

# Possible $\alpha_s$ extraction in the unexpected post-confinement regime of EEC

刘晓辉

北京师范大学

第八届全国重味物理与量子色动力学研讨会

重庆, 2026

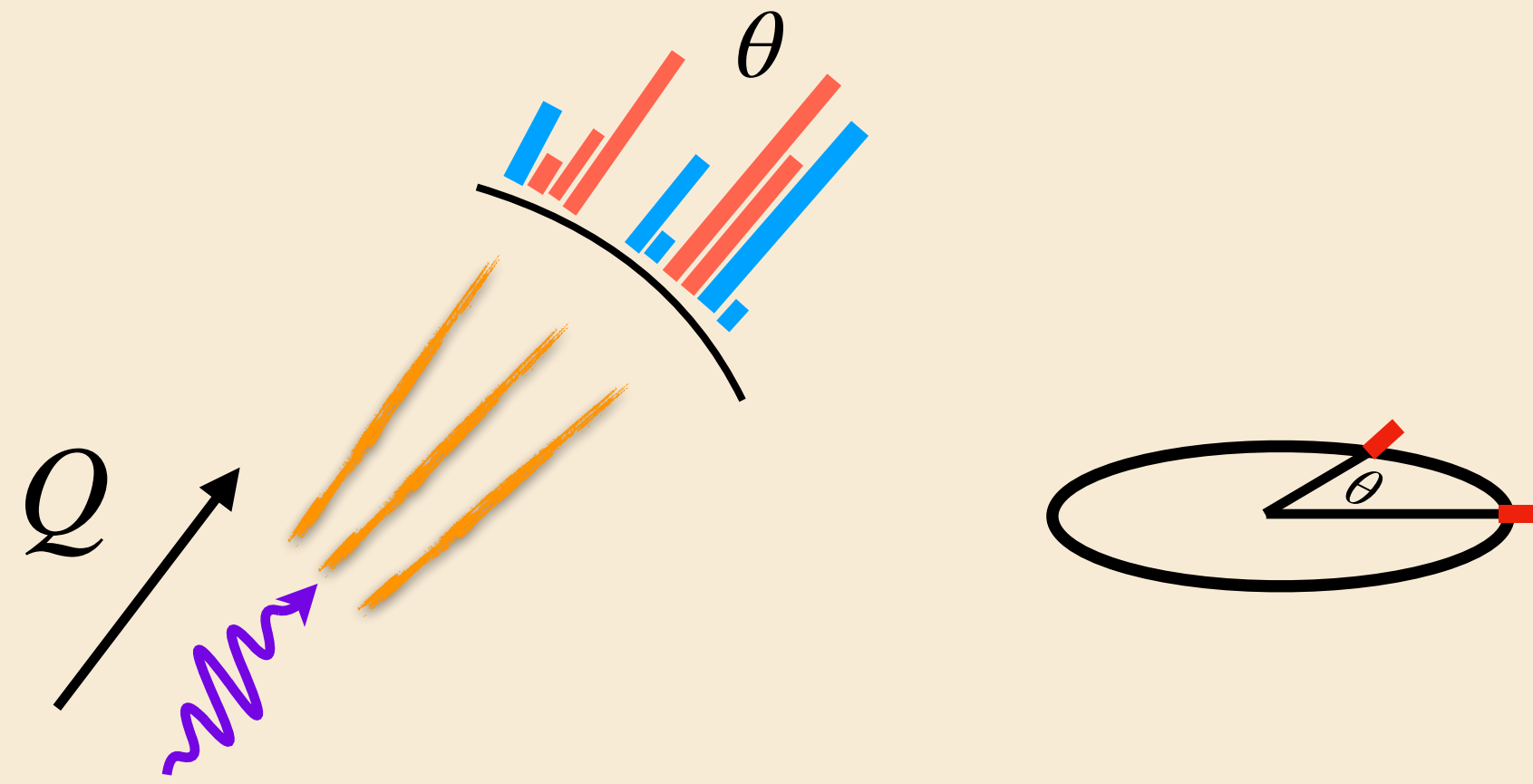
PRL 134 (2025), PRL 136 (2026)

Zhan Wang ,et al. In preparation

# Outline

- Scaling behavior of the post-confinement EEC
  - light-ray OPE Analysis
- Phenomenology
  - Comparison with current data
  - $\alpha_s$  extraction
- Conclusion

# Energy Energy Correlators

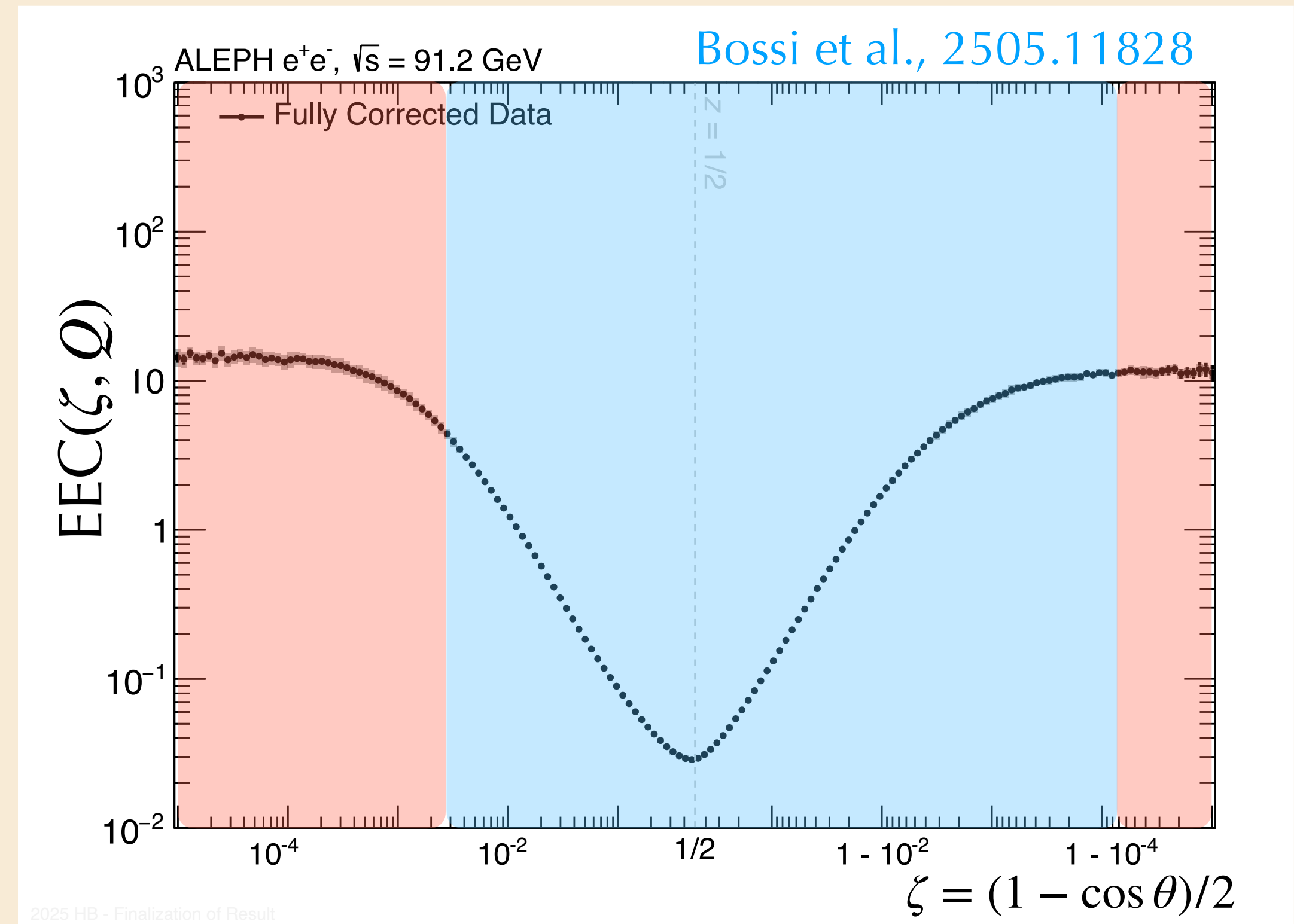


$$EEC = \frac{1}{\sigma} \int d\sigma \sum_{ij} \frac{E_i E_j}{Q^2} \delta(\theta - \theta_{ij})$$

$$= \frac{\langle \mathcal{E}(n_i) \mathcal{E}(n_j) \rangle}{Q^2} \quad \mathcal{E}(n) = \lim_{\bar{n} \cdot x \rightarrow \infty} (\bar{n} \cdot x)^{4-2} \int dn \cdot x T_{\mu\nu} \bar{n}^\mu \bar{n}^\nu$$

Sterman, 1975, Bashman, et al. 1978

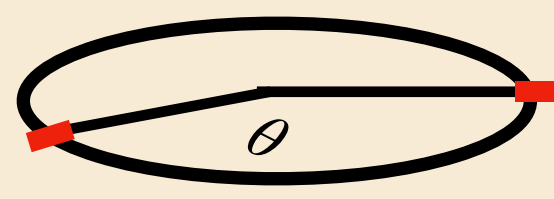
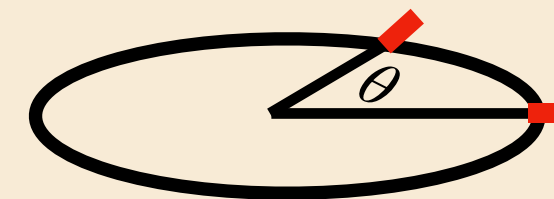
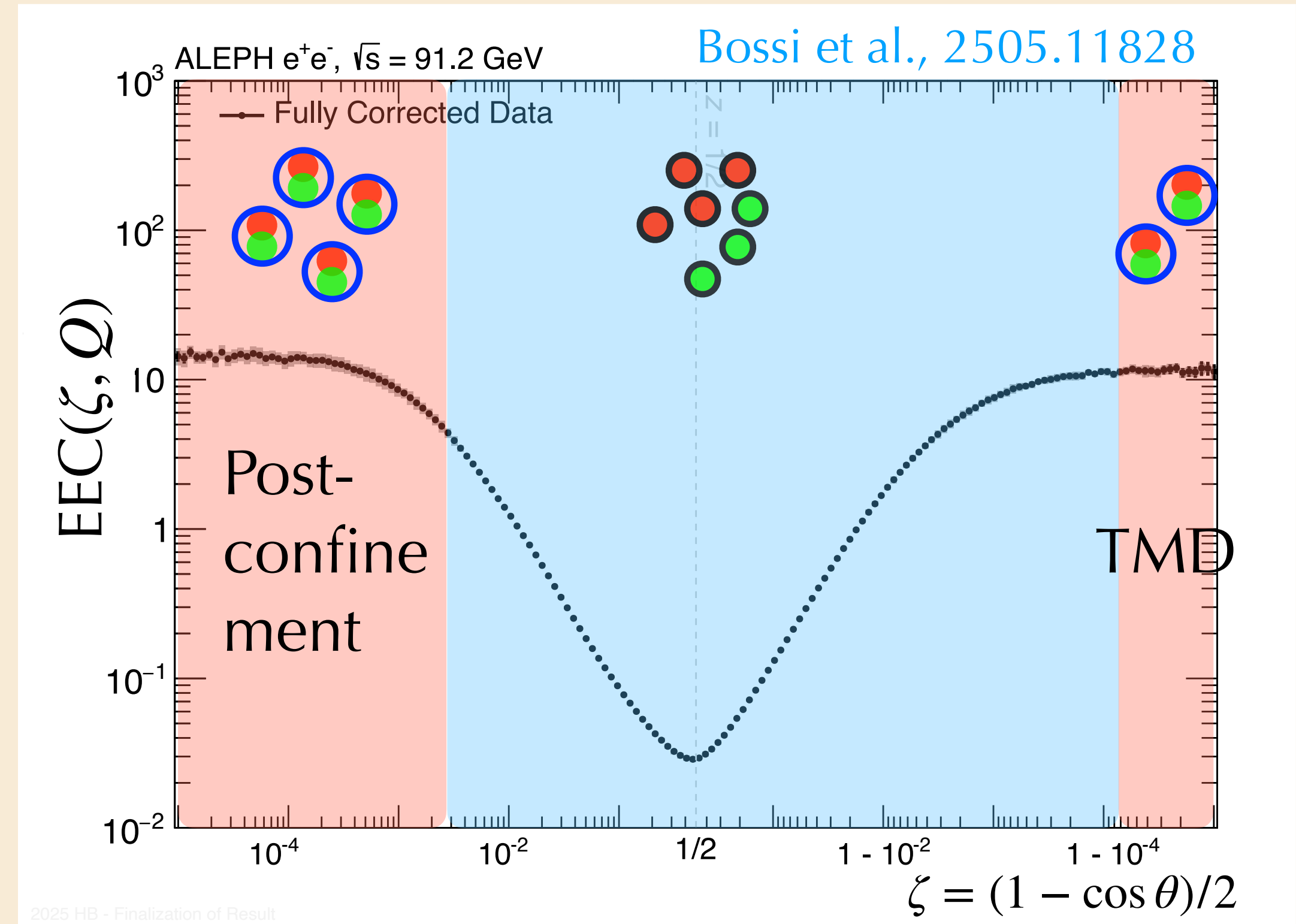
Moult, Zhu, 2506.09119 for reivew



