Global Analyses of Transverse Momentum Dependent Parton Distributions

第八届全国重味物理与量子色动力学研讨会
2026年4月25日 @ 重庆

Tianbo Liu (刘天博)

Shandong University
Southern Center for Nuclear-Science Theory, IMP, CAS

Transverse Nucleon Tomography Collaboration:
Hongxin Dong, TL, Bo-Qiang Ma, Peng Sun, Ke Yang, Chunhua Zeng, Yuxiang Zhao

山东大学
SHANDONG UNIVERSITY

SCNT
Southern Center for Nuclear-Science Theory

Lepton-Hadron Deep Inelastic Scattering

Inclusive DIS at a large momentum transfer:

$Q \gg \Lambda_{\rm QCD}$

Modern "Rutherford" experiment.

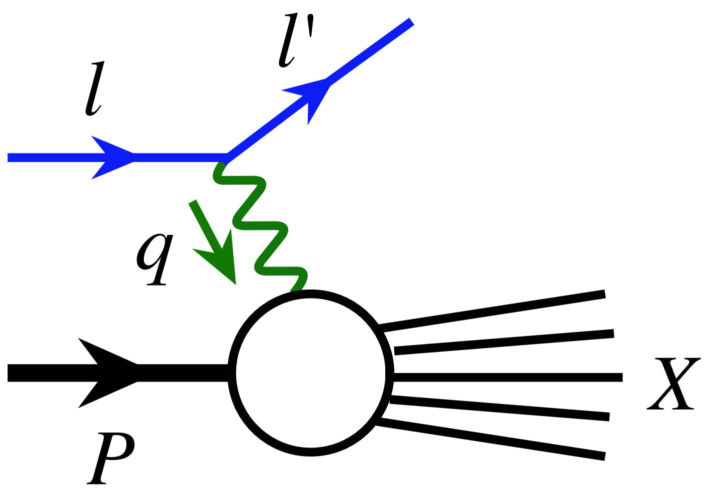
- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale ~ 1/fm
- collinear factorization:

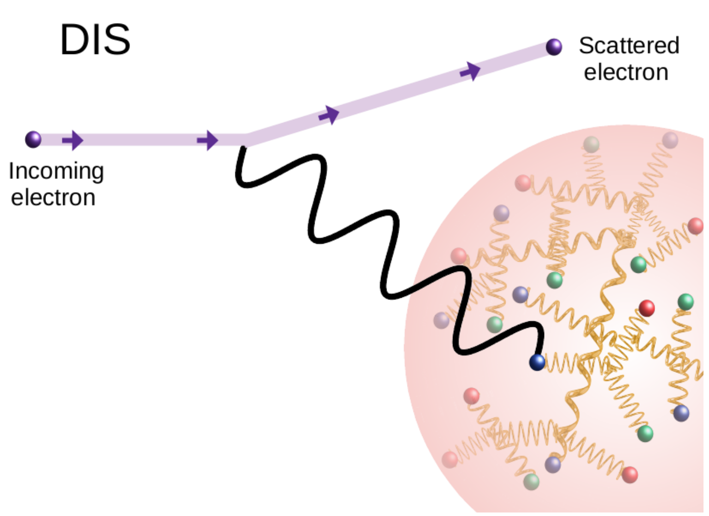
$$\sigma \propto H(Q) \otimes f_{i/P}(x, \mu^2)$$

- overall corrections suppressed by $1/Q^n$
- indirectly "see" quarks, gluons and their dynamics
- predictive power relies on

— precision of the probe

— universality of $f_{i/P}(x, \mu^2)$





[Figure from DESY-21-099]

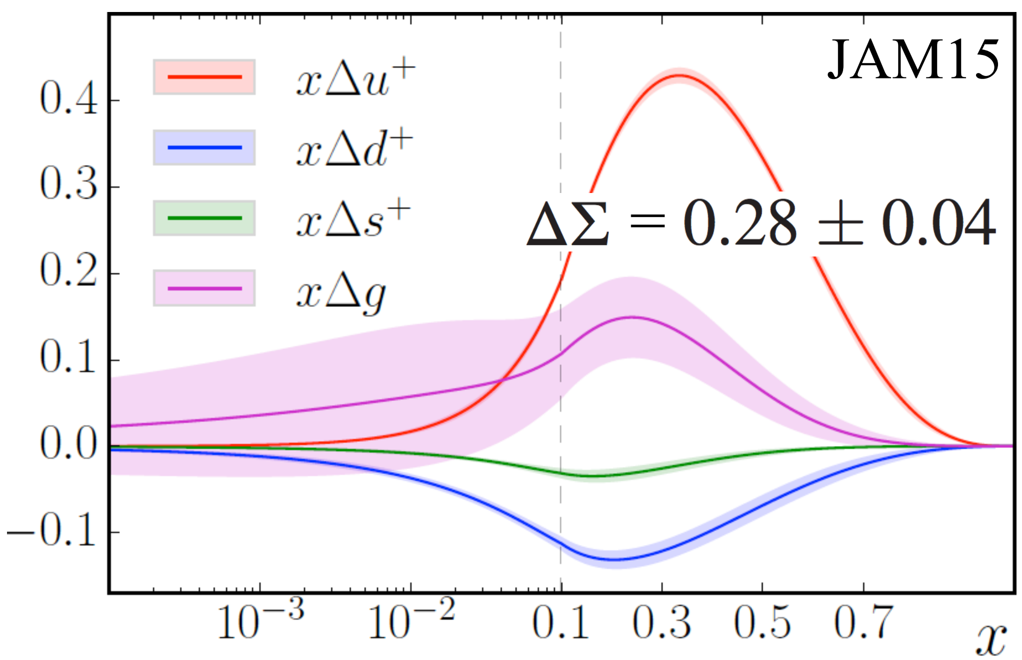
Nucleon Spin Structure

Proton spin puzzle

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \sim 0.3$$

Spin decomposition

$$J = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$



JAM Collaboration, PR D 93, 074005 (2016).

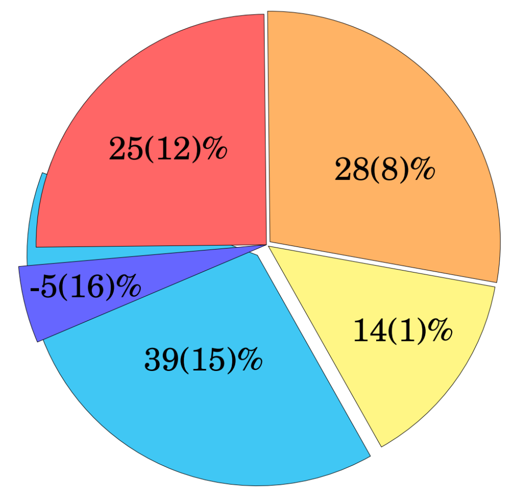
JAM17: $\Delta\Sigma = 0.36 \pm 0.09$

JAM Collaboration, PRL 119, 132001 (2017).

Quark spin only contributes a small fraction to the nucleon spin.

J. Ashman *et al.*, PLB 206, 364 (1988); NP B328, 1 (1989).

Lattice QCD (kinetic decomposition)



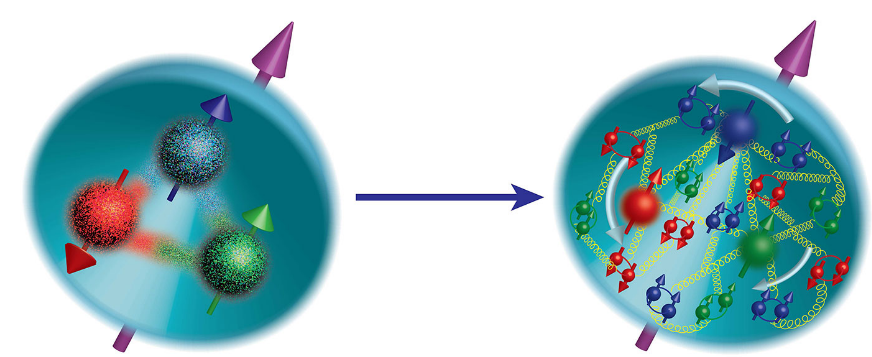
- L^u (CI + DI)
- L^d (CI + DI)
- L^s (DI)
- J^g
- $\frac{\Delta\Sigma}{2}\Big|^{u+d+s}$

χQCD Collaboration, PR D 91, 014505 (2015).

Gluon spin from LQCD: $S_g = 0.251(47)(16)$

50% of total proton spin

Y.-B. Yang *et al.* (χQCD Collaboration), PRL 118, 102001 (2017).



Wigner Rotation Effect

Melosh-Wigner rotation

quark spin in a rest proton $\neq$ quark spin in a moving proton

If applying a kinetic boost, one may relate the spin states in *proton rest frame* to the spin states in *infinite momentum frame*

$$\chi_T^{\uparrow} = w\left[\left(k^+ + m\right)\chi_F^{\uparrow} - \left(k^1 + ik^2\right)\chi_F^{\downarrow}\right]$$

$$\chi_T^{\downarrow} = w\left[\left(k^+ + m\right)\chi_F^{\downarrow} + \left(k^1 - ik^2\right)\chi_F^{\uparrow}\right]$$

$$k^+ = k^0 + k^3$$

$$w = \left[2k^+\left(k^0 + m\right)\right]^{-1/2}$$

E.P. Wigner, Ann. Math 40 (1939) 149; H.J. Melosh, Phys. Rev. D 9 (1974) 1095.

The effect on quark polarization

$$\Delta q = \int \mathrm{d}^3\mathbf{k}\mathscr{M}\left[q^{\uparrow}(k) - q^{\downarrow}(k)\right]$$

$$\mathscr{M} = \frac{(k^+ + m)^2 - k_T^2}{2k^+(k^0 + m)}$$

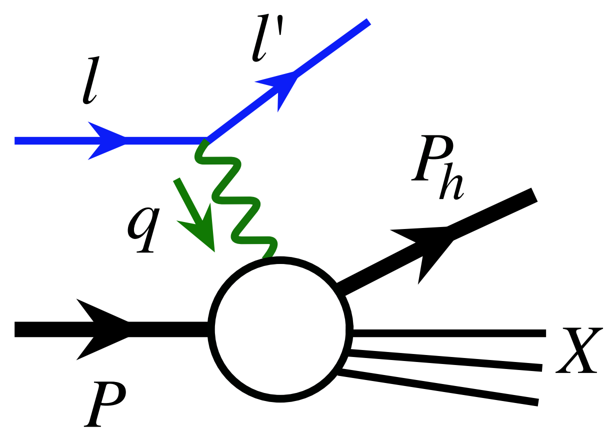
B.-Q. Ma, J. Phys. G 17 (1991) L53-L58; B.-Q. Ma, Q.-R. Zhang, Z. Phys. C 58 (1993) 479.

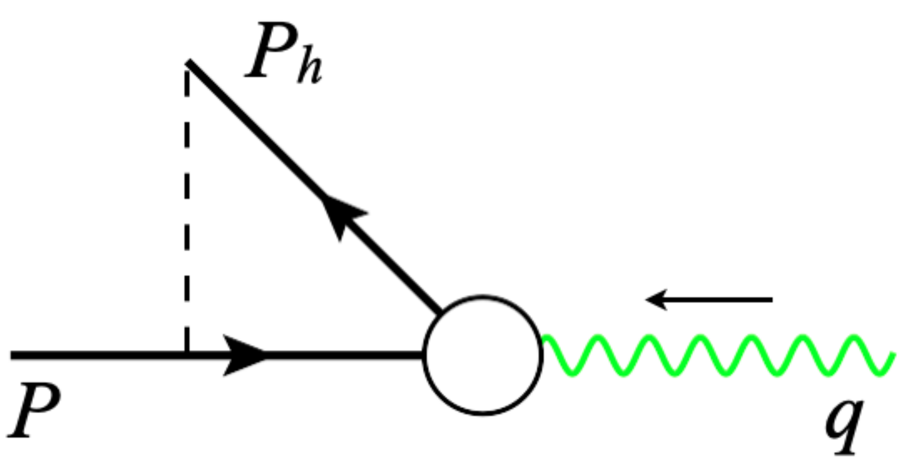
It predicts decreasing polarization with k_T, which should be tested by data.
This interpretation is based on a kinetic boost, but a complete boost including QCD dynamics is challenging.

Semi-inclusive Deep Inelastic Scattering

Semi-inclusive DIS: a final state hadron (P_h) is identified

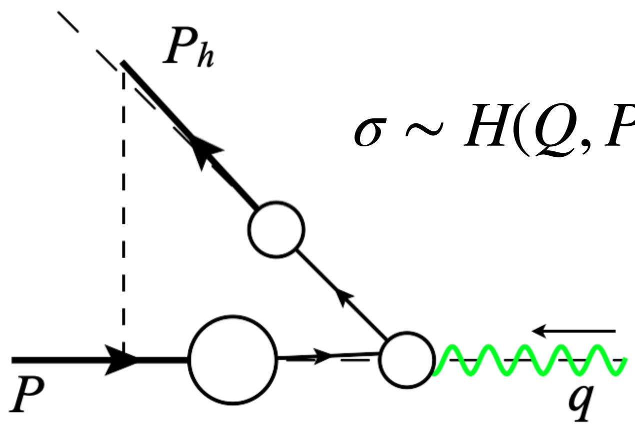
- enable us to explore the emergence of color neutral hadrons from colored quarks/gluons
- flavor dependence by selecting different types of observed hadrons: pions, kaons, …
- a large momentum transfer Q provides a short-distance probe
- an additional and adjustable momentum scale P_{hT}
- multidimensional imaging of the nucleon





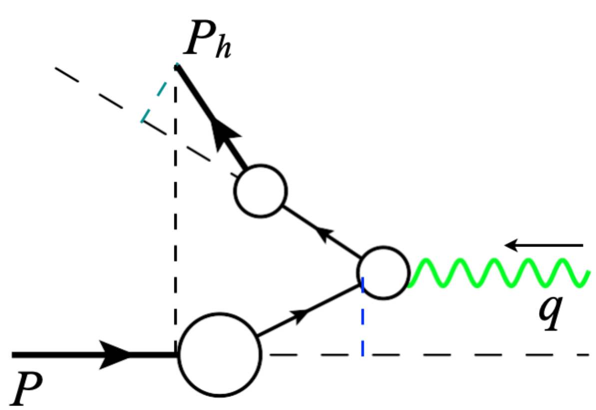
SIDIS Kinematic Regions

Sketch of kinematic regions of the produced hadron



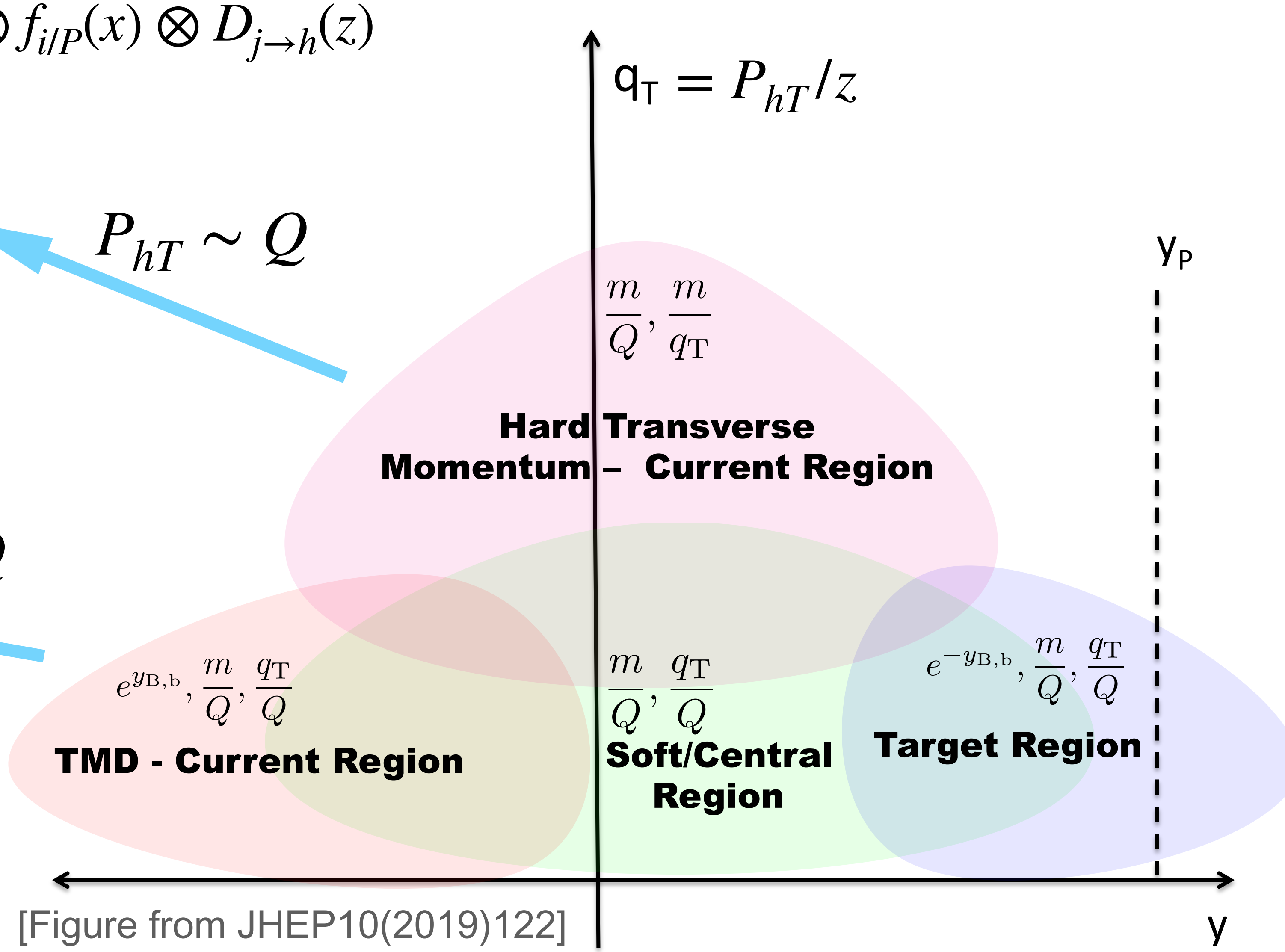
$\sigma \sim H(Q, P_{hT}) \otimes f_{i/P}(x) \otimes D_{j\to h}(z)$

$P_{hT} \sim Q$



$P_{hT} \ll Q$

$\sigma \sim H(Q) \otimes f_{i/P}(x, k_T) \otimes D_{j\to h}(z, p_T)$



[Figure from JHEP10(2019)122]

P_{hT} is defined in the photon-hadron frame

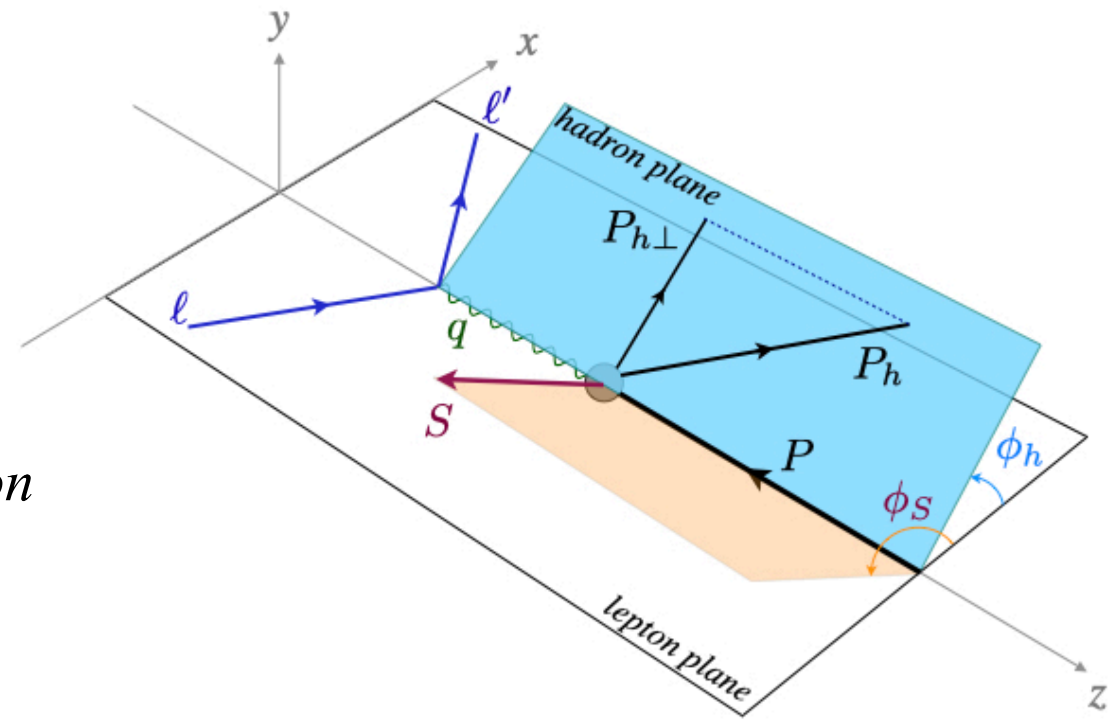
Structure Functions of SIDIS

SIDIS differential cross section

in terms of 18 structure functions

$$F_{AB,C}(x_B, z, P_{hT}^2, Q^2)$$

A: lepton polarization
B: nucleon polarization
C: virtual photon polarization



$$\frac{d\sigma}{dx_B\, dy\, dz\, dP_{hT}^2\, d\phi_h\, d\phi_S}$$

$$= \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right)$$

$$\times \Big\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} F_{UU}^{\cos\phi_h} \cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h + \lambda_e \sqrt{2\epsilon(1-\epsilon)} F_{LU}^{\sin\phi_h} \sin\phi_h$$

$$+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} F_{UL}^{\sin\phi_h} \sin\phi_h + \epsilon F_{UL}^{\sin 2\phi_h} \sin 2\phi_h\right] + \lambda_e S_L \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} F_{LL}^{\cos\phi_h} \cos\phi_h\right]$$

$$+ S_T \Big[\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \sin(\phi_h - \phi_S) + \epsilon F_{UT}^{\sin(\phi_h+\phi_S)} \sin(\phi_h + \phi_S)$$

$$+ \epsilon F_{UT}^{\sin(3\phi_h-\phi_S)} \sin(3\phi_h - \phi_S) \quad + \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin\phi_S} \sin\phi_S + \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin(2\phi_h-\phi_S)} \sin(2\phi_h - \phi_S)\Big]$$

$$+ \lambda_e S_T \Big[\sqrt{1-\epsilon^2} F_{LT}^{\cos(\phi_h-\phi_S)} \cos(\phi_h - \phi_S)$$

$$+ \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos\phi_S} \cos\phi_S + \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos(2\phi_h-\phi_S)} \cos(2\phi_h - \phi_S)\Big]\Big\}$$

$$x_B = \frac{Q^2}{2P\cdot q}$$

$$y = \frac{P\cdot q}{P\cdot l}$$

$$z = \frac{P\cdot P_h}{P\cdot q}$$

$$\gamma = \frac{2x_B M}{Q}$$

Leading Twist TMDs

		Quark Polarization		
		U	L	T
Nucleon Polarization	U	f_1 unpolarized		$h_1^{\perp}$ Boer-Mulders
	L		g_{1L} helicity	$h_{1L}^{\perp}$ longi-transversity (worm-gear)
	T	$f_{1T}^{\perp}$ Sivers	g_{1T} trans-helicity (worm-gear)	h_1 transversity $h_{1T}^{\perp}$ pretzelosity

Longitudinal Double Spin Asymmetry

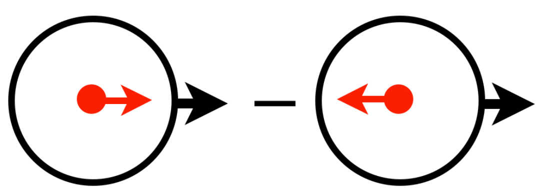
Longitudinal DSA in SIDIS

$$A_{LL} \equiv \frac{\sigma_{++} - \sigma_{+-} + \sigma_{--} - \sigma_{-+}}{\sigma_{++} + \sigma_{+-} + \sigma_{--} + \sigma_{-+}} = \frac{\sqrt{1-\varepsilon^2} F_{LL}\left(x, z, P_{hT}^2, Q^2\right)}{F_{UU}\left(x, z, P_{hT}^2, Q^2\right)}$$

In TMD region:

$$F_{LL}\left(x, z, P_{hT}^2, Q^2\right) \sim g_{1L}(x, k_T^2) \otimes D_1(z, p_T^2)$$

$$F_{UU}\left(x, z, P_{hT}^2, Q^2\right) \sim f_1(x, k_T^2) \otimes D_1(z, p_T^2)$$

g_{1L} 

(both collinear and TMD)

Several global analyses of collinear helicity but TMD helicity was missing for a long time!

P_{hT} dependent DSA measurements

HERMES: proton (H_2) and deuteron (D_2) targets

HERMES Collaboration, Phys. Rev. D 99 (2019) 112001.

JLab CLAS: proton (NH_3) target

CLAS Collaboration, Phys. Lett. B 782 (2018) 662.

