

热背景下真空衰变的数值模拟

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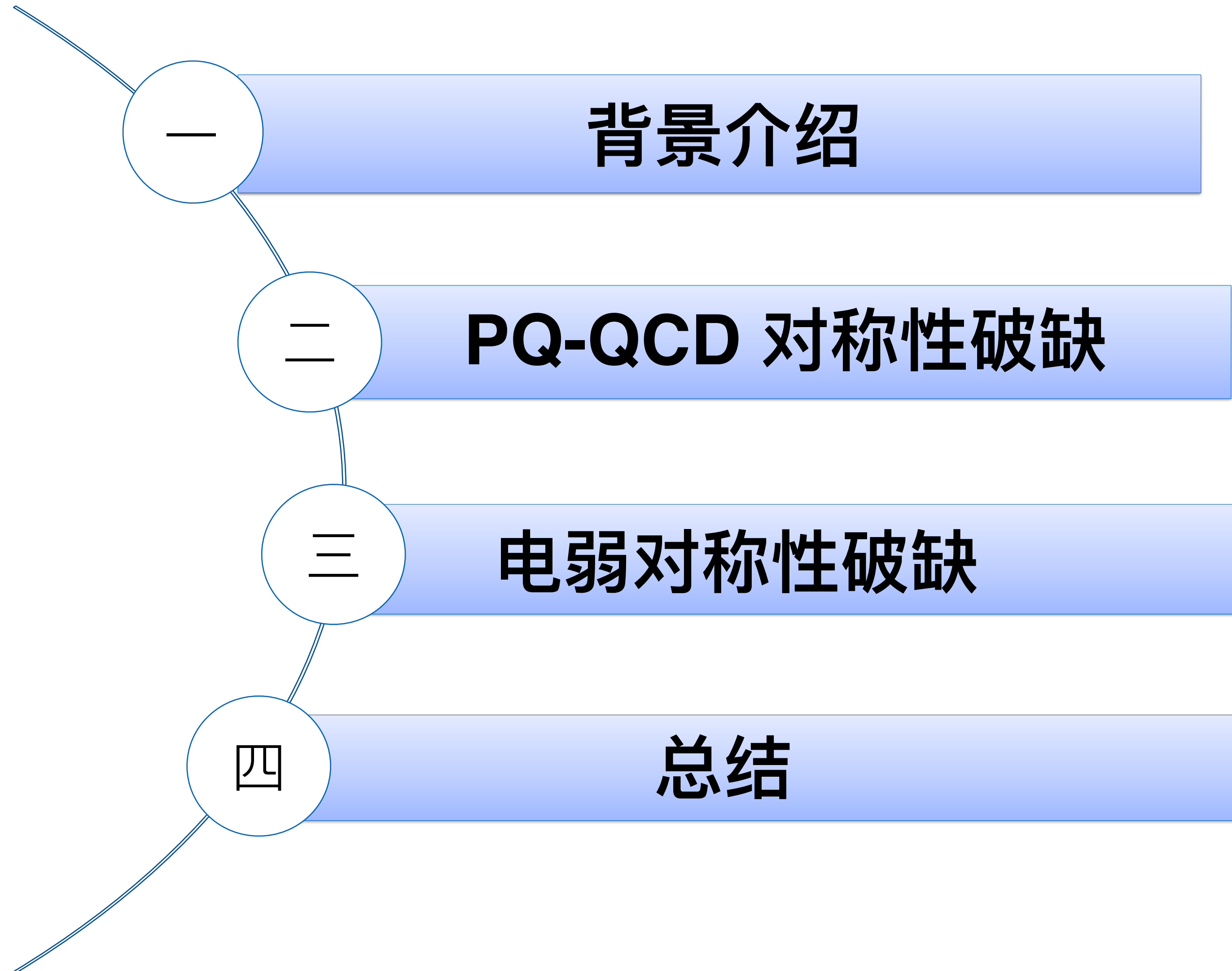
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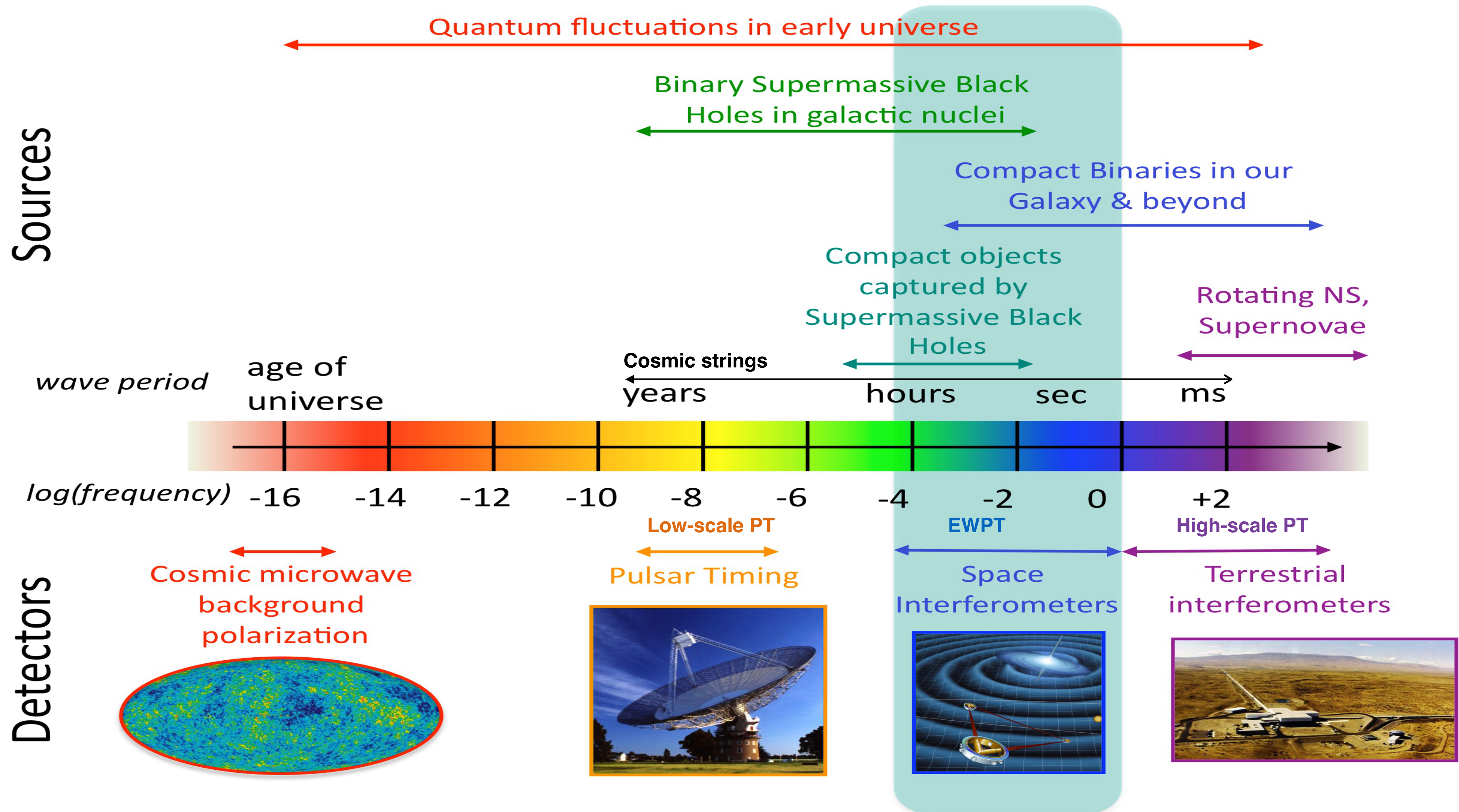
第八届全国重味物理与量子色动力学研讨会