- Jet w/o jet algorithms, a portal to hadrons
- Fundamental object in QFT

# Energy Energy Correlators

- Different angles probe physics at different scales  
 $\sim Q \min(\sqrt{\zeta}, \sqrt{1-\zeta})$
- Most regions are well-understood
- The post-confinement region,  $\zeta \rightarrow 0 \lesssim \Lambda_{\text{QCD}}/Q$ , remains challenging



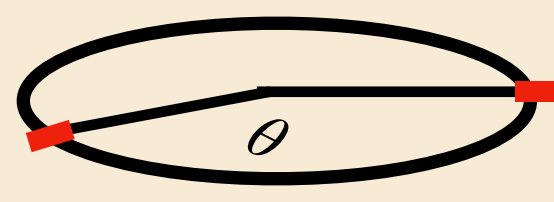
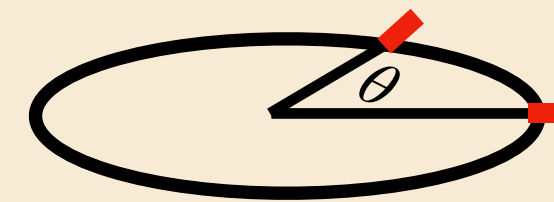
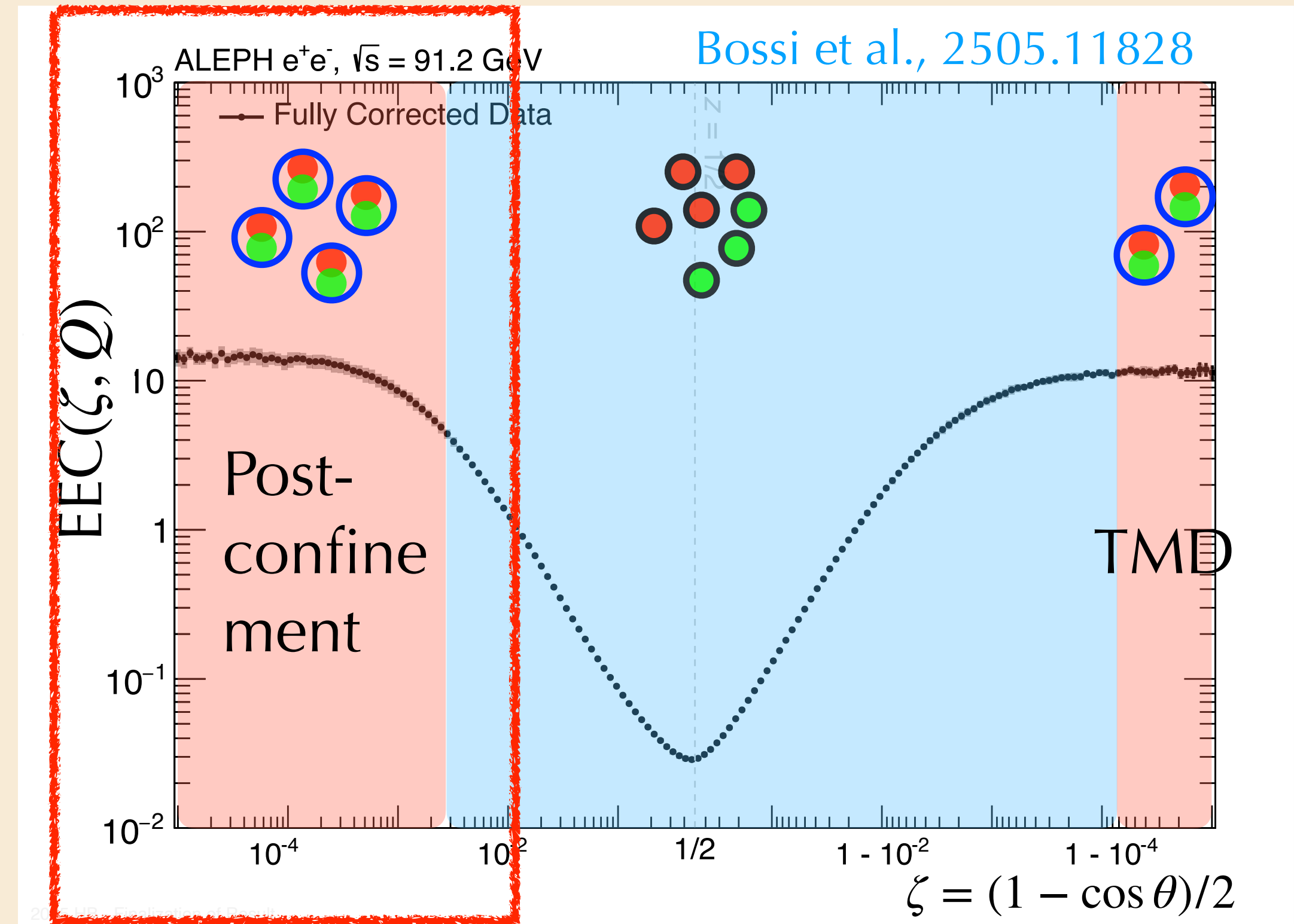
# Energy Energy Correlators

- The post-confinement region,  $\zeta \rightarrow 0 \lesssim \Lambda_{\text{QCD}}/Q$

AdS/CFT: [Hofman, Maldacena, 2008](#)

$$\text{EEC}(\zeta) \sim 1 + \frac{6\pi^2}{\lambda} \left( \frac{2}{3} - 4\zeta(1 - \zeta) \right) + \dots$$

- Height of the plateau  $\propto Q^0$
- The shape is only a function of  $\zeta$  or  $\theta$



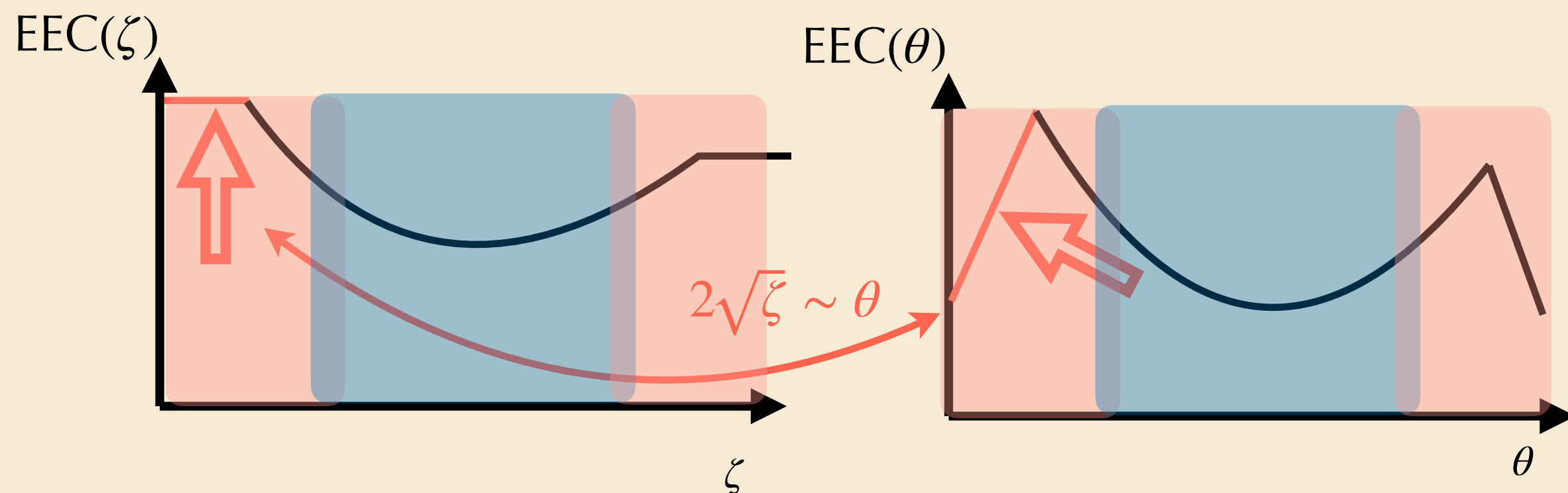
# Energy Energy Correlators

- The post-confinement region,  $\zeta \rightarrow 0 \lesssim \Lambda_{\text{QCD}}/Q$

AdS/CFT: Hofman, Maldacena, 2008

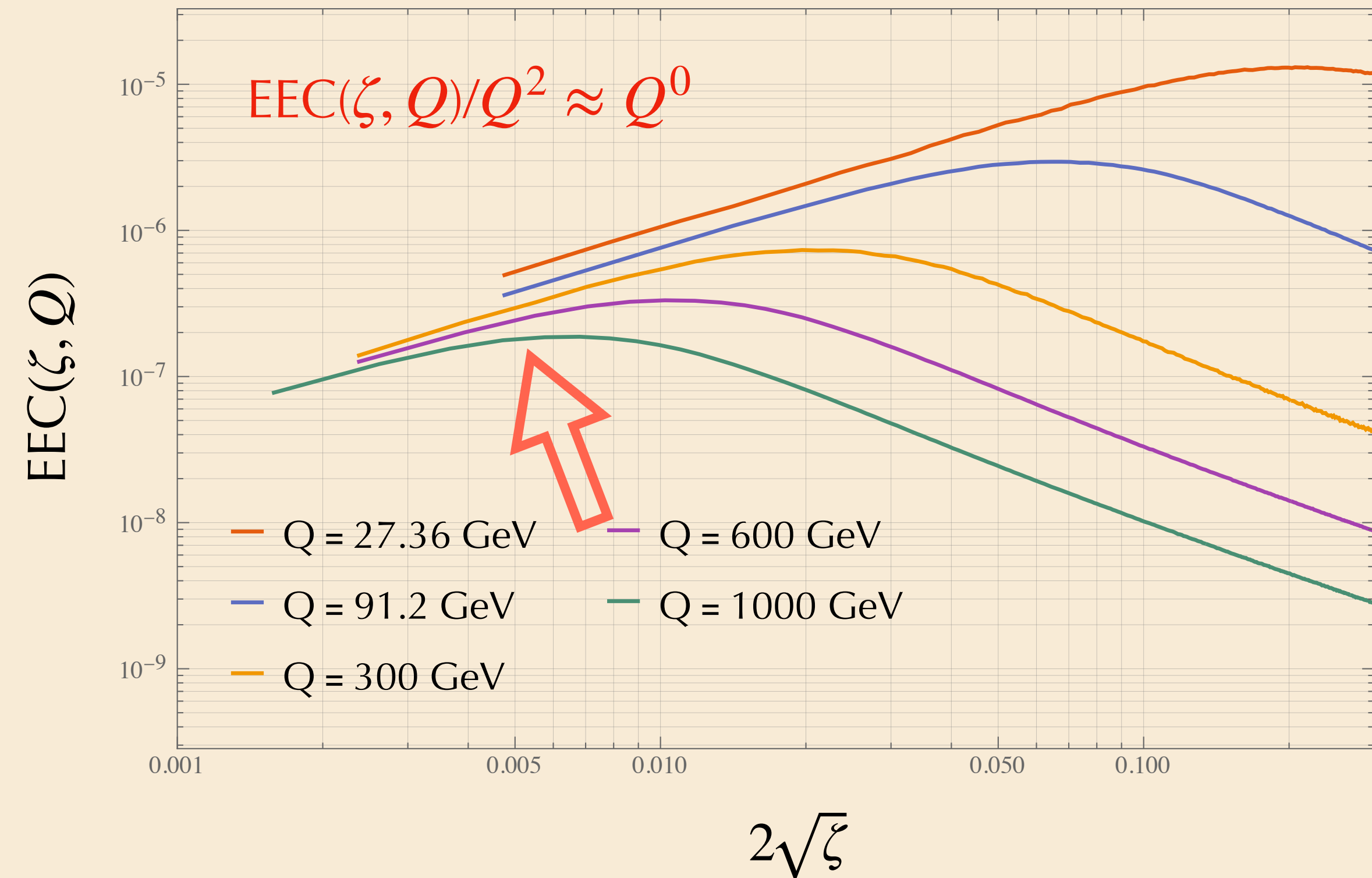
$$\text{EEC}(\zeta) \sim 1 + \frac{6\pi^2}{\lambda} \left( \frac{2}{3} - 4\zeta(1 - \zeta) \right) + \dots$$

