Detector operators in High Energy QCD

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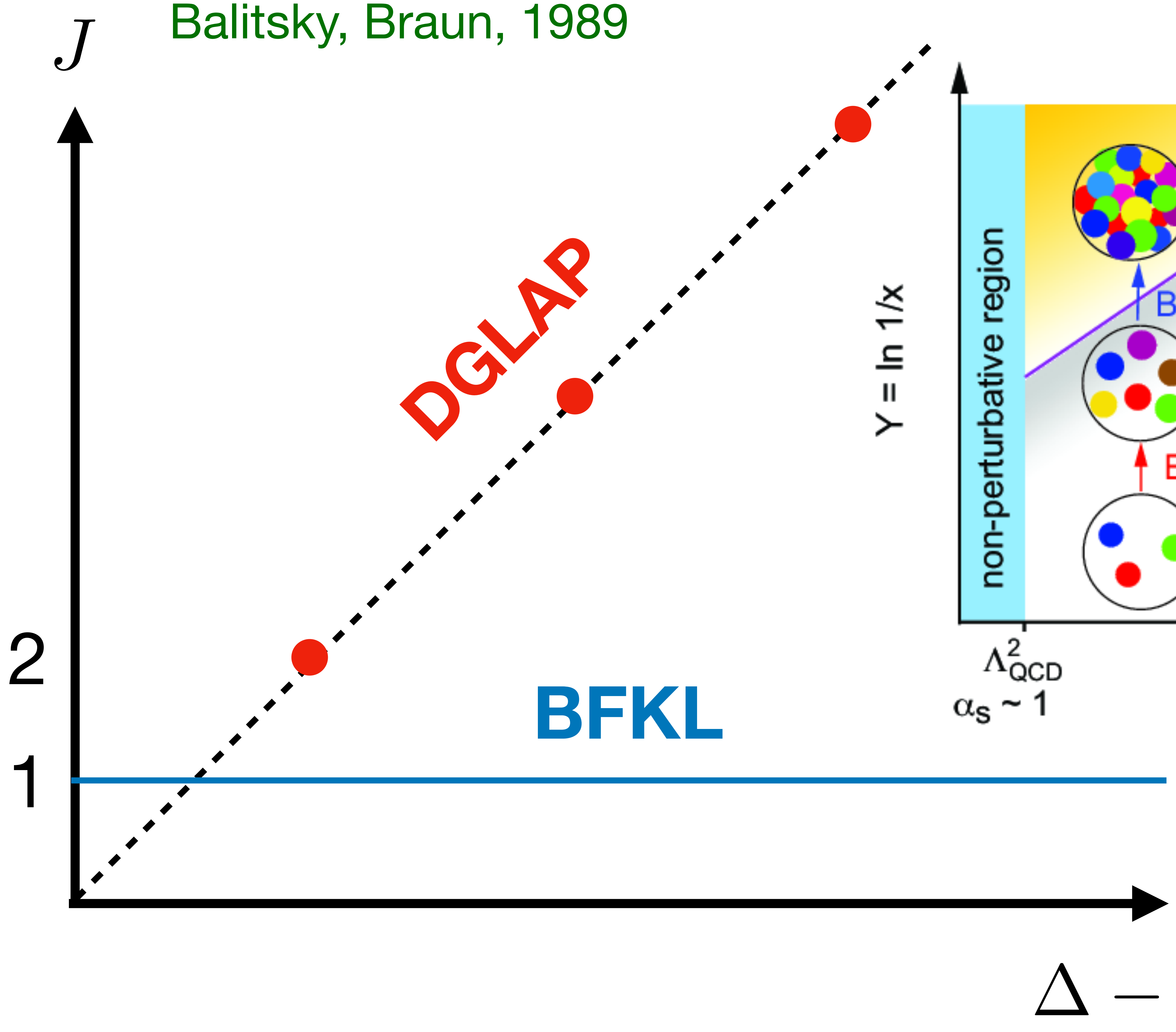
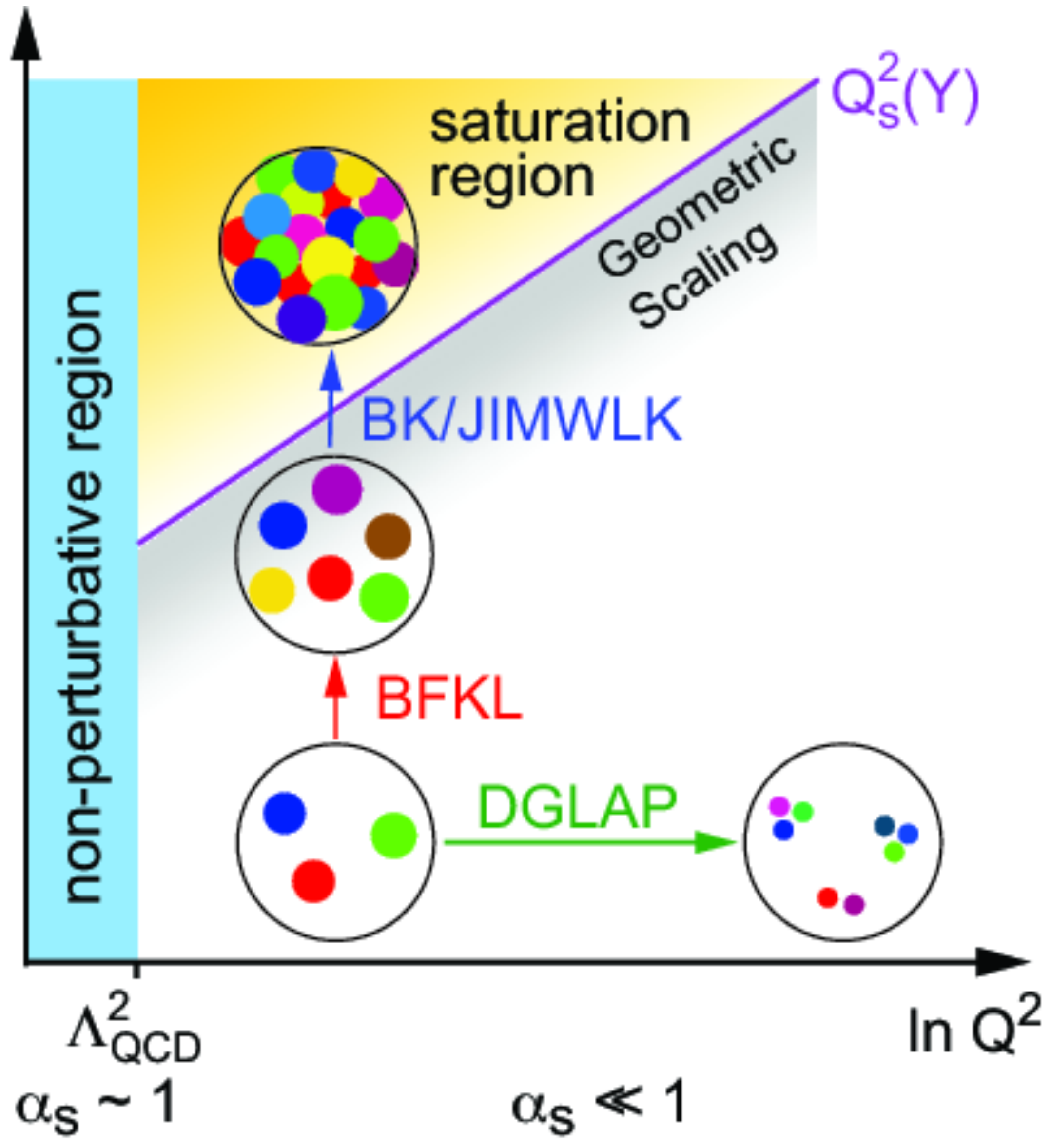
The Chew-Fraustri plot in high energy QCD

connecting operator into lines

$$\mathcal{O}_g^{\mu_1\dots\mu_J} = S\left[\mathrm{Tr}(G^{\mu_1\alpha}D^{\mu_2}\dots D^{\mu_{J-1}}G^{\mu_J}{}_{\alpha})\right] - \text{traces}$$

Balitsky, Braun, 1989





twist-2 spin-J local operator

Gross, Wilczek, 1974

Gribov, Lipatov; Lipatov; Altarelli, Parisi; Dokshitzer; 1972-1977

Curci, Furmanski, Petronzio,1980

Moch, Vermaseren, and Vogt, 2004

4-loop frontier:

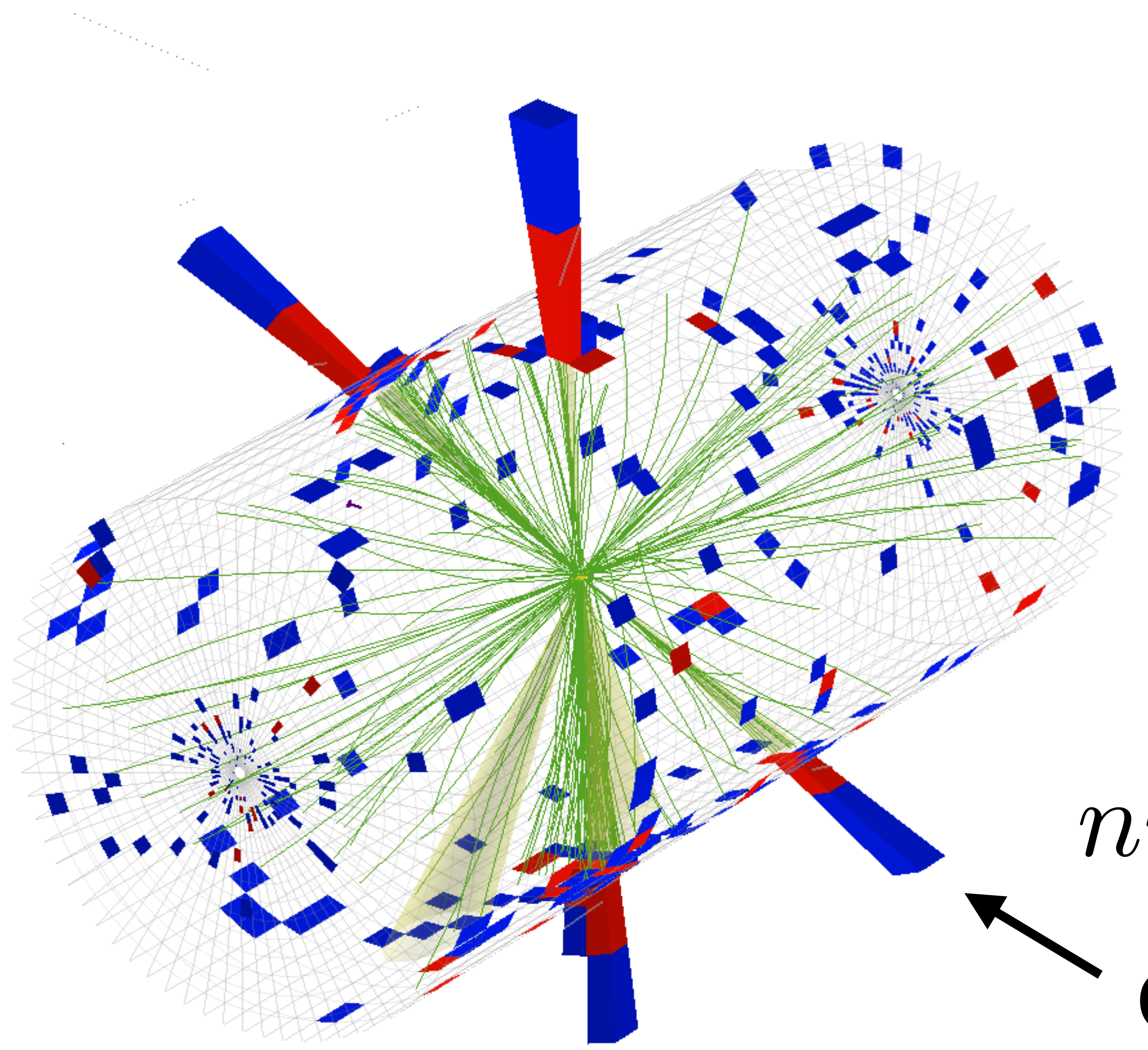
finite # of moment: Moch, Ruijl, Ueda, Vermaseren, Vogt, 2018