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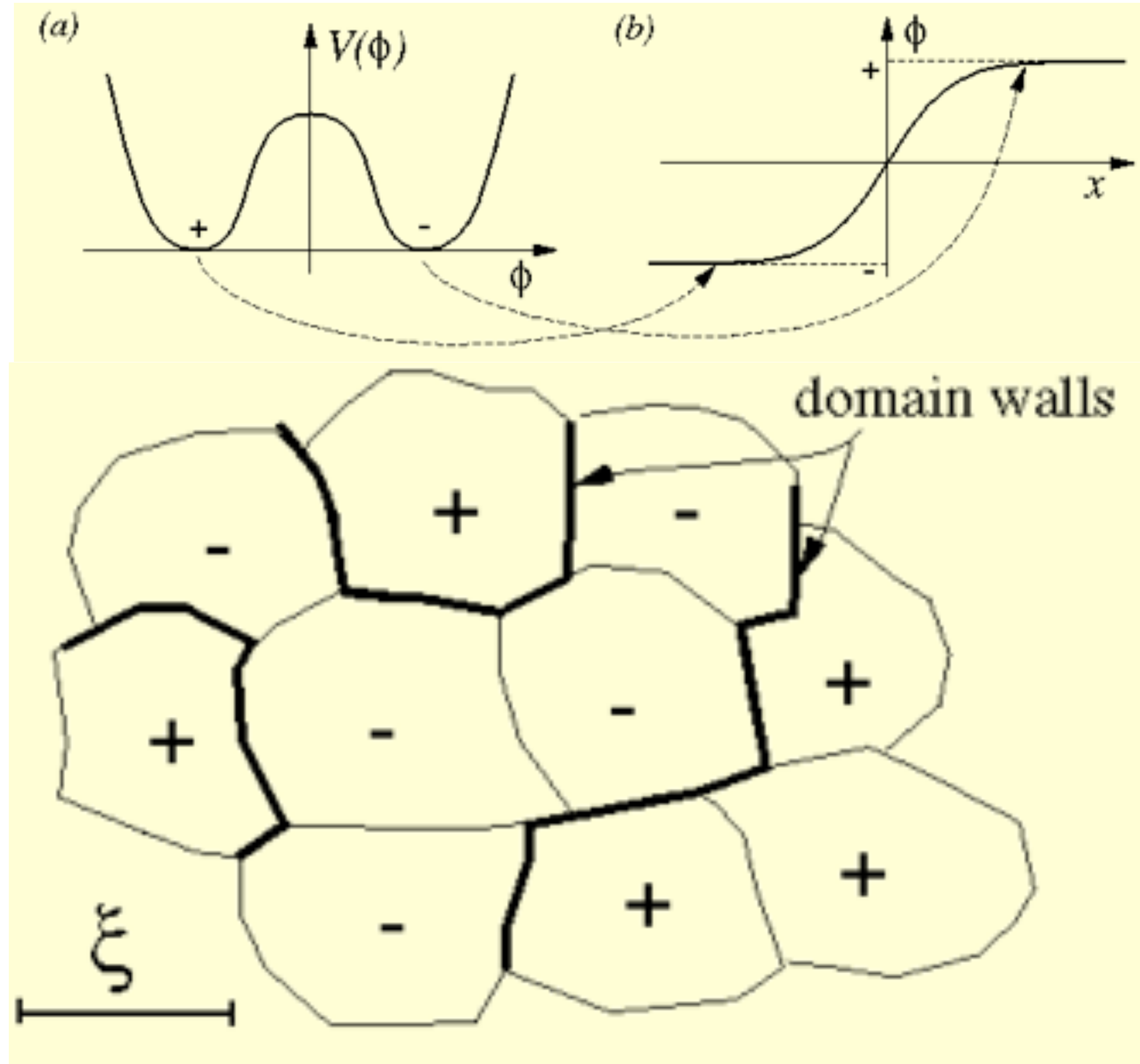
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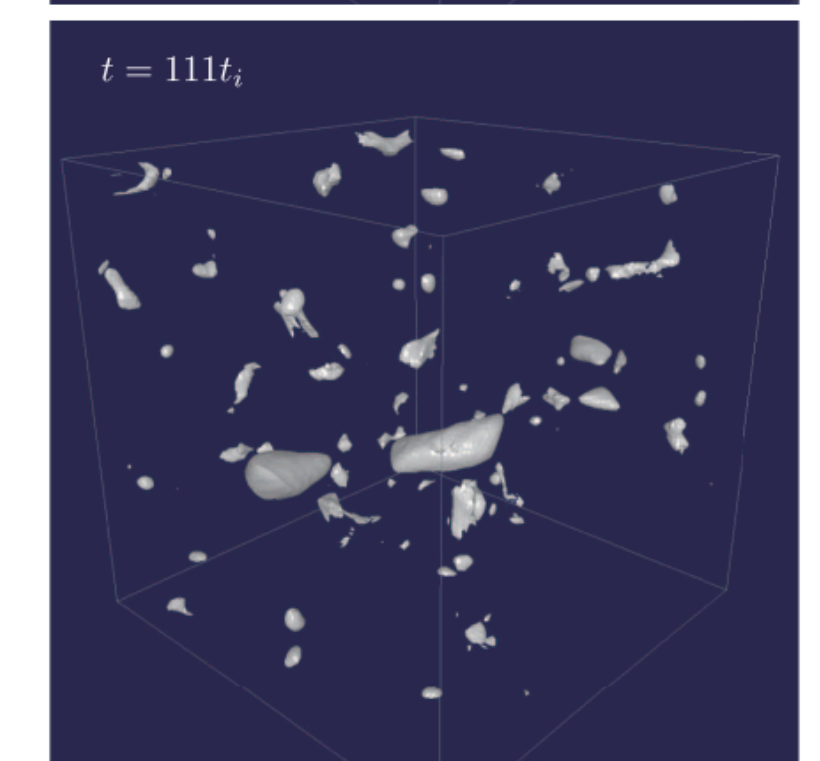
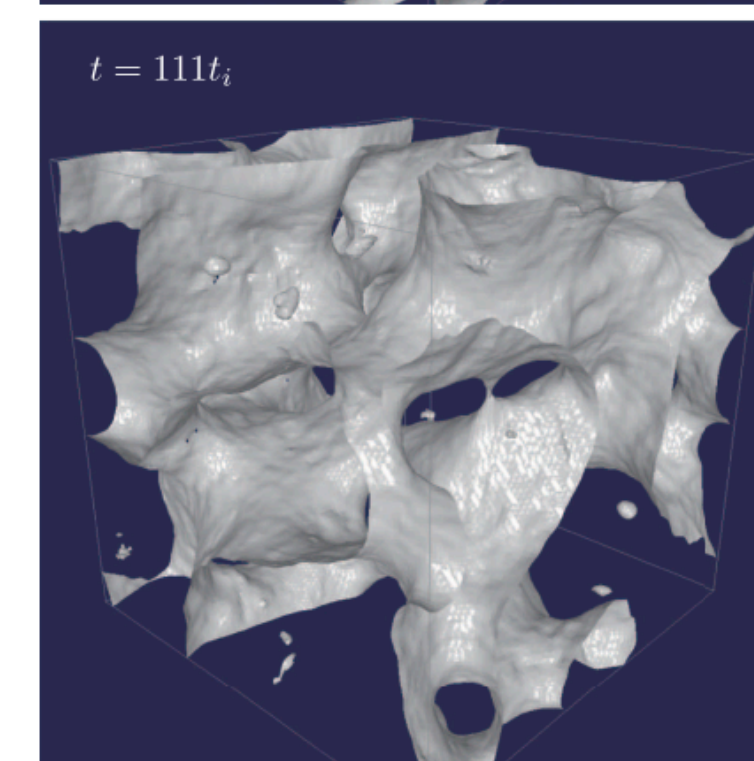
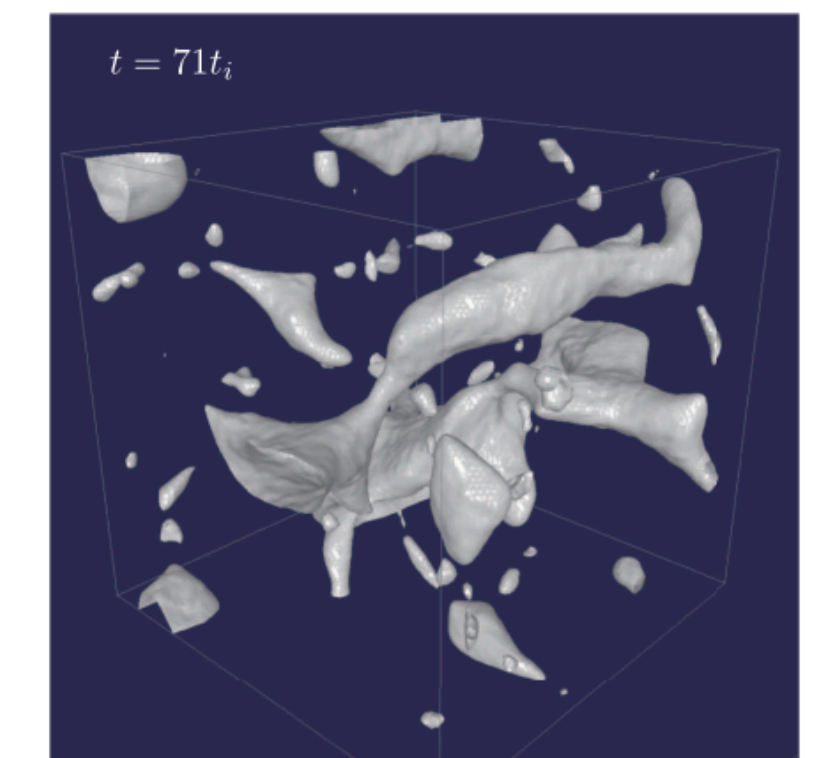
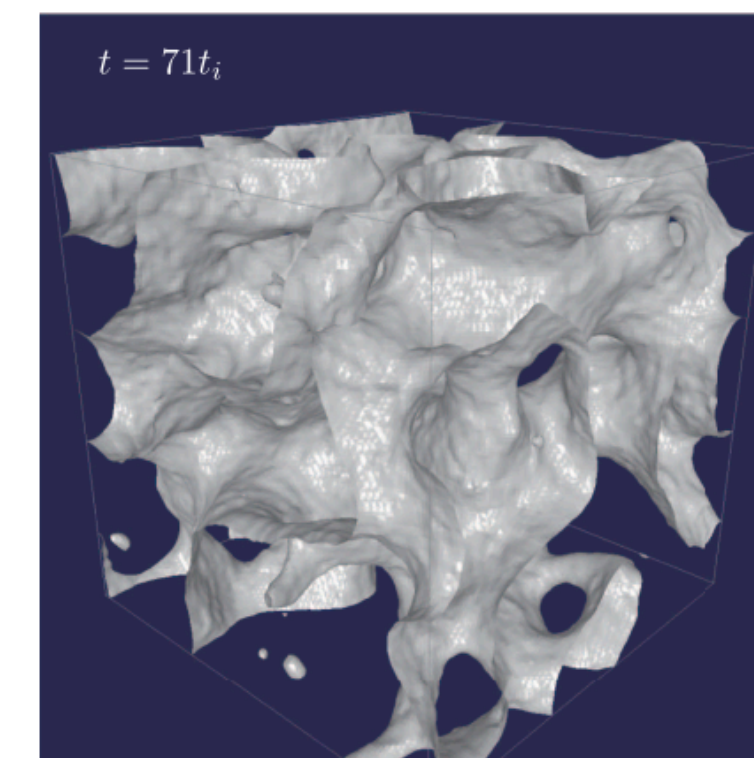
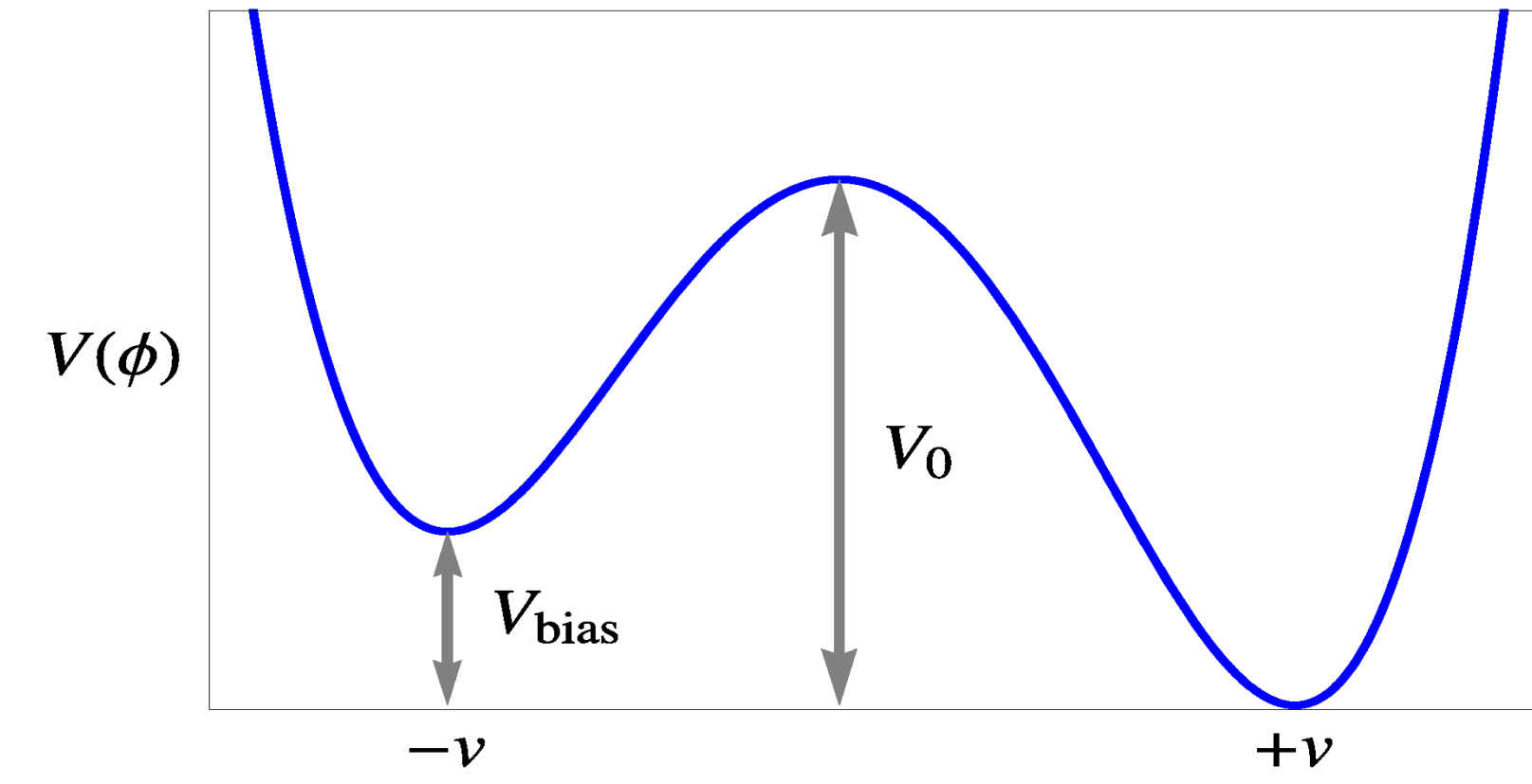
The Gravitational Wave Spectrum



