4.3762}$	31.4 ± 3.0	0.814
$\bar{B}^0 \to \pi^0\pi^0$	$33.4359^{+7.1074}_{-7.2281}$	23.0 ± 18.0	0.538
$\bar{B}^0 \to K^+K^-$	$-30.9353^{+55.4875}_{-46.6351}$	–	–
$\bar{B}^0 \to K^0\bar{K}^0$	$-4.3341^{+3.7669}_{-3.9982}$	-60.0 ± 70.0	0.794
$\bar{B}_s \to \pi^0K^0$	$42.0670^{+4.0089}_{-3.8294}$	–	–
$\bar{B}_s \to \pi^-K^+$	$21.5213^{+2.0595}_{-2.0289}$	22.4 ± 1.2	0.369
$B^- \to \pi^0K^-$	$2.7391^{+2.8502}_{-2.7146}$	2.70 ± 1.20	0.013
$B^- \to \pi^-\bar{K}^0$	$-0.3975^{+0.8215}_{-0.8240}$	-2.30 ± 1.40	1.171
$\bar{B}^0 \to \pi^+K^-$	$-8.3535^{+0.7598}_{-0.7682}$	-8.31 ± 0.31	0.053
$\bar{B}^0 \to \pi^0\bar{K}^0$	$-20.4839^{+1.6261}_{-1.6508}$	-1.00 ± 13.0	1.487
$\bar{B}_s \to \pi^+\pi^-$	$2.5832^{+4.8704}_{-4.6516}$	–	–
$\bar{B}_s \to \pi^0\pi^0$	$2.5832^{+4.8704}_{-4.6516}$	–	–
$\bar{B}_s \to K^+K^-$	$-11.7899^{+2.0334}_{-2.3355}$	-16.2 ± 3.5	1.048
$\bar{B}_s \to K^0\bar{K}^0$	$0.4511^{+0.4045}_{-0.3964}$	–	–

- **Our predictions for some unmeasured observables can be tested by future data**



Mixing-Induced CP Asymmetries I			
Channel	Fit result $[10^{-2}]$	Experiment $[10^{-2}]$	Pull $[\sigma]$
$\bar{B}^0 \to \pi^+\pi^-$	$-65.4123^{+5.2767}_{-4.9069}$	-66.6 ± 2.9	0.363
$\bar{B}^0 \to \pi^0\pi^0$	$48.3618^{+7.6002}_{-7.4294}$	–	–
$\bar{B}^0 \to K^+K^-$	$70.7086^{+21.4976}_{-53.3108}$	–	–
$\bar{B}^0 \to K^0\bar{K}^0$	$-13.1626^{+11.5952}_{-11.2942}$	-80.0 ± 50.0	1.336
$\bar{B}_s \to \pi^0K^0$	$36.0626^{+6.3772}_{-6.7687}$	–	–
$\bar{B}_s \to \pi^-K^+$	$-97.5845^{+0.4910}_{-0.4304}$	–	–
$\bar{B}^0 \to \pi^+K^-$	$-56.4665^{+1.1926}_{-1.1926}$	–	–
$\bar{B}^0 \to \pi^0\bar{K}^0$	$77.5335^{+2.4528}_{-2.2164}$	64.0 ± 13.0	1.039
$\bar{B}_s \to \pi^+\pi^-$	$-6.5202^{+5.1912}_{-5.5795}$	–	–
$\bar{B}_s \to \pi^0\pi^0$	$-6.5202^{+5.1912}_{-5.5795}$	–	–
$\bar{B}_s \to K^+K^-$	$16.0802^{+1.7755}_{-1.6590}$	14.0 ± 3.0	0.729
$\bar{B}_s \to K^0\bar{K}^0$	$0.1429^{+0.1732}_{-0.1284}$	–	–
$\bar{B}_s \to \eta'K^0$	$31.9321^{+8.5757}_{-8.9752}$	–	–
$\bar{B}^0 \to \eta'\bar{K}^0$	$70.1850^{+1.1782}_{-1.1808}$	64.0 ± 5.0	1.239
$\bar{B}_s \to \eta'\pi^0$	$-2.0753^{+12.0910}_{-12.5058}$	–	–

  - ➢ **More data on $B \to \pi K, KK, \eta' K$ welcome**



Mixing-Induced CP Asymmetries III			
Channel	Fit result $[10^{-2}]$	Experiment $[10^{-2}]$	Pull $[\sigma]$
$\bar{B}^0 \to \eta\eta$	$-67.4530^{+34.0045}_{-21.4451}$	–	–
$\bar{B}_s \to \eta\eta$	$-14.6707^{+\ 7.3128}_{-10.6542}$	–	–

Summary

- ☐ **Hadronic B decays important for testing precisely the SM and probing NP beyond it**
- ☐ **Within QCDF/SCET, NNLO QCD corrections to color-allowed, color-suppressed tree & leading-power penguin amplitudes complete, factorization established up to 2-loop**
- ☐ **Due to delicate cancellation, NNLO corrections found small; some puzzles still remain:**
  - ➢ long-standing $\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(B^- \to \pi^0 K^-) - A_{CP}(\bar{B}^0 \to \pi^+ K^-)$
  - ➢ sub-leading power corrections in QCDF/SCET need be considered, and progress already achieved
- ☐ **Enough data allow to perform global fit based on flavor SU(3) symmetry**
  - ➢ dominant SU(3)-breaking effects from transition form factors, decay constants, and phase space
  - ➢ information from QCDF dynamics helps to reduce TDA parameters from 20 to 17 or 16
  - ➢ very good fit to all data achieved and resemble many dynamical features obtained in QCDF
  - ➢ many dynamical features in QCDF/SCET reproduced, but large EWP amplitudes needed

Thank You for your attention!