Full non-singlet: Gehrmann, von Manteuffel, Sotnikov, T.Z. Yang, 2026

$$\gamma_{GG}^{\mathrm{LL}x}(J,\alpha_s) = -2\frac{\lambda}{J-1} - 4\zeta(3)\frac{\lambda^4}{(J-1)^4} - 4\zeta(5)\frac{\lambda^6}{(J-1)^6} + \cdots$$

Jaroszewicz, 1982

Detector operators



$n^\mu = (1, \boldsymbol{n})$

detector

Measurement of energy/charge flow

Sterman, 1975
Sveshnikov, Tkachov, 1995
Hofman, Maldacena, 2008
Kravchuk, Simmons-Duffin, 2018
Caron-Huot, Kologlu, Kravchuk, Meltzer, Simmons-Duffin, 2022

energy operator: $$\mathcal{E}(n) = \int \frac{E^2 dE}{(2\pi)^3 2E} \, E a^\dagger_{h,En} a_{h,En}$$

charge operator: $$\mathcal{Q}(n) = \int \frac{E^2 dE}{(2\pi)^3 2E} \, Q_h a^\dagger_{h,En} a_{h,En}$$

$$\langle \mathcal{E}(n) \rangle$$

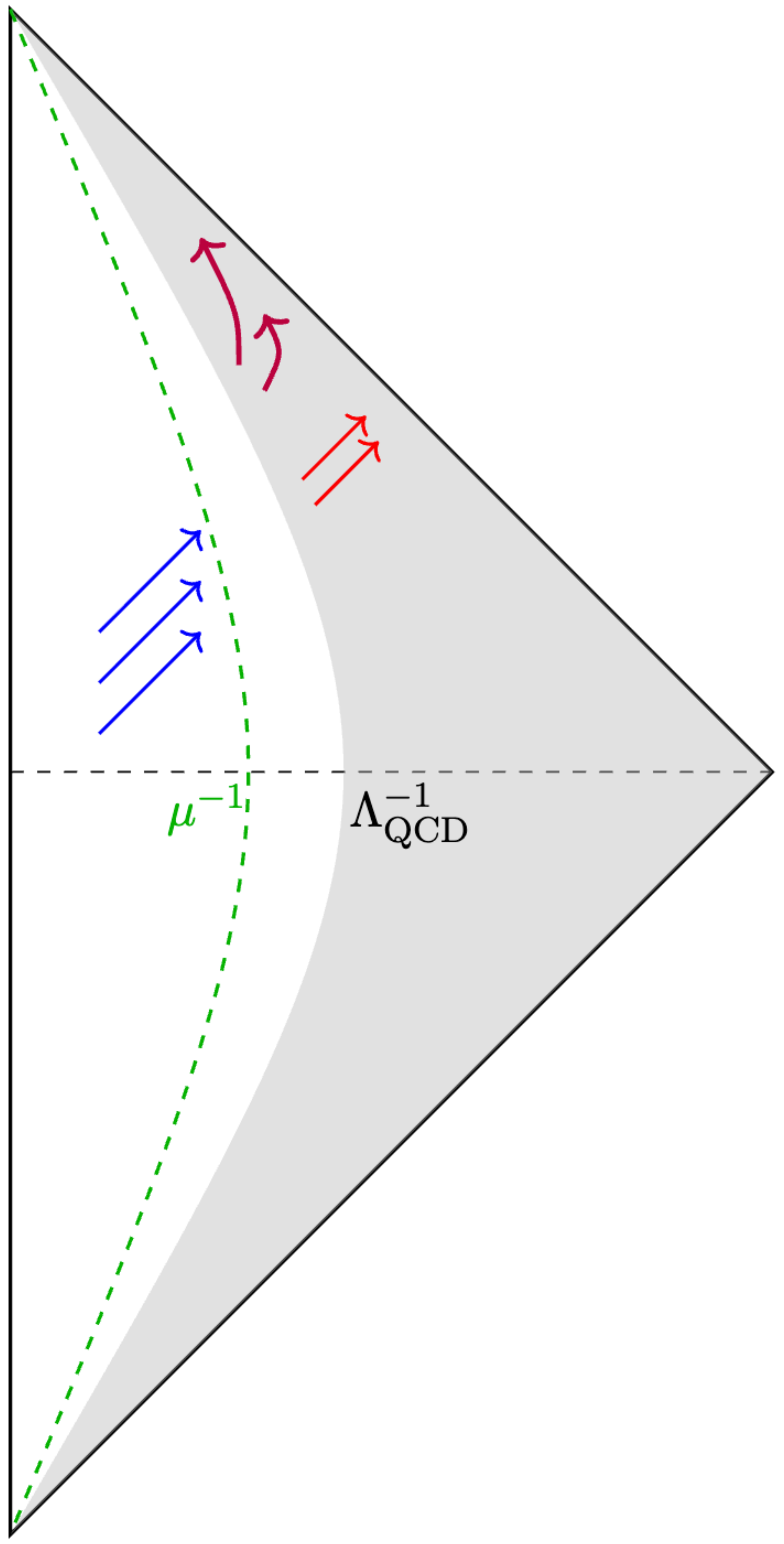
$$\langle \mathcal{E}(n_1) \mathcal{E}(n_2) \rangle$$

$$\langle \mathcal{E}(n_1) \mathcal{E}(n_2) \mathcal{E}(n_3) \rangle$$

see 2506.09119 for a review

Detector operator matching

C.-H. Chang, H. Chen, Simmons-Duffin, HXZ, 2025



Penrose diagram

hadron detector $\mathbb{N}_{J_L}(n) = \int \frac{E^2 dE}{(2\pi)^3 2E} E^{-J_L-2} a^\dagger_{h,En} a_{h,En}$

IR to UV matching $\mathbb{N}_{J_L}(z) \simeq \sum_k C_k(J_L,\mu)\,[\mathcal{D}_{J_L,k}]_R(z;\mu)$

C_k: matching coeff.

$\mathcal{D}_{J_L,k}$: parton detector

$\langle \mathbb{N}_L \rangle \sim Q^{-\Delta_L}$

$$\mathcal{D}^{\rm DGLAP}_{J_L}(n) = \sum_\lambda \int \frac{E^2 dE}{(2\pi)^3 2E} E^{-J_L-2} a^\dagger_{\lambda,En} a_{\lambda,En}$$

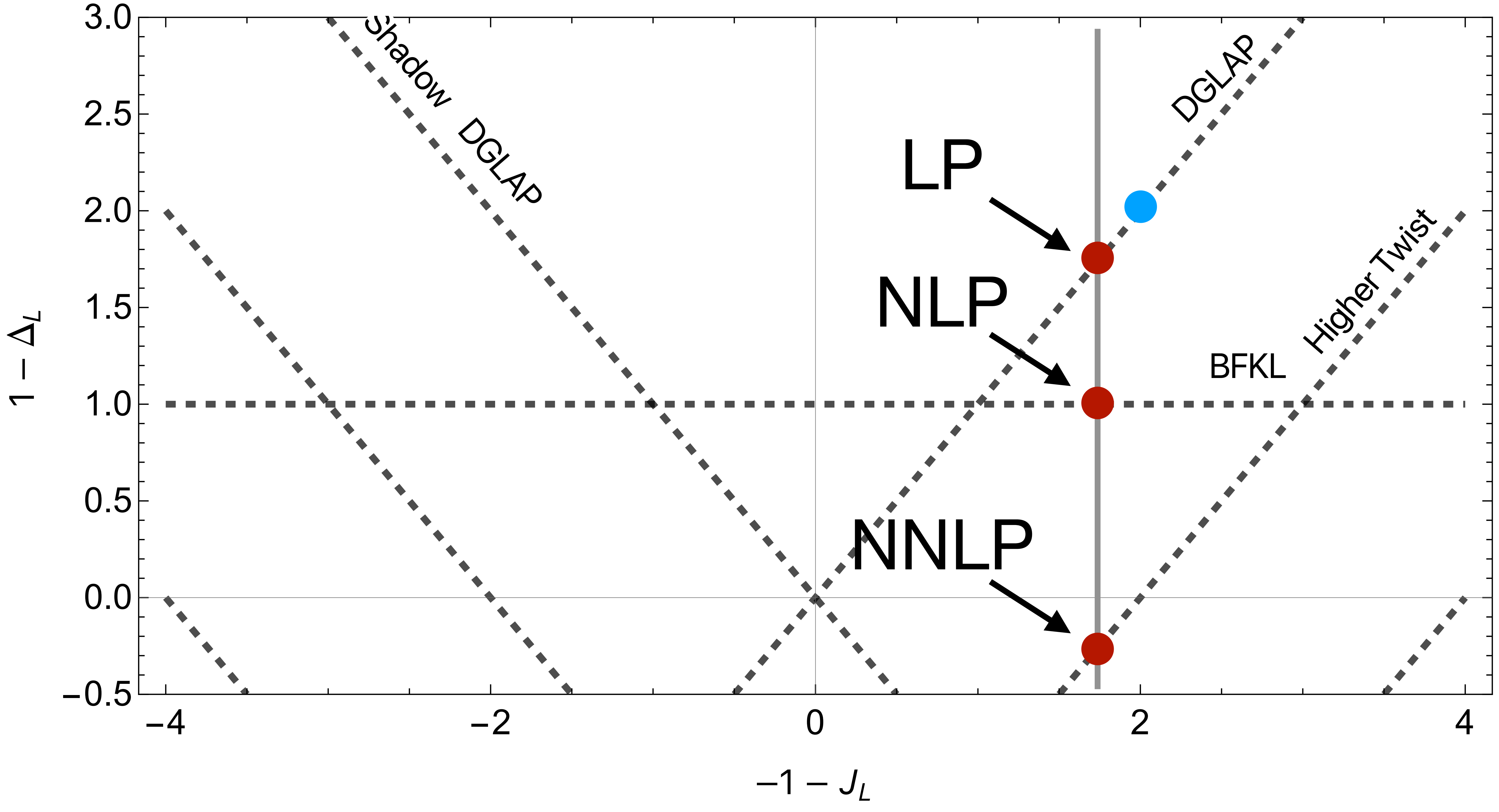
$\sum_\lambda$: sum over parton flavor, spin…

$(-\Delta_L, J_L)$

scaling dimension — spin

DGLAP trajectory

$$\mathcal{D}^{\mathrm{DGLAP}}_{J_L}(n) = \sum_\lambda \int \frac{E^2 dE}{(2\pi)^3 2E} E^{-J_L-2} a^\dagger_{\lambda,En} a_{\lambda,En}$$



J_L is a sliding parameter controlled by experimentalist

At J_L=-3 collapse to energy operator

$$\mathcal{E}(n) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, T_{0\boldsymbol{n}}(t, r\boldsymbol{n})$$