Domain wall



Kibble mechanism



► Cosmic string simulation: the DFSZ axion model

The action in expanding universe with spatially flat FLRW metric:

$$S = -\int d^4x \sqrt{-g} \left(g^{\mu\nu} \frac{1}{2} \partial_\mu \varphi^* \partial_\nu \varphi + V(\varphi) \right)$$

PQ complex scalar: $\varphi = \phi_1 + i\phi_2$

$$V(\varphi) = \underbrace{\frac{1}{4} \lambda (|\varphi|^2 - v^2)^2 + \frac{\lambda}{6} T^2 |\varphi|^2}_{\text{PQ era}} + \underbrace{\frac{m^2(T)v^2}{N_{DW}^2} (1 - \cos(N_{DW}\theta)) - \Xi v^3 (\varphi e^{-i\delta} + \text{h.c.})}_{\text{QCD era}}$$

$\min \left[\frac{\alpha_a \Lambda^4}{f_a^2 (T/\Lambda)^{6.68}}, m_a^2 \right]$
axion field
bias term

PQ era, PQ symmetry broken, second order phase transition, $T_c \sim 10^9\text{-}10^{11}\text{GeV}$

QCD era, axion acquires a non-zero mass due to QCD non-perturbative effect, $T \sim 100\text{MeV}$

Axion(global) strings form and enters the scaling regime

String-domain wall hybrid networks form and eventually decay

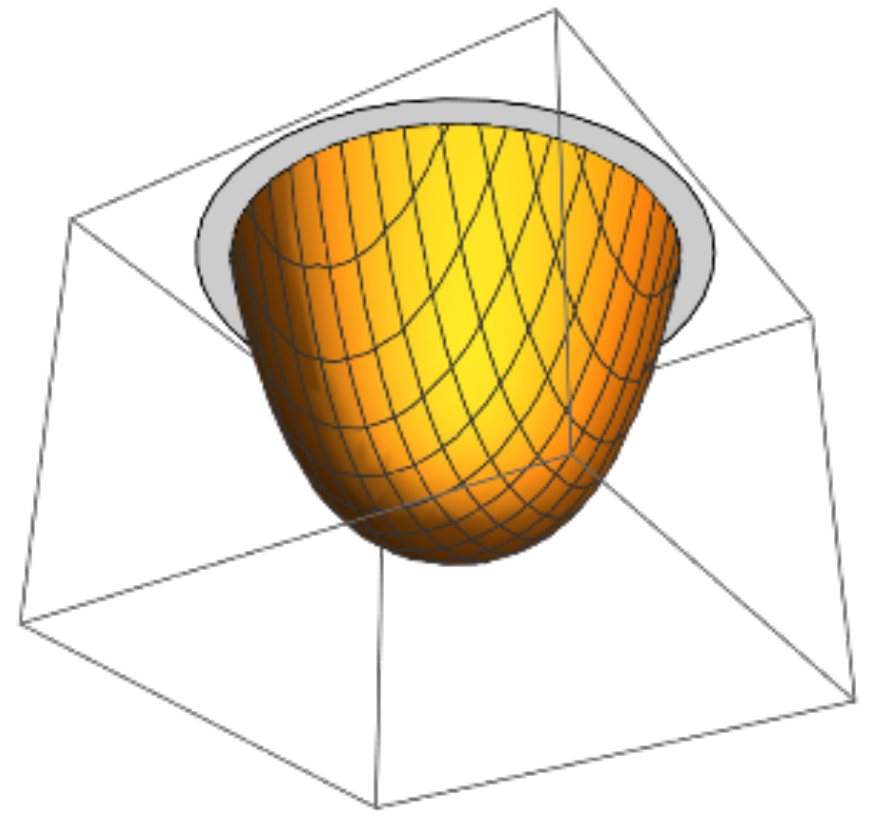
Gravitational waves and axion radiated by topological defects of two eras

→ Detection of axion dark matter

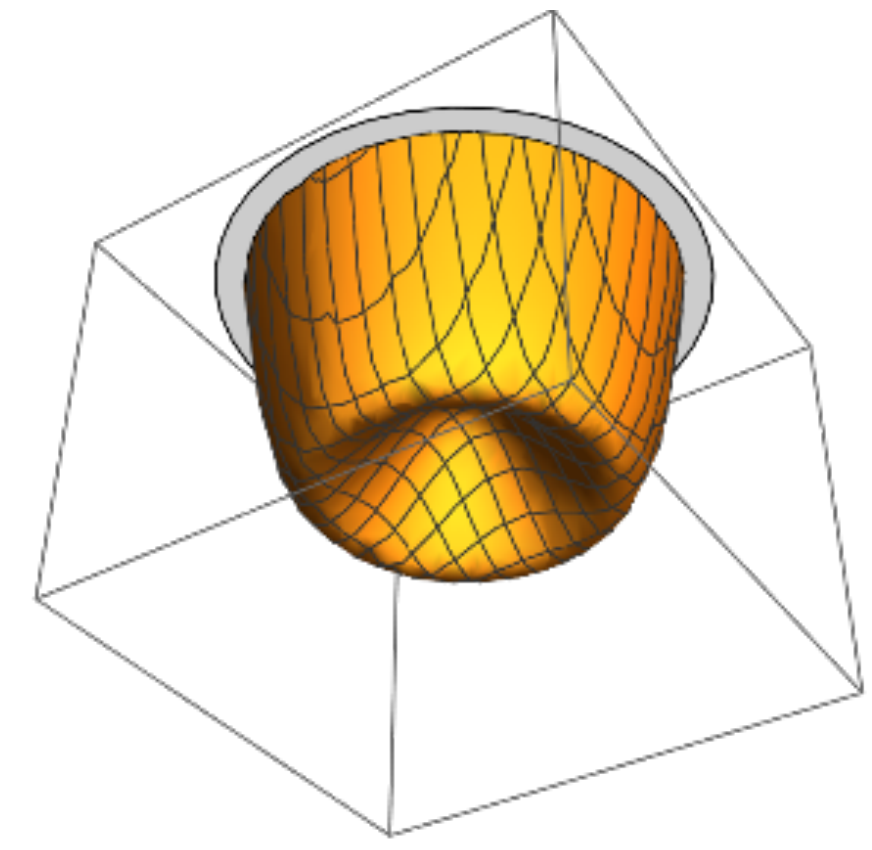
► Shape of the potential

PQ era:

before PQ transition

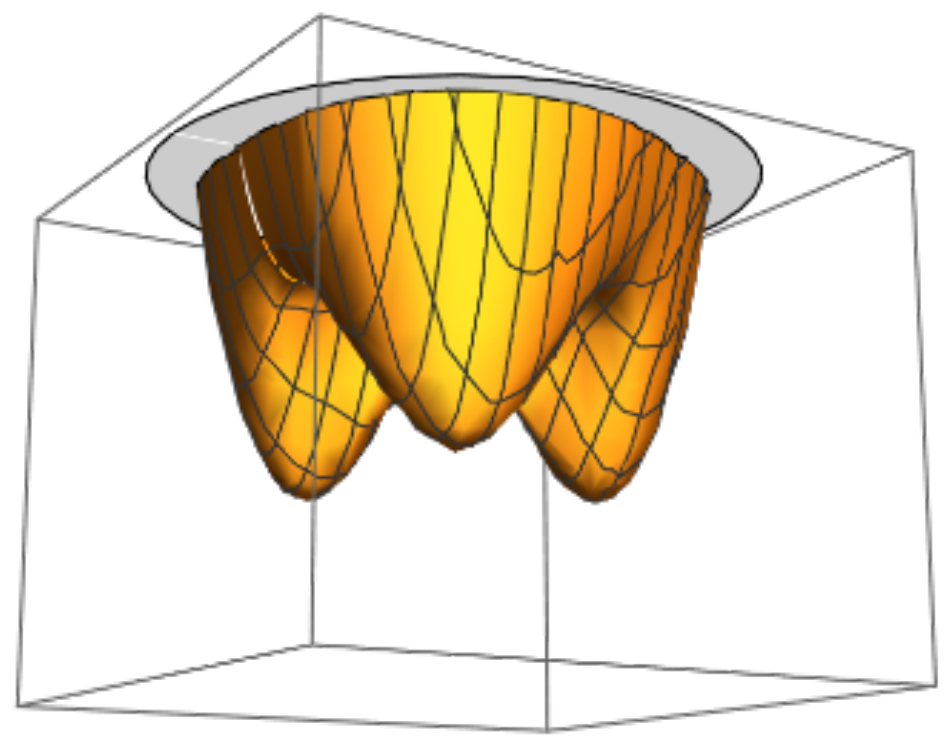


after PQ transition

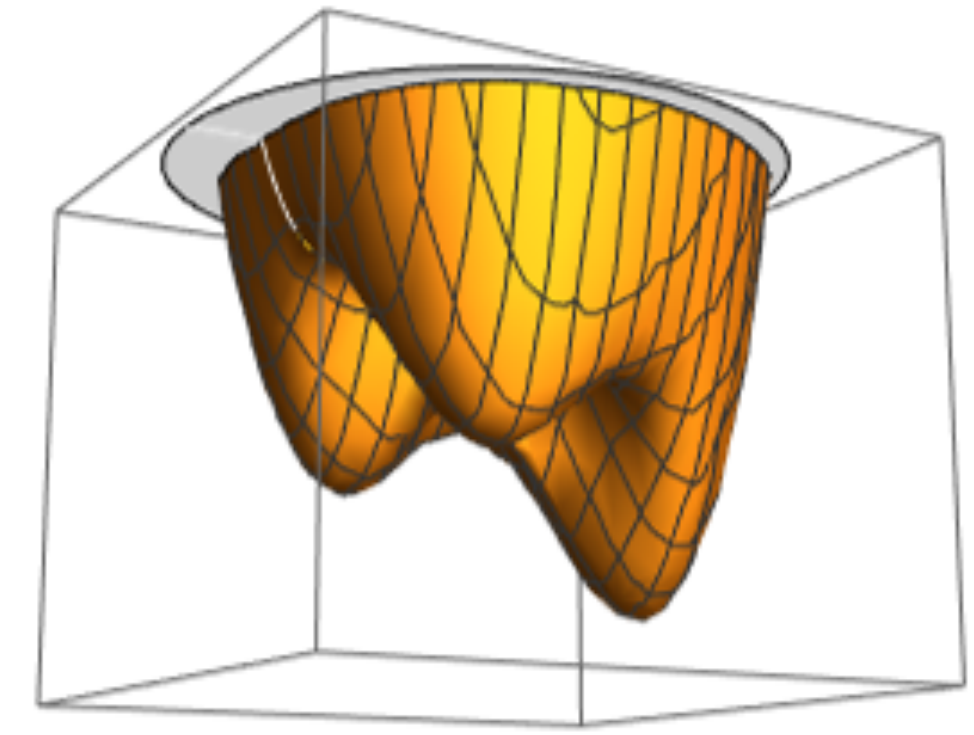


QCD era:

with nonzero axion mass
without bias term



with bias term



$N_{DW}=3$

Equations of motion

PQ era-the first stage

$$\begin{cases} \phi_1'' + 2\frac{a'}{a}\phi_1' - \nabla^2\phi_1 = -a^2[\lambda\phi_1(\phi_1^2 + \phi_2^2 - v^2 + \frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\text{DW}}^2}(\cos\theta \cos N_{\text{DW}}\theta + N_{\text{DW}} \sin\theta \sin N_{\text{DW}}\theta) - 2\Xi v^3 \cos\delta] \\ \phi_2'' + 2\frac{a'}{a}\phi_2' - \nabla^2\phi_2 = -a^2[\lambda\phi_2(\phi_1^2 + \phi_2^2 - v^2 + \frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\text{DW}}^2}(\sin\theta \cos N_{\text{DW}}\theta - N_{\text{DW}} \cos\theta \sin N_{\text{DW}}\theta) - 2\Xi v^3 \sin\delta] \end{cases}$$

