Vtb from tW Production

HIGH-PRECISION PREDICTION FOR SINGLE TOP QUARK PRODUCTION AT THE LHC

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Based on arXiv: 2512.10711 with Jia-Le Ding and Hai Tao Li

Heavy flavor physics and QCD

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目录

CONTENTS

01

Background & Motivation

Top quark physics and tW production channel

1.1 Top quark and tW production at the LHC

1.2 Two obstacles to N2LO predictions

Top Quark and tW Production at the LHC

- **Heaviest elementary particle:** Top quark has close connection to new physics (e.g., supersymmetry)
- **Top quark pair production:** dominant channel, but insensitive to top quark electroweak interaction
- **Decay channel:** $t \to Wb$ almost exclusively; width predicted at N^{3}LO [Chen, Li, Li, JW, Wang, Wu, 2309.00762; Chen, Chen, Guan, Ma, 2309.01937]; determines V_{tb} only assuming CKM unitarity
- **Single top production:** Cross section ~1/3 of pair production; directly probes V_{tb} without unitarity assumption; s- and t-channels computed at N^{2}LO [Brucherseifer, Caola, Melnikov, 1404.7116; Berger, Gao, Yuan, Zhu, 1606.08463; Liu, Gao, 1807.03835; Campbell, Neumann, Sullivan, 2012.01574]
- **tW channel:** $pp \to tW$ is the 2nd largest single top channel at the LHC; theoretical prediction remained at NLO for 20+ years [Geile, Keller, Laenen, hep-ph/9511449; Zhu, hep-ph/0109269; Campbell, Tramontano, hep-ph/0506289; Cao, 0801.1539]; higher-order soft gluon effects [Kidonakis, 06-19, Li, Li, Shao, JW, 1903.01646]

Motivation: Precise tW predictions are essential for V_{tb} determination and new physics searches at the LHC.

Two Obstacles to N²LO Predictions

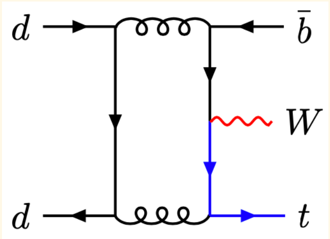
Obstacle 1: Two-Loop Corrections

- Challenging two-loop virtual corrections
- Multiple scales in loop diagrams
- Leading color: analytical results [Chen, Dong, Li, Li, JW, Wang, 2208.08786]
- Full color: numerical results [Chen, Dong, Li, Li, JW, Wang, 2212.07190]

Status: **Recently solved**

Obstacle 2: $t\bar{t}$ Interference

- tW and $t\bar{t}$ interfere at higher orders
- Tree-level subtraction: fully developed [Belyaev, Boos, Dudko, hep-ph/9806332, Tait, hep-ph/9909352, Frixione, Laenen, Motylinski, Webber, White, 0805.3067]
- One-loop subtraction: proposed recently [Dong, Li, Li, JW, 2411.07455]
- $tW\bar{b}g$ final state: still missing



Status: **Partially solved**

Our strategy: Focus on soft radiation region where $t\bar{t}$ interference is kinematically forbidden, circumventing Obstacle 2.

02

Theoretical Framework

Factorization and threshold resummation

2.1 Factorization formula in the threshold limit

2.2 Hard and soft functions, RG evolution

2.3 Resummation-improved cross section

Factorization Formula in the Threshold Limit

$\boldsymbol{p(P_1) + p(P_2) \to t(p_3) + W(p_4) + X(p_X)}$

- $s = (P_1 + P_2)^2,\ Q^2 = (p_3 + p_4)^2,\ \tau = Q^2/s$
- $p_X^0 = \frac{\sqrt{s}(1-\tau)}{2} \to 0$: hadronic threshold limit

Factorized Differential Cross Section

$$\frac{d\sigma}{dQ^2 d\Phi_2} = \frac{1}{s}\int_\tau^1 \frac{dz}{z}\mathcal{L}\left(\frac{\tau}{z},\mu\right)\frac{1}{2Q^2}H(\beta_t, y, \mu)\,\mathcal{S}(\bar{z}, \beta_t, y, \mu)$$

- H: hard function (virtual corrections)
- S: soft function (soft radiation)
- L: parton luminosity (PDF convolution)

Key Concepts

Partonic process: $b(p_1) + g(p_2) \to t(p_3) + W(p_4)$

at LO with $z = \frac{Q^2}{\hat{s}}, \bar{z} = 1 - z$

Threshold limit: Soft radiation induces large logs

$\ln^{2n-i}(\bar{z})/\bar{z}$ dominating higher-order corrections

Dynamical enhancement: PDF suppression at $x \to 1$ enhances parton luminosity $z \to 1$ [Becher, Neubert, Xu, 0710.0680]

The soft region captures dominant real corrections and avoids the $t\bar{t}$ interference problem.

