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Transverse momentum resummation for fully charm tetraquark production at hadron colliders

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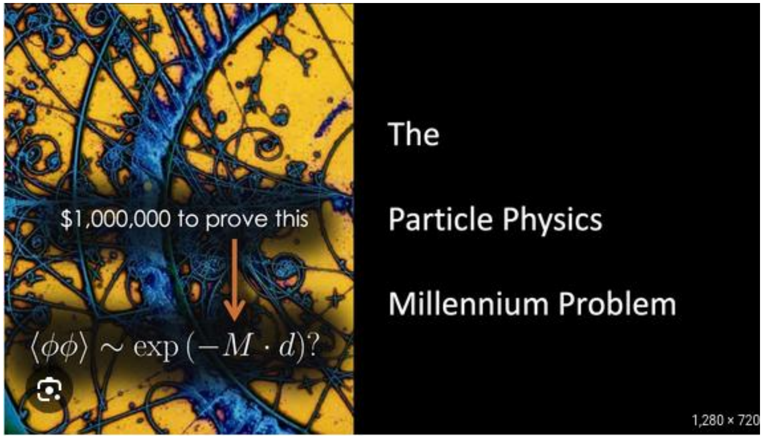
Color Confinement

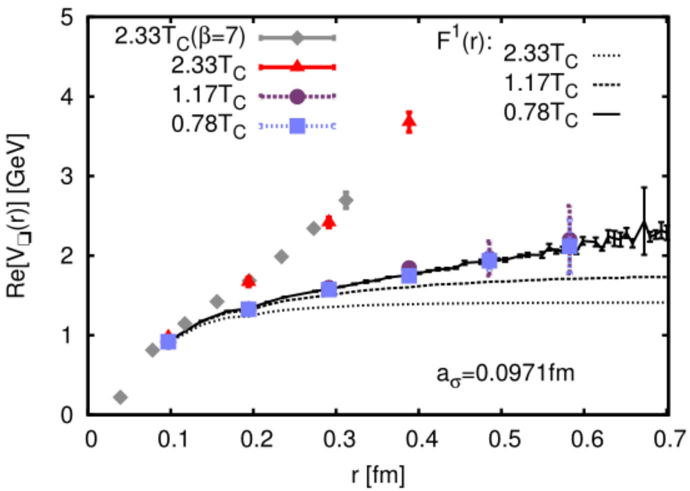
- **One of Seven Millennium Prize Problems**

- **QCD as fundamental theory of Strong interaction but not been fully understood**

  1)Quark confining potential is not derived analytically

  2)Heavy quark system provides a vivid picture from the spectrum to the confining mechanism; support from Lattice QCD

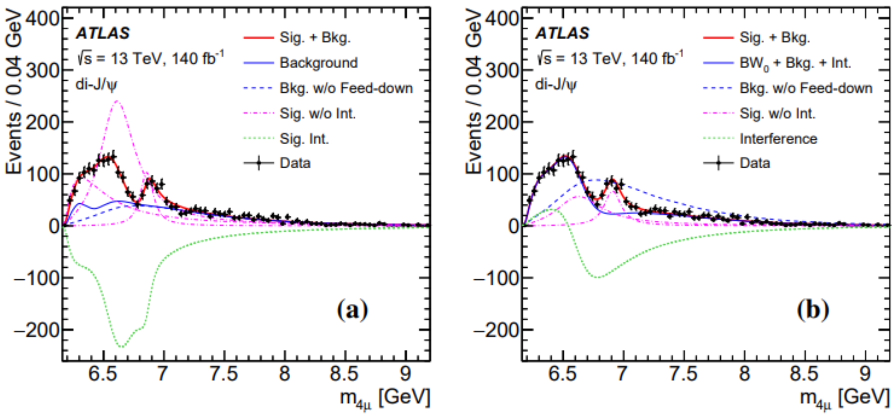




arXiv, 1108.1579

Fully charm tetraquark candidates

Exp.	Fit	M_{BW_1}	Γ_{BW_1}	$M_{X(6900)}$	$\Gamma_{X(6900)}$	M_{BW_3}	Γ_{BW_3}
LHCb	No interf.	–	–	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	–	–
CMS	No interf.	$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 184$	$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$
LHCb	Interf.	6741 ± 6	288 ± 16	$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	–	–
CMS	Interf.	6638^{+43+16}_{-38-31}	$440^{+230+110}_{-200-240}$	6847^{+44+48}_{-28-20}	191^{+66+25}_{-49-17}	7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}
ATLAS	Fit-A	$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$	$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$	$7220 \pm 30^{+10}_{-40}$	$90 \pm 60^{+60}_{-50}$
ATLAS	Fit-B	$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$	$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$	–	–



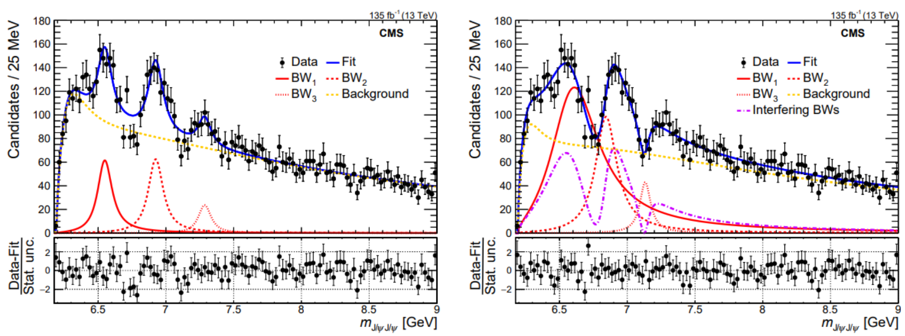
LHCb, 2006.16957; 9fb-1

rapidity range $2.0 < y^{J/\psi} < 4.5$

ATLAS, 2304.08962; 140fb-1 data;

P_t(mu1,2,3,4)>4,4,3,3GeV; |eta(mu1,2,3,4)|<2.5;

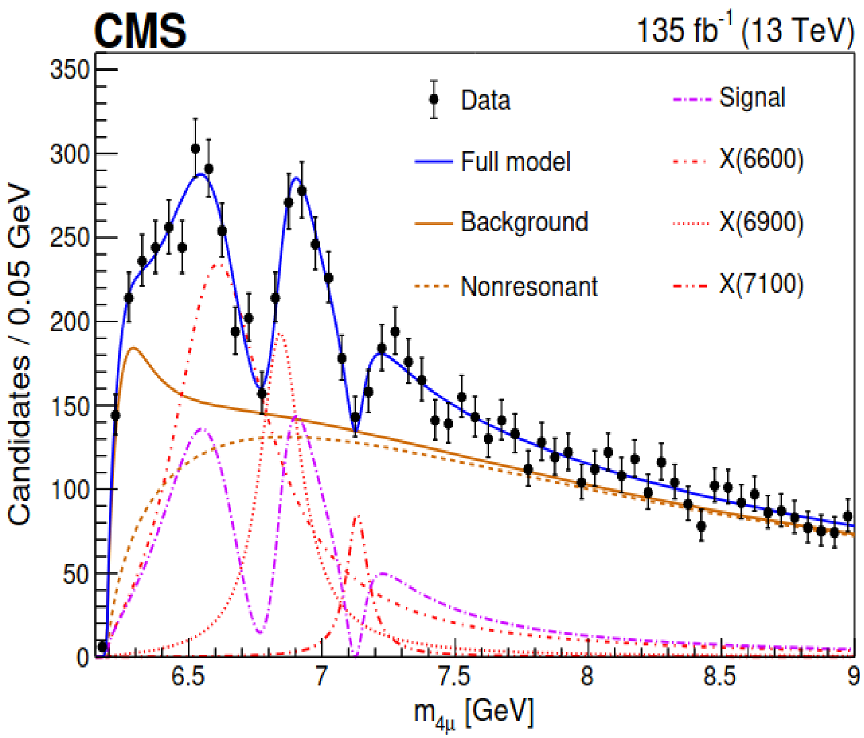
2.94(3.56)GeV<M(dimuon)<3.25(3.80)GeV

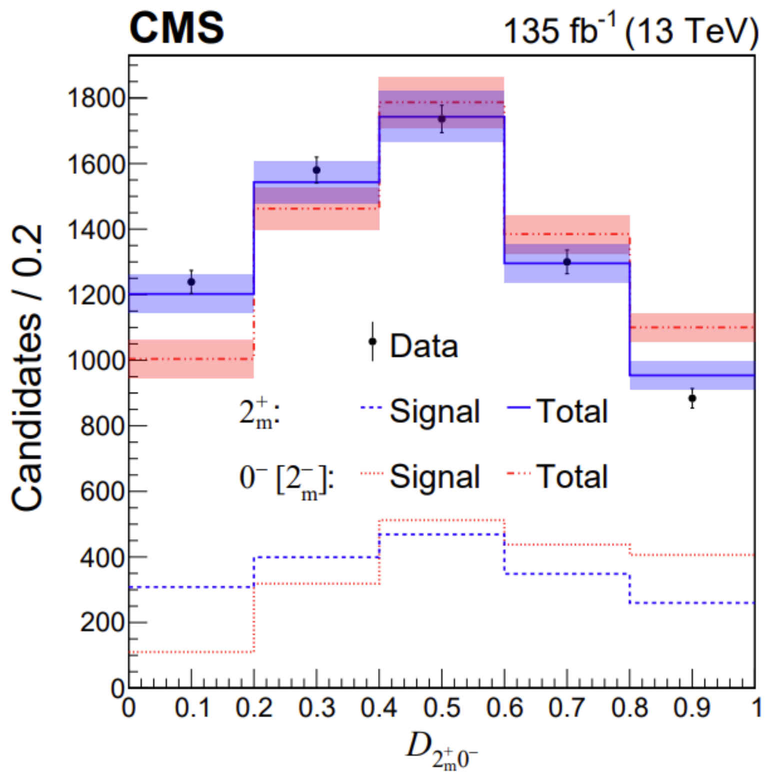


CMS, 2306.07164; 135fb-1 data;