TMD Evolution

Evolution equations

$$\mu^2 \frac{dF(x,b;\mu^2,\zeta)}{d\mu^2} = \frac{\gamma_F(\mu,\zeta)}{2} F(x,b;\mu^2,\zeta)$$

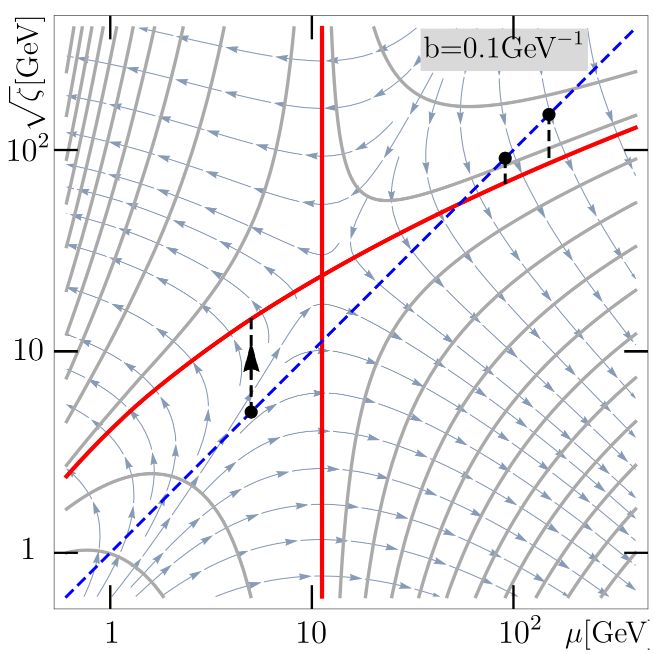
$$\zeta \frac{dF(x,b;\mu^2,\zeta)}{d\zeta} = -\mathcal{D}(\mu,b) F(x,b;\mu^2,\zeta)$$

$$-\zeta \frac{d\gamma_F(\mu,\zeta)}{d\zeta} = \mu \frac{d\mathcal{D}(\mu,b)}{d\mu} = \Gamma_{\rm cusp}(\mu)$$

$$\gamma_F(\mu,\zeta) = \Gamma_{\rm cusp}(\mu) \ln\frac{\mu^2}{\zeta} - \gamma_V(\mu)$$

$$F\left(x,b;\mu_f,\zeta_f\right) = \exp\left[\int_P \left(\gamma_F(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}(\mu,b)\frac{d\zeta}{\zeta}\right)\right] F\left(x,b;\mu_i,\zeta_i\right)$$

ζ-prescription



equipotential lines:

$$\frac{d\ln\zeta_\mu(\mu,b)}{d\ln\mu^2} = \frac{\gamma_F\left(\mu,\zeta_\mu(\mu,b)\right)}{2\mathcal{D}(\mu,b)}$$

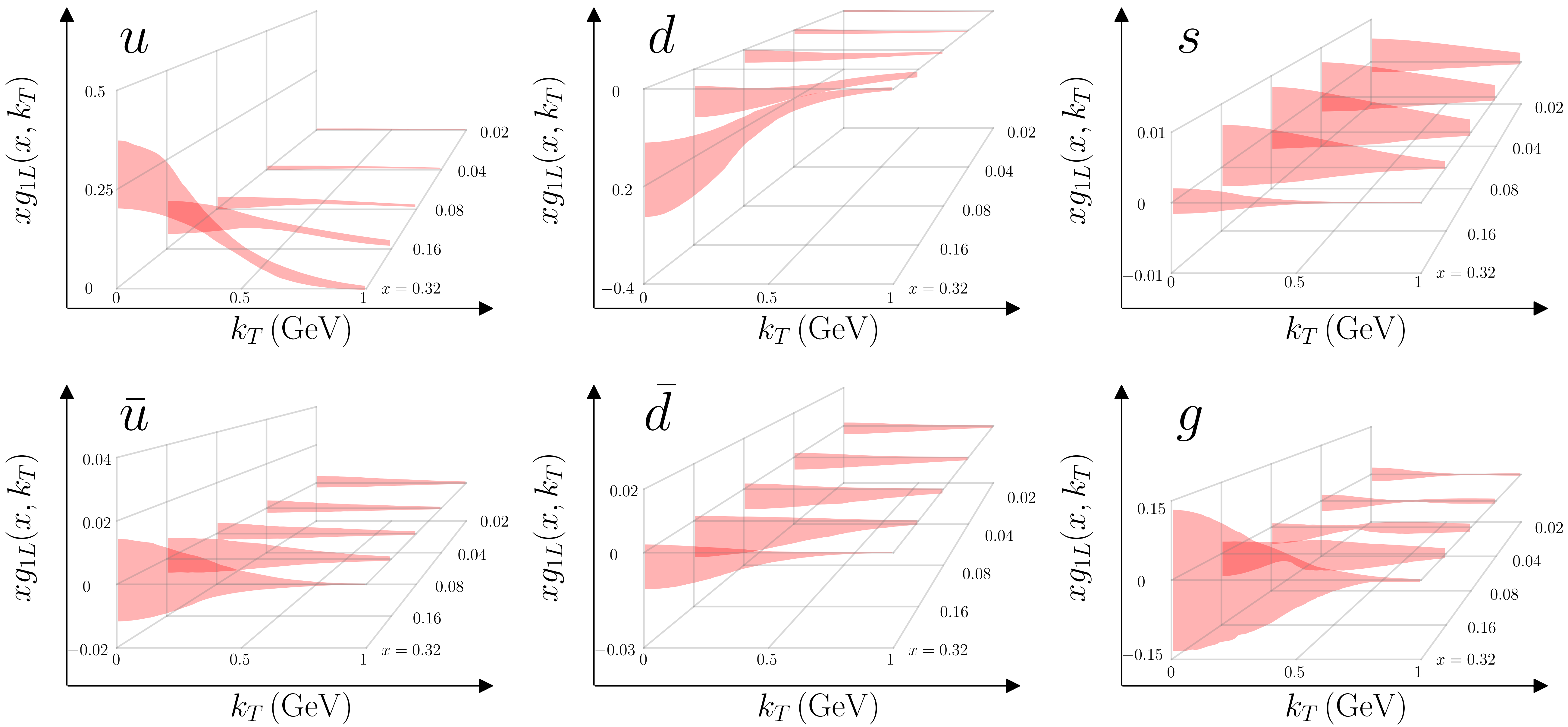
$$\mathcal{D}\left(\mu_0,b\right) = 0, \quad \gamma_F\left(\mu_0,\zeta_\mu\left(\mu_0,b\right)\right) = 0$$

$$F\left(x,b;Q,Q^2\right) = \left(\frac{Q^2}{\zeta_Q(b)}\right)^{-\mathcal{D}(Q,b)} F(x,b), \quad \mu_f^2 = \zeta_f = Q^2$$

I. Scimemi, A. Vladmirov, JHEP 06 (2020) 137.

First Extraction of TMD Helicity

NLO+NNLL analysis results

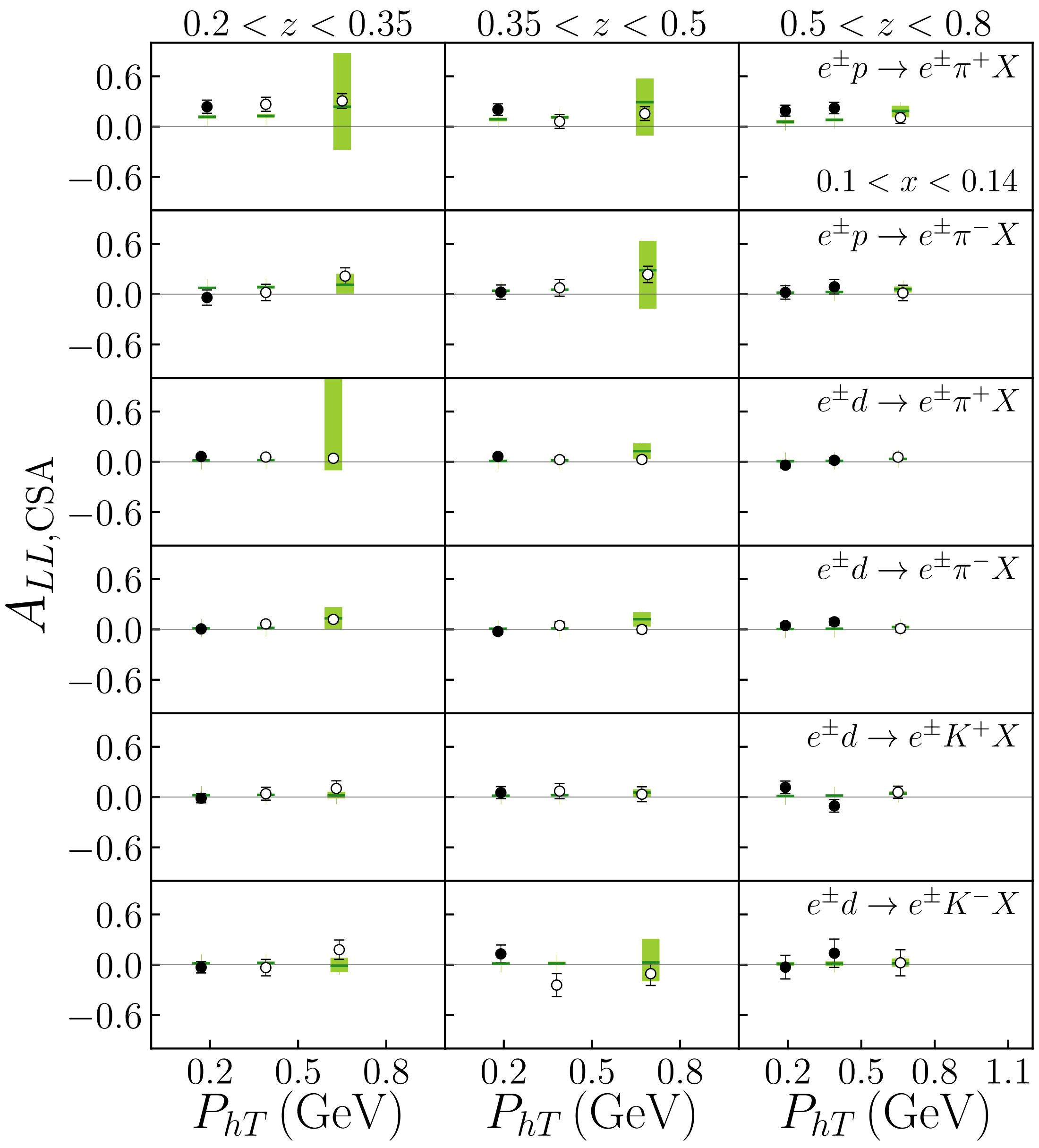


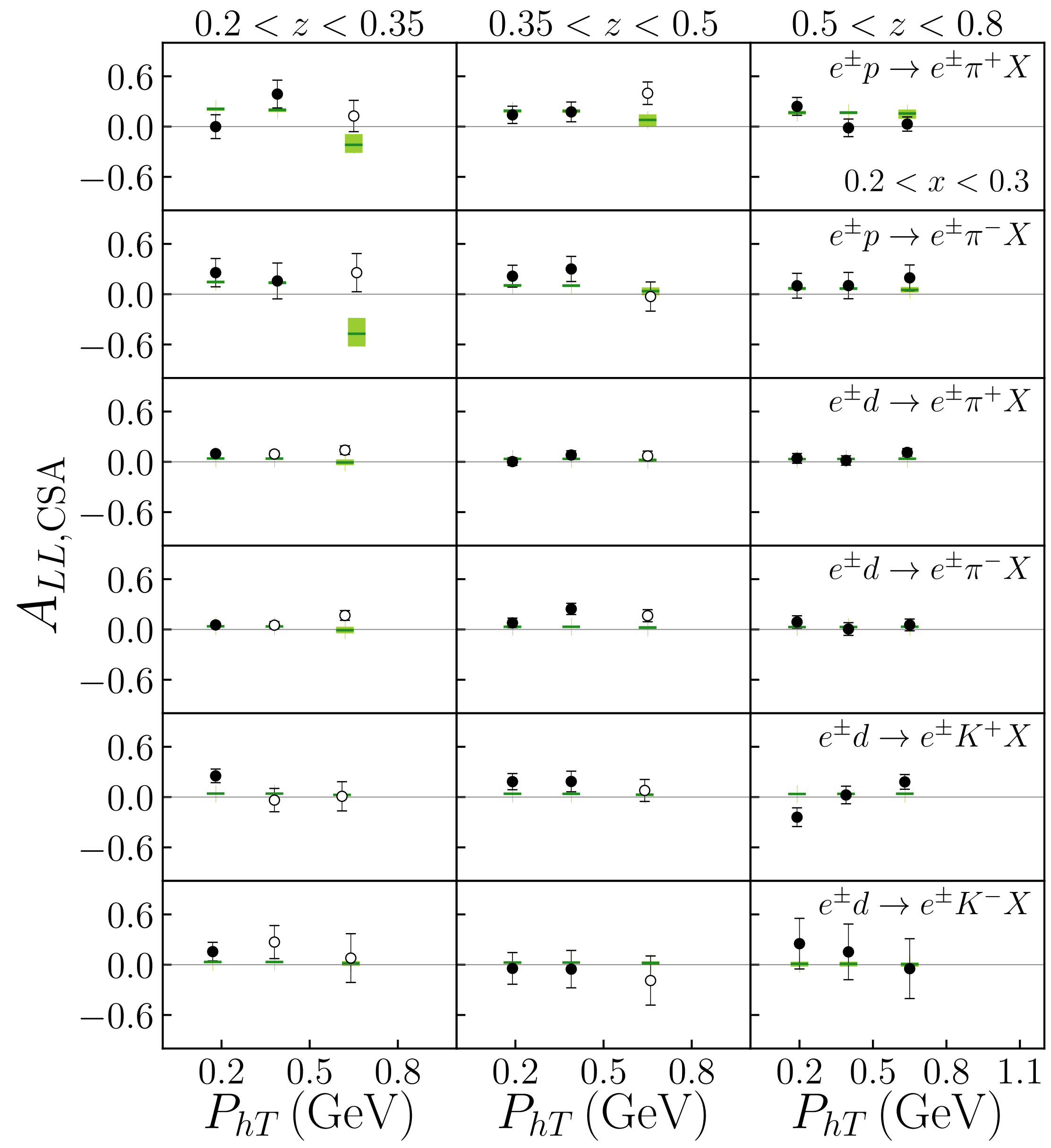
Nonzero signals for *u* and *d* quarks, while sea quarks and gluons are loosely constrained.