Hard and Soft Functions, RG Evolution

Hard Function H

- Process-dependent virtual correction
- Function of $y = -cos\theta$ and β_t
- Intrinsic scale: $\mu_h \sim \sqrt{\hat{s}}$
- $H = \sum_n \left(\frac{\alpha_s}{4\pi}\right)^n H^{(n)}$
- Calculated at N²LO [Chen, Dong, Li, Li, JW, Wang, 2208.08786, 2212.07190]

Soft Function S

- Encodes all soft radiation effects
- Logarithms: $\ln^{2n-i}(\bar{z})/\bar{z}$
- Intrinsic scale: $\mu_s \sim \sqrt{\hat{s}} - Q$
- Calculated at N²LO [Ding, Li, JW, 2502.18648]
- Laplace transform: $\tilde{S}\left(\ln\left(\frac{\hat{s}\xi^2}{\mu^2}\right)\right) = \int_0^\infty d\bar{z}\, e^{-\frac{\bar{z}}{\xi}} S(\bar{z})$

RG Evolution Equations

Soft: $\frac{d\tilde{S}}{dln\,\mu} = \gamma_s \tilde{S}$, Hard: $\frac{dln\,H}{dln\,\mu} = -\frac{dln\,\tilde{f}_q}{dln\,\mu} - \frac{dln\,\tilde{f}_g}{dln\,\mu} - \frac{dln\,\tilde{S}}{dln\,\mu}$

Anomalous dimensions: known to 3 loops [Liu, Schalch, 2207.02864]; a new color structure $T_{1233} = \frac{1}{2} f^{ade} f^{bce} T_1^a T_2^b (T_3^c T_3^d + T_3^d T_3^c)$

Resummation-Improved Cross Section

Resummation Formula

$$\frac{d\sigma}{dQ^2 d\Phi_2} = \frac{1}{s}\int_\tau^1 \frac{dz}{z} L\left(\frac{\tau}{z}, \mu_f\right) \frac{1}{2Q^2} H\left(\beta_t, y, \mu_h\right) U(\mu_h, \mu_s, \mu_f) \tilde{S}\left(\partial_\eta, \beta_t, y\right) \frac{1}{\bar{z}} \left(\frac{\bar{z}^2 Q^2}{z\mu_s^2}\right)^\eta \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$

$U(\mu_h, \mu_s, \mu_f)$: evolution factor; η: depends on hard/soft scales.

Choose $\mu_h \sim \sqrt{\hat{s}}$, $\mu_s \sim \sqrt{\hat{s}} - Q$ to resum all logarithms.

Set $\mu_h = \mu_s = \mu_f = \mu$ for fixed order ($U = 1, \eta = 0$).