P_t(muon)>2GeV;|eta(muon)|<2.4;

P_t(di muon)>3.5GeV;

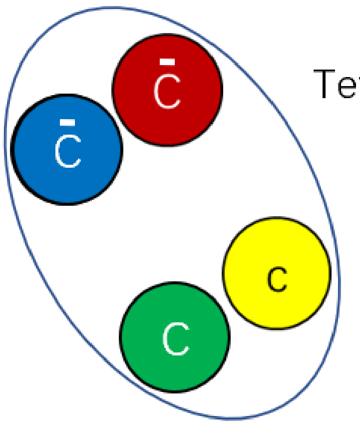




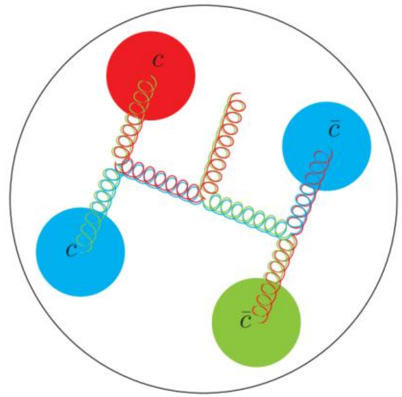
The spin-parity for three states is determined as 2^{++}

CMS,2506.07944 (Nature), also see in talks by Jingqing Zhang and Yilin Zhou and Liangliang Chen

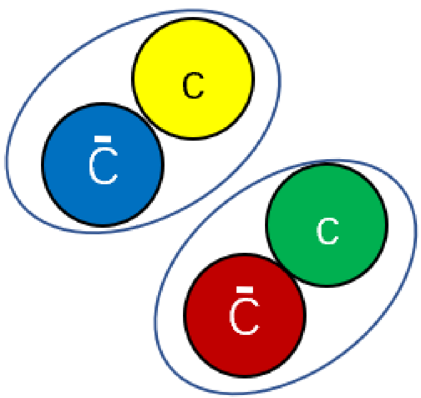
How to explain these exotic states?



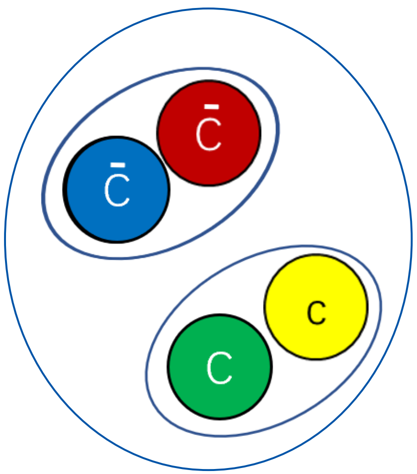
Tetraquark



Gluonic Tetracharm Hybrid

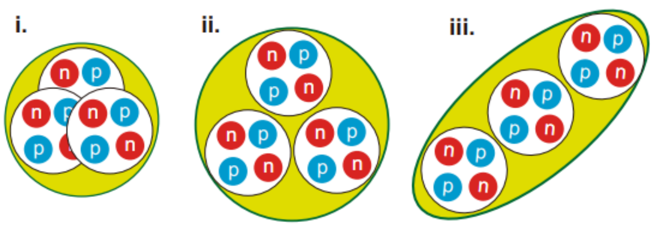


Charmonia Molecule



Diquark-antidiquark

c. Cluster model for C-12 nucleus



Similar to alpha cluster;
Different in diquark-antidiquark (color-confining) tetraquark

Previous theoretical studies

Y. Iwasaki, Phys. Rev. Lett. 36, 1266 (1976)
K. T. Chao, Z. Phys. C 7, 317 (1981)
J. P. Ader, J. M. Richard and P. Taxil, Phys. Rev. D 25, 2370 (1982)
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W. Chen, H. X. Chen, X. Liu, T. G. Steele and S. L. Zhu, Phys. Lett. B 773, 247-251 (2017)
Z. G. Wang, Eur. Phys. J. C 77, no.7, 432 (2017)
M. Karliner, S. Nussinov and J. L. Rosner, PRD 95, 034011 (2017)
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M. N. Anwar, J. Ferretti, F. K. Guo, E. Santopinto and B. S. Zou, Eur. Phys. J. C 78, no.8, 647 (2018)
M. S. Liu, Q. F. Lu, X. H. Zhong and Q. Zhao, PRD 100, 016006 (2019)
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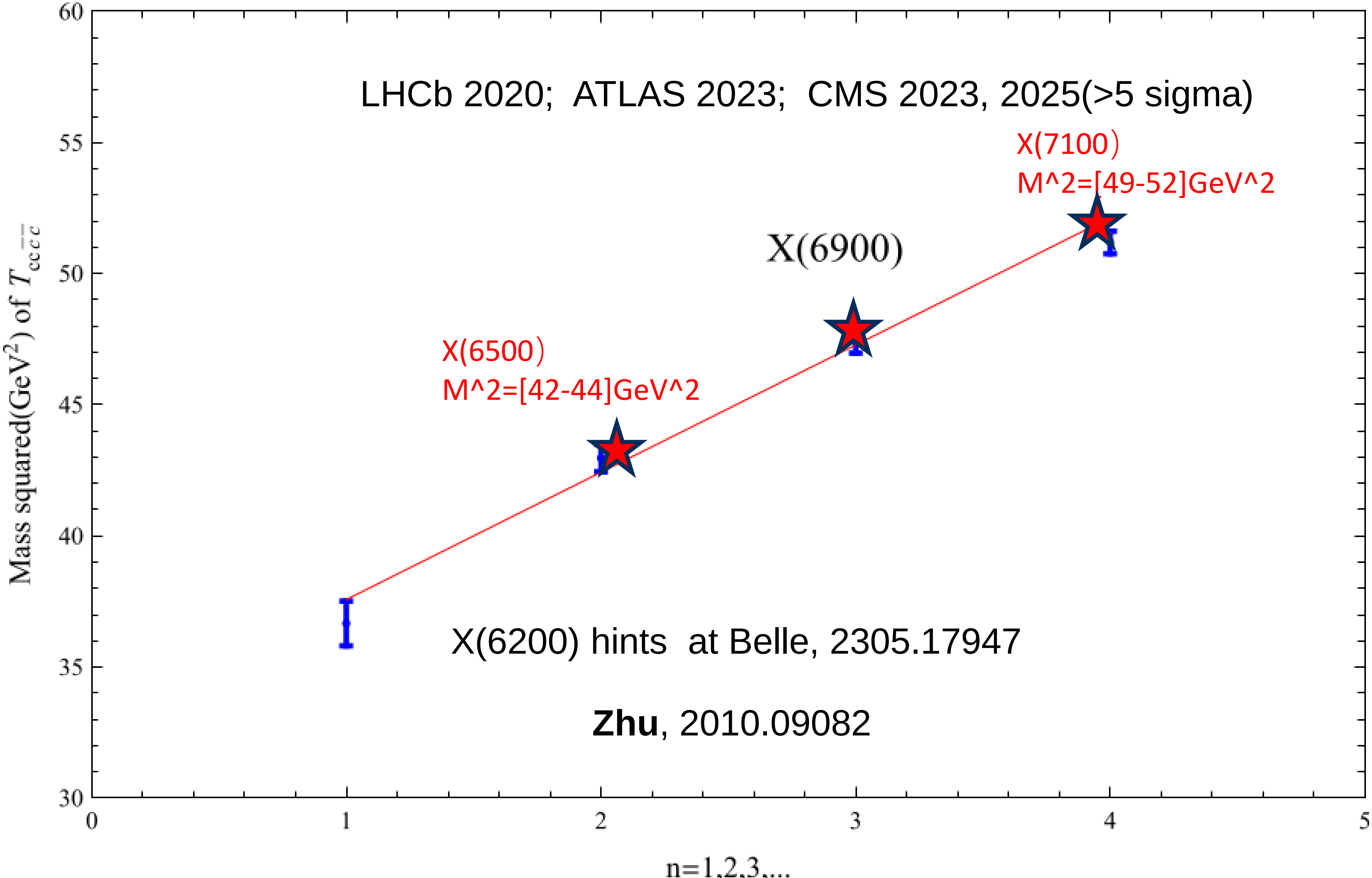
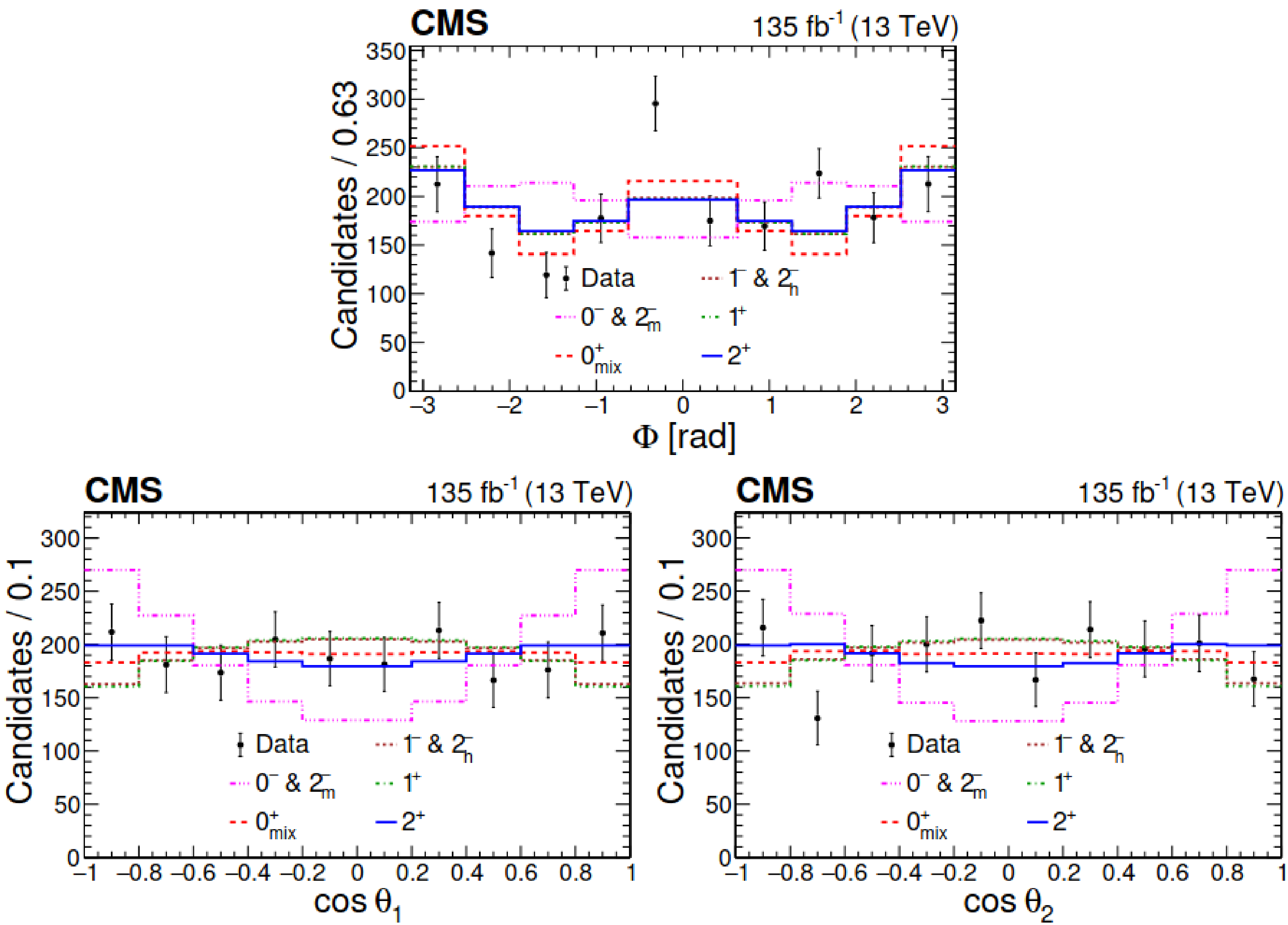
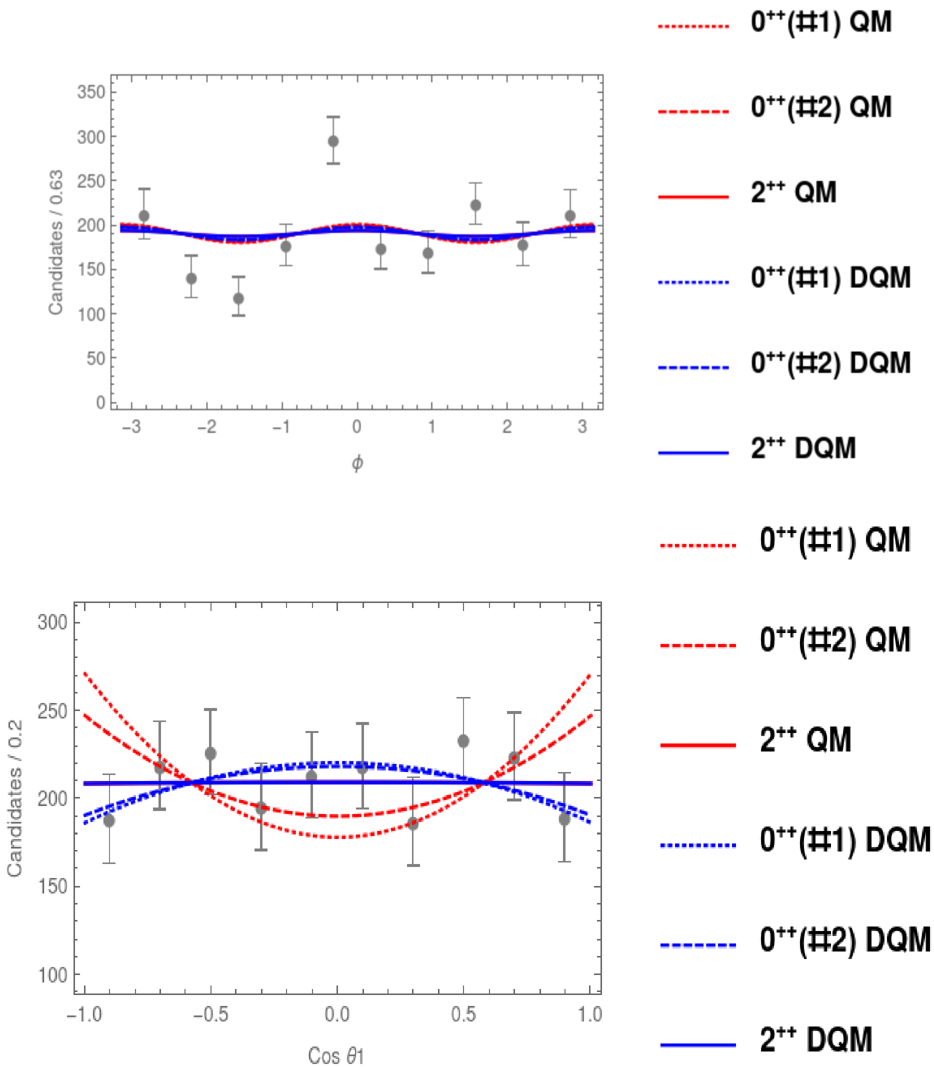
Feng, Huang, Jia,Sang, Xiong and Zhang,arXiv:2009.08450
Ma, Sang, Zhang, arXiv:2009.08376； **Zhu**,2010.09082
Zhuang, Zhang,Ma,Wang,2111.14028 ……
F. Feng, Y. Huang, Y. Jia, W. L. Sang, X. Xiong and J. Y.Zhang, arXiv:2009.08450; F. Feng et al, 2304.11142