K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

Compare with HERMES data

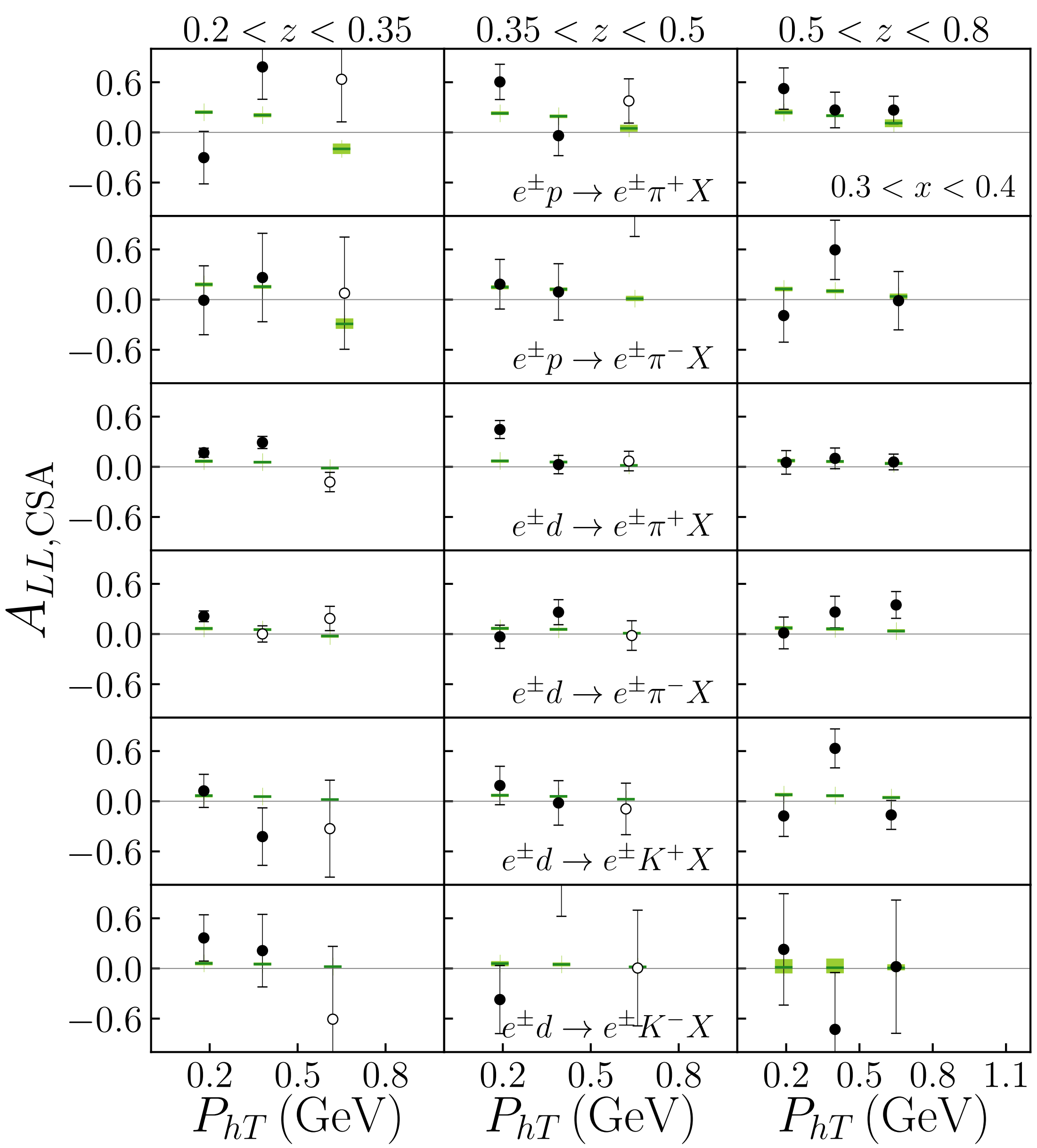


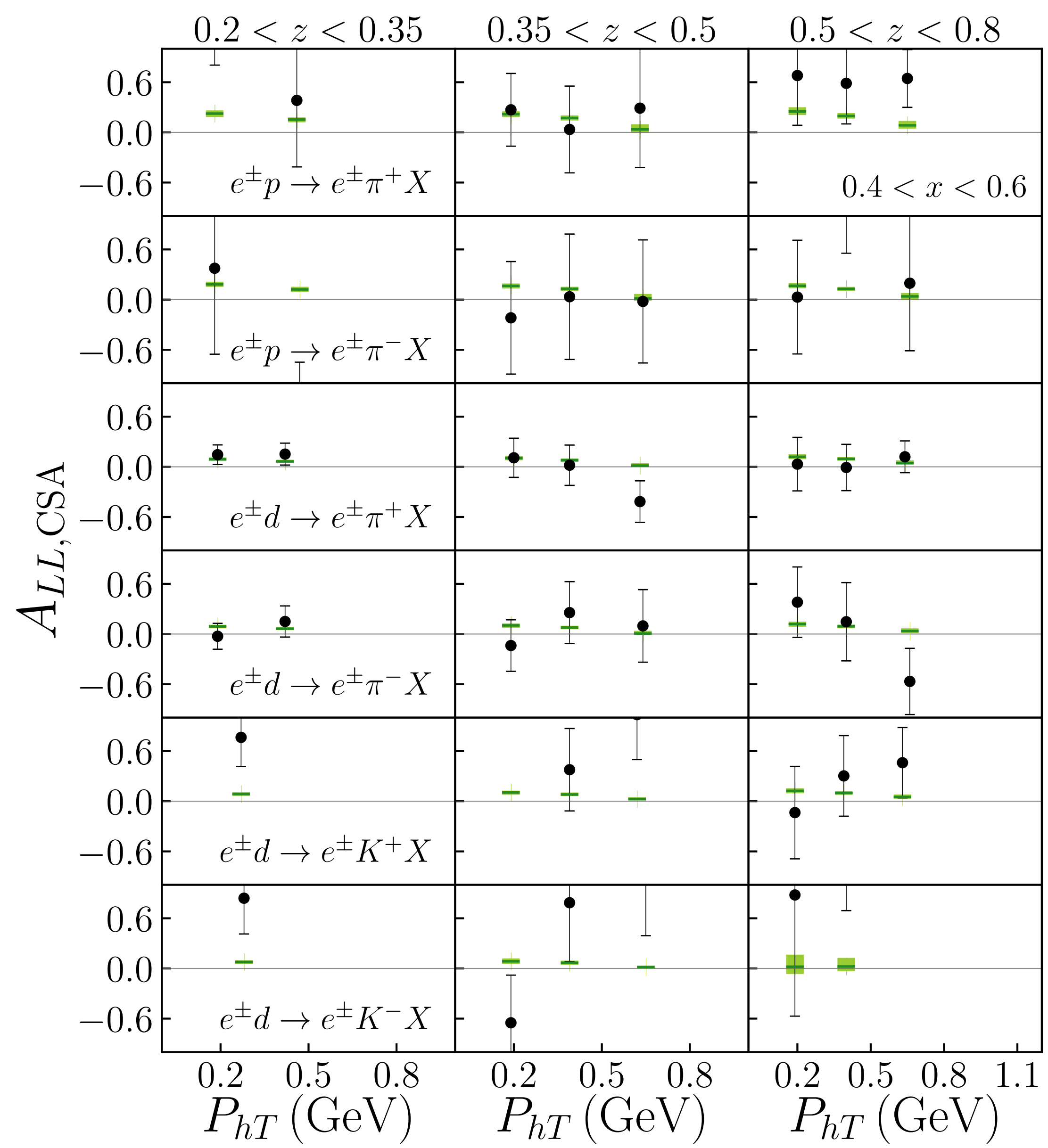


K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

Compare with HERMES data

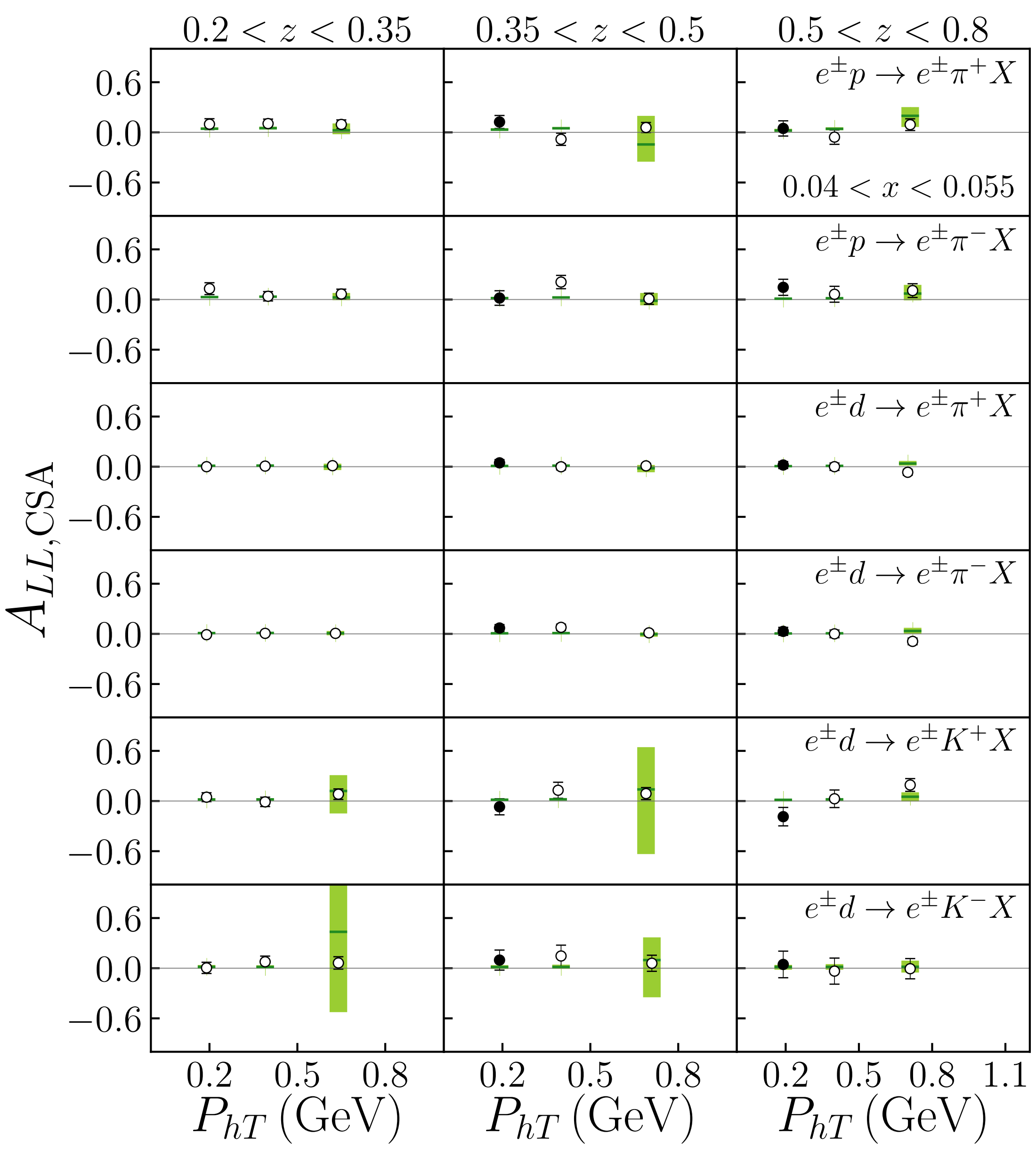


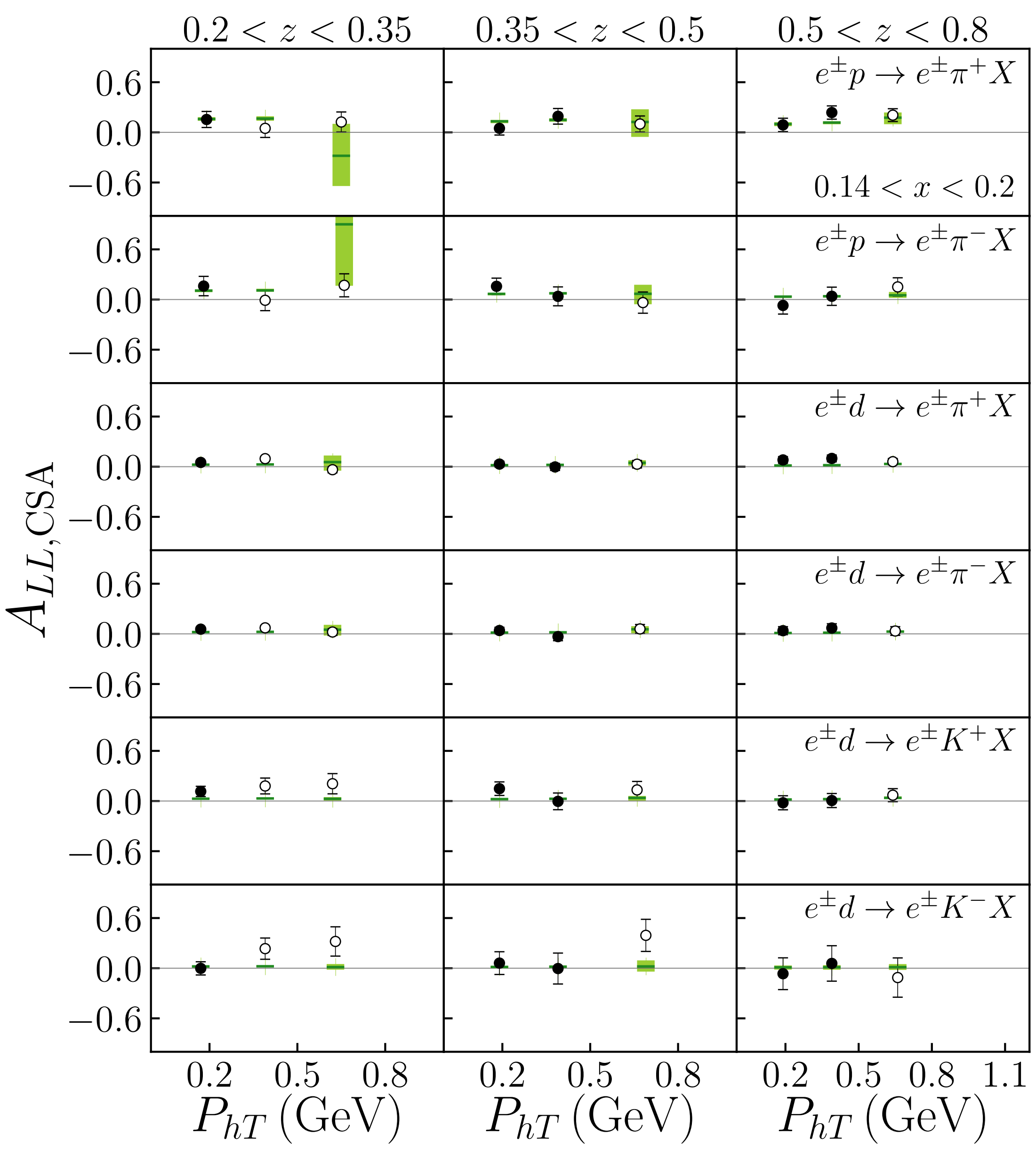


K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

Compare with HERMES data

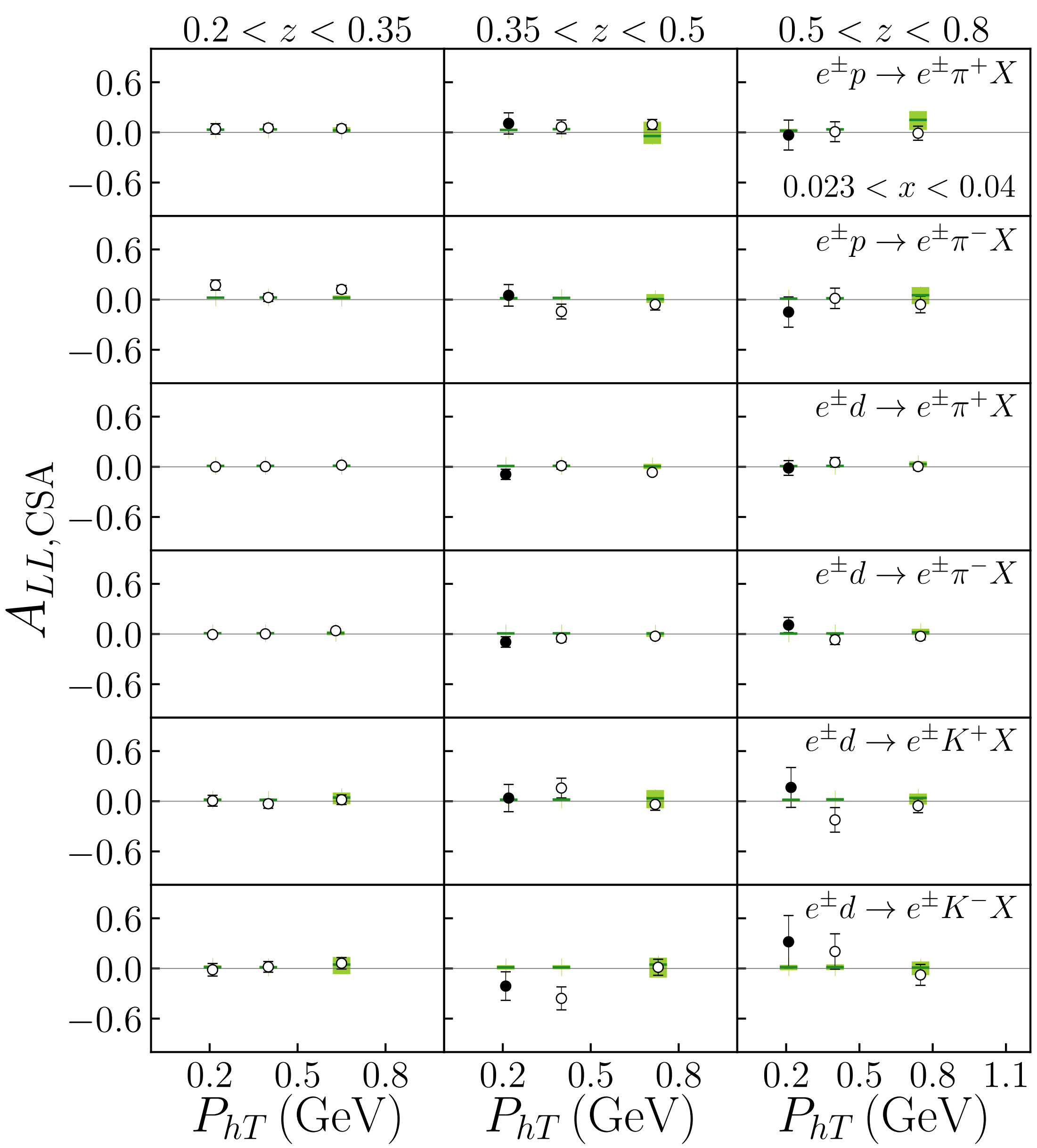


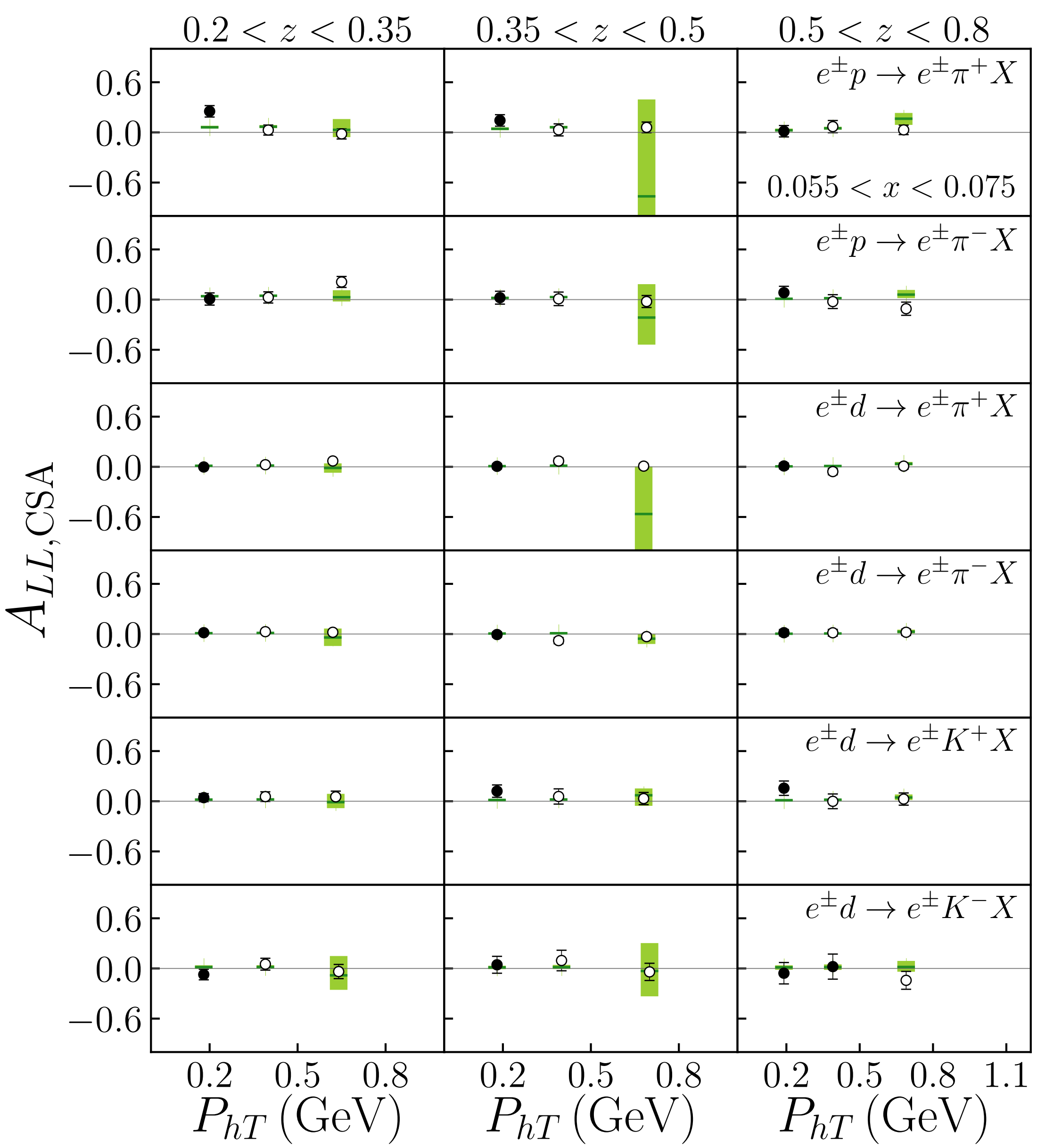


K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

Compare with HERMES data

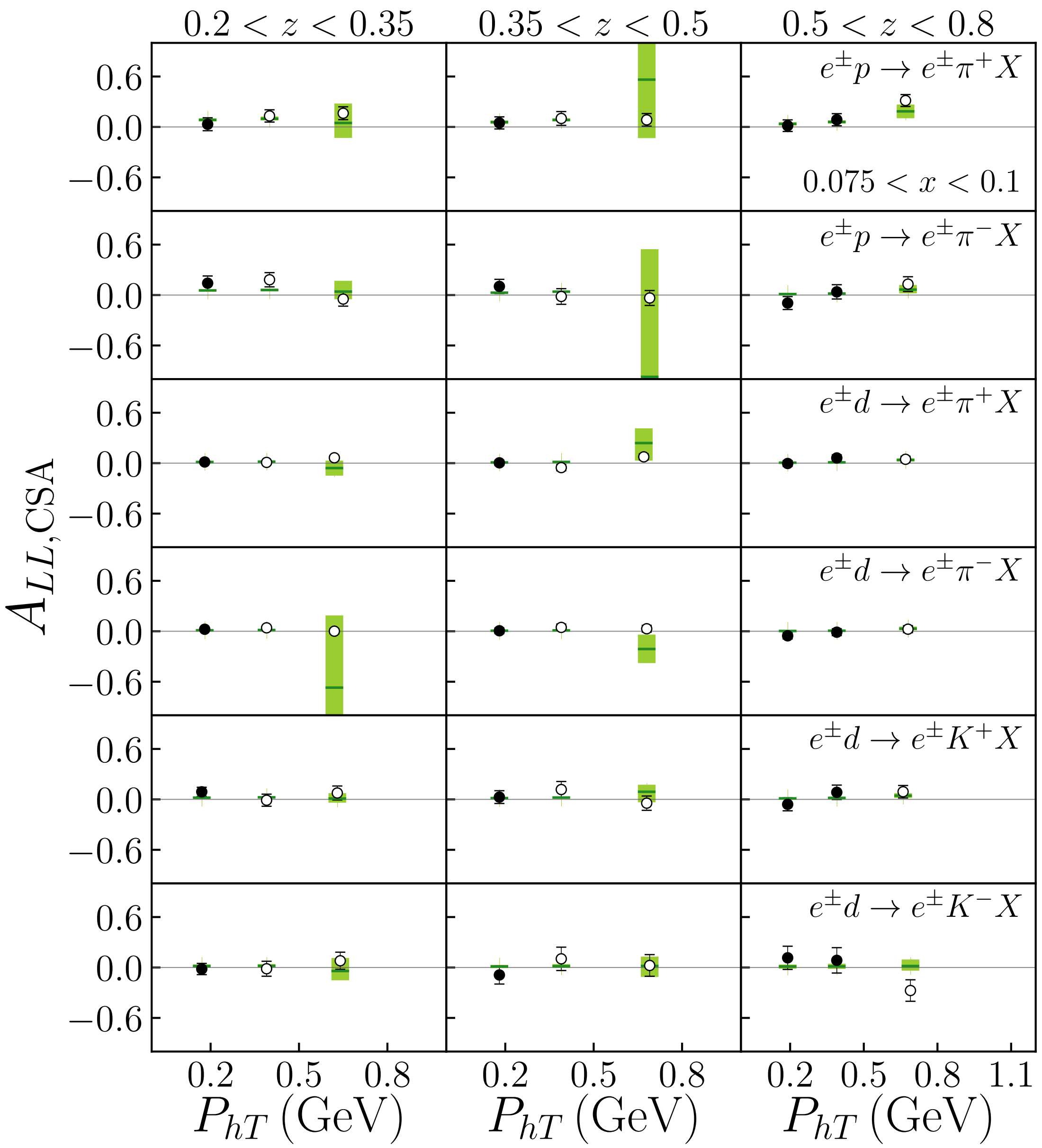


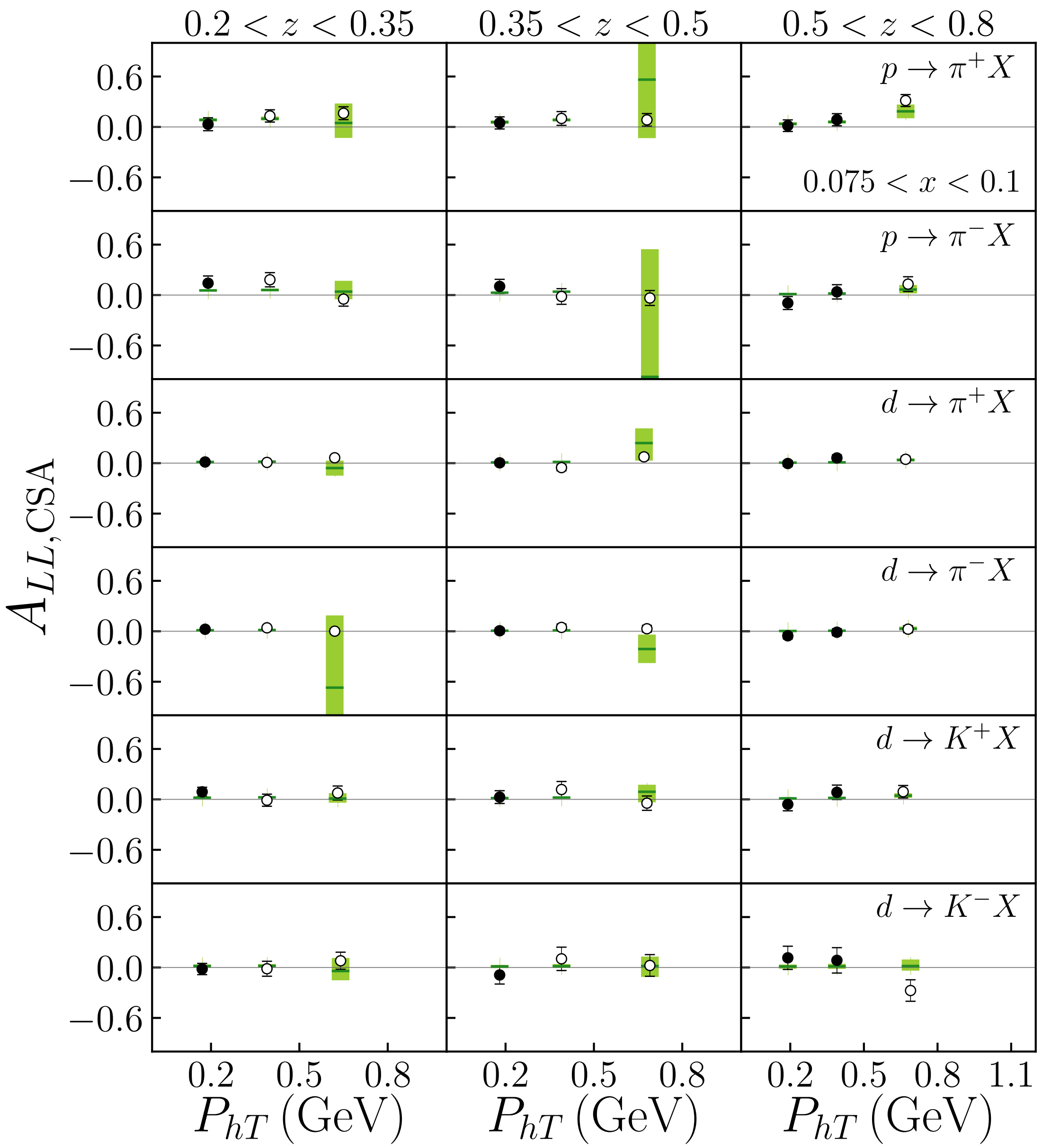


K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

Compare with HERMES data

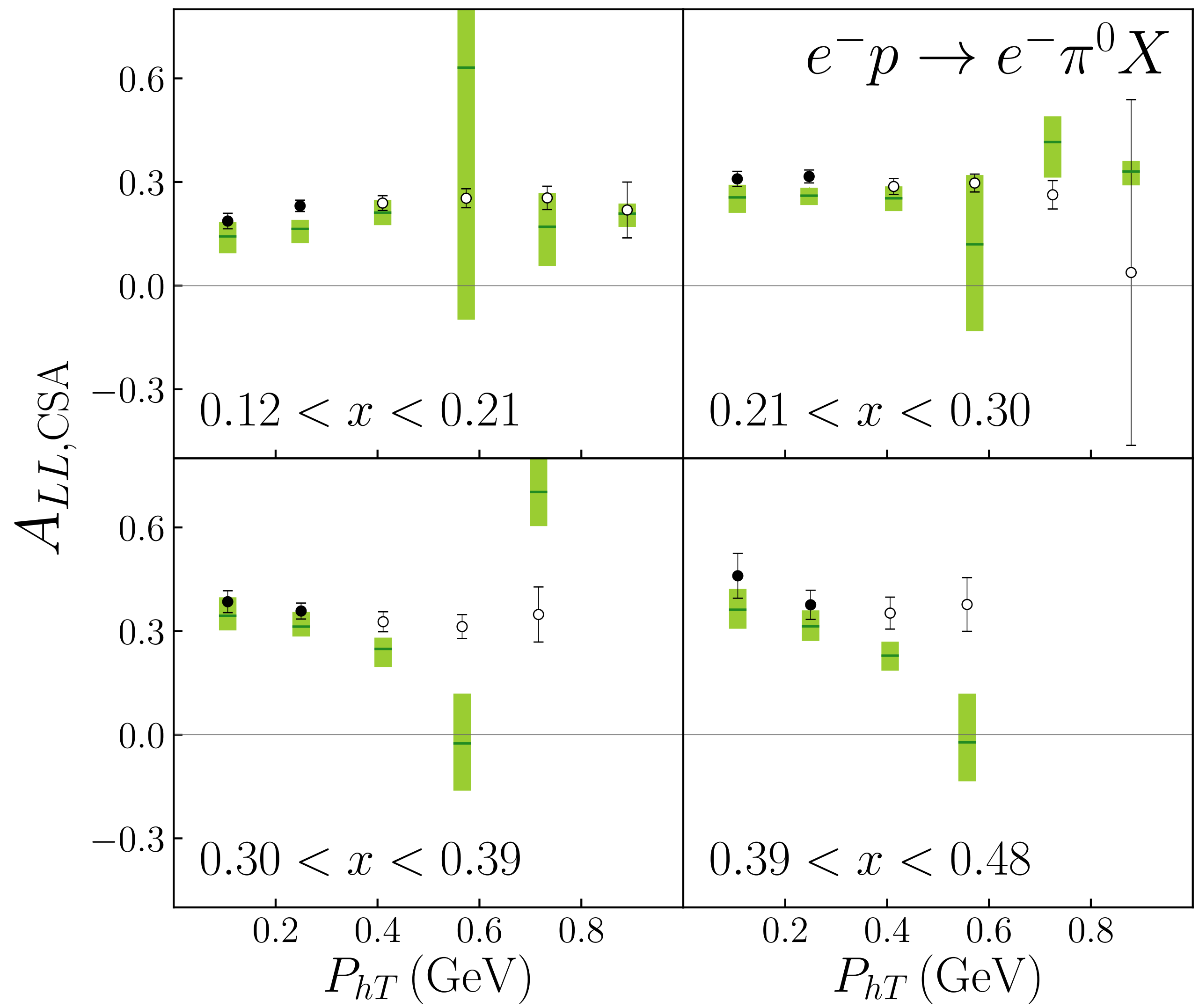




K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

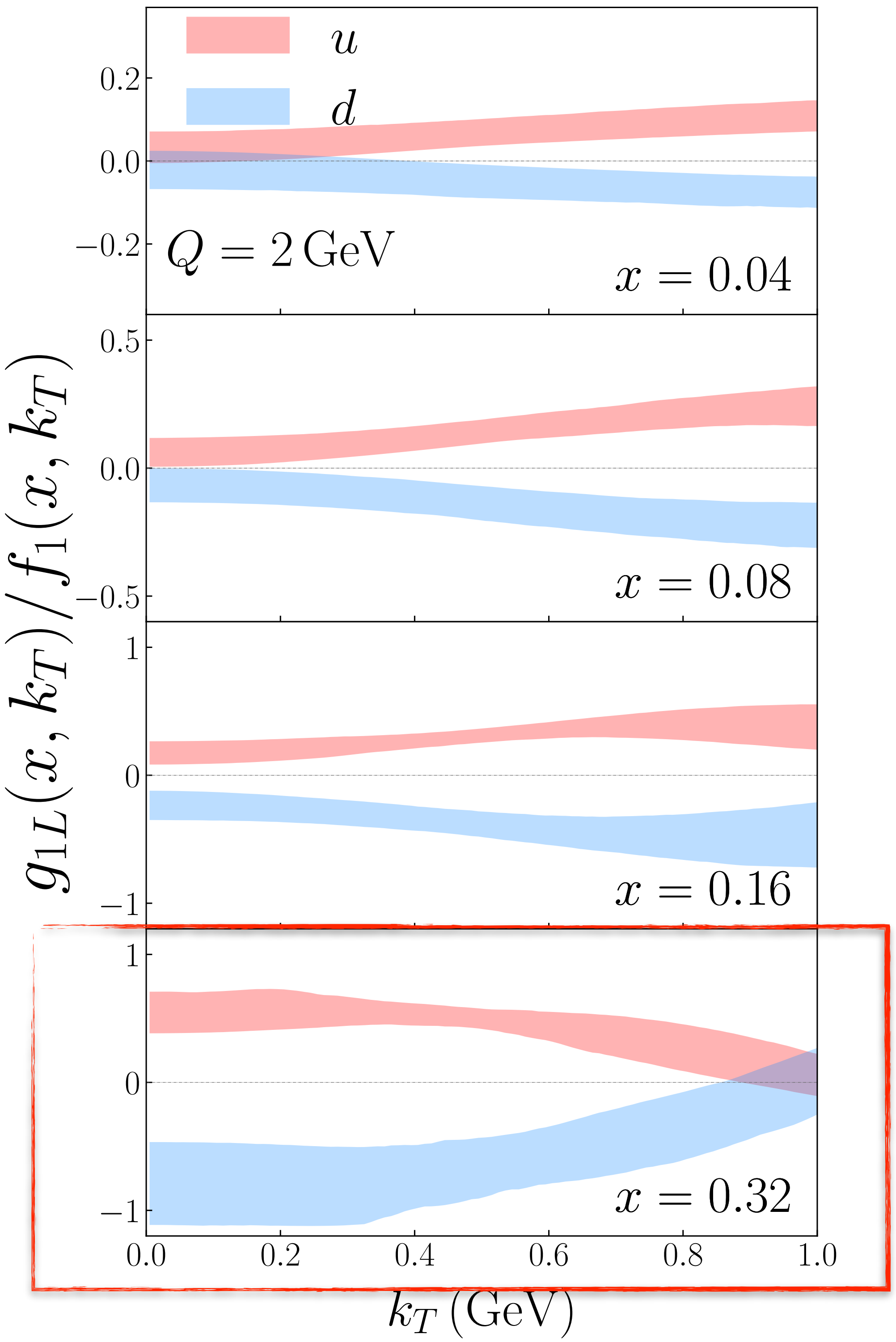
Comparison with Data

Compare with CLAS data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Transverse Momentum Dependent Polarization



$g_{1L}(x, k_T^2)$ gives the absolute number density difference between spin-parallel and spin-antiparallel quarks.

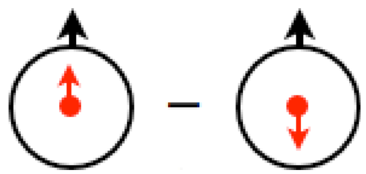
The ratio $g_{1L}(x, k_T^2)/f_1(x, k_T^2)$ measures the polarization rate of quarks.

- At large x, where valence components dominate, the polarization decreases with increasing k_T
  Qualitatively consistent with kinetic Wigner rotation effects
- At low x, where the valence component is no longer adequate, distributions are highly driven by complex QCD dynamics
  The polarization is found increasing with k_T

K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Transversity Distribution

Transversity distribution

h_1  (Collinear & TMD)

A transverse counter part to the longitudinal spin structure: helicity g_{1L}, but NOT the same.

