New bootstrap method for perturbative QCD

The 8th Heavy Flavor Physics and QCD Conference
Apr. 25, 2026

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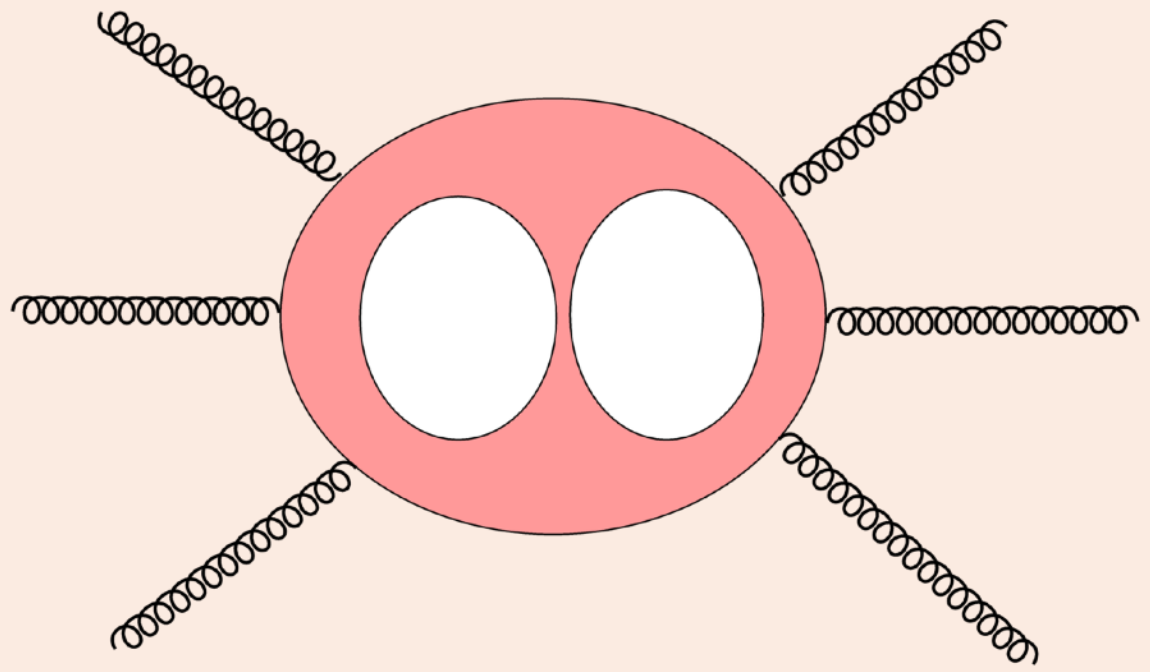
Based on



✓ First bootstrap computation of 2loop 6point QCD amplitudes

ACCEPTED PAPER

Bootstrapping Six-Gluon QCD Amplitudes

Sérgio Carrôlo and Dmitry Chicherin and Johannes Henn and Qinglin Yang and Yang Zhang