Fully charm tetraquark family: Linear Regge trajectories?





CMS data supports spin-parity 2^++;
However, derivation from the data and MC

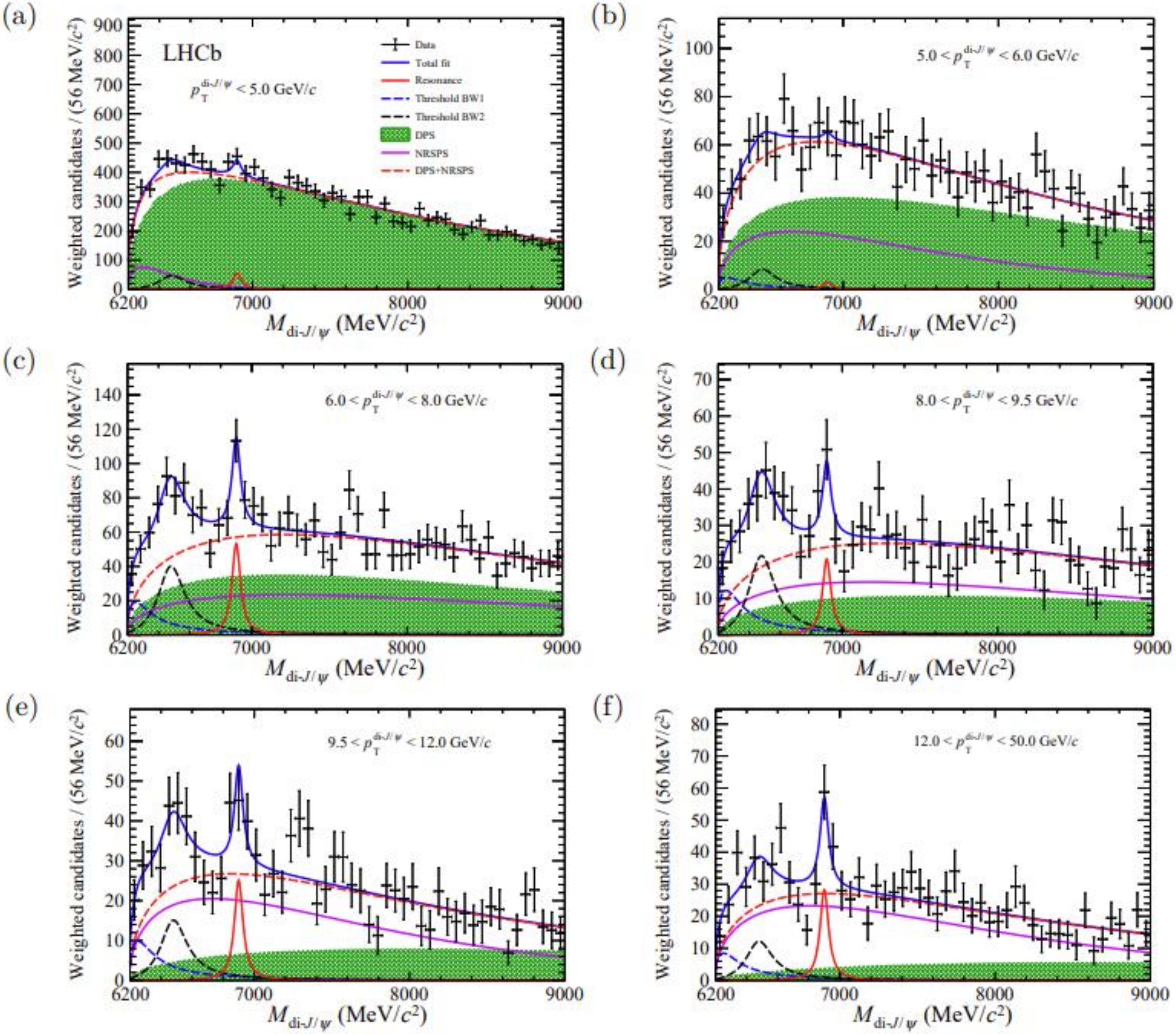


$$\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\Phi}$$

$$= \frac{9P_H}{256\pi^2 M_T^2}\Big\{h_{00}^{00}\sin^2\theta_1\sin^2\theta_2 + \frac{1}{2}h_{11}^{11}(1+\cos^2\theta_1)(1+\cos^2\theta_2) + \frac{1}{8}h_{11}^{11}\sin^2\theta_1\sin^2\theta_2\cos 2\Phi + \frac{1}{4}h_{00}^{11}\sin 2\theta_1\sin 2\theta_2\cos\Phi\Big\}. \quad (8)$$