- Height of the plateau  $\propto Q^0$
- The shape is only a function of  $\zeta$  or  $\theta$



## Real World:

XL, Vogelsang, Yuan, Zhu, PRL 134 (2025)



- Height of the plateau  $\propto Q^2 + \text{corrections}$
- The shape also depends on  $Q$
- $\text{EEC} \sim Q^2/\Lambda_{\text{QCD}}^2$

# Scaling behavior by light-ray OPE

Chang, Chen, **XL**, Simmons-Duffin, Yuan, Zhu, PRL 136 (2026) 8, 081903

Approach using dihadron picture see: Lee, Stewart, PRL 2026

Kang, et al., PRL 2026

Guo, et al., PRL 2026

Scaling behavior by ~~light-ray OPE~~

*primary school math*

Chang, Chen, XL, Simmons-Duffin, Yuan, Zhu, PRL 136 (2026) 8, 081903

Approach using dihadron picture see: Lee, Stewart, PRL 2026

Kang, et al., PRL 2026

Guo, et al., PRL 2026

# Scaling behavior by light-ray OPE

Scaling under  
 $n \rightarrow \rho n, \bar{n} \rightarrow \rho^{-1} \bar{n}$   
 $\mathbb{O}(\rho n) = \rho^{J_L} \mathbb{O}(n)$

1905.01311

$\mathbb{O}^{-\Delta_L, J_L} \sim$   
 Light-ray operator

$$\lim_{\bar{n} \cdot x \rightarrow \infty} (\bar{n} \cdot x)^{\Delta - J} \int dn \cdot x$$

Light transform

$$\mathcal{O}_{\mu_1 \dots \mu_J}^{\Delta, J}(x) \bar{n}^{\mu_1} \dots \bar{n}^{\mu_J}$$

Local operator

Dimension,  
 classic scaling

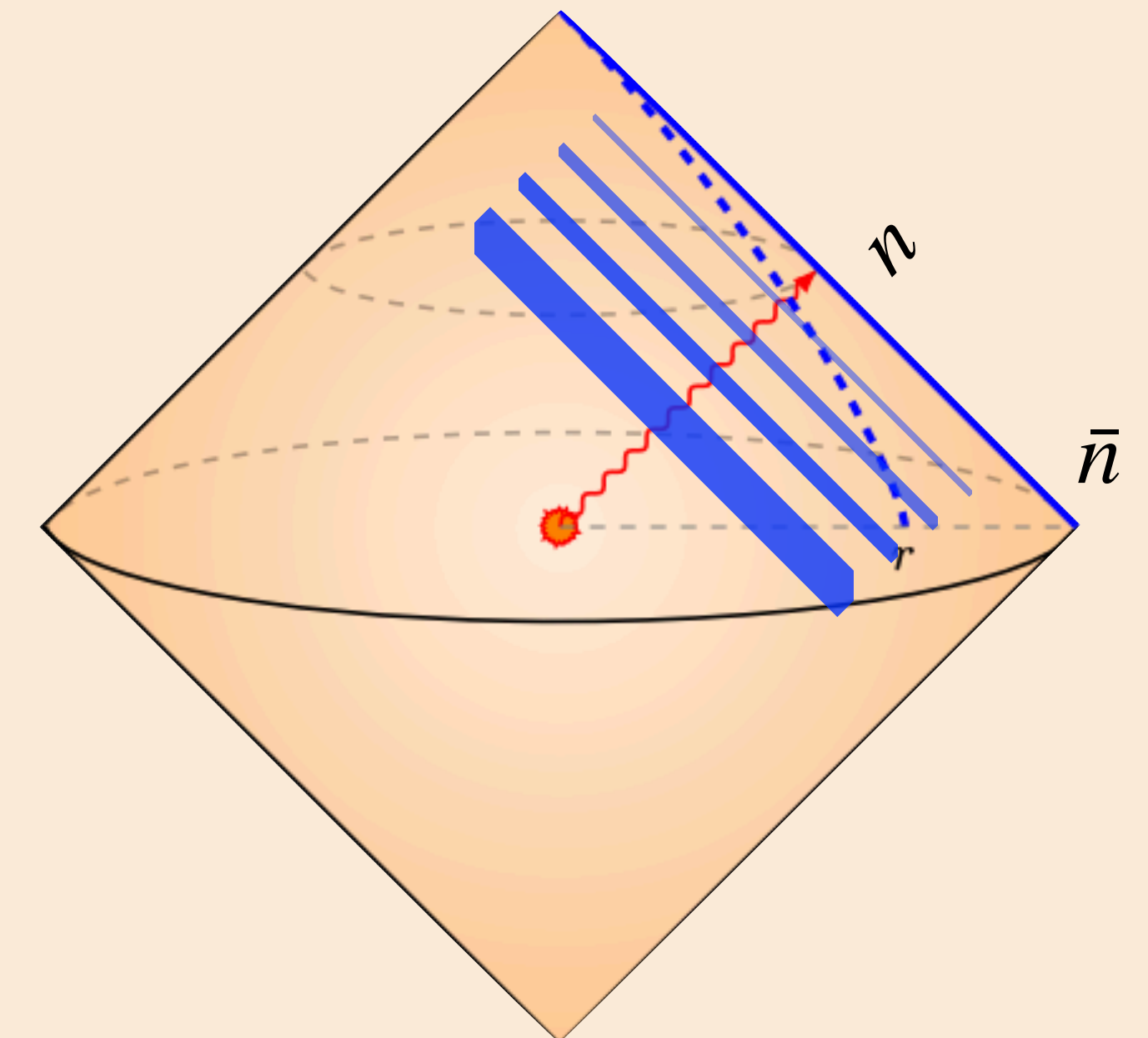
Spin

e.g.  $T_{\mu\nu}$

$$\bar{\psi} \gamma_{\mu_1} iD_{\mu_2} \dots iD_{\mu_J} \psi$$

$$-\Delta_L = \Delta - 1 - \Delta + J = J - 1 \quad J_L = -\Delta + J + 1 - J = -\Delta + 1$$

$$\Delta_L - J_L = \Delta - J = \tau \quad \text{Conserves twist}$$



# Scaling behavior by light-ray OPE

Scaling under  
 $n \rightarrow \rho n, \bar{n} \rightarrow \rho^{-1} \bar{n}$   
 $\mathbb{O}(\rho n) = \rho^{J_L} \mathbb{O}(n)$

1905.01311

$\mathbb{O}^{-\Delta_L, J_L}$   
 Light-ray operator

$$\lim_{\bar{n} \cdot x \rightarrow \infty} (\bar{n} \cdot x)^{\Delta - J} \int dn \cdot x$$

Light transform

$$\mathcal{O}_{\mu_1 \dots \mu_J}^{\Delta, J}(x) \bar{n}^{\mu_1} \dots \bar{n}^{\mu_J}$$

Local operator

Dimension,  
 classic scaling

Spin

$$\mathcal{E} : \quad -\Delta_L = 2 - 1 = 1, \quad J_L = -4 + 1 = -3 \quad T_{\mu\nu}$$

$$\sqrt{\zeta} : \quad -\Delta_L = 0, \quad J_L = 1 \quad (p_{\perp} \sim \bar{n} \cdot P \sqrt{\zeta} \text{ unchanged under boost})$$

$\mathcal{D}_{\text{DGLAP}}$ , Light-ray transformation of the local twist-2 operators:

$$\tau = 2 = \Delta - J = \Delta_L - J_L$$

$$\frac{d\mathcal{D}_{\text{DGLAP}}}{d \ln \mu^2} = \hat{\gamma}(J) \mathcal{D}_{\text{DGLAP}}$$

$$\bar{\psi} \gamma_{\mu_1} iD_{\mu_2} \dots iD_{\mu_J} \psi$$

# Scaling behavior by light-ray OPE

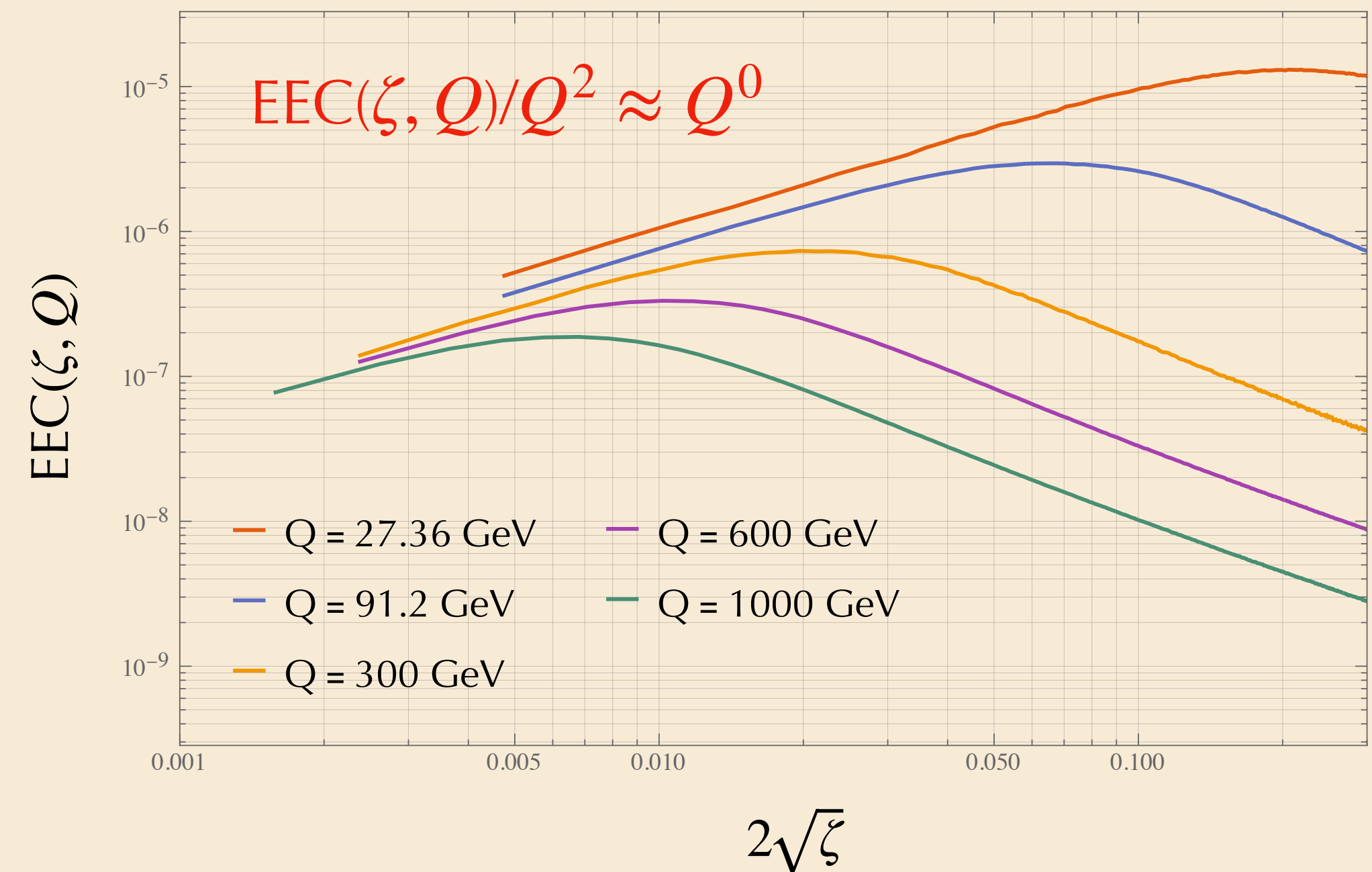
$$\lim_{n_1 \rightarrow n_2} \mathcal{E}(n_1) \mathcal{E}(n_2) \approx z^0 \frac{1}{\Lambda_{\text{QCD}}^2} \mathcal{F}(\mu^2) \times \mathcal{D}_{\text{DGLAP}}^{-\Delta_L = -(2-6)=4, J_L = -6}(\mu^2) + \mathcal{O}(z, \Lambda_{\text{QCD}}^{-1})$$

$$-\Delta_{L, \mathcal{E}\mathcal{E}} = 2 \quad J_{L, \mathcal{E}\mathcal{E}} = -6$$