Initial condition

thermal spectrum

$$\mathcal{P}_{\phi_1}(k) = \mathcal{P}_{\phi_2}(k) = \frac{n_k}{w_k} = \frac{1}{w_k} \frac{1}{e^{w_k/T} - 1}, \quad \mathcal{P}_{\dot{\phi}_1}(k) = \mathcal{P}_{\dot{\phi}_2}(k) = n_k w_k = \frac{w_k}{e^{w_k/T} - 1}$$

$$w_k = \sqrt{k^2/R^2 + m_{\text{eff}}^2} \quad m_{\text{eff}}^2 = \lambda(T^2/3 - v^2)$$

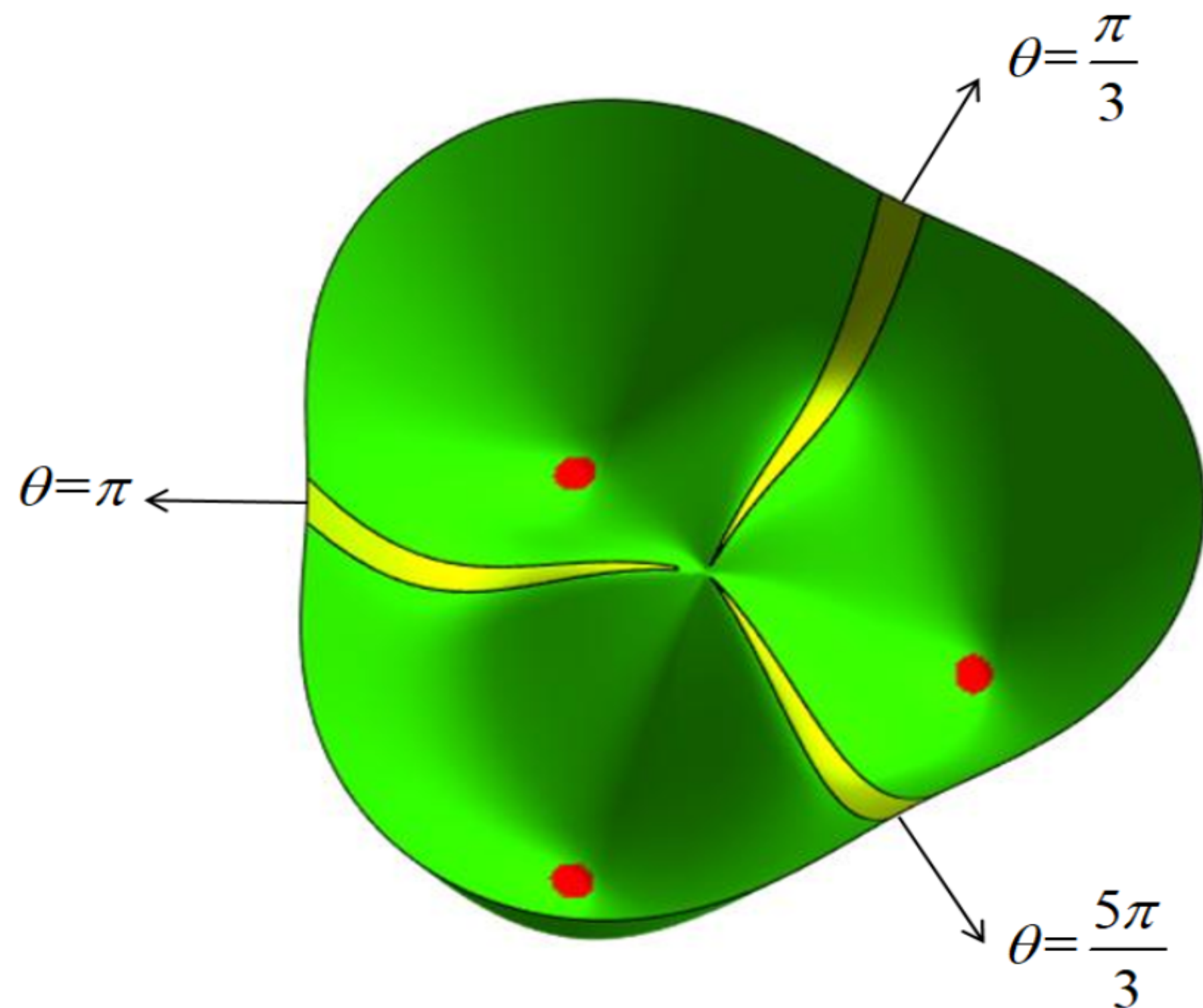
two-point correlation functions

$$\begin{aligned} \langle \phi_i(\mathbf{k})\phi_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_\phi(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \dot{\phi}_i(\mathbf{k})\dot{\phi}_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_{\dot{\phi}}(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \phi_i(\mathbf{k})\dot{\phi}_j(\mathbf{k}') \rangle &= 0. \end{aligned}$$

$$\begin{aligned} \langle |\phi_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\phi_i}(k), & \langle \phi_i(\mathbf{k}) \rangle &= 0, \\ \langle |\dot{\phi}_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\dot{\phi}_i}(k), & \langle \dot{\phi}_i(\mathbf{k}) \rangle &= 0, \end{aligned}$$

► Domain wall's identification

Top view of the shape of potential energy



Comoving area density

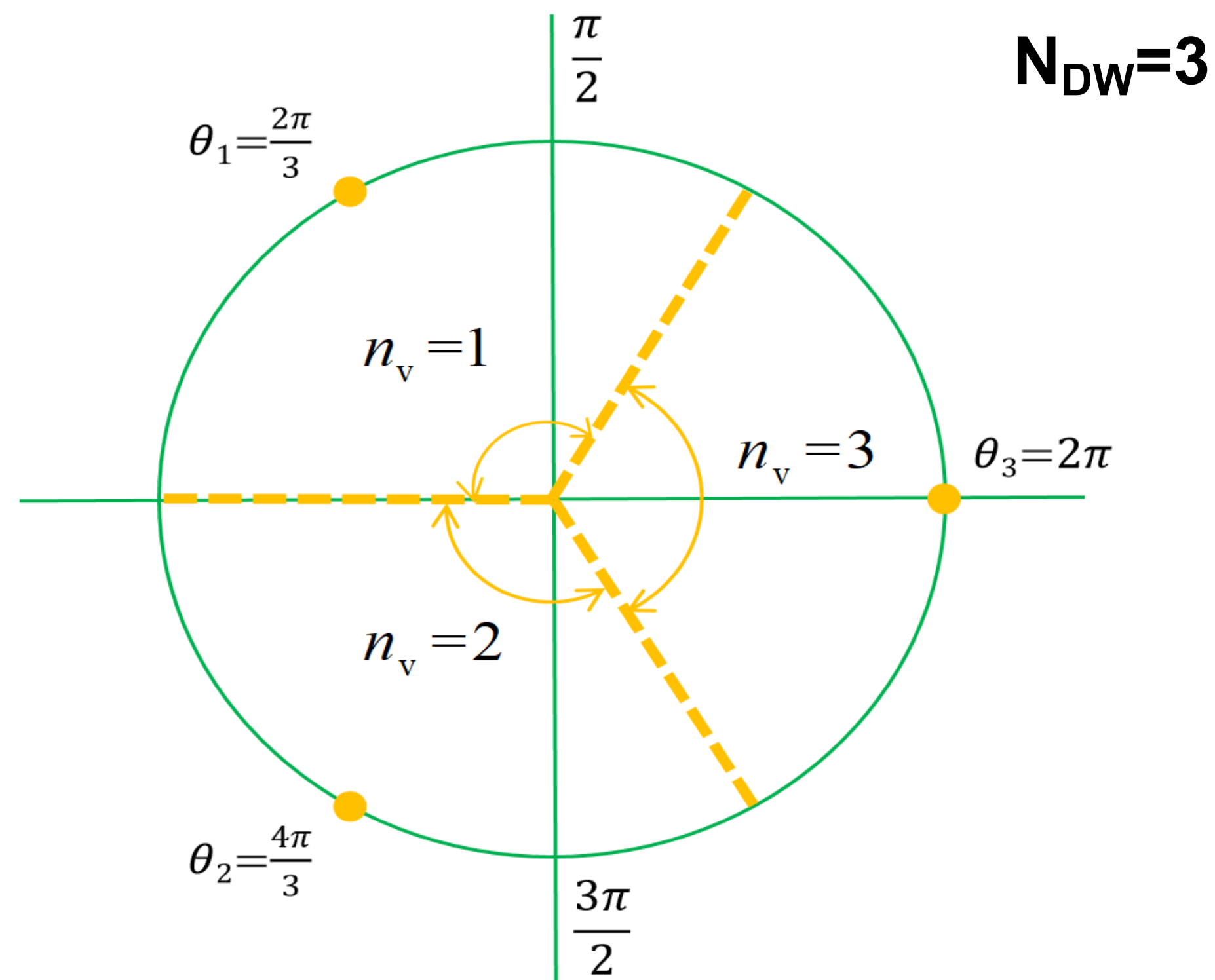
$$A/V = C \sum_{\text{links}} \delta \frac{|\nabla\theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

$\theta_{,i}$ ($i = x, y, z$) : spatial derivatives of the dimensionless axion field $\theta(x)$

Area parameter A of DW (scaling parameter of DW)

$$A = \Delta A \sum_{\text{links}} \delta \frac{|\nabla\theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