Scale Structure and Resummation

Hard Scale
$\mu_h \sim \sqrt{\hat{s}}$
Calculate H, no large logs

→

RG Evolution
$U(\mu_h, \mu_s, \mu_f)$
Resum large logarithms

←

Soft Scale
$\mu_s \sim \sqrt{\hat{s}} - Q$
Calculate $\tilde{S}$, no large logs

03

Higher-Order Corrections

N2LO functions, N3LO terms, and fixed-order coefficients

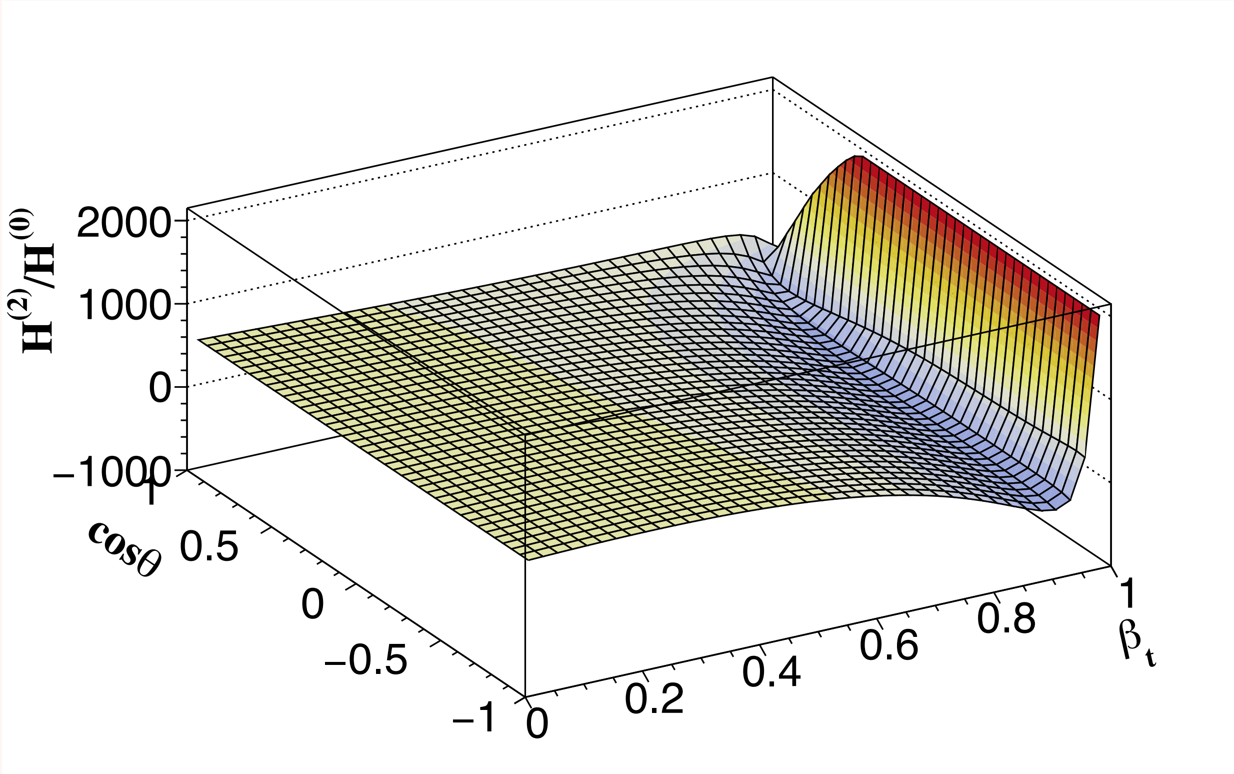
3.1 N2LO hard and soft functions

3.2 N3LO scale-dependent terms

3.3 Fixed-order partonic cross section

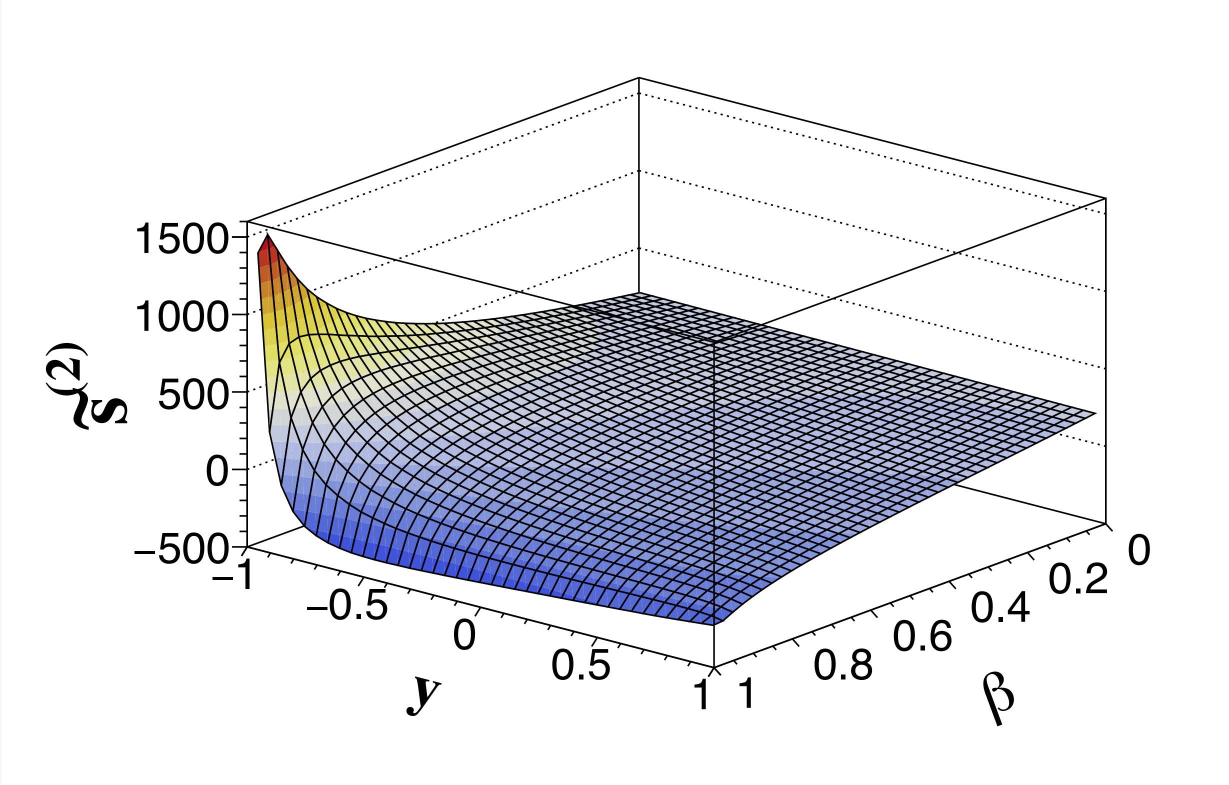
N^2LO Hard and Soft Functions

Hard function at N^2LO



[Chen, Dong, Li, Li, JW, Wang, 2208.08786, 2212.07190]

Soft function at N^2LO



[Ding, Li, JW, 2502.18648]

Key ingredients: N^2LO hard + soft functions + 3-loop anomalous dimensions enable approximate N^2LO and N^3LO predictions.