A light-transform to local operator

$$(\Delta, J) \to (\Delta_L = 1 - J, J_L = 1 - \Delta)$$

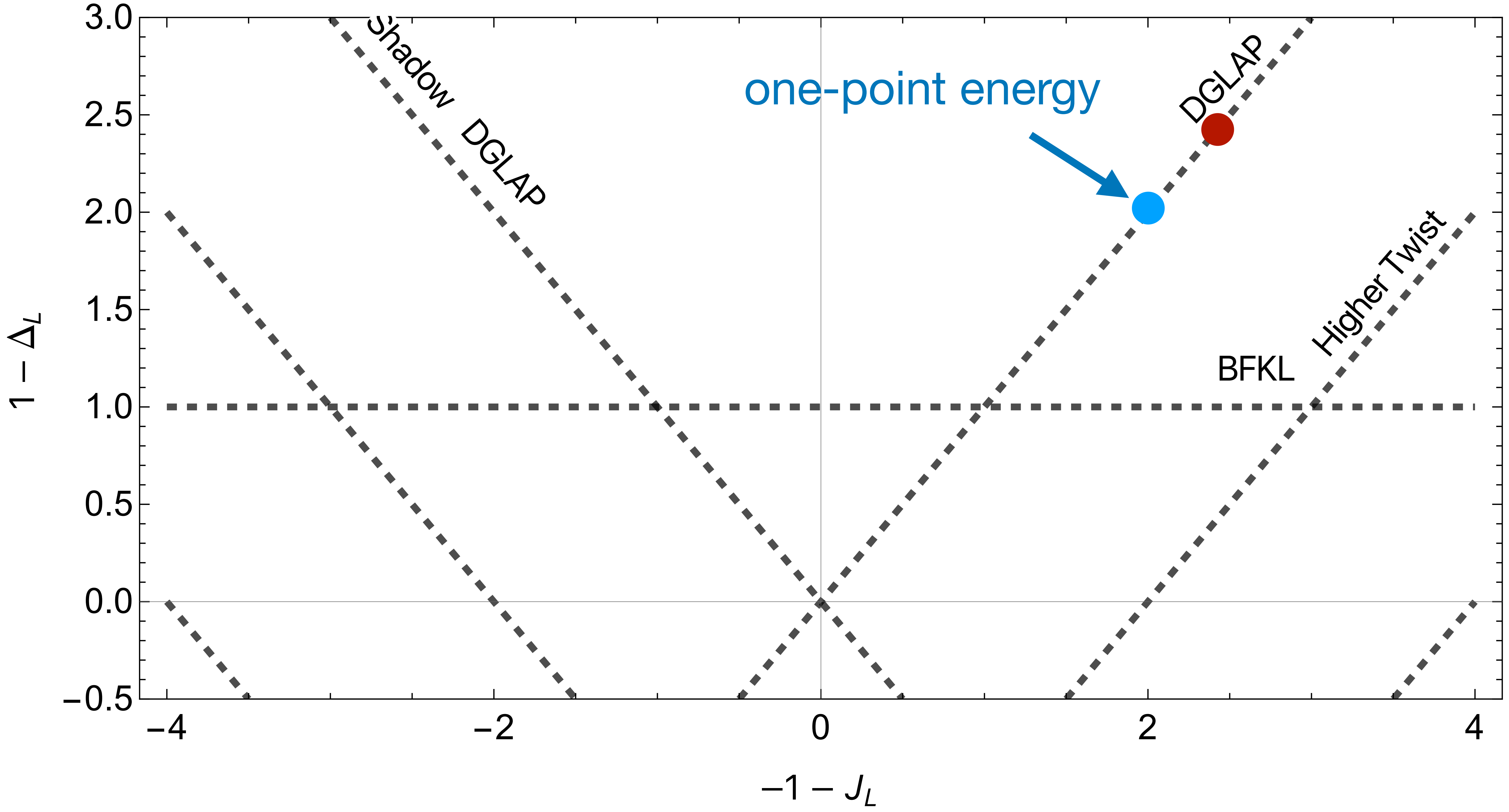
$$\langle D_{J_L}(n)\rangle \sim \sum_i E_i^{-J_L-2} \delta^2(\hat{p}_i - \boldsymbol{n})$$

Power corrections in Λ / Q:

H. Chen, Monni, Z. Xu, HXZ, 2024
C.-H. Chang, H. Chen, Simmons-Duffin, HXZ, 2025
H. Chen, Y.B. Li, 2026

One-point energy correlator

$$\mathcal{D}_{J_L}^{\mathrm{DGLAP}}(n) = \sum_{\lambda} \int \frac{E^2 dE}{(2\pi)^3 2E} E^{-J_L-2} a^{\dagger}_{\lambda,En} a_{\lambda,En}$$



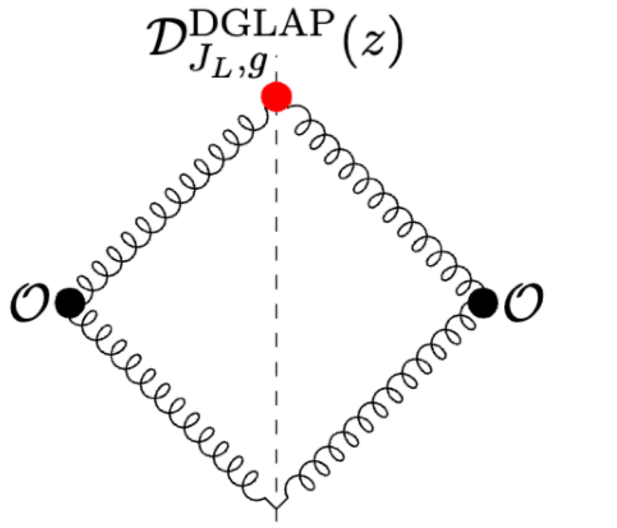
$$\mathbb{N}_{J_L}(z) \simeq \sum_k C_k\left(J_L, \mu\right)\left[\mathcal{D}_{J_L,k}\right]_R(z;\mu)$$

$\langle \mathbb{N}_L \rangle \sim Q^{-\Delta_L}$ (ignoring running coup.)

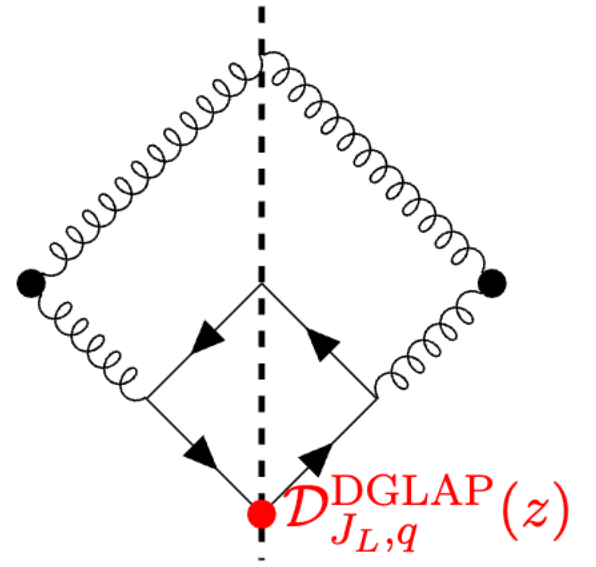
To obtain the anomalous dimension, compute vacuum matrix element:

$$\int d^4x\, e^{ixq} \langle \mathcal{O}(x) \mathcal{D}_{J_L}(z) \mathcal{O}(0) \rangle$$

tree level:



one-loop:



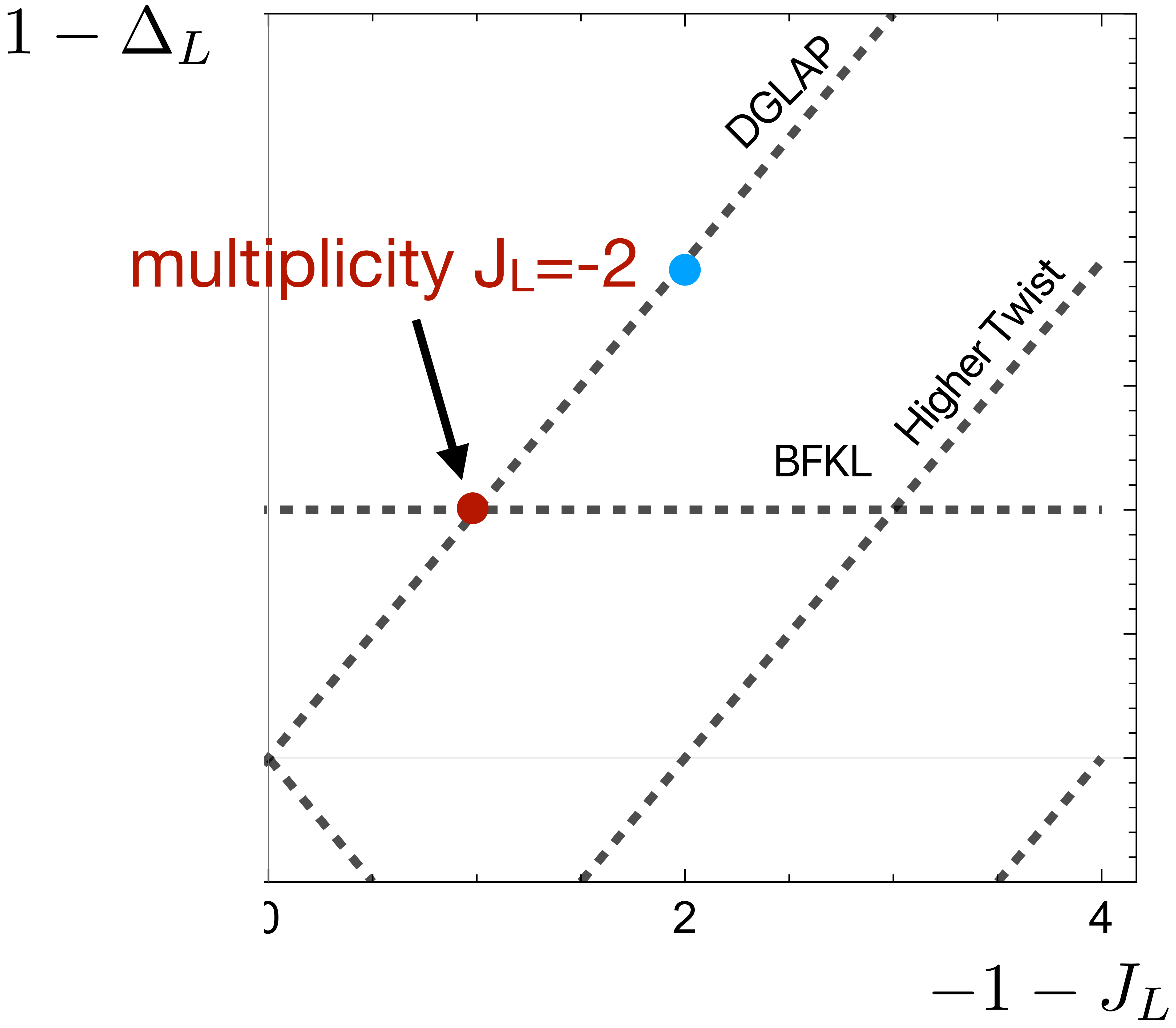
+ ...