Phys. Rev. Lett. - **Accepted** 20 March, 2026

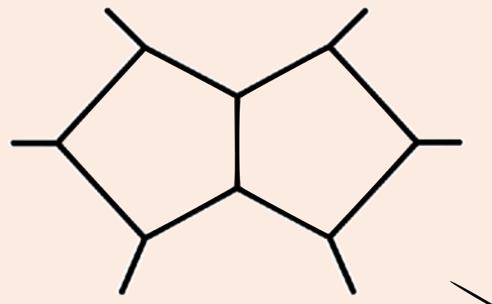
DOI: https://doi.org/10.1103/k237-8ccq

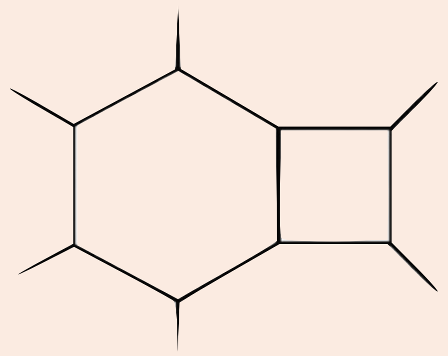
Carrôlo, Chicherin, Henn, Yang, YZ

arXiv:2510.20565, Phys. Rev. Lett. accepted

arXiv:2602.02783

✓ analytic computation of all 2loop 6point planar massless integrals





Henn, Matijasic, Miczajka, Peraro, Xu, YZ, Phys. Rev. Lett. 135 (2025) 3, 031601

Henn, Matijasic, Miczajka, Peraro, Xu, YZ, JHEP 08(2024) 027

Liu, Matijasic, Peraro, Xu, Yang, YZ, arXiv. 2603.16831

Outline

Bootstrap, then and now

Analytic Feynman integral

Two-loop Six-point QCD amplitude bootstrap

Summary

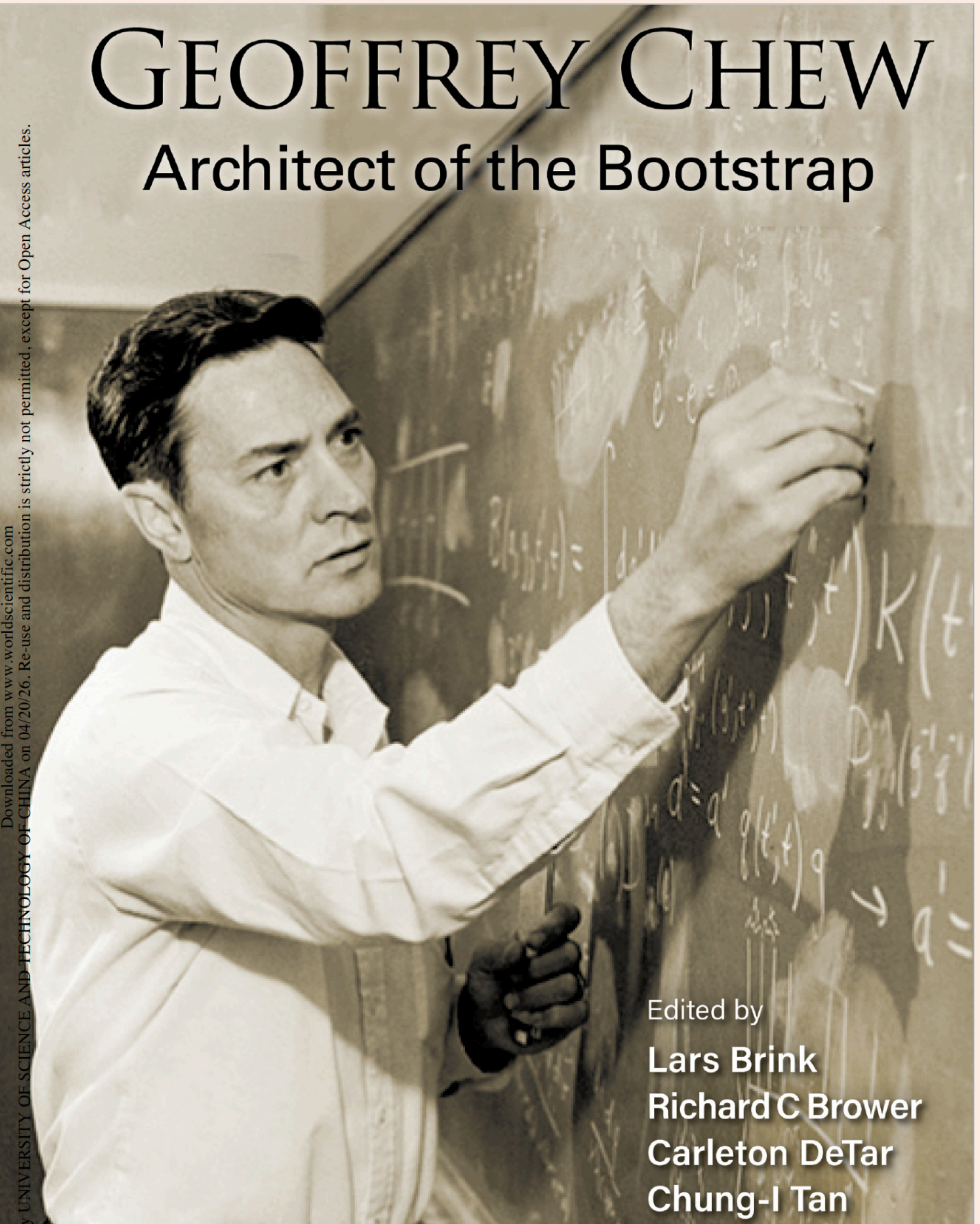
Bootstrap then and now

“pull yourself up by your bootstraps”

Bootstrap

S-matrix bootstrap

Chew and Mandelstam, 1960s



Construct the observable, S-matrix from a few general principles

Unitarity

Analyticity

Cross symmetry

Regge limit

different approach from QFT;

less popular after the triumph of QCD

Amplitude Bootstrap

maximally supersymmetric-Yang-Mills

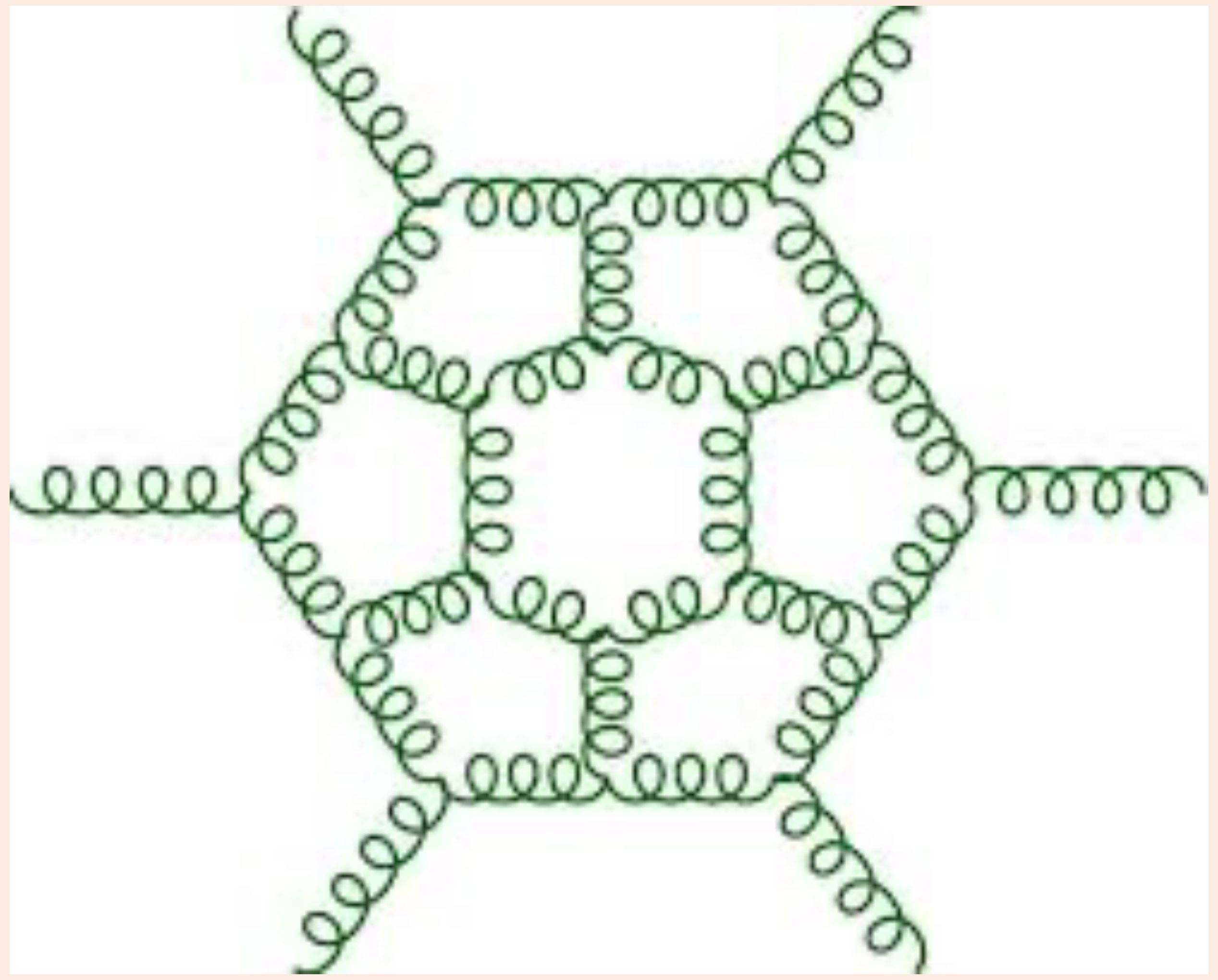
However, our world
is not supersymmetric, yet …

dual conformal letters → Integrable symbols → Ansatz → Amplitude

supersymmetry + dual conformal symmetry
only 9 letters for six-particle scattering

with Steinmann relation

collinear/Regge limit, dihedral symmetry



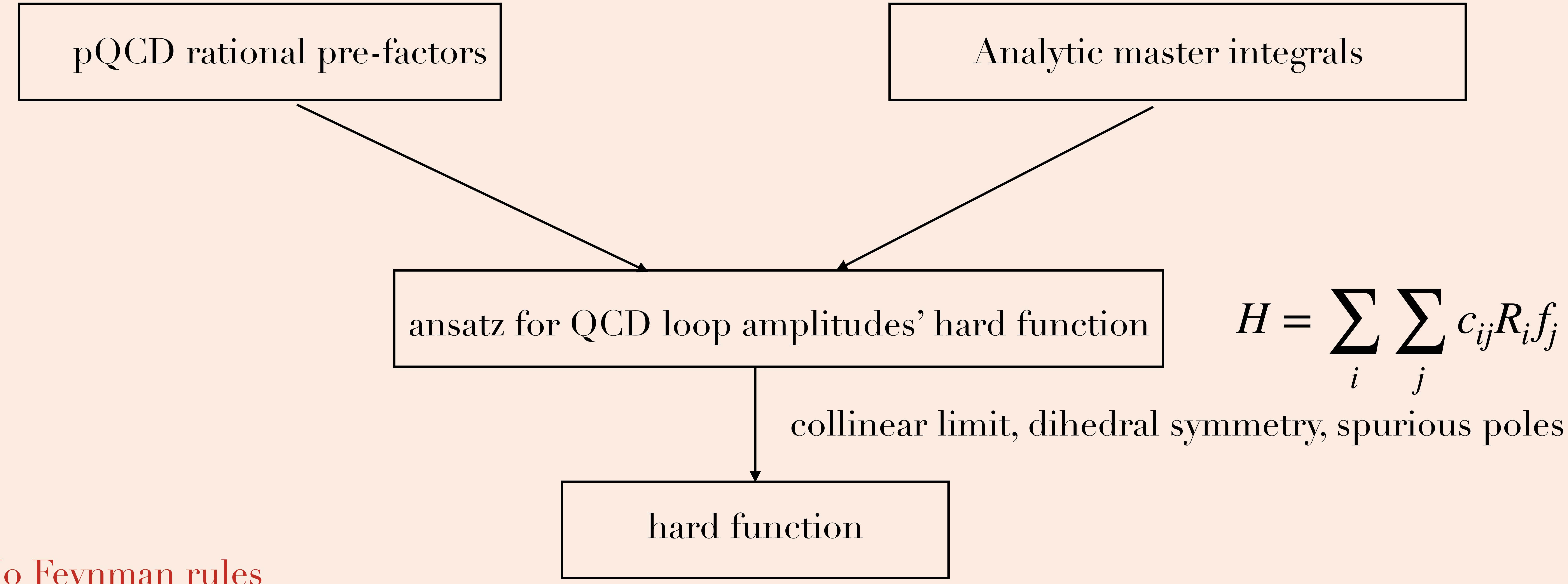
Beautiful 7-loop 6-point diagram
by Lance Dixon, Amplitudes 2016