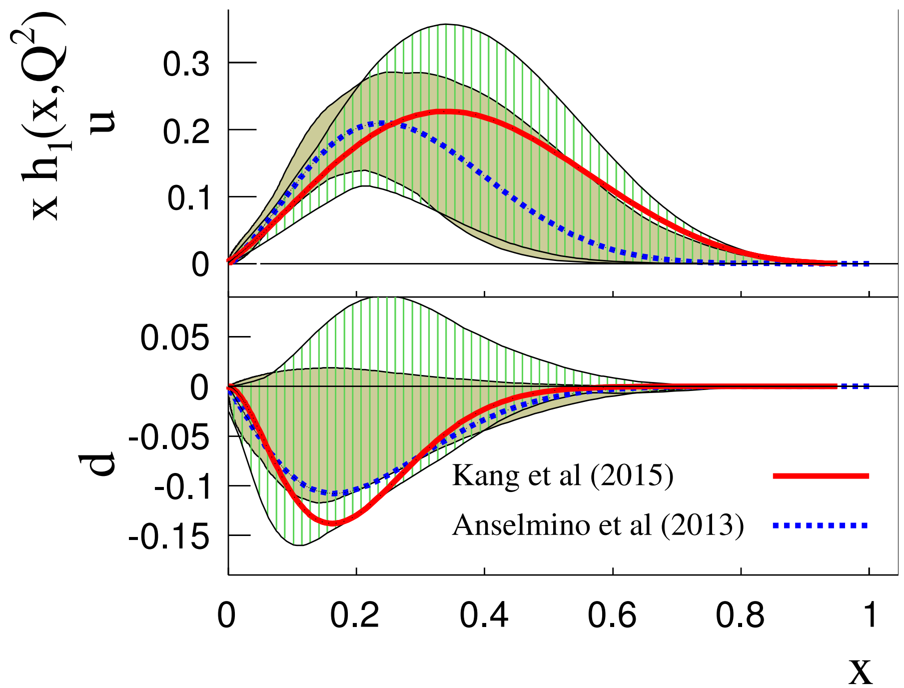
Chiral-odd:

No mixing with gluons
Valence dominant
Couple to another chiral-odd function.

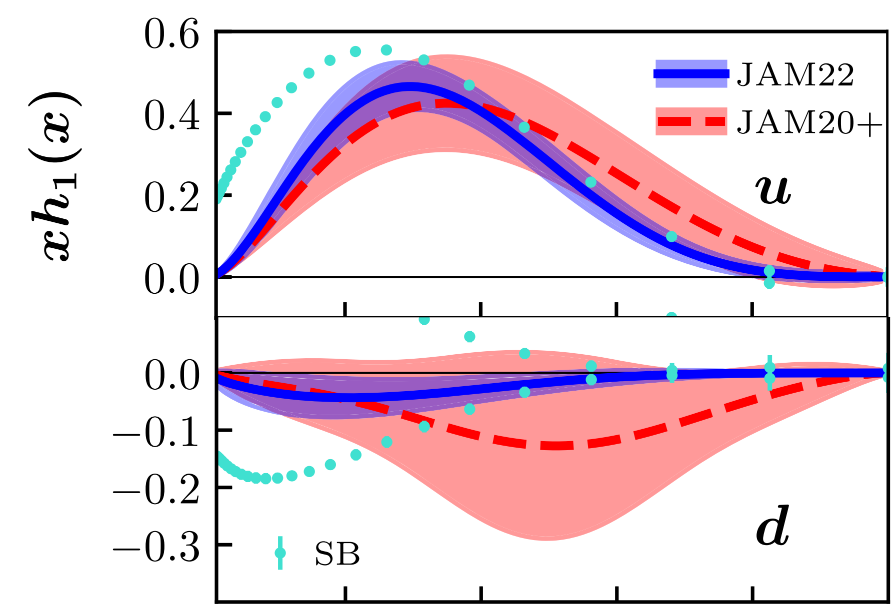
Effect in SIDIS:
transverse single spin asymmetry (Collins asymmetry)

$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1(x,k_T^2) \otimes H_1^\perp(z,p_T^2)$$

Phenomenological extractions



Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, PRD 93, 014009 (2016).

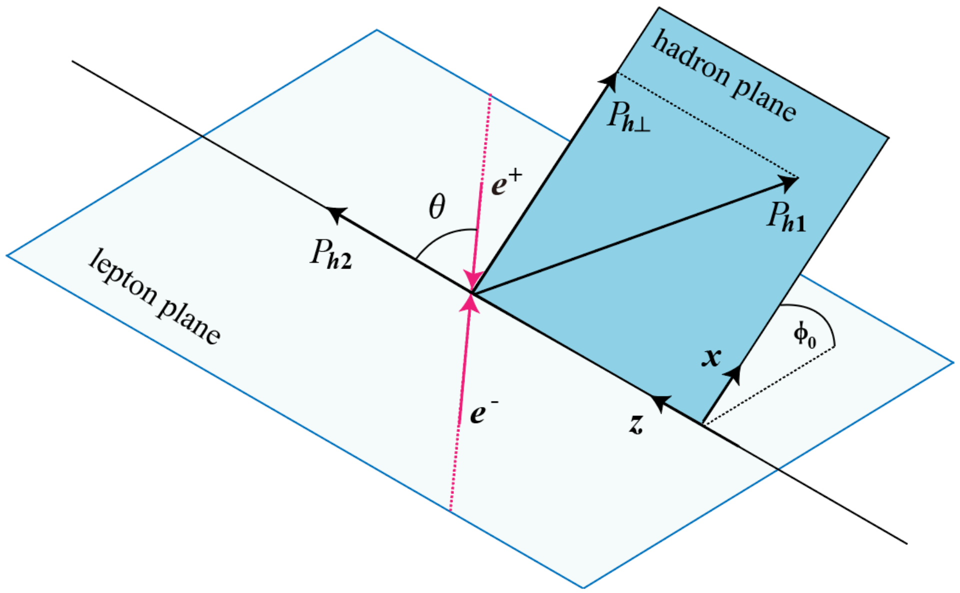


JAM Collaboration, PRD 104, 034014 (2022).

Assuming vanishing transverse polarization of sea quarks!

Complementary Process

Semi-inclusive e^+e^- annihilation: $e^+e^- \to h_1 h_2 X$



$$\frac{d^5\sigma}{dz_1 dz_2 d^2\boldsymbol{P}_{h\perp} d\cos\theta} = \frac{3\pi\alpha^2}{2Q^2} z_1^2 z_2^2 \left[\left(1+\cos^2\theta\right) F_{UU}^{h_1h_2} + \sin^2\theta \cos\left(2\phi_0\right) F_{\text{Collins}}^{h_1h_2} \right]$$

In TMD region: h_1 and h_2 are near back-to-back, $P_{hT} \ll Q$ $\quad F_{\text{Collins}}^{h_1h_2} \sim H_1^{\perp h_1} \otimes H_1^{\perp h_2}$

Experimental measurements:

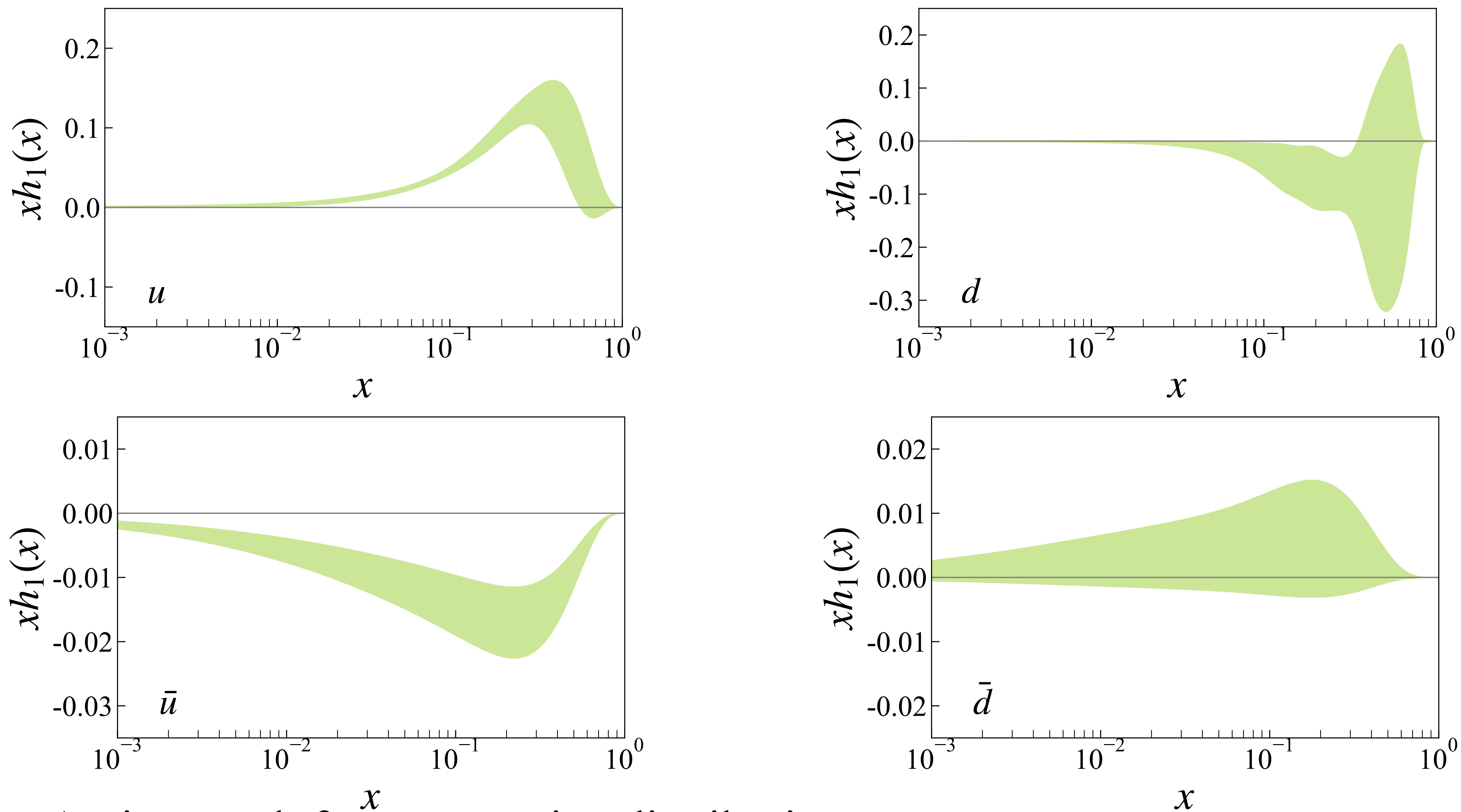
Belle: $\sqrt{s} = 10.58\,\text{GeV}$ Phys. Rev. D 78 (2008) 032011; 86 (2012) 039905(E).

BaBar: $\sqrt{s} = 10.6\ \text{GeV}$ Phys. Rev. D 90 (2014) 052003; Phys. Rev. D 92 (2015) 111101.

BESIII: $\sqrt{s} = 3.68\ \text{GeV}$ Phys. Rev. Lett. 116 (2016) 042001.

Sea Quark Transversity

First determination of sea quark transversity, including TMD evolution



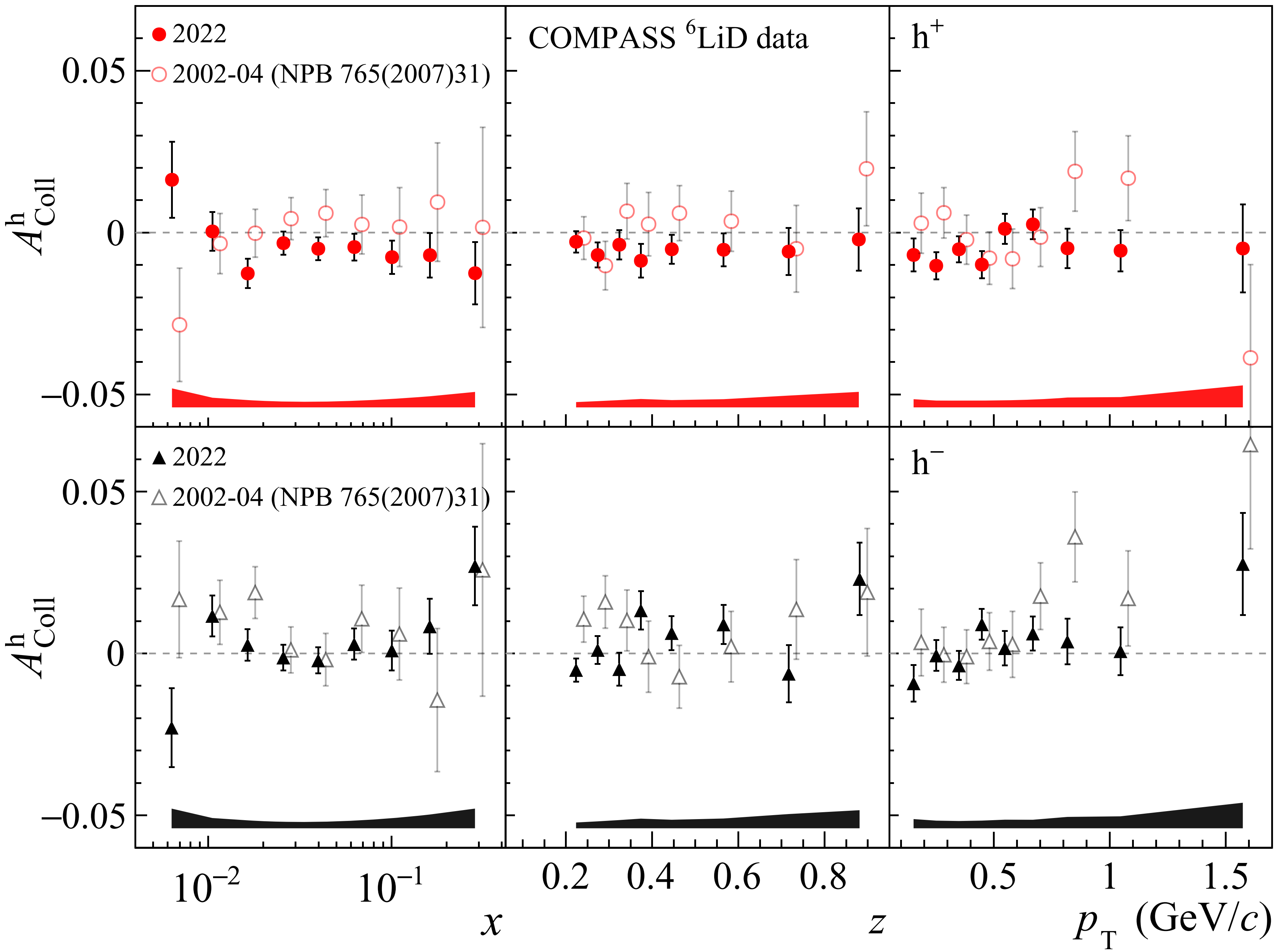
Anti-u quark favors negative distribution

Anti-d quark consistent with zero with current precision

C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, PRD 109 (2024) 056002.

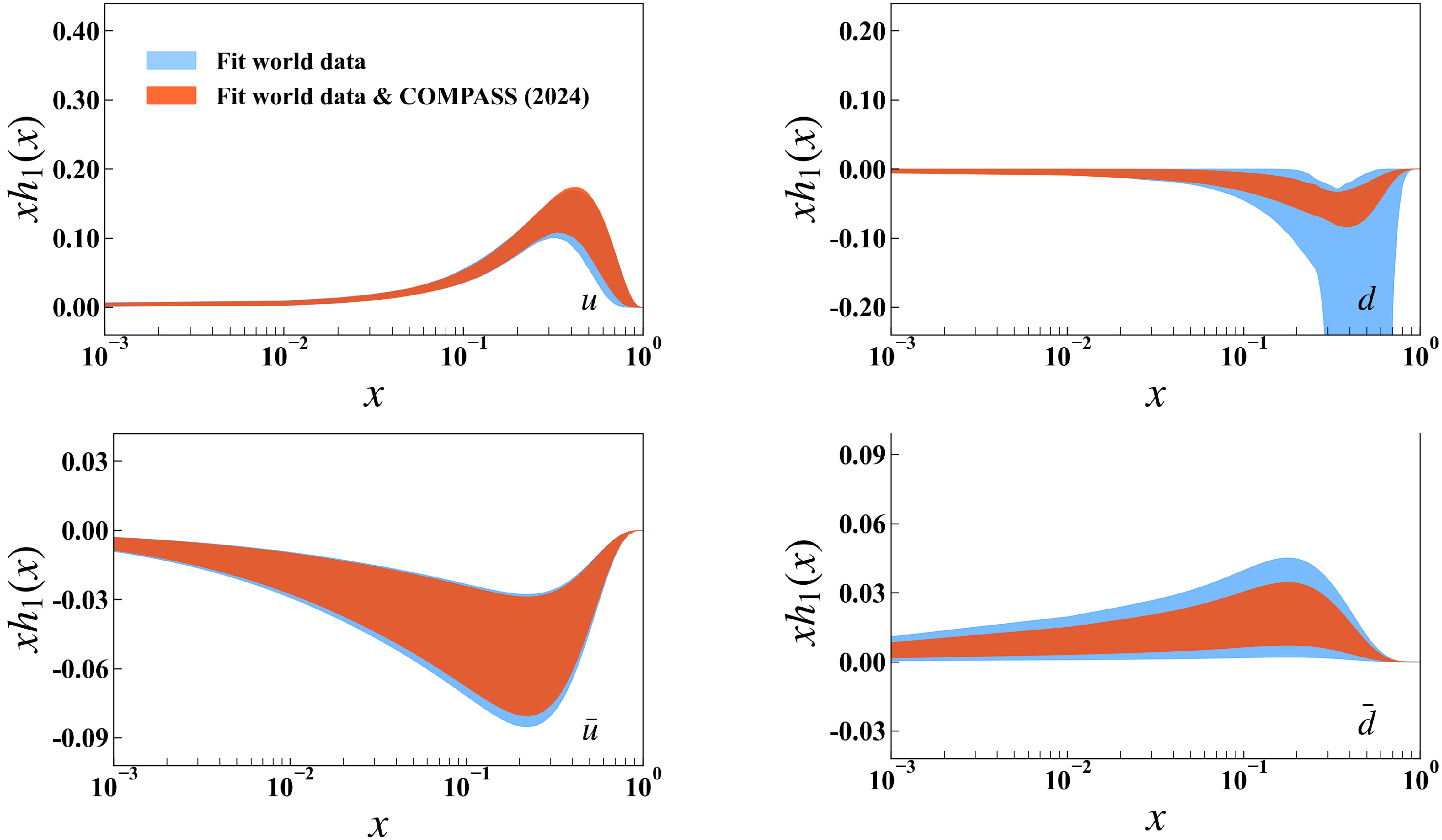
New COMPASS Data

SIDIS on transversely polarized deuteron target



COMPASS Collaboration, Phys. Rev. Lett. 133 (2024) 101903.

Transversity Distributions

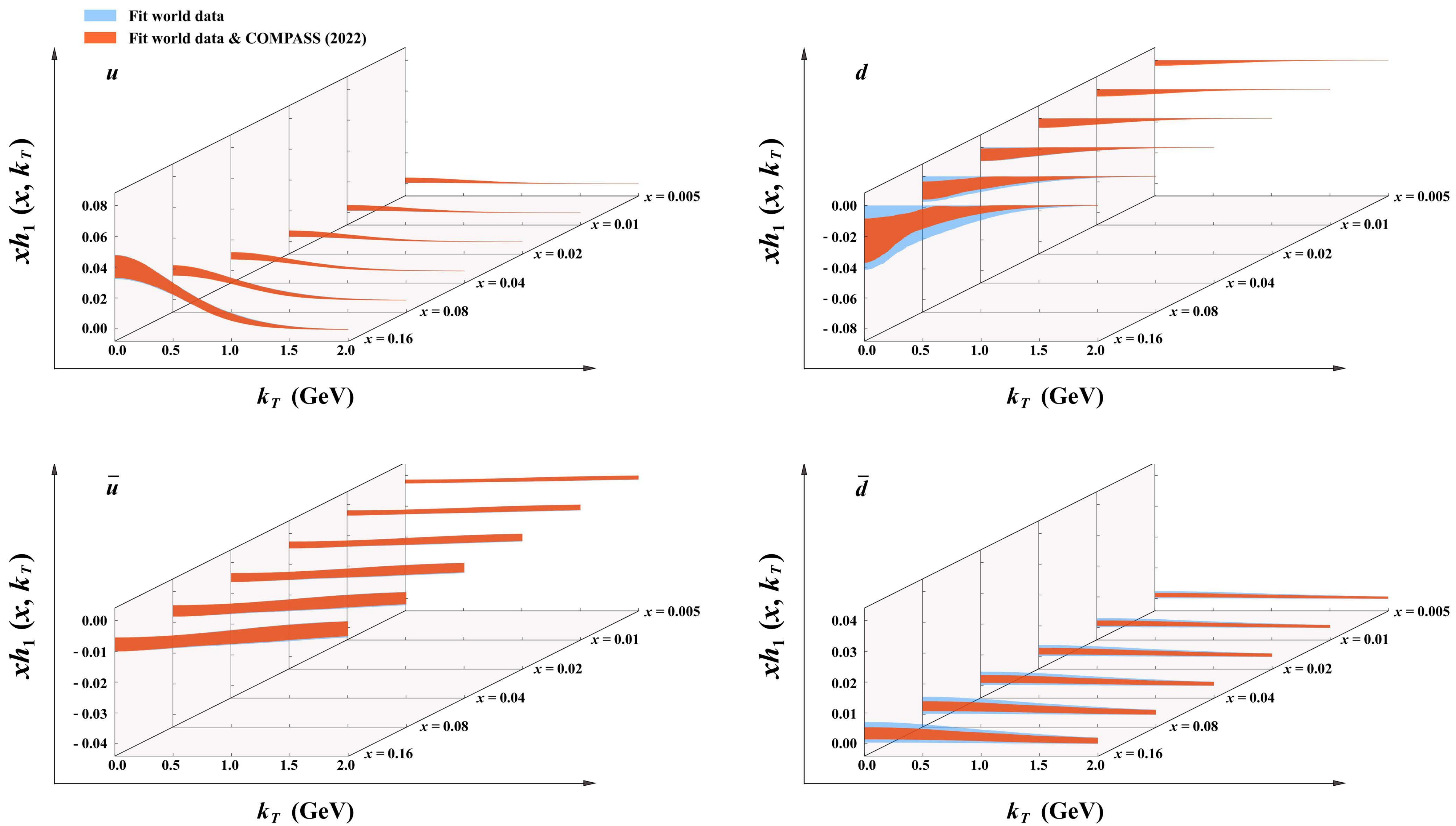


Reconfirmed by new COMPASS data.

Significant improvement on d and $\bar{d}$ distributions.