N³LO Scale-Dependent Terms

N³LO Soft Function from RG

$$\tilde{S}(L_\mu) = \tilde{S}(0)\exp\left(\int_{\ln\mu_s}^{\ln\mu}\gamma_s d\ln\mu\right)$$

with $\mu_s = \sqrt{\hat{s}}\xi$ and $\tilde{S} = \sum_n \left(\frac{\alpha_s}{4\pi}\right)^n \tilde{S}^{(n)}$

· Scale-dependent terms: fully determined by the exponential function

· Only missing: $\tilde{S}^{(3)}(0)$ (scale-independent)

· Same structure for hard function

· Anomalous dimensions known to 3 loops [Liu, Schalch, 2207.02864]

Fixed-Order Partonic Cross Section

$$P_m = \left[\frac{1}{\bar{z}}\left(\ln\frac{\bar{z}^2 Q^2}{z\mu_f^2}\right)^m\right]_+$$

$$\frac{d\hat{\sigma}}{dQ^2 d\Phi_2} = \frac{d\hat{\sigma}_{\rm LO}}{dQ^2 d\Phi_2}\sum_{n=0}\left(\frac{\alpha_s(\mu_r)}{4\pi}\right)^n\left(\sum_{m=-1}^{2n-1} C_{n,m}P_m\right)$$

$$C_{n,2n-1} = \frac{n}{4^{n-1}n!}(-\gamma_{s,d}^{(0)})^n$$

$$C_{n,2n-2} = -\frac{n(2n-1)}{4^{n-1}n!}(-\gamma_{s,d}^{(0)})^{n-1}\left[\gamma_{s,c}^{(0)} + \frac{2(n-1)}{3}\beta_0\right]$$

Leading power in $\bar{z}$ expansion, denoted by $\mathrm{N}^n\mathrm{LO}_{\mathrm{LP}}$

Subleading power: $O(\bar{z}^i), i \geq 0$, from other partonic channels and higher-power in gb channel — included at NLO

Approximate NⁿLO:

$$d\sigma(\mathrm{aN}^n\mathrm{LO}) = d\sigma(\mathrm{N}^n\mathrm{LO}_{\mathrm{LP}}) + d\sigma(\mathrm{NLO}) - d\sigma(\mathrm{NLO}_{\mathrm{LP}})$$