The distribution of fields in phase space



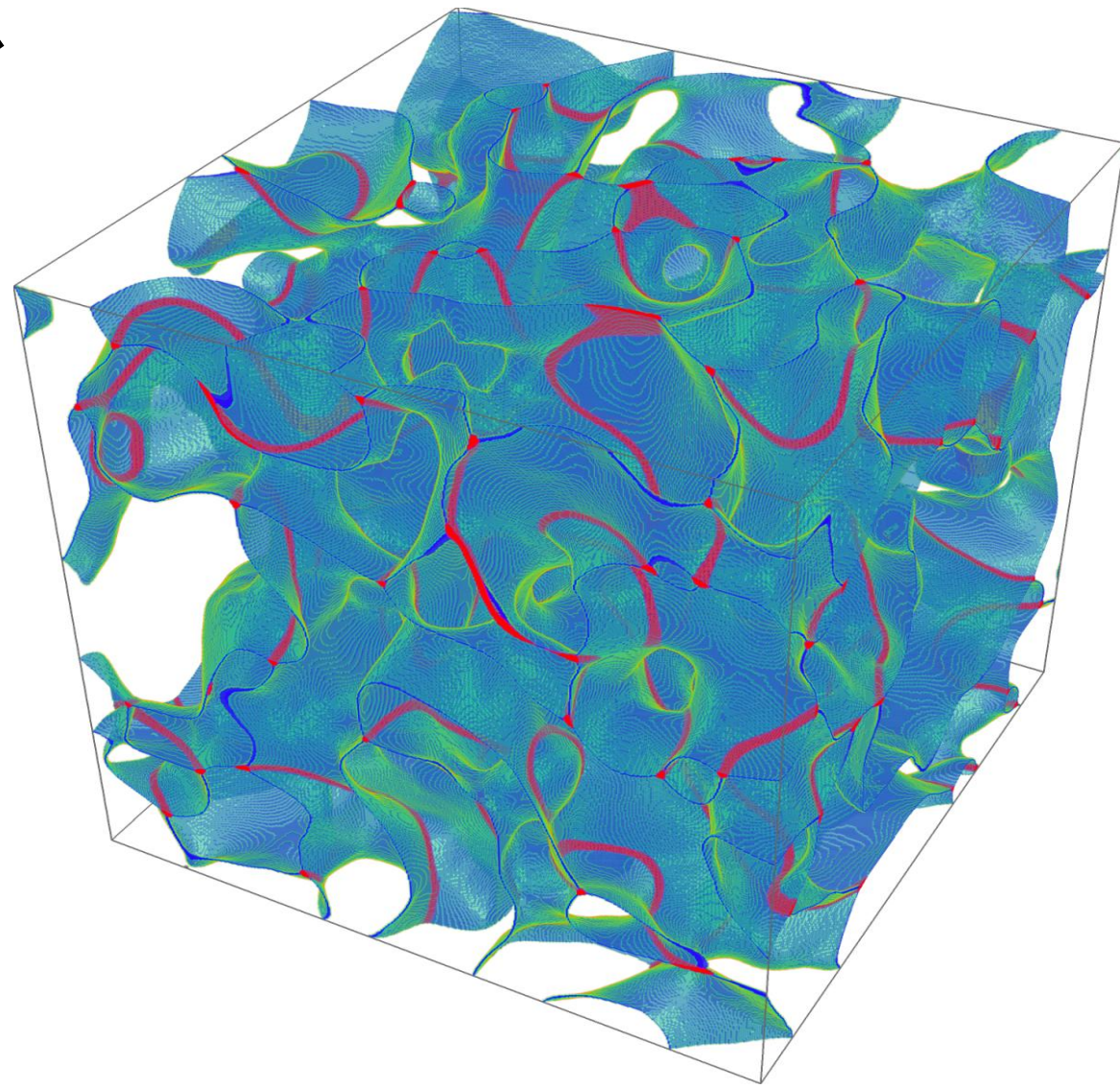
$$\xi_{dw} \equiv \mathcal{A} = \frac{\rho_{wall}}{\sigma_{wall}} t, \quad \text{with } \rho_{wall} = \frac{\sigma_{wall} A}{R(t)V}$$

$\Delta A = (\delta x)^2$ is the comoving area of one grid surface

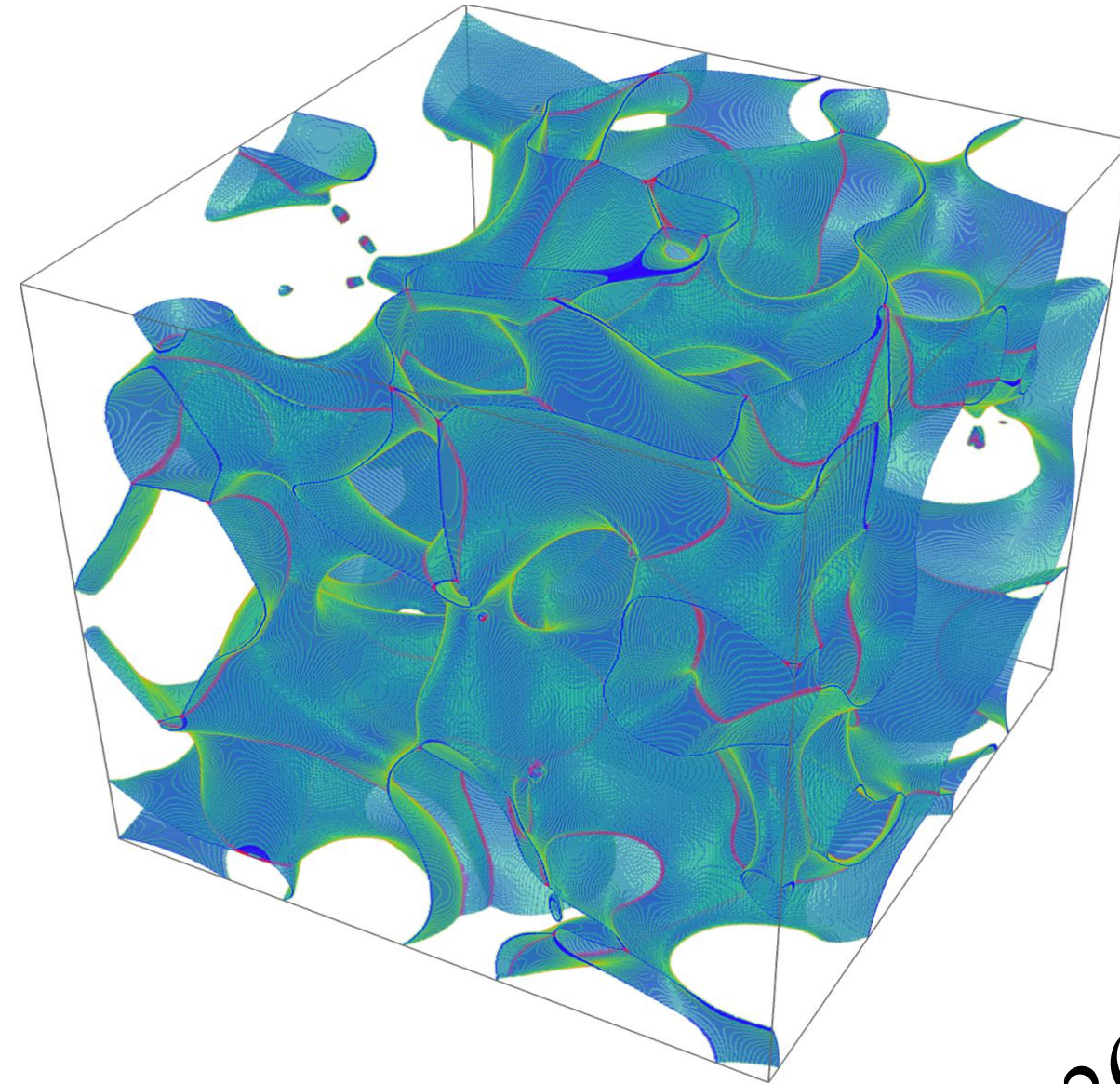
String-wall evolution

Axion string-domain wall hybrid network destruction with gravitational waves and axions emission