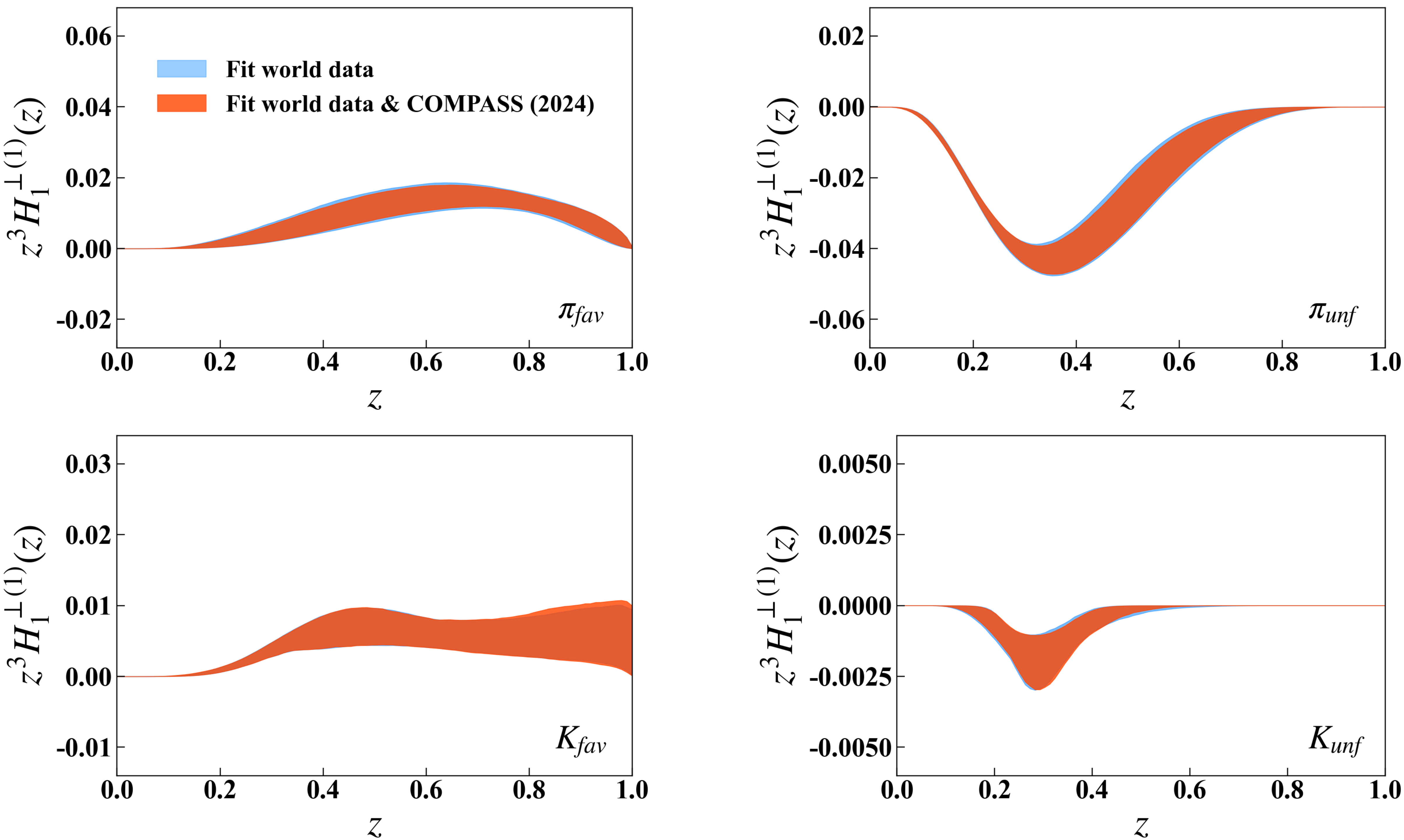
C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Transversity Distributions



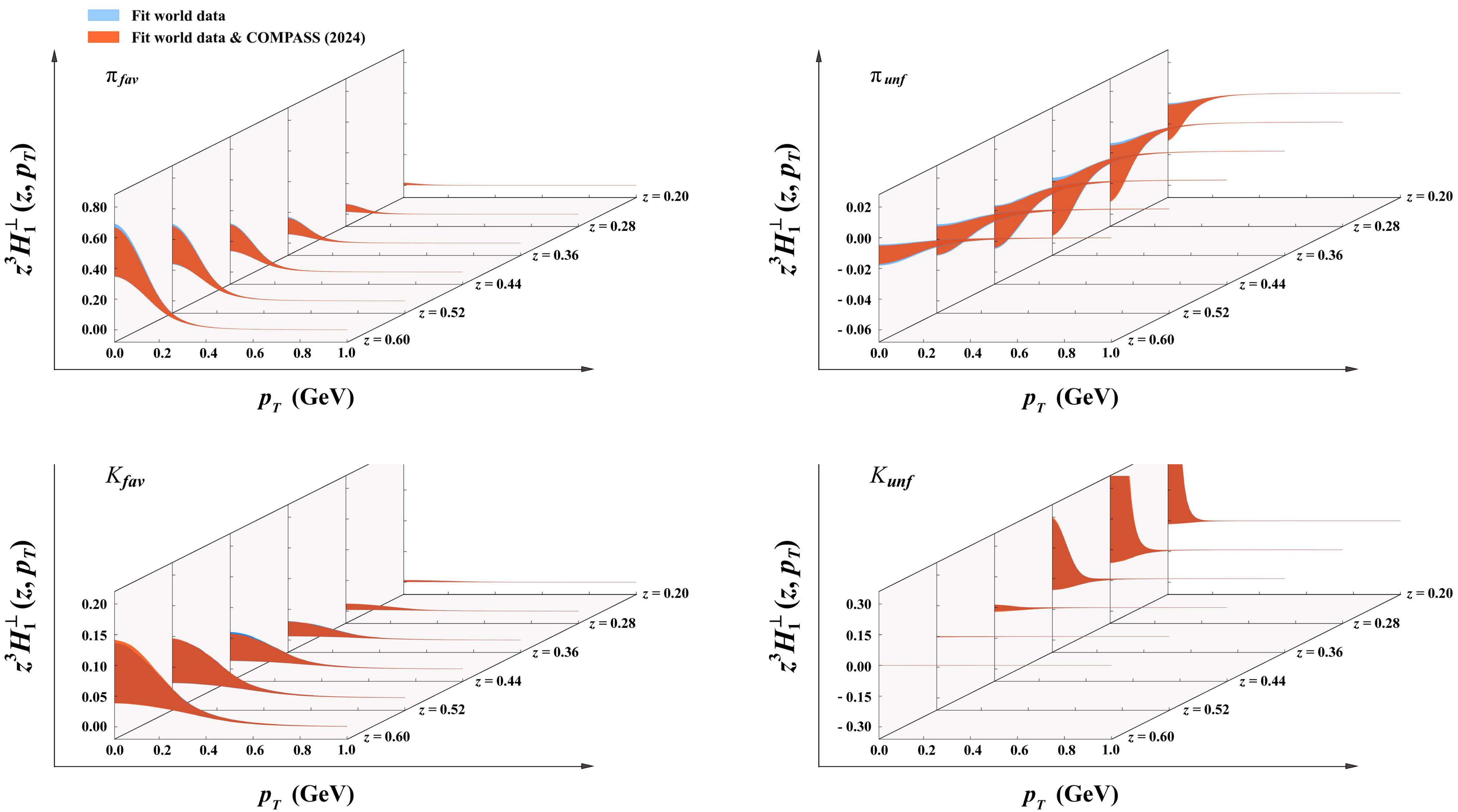
C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Collins Fragmentation Functions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

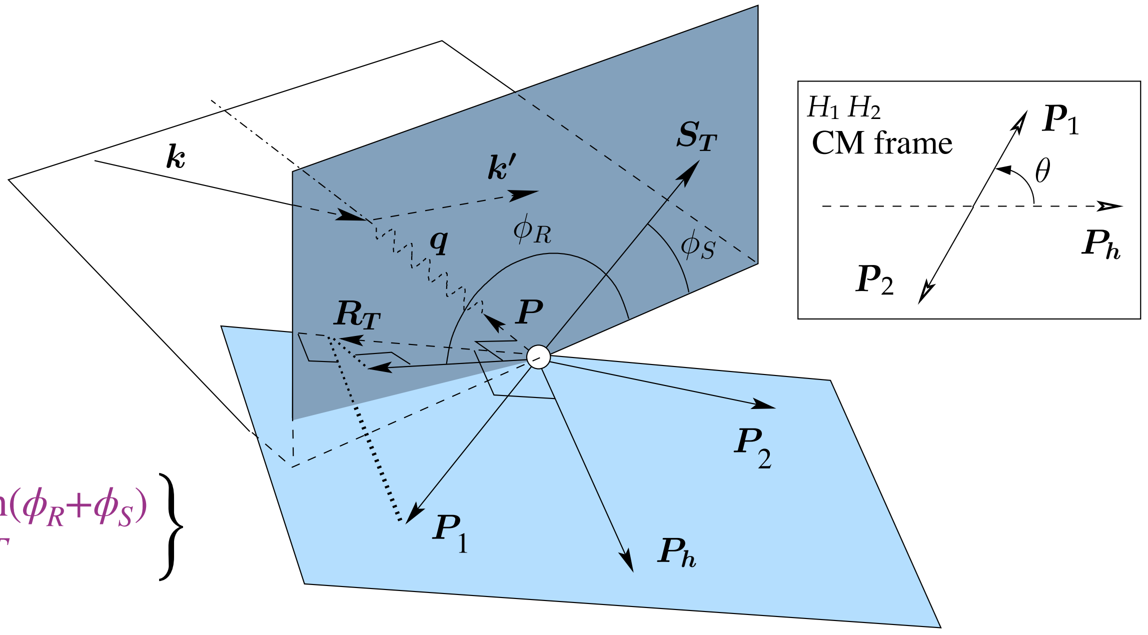
Collins Fragmentation Functions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

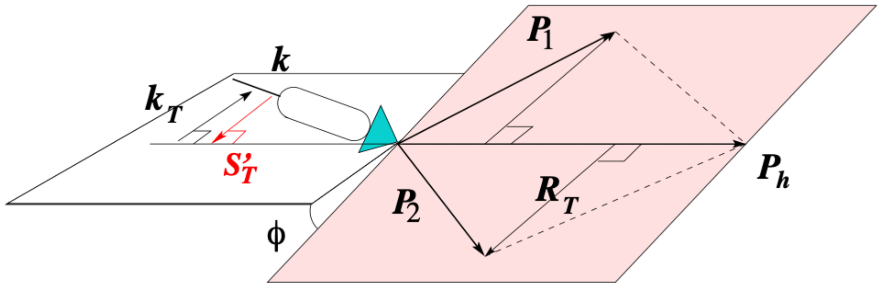
Collinear Transversity Distributions

Dihadron SIDIS



$$\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz\, d\phi_R\, dM_h^2\, d\cos\theta} = \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\left\{F_{UU} + S_T\,\varepsilon\,\sin(\phi_R+\phi_S)F_{UT}^{\sin(\phi_R+\phi_S)}\right\}$$

$$F_{UT}^{\sin(\phi_R+\phi_S)} \sim h_1 \otimes H_1^{\sphericalangle}$$



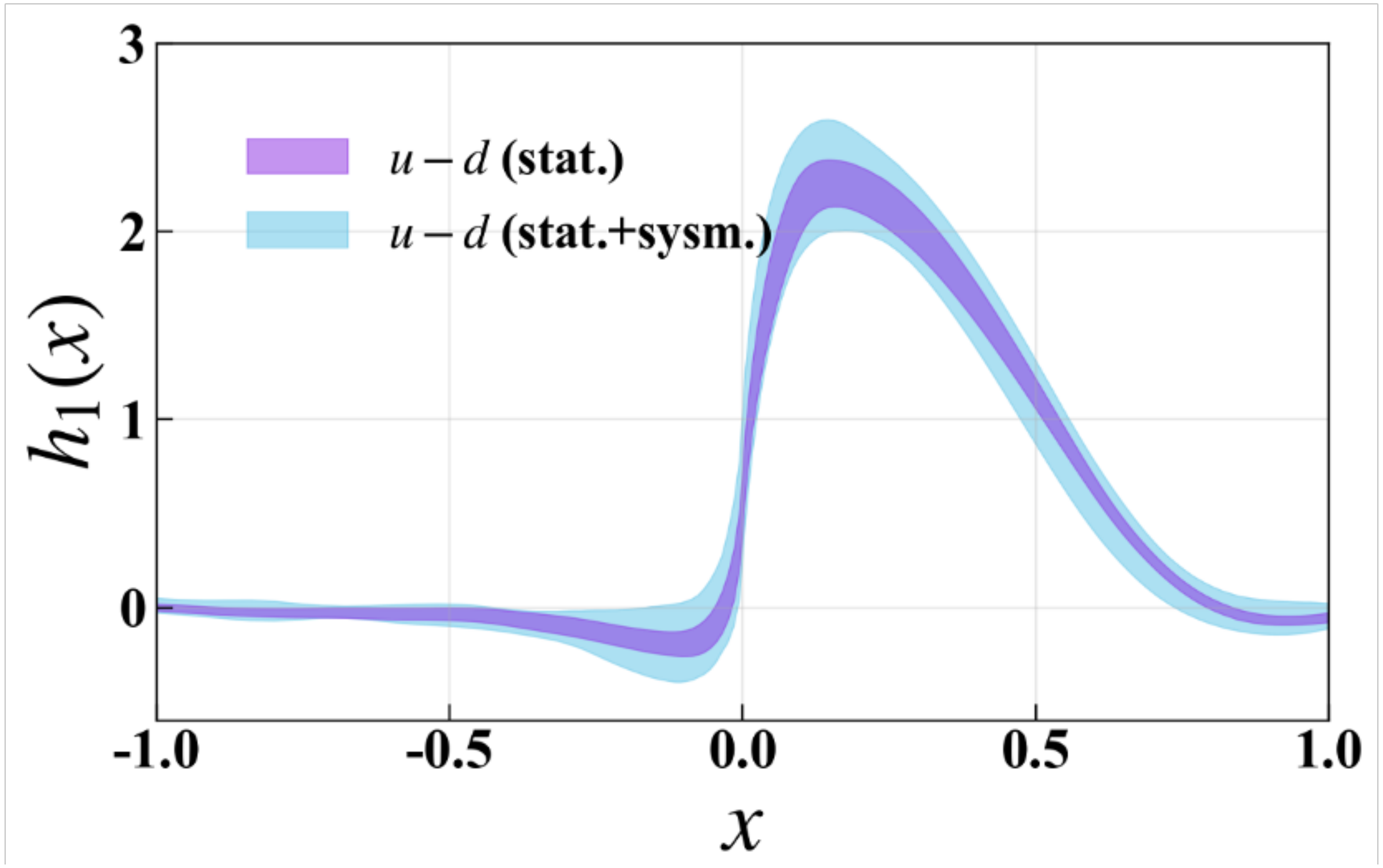
h_1: collinear transversity distribution function

$H_1^{\sphericalangle}$: a chiral-odd dihadron fragmentation function

describing the correlation between quark transverse polarization and the azimuthal orientation of the hadron pair plane

M. Radici, R. Jakob, and A. Bianconi, Phys. Rev. D 65 (2002) 074031;
A. Bacchetta, A. Courtoy, M. Radici, JHEP 03 (2013) 119.

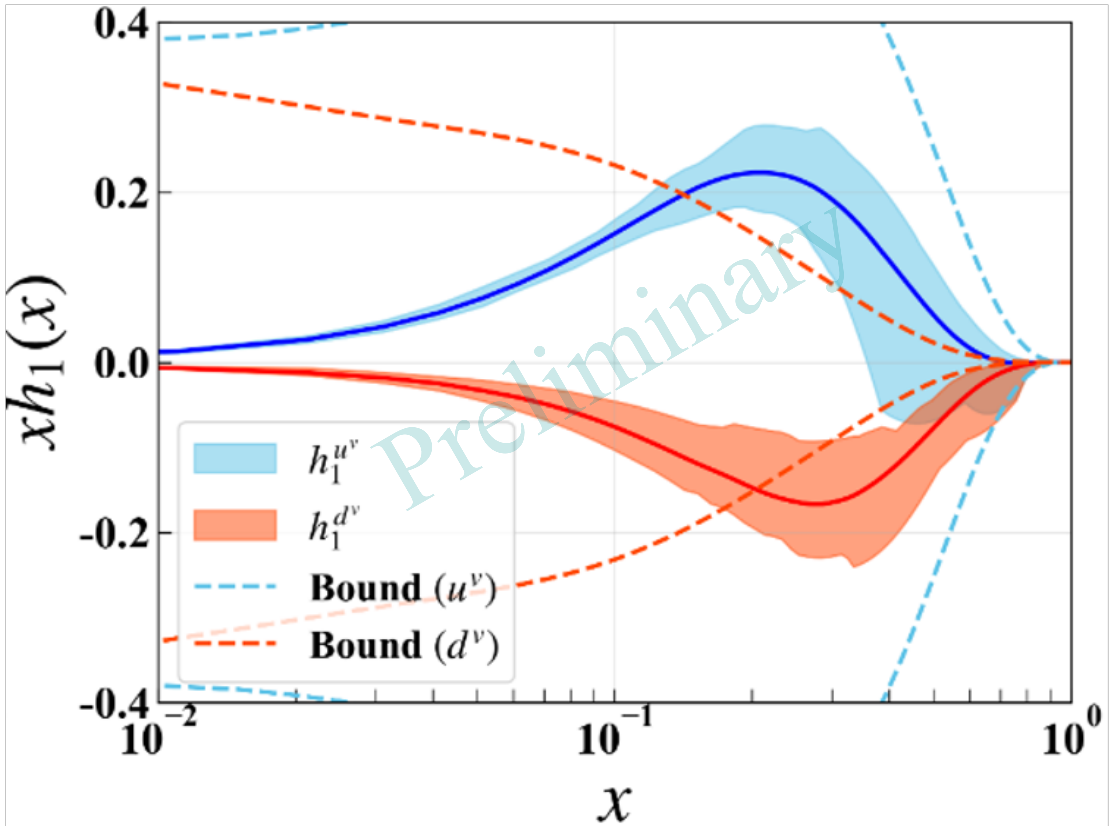
Transversity Distribution from Lattice QCD



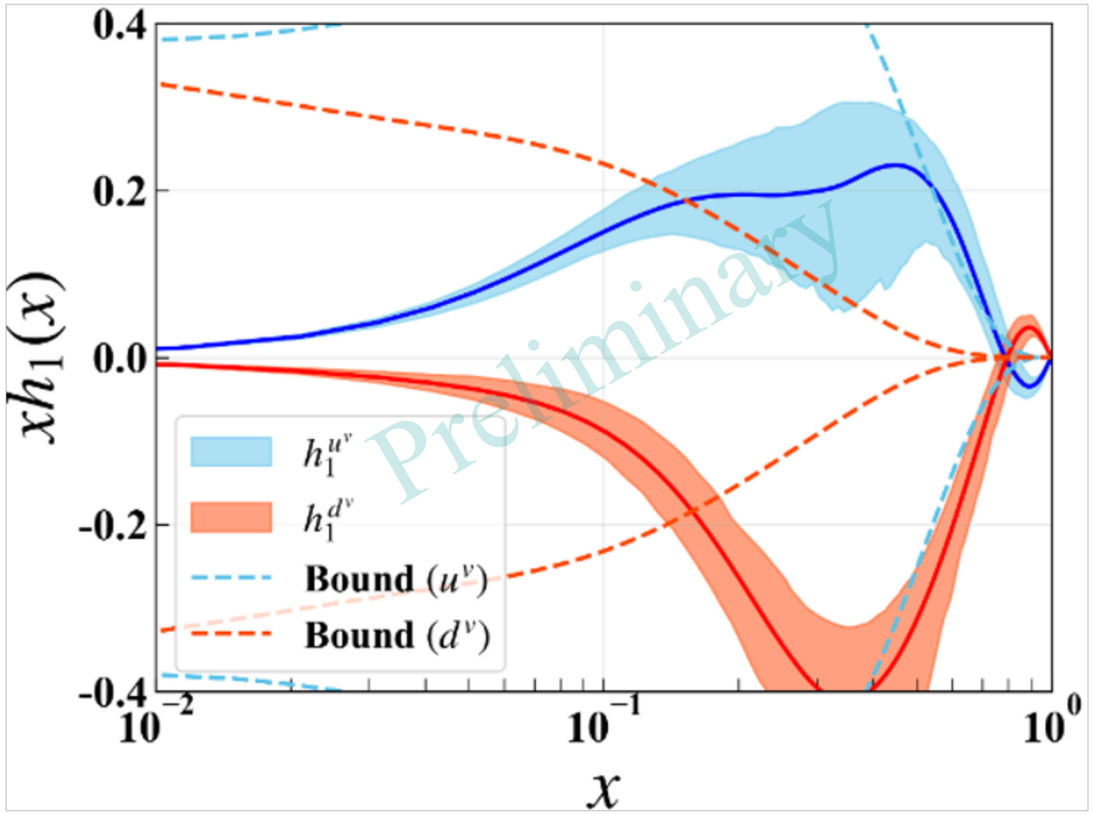
Lattice Parton Collaboration, Phys. Rev. Lett. 131 (2023) 261901.

A Combined Analysis of Transversity

SIDIS + e^+e^- dihadron data



SIDIS + e^+e^- + LQCD

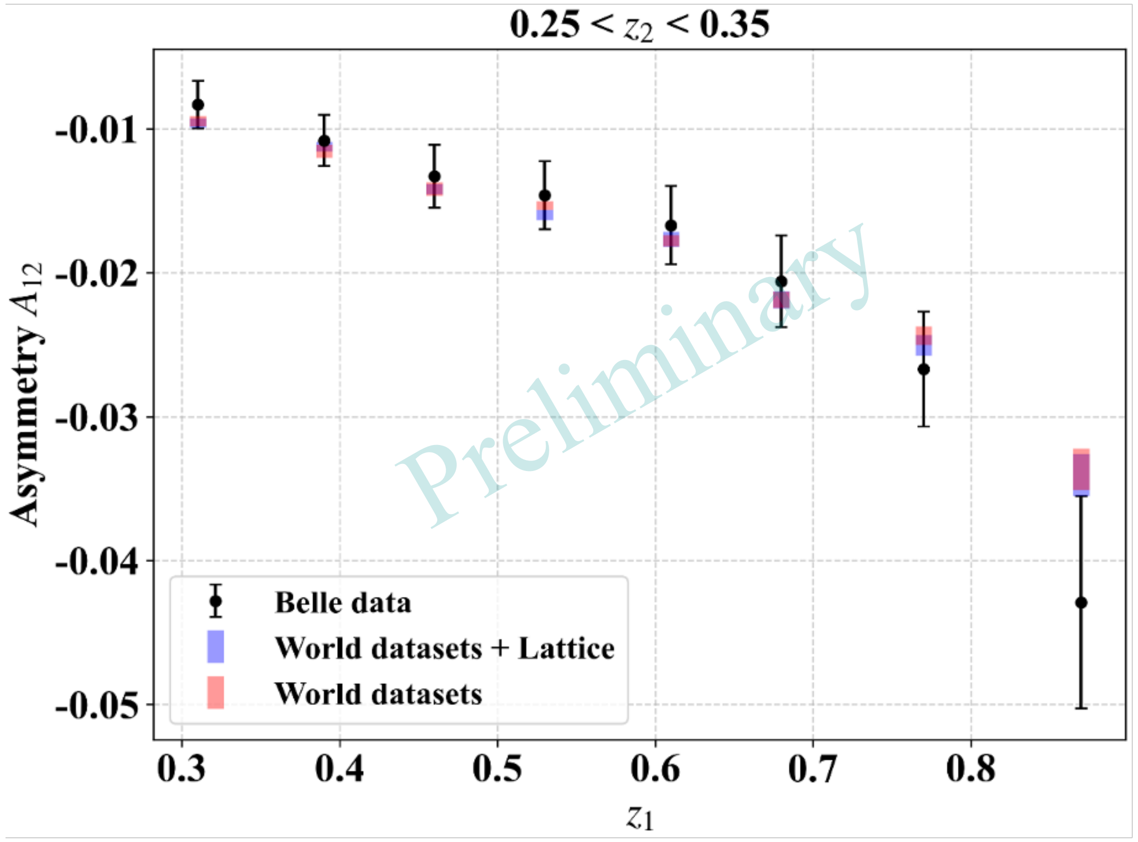


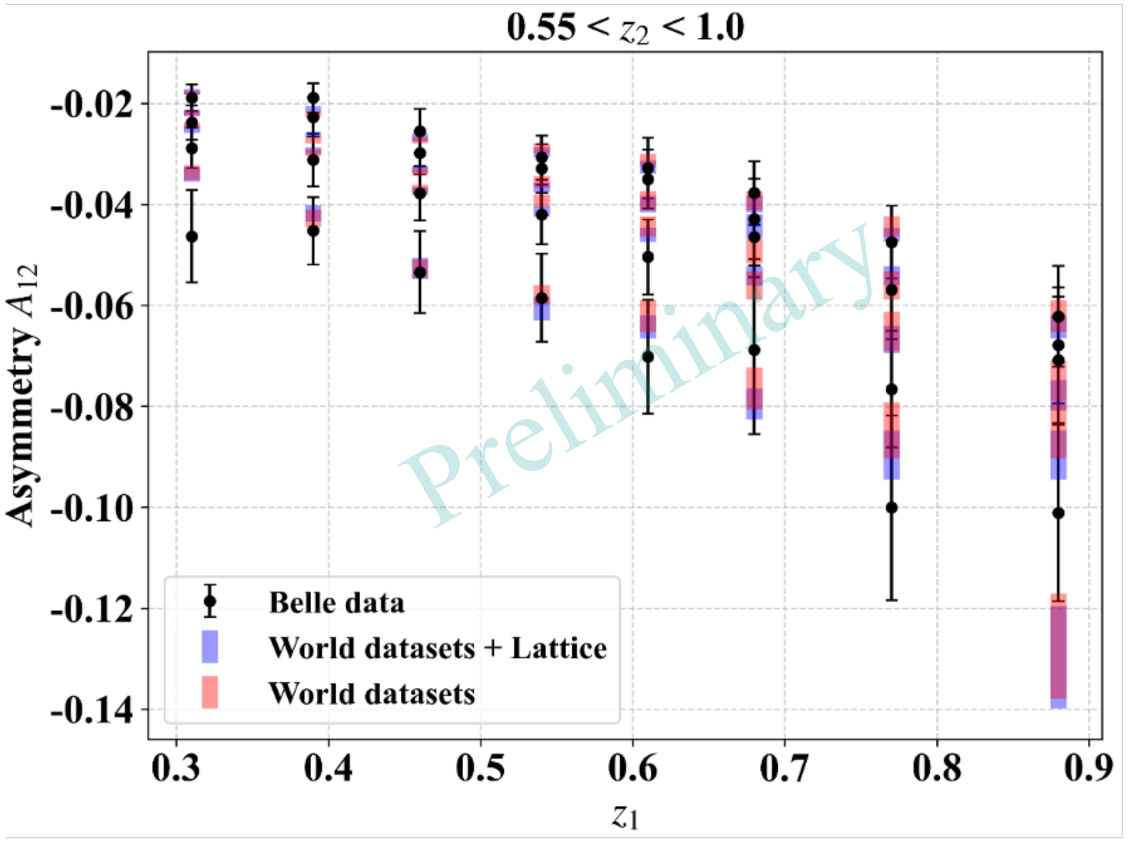
χ^2/N	Belle (dσ)	Belle (Asym)	HERMES (p)	COMPASS(p)	COMPASS(d)
without LQCD	1.186	0.785	0.853	0.934	0.780
with LQCD	0.967	0.807	0.845	1.085	0.976

H. Dong, TL, P. Sun, Y. Zhao, 2026

A Combined Analysis of Transversity

Comparison with Belle data



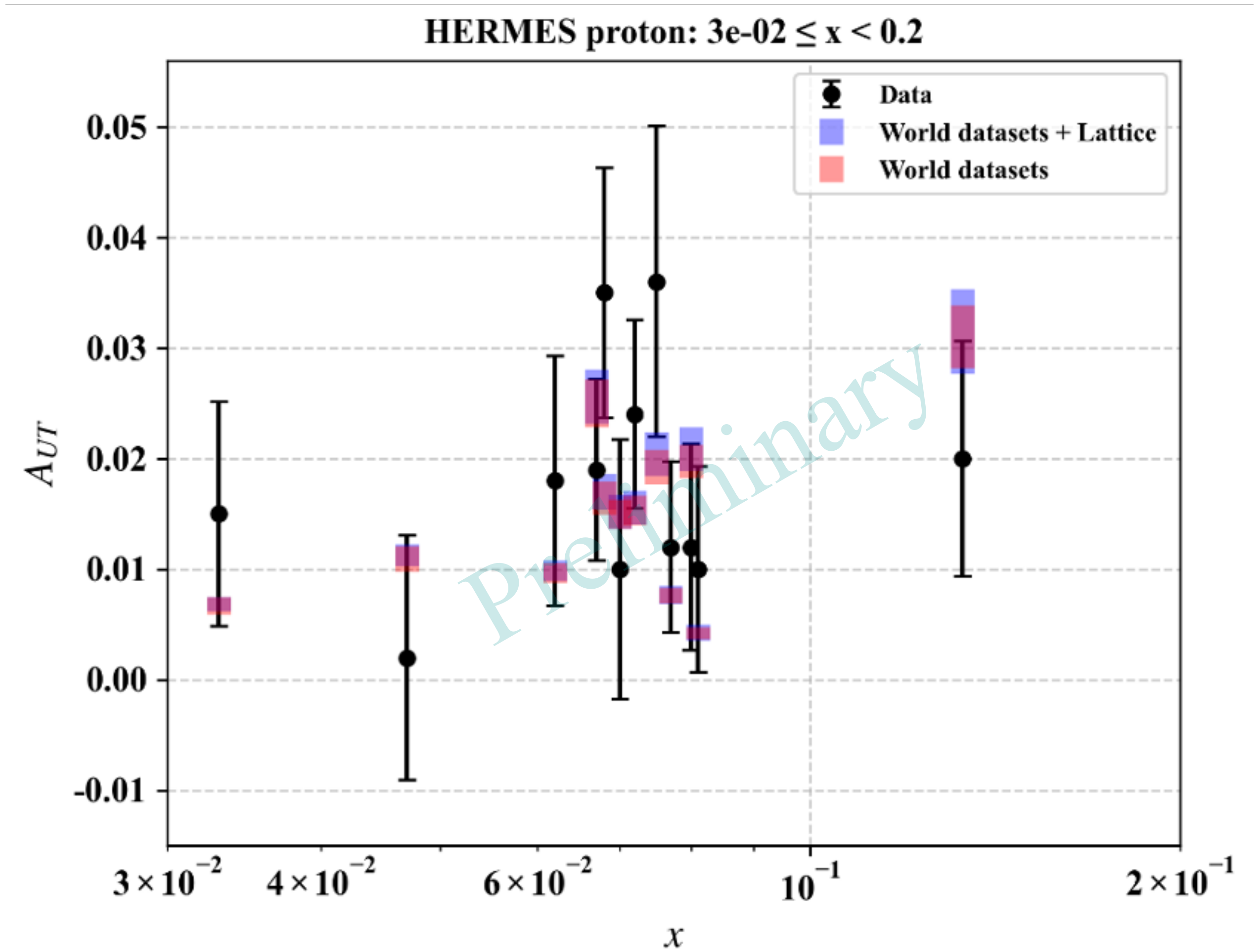


Data: Belle Collaboration, Phys. Rev. Lett. 107 (2011) 072004.