the derivation appears from the data and (diquark)quark model

Chen-Liu-Zhao-Zhong-**Zhu**-Zou:2412.13455



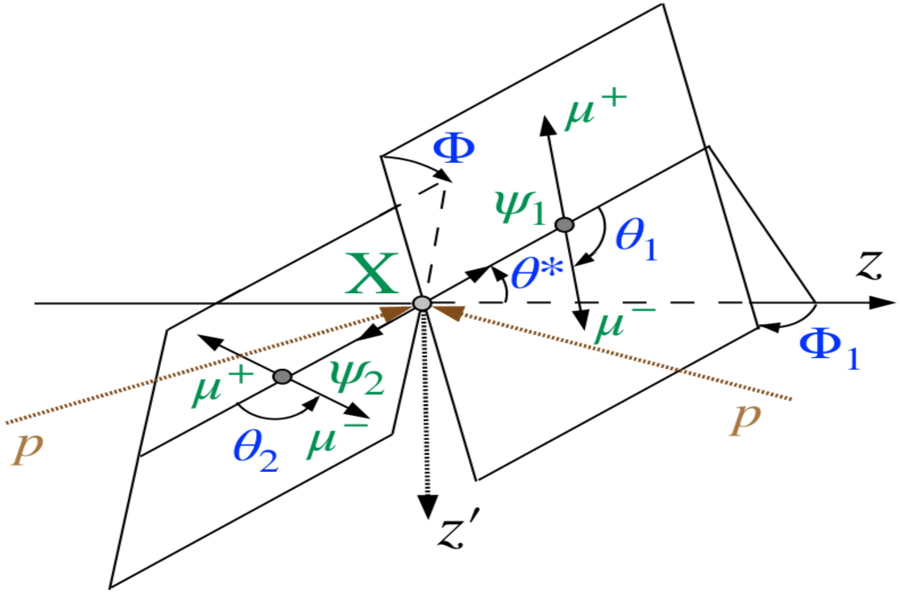
Significance	
$p_T^{di-J/\psi}$-threshold	$p_T^{di-J/\psi}$-binned
3.4 σ	6.0 σ
6.4 σ	6.9 σ
6.0 σ	6.5 σ
5.1 σ	5.4 σ

the signal significance is different for different transverse momentum

Outline

- Transverse momentum resummation formulae for fully charm tetraquarks production at hadron-hadron colliders
- Extraction of LDMEs and comparison with experimental data
- Summary and Outlook

(1)Transverse momentum resummation formulae for fully charm tetraquarks production at hadron-hadron colliders



Hadronic production cross-section

QCD factorization:

$$\sigma\left(p(K_1)+p(K_2)\to T_{4c}(P)+X\right)=\sum_{ij}\int_0^1 dx_1\int_0^1 dx_2 f_{i/p}(x_1,\mu_F)f_{j/p}(x_2,\mu_F)$$
$$\times\,\hat{\sigma}_{i+j\to T_{4c}+X}(\mu_R,\mu_F,x_1,x_2,\hat{s}=x_1x_2S),$$

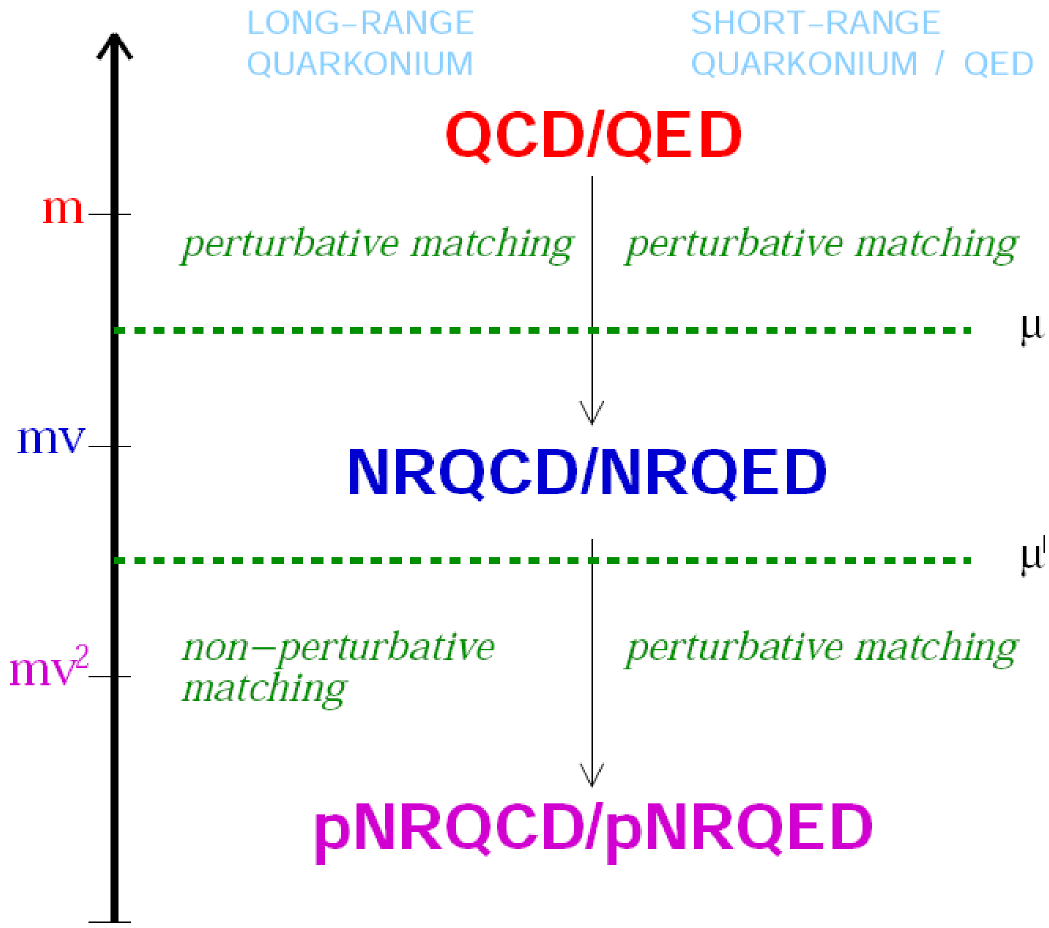
Partonic processes:

LO+NLO virtual: $g+g\to T_{4c}$ and $q+\bar{q}\to T_{4c}$.

NLO real: $g+g\to T_{4c}+g,\ q+\bar{q}\to T_{4c}+g$

$q+g\to T_{4c}+q,\ \bar{q}+g\to T_{4c}+\bar{q}$

NRQCD/pNRQCD factorization



$$\alpha_s(mv) \sim v$$

$v^2 \approx 0.1$ for the Υ

Bodwin-Braaten-Lapage
1995

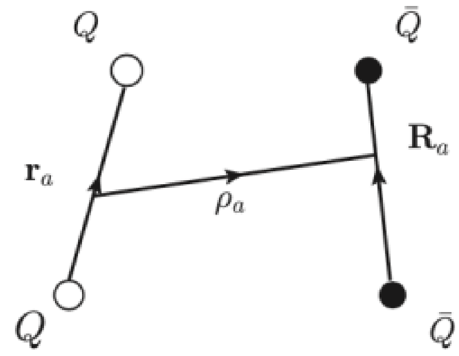
Pineda-Soto-Brambilla-Vairo
2000

Quark configurations from symmetry

Color-antisymmetry-symmetry basis:

$$\bar{\mathbf{3}} \otimes \mathbf{3} \to \mathbf{1} + \cdots \text{ and } \mathbf{6} \otimes \bar{\mathbf{6}} \to \mathbf{1} + \cdots .$$

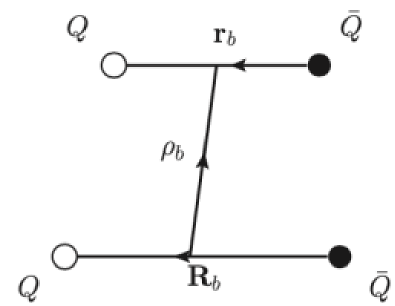
$$|\bar{\mathbf{3}}, \mathbf{3}\rangle = |(Q_1 Q_2)_{\bar{\mathbf{3}}}; (\bar{Q}_3 \bar{Q}_4)_{\mathbf{3}}\rangle,$$
$$|\mathbf{6}, \bar{\mathbf{6}}\rangle = |(Q_1 Q_2)_{\mathbf{6}}; (\bar{Q}_3 \bar{Q}_4)_{\bar{\mathbf{6}}}\rangle,$$



Color-singlet-octet basis:

$$\mathbf{1} \otimes \mathbf{1} \to \mathbf{1} + \cdots \text{ and } \mathbf{8} \otimes \mathbf{8} \to \mathbf{1} + \cdots .$$

$$|\mathbf{1}, \mathbf{1}\rangle = |(Q_1 \bar{Q}_3)_{\mathbf{1}}; (Q_2 \bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{8}, \mathbf{8}\rangle = |(Q_1 \bar{Q}_3)_{\mathbf{8}}; (Q_2 \bar{Q}_4)_{\mathbf{8}}\rangle,$$



1)They are equivalent according to Fierz Transformation when deal with local observables.