04

Numerical Results

Cross sections, experimental comparison, and Vtb extraction

4.1 Scale dependence and total cross sections

4.2 Comparison with ATLAS and CMS data

4.3 Kinematic distributions

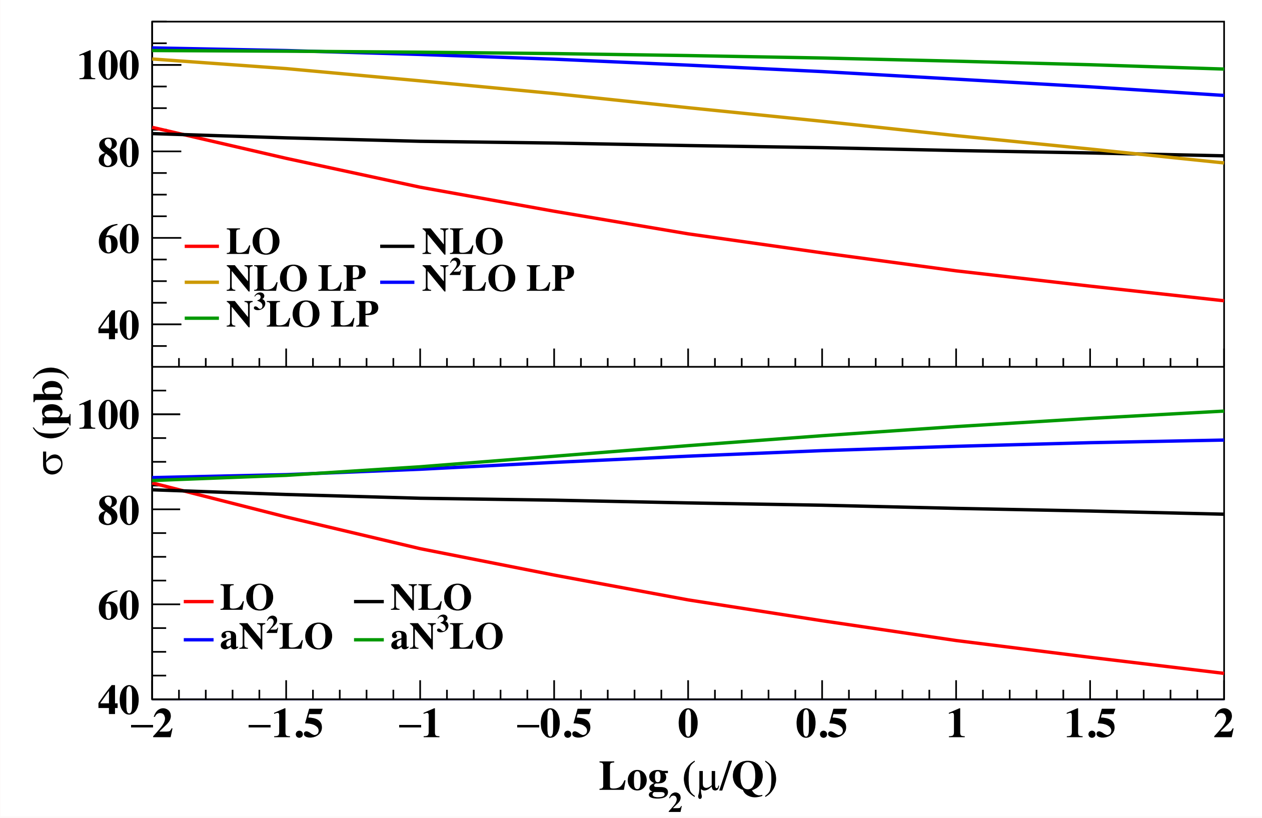
Scale Dependence and Total Cross Sections

+34%
NLO correction at 13 TeV

+12%
aN^2LO beyond NLO

+2%
aN^3LO beyond aN^2LO

1/3
Scale-independent at $O(\alpha_s^2)$



- Scale uncertainty reduced at higher orders
- NLO LP ≈ full NLO when μ > Q

$\sqrt{s}$	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO	$10.7^{+2.3}_{-1.8} \pm 0.3$	$15.4^{+3.2}_{-2.5} \pm 0.4$	$51.4^{+9.3}_{-7.3} \pm 1.1$	$57.1^{+10.2}_{-8.1} \pm 1.2$	$60.9^{+10.8}_{-8.6} \pm 1.2$
NLO	$14.6^{+0.4}_{-0.5} \pm 0.5$	$20.9^{+0.6}_{-0.7} \pm 0.7$	$68.7^{+2.5}_{-2.9} \pm 1.4$	$76.1^{+2.7}_{-3.2} \pm 1.5$	$81.3^{+2.8}_{-3.5} \pm 1.6$
N^2LO_{LP}	$18.5^{+0.6}_{-0.8} \pm 0.5$	$26.3^{+0.8}_{-1.0} \pm 0.7$	$84.8^{+2.3}_{-2.8} \pm 1.7$	$93.7^{+2.5}_{-3.1} \pm 1.8$	$99.9^{+2.6}_{-3.1} \pm 1.9$
N^3LO_{LP}	$19.1^{+0.2}_{-0.3} \pm 0.5$	$27.1^{+0.2}_{-0.4} \pm 0.6$	$86.8^{+0.5}_{-1.0} \pm 1.7$	$95.9^{+0.6}_{-1.1} \pm 1.9$	$102.2^{+0.6}_{-1.2} \pm 1.9$
aN^2LO	$16.6^{+0.4}_{-0.5} \pm 0.5$	$23.8^{+0.6}_{-0.7} \pm 0.6$	$77.2^{+1.8}_{-2.2} \pm 1.6$	$85.4^{+2.1}_{-2.5} \pm 1.7$	$91.2^{+2.1}_{-2.8} \pm 1.8$
aN^3LO	$17.2^{+0.8}_{-0.9} \pm 0.5$	$24.5^{+1.1}_{-1.3} \pm 0.7$	$79.2^{+3.5}_{-3.8} \pm 1.6$	$87.6^{+3.9}_{-4.2} \pm 1.7$	$93.5^{+4.1}_{-4.6} \pm 1.8$

- aN^3LO within aN^2LO scale band
- Excellent perturbative convergence
- Hard function: 24% of $O(\alpha_s^2)$
- Soft function: 10% of $O(\alpha_s^2)$
- Tripole: 4% of $O(\alpha_s^3)$
- PDF uncertainties: ~2% (independent of perturbative order)