For recent application of one-point correlator:
Riembau, Son, 2025, PRL
Y.K. Song, S.Y. Wei, L. Yang, J. Zhou, 2025, PRL
M.S. Gao, Z.B. Kang, W.C. Li, D.Y. Shao, 2025, PRL

Signal of intersection

$$\mathcal{D}^{\rm DGLAP}_{J_L}(n) = \sum_\lambda \int \frac{E^2 dE}{(2\pi)^3 2E} E^{-J_L-2} a^\dagger_{\lambda,En} a_{\lambda,En}$$

The renormalization factor must take into account the pole due to mixing with other trajectory



$$\mathcal{Z}_{J_L} = 1 + \begin{pmatrix} \frac{4C_A}{\epsilon(J_L+2)} + \frac{\widetilde{R}_1(\epsilon)}{J_L+2} + \frac{\gamma_1(J_L)}{\epsilon} & ? \\ ? & ? \end{pmatrix}$$

pure Yang-Mills

To solve the mixing problem:

Must define the trajectory mixed with DGLAP at J_L=-2, and compute matrix elements for it

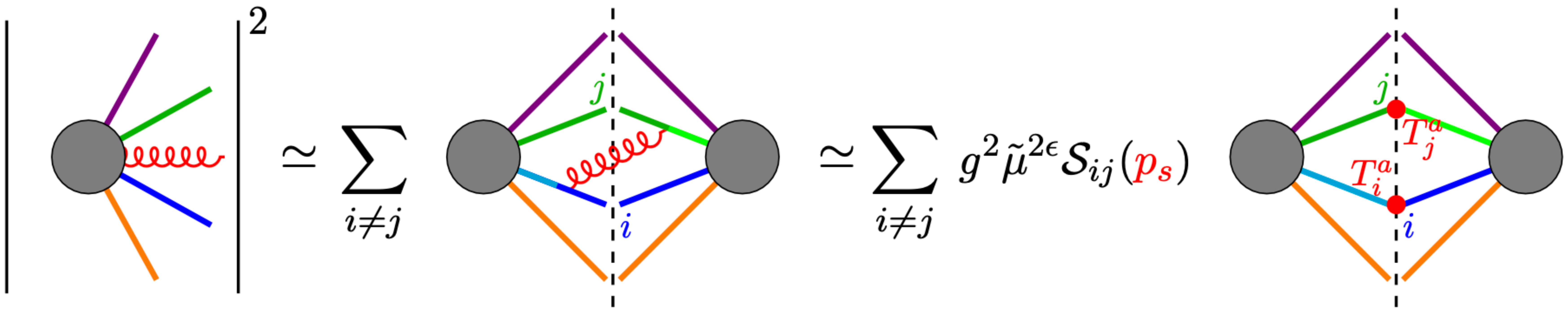
Must redefine a non-degenerate basis at the intersection point

$$H = \begin{pmatrix} 1-j & \alpha \\ 0 & 0 \end{pmatrix} \quad v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad v_2 = \begin{pmatrix} \frac{a}{j-1} \\ 1 \end{pmatrix}$$

The origin of J_L=-2 singularity

At J_L=-2, sensitive to soft gluon emission

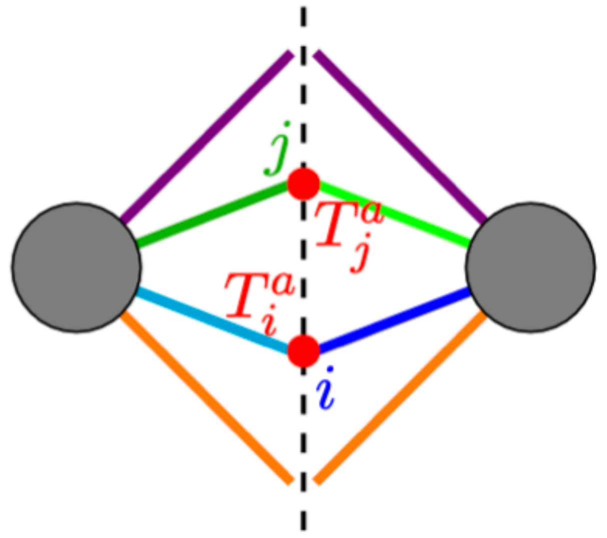
$$\mathcal{D}_{J_L}^{\mathrm{DGLAP}}(n) = \sum_\lambda \int \frac{E^2 dE}{(2\pi)^3 2E} E^{-J_L-2} a^\dagger_{\lambda,En} a_{\lambda,En}$$

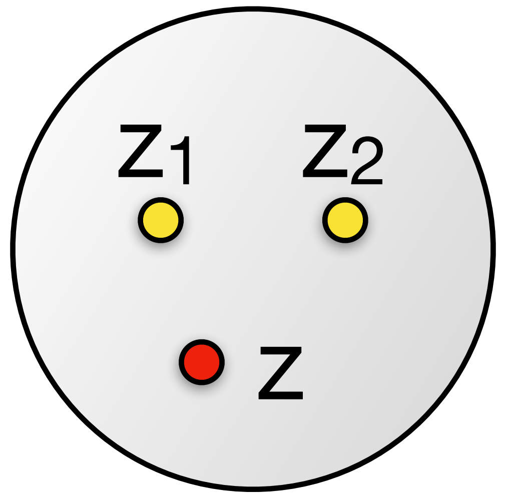


$$\mathcal{S}_{ij}(p_s) = \frac{p_i \cdot p_j}{(p_s \cdot p_i)(p_s \cdot p_j)}$$

The BFKL detector

$$\sum_{i\neq j} g^2 \tilde{\mu}^{2\epsilon} \mathcal{S}_{ij}(p_s)$$





$$\mathcal{D}^{\mathrm{BFKL}}_{J_L,g}(z) \equiv \mathcal{D}_{2-d,2-d;J_L,g}(z)$$

$$= \frac{\Gamma(d-2+J_L)}{\Gamma(\frac{d-2+J_L}{2})^2} \int D^{d-2}z_1 D^{d-2}z_2 \underbrace{\left(\frac{2z_1\cdot z_2}{(2z_1\cdot z)(2z_2\cdot z)}\right)^{-\frac{J_L}{2}}}_{\text{soft factor}} \underbrace{:\mathcal{N}_g^c(z_1)\mathcal{N}_g^c(z_2):}_{\text{color-color correlator}}\,.$$

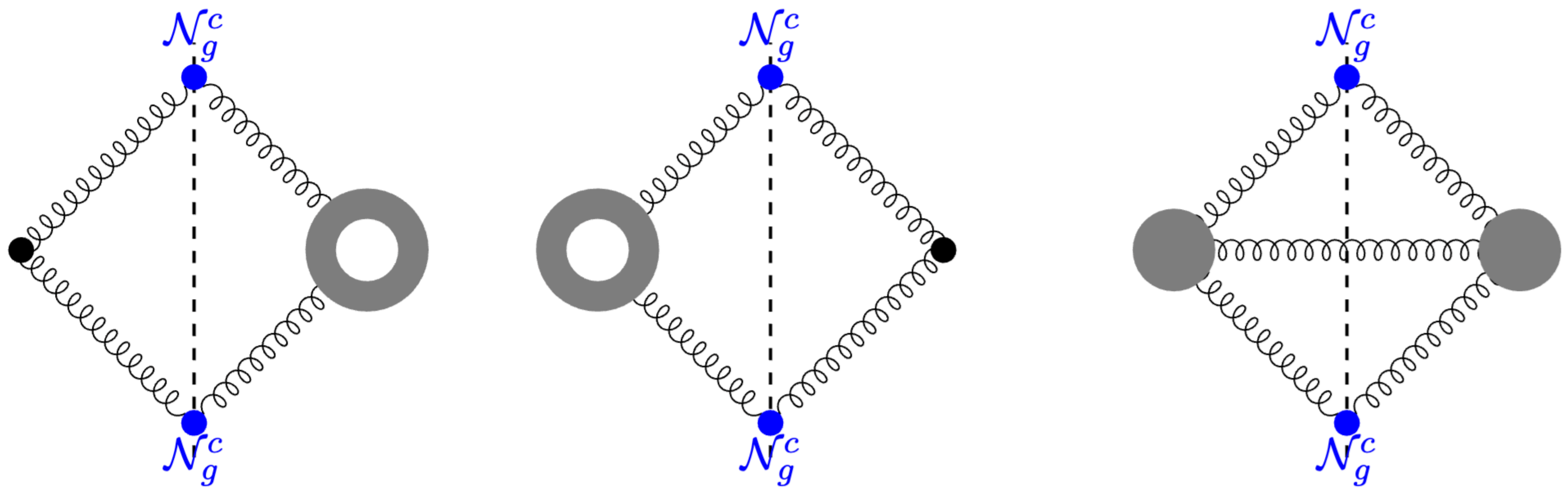
$$\mathcal{N}_g^a(z) = i f^{abc} \sum_\lambda \int_0^\infty \frac{E^{d-2}dE}{(2\pi)^{d-1}2E} \left[a^\dagger_{\lambda,c}(p) a_{\lambda,b}(p)\right]\Big|_{p=Ez}$$

soft factor

color-color correlator

quantum number $(0, J_L)$

One-loop matrix element:

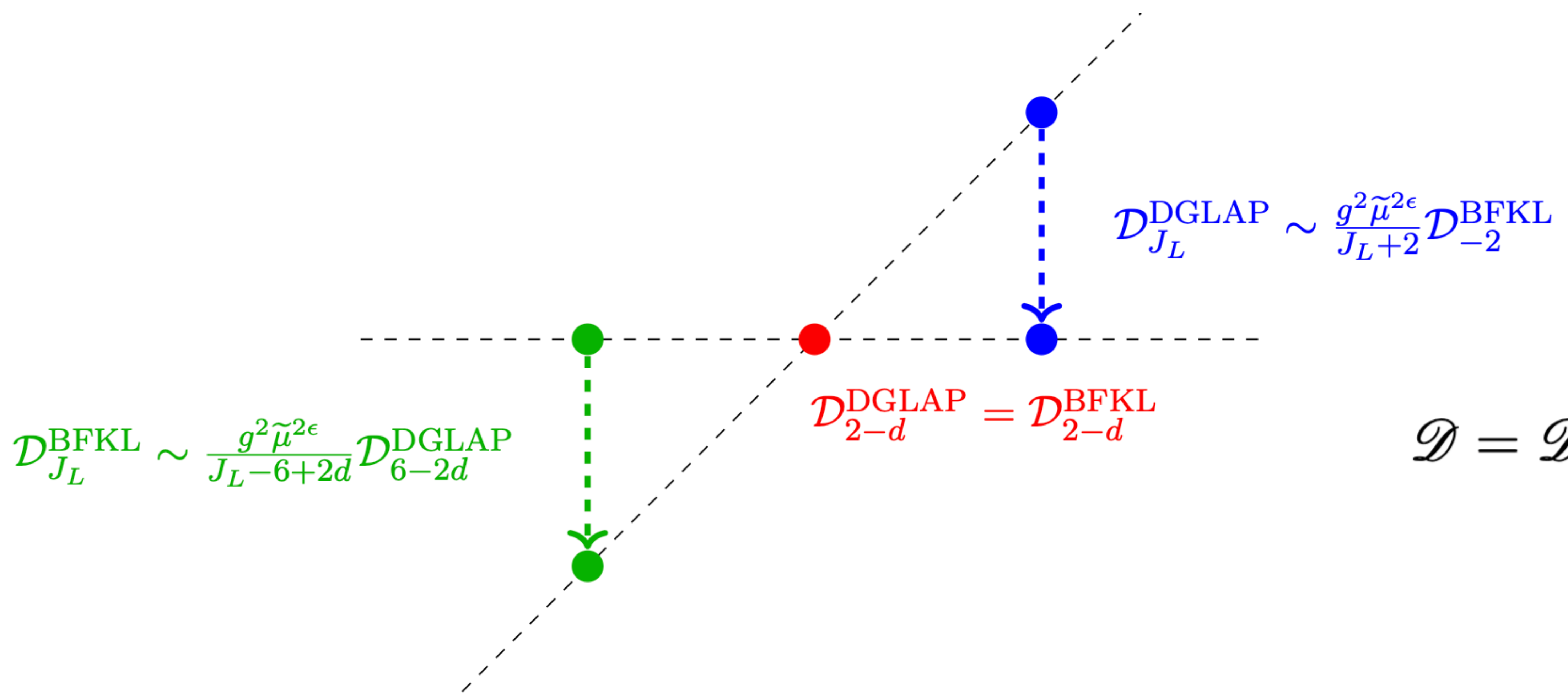


Full mixing structure

A non-degenerate basis

$$\mathbb{D}_{J_L} = U_1 \begin{pmatrix} \mu^{J_L+2-2\epsilon}\mathcal{D}^{\mathrm{DGLAP}}_{J_L,g} \\ \mathcal{D}^{\mathrm{BFKL}}_{J_L,g} \end{pmatrix}, \quad U_1 = \begin{pmatrix} -\frac{C_A\pi^{1-\epsilon}}{\Gamma(1-\epsilon)} & 0 \\ \frac{\frac{C_A\pi^{1-\epsilon}}{\Gamma(1-\epsilon)}}{J_L+2-2\epsilon} & \frac{1}{J_L+2-2\epsilon} \end{pmatrix}$$