2)Heavy diquark is very different to light diquark(isospin symmetry):
good diquark is 1^+ for heavy sector but 0^+ for light sector

Amplitudes calculation in NRQCD

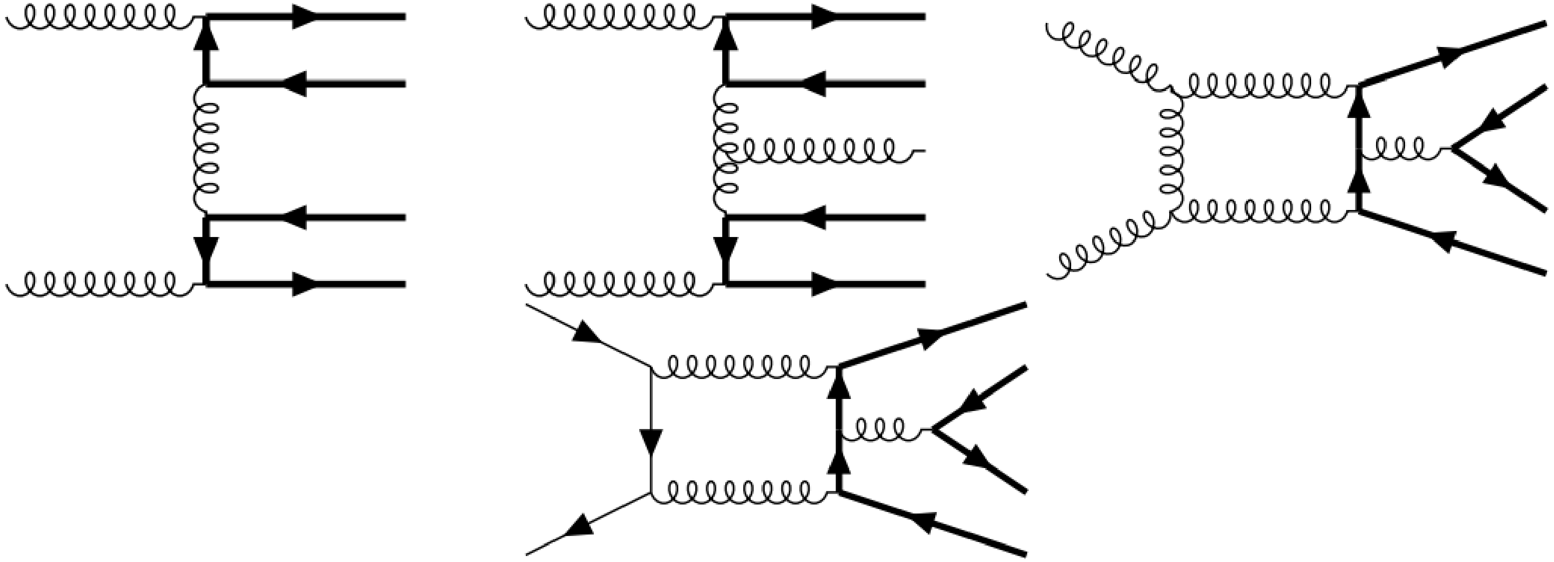
$$
\begin{aligned}
\mathcal{A} &= \sum_n \langle T_{4c} + X | c\bar{c}c\bar{c}[n] \rangle \langle c\bar{c}c\bar{c}[n] | \overline{\psi}_i \psi_j \overline{\psi}_k \psi_l | 0 \rangle \mathcal{T}_{ijkl} \\
&\equiv \sum_n \langle T_{4c} + X | c\bar{c}c\bar{c}[n] \rangle \mathrm{Tr}\left[\mathcal{T} \mathcal{P}^{(n)} \right].
\end{aligned}
$$

$$
\begin{aligned}
|\mathcal{A}|^2 &= \sum_X | \sum_n \langle T_{4c} + X | c\bar{c}c\bar{c}[n] \rangle |^2 | \mathrm{Tr}\left[\mathcal{T} \mathcal{P}^{(n)} \right] |^2 \\
&= \sum_{m,m'} \langle 0 | O^{4c}_{m,n'} | 0 \rangle \left| \mathcal{M}^m \mathcal{M}^{m'\,\dagger} \right|,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} &= -\frac{1}{\sqrt{3}} \left[\psi_a^T \left(i\sigma^2 \right) \sigma^i \psi_b \right] \left[\chi_c^\dagger \sigma^i \left(i\sigma^2 \right) \chi_d^* \right] \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}, \\
\mathcal{O}^{(2;ij)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} &= \left[\psi_a^T \left(i\sigma^2 \right) \sigma^m \psi_b \right] \left[\chi_c^\dagger \sigma^n \left(i\sigma^2 \right) \chi_d^* \right] \Gamma^{ij;mn} \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}, \\
\mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} &= \left[\psi_a^T \left(i\sigma^2 \right) \psi_b \right] \left[\chi_c^\dagger \left(i\sigma^2 \right) \chi_d^* \right] \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}},
\end{aligned}
$$

Feng, Huang, Jia, Sang, Xiong and Zhang, arXiv:2009.08450;
Zhu, 2010.09082

Complete NLO calculation



LO:64(gg)，4(qqbar)

NLO Virtual:2008(gg)，170(qqbar)

NLO Real: 618(gg)， 98(qqbar, qg)

Exact IR cancellation and no additional renormalization at NLO

$$K^{\rm LH3}_{\rm gg,virtual} = \frac{3}{\epsilon^2} - \frac{1}{\epsilon}\left(3\log\left(\frac{\mu}{4m_c}\right)^2 - \frac{n_l}{3} + \frac{11}{2}\right) - \frac{3}{2}\log^2\left(\frac{\mu}{4m_c}\right)^2$$
$$+\left(\frac{11}{2} - \frac{2n_h+n_l}{3}\right)\log\left(\frac{\mu}{m_c}\right)^2 + \frac{4719}{256}\left(\mathrm{Li}_2\left(2\sqrt{2}-2\right) + \mathrm{Li}_2\left(-2\sqrt{2}-2\right)\right)$$
$$+\cdots$$

There are soft divergences when $k_g = 0$ or $z = 1$, and collinear divergences when $y_\theta = \cos\theta_{k_n k_g} = \pm 1$ with $k_n = k_1, k_2, k_T$.

$$\hat{\sigma}_{\rm soft}\ (i+j\to T_{4c}+k) = -C\frac{1}{2\epsilon_{\rm IR}}\delta(1-z)\frac{4^{-\epsilon}\Gamma(1-\epsilon)\Gamma(-\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{B}_{ij}\left(z=1,y_\theta\right)$$
$$=\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^{\epsilon}\frac{\alpha_s}{\pi}\hat{\sigma}^{(0)}_{ij}\delta(1-z)\left(\frac{1}{\epsilon_{\rm IR}^2} - \frac{\pi^2}{3}\right)C_{ij}$$

$$\hat{\sigma}_{\text{hard col. } y_\theta=\pm1}\ (i+j\to T_{4c}+k)$$
$$= -C\frac{4^{-\epsilon}}{2\epsilon_{\rm IR}}\left[\left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\log(1-z)}{1-z}\right)_+\right]\mathcal{B}_{ij}\left(z,y_\theta=\pm1\right)$$
$$= -C\frac{4^{-\epsilon}}{2\epsilon_{\rm IR}}\left[\left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\log(1-z)}{1-z}\right)_+\right]\mathcal{B}_{ij}\left(z=1,y_\theta\right)\frac{b_{ij}^{\rm collinear}}{z^3}$$
$$=\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^{\epsilon}\frac{\alpha_s}{\pi}\hat{\sigma}^{(0)}_{ij}\delta(1-z)\left[-2C_{ij}b_{ij}^{\rm collinear}\left(\left(\frac{1}{1-z}\right)_+\frac{1}{\epsilon_{\rm IR}} - 2\left(\frac{\log(1-z)}{1-z}\right)_+\right)\right]$$

$$\hat{\sigma}^{\rm AP-CT} = \frac{1}{\epsilon}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu_R^2}{\mu_F^2}\right)^{\epsilon}\Gamma(1+\epsilon)\hat{\sigma}^{(0)}zP_{ij}(z),$$

The renormalization constant for four heavy quarks in a color singlet is 1 at NLO

Soft and collinear gluon radiation produces singularities

$$
\begin{aligned}
\frac{d\sigma}{dydP_\perp^2}\Big|^{NLO,R}_{Singular} &= \left(\frac{d\sigma}{dydP_\perp^2}\Big|^{NLO,R}\right)_{P_\perp\to 0} \\
&= \int\frac{dx_1}{x_1}\int\frac{dx_2}{x_2} f_{i'}(x_1,\mu_F)f_{j'}(x_2,\mu_F)\frac{M^2}{S}\hat{\sigma}^{(0)}_{ij}\frac{\alpha_s(\mu_F)}{2\pi} \\
&\times\frac{1}{P_\perp^2}\Big[P_{jj'}(\xi_2)\delta_{ii'}\delta(1-\xi_1)+P_{i'i}(\xi_1)\delta_{j'j}\delta(1-\xi_2) \\
&-2C_{i'j'}\delta_{i'i}\delta_{j'j}\ln\left(\frac{P_\perp^2}{M^2}\right)\delta(1-\xi_1)\delta(1-\xi_2)\Big].
\end{aligned}
$$

At low transverse momentum P_\perp, the soft and collinear gluon radiations generate the divergence, which should be resumed for a reliable prediction.