Comparison with Previous Results

σ (pb)	13 TeV	13.6 TeV
Ref. [27]	$79.3^{+1.9}_{-1.8} \pm 2.2$	$87.9^{+2.0}_{-1.9} \pm 2.4$
$\mathrm{aN^3LO_1}$	$68.6^{+4.0}_{-3.1} \pm 1.5(\pm 2.5)$	$75.8^{+4.5}_{-3.3} \pm 1.7(\pm 2.9)$
$\mathrm{aN^3LO_2}$	$75.3^{+4.0}_{-4.2} \pm 1.6(\pm 2.7)$	$83.2^{+4.6}_{-4.3} \pm 1.7(\pm 2.9)$
$\mathrm{aN^3LO_3}$	$75.2^{+4.0}_{-4.3} \pm 1.6(\pm 2.7)$	$83.1^{+4.5}_{-4.6} \pm 1.7(\pm 2.9)$
$\mathrm{aN^3LO_4}$	$77.8^{+3.7}_{-4.2} \pm 1.6(\pm 2.7)$	$86.0^{+4.2}_{-4.5} \pm 1.7(\pm 2.8)$
$\mathrm{aN^3LO_5}$	$79.2^{+3.5}_{-3.8} \pm 1.6(\pm 2.9)$	$87.5^{+3.9}_{-4.3} \pm 1.7(\pm 2.9)$
$\mathrm{aN^3LO_6}$	$79.2^{+3.5}_{-3.8} \pm 1.6(\pm 2.9)$	$87.6^{+3.9}_{-4.2} \pm 1.7(\pm 2.9)$

Ref. [27]: Kidonakis, 2408.13870.

16% lower than [27] with the same PDF, scales, m_t .

+10% with the scales changed from m_t to Q.

marginal effect with $m_t = 172.5$ GeV to $m_t = 172.57$ GeV.

$\mathrm{N^2LO}$ hard function included, +3%

$\mathrm{N^2LO}$ soft function included, +2%

$\mathrm{N^3LO}$ tripole correlation in soft function included

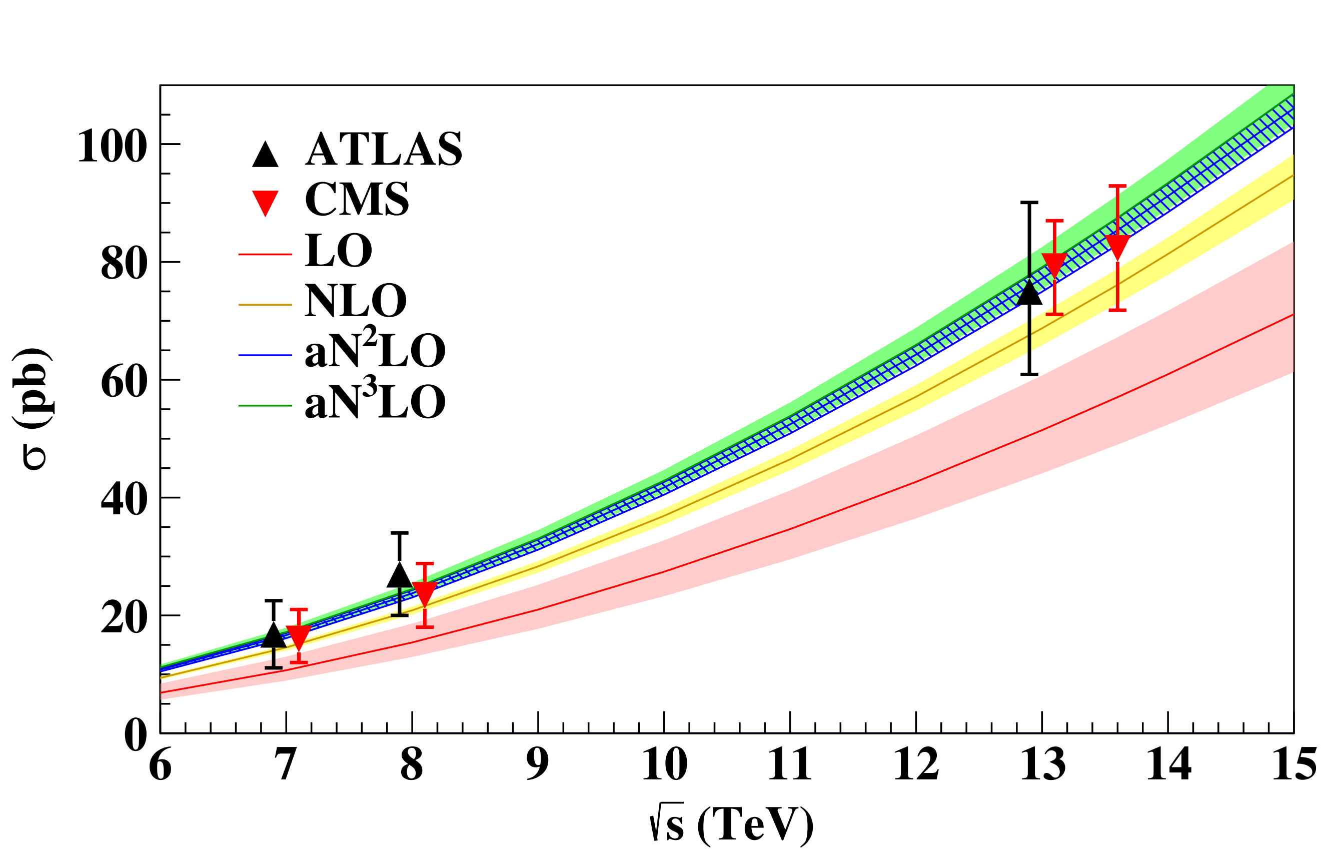
Sources of Difference with Ref. [27]