(5loop, 6point MHV) Caron-Huot, Dixon, McLeod, von Hippel, Phys.Rev.Lett. 117 (2016) 24, 241601
(4loop, 7point MHV) Dixon, Drummond, Harrington, J. McLeod, Papathanasiou, Spradlin, JHEP 02 (2017) 137
(6loop NMHV & 7loop MHV, 6point) Caron-Huot, Dixon, Dulat, von Hippel, McLeod, Papathanasiou, *JHEP* 08 (2019) 016

QCD Bootstrap (our approach)

QCD is not supersymmetric, not dual conformal invariant …

N=4 Super-Yang-Mills six-point amplitudes have 9 letters to all loop order,
however, for QCD, two-loop six-point Feynman integrals contains 167 letters. The integrable symbol approach will not work.

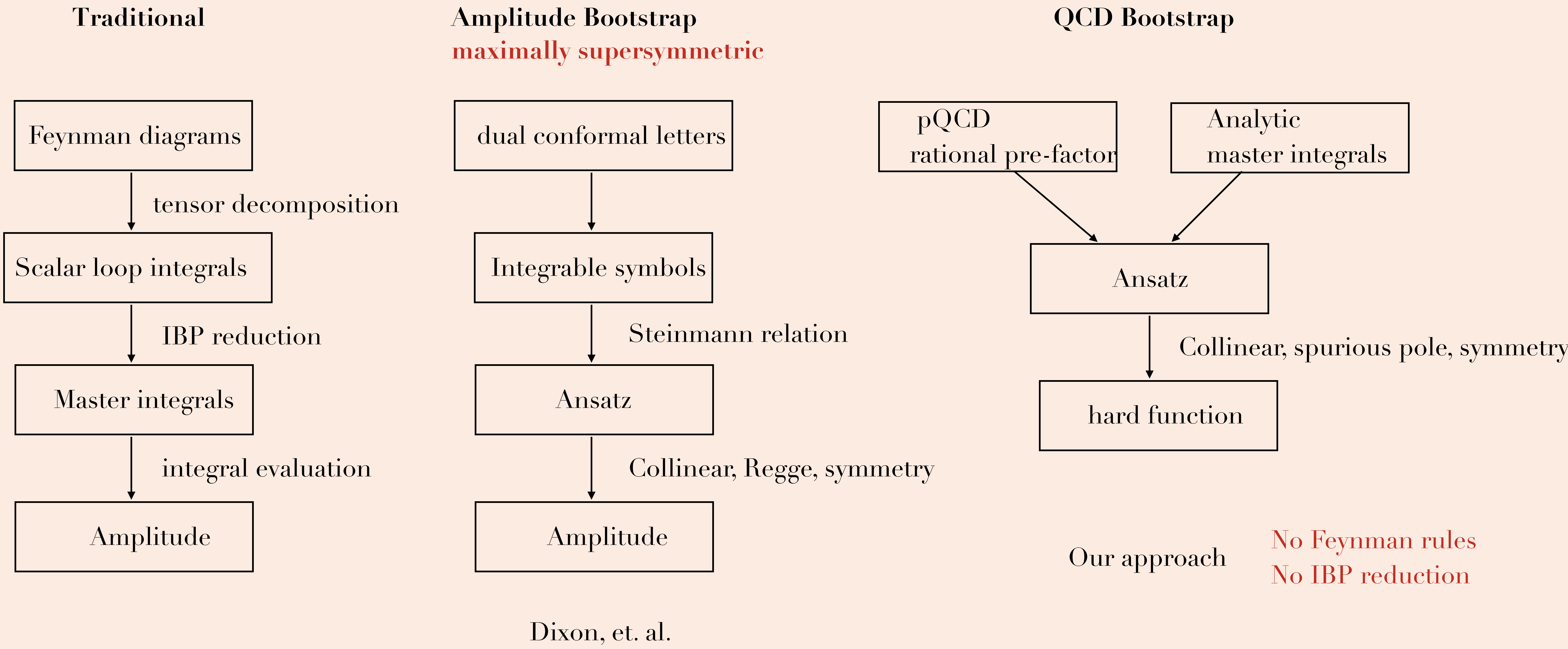


$$H = \sum_i \sum_j c_{ij} R_i f_j$$

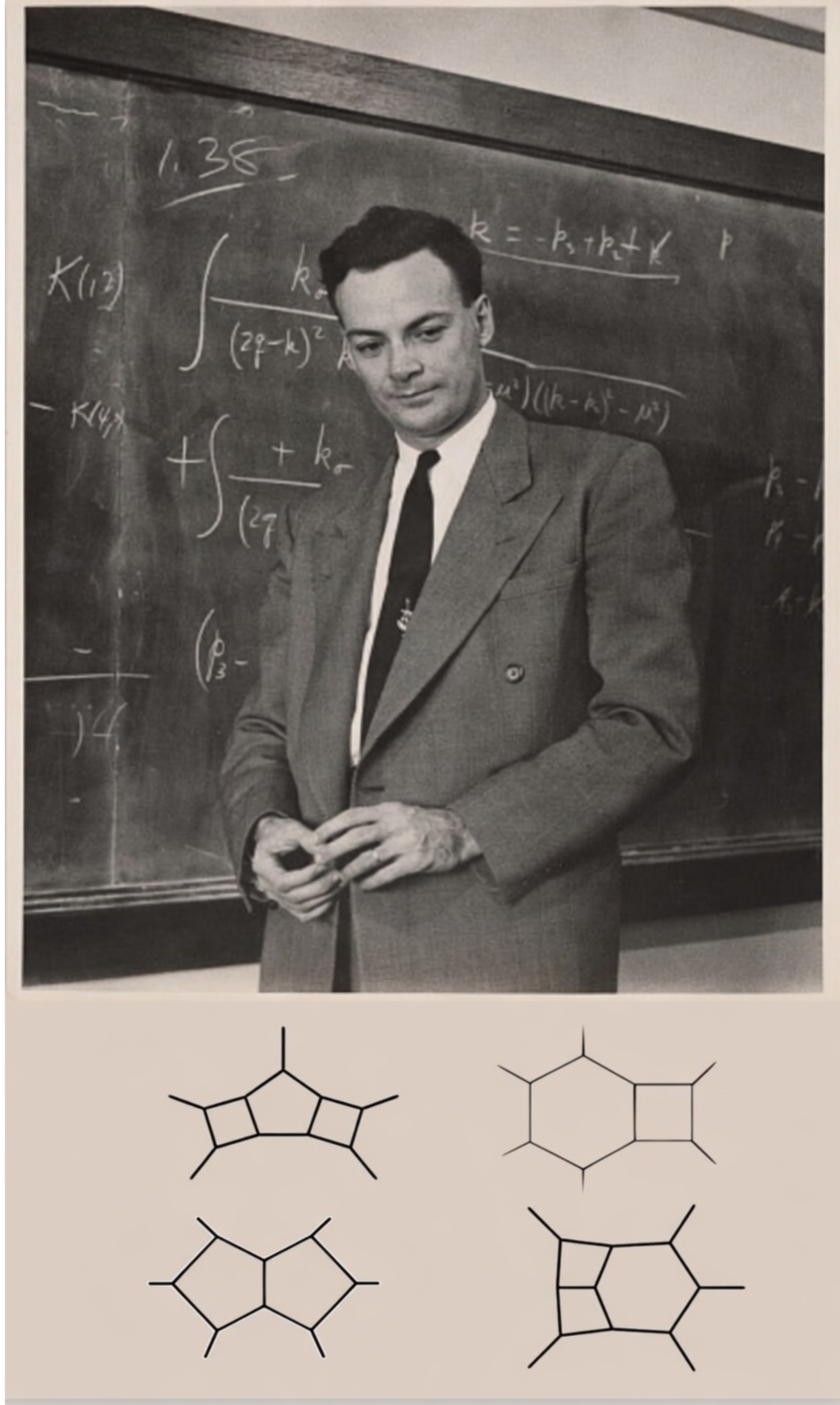
No Feynman rules
No IBP reduction

Carrôlo, Chicherin, Henn, Yang, YZ
arXiv:2510.20565, Phys. Rev. Lett. accepted
arXiv:2602.02783

A comparison



Analytic Feynman integrals



Current status of analytic Feynman integral computation

also an advertisement of our USTC group

Planar massless —> nonplanar massless —> massive

we can calculate analytic Feynman integrals for you!

	4-point	5-point	6-point	7-point	8-point
One-loop	known	known	known	known	known
Two-loop	most known	some results	?	?	?
Three-loop	some results	?	?	?	?
Four-loop	some results	?	?	?	?

with dimensional regulation

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with dimensional regulation

Frontier

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with dimensional regulation

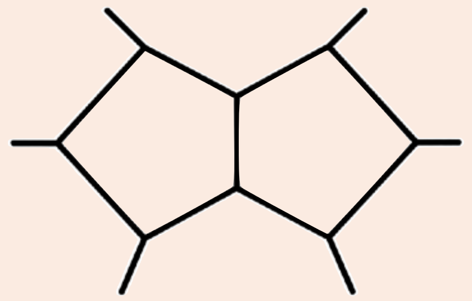
Frontier

Liu, Matijasic, Miczajka, Xu, Xu, YZ,
Phys.Rev.D 112 (2025) 1, 016021, editors' suggestion
Chicherin, Wu, Wu, Xu, Zhang, YZ,
arXiv 2512.17330

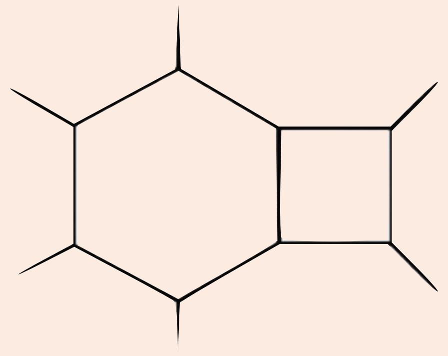
Henn, Matijasic, Miczajka, Peraro, Xu, YZ, PRL. 135, 031601
Henn, Matijasic, Miczajka, Peraro, Xu, YZ, JHEP 08(2024) 027