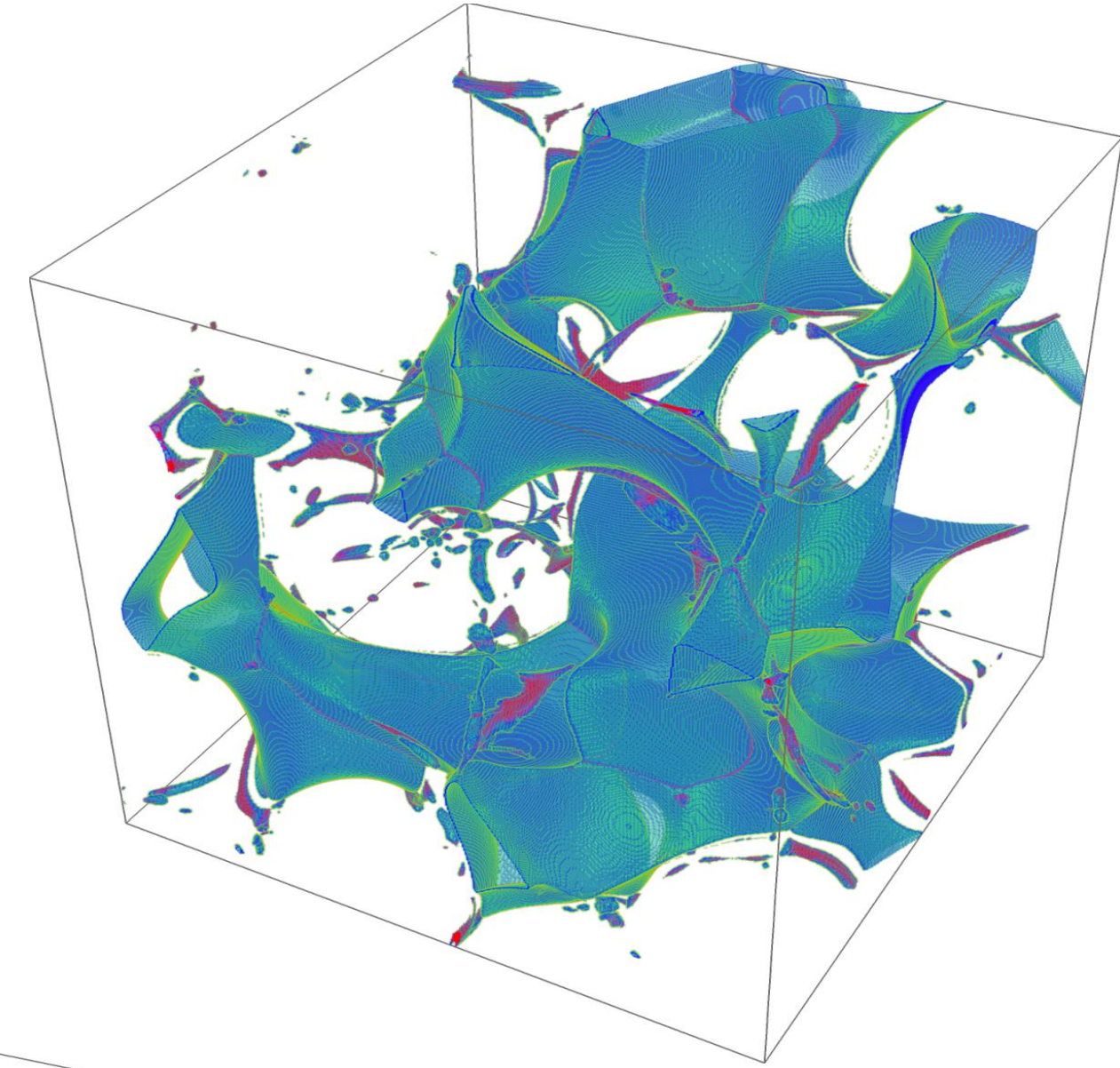
$\eta=4.6$



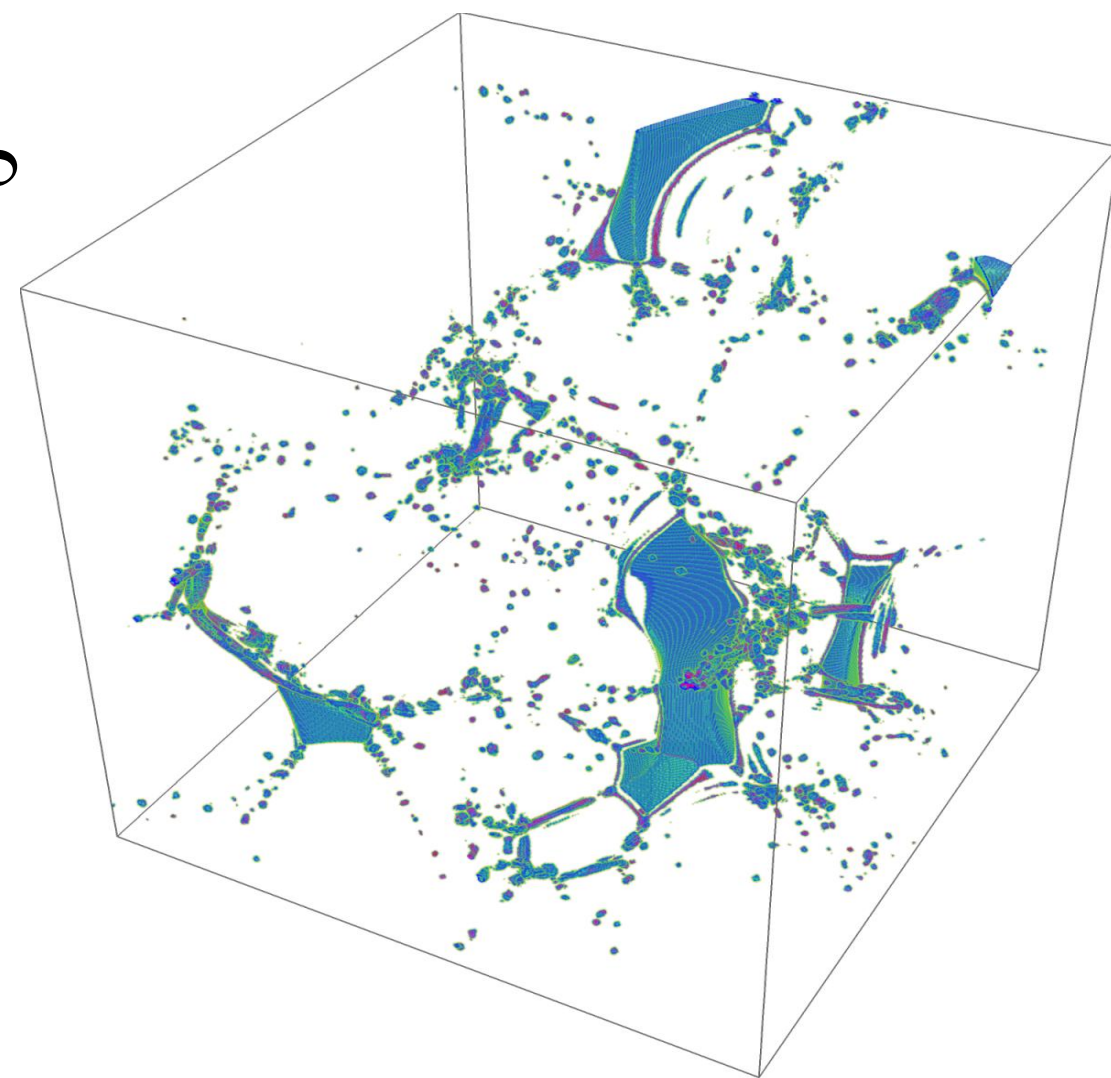
$\eta=10$



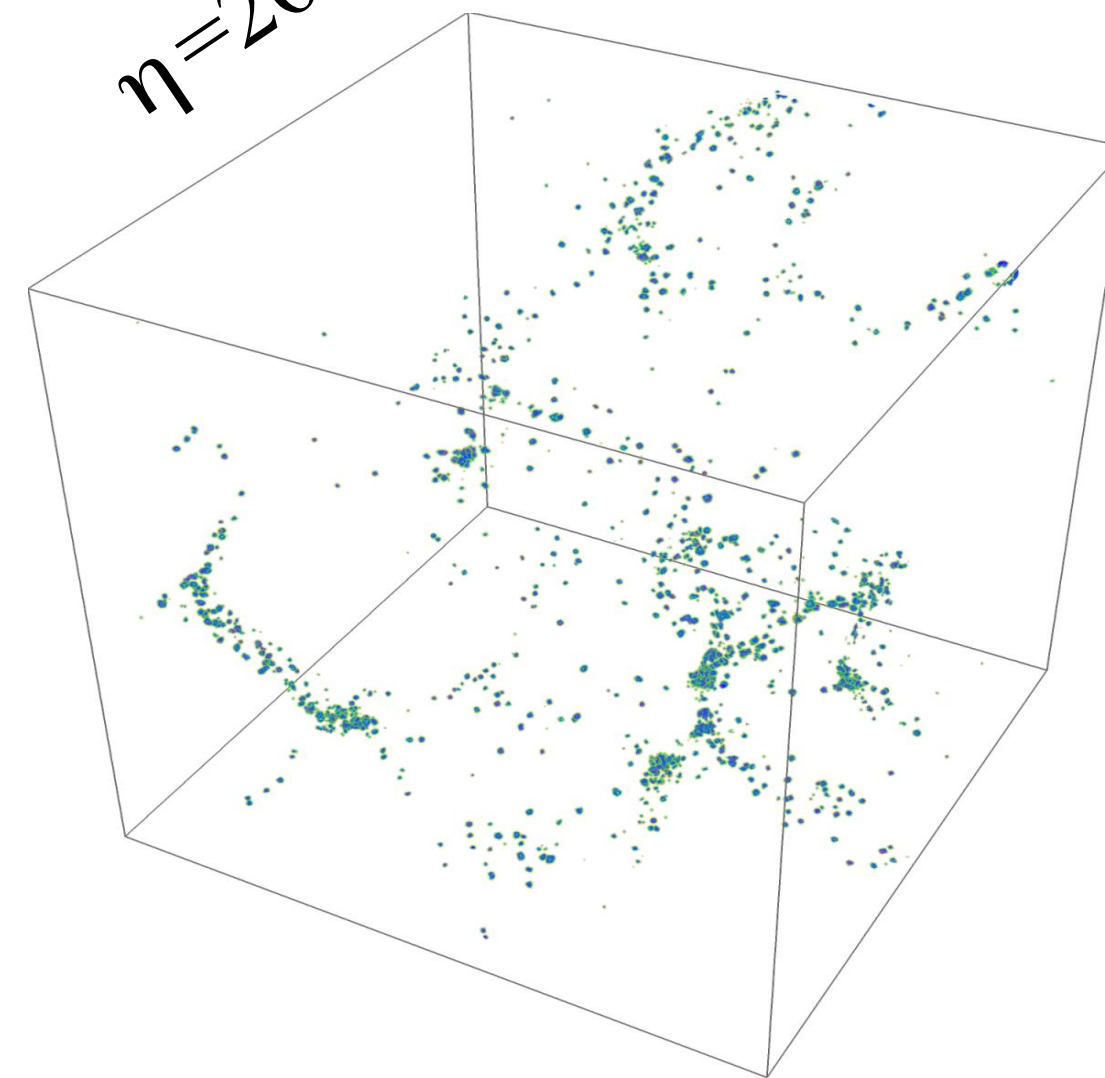
$\eta=15.4$



$\eta=20.8$