1. **Different threshold variable:** Our $\bar{z}$ vs. $s_4 = 2p_4 \cdot p_X$ in [27]

2. **$\mathrm{N^2LO}$ scale-independent parts:** Included in our calculation, not in previous works

3. **Tripole correlation:** Our three-loop soft anomalous dimensions include massive massless tripole terms

Comparison with ATLAS and CMS Data



$|V_{tb}|$ Extraction

$\sigma \propto |V_{tb}|^2$, single-process direct determination

$$V_{tb} = 0.99 \pm 0.03(exp.) \pm 0.03(theo.)$$

Comparable precision to world average

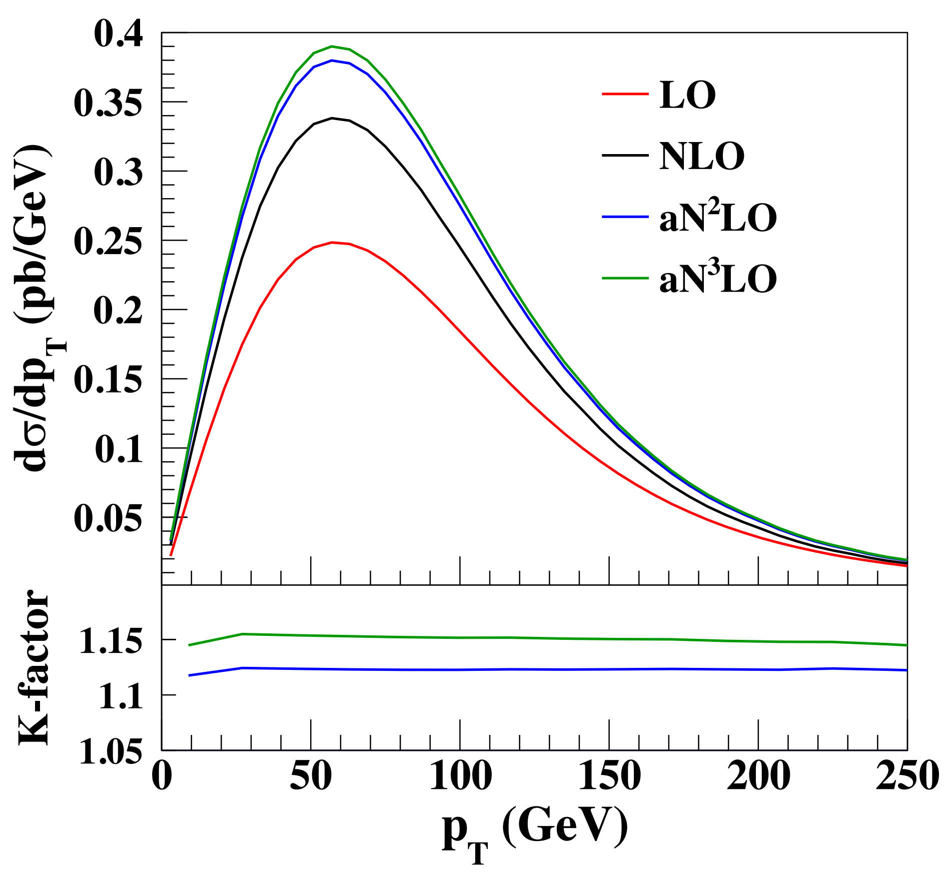
1.010 ± 0.027

Key Findings from Data Comparison

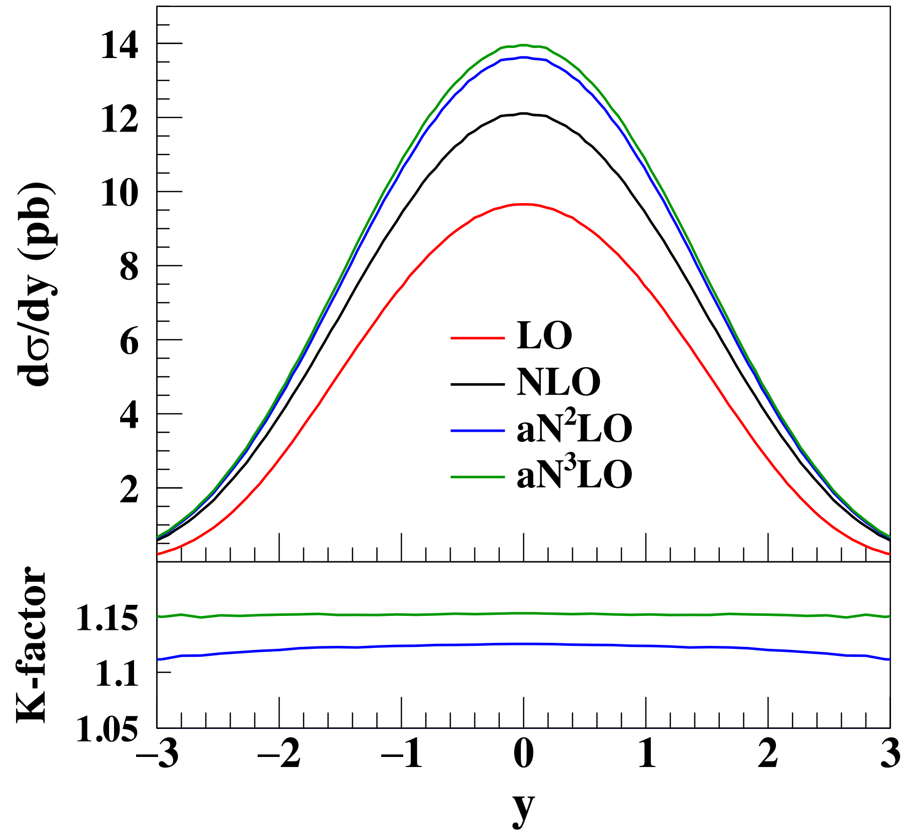
- aN^2LO and aN^3LO central values agree better with experimental data than NLO predictions
- aN^3LO scale uncertainty band overlaps with the aN^2LO one— excellent convergence

Kinematic Distributions at 14 TeV LHC

Top Quark p_T Distribution



Top Quark Rapidity Distribution



K-Factor Analysis

- aN^2LO and aN^3LO corrections are sizable across full kinematic regions
- Weak kinematic dependence → universal K-factor ($K = \sigma/\sigma_{NLO}$) (1.10–1.15)

05

Summary & Outlook

Key achievements and future directions

5.1 Key results and significance

5.2 Future work: complete N2LO calculation

Key Results and Significance

Higher-Order QCD Corrections

- Approximate N^2LO and N^3LO corrections for tW production at the LHC
- Incorporate N^2LO hard and soft functions + complete N^3LO scale-dependent terms
- These dominate the full perturbative predictions
- Increase NLO cross section by more than **10%**

CKM Matrix Element $|V_{tb}|$