○ Light-ray OPE conserves quantum numbers

○  $\langle \mathcal{D}_{\text{DGLAP}}^{-\Delta_L=4, J_L=-6}(\mu^2) \rangle \sim \langle \mathbb{L}[\bar{\psi} \gamma^+ (iD_+)^{J-1=5-1=4} \psi] \rangle \sim Q^4$ ,  
twist-3  $\sim Q^3$

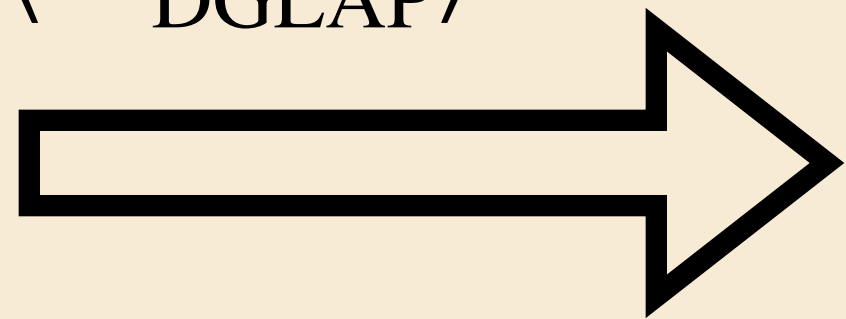
○  $\text{EEC}|_{z \rightarrow 0} = \lim_{n_1 \rightarrow n_2} \langle \mathcal{E}(n_1) \mathcal{E}(n_2) \rangle / Q^2 \sim Q^2$ , produces the classic scaling



# Scaling behavior by light-ray OPE

$$H(Q) \equiv \lim_{n_1 \rightarrow n_2} \langle \mathcal{E}(n_1) \mathcal{E}(n_2) \rangle / Q^4 = \frac{1}{\Lambda_{\text{QCD}}^2} \mathcal{F}(\mu^2) \langle \mathcal{D}_{\text{DGLAP}}^{4,-6}(\mu^2) \rangle / Q^4 + \dots$$

$\mu$  cancellation  
between  $\mathcal{F}$  and  
 $\langle \mathcal{D}_{\text{DGLAP}} \rangle$



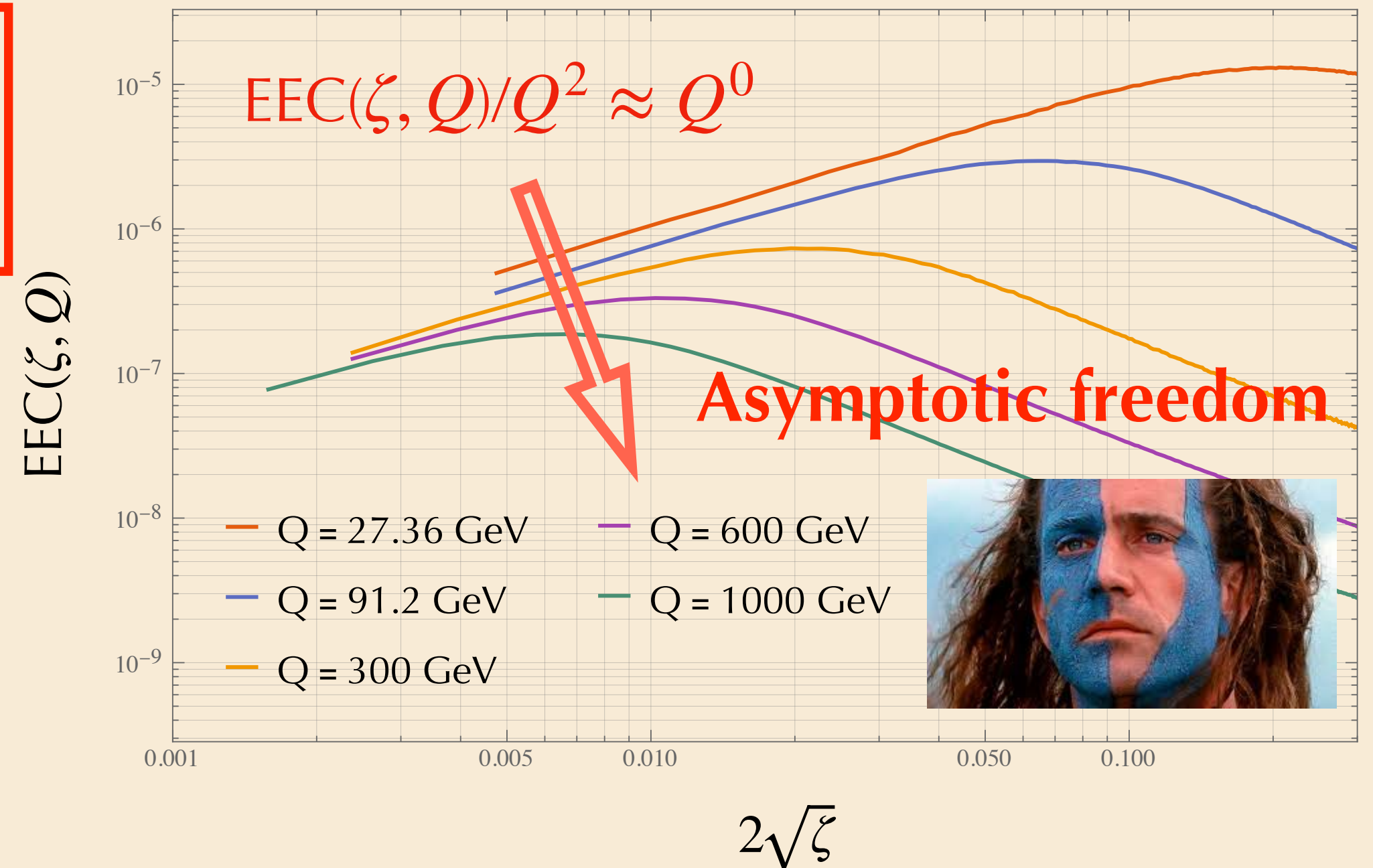
$$\frac{d}{d \ln \mu} \mathcal{D}_{\text{DGLAP}}^{-\Delta_L=4, J_L=-6} = \gamma(J=5) \mathcal{D}_{\text{DGLAP}}^{-\Delta_L=4, J_L=-6}$$

$$\frac{H(Q_2)}{H(Q_1)} \sim \left[ \frac{\alpha_s(Q_2)}{\alpha_s(Q_1)} \right]^{\gamma(J=5)/\beta_0}$$



The height difference governed by  $\alpha_s$

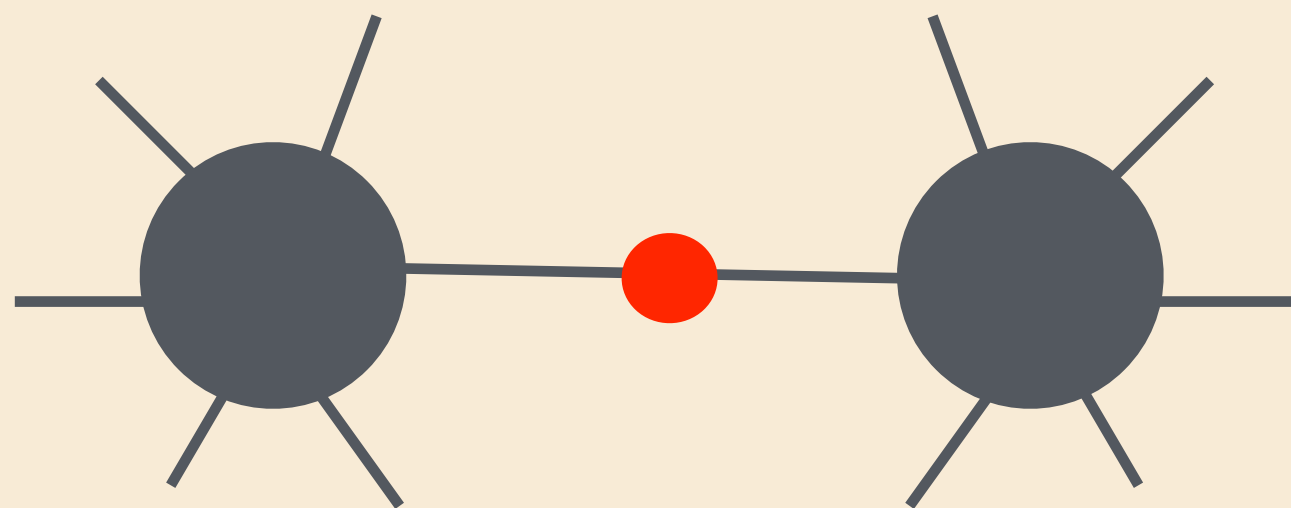
Possible for  $\alpha_s$  extraction with limited NP inputs



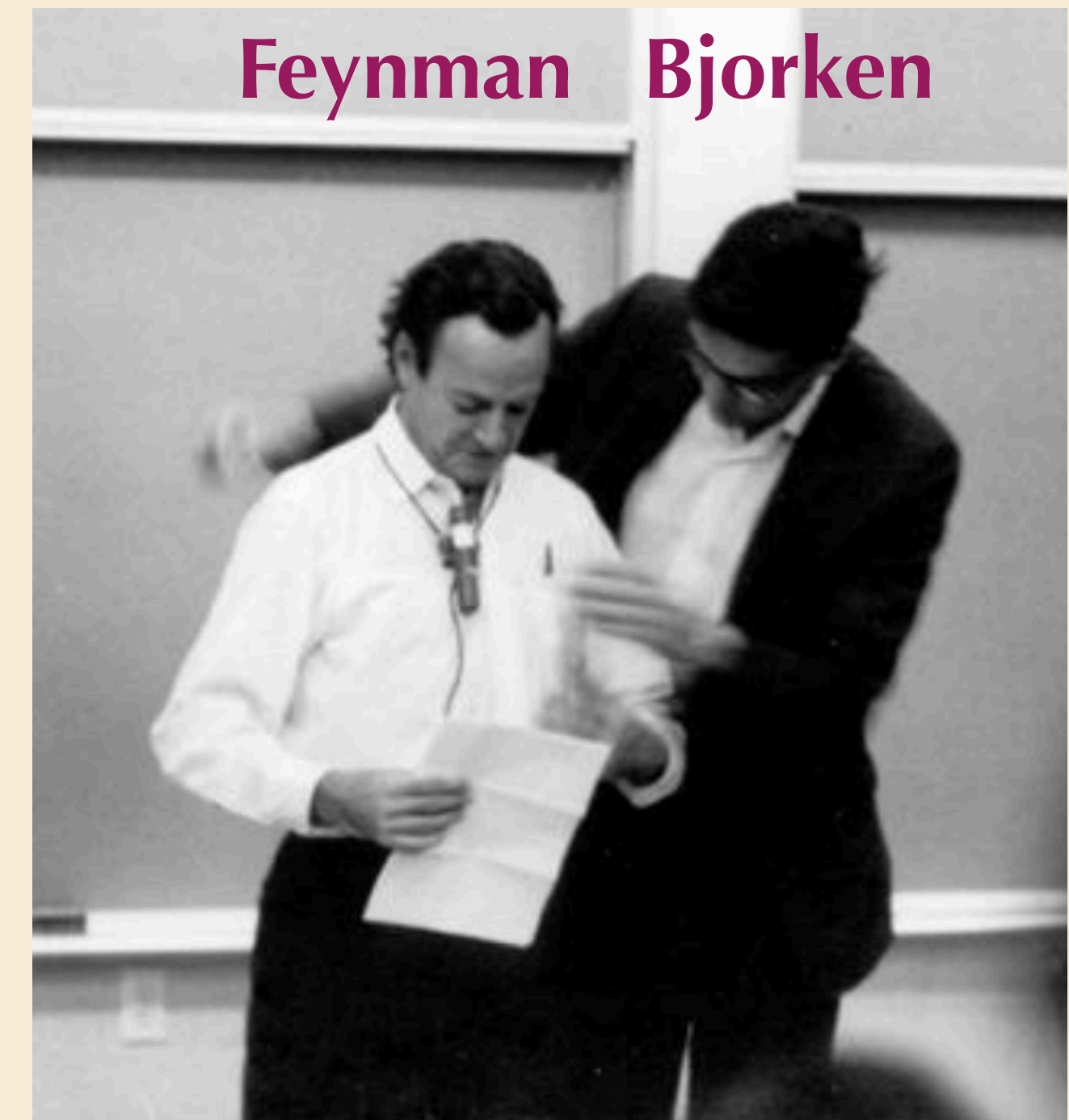
# Scaling behavior by light-ray OPE

$$H(Q) \equiv \lim_{n_1 \rightarrow n_2} \langle \mathcal{E}(n_1) \mathcal{E}(n_2) \rangle / Q^4 = \frac{1}{\Lambda_{\text{QCD}}^2} \mathcal{F}(\mu^2) \langle \mathcal{D}_{\text{DGLAP}}^{4,-6}(\mu^2) \rangle / Q^4 + \dots$$