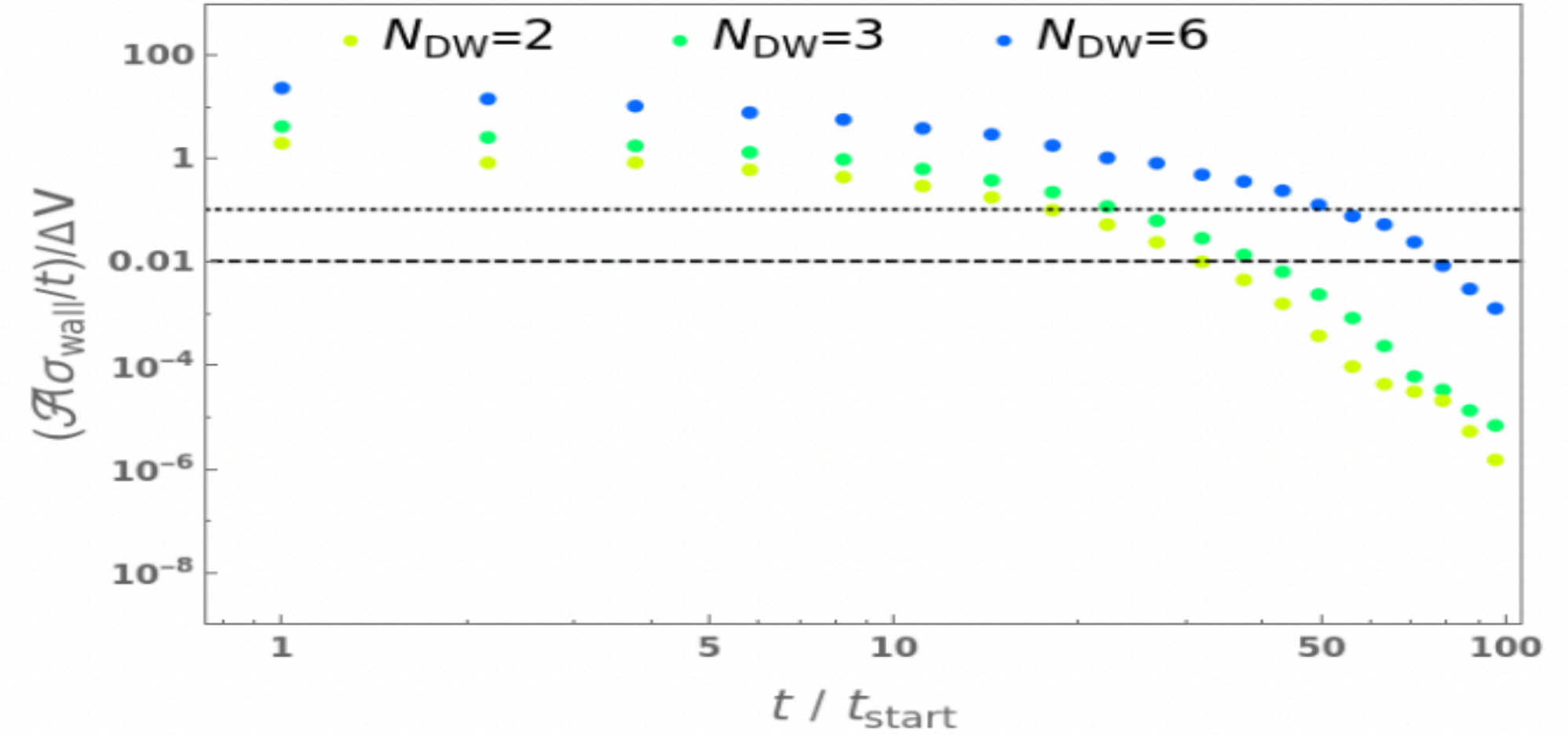
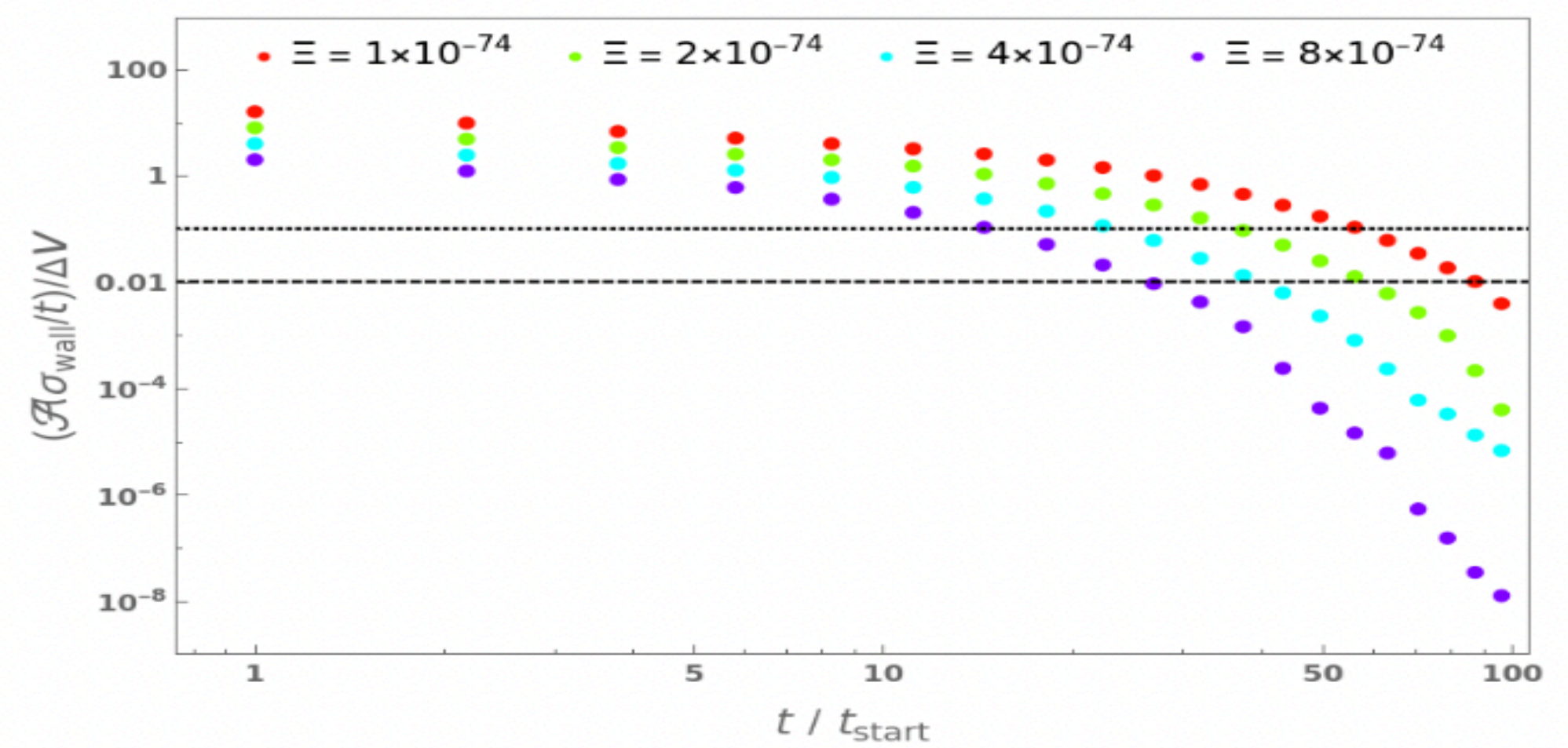
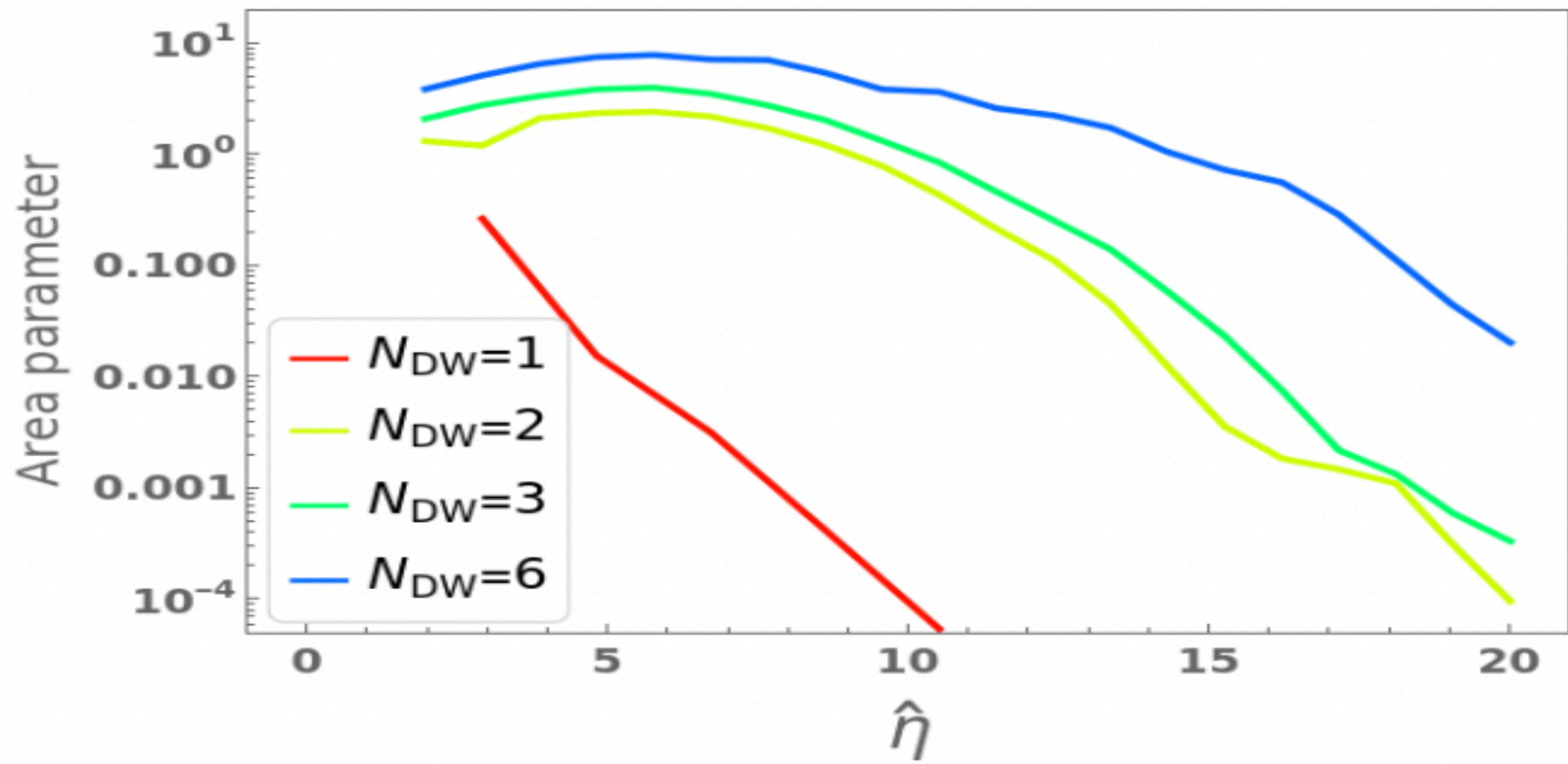
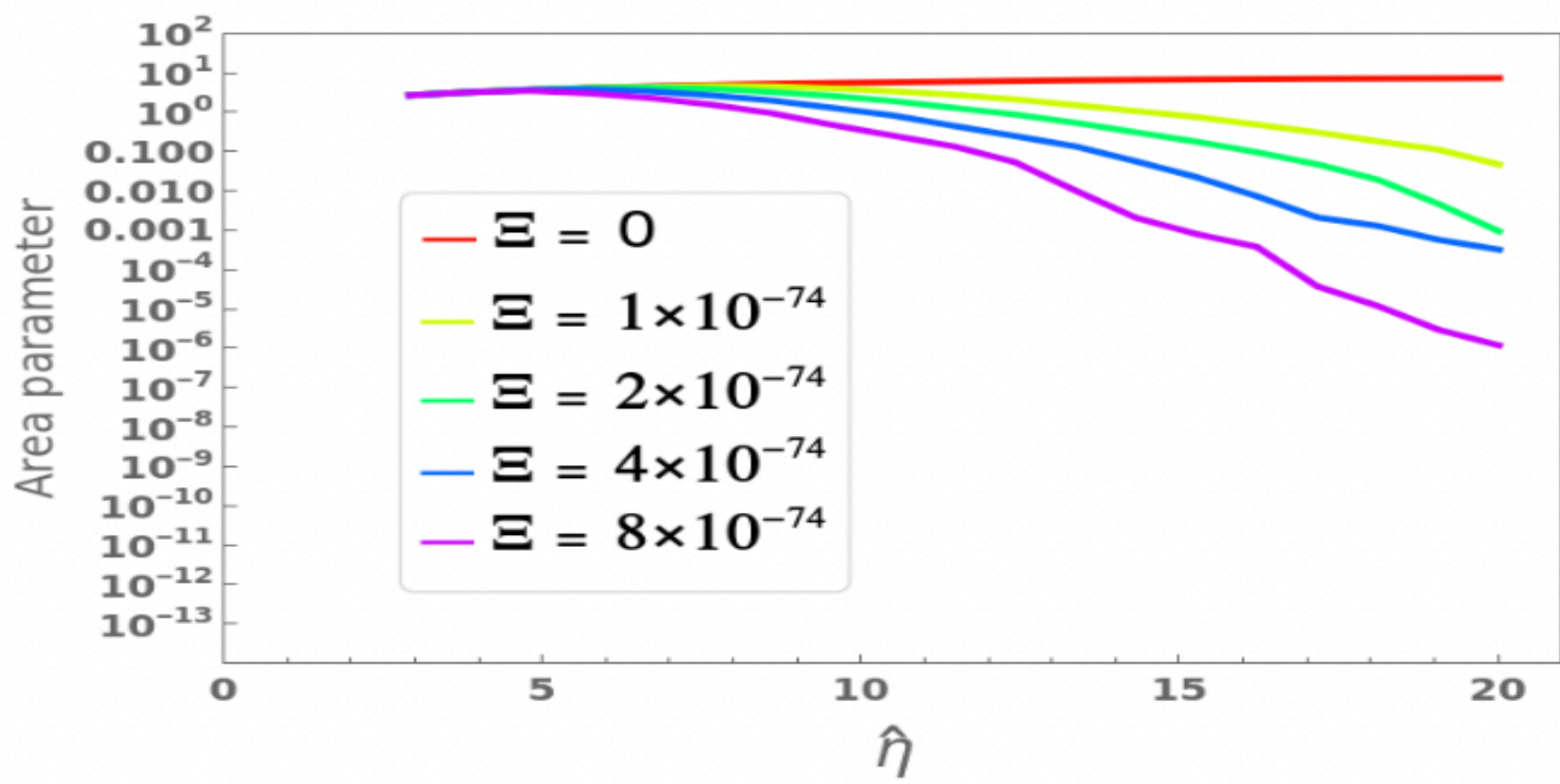