Henn, Peraro, Xu, YZ, JHEP 03 (2022) 056
Abreu, Monni, Page, Usovitsch JHEP 06 (2025) 112

2loop 6point Feynman integrals

NNLO production of 4 jets, 4 photons ...



267 master integrals



202 master integrals

+ dihedral permutations

Analytically calculated by canonical differential equations

167 letters to $O(\epsilon^0)$, 269 letters to all orders of ϵ

156 + 3 Even letters

$$s_{12}, \quad s_{123}$$
$$s_{12} - s_{123}$$
$$\dots$$
$$-s_{12}s_{45} + s_{123}s_{345}$$
$$\dots$$
$$\sqrt{\lambda(s_{12}, s_{34}, s_{56})}, \quad \epsilon_{ijkl}$$
$$\sqrt{\lambda(s_{45}\epsilon_{1236}, s_{46}\epsilon_{1235}, s_{56}\epsilon_{1234})}$$

79 + 18 Odd letters

$$\frac{s_{12} + s_{34} - s_{56} - \sqrt{\lambda(s_{12}, s_{34}, s_{56})}}{s_{12} + s_{34} - s_{56} + \sqrt{\lambda(s_{12}, s_{34}, s_{56})}}$$
$$\dots$$
$$\frac{s_{12}s_{23} - s_{23}s_{34} + s_{23}s_{56} + s_{34}s_{123} - s_{234}(s_{12} + s_{123}) - \epsilon_{1234}}{s_{12}s_{23} - s_{23}s_{34} + s_{23}s_{56} + s_{34}s_{123} - s_{234}(s_{12} + s_{123}) + \epsilon_{1234}},$$
$$\dots$$
$$\frac{-s_{12}s_{45}s_{234} + s_{34}s_{61}s_{123} + s_{345}(-s_{23}s_{56} + s_{123}s_{234}) - \Delta_6}{-s_{12}s_{45}s_{234} + s_{34}s_{61}s_{123} + s_{345}(-s_{23}s_{56} + s_{123}s_{234}) + \Delta_6}$$
$$\frac{-\sqrt{\lambda(s_{45}\epsilon_{1236}, s_{46}\epsilon_{1235}, s_{56}\epsilon_{1234})} + Q_i}{\sqrt{\lambda(s_{45}\epsilon_{1236}, s_{46}\epsilon_{1235}, s_{56}\epsilon_{1234})} + Q_i}$$

Pink letters

only for high ϵ orders

10 double-square letters

$$\frac{P - \epsilon_{1234}\sqrt{\lambda(s_{12}, s_{34}, s_{56})}}{P + \epsilon_{1234}\sqrt{\lambda(s_{12}, s_{34}, s_{56})}}$$

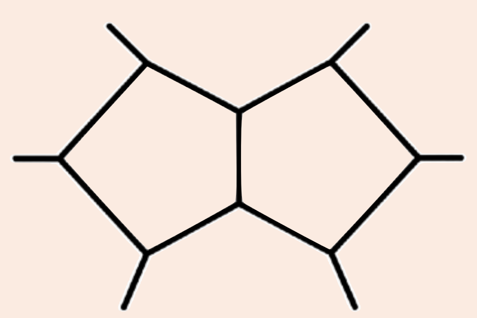
Liu, Matijasic, Peraro, Xu, Yang, YZ, *arXiv. 2603.16831*

Henn, Matijasic, Miczajka, Peraro, Xu, YZ, *Phys. Rev. Lett. 135 (2025) 3, 031601*