H. Dong, TL, P. Sun, Y. Zhao, 2026

A Combined Analysis of Transversity

Comparison with HERMES data

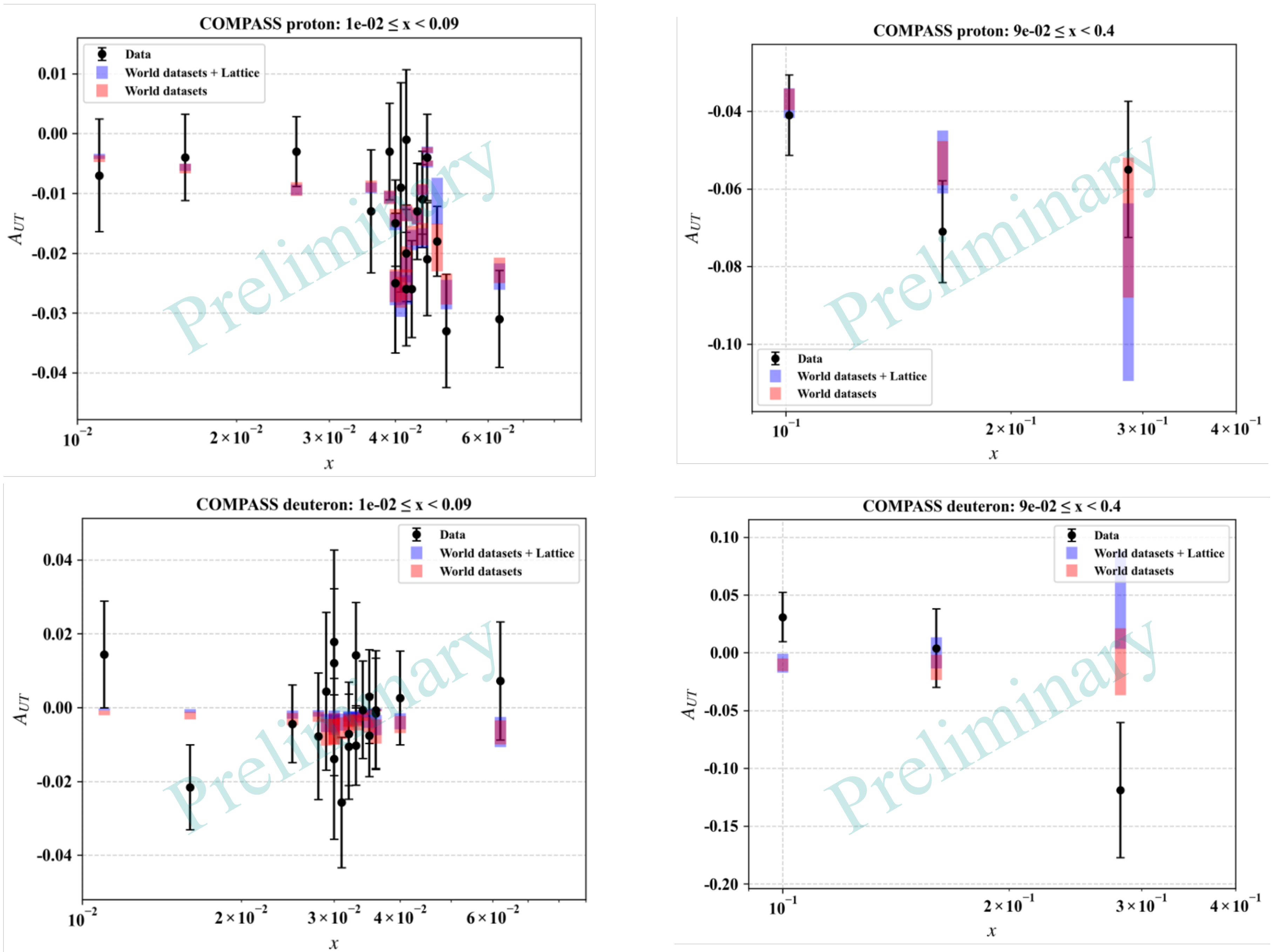


Data: HERMES Collaboration, JHEP 06 (2008) 017.

H. Dong, TL, P. Sun, Y. Zhao, 2026

A Combined Analysis of Transversity

Comparison with COMPASS data



Data: COMPASS Collaboration, Phys. Lett. B 736 (2014) 124; Phys. Lett. B 845 (2023) 138155.

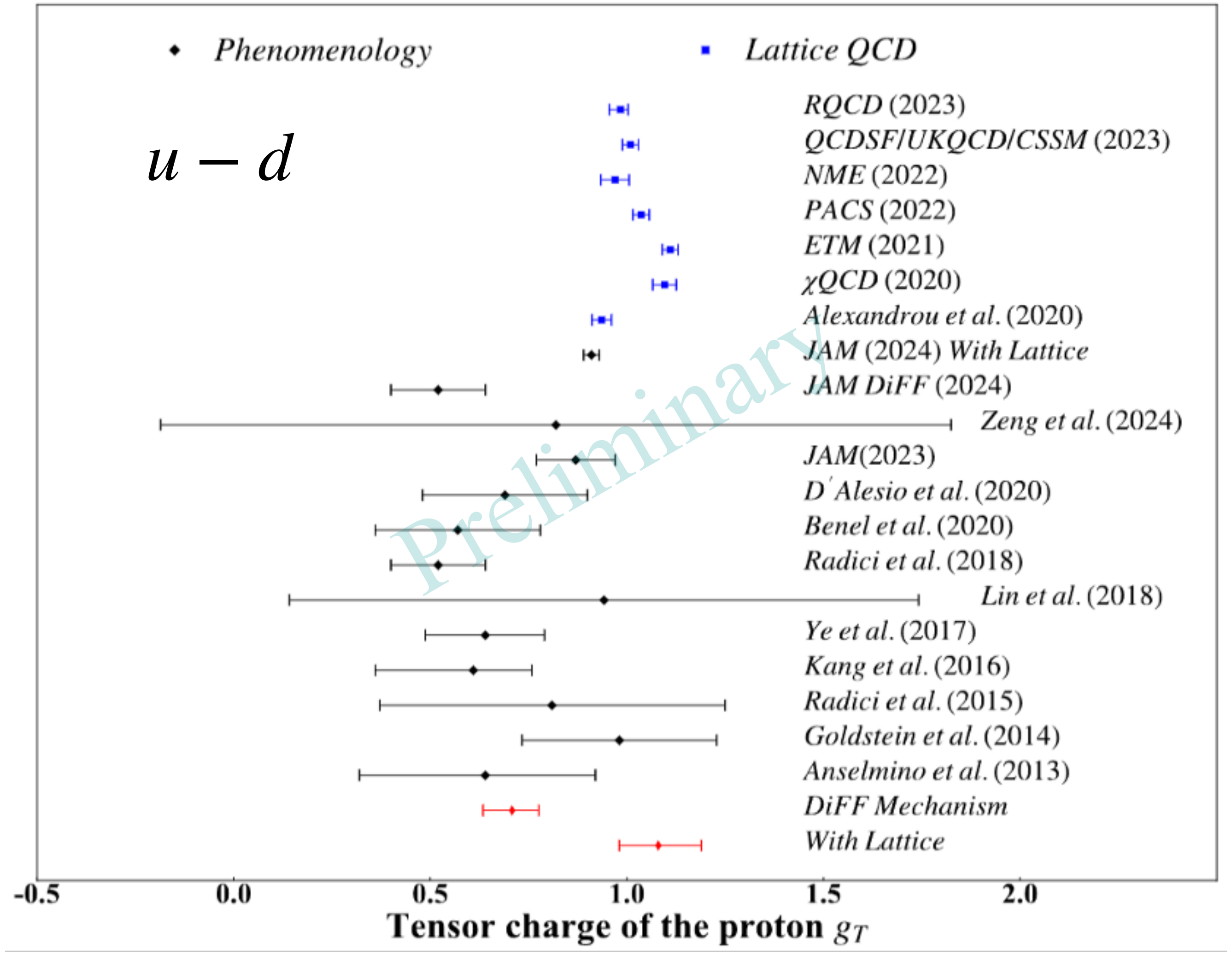
H. Dong, TL, P. Sun, Y. Zhao, 2026

Tensor Charge

Tensor charge

$$\langle P,S|\bar{\psi}^q i\sigma^{\mu\nu}\gamma_5\psi^q|P,S\rangle = g_T^q \bar{u}(P,S) i\sigma^{\mu\nu}\gamma_5 u(P,S) \qquad g_T^q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)]\,dx$$

- A fundamental QCD quantity: matrix element of local operators.
- Moment of the transversity distribution: valence quark dominant.
- Calculable in lattice QCD.

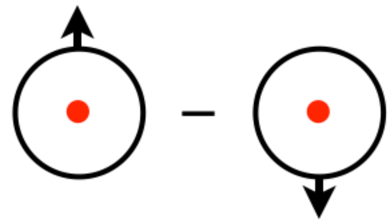


H. Dong, TL, P. Sun, Y. Zhao, 2026

The Sivers Function

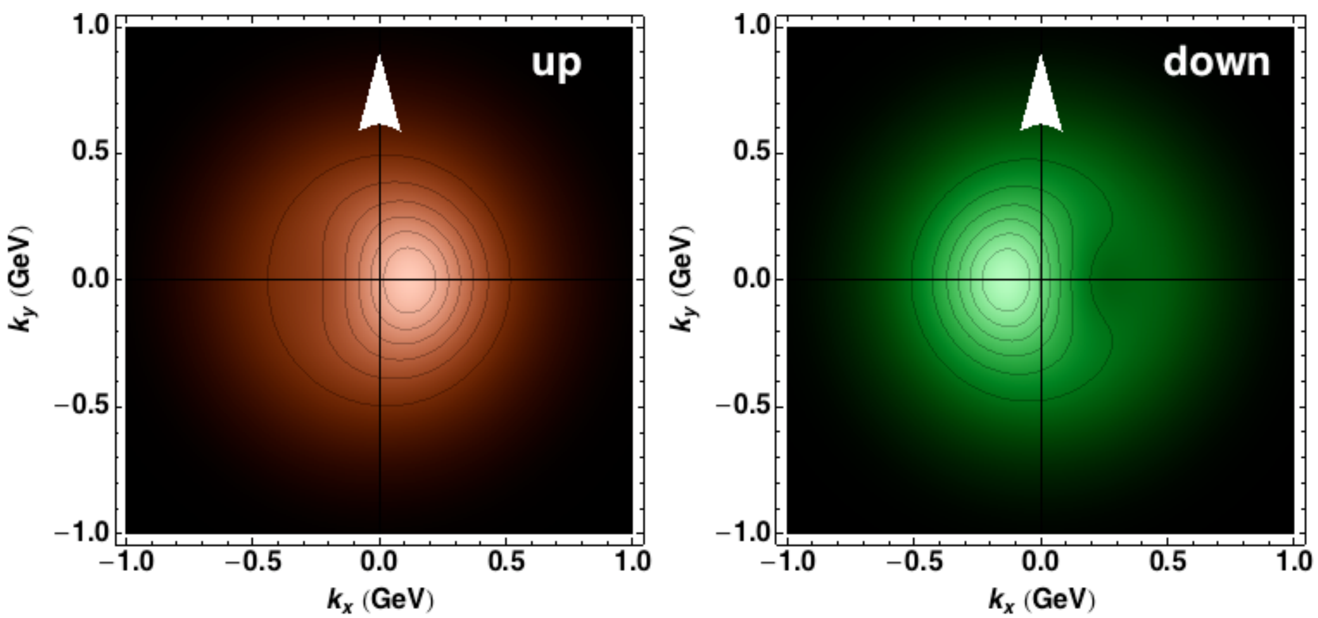
Sivers TMD distribution function

$$\frac{\epsilon_{ij} k_T^i S_T^j}{M} f_{1T}^{\perp}(x, k_T^2)$$



A naive T-odd distribution function

Transverse momentum distribution distorted by nucleon transverse spin



[Figure from A. Bacchetta]

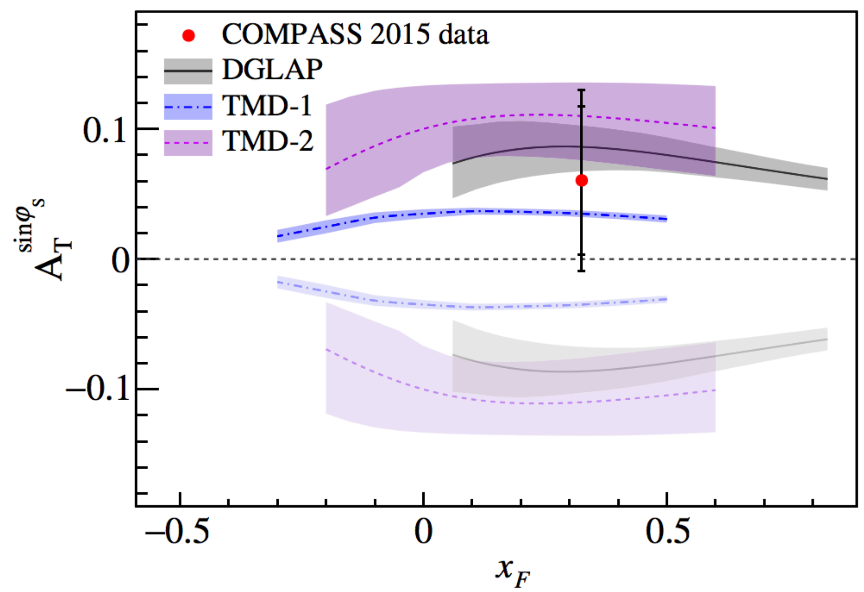
Effect in SIDIS:
transverse single spin asymmetry (Sivers asymmetry)

$$A_{UT}^{\sin(\phi_h - \phi_S)} \sim f_{1T}^{\perp} \otimes D_1$$

sizable Sivers asymmetry observed by HERMES, COMPASS, JLab

Sign change prediction:

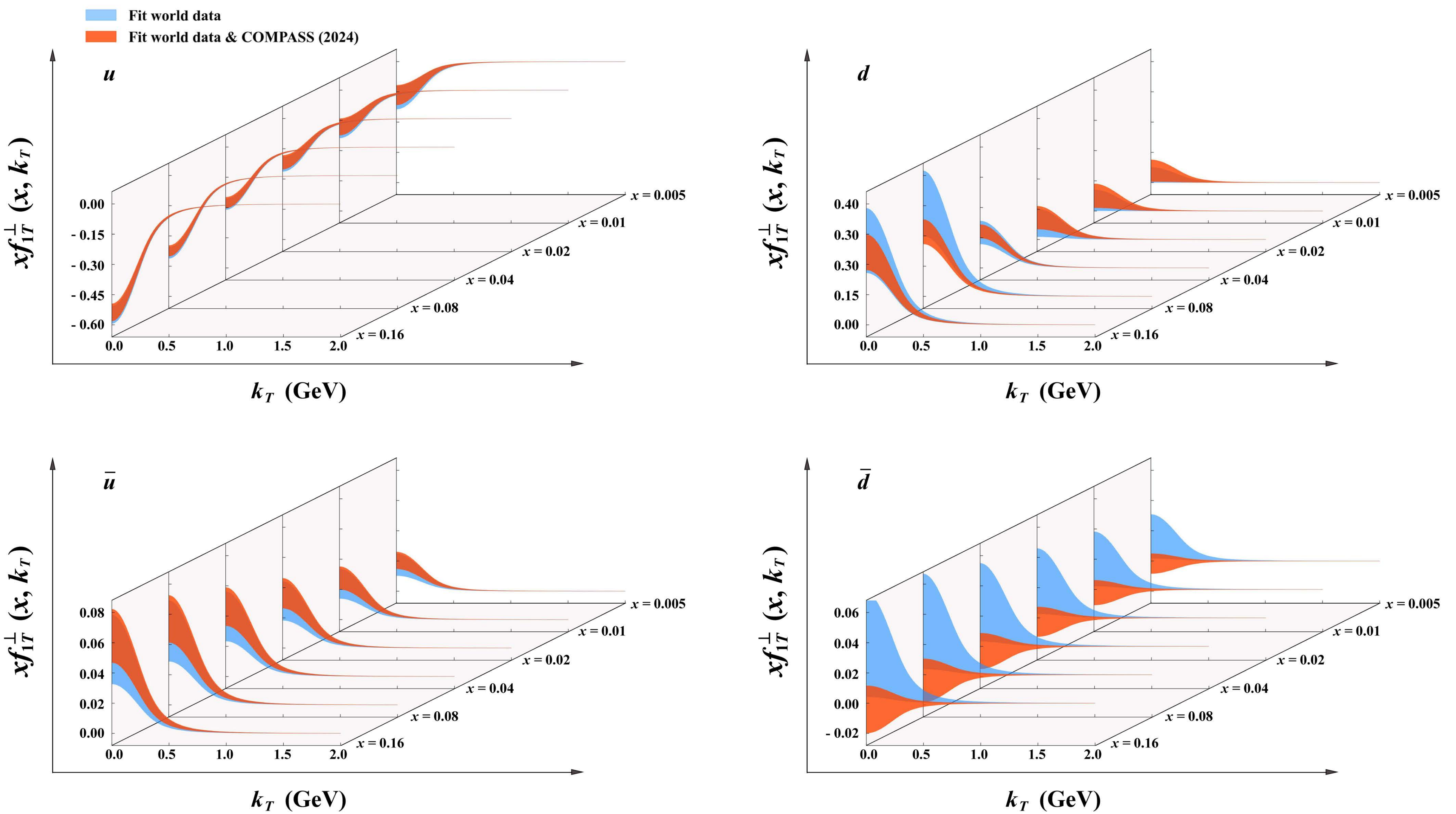
$$f_{1T}^{\perp}(x, k_T^2)\,|_{\text{SIDIS}} = -f_{1T}^{\perp}(x, k_T^2)\,|_{\text{DY}}$$



COMPASS Collaboration, PRL 119, 112002 (2017).

Sivers Functions

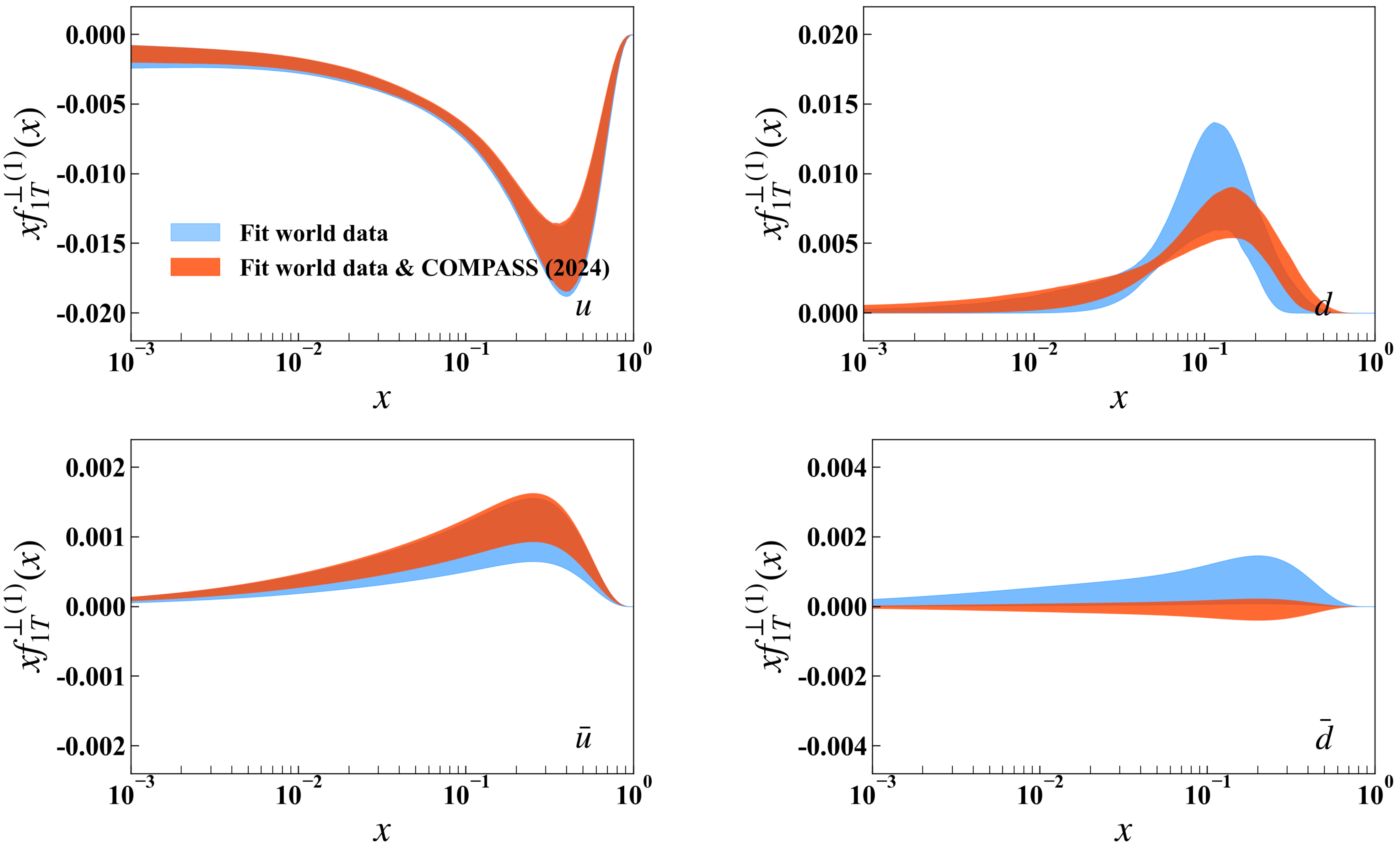
Global analysis of SIDIS, Drell-Yan, $W^{\pm}/Z^0$ production data



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Sivers Functions

Global analysis of SIDIS, Drell-Yan, $W^{\pm}/Z^{0}$ production data

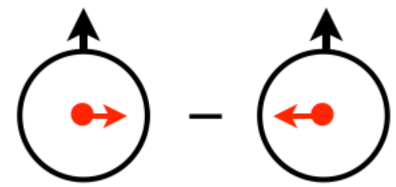


C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Double Spin Asymmetry and Worm-gear