- $\mathcal{F} = (\mathcal{F}_q, \mathcal{F}_g)$  Non-pert. **Numbers** (diFF moments)
- $\langle \mathcal{D}_{\text{DGLAP}} \rangle \sim \langle \mathbb{L}[\bar{\psi} \gamma^+ (iD_+)^{J-1} \psi] \rangle$  calculable in pQCD, the 5-th moment of **single parton inclusive production cross section**



$$\frac{(iD_+)^{J-1}}{Q^{J-1}} = z^{J-1}$$



# Phenomenology

Zhan Wang ,et al. In preparation

# Phenomenology

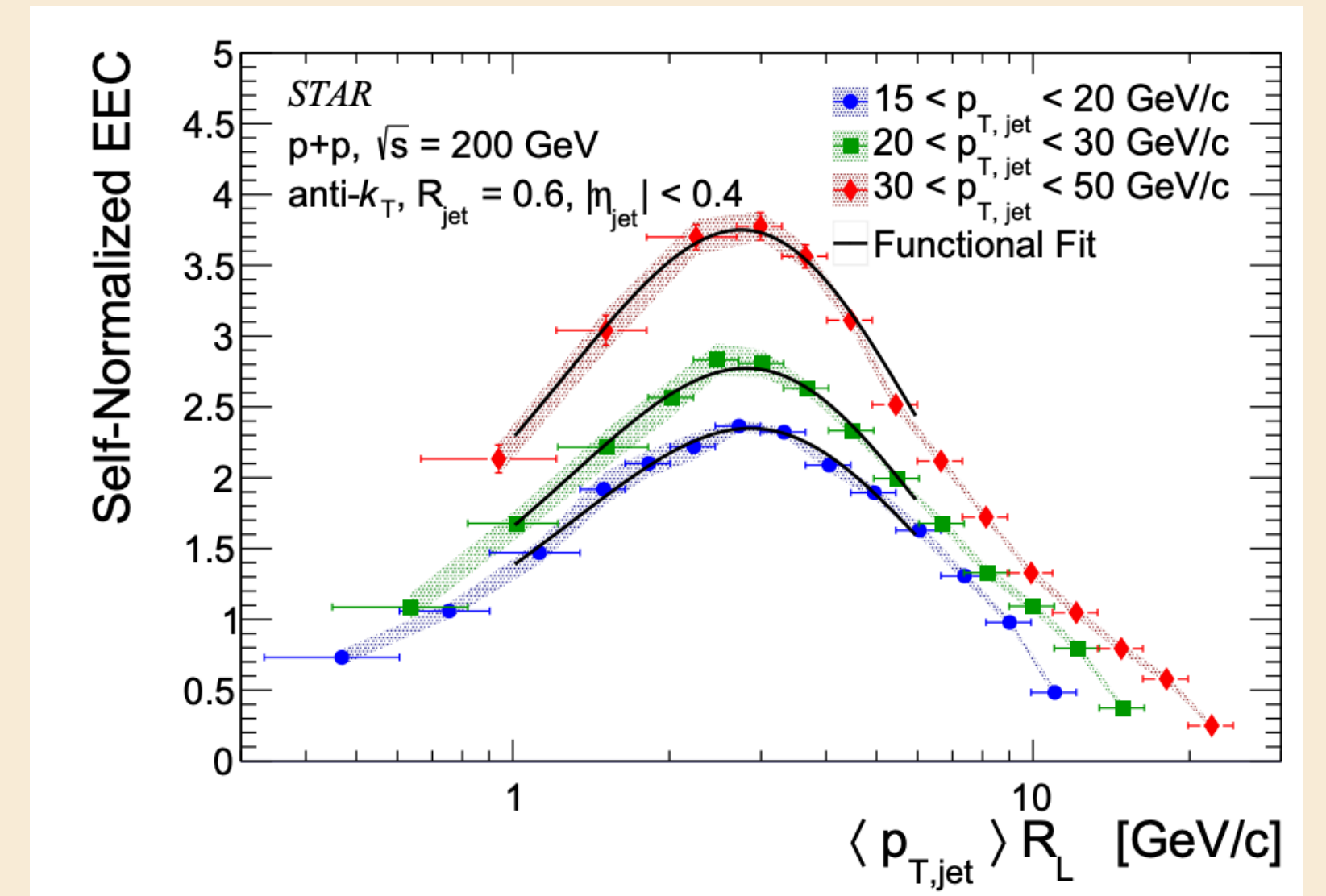
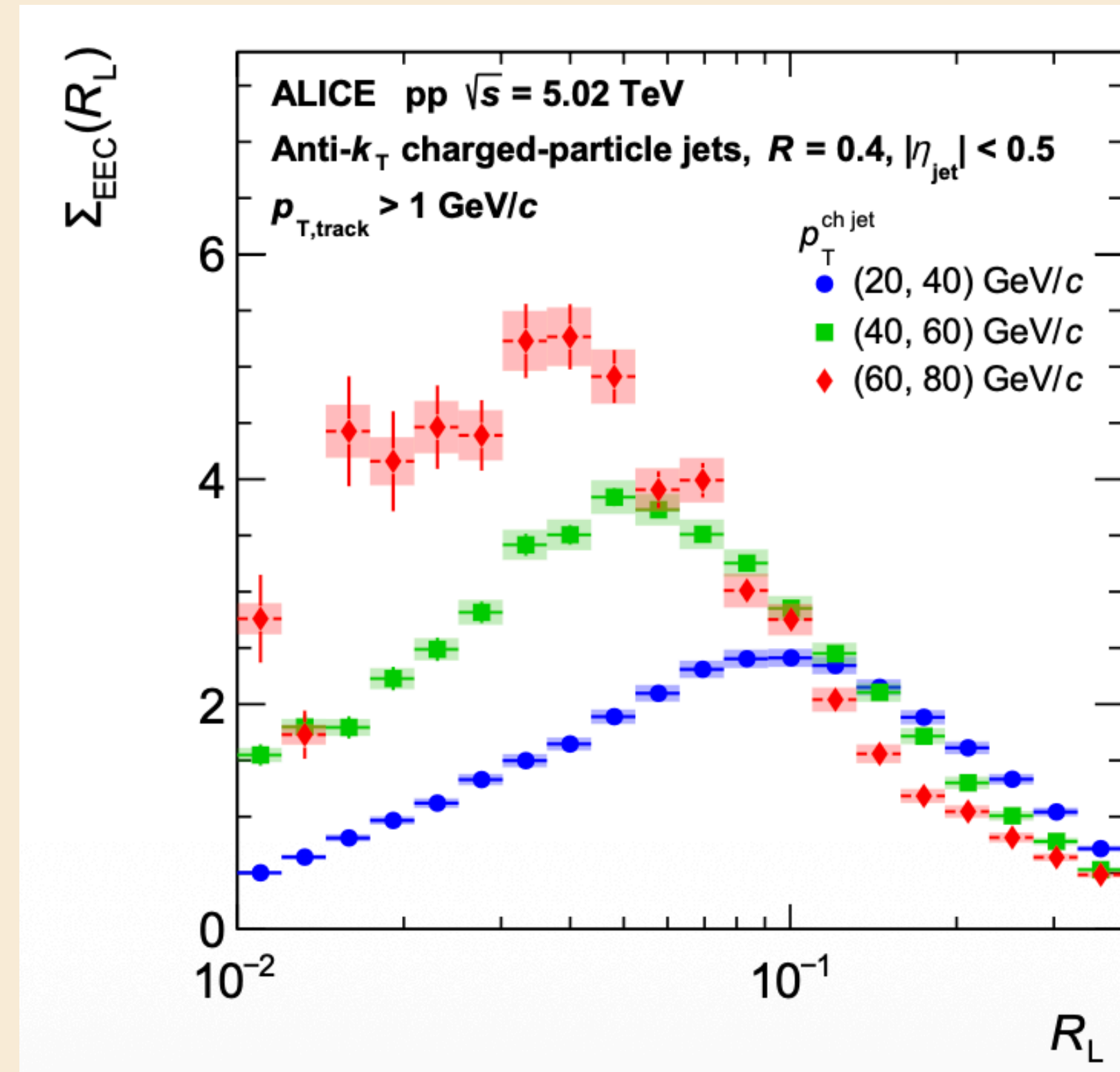
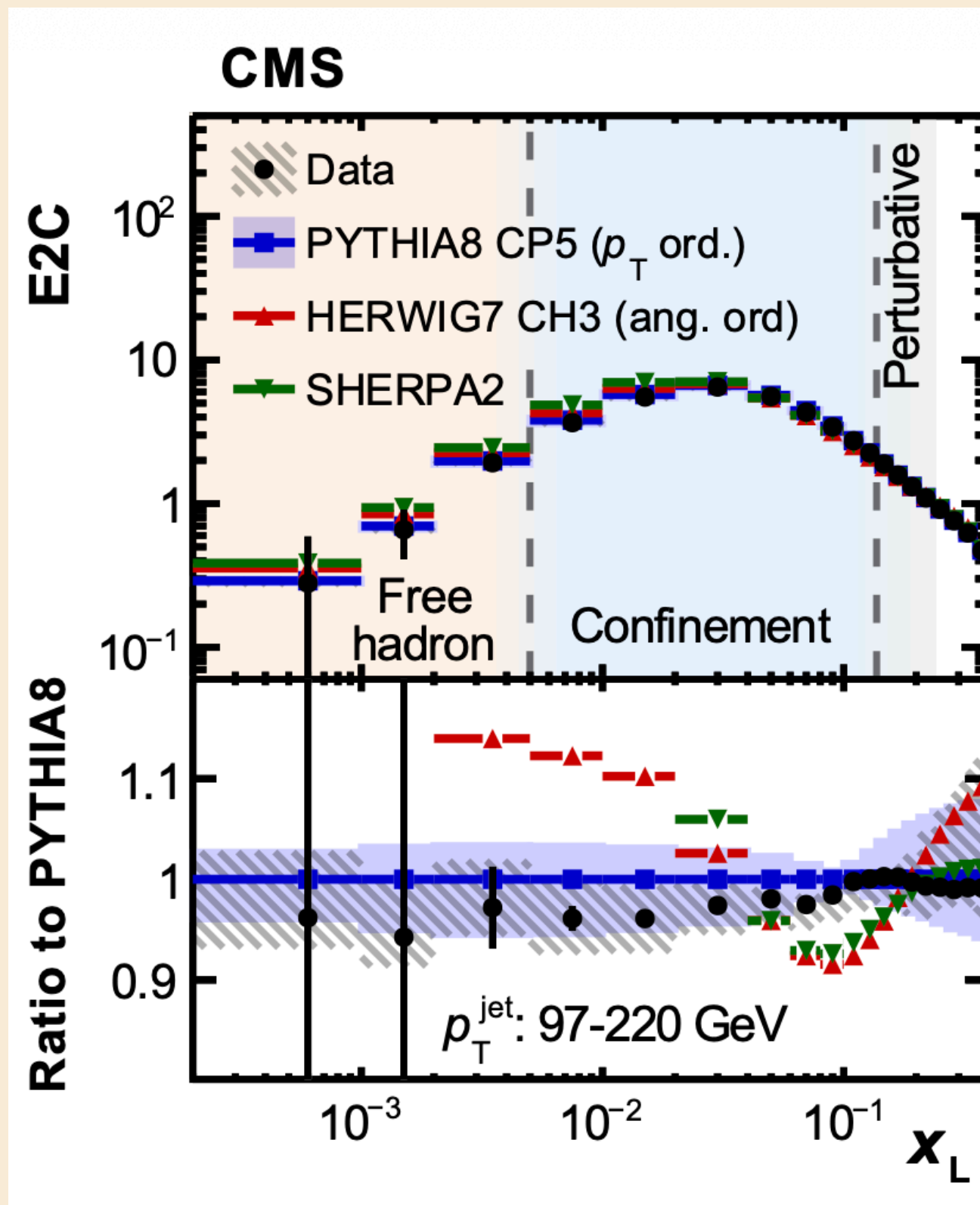
## Precision budget:

- N3LO + NNLL for  $e^+e^-$ , N3LO adapted from hadron leptonic production [He, Xing, Yang, Zhu PRL 2025](#)
- NLO + NNLL for  $pp$ , NLO by hacking FMNLO [C. Liu, X. Shen, B. Zhou, and J. Gao, JHEP 2023](#)
- NNLO + NNLL for  $pp$  in principle achievable [Czakon, et al., 2025; NNLOJET group, 2025](#)
- N3LO + NNLL for ep in principle achievable [Dong, Fang, Gao, Li, Shao, Zhu, Zhu, 2603.29673](#)
- N3LL needs 4-loop splitting function, now known for space-like non-singlet  
[Gehrmann, Manteuffel, Sotnikov, Yang, 2604.09534](#)

# Phenomenology

## Experiment status:

- CMS, ALICE, STAR

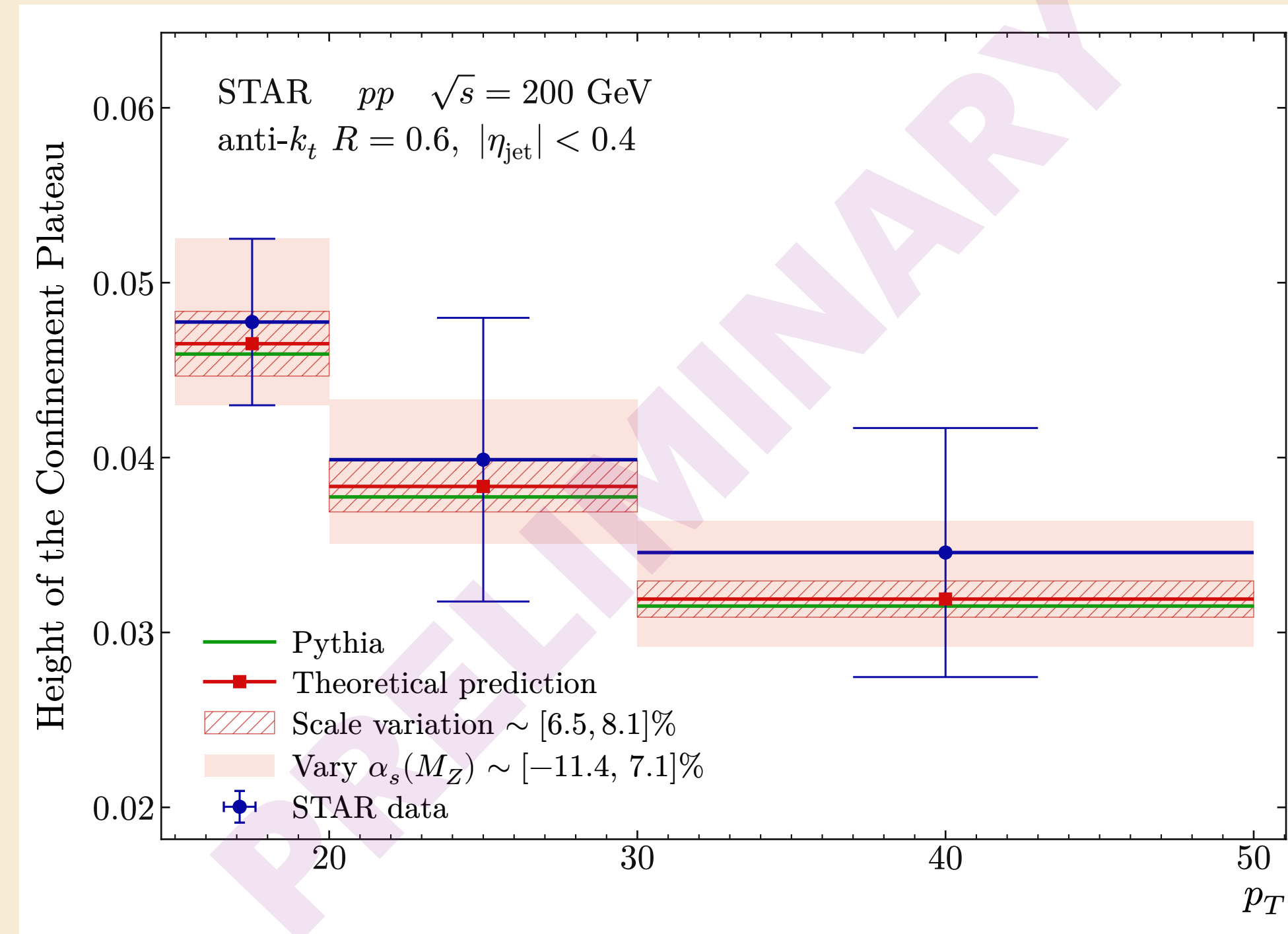
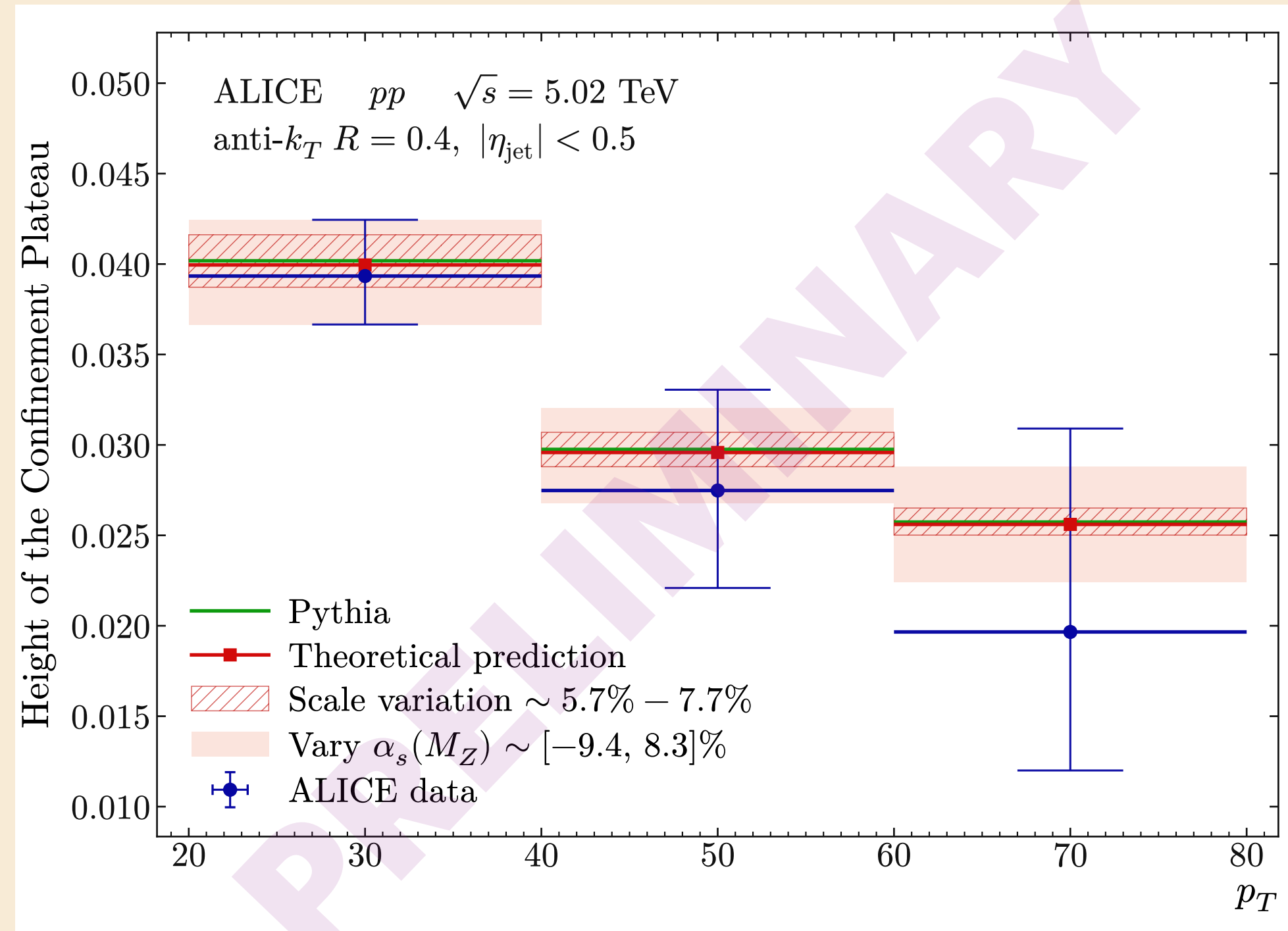


- All measure EEC and the data cover the post-confinement region
- Not directly the height of the  $\text{EEC}/Q^2$ , less sensitive to quantum scaling currently
- Linear fit