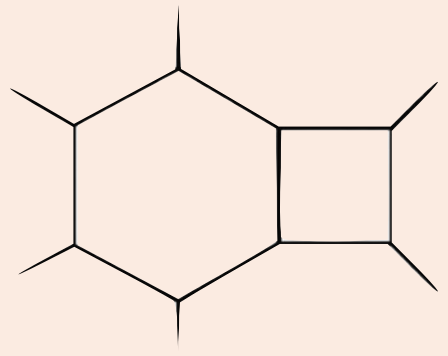
Henn, Matijasic, Miczajka, Peraro, Xu, YZ, *JHEP 08(2024) 027*

Abreu, Monni, Page, Usovitsch *JHEP 06 (2025) 112*

2loop 6point Feynman integrals



267 master integrals



202 master integrals

+ dihedral permutations

NNLO production of 4 jets, 4 photons …

Surprisingly small number of functions

As a comparison, 3841 integrable weight-4 symbols,
the N=4 super-Yang-Mills bootstrap does not work.

weight	1	2	3	4
2L6P symbols	9	59	266	639
2L5P1mass symbols	9	59	263	594
1L6P symbols	9	26	32	32
(1+2)L6P symbols	9	59	271	652

the small function space
is the foundation of our QCD bootstrap method

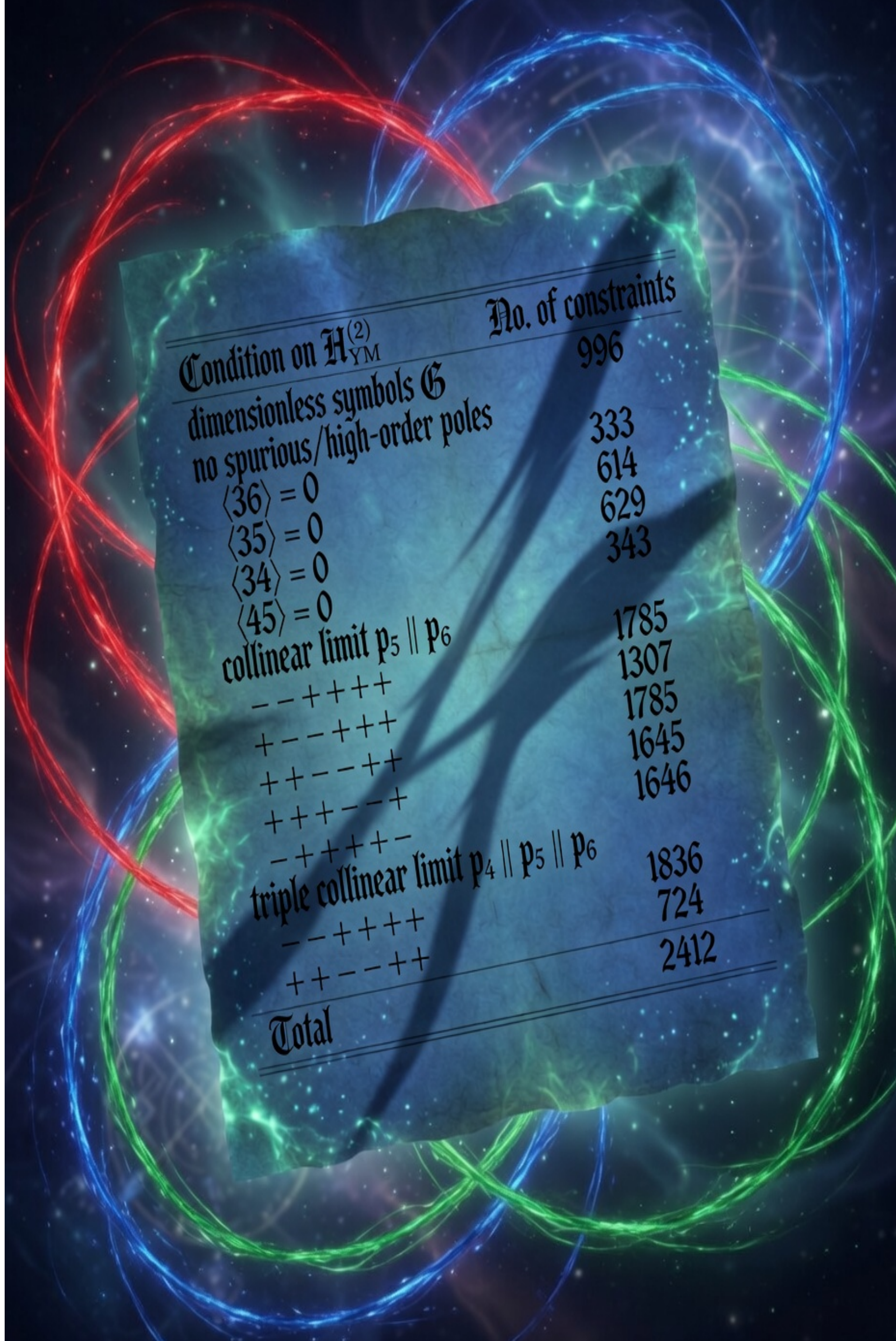
Liu, Matijasic, Peraro, Xu, Yang, YZ, *arXiv. 2603.16831*

Henn, Matijasic, Miczajka, Peraro, Xu, YZ, *Phys. Rev. Lett. 135 (2025) 3, 031601*

Henn, Matijasic, Miczajka, Peraro, Xu, YZ, *JHEP 08(2024) 027*

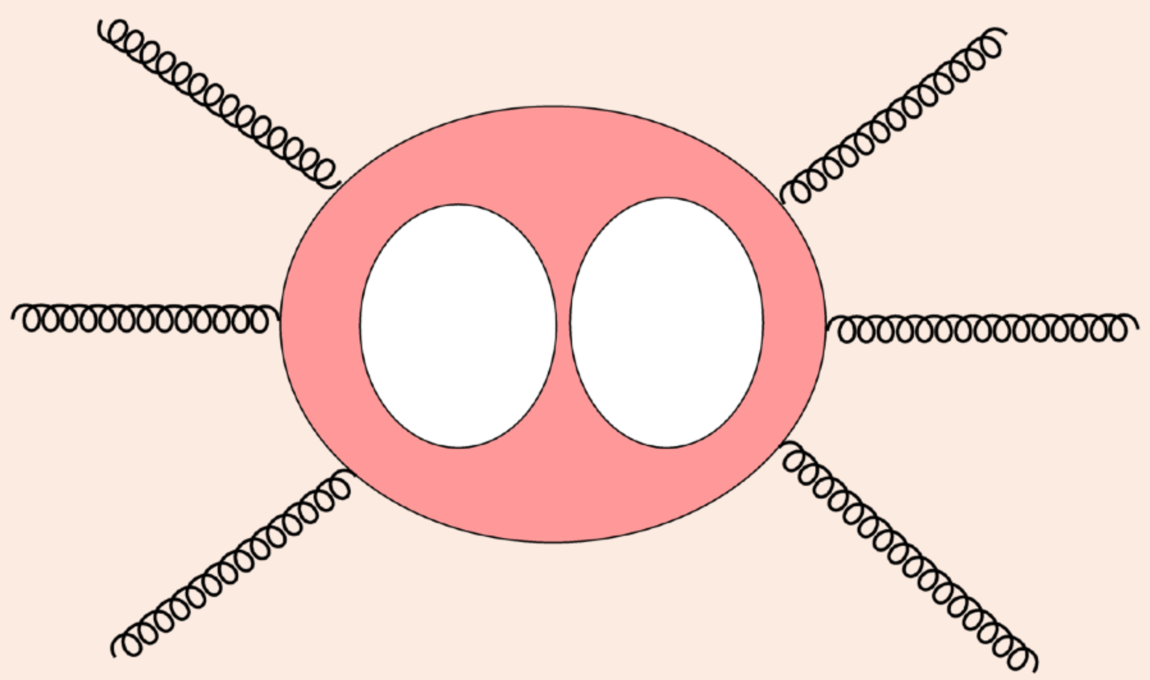
Abreu, Monni, Page, Usovitsch *JHEP 06 (2025) 112*