Trans-helicity worm-gear distribution

$$\frac{k_T \cdot S_T}{M} g_{1T}^{\perp}(x, k_T^2)$$

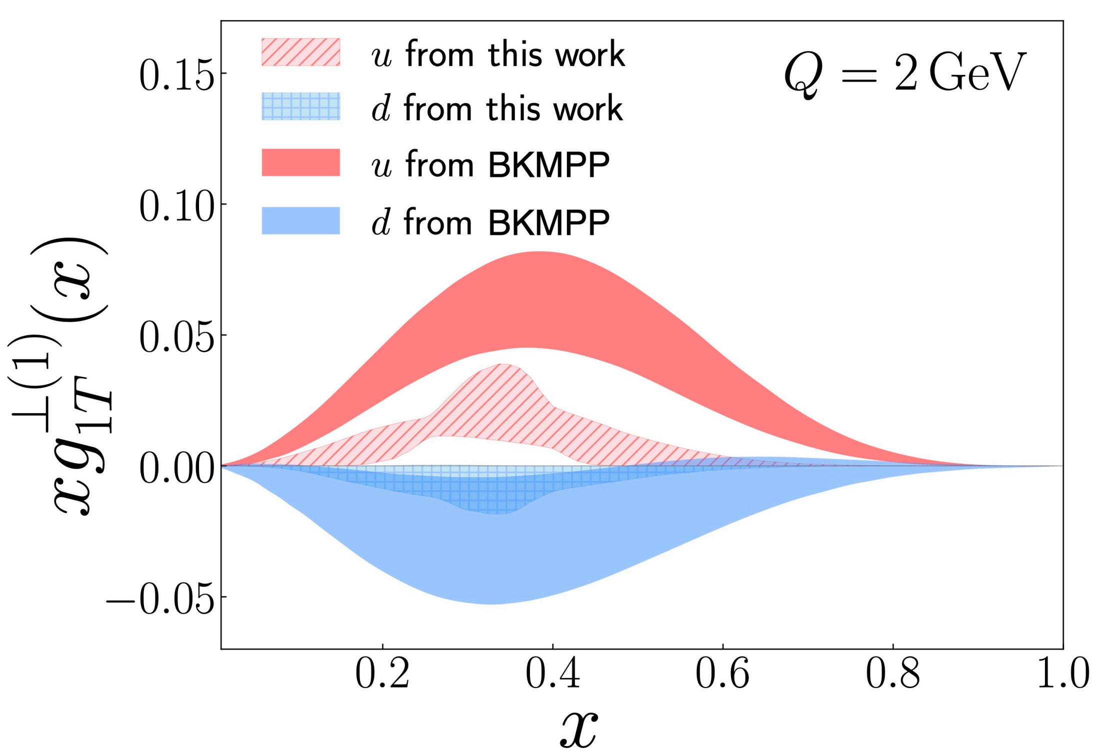


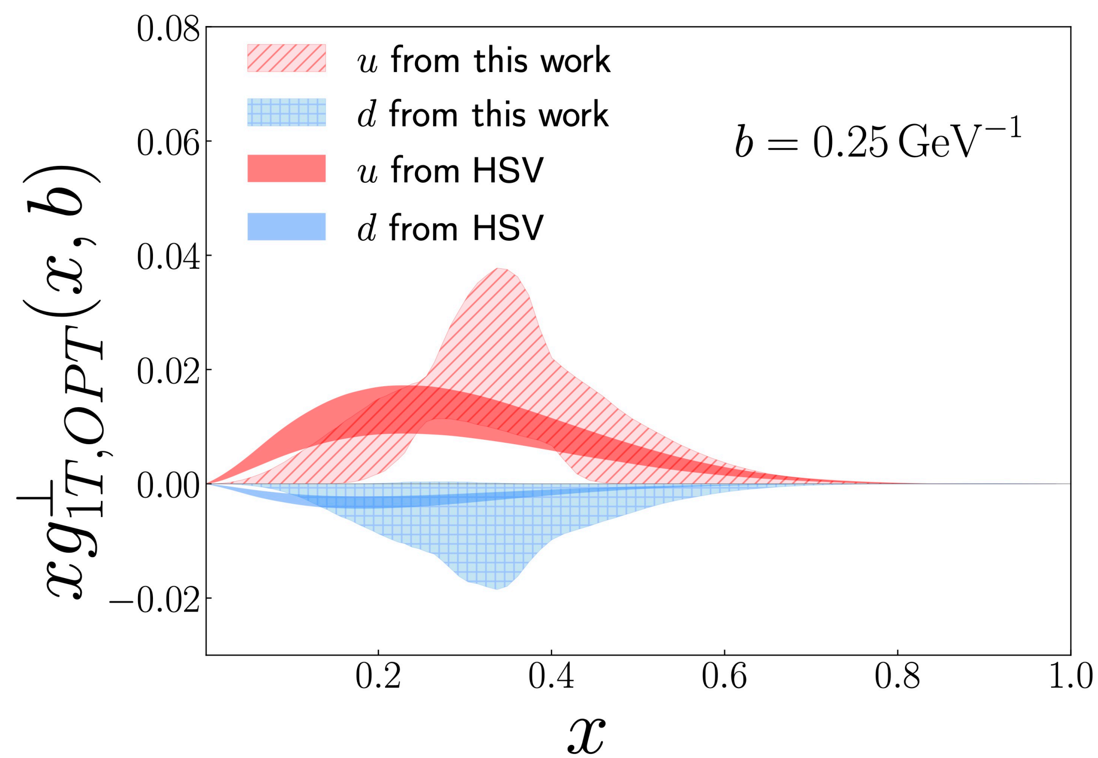
Effect in SIDIS:
A longitudinal-transverse double spin asymmetry

- Longitudinally polarized quark density in a transversely polarized nucleon
- Overlap between wave functions differing by one unit of orbital angular momentum

$$A_{LT}^{\cos(\phi_h - \phi_S)} \sim g_{1T}^{\perp} \otimes D_1$$

Phenomenological extraction



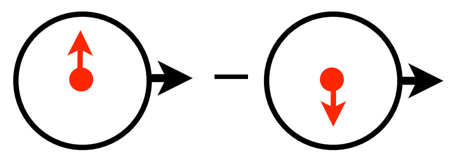


K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. D 110 (2024) 034036.

Longitudinal SSA and Worm-gear

Longi-transversity worm-gear distribution

$$\frac{S_L k_T^\alpha}{M} h_{1L}^\perp(x, k_T^2)$$

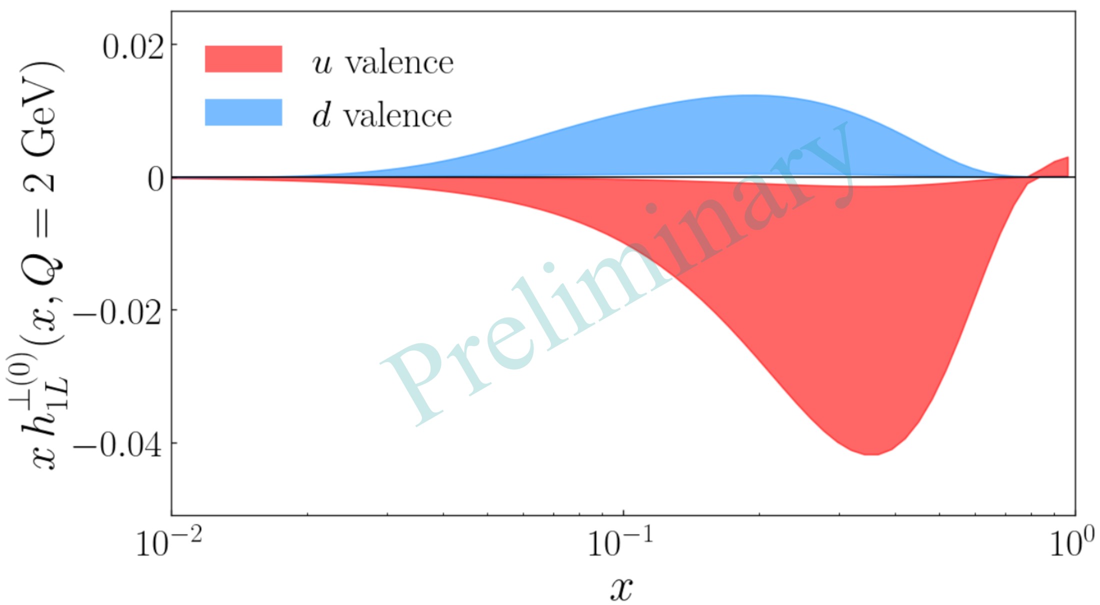


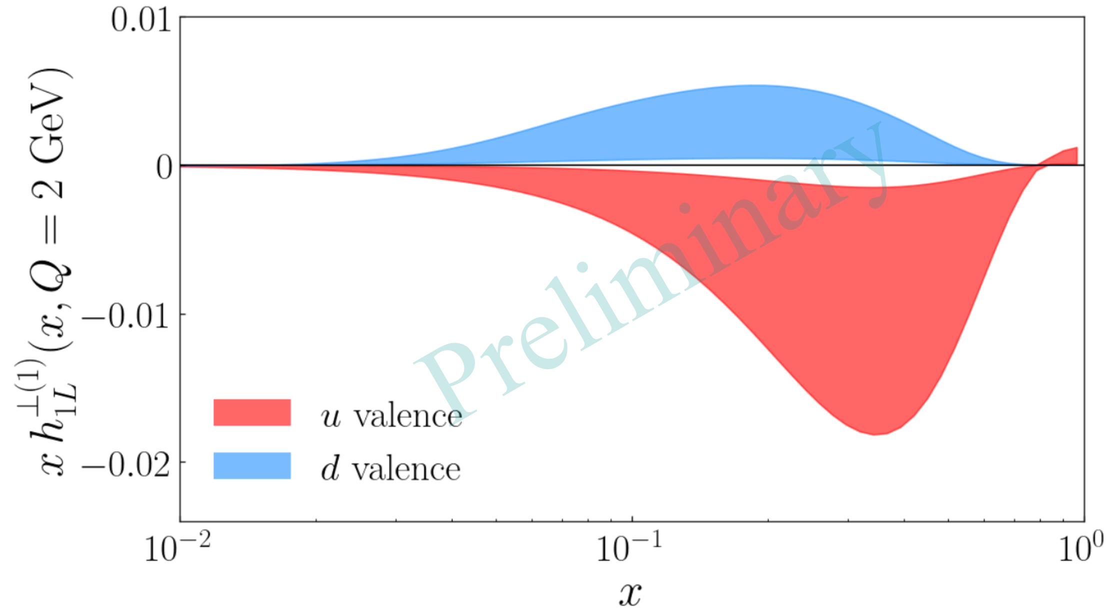
Effect in SIDIS:
A longitudinal target single spin asymmetry

$$A_{UL}^{\sin 2\phi_h} \sim h_{1L}^\perp \otimes H_1^\perp$$

- Transversely polarized quark density in a longitudinally polarized nucleon
- Overlap between wave functions differing by one unit of orbital angular momentum

First extraction





W. Chen, K. Yang, TL, P. Sun, Y. Zhao, 2026

Summary

- Nucleon spin structure is still not well understood
- Rich information is contained in TMDs
  - helicity: quark polarization has nontrivial dependence on transverse momentum;
  - transversity: sea quarks may have nonzero transverse polarization, suggest intrinsic sea;
  - Sivers: quark transverse momentum is distorted by the nucleon transverse spin;
  - worm-gears: correlation between quark and nucleon longtudinal/transverse and transverse/ longitudinal polarizations
  - …
- SIDIS with polarized beam and target is a main process to study polarized TMDs
- Electron-positron annihilation is also important to constrain TMDs and to understand the role of spin in hadronization process
- Lattice QCD can provide complementary information to determine the distributions, especially when lacking precise data
- Opportunities from existing experiments at JLab12, BESIII, BelleII, and future facilities, EIC, EicC, STCF, to understand nucleon spin structures and fragmentation functions.

Thank you!

Backup

Imposing a Bound?

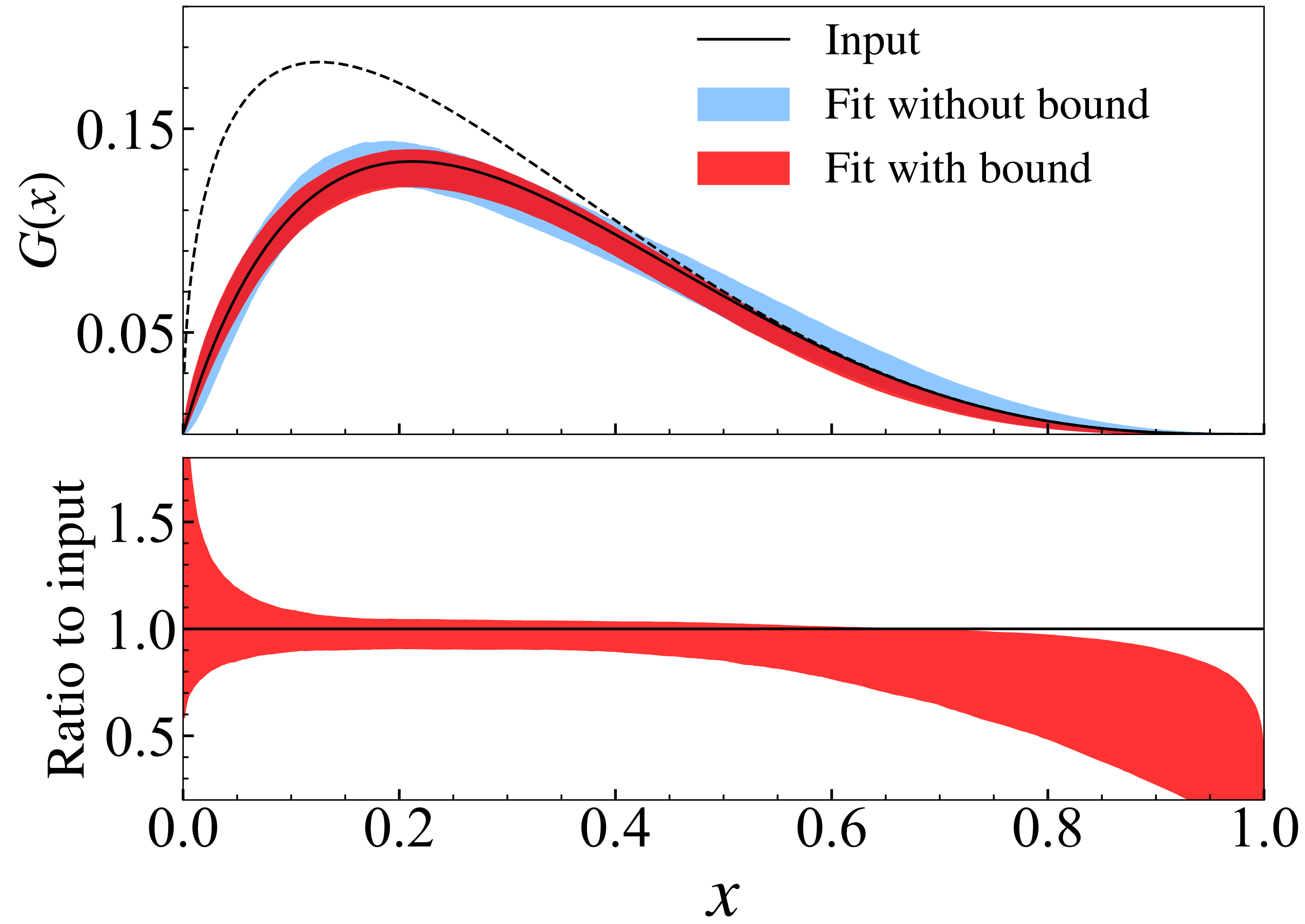
Question:

Whether we should impose the bound $|g_1(x)| \leq f_1(x)$ in the fit?
Similar situations in many other quantities.

A toy model test:

Input some "pol" function $G(x)$
and "unpol" function $F(x)$
Generate asymmetry "data"

Fit the "data" with or without
imposing the bound $G(x) \leq F(x)$



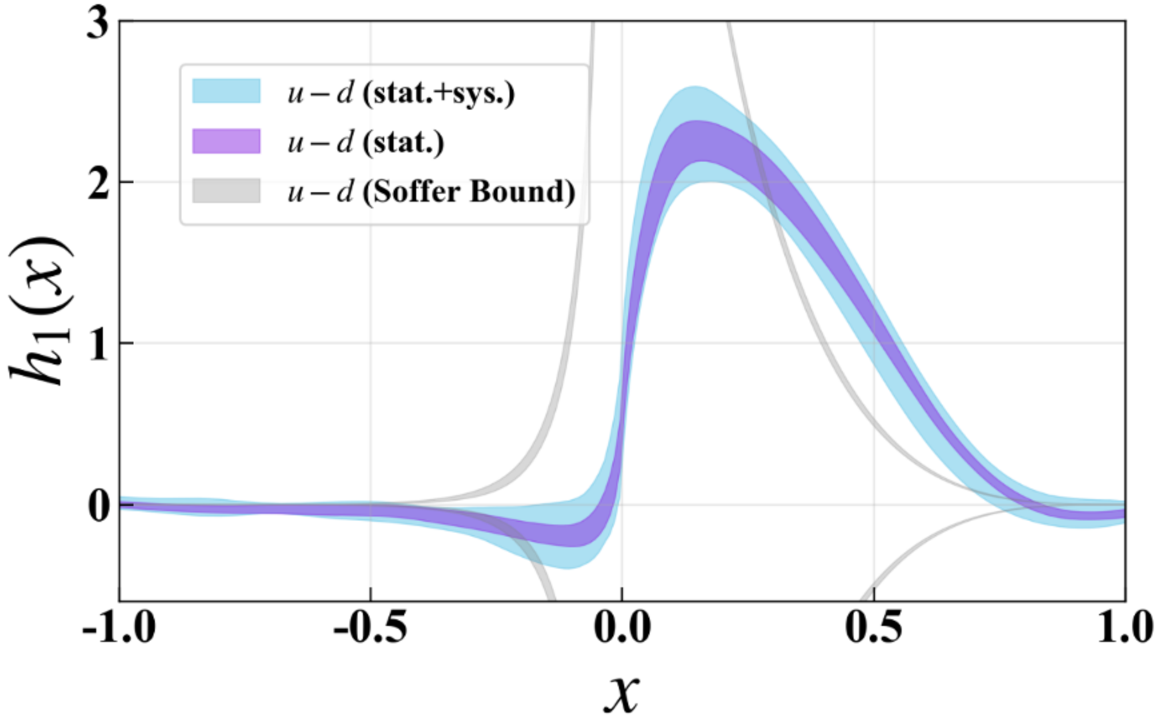
Imposing a bound in the fit may bias the fit result!

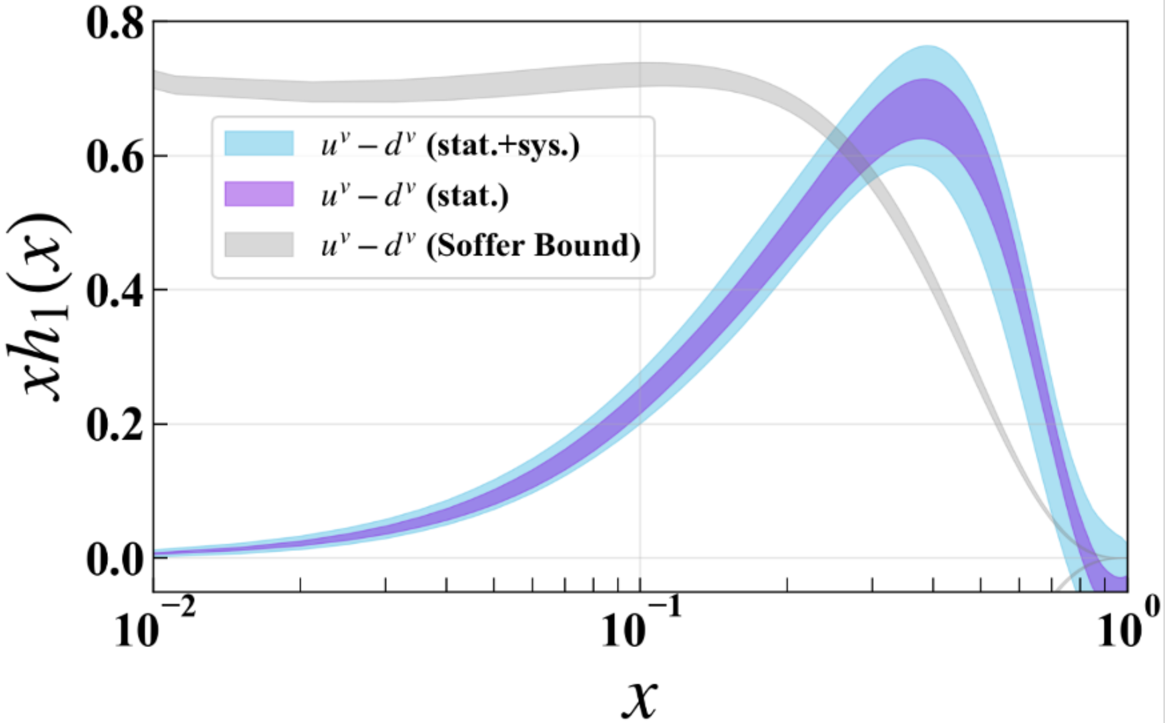
About Soffer's Bound

Soffer's bound on transversity distributions

$$|h_1(x)| \leq \frac{1}{2}[f_1(x) + g_1(x)]$$

It is broken in existing lattice QCD calculations

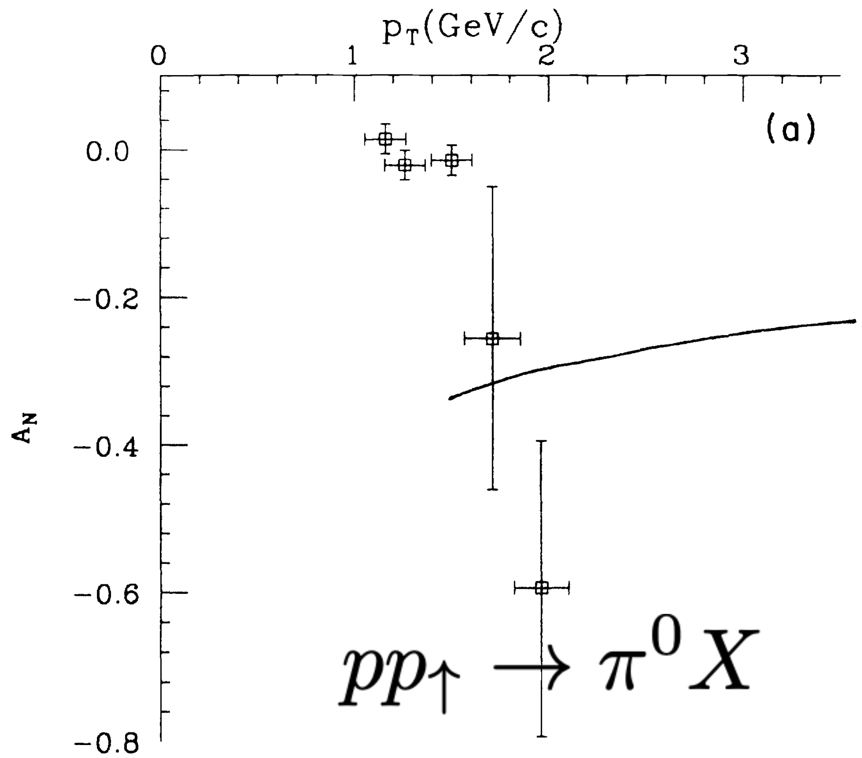




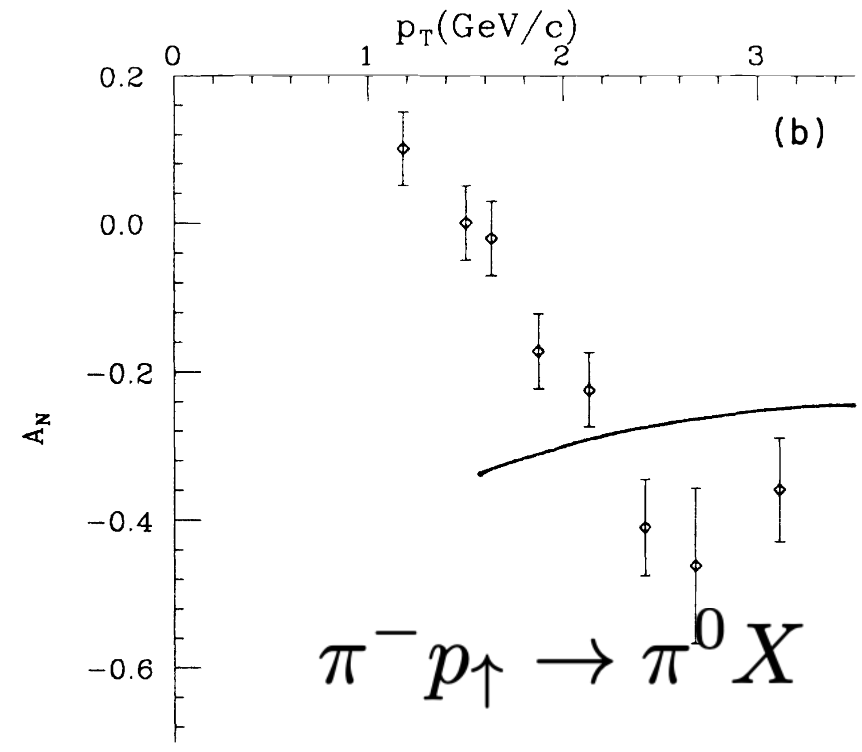
Lattice results from: LPC Collaboration, Phys. Rev. Lett. 131 (2023) 261901.

The Sivers Function: Early Story

Transverse single spin asymmetry observed in experiments



Data: J. Antille *et al*., Phys. Lett B94 (1980) 523.



Data: 7th Symposium on High Energy Spin Physics (1986).

D. Sivers proposed to explain such SSA a new distribution function

Sivers function $\Delta^N G_{a/p(\uparrow)}(x, \mathbf{k}_T; \mu^2)$ D. Sivers, Phys. Rev. D 41 (1990) 83.

However it was soon shown this function was T-odd and prohibited by QCD

J. Collins, Nucl. Phys. B 396 (1993) 161.

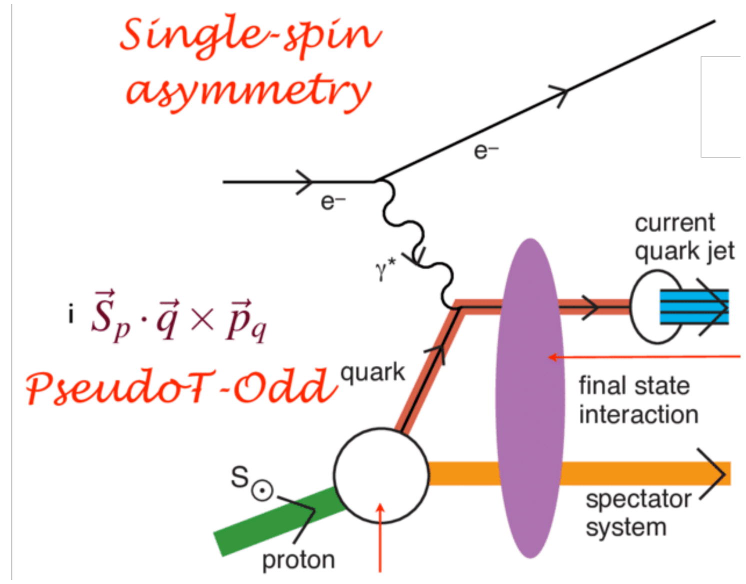
For the next decade, the "Sivers effect" was thought to vanish.