$\eta=26.2$



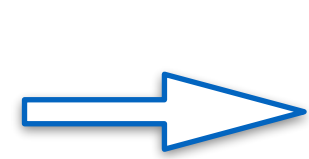
$$N_{\text{DW}} = 3$$

Domain wall Area parameter



DW decay

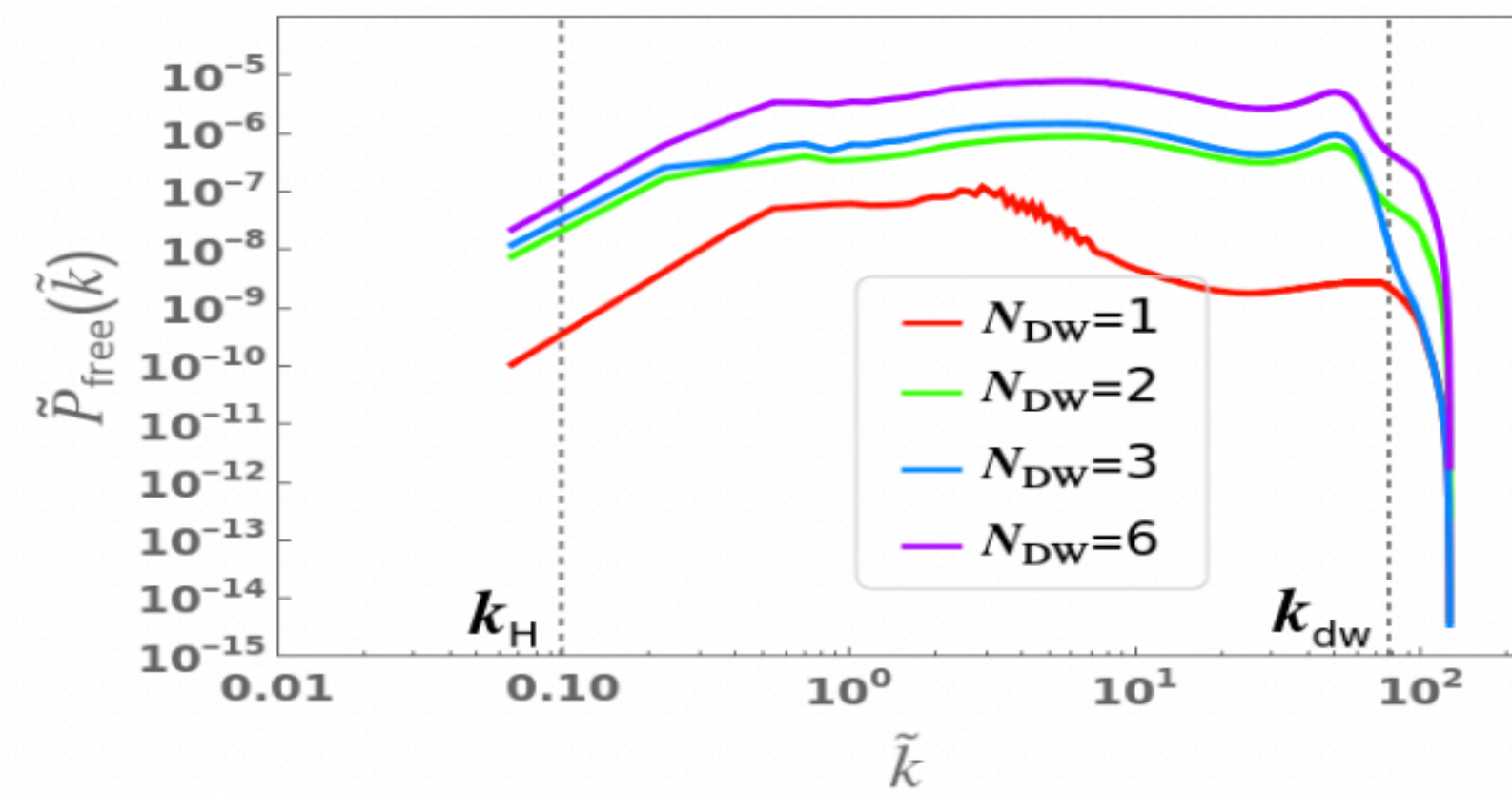
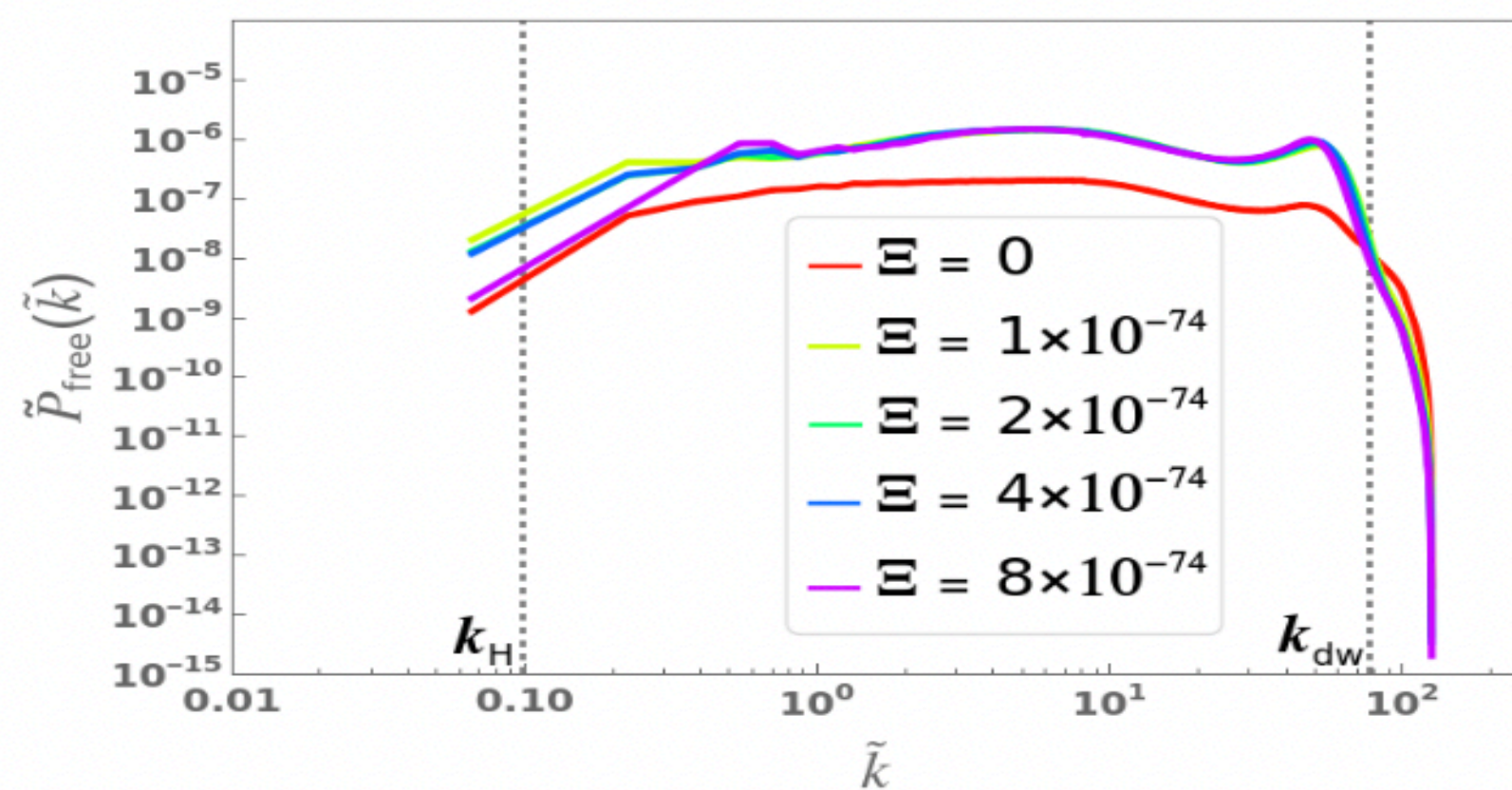
$$\Delta V = 2\Xi v^4(1 - \cos(2\pi/N_{DW})) \sim \rho_{\text{wall}} = \mathcal{A}\sigma_{\text{wall}}/t$$



$$t_{\text{dec}} = t_{\text{form}} \left(\frac{C_d \mathcal{A}_{\text{form}} \sigma_{\text{wall}}}{t_{\text{form}} \Xi N_{\text{DW}}^4 f_a^4 (1 - \cos(2\pi/N_{\text{DW}}))} \right)^{1/p}$$

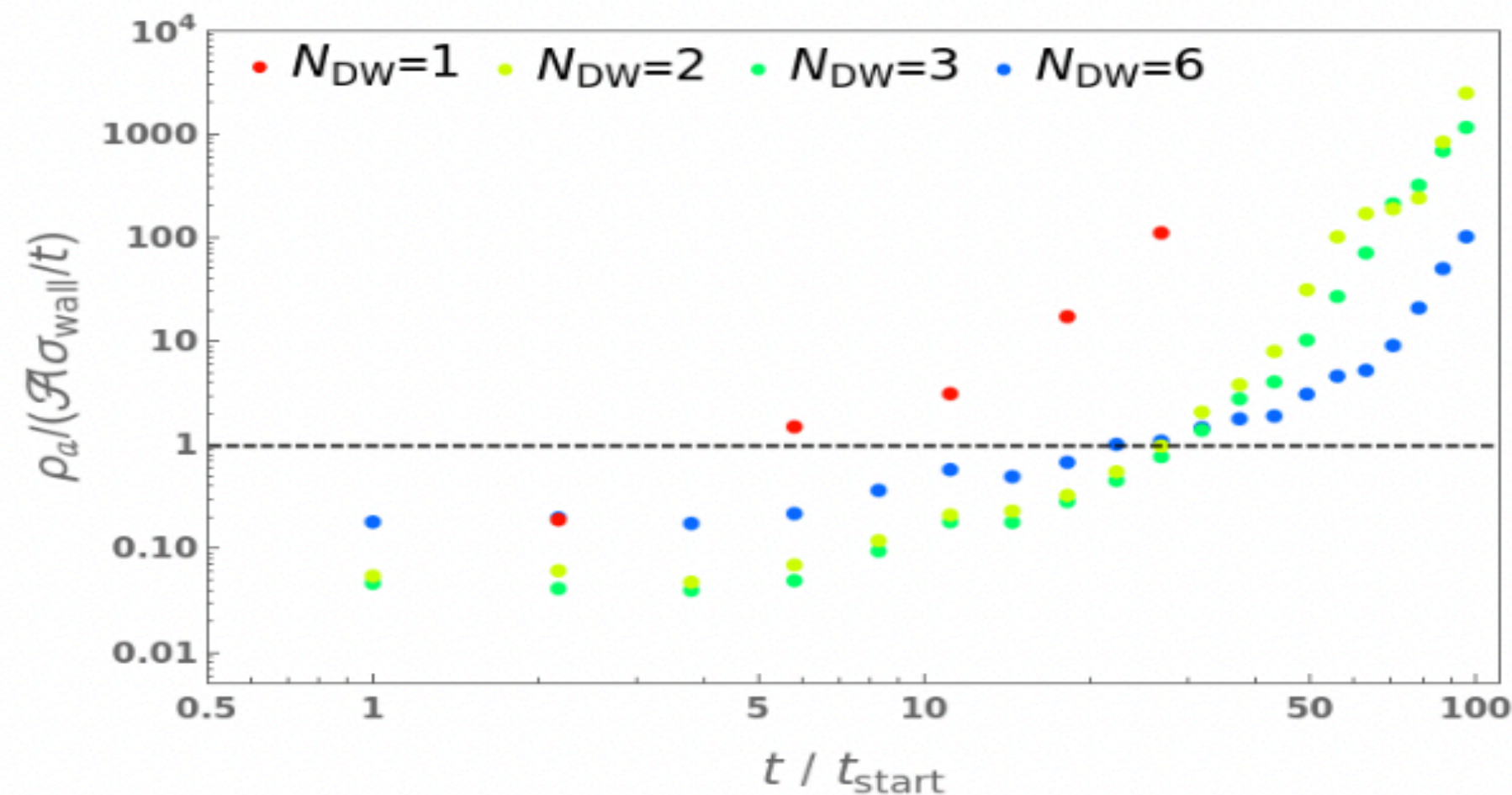