QCD Bootstrap



Two-loop six-point QCD amplitude

maximal transcendental part (weight-4), bootstrap



rational pre-factor

transcendental functions from master integrals

$$A = \sum_i \sum_j c_{ij} R_i f_j$$

"Bootstrapping Six-Gluon QCD Amplitudes"
Carrôlo, Chicherin, Henn, Yang, YZ, PRL accepted

$--++++$ helicity, with fermion loops, weight-4, symbol level

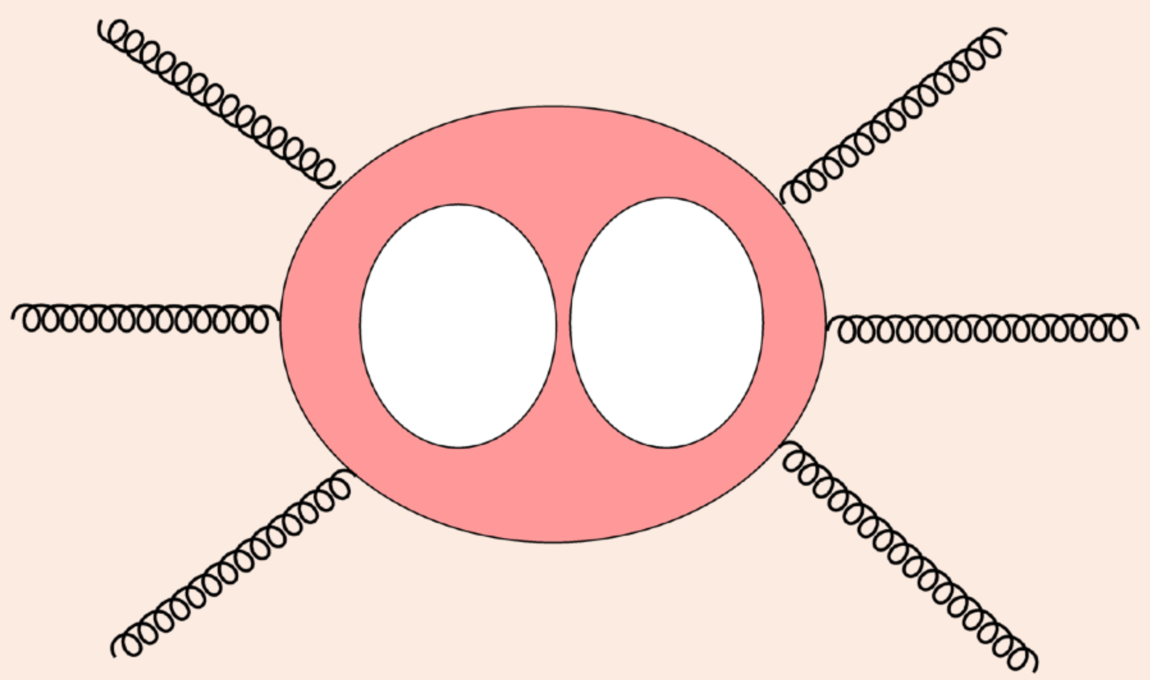
"QCD Scattering Amplitudes and Prescriptive Unitarity"
Carrôlo, Chicherin, Henn, Yang, YZ, arXiv: 2602.02783
other MHV helicities

We work out weight-4 part of the amplitude first, the "most complicated" part …

transcendental weight	rational prefactor	transcendental function
4	easy	hard
…	…	…
0	hard	easy

Two-loop six-point QCD amplitude

maximal transcendental part (weight-4), bootstrap



It is well known that the maximal transcendentality principle may not work for QCD amplitudes

$$L.S.(A^{\text{YM, 1-loop}}_{-+-+}) \neq L.S.(A^{\text{N=4, 1-loop}}_{-+-+})$$

Consider hard function instead of amplitude, to avoid D-dimensional leading singularity

$$\mathcal{H}^{(2)} = \lim_{\epsilon\to 0}\left[\mathcal{A}^{(2)} - I^{(1)}\mathcal{A}^{(1)} + \frac{1}{2}\left(I^{(1)}\right)^2 \mathcal{A}^{(0)}\right],$$

rational pre-factor

transcendental functions from master integrals

$$H = \sum_i \sum_j c_{ij} R_i f_j$$

maximal cut residue of the integrand

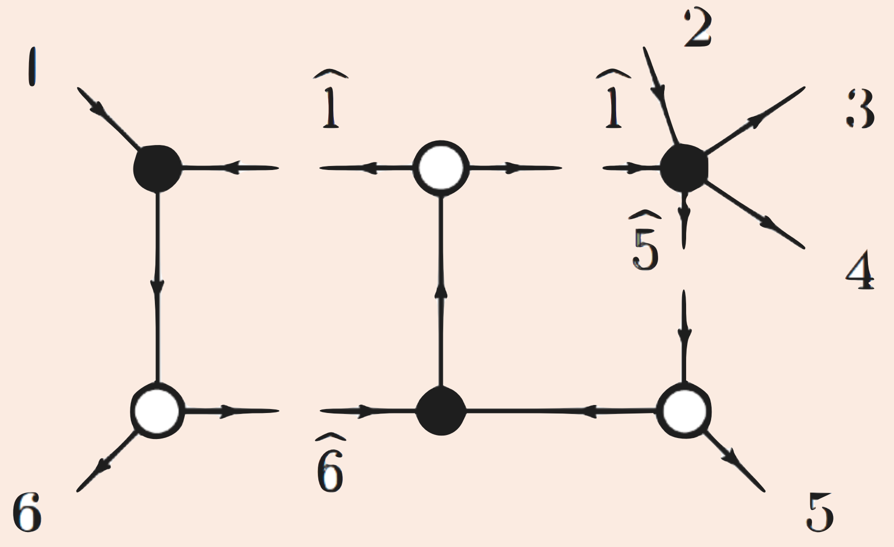
Generalized unitarity suggests that for the maximal weight part, rational pre-factors come from 4D leading singularity.