Transverse momentum resummation in various processes

$$\frac{d\sigma}{dM^2 dy dp_t^2} = \frac{8}{27}\frac{\alpha^2\alpha_s}{M^2}\frac{1}{p_T^2}\int_{x_a^{min}}^{1} dx_a P^{DIS}\frac{1}{x_a - x_1}\left(1 + \frac{\tau^2}{(x_a x_b)^2} - \frac{x_T^2}{2x_a x_b}\right)$$

- with

$$x_a^{min} = \frac{x_a x_2 - \tau}{x_a - x_1}$$

$$p_t^2 = \frac{\hat{t}\hat{u}}{\hat{s}}$$

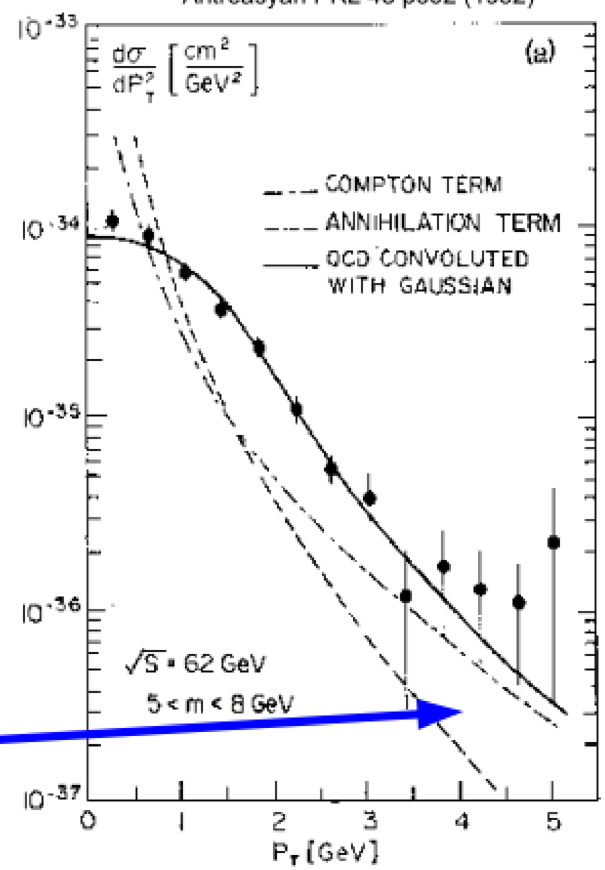
$$x_t = \frac{2p_t}{\sqrt{s}}$$

$$P^{DIS} = \sum e_q^2 \left(q_i(x_a, Q^2)\bar{q}_i(x_b, Q^2) + \bar{q}_i(x_a, Q^2) q_i(x_b, Q^2)\right)$$

- large pt

R. Field, Appl. of pQCD, p195 ff

Antreasyan PRL 48 p302 (1982)



Drell-Yan, vector boson production, DIS processes, Higgs production,…
Collins, Soper 81; Collins, Soper, Sterman 85
CP Yuan 1992; Ji-Ma-Yuan 2004; CS Li, HX Zhu, DY Shao, P Sun, Gao, Wang, Cao, Liu, Li, Yan, et al, 2010s,…….

Fourier transformation to impact-parameter b space

$$W(b)|^{NLO,R}_{Singular} = \int d^2P_\perp e^{-i\overrightarrow{P_\perp}\cdot\vec{b}} \frac{d\sigma}{dyd^2P_\perp}|^{NLO,R}_{Singular}$$

$$= \int_{x_A}^{1} \frac{d\xi_1}{\xi_1} \int_{x_B}^{1} \frac{d\xi_2}{\xi_2} f_{i'}(\frac{x_A}{\xi_1}, \mu_F) f_{j'}(\frac{x_B}{\xi_2}, \mu_F) \frac{M^2}{S} \hat{\sigma}^{(0)}_{ij} \frac{\alpha_s(\mu_F)}{2\pi} \Gamma(1+\epsilon)(4\pi)^\epsilon \pi$$

$$\times \Big[P_{jj'}(\xi_2)\delta_{i'i}\delta(1-\xi_1)\left(-\frac{1}{\epsilon} + \ln\left(\frac{4e^{-2\gamma_E}}{b^2\mu_R^2}\right)\right) + (\xi_1 \leftrightarrow \xi_2, i \leftrightarrow j, i' \leftrightarrow j')$$

$$+2C_{i'j'}\delta_{i'i}\delta_{j'j}\delta(1-\xi_1)\delta(1-\xi_2)\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\left(\frac{\mu_R^2}{M^2}\right) + \frac{1}{2}\ln^2\left(\frac{\mu_R^2}{M^2}\right)\right.$$

$$\left. - \frac{\pi^2}{6} - \frac{1}{2}\ln^2\left(\frac{M^2b^2e^{2\gamma_E}}{4}\right)\right)\Big]. \quad ($$

$$W(b)|^{NLO,V} = \int d^2P_\perp e^{-i\overrightarrow{P_\perp}\cdot\vec{b}} \frac{d\sigma}{dydP_\perp^2}|^{NLO,V}$$

$$= \int_{x_A}^{1} \frac{d\xi_1}{\xi_1} \int_{x_B}^{1} \frac{d\xi_2}{\xi_2} f_i(\frac{x_A}{\xi_1}, \mu_F) f_j(\frac{x_B}{\xi_2}, \mu_F) \frac{M^2}{S} \hat{\sigma}^{(0)}_{ij} \frac{\alpha_s(\mu_F)}{\pi} \Gamma(1+\epsilon)(4\pi)^\epsilon \pi$$

$$\times K^{V,(J)}_{ij,g_n} \delta(1-\xi_1)\delta(1-\xi_2).$$

Collins-Soper evolution equation

$$
\begin{aligned}
W(b) =& W(b)|^{NLO,R}_{Singular} + W(b)|^{NLO,V} + W(b)|^{PDFs}_{CT} \\
=& \int_{x_A}^{1}\frac{d\xi_1}{\xi_1}\int_{x_B}^{1}\frac{d\xi_2}{\xi_2} f_{i'}(\frac{x_A}{\xi_1},\mu_F) f_{j'}(\frac{x_B}{\xi_2},\mu_F)\frac{M^2}{S}\hat{\sigma}^{(0)}_{ij}\frac{\alpha_s(\mu_F)}{2\pi}\Gamma(1+\epsilon)(4\pi)^{\epsilon}\pi \\
&\times\left[P_{jj'}(\xi_2)\ln\left(\frac{4e^{-2\gamma_E}}{b^2\mu_F^2}\right)\delta_{i'i}\delta(1-\xi_1) + P_{ii'}(\xi_1)\ln\left(\frac{4e^{-2\gamma_E}}{b^2\mu_F^2}\right)\delta_{j'j}\delta(1-\xi_2)\right. \\
&\left. +2C_{ij}\delta_{i'i}\delta_{j'j}\delta(1-\xi_1)\delta(1-\xi_2)\left(-\frac{\pi^2}{6}-\frac{1}{2}\ln^2\left(\frac{M^2b^2e^{2\gamma_E}}{4}\right)+Fin^{V,(J)}_{ij,g_n}\right)\right],
\end{aligned}
$$

$$
\frac{\partial}{\partial \ln M^2}W(b,M) = \left[K(b\mu,\alpha_s(\mu)) + G\left(\frac{M}{\mu},\alpha_s(\mu)\right)\right]W(b,M)
$$

$$
\mu\frac{d}{d\mu}K(b\mu,\alpha_s(\mu)) = -\gamma_K(\alpha_s(\mu)),
$$

$$
\mu\frac{d}{d\mu}G(b\mu,\alpha_s(\mu)) = +\gamma_K(\alpha_s(\mu)).
$$

Separation of scales around 1/b and M respectively

Collins, Soper 81
Collins, Soper, Sterman 85

Sukakov form factor

$$\frac{\partial W(b,M)}{\partial \ln M^2} = \left[-\frac{1}{2}\int_{C_1^2/b^2}^{C_2^2M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\gamma_K(\alpha_s(\bar{\mu})) + K(C_1,\alpha_s(C_1/b)) + G(1/C_2,\alpha_s(C_2M))\right]W(b,M).$$

$$\ln\frac{W(M,b)}{W(Q_0,b)} = \int_{Q_0^2}^{Q^2} d\ln q^2 \left[\int_{C_1^2/b^2}^{C_2^2q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}A(\alpha_s(\bar{\mu})) + B(\alpha_s(C_2q))\right]$$

$$W(b,M) = e^{-S(b,M,C_1,C_2)}W\left(b,\frac{C_1}{C_2b}\right),$$

where the Sudakov exponent is

$$S(b,M,C_1,C_2) = \int_{C_1^2/b^2}^{C_2^2M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}))\ln\left(\frac{C_2^2M^2}{\bar{\mu}^2}\right) + B(\alpha_s(\bar{\mu}))\right]$$

Nonperturbative Sukakov form factor

$$W(b,M) = e^{-S(b,M,C_1,C_2)} W\left(b, \frac{C_1}{C_2 b}\right),$$

where the Sudakov exponent is

$$S(b,M,C_1,C_2) = \int_{C_1^2/b^2}^{C_2^2 M^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}))\ln\left(\frac{C_2^2 M^2}{\bar{\mu}^2}\right) + B(\alpha_s(\bar{\mu}))\right]$$

Solve the nonperturbative problem when $b \gg 1/\Lambda_{\rm QCD}$

$$W(b,M) = W(b_*,M) W^{NP}(b,M) = W(b_*,M) e^{-S_{NP}(b,M)},$$

$$b_* = \frac{b}{\sqrt{1+(b/b_{\rm max})^2}}.$$

$$S_{NP}^{SIYY-g}(b,M) = g_1 b^2 + g_2 b^2 \ln(M/(2Q_0)) + g_3 b^2 \ln(100 x_1 x_2).$$