- Direct extraction without assuming unitarity
- $|V_{tb}| = 0.99 \pm 0.03(\text{exp.}) \pm 0.03(\text{theo.})$
- Comparable precision to current world average
- Improved agreement between theory and data

Additional Achievements

- Sizable impacts on kinematic distributions (top quark p_T and rapidity) across full kinematic regions
- Universal K-factor applicable due to weak kinematic dependence of enhancements
- Valuable theoretical input for ongoing and future LHC experimental analyses
- tW production is one of the main backgrounds in Higgs pair searches at the LHC

Future Work: Complete N^2LO Calculation

Remaining Challenge

A complete N^2LO calculation is essential to further reduce theoretical uncertainties. Two key obstacles must be overcome:

- **Obstacle 1 (Solved):** Two-loop virtual corrections — analytical (leading color) and numerical (full color) results available
- **Obstacle 2 (Open):** Proper subtraction of top-quark pair production interference at higher orders

Path Forward

1. **Subtraction scheme development:** Robust method for tWbg final state interference with $t\bar{t}$ production
2. **Coherent combination of corrections:**
   - Double-virtual corrections (two-loop)
   - Virtual-real corrections (one-loop + real radiation)
   - Double-real corrections (tree-level + double radiation)
3. **Current workaround:** Focusing on soft radiation region where $t\bar{t}$ interference is kinematically forbidden

Thank You

THANK YOU FOR YOUR ATTENTION

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Questions and comments are welcome

Q & A