Conjectured by Henn and Bobadilla JHEP 03 (2022) 174

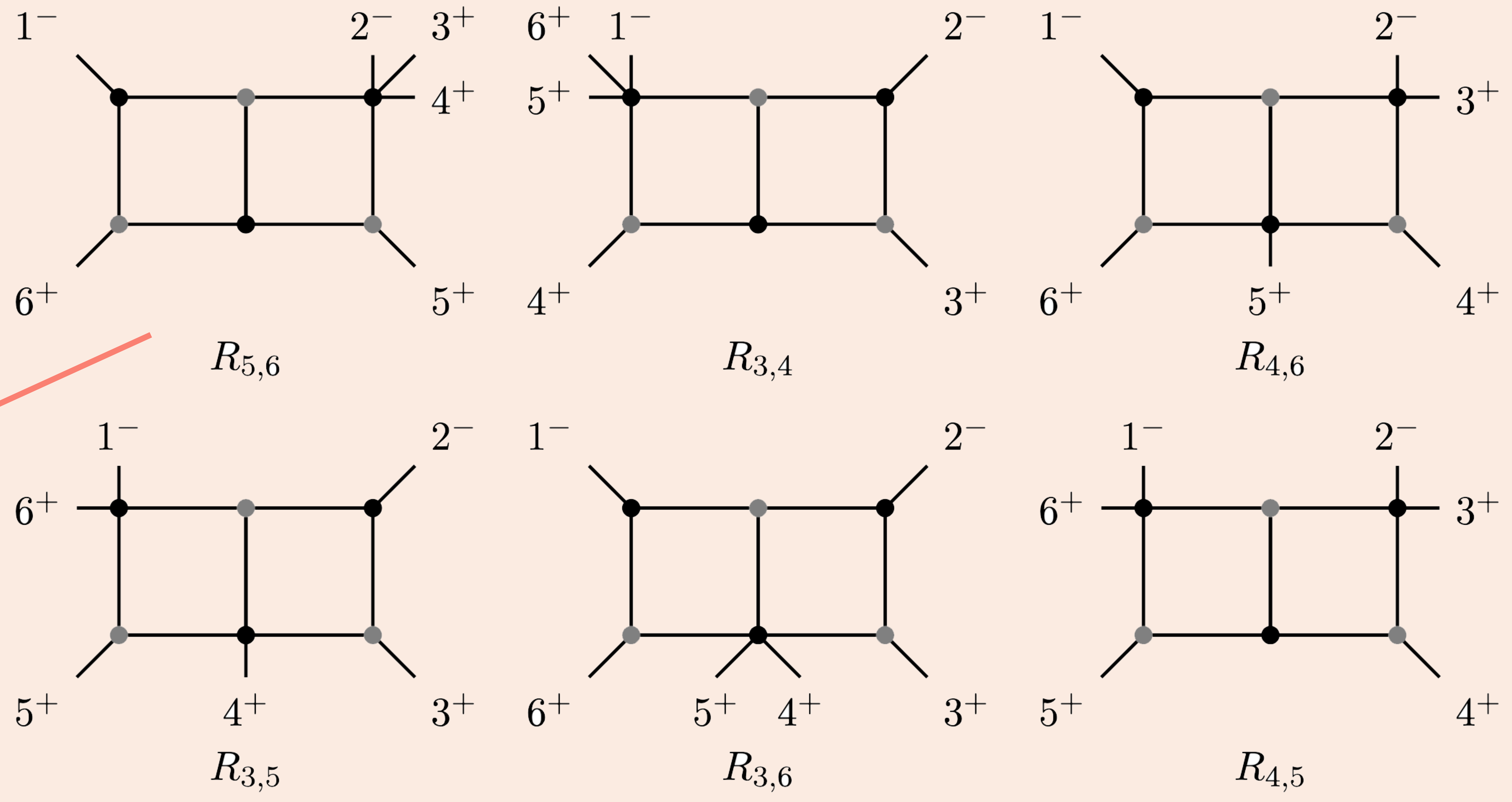
Two-loop six-point QCD amplitude

4D leading singularity

On shell diagram
as a product of tree amplitudes,
calculated by
Britto-Cachazo-Feng-Witten shifts



$$R_{5,6} = R_1 \times \oint dz \frac{(\langle 12\rangle + z\langle 26\rangle)^3}{\langle 12\rangle^4(\langle 15\rangle + z\langle 56\rangle)^4}$$



for $--++++$ weight-4, six rational pre-factors are needed

$$\mathcal{H}^{(2)}_{\rm YM} = R_1 G_1 + \sum_{2<i<j\leq 6} R_{i,j} G_{i,j}.$$

Parke-Taylor

$$R_{i,j}/R_1 = -1 + 12u_{i,j} - 30u_{i,j}^2 + 20u_{i,j}^3\,, \qquad u_{i,j} = \frac{\langle 1i\rangle\langle 2j\rangle}{\langle 12\rangle\langle ij\rangle}$$

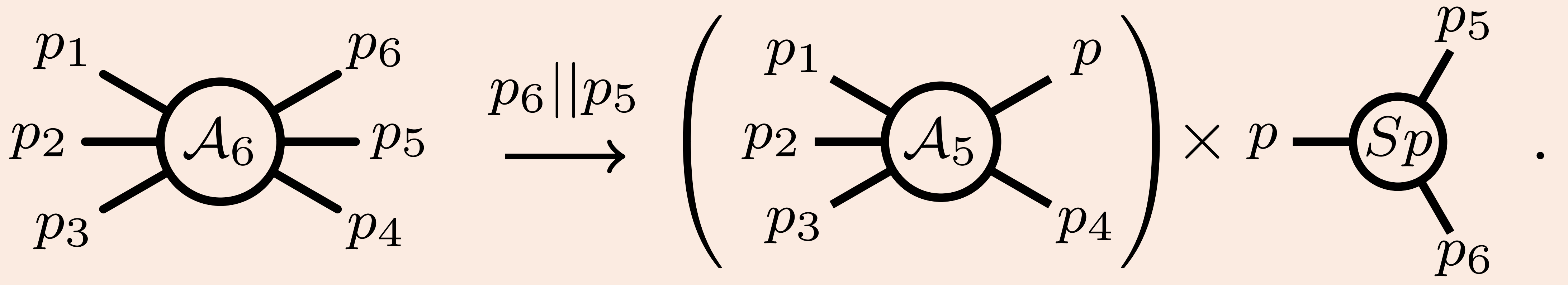
Two-loop six-point QCD bootstrap

linear combination of weight-4 function
from 216p master integrals, and IR subtraction

Condition on $\mathcal{H}^{(2)}_{\mathrm{YM}}$	No. of constraints
dimensionless symbols G	996
no spurious/high-order poles	
$\langle 36\rangle = 0$	333
$\langle 35\rangle = 0$	614
$\langle 34\rangle = 0$	629
$\langle 45\rangle = 0$	343
collinear limit $p_5 \parallel p_6$	
$--++++$	1785
$+--+++$	1307
$++--++$	1785
$+++--+$	1646
$-++++-$	1646
triple collinear limit $p_4 \parallel p_5 \parallel p_6$	
$--++++$	1836
$++--++$	724
Total	2412

$$\mathcal{H}^{(2)}_{\mathrm{YM}} = R_1 G_1 + \sum_{2<i<j\leq 6} R_{i,j} G_{i,j}.$$

Six rational "R", 712 weight-4 symbols, with reflection symmetry
2412 unknown coefficients to bootstrap



$$\mathcal{H}^{(2)}_6/\mathcal{A}^{(0)}_6 \xrightarrow{p_5\|p_6} \mathcal{H}^{(2)}_5/\mathcal{A}^{(0)}_5 + C^{h,(1)}_{h_5 h_6}\mathcal{H}^{(1)}_5/\mathcal{A}^{(0)}_5 + \frac{1}{2}\left(C^{h,(1)}_{h_5 h_6}\right)^2,$$

for this helicity,
$++ \to -$ splitting function does not contribute to weight-4

collinear limit for all adjacent pairs

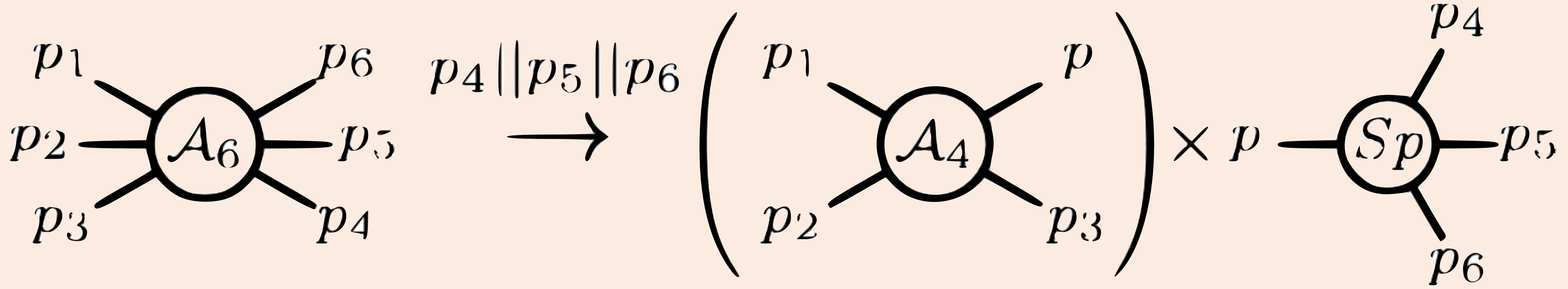
match with known splitting function and five-point amplitudes