$$\mathcal{Z}_{J_L} = 1 + \begin{pmatrix} \frac{4C_A}{\epsilon(J_L+2)} + \frac{\widetilde{R}_1(\epsilon)}{J_L+2} + \frac{\gamma_1(J_L)}{\epsilon} & -\frac{2\left(4C_A+\epsilon\widetilde{R}_1(\epsilon)\right)}{J_L+2} \\ -\frac{8C_A}{\epsilon(J_L+2)(J_L+2-4\epsilon)} + \frac{\widetilde{\mathcal{R}}_1(\epsilon)}{2\epsilon(J_L+2)} + \frac{\widetilde{\mathcal{R}}_2(\epsilon)}{2\epsilon(J_L+2-4\epsilon)} + \frac{\gamma_{21}(J_L)}{\epsilon} & -\frac{4C_A}{\epsilon(J_L+2)} - \frac{\widetilde{\mathcal{R}}_1(\epsilon)}{J_L+2} + \frac{\gamma_2(J_L)}{\epsilon} \end{pmatrix}$$



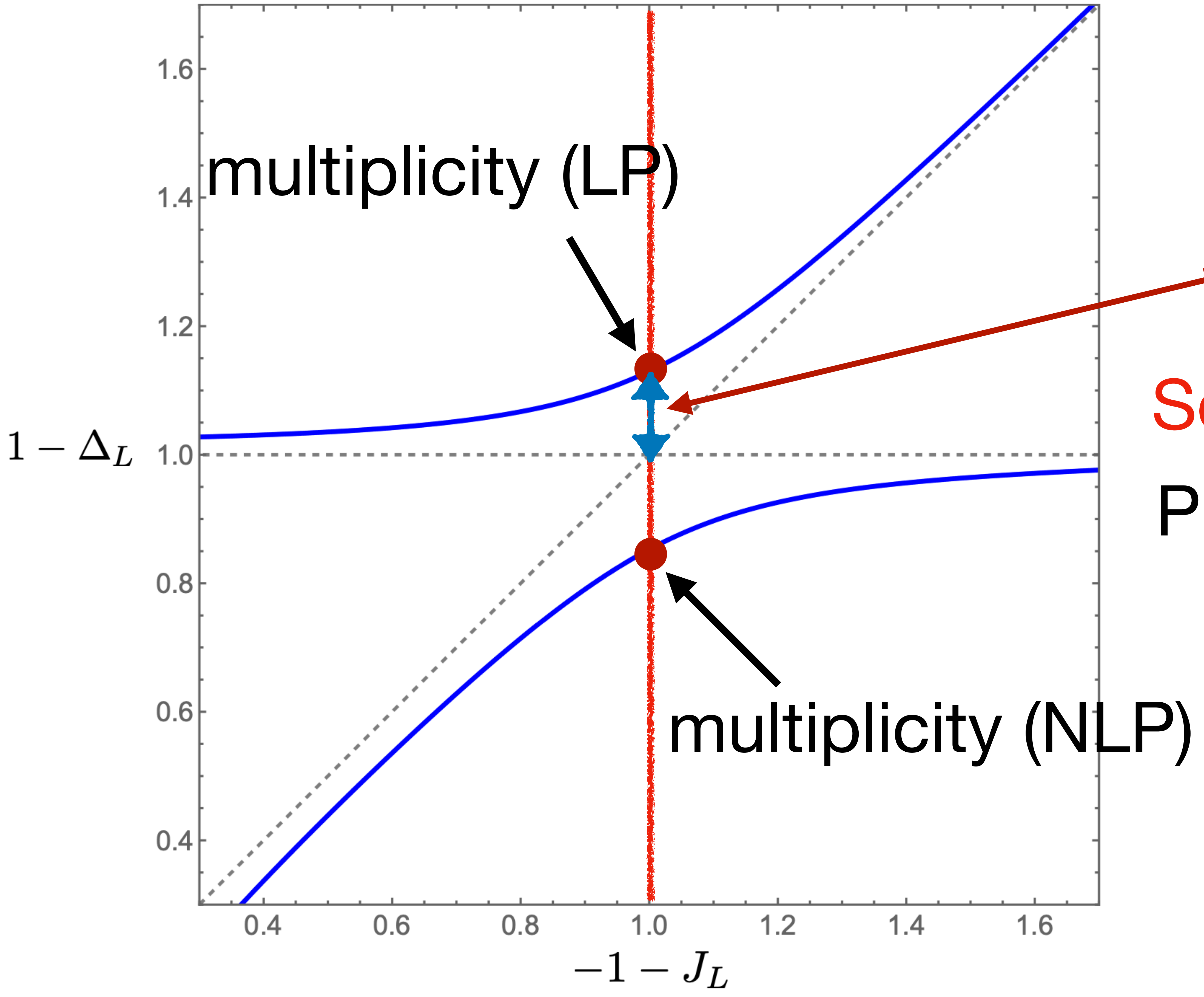
$$\mathscr{D} = \mathcal{Z}^{-1}_{J_L}\left(\mathscr{D}_0 + \beta(g)\frac{\partial}{\partial g}\right)\mathcal{Z}_{J_L}$$

$$\mathscr{D} = \mathscr{D}_0 + \frac{\alpha_s}{2\pi}\begin{pmatrix} -\gamma_1(J_L) & 4C_A + \epsilon\widetilde{\mathcal{R}}_1(\epsilon) \\ \frac{(4+2J_L-8\epsilon)\gamma_{21}(J_L)+2\gamma_1(J_L)-2\gamma_2(J_L)+\widetilde{\mathcal{R}}_1(\epsilon)+\widetilde{\mathcal{R}}_2(\epsilon)}{4\epsilon} & -\gamma_2(J_L) \end{pmatrix} + O(\alpha_s^2).$$

finite in ε

Level repulsion in QCD

characteristic equation: $$\det(\mathscr{D} + \Delta_L) = 0$$



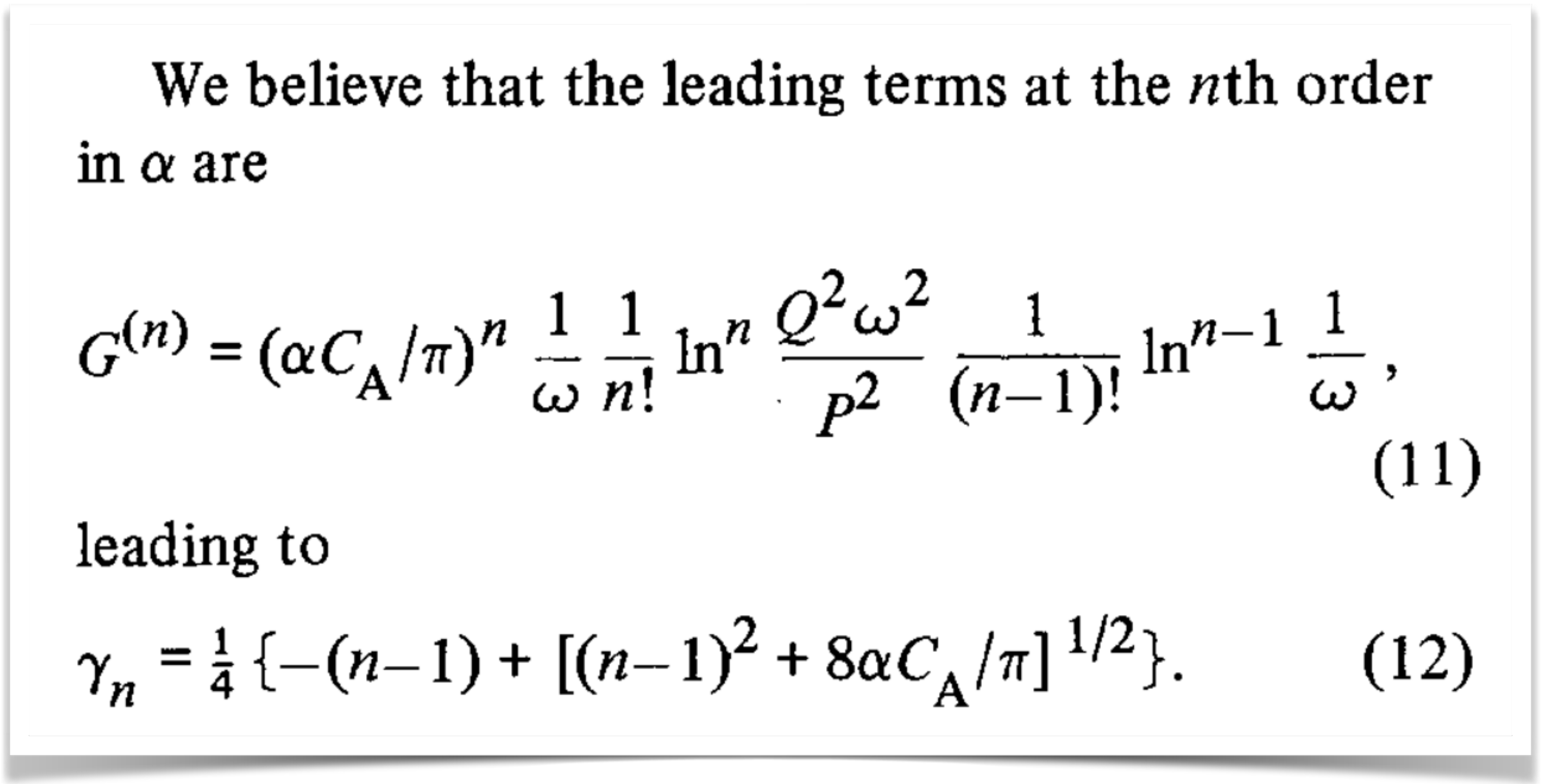
$$\frac{d}{d\log Q}\log\langle \mathbb{N}_{J_L}(\vec{n})\rangle_Q = -\Delta_{L,i_{\min}}(J_L, \alpha_s(Q)) + \ldots .$$

$$-\Delta_L(J_L = -2; \alpha_s) = \sqrt{\frac{2C_A}{\pi}\alpha_s} + O(\alpha_s).$$

Square root corrections in α_s by level recombination

Previously computed in QCD by resummation

Mueller, 1981


We believe that the leading terms at the nth order in α are

$$G^{(n)} = (\alpha C_A/\pi)^n \frac{1}{\omega}\frac{1}{n!}\ln^n \frac{Q^2\omega^2}{p^2}\frac{1}{(n-1)!}\ln^{n-1}\frac{1}{\omega}, \qquad (11)$$

leading to

$$\gamma_n = \tfrac{1}{4}\{-(n-1) + [(n-1)^2 + 8\alpha C_A/\pi]^{1/2}\}. \qquad (12)$$