Transverse momentum resummation formula for T4c

$$\frac{d\sigma(p+p\to T_{4c}+X)}{dydP_\perp^2}$$

$$=\int\frac{d^2b}{(2\pi)^2}e^{i\overrightarrow{P_\perp}\cdot\vec{b}}W(b,M)+\frac{d\sigma}{dydP_\perp^2}|_{Regular}$$

$$=\int\frac{d^2b}{(2\pi)^2}e^{i\overrightarrow{P_\perp}\cdot\vec{b}}\sum_{ij}\int_{x_A}^1\frac{d\xi_1}{\xi_1}\int_{x_B}^1\frac{d\xi_2}{\xi_2}f_i(\frac{x_A}{\xi_1},\mu_F)f_j(\frac{x_B}{\xi_2},\mu_F)\frac{M^2}{S}\hat{\sigma}^{(0)}_{i'j'}\pi$$

$$\times W^{NP}(b,M)e^{-\int_{C_1^2/b_*^2}^{C_2^2M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A_{ij}(\alpha_s(\bar{\mu}))\ln\left(\frac{C_2^2M^2}{\bar{\mu}^2}\right)+B_{ij}(\alpha_s(\bar{\mu}))\right]}C_{i'i}\left(\xi_1,b_*,\frac{C_1}{C_2},\mu\right)$$

$$\times C_{j'j}\left(\xi_2,b_*,\frac{C_1}{C_2},\mu\right)+Y(P_\perp,M,x_A,x_B),$$

and

$$Y(P_\perp,M,x_A,x_B)=\frac{d\sigma}{dydP_\perp^2}|_{Regular}$$

$$=\sum_{ij}\int_{x_A}^1\frac{d\xi_1}{\xi_1}\int_{x_B}^1\frac{d\xi_2}{\xi_2}f_i(\frac{x_A}{\xi_1},\mu_F)f_j(\frac{x_B}{\xi_2},\mu_F)R_{ij}(P_\perp,M,x_A,x_B).$$

(2)Extraction of LDMEs and comparison with experimental data

Extraction of LDMEs

Table 5: Differential cross-sections $d\sigma/dp_T^{\text{di-}J/\psi}$ of di-J/ψ production. The first uncertainties are statistical, and the second systematic.

$p_T^{\text{di-}J/\psi}$ [GeV/c]	$d\sigma/dp_T^{\text{di-}J/\psi}$ [nb/(GeV/c)]
$0-1$	$1.408 \pm 0.089 \pm 0.083$
$1-2$	$2.523 \pm 0.126 \pm 0.150$
$2-3$	$2.858 \pm 0.158 \pm 0.164$
$3-4$	$2.542 \pm 0.110 \pm 0.146$
$4-5$	$2.017 \pm 0.081 \pm 0.120$
$5-6$	$1.527 \pm 0.073 \pm 0.091$
$6-7$	$1.085 \pm 0.048 \pm 0.065$
$7-8$	$0.738 \pm 0.038 \pm 0.044$
$8-10$	$0.424 \pm 0.018 \pm 0.027$
$10-12$	$0.200 \pm 0.011 \pm 0.013$
$12-14$	$0.093 \pm 0.007 \pm 0.005$
$14-24$	$0.017 \pm 0.001 \pm 0.001$

LHCb, 2311.14085

$$\mathcal{R} \equiv \sigma(T_{4c})/\sigma(2J/\psi)\mathcal{B}(T_{4c} \to 2J/\psi)$$

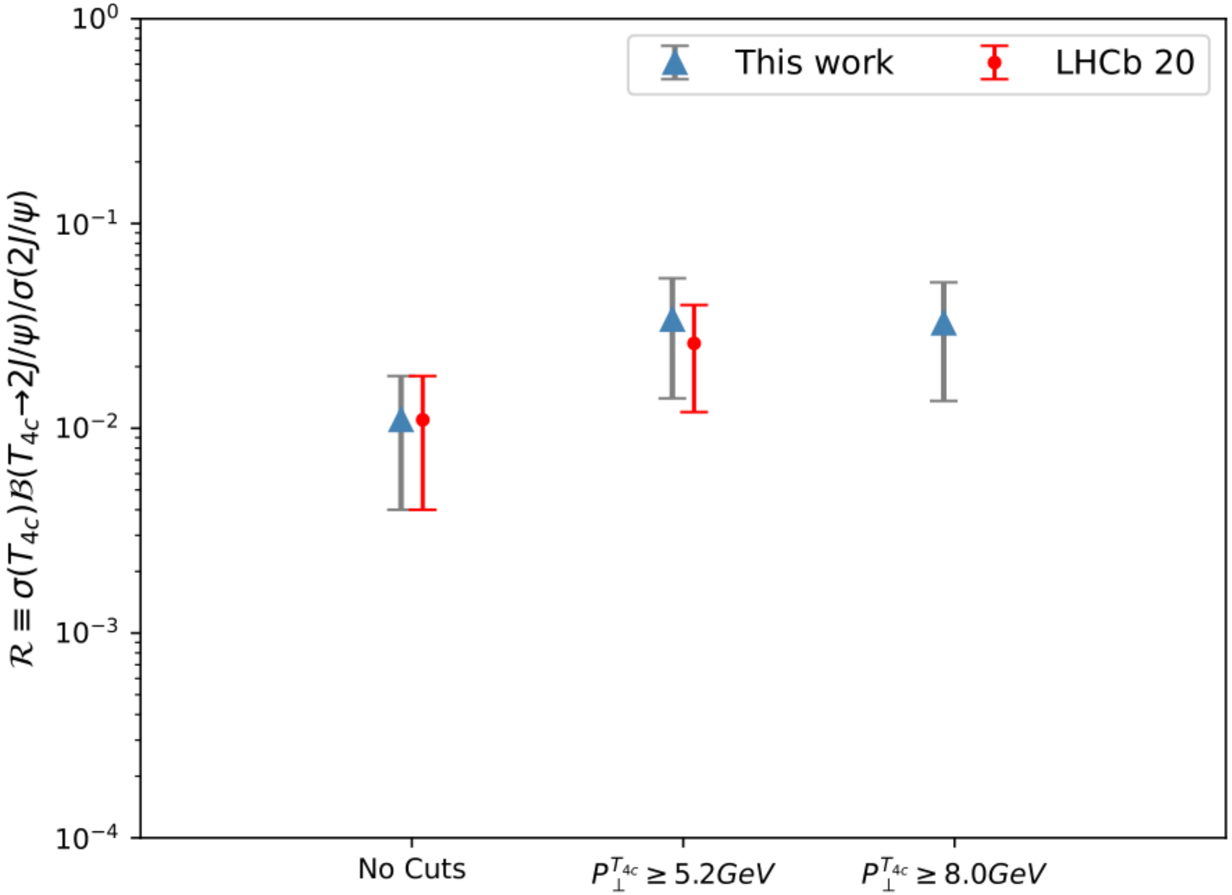
LHCb, 2006.16957

$$\mathcal{R}^{\text{LHCb}} = [1.1 \pm 0.4(\text{stat}) \pm 0.3(\text{syst})]\%.$$

Using the 2J/\psi cross section and R-value data from LHCb, we extracted the LDMEs based on the NLO+NLL QCD formula

$$\begin{aligned}\langle \mathcal{O}_{3\bar{3}}^{T4c} \rangle \left({}^5S_2, 2^{++}\right) \mathcal{B}(T_{4c}) &= \langle 0 | O^2_{\bar{\mathbf{3}}\otimes\mathbf{3},\bar{\mathbf{3}}\otimes\mathbf{3}} | 0 \rangle \mathcal{B}(T_{4c}) \\ &= (2.22 \pm 0.80^{+1.29+0.62}_{-0.57-0.00}) \times 10^{-4} GeV^9,\end{aligned}$$

A test of the predictive power of our formula



$P_\perp^{2J/\psi} > 5.2GeV$ case

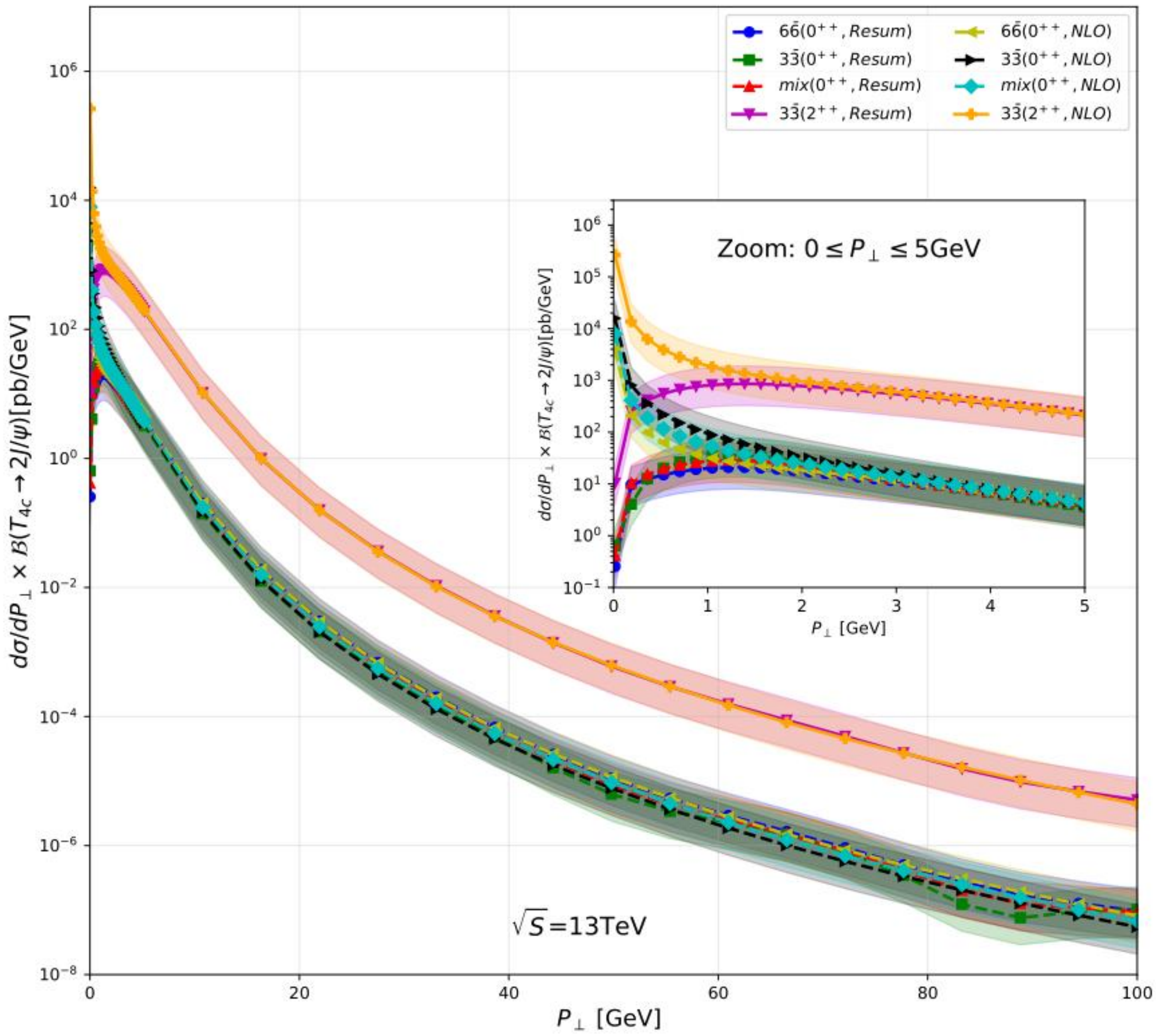
$$\mathcal{R}^{Theo.} = [3.2 \pm 2.0]\%,$$