We can also consider triple collinear limit …

Two-loop six-point QCD bootstrap

Condition on $\mathcal{H}^{(2)}_{\text{YM}}$	No. of constraints
dimensionless symbols G	996
no spurious/high-order poles	
$\langle 36 \rangle = 0$	333
$\langle 35 \rangle = 0$	614
$\langle 34 \rangle = 0$	629
$\langle 45 \rangle = 0$	343
collinear limit $p_5 \parallel p_6$	
$- - + + + +$	1785
$+ - - + + +$	1307
$+ + - - + +$	1785
$+ + + - - +$	1646
$- + + + + -$	1646
triple collinear limit $p_4 \parallel p_5 \parallel p_6$	
$- - + + + +$	1836
$+ + - - + +$	724
Total	2412

triple collinear limit ...



$$\mathcal{H}_6^{(2)}/\mathcal{A}_6^{(0)} \xrightarrow{p_4\|p_5\|p_6} \mathcal{H}_4^{(2)}/\mathcal{A}_4^{(0)} + C^{h,(1)}_{h_4h_5h_6}\mathcal{H}_4^{(1)}/\mathcal{A}_4^{(0)} + \frac{1}{2}\left(C^{h,(1)}_{h_4h_5h_6}\right)^2 + C^{h,(2)}_{h_4h_5h_6}.$$

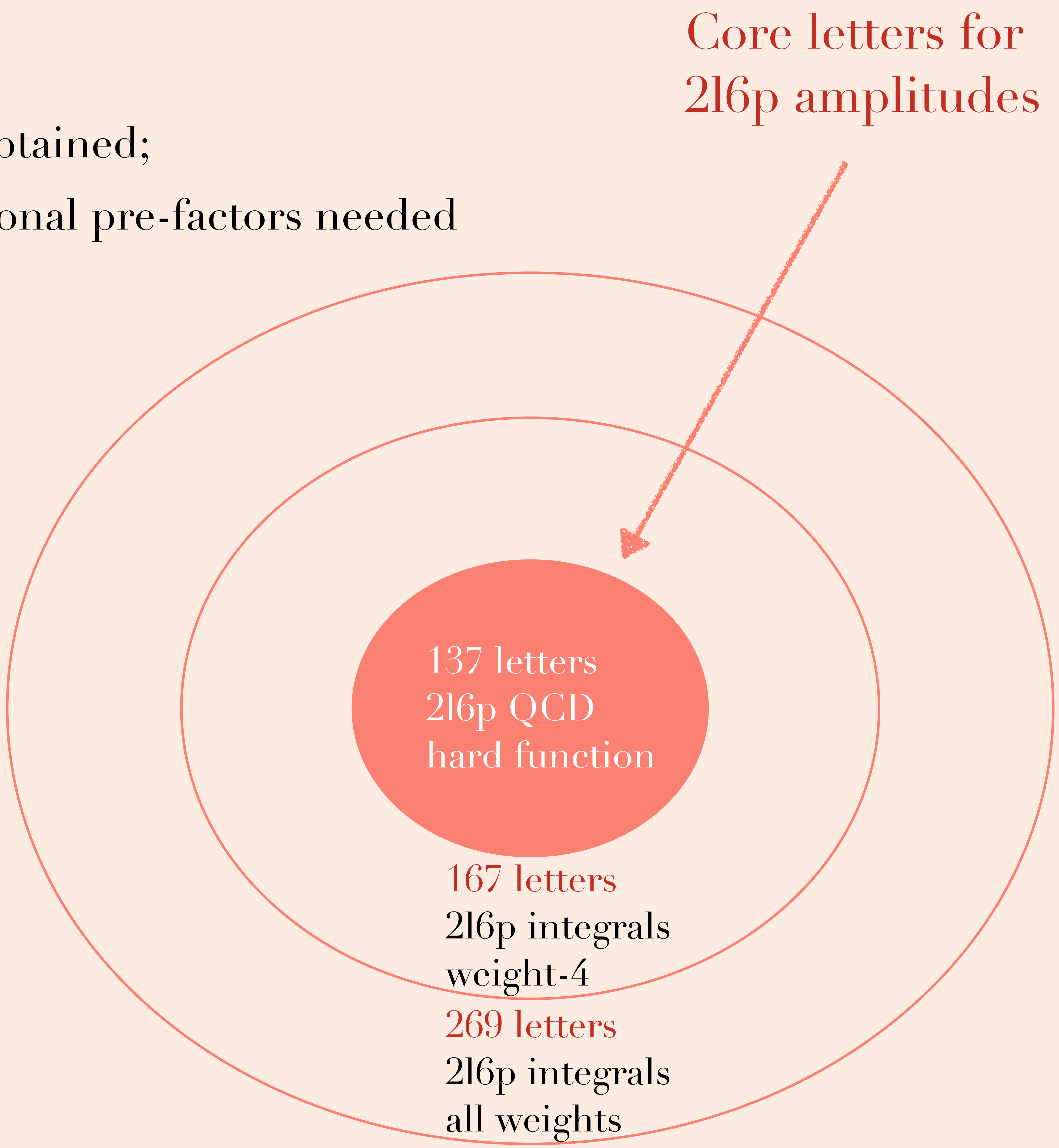
two-loop QCD triple collinear function was not known before.
However, this factorization form already provides more constraints.

All 2412 coefficients are uniquely fixed.

Two-loop six-point QCD bootstrap

Maximal weight two-loop six-point $--++++$ Amplitude obtained;

Fermion part is also included with bootstrap, anther six rational pre-factors needed



A byproduct

two-loop QCD triple splitting function, weight-4 part

$$C^{+,(2)}_{+++} = C^{(2),\mathrm{sYM}}_{N=4},$$

$$C^{-,(2)}_{+++} = C^{+,(2)}_{+++} + \left(r^{-}_{+++} + \frac{N_f}{N_c} s^{-}_{+++} \right) f.$$

↑ rational factor ↑ transcendental function

Summary and Outlook

New bootstrap method for QCD amplitudes,
based on analytic Feynman integrals and leading singularity

no Feynman rules

all 2loop 6point /3loop 5point planar massless integrals are calculated;

Cutting-edge perturbative QCD computation
leading colour, leading weight, two-loop $2 \to 4$ gluon scattering

"QCD Scattering Amplitudes and Prescriptive Unitarity"

other 2loop 6point MHV helicites, by the same authors, arXiv: 2602.02783

New applications are coming …

四川美术学院

Thanks