Going beyond one loop

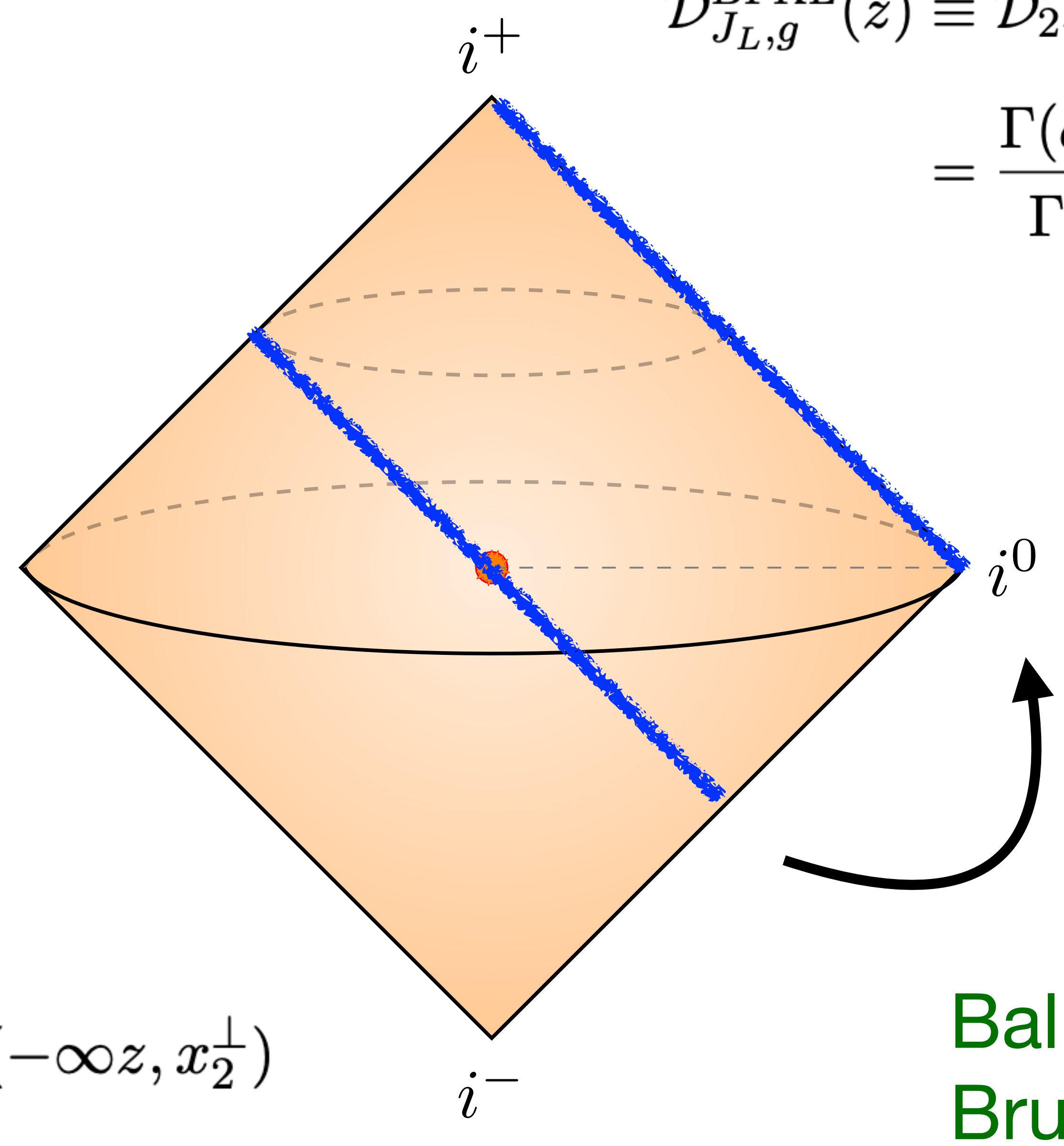
- Why do we call the horizontal trajectory BFKL detector?

$$\gamma_{\mathrm{BFKL}}(J_L) = 2C_A\left(2\gamma_E + \psi\left(\frac{2+J_L}{2}\right) + \psi\left(-\frac{J_L}{2}\right)\right)$$

ε pole of $\langle \mathcal{D}_{J_L}^{\mathrm{BFKL}}(z)\rangle$

- Does it holds beyond one loop? Yes, up to a conformal transformation

$$\mathcal{D}_{J_L,g}^{\mathrm{BFKL}}(z) \equiv \mathcal{D}_{2-d,2-d;J_L,g}(z) = \frac{\Gamma(d-2+J_L)}{\Gamma(\frac{d-2+J_L}{2})^2}\int D^{d-2}z_1 D^{d-2}z_2 \left(\frac{2z_1\cdot z_2}{(2z_1\cdot z)(2z_2\cdot z)}\right)^{-\frac{J_L}{2}} :\mathcal{N}_g^c(z_1)\mathcal{N}_g^c(z_2):.$$



conformal transformation

$$\mathcal{H}_{J_L,\mathrm{BFKL}}^{\mathrm{dis}}(-\infty z, x^\perp) \sim \int d^{d-2}x_1^\perp d^{d-2}x_2^\perp \langle \varphi_{-J_L}(x^\perp)\varphi_1(x_1^\perp)\varphi_1(x_2^\perp)\rangle \mathbf{L}[\phi^2](-\infty z, x_1^\perp)\mathbf{L}[\phi^2](-\infty z, x_2^\perp)$$

Y.Z. Li, Simmons-Duffin, 2025

Balitsky, Chirilli, 2024
Brunello, Caron-Huot, Crisanti, Giroux, Smith, 2025

Two-loop calculation

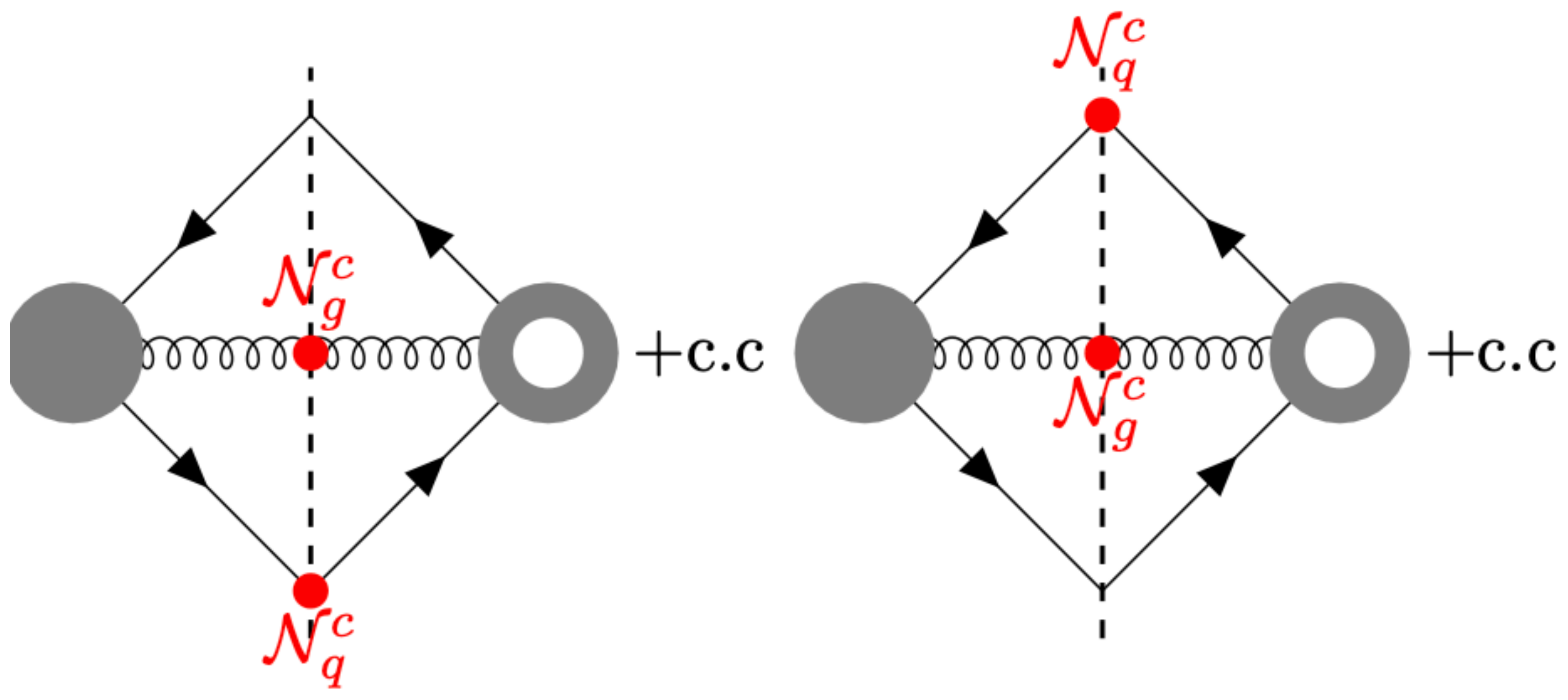
$$\langle \mathcal{D}^{\rm BFKL}_{J_L}(z)\rangle \propto \int_0^1 d\zeta \zeta^{\frac{d-4-J_L}{2}}(1-\zeta)^{\frac{d-4}{2}} {}_2F_1\left(-\frac{J_L}{2},-\frac{J_L}{2},\frac{d-2}{2},1-\zeta\right) F_{ab}(\zeta)$$

regularization factor

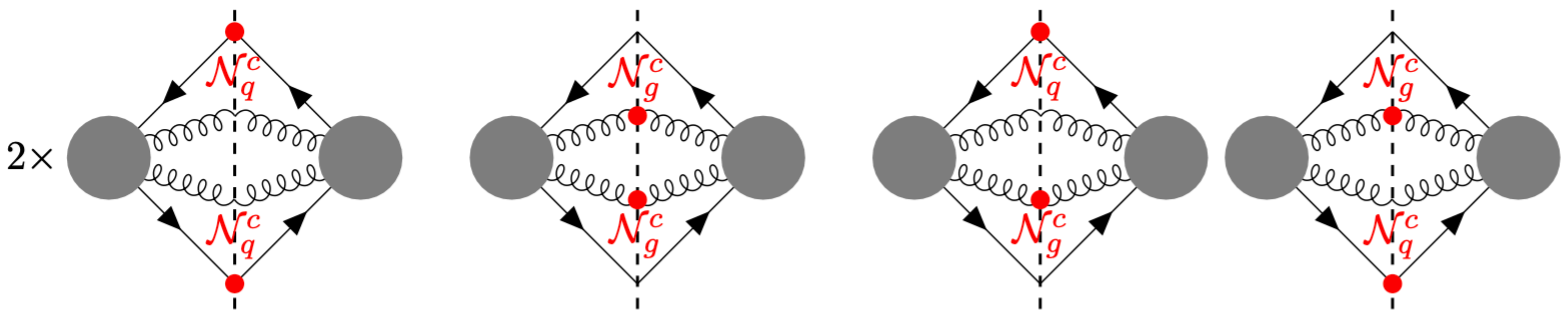
EEC: Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 2018

NLO color-color correlator

contains both coll. and b2b div.