# Phenomenology

## CMS, ALICE, STAR

Zhan Wang ,et al. In preparation

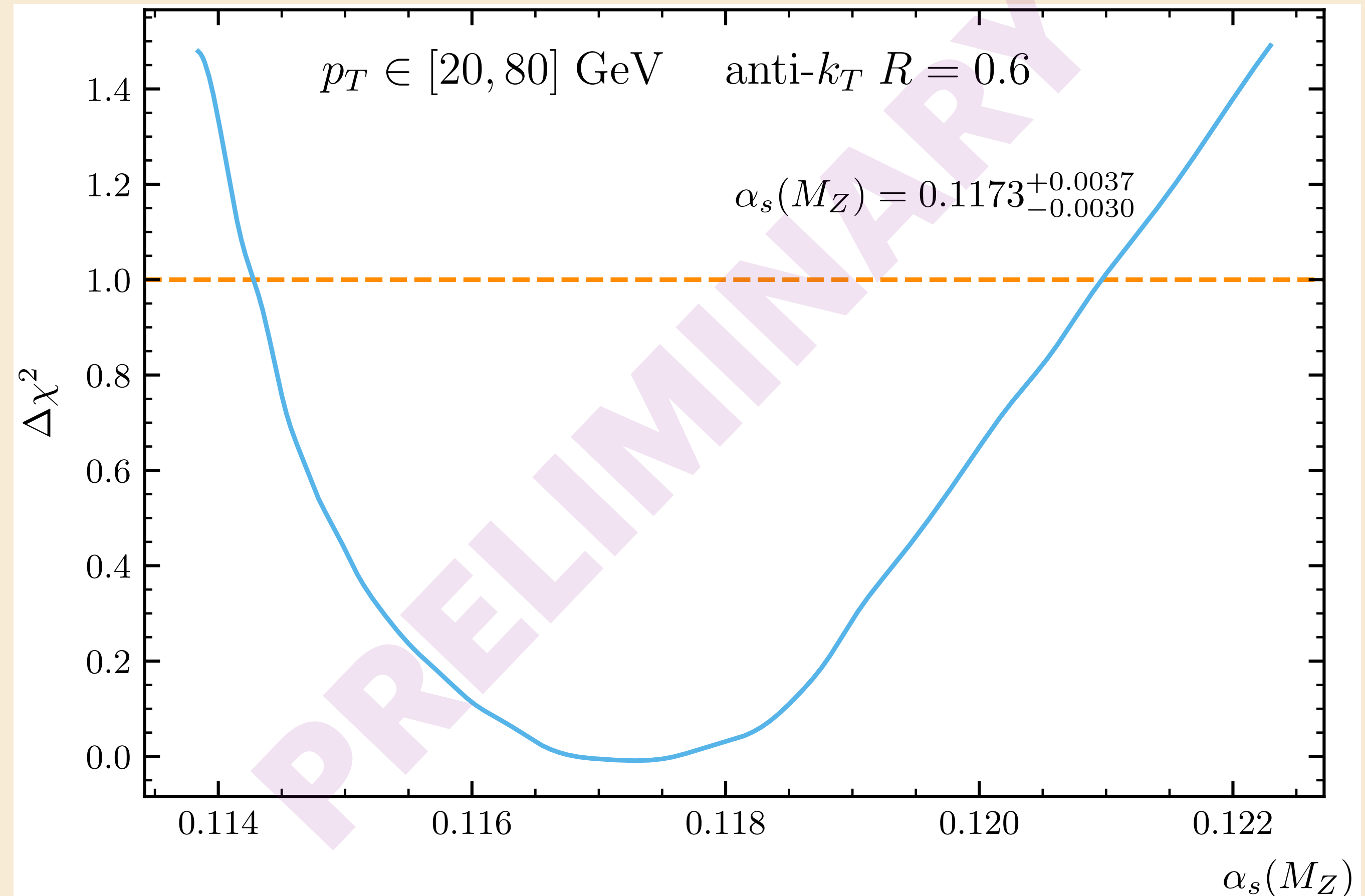


o Overall good agreement with NLO + NNLL @ LHC

# Phenomenology

## CMS, ALICE, STAR

- $\alpha_s$  extraction study using Pythia pseudo-data
  - Vary  $\alpha_s$  while fitting the NP parameters (height)
  - $\sim \pm 5\%$  precision in  $\alpha_s$  extraction, better than other jet-substructure approaches

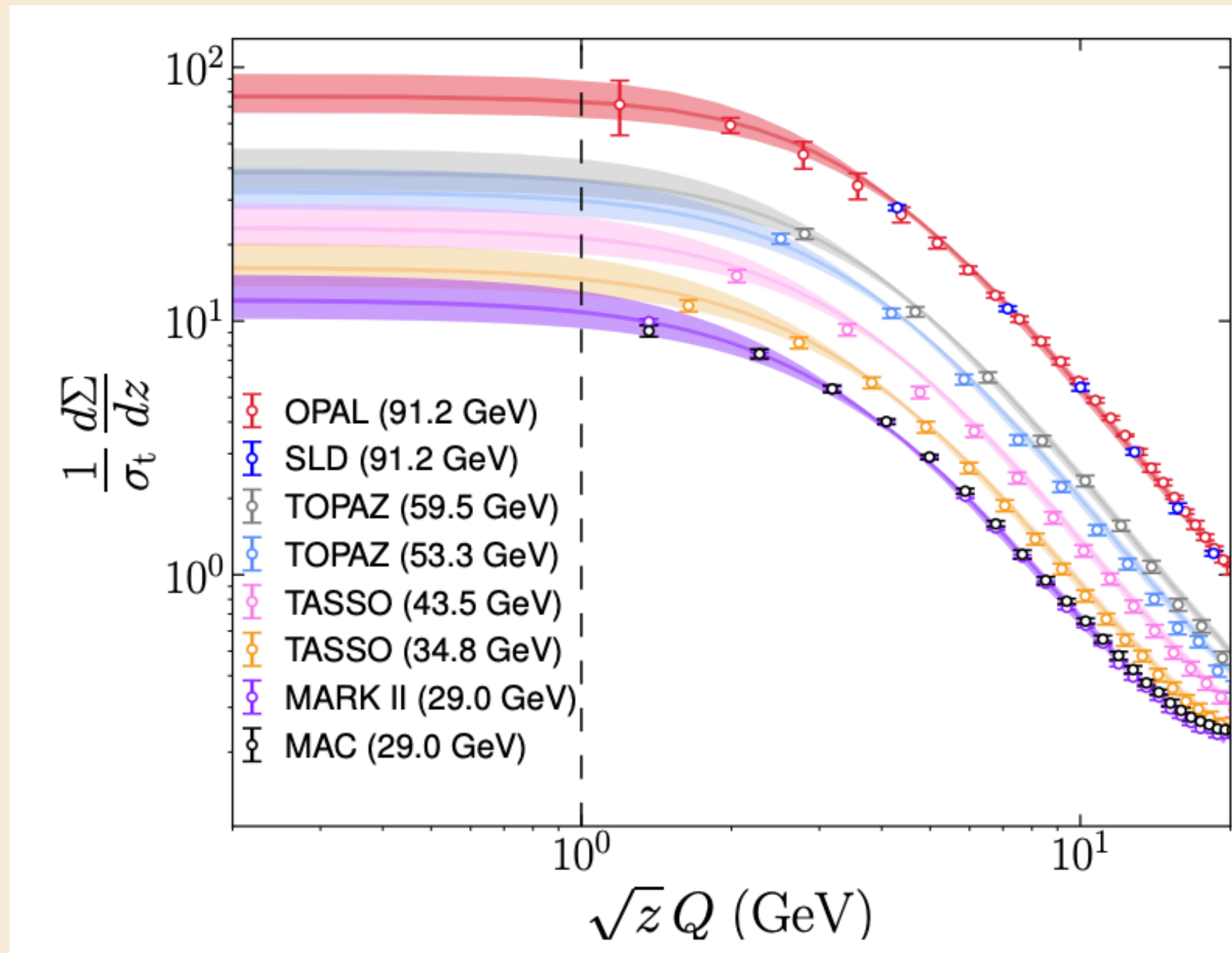


Zhan Wang ,et al. In preparation

# Phenomenology

## Experiment status:

○  $e^+e^-$

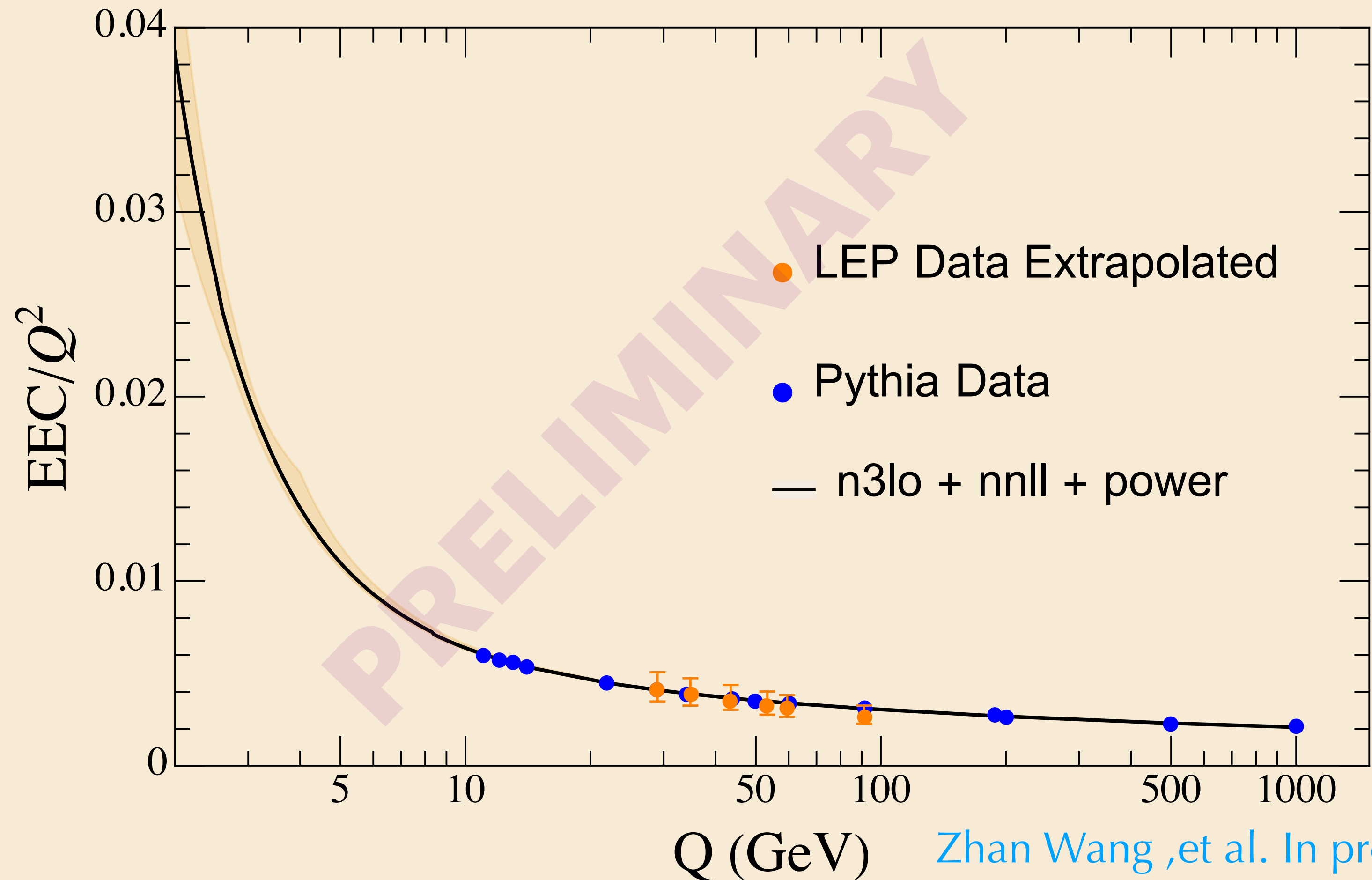


- Mainly old data, do not cover the post-confinement region
- Extrapolation

[Herrmann et al., 2507.17704](#)