$$\mathcal{R}^{LHCb} =$$

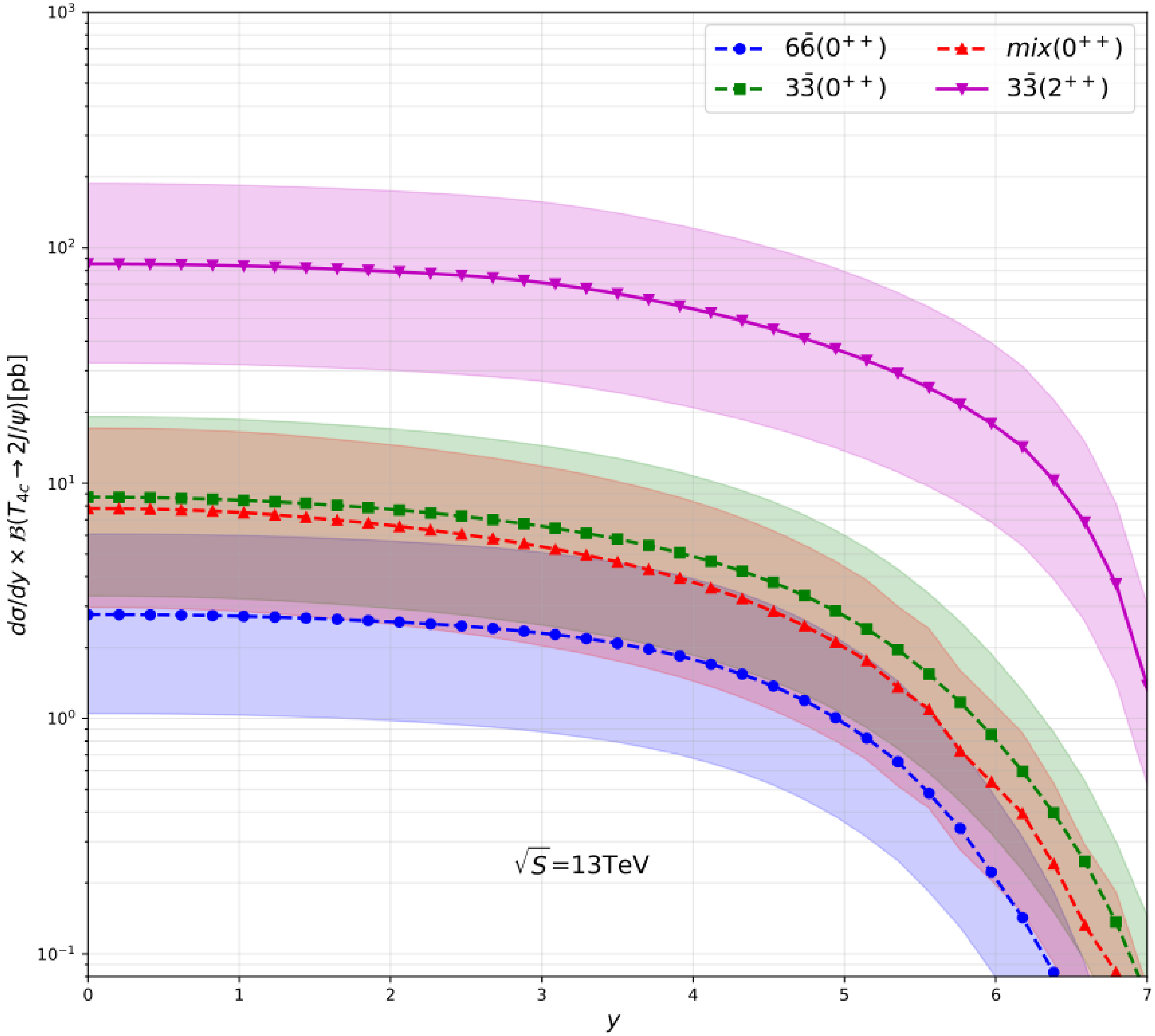
$$[2.6 \pm 0.6(stat) \pm 0.8(syst)]\%$$

Well agreement with LHCb measurement

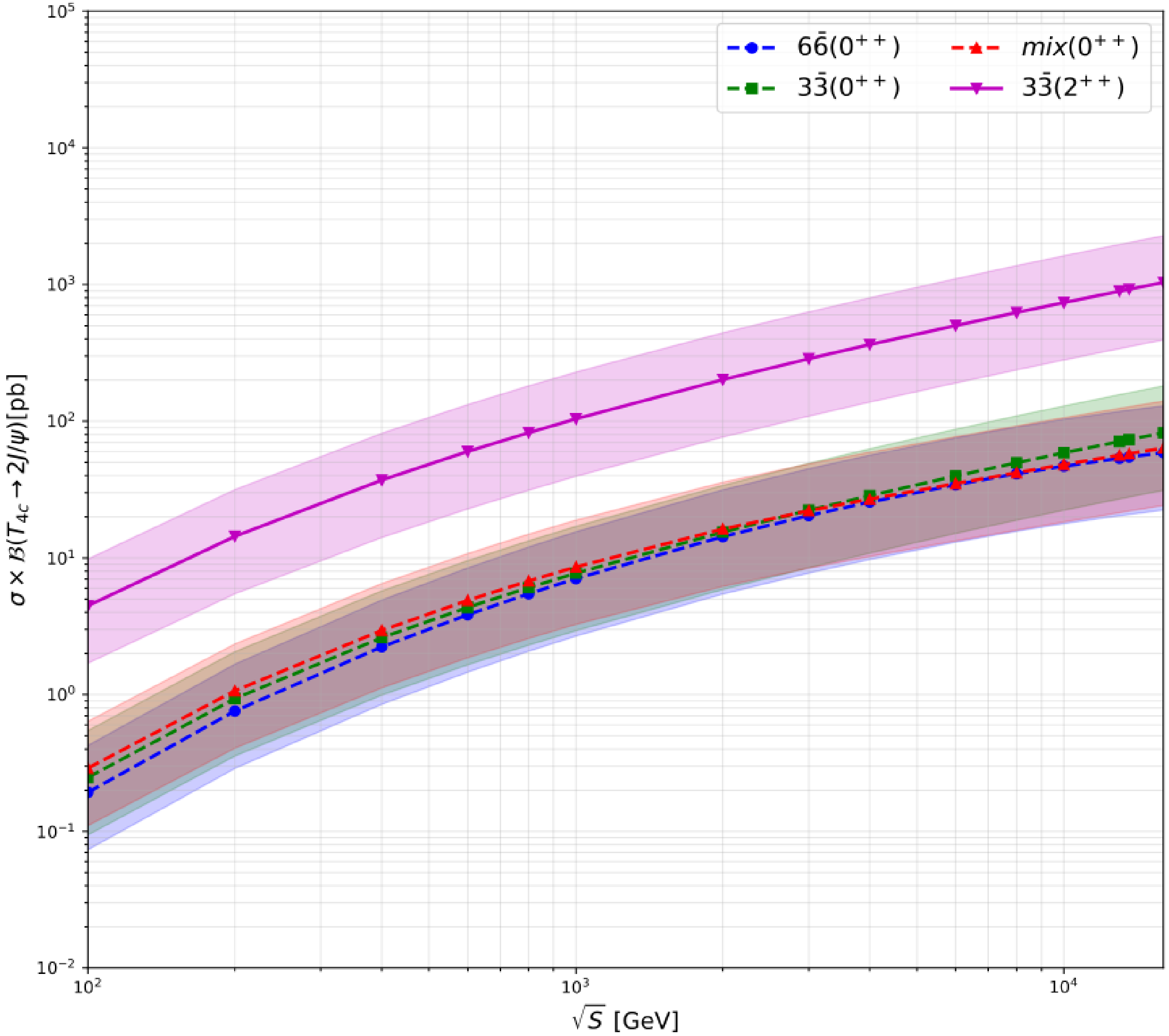
Transverse momentum distribution results at NLO+NLL



Rapidity distribution results



Total cross section dependence on center-of-mass energy



Summary and outlook

- ✓ QCD resummation formulae for T4c production at LHC are established.
- ✓ The LDMEs are extracted from the LHCb data, then the transverse momentum, rapidity distributions are predicted.

Outlook: measuring the (differential) cross section (or) and the decay angular distribution shall tell us the inner structure of fully charm tetraquarks; a lot of tasks in there

Thank you a lot!