real-virtual



double real

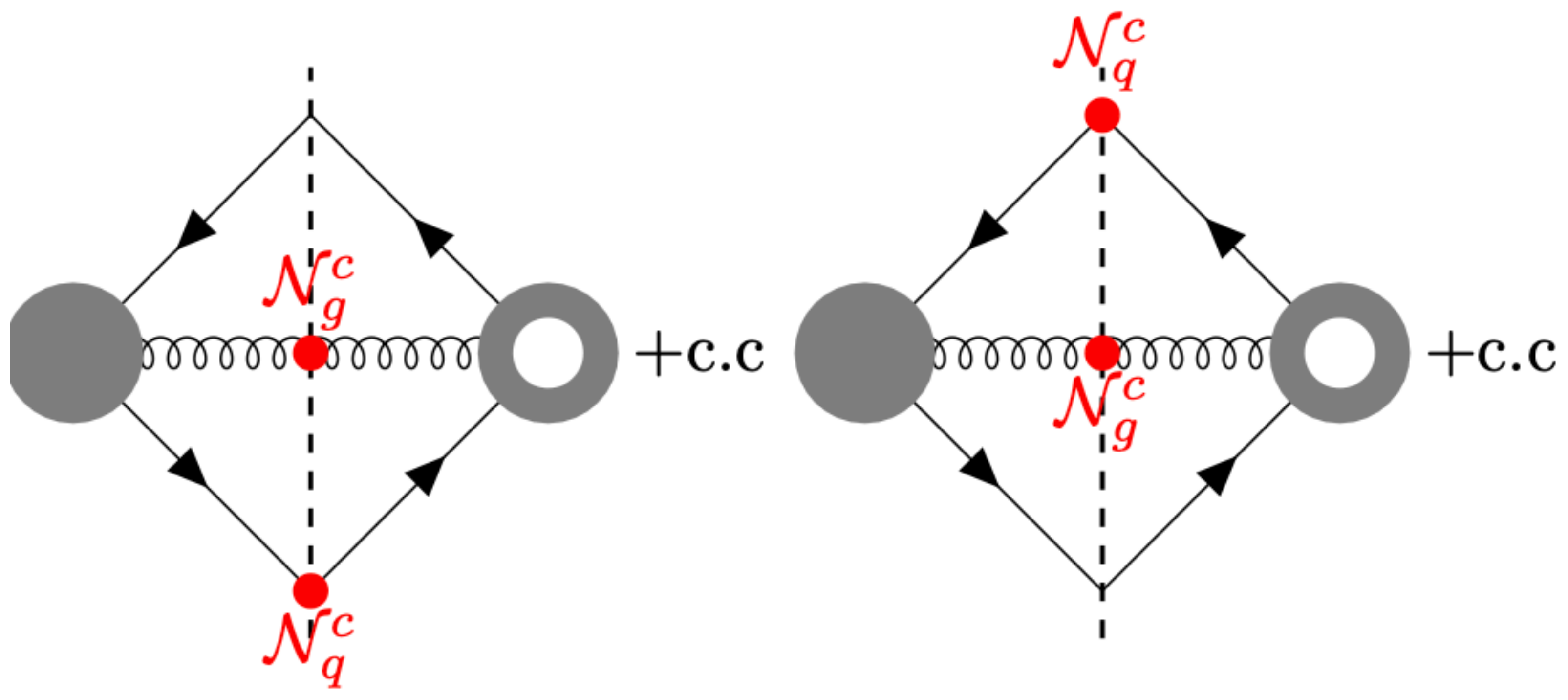
Two-loop calculation

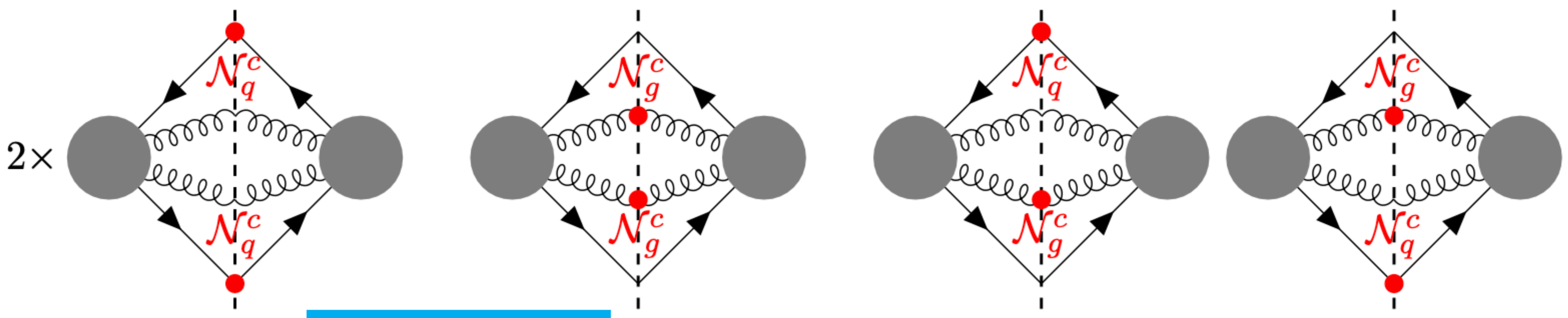
$$\langle \mathcal{D}^{\mathrm{BFKL}}_{J_L}(z)\rangle \propto \int_0^1 d\zeta\, \underbrace{\zeta^{\frac{d-4-J_L}{2}}(1-\zeta)^{\frac{d-4}{2}}}_{\text{regularization factor}}\, {}_2F_1\left(-\frac{J_L}{2}, -\frac{J_L}{2}, \frac{d-2}{2}, 1-\zeta\right) F_{ab}(\zeta)$$

regularization factor

NLO color-color correlator

contains both coll. and b2b div.





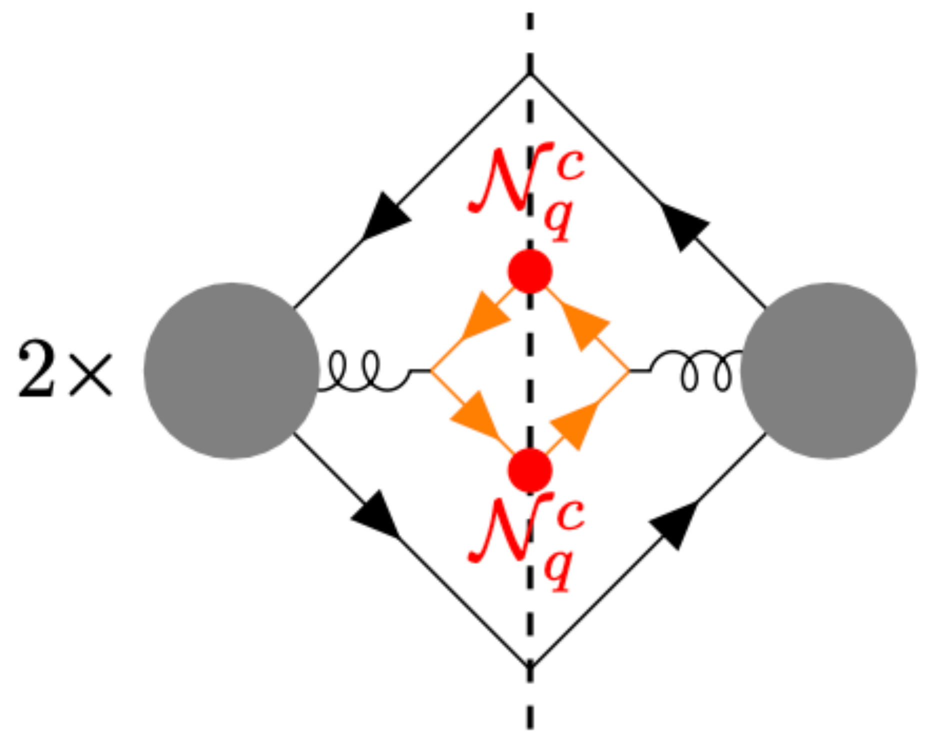
real-virtual

Loops + legs = 5!

ENERGY STAR

double real

An example of nf contribution

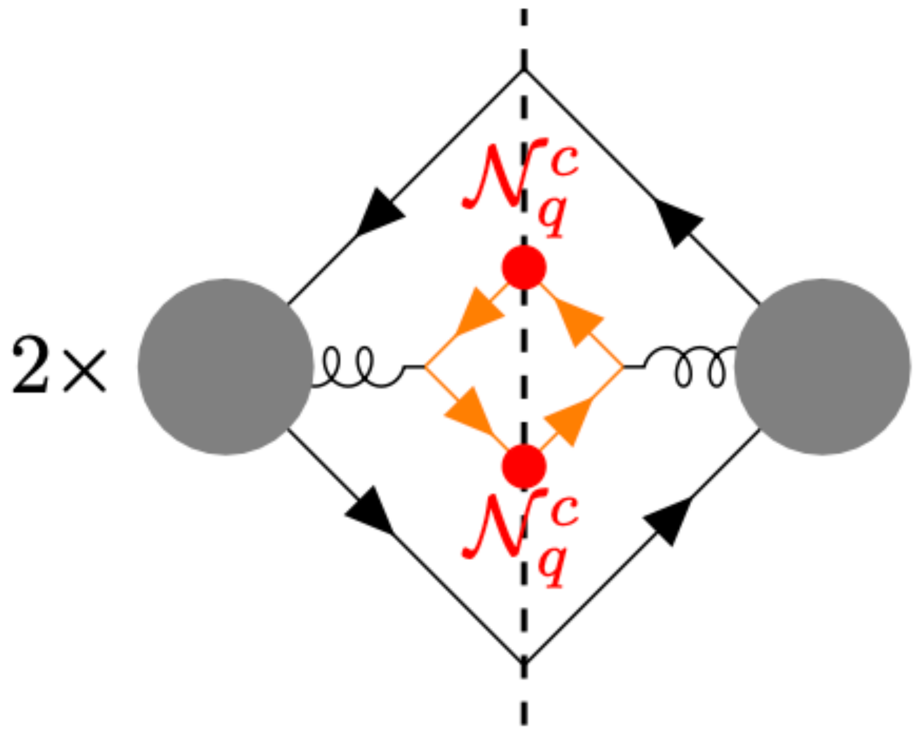


$$\int_0^1 d\zeta \zeta^{-\frac{J_L}{2}} {}_2F_1\left(-\frac{J_L}{2}, -\frac{J_L}{2}, 1, 1-\zeta\right) F_{\mathrm{sqrt}}(\zeta)$$

$$F_{\mathrm{sqrt}}(\zeta) = \left[\frac{2-\log\zeta}{\zeta^2} + \frac{1-3\zeta}{2\zeta^{5/2}}\left(\mathrm{Li}_2(\zeta) - 4\,\mathrm{Li}_2(\sqrt{\zeta}) + \log\frac{1+\sqrt{\zeta}}{1-\sqrt{\zeta}}\log\zeta\right)\right]$$