# Phenomenology

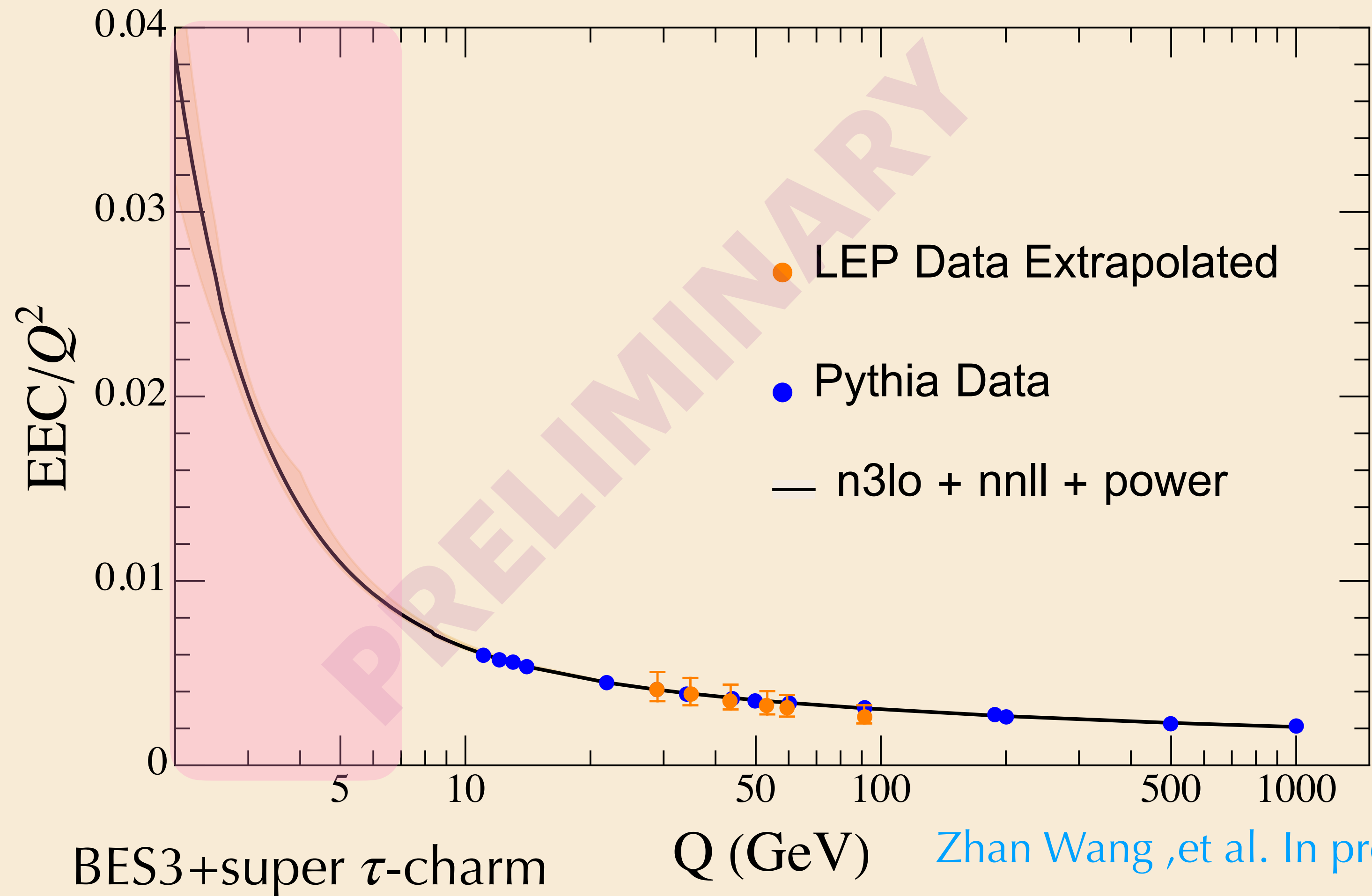
$e^+e^-$



Zhan Wang, et al. In preparation

# Phenomenology

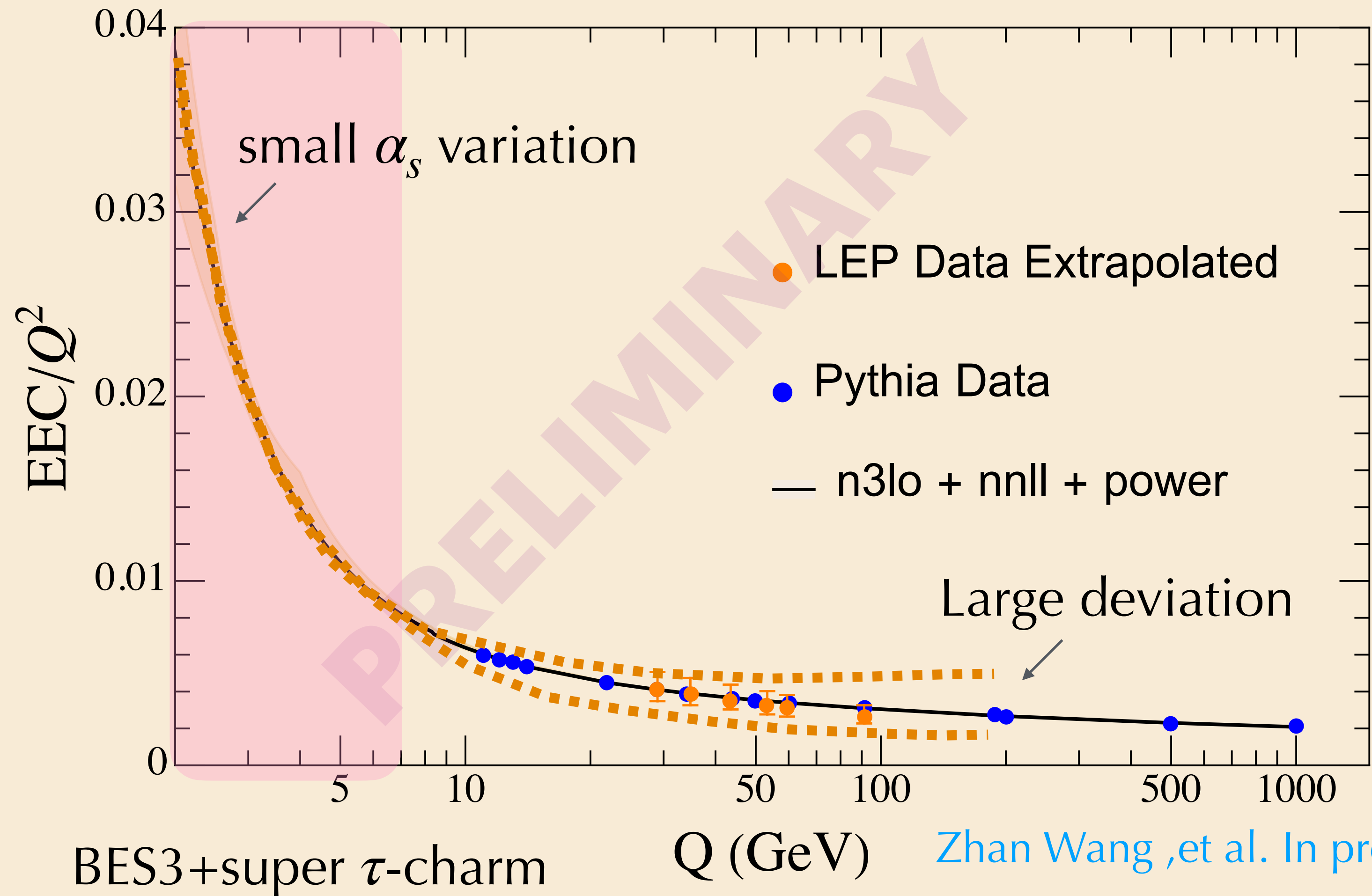
$e^+e^-$



- Overall good agreement with N3LO + NNLL @ LEP kinematics
- Important future check (NP power corrections) within the BES3+super  $\tau$ -charm region

# Phenomenology

$e^+e^-$



- Possible precision  $\alpha_s$  extraction with BES3/super  $\tau$ -charm + LEP



# Conclusion

- Quantum scaling behavior of the post-confinement EEC governed by DGLAP  $J = 5$
- Realizing N3LO+NNLL for  $e^+e^-$  and NLO+NNLL for  $pp$ , finding good agreement with current data and Pythia
- Similar Idea applicable to EIC, (EEC or NEEC)
- Further pushing the theory precision and possible  $\alpha_s$  extraction in the future

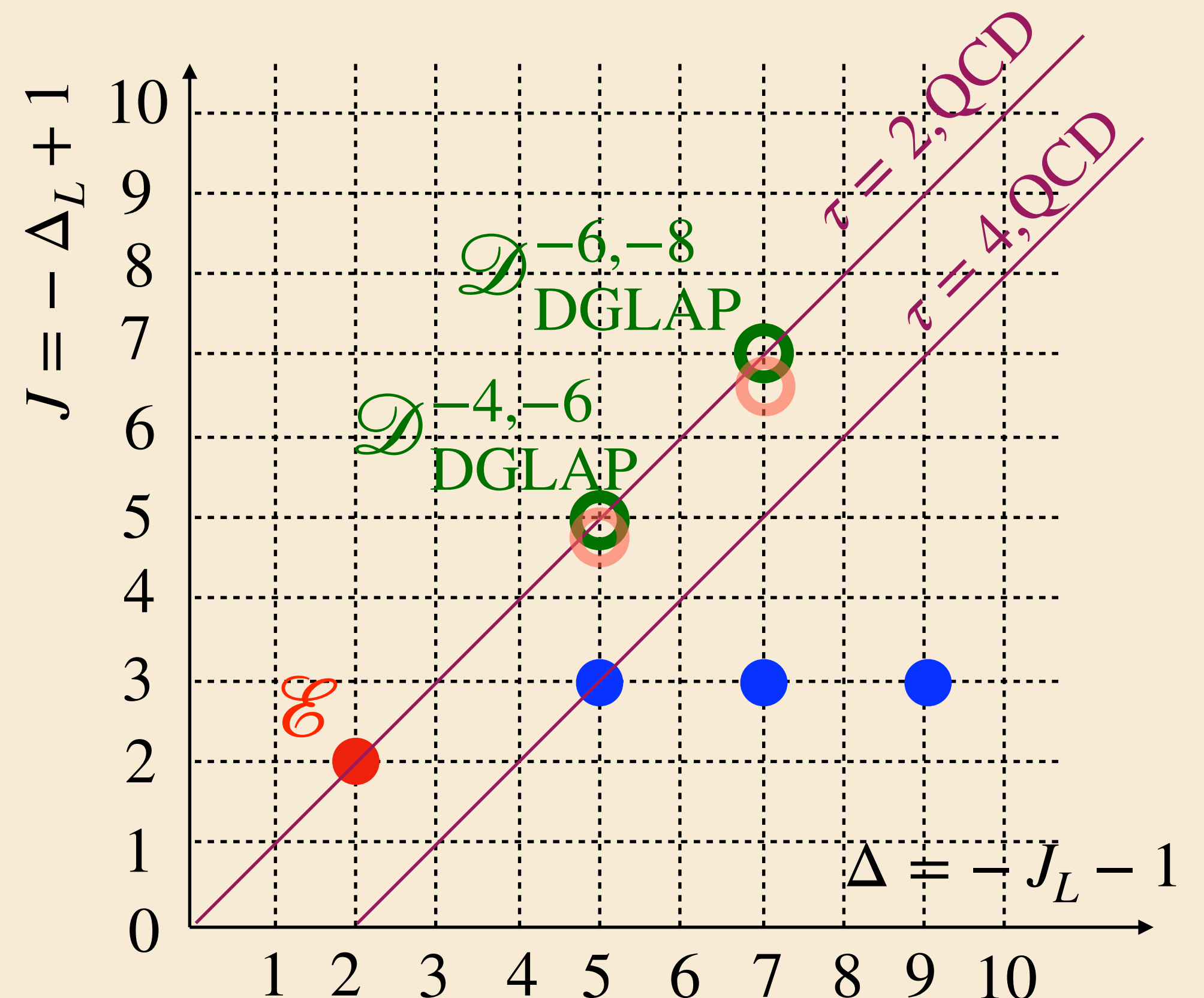
Thanks

# Backup: Two Step Matching-1. onto hadronic operators

$$\begin{aligned}
 -\Delta_{L,\mathcal{E}\mathcal{E}} &= 2 \\
 J_{L,\mathcal{E}\mathcal{E}} &= -6
 \end{aligned}
 \quad
 \mathcal{E}(n_1)\mathcal{E}(n_2) = \sum_{J_L} A_{J_L} \sqrt{\zeta}^{-6-J_L} \mathbb{O}_H^{-2,J_L}(n_2) = A_0 \zeta^0 \mathbb{O}_H^{-2,-6} + \dots$$

$-\Delta_L = 2$   
 $-J_L = 6$

- Light-ray OPE in the small  $\zeta$  limit with a hadron EFT ((boost)-Chiral? ), in the free hadron regime, assume zero hadron mass
- Equate the Lorentz dimension and Lorentz spin on both sides of the expansion. Keep the dimension unchanged, All  $\mathbb{O}_H^{\Delta_L, J_L}$  have the same  $\Delta_L = -2$
- Both  $A_{J_L}$  and  $\mathbb{O}_H$  can be calculated within the hadron EFT



# Backup: Two Step Matching-2. onto partonic operators

$$\begin{aligned}
 -\Delta_{L,\mathcal{E}\mathcal{E}} &= 2 \\
 J_{L,\mathcal{E}\mathcal{E}} &= -6
 \end{aligned}
 \quad
 \mathcal{E}(n_1)\mathcal{E}(n_2) = \sum_{J_L} A_{J_L} \sqrt{\zeta}^{-6-J_L} \mathbb{O}_H^{-2,J_L}(n_2) = A_0 \zeta^0 \mathbb{O}_H^{-2,-6} + \dots$$

$-\Delta_L = 2$   
 $-J_L = 6$

○ Match onto quark/gluon operators

$$\mathbb{O}_H^{-2,-6} = \frac{B_{-4}}{\Lambda_{\text{QCD}}^2} \mathcal{D}_{\text{DGLAP}}^{-4,-6}(\mu^2) + \text{higher twist}$$

$J_L$  keeps fixed while  $\Delta_L$  could change due to  $\Lambda_{\text{QCD}}$