The Sivers Function: Early Story

Until an explicit model calculation showing …

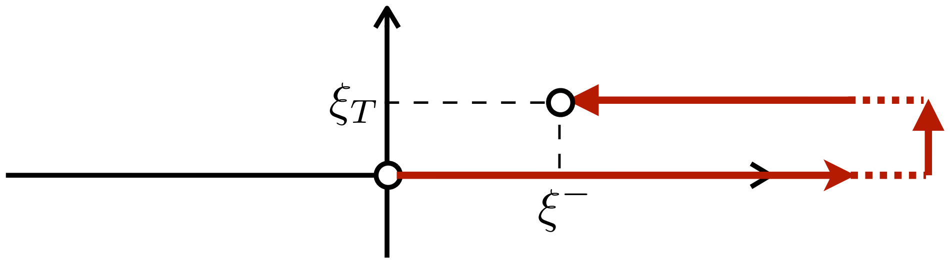
nonzero Sivers effects exist at leading twist
due to final-state interactions

S.J. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett. B 530 (2002) 99.



Sivers function can exist due to nontrivial gauge link

$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\,\mathcal{U}^{n_-}_{(0,+\infty)}\,\mathcal{U}^{n_-}_{(+\infty,\xi)}\,\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$



J.C. Collins, Phys. Lett. B 536 (2002) 43.

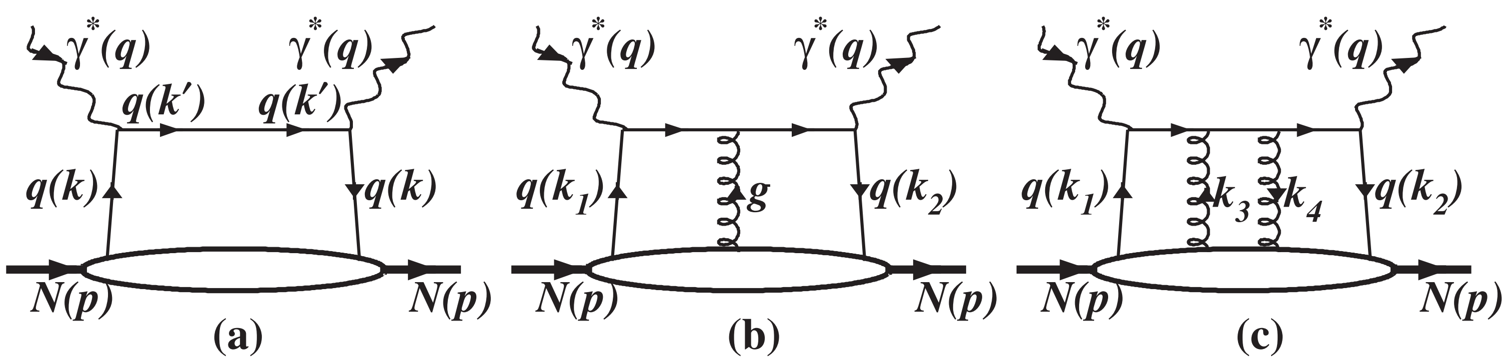
[Figure from A. Bacchetta]

This gauge link effect cannot be removed by choosing light-cone gauge $A^+ = 0$

X. Ji and F. Yuan, Phys. Lett. B 543 (2002) 66.

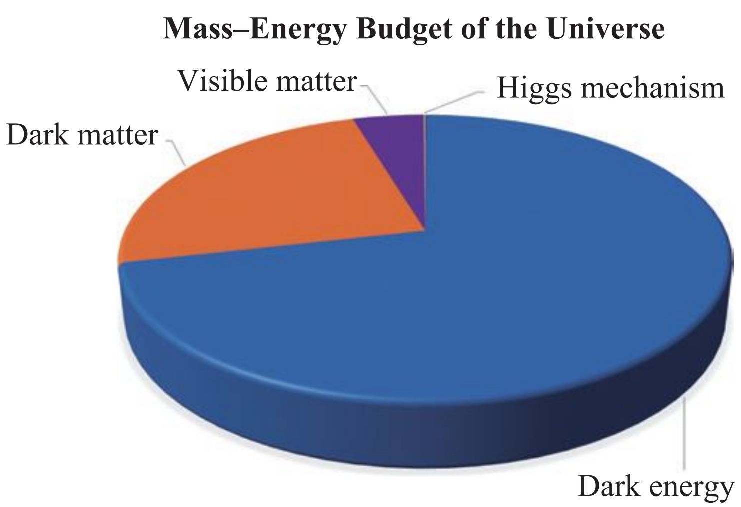
Collinear expansion

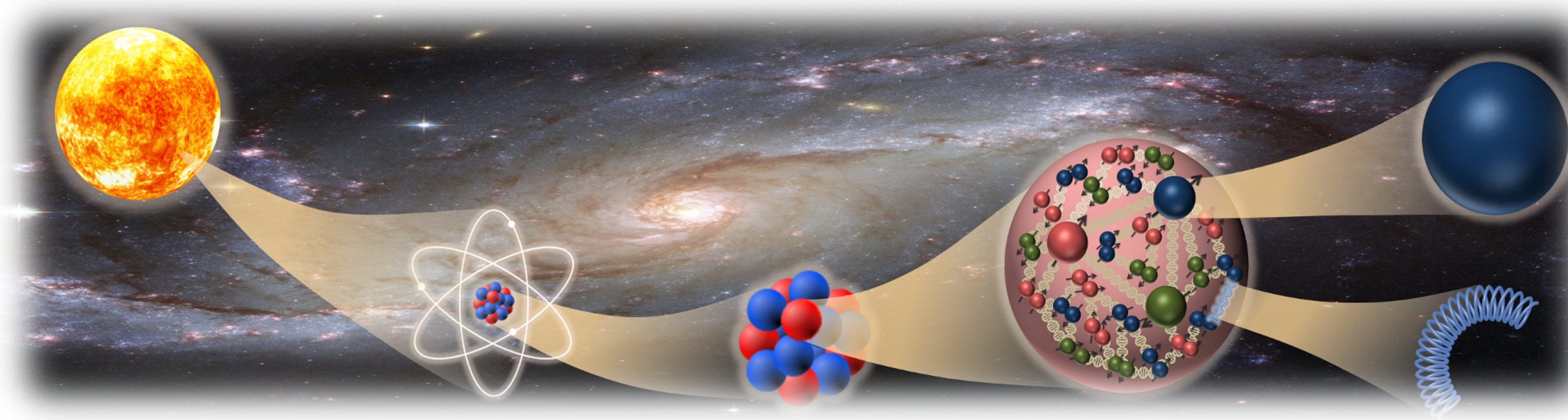
Z.T. Liang and X.N. Wang,
Phys. Rev. D 75 (2007) 094002.

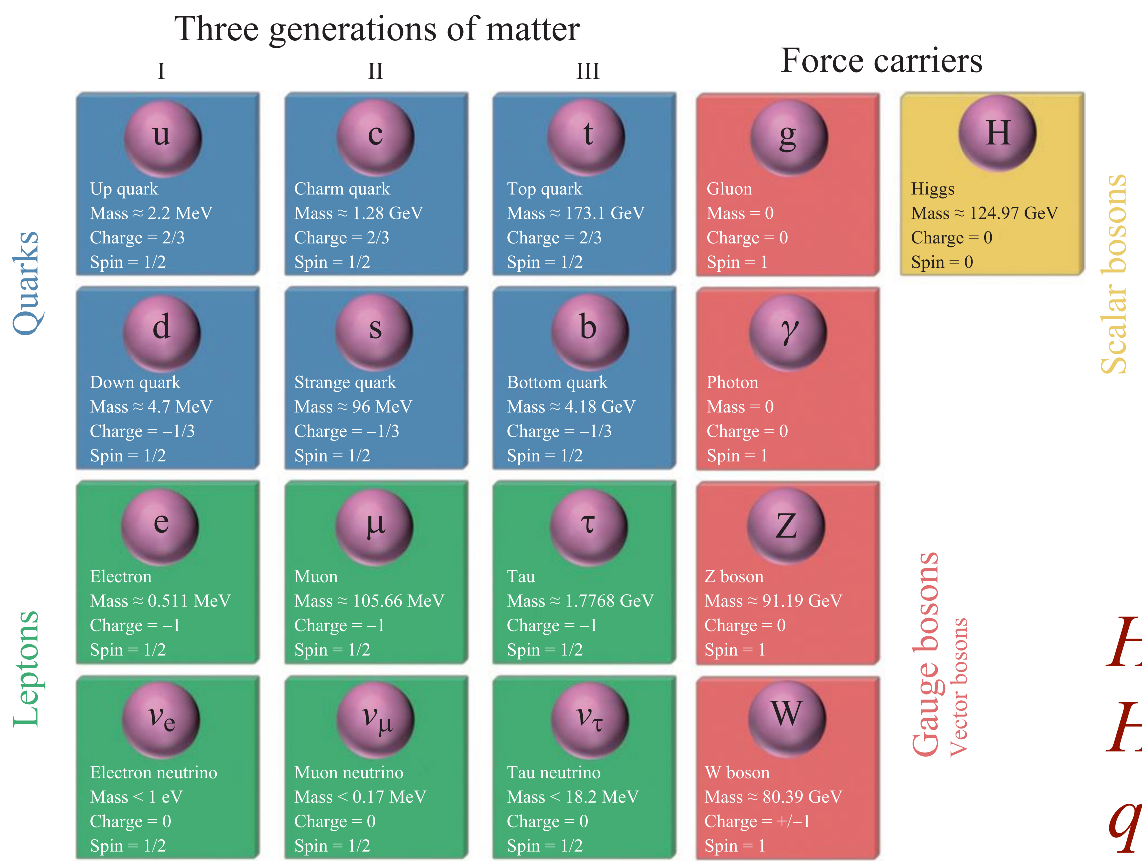


山东大学（青岛） SHANDONG UNIVERSITY, QINGDAO

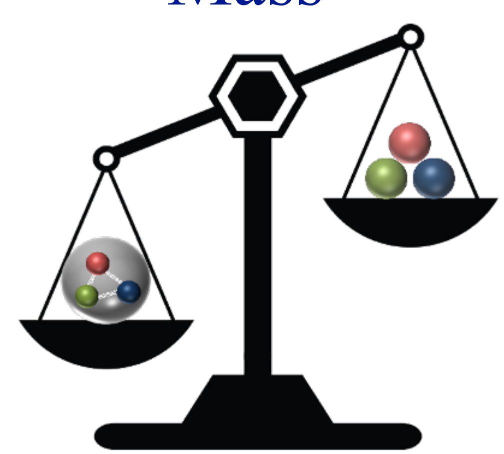
How much do we understand our world?



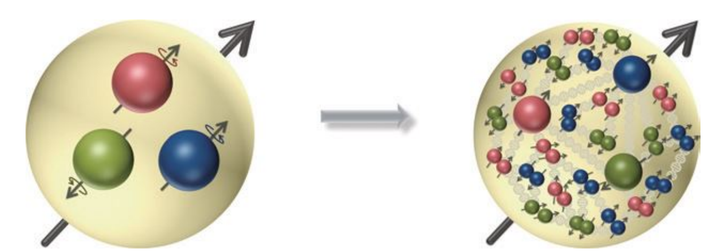




Mass



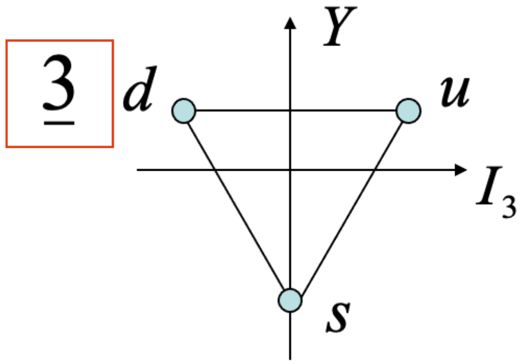
Spin

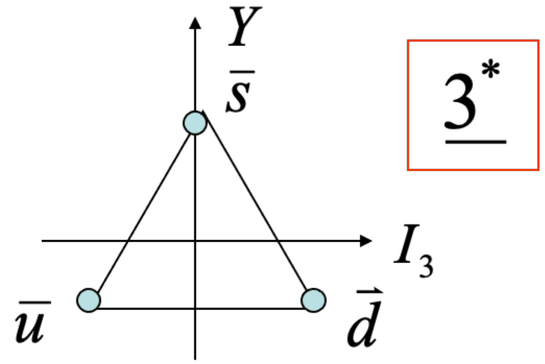


How do quarks and gluons make up a nucleon?
How can nucleon properties be explained at quarks and gluons degrees of freedom?

Proton Spin Structure in Naïve Quark Model

Quark model:

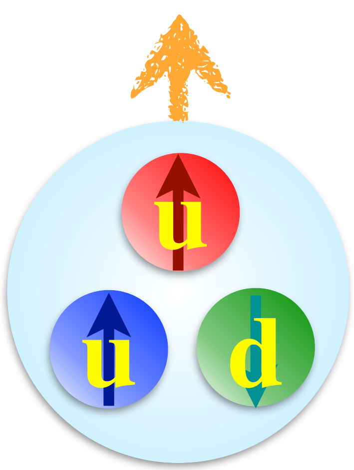




M. Gell-Mann, Phys. Lett. 8, 214 (1964);
G. Zweig, CERN Report No. TH-401 (1964).

ordinary baryons: $|qqq\rangle$, mesons: $|q\bar{q}\rangle$

Spin-flavor wave function of the proton:



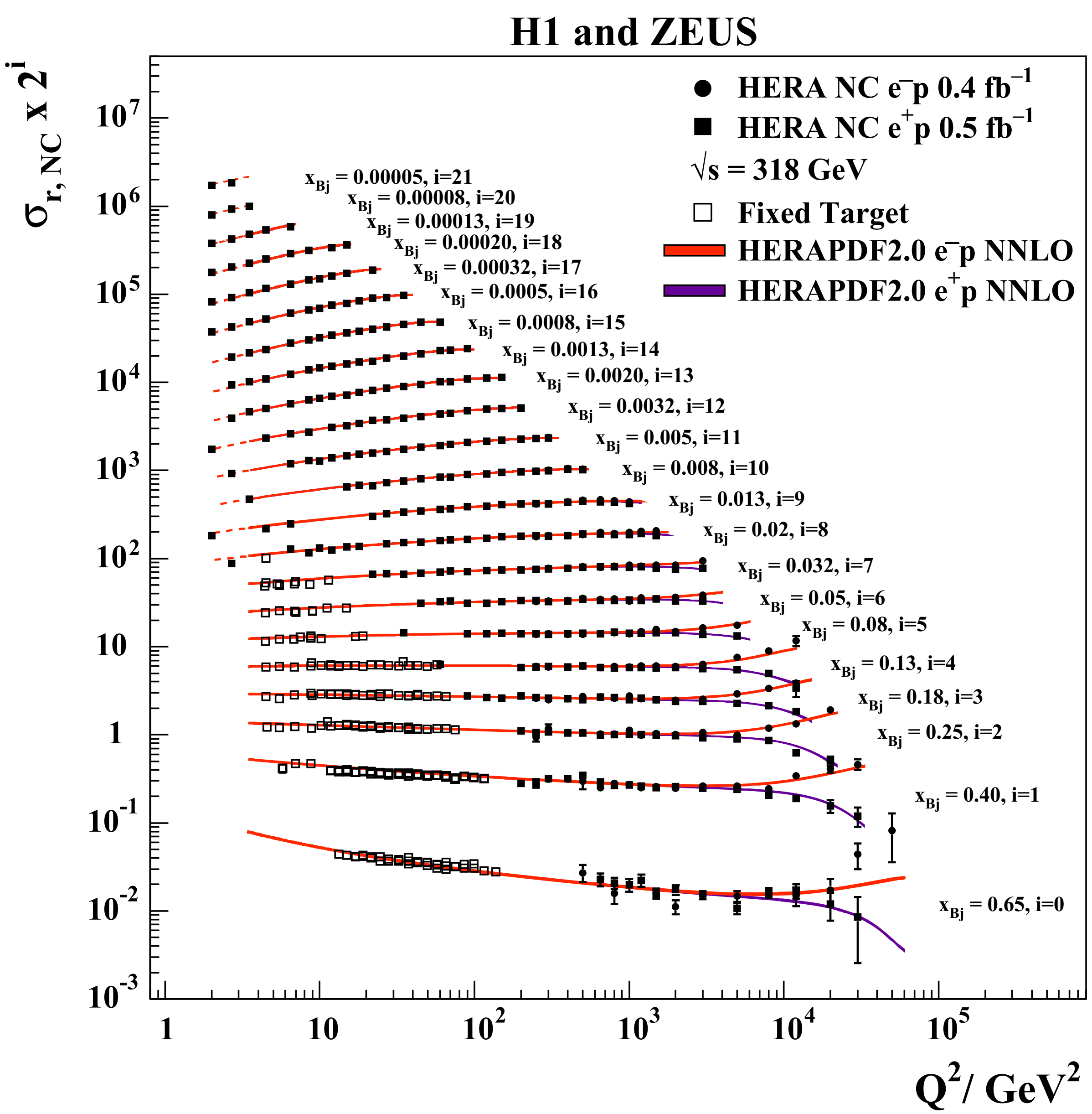
$$|p_\uparrow\rangle = \frac{1}{\sqrt{18}}\Big[2\,|u_\uparrow d_\downarrow u_\uparrow\rangle + 2\,|u_\uparrow u_\uparrow d_\downarrow\rangle + 2\,|d_\downarrow u_\uparrow u_\uparrow\rangle - |u_\uparrow u_\downarrow d_\uparrow\rangle$$

$$-|u_\uparrow d_\uparrow u_\downarrow\rangle - |u_\downarrow d_\uparrow u_\uparrow\rangle - |d_\uparrow u_\downarrow u_\uparrow\rangle - |d_\uparrow u_\uparrow u_\downarrow\rangle - |u_\downarrow u_\uparrow d_\uparrow\rangle\Big].$$

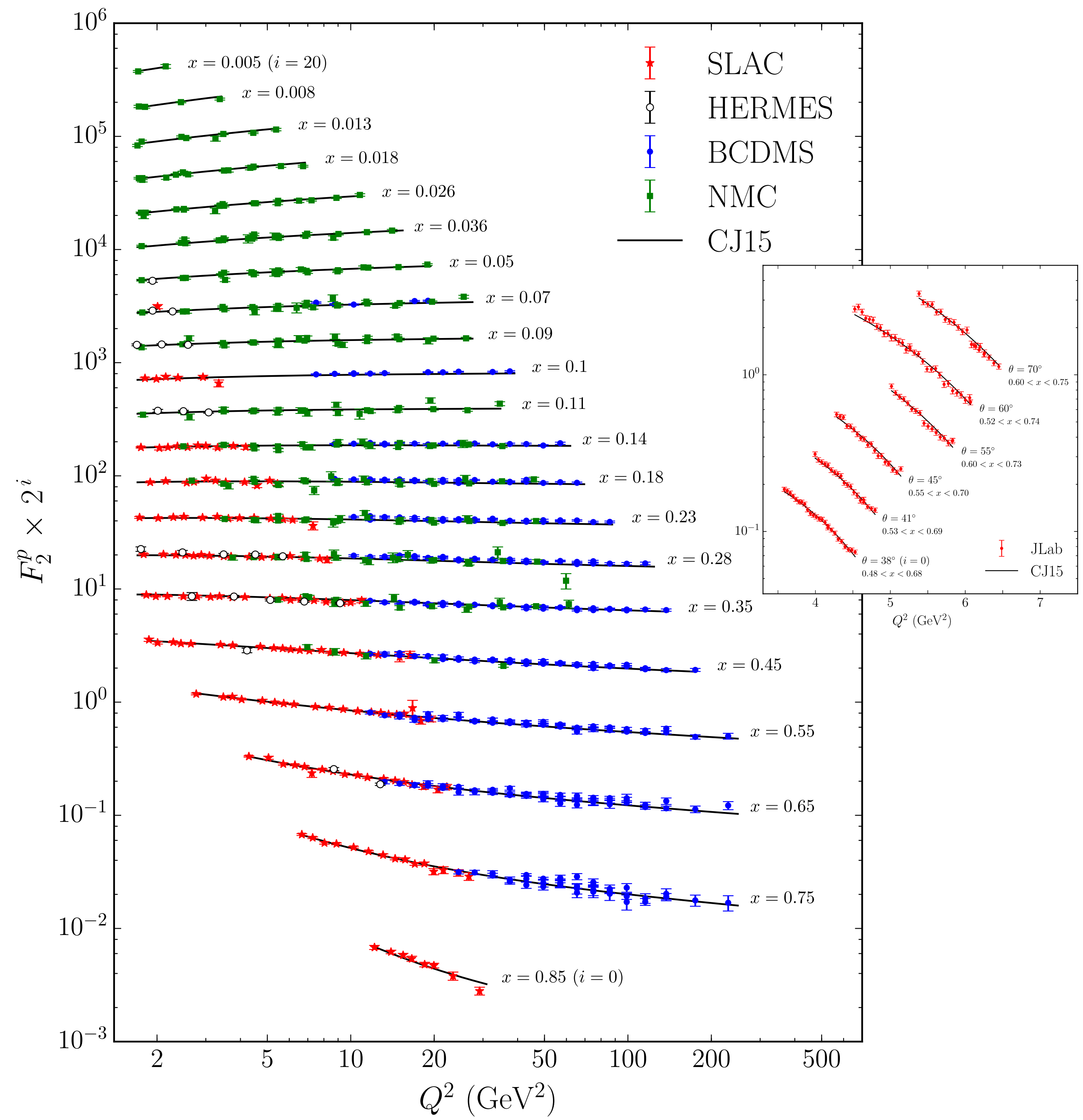
$$\Delta u = u_\uparrow - u_\downarrow = \frac{4}{3} \qquad \Delta d = d_\uparrow - d_\downarrow = -\frac{1}{3}$$

The sum of quark spins gives the proton spin.

Lepton-Hadron Deep Inelastic Scattering

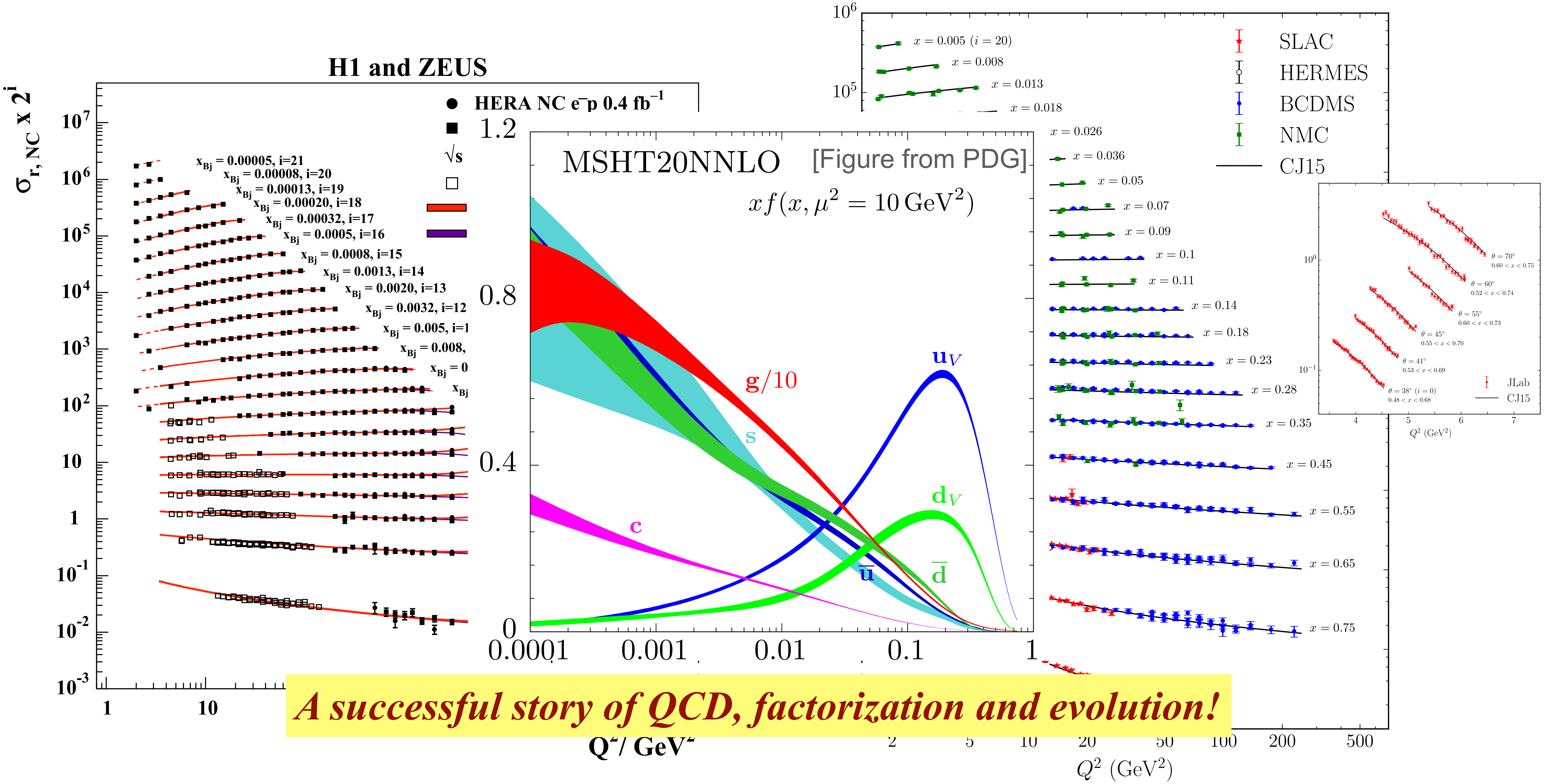


H. Abramowicz *et al.*, EPJC 78, 580 (2015).



A. Accardi *et al.*, PRD 93, 114017 (2016).

Lepton-Hadron Deep Inelastic Scattering



A successful story of QCD, factorization and evolution!

H. Abramowicz *et al.*, EPJC 78, 580 (2015).

A. Accardi *et al.*, PRD 93, 114017 (2016).