My collaborators can do it, but I decided to give Gemini a try

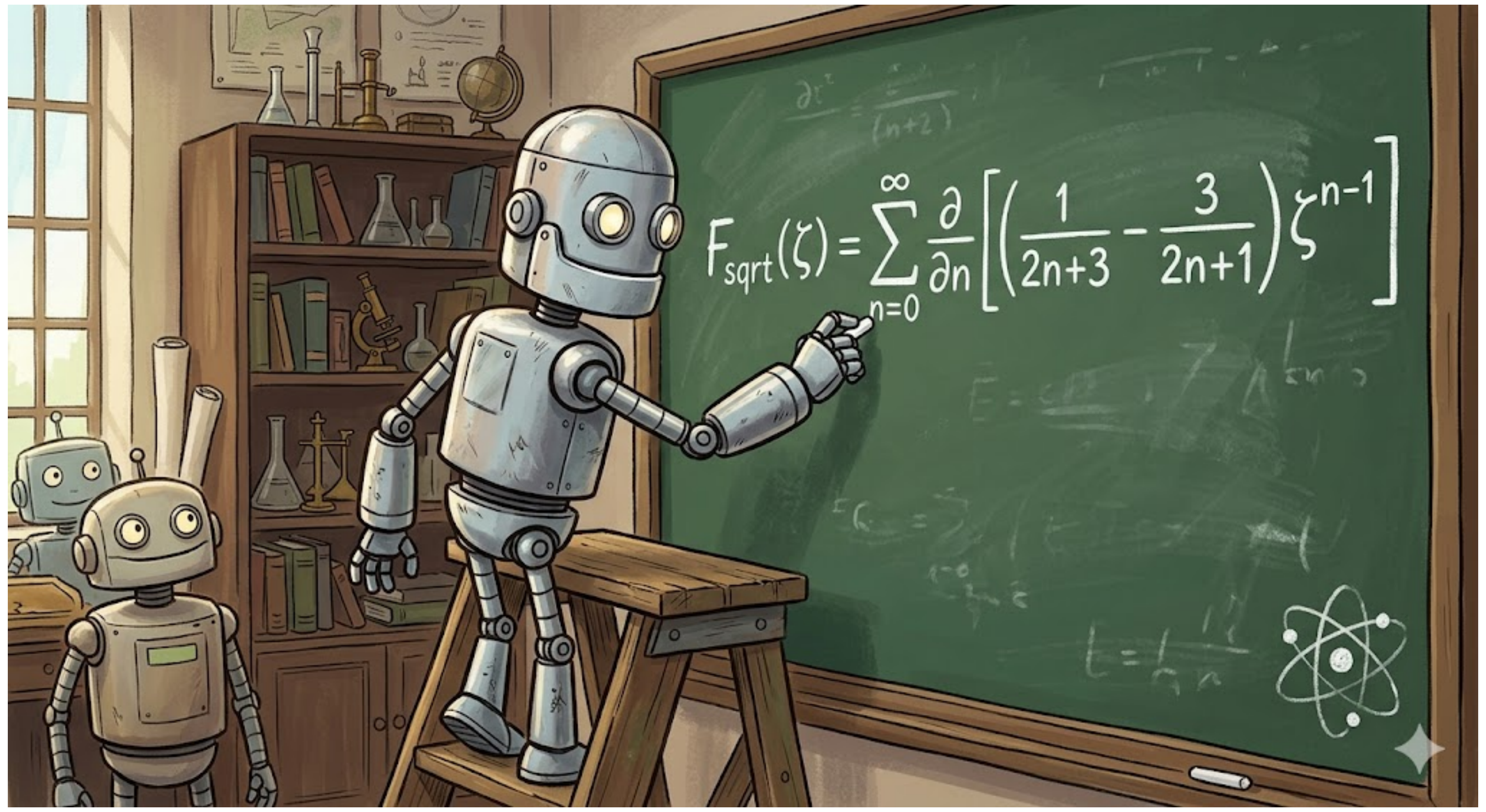
An example of nf contribution



$$\int_0^1 d\zeta \zeta^{-\frac{J_L}{2}} {}_2F_1\left(-\frac{J_L}{2}, -\frac{J_L}{2}, 1, 1-\zeta\right) F_{\text{sqrt}}(\zeta)$$

$$F_{\text{sqrt}}(\zeta) = \left[\frac{2-\log\zeta}{\zeta^2} + \frac{1-3\zeta}{2\zeta^{5/2}}\left(\text{Li}_2(\zeta) - 4\,\text{Li}_2(\sqrt{\zeta}) + \log\frac{1+\sqrt{\zeta}}{1-\sqrt{\zeta}}\log\zeta\right)\right]$$

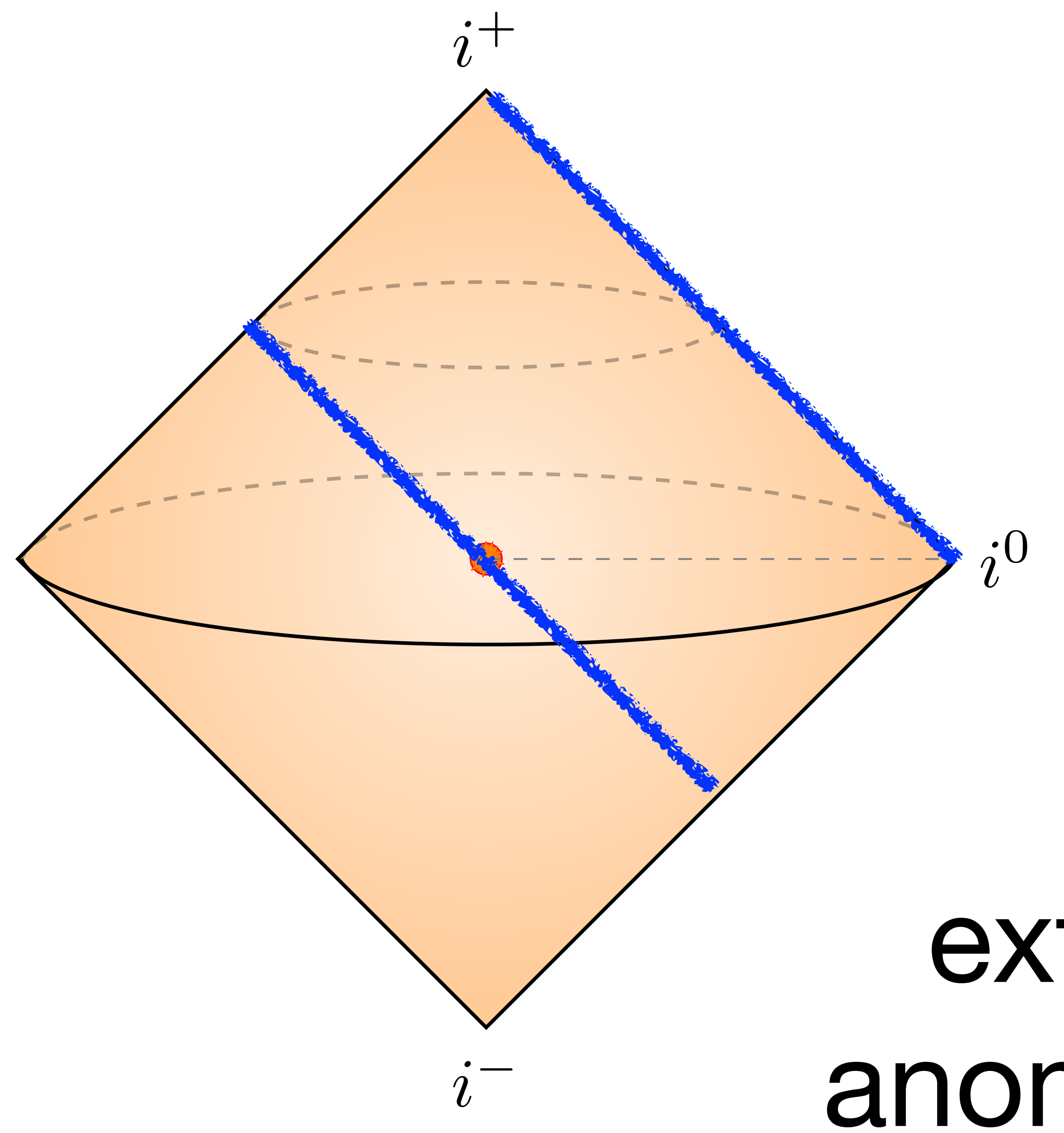
My collaborators can do it, but I decided to give Gemini a try



$$I = \frac{\pi^2 \sin(\pi\nu)}{8\nu\cos^2(\pi\nu)} \frac{11-12\nu^2}{1-\nu^2}\bigg|_{\nu=-(J_L+1)/2}$$

Fadin, Lipatov, 1998

The Wilson-Fisher fixed point

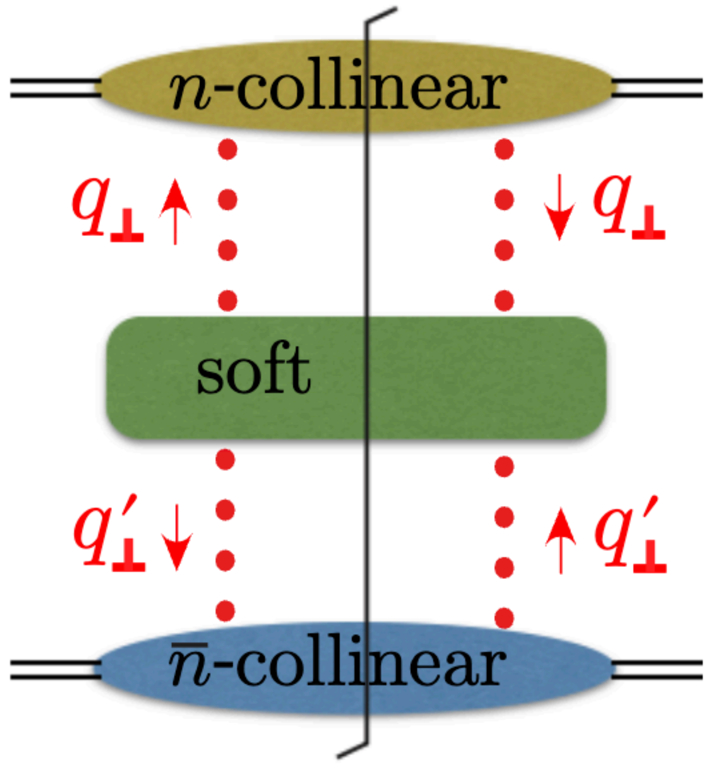


$$\beta(g,\epsilon) = \frac{dg}{d\ln\mu} = -\epsilon^* g + \beta(g) = 0$$

$$\gamma^{\mathrm{BFKL}}(J_L,\epsilon^*)\Big|_{\mathrm{bulk}} = \gamma^{\mathrm{BFKL}}(J_L)\Big|_{\mathrm{boundary}}$$

extracted as rapidity anomalous dimension in d=4-2ε* dim.

extracted from BFKL detector operator



$$\nu\frac{d}{d\nu}S_G(q_\perp,q'_\perp,\nu) = \int d^2k_\perp\,\gamma_{S_G}(q_\perp,k_\perp)\,S_G(k_\perp,q'_\perp,\nu)$$

Rothstein, Stewart, 2016

BFKL eigenvalue at NLO

$$\delta(\gamma) = -\left[\left(\frac{11}{3} - \frac{2n_f}{3N_c}\right)\frac{1}{2}\left(\chi^2(\gamma) - \psi'(\gamma) + \psi'(1-\gamma)\right) - \left(\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{n_f}{N_c}\right)\chi(\gamma)\right.$$
$$-6\zeta(3) + \frac{\pi^2\cos(\pi\gamma)}{\sin^2(\pi\gamma)(1-2\gamma)}\left(3 + \left(1 + \frac{n_f}{N_c^3}\right)\frac{2+3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)}\right)$$
$$\left. -\psi''(\gamma) - \psi''(1-\gamma) - \frac{\pi^3}{\sin(\pi\gamma)} + 4\phi(\gamma)\right]. \qquad (14)$$

The function $\phi(\gamma)$ is

$$\phi(\gamma) = -\int_0^1 \frac{dx}{1+x}\left(x^{\gamma-1} + x^{-\gamma}\right)\int_x^1 \frac{dt}{t}\ln(1-t)$$
$$= \sum_{n=0}^{\infty}(-1)^n\left[\frac{\psi(n+1+\gamma)-\psi(1)}{(n+\gamma)^2} + \frac{\psi(n+2-\gamma)-\psi(1)}{(n+1-\gamma)^2}\right]. \qquad (15)$$

Fadin, Lipatov, 1998

Reproduced with a Loops + Legs = 5 detector operator calculation

No additional diagonalization needed

Pure 1/ε pole, no rapidity regularization needed on the detector side

Conclusion

- Detector operators and correlators have provide new insight to QCD, collider experiments, and quantum field theory in general
- Phenomena of level repulsion in perturbative QCD: new way to compute multiplicity distribution
- The BFKL operator can be related to BFKL kernel for high energy scattering through Wilson-Fisher fixed point. **A new approach to compute BFKL.**
- Higher loop (NNLO BFKL)? More legs (Multi-Reggion exchange)?
- Other trajectories in the Chew-Fraustri plot? Relation to factorization and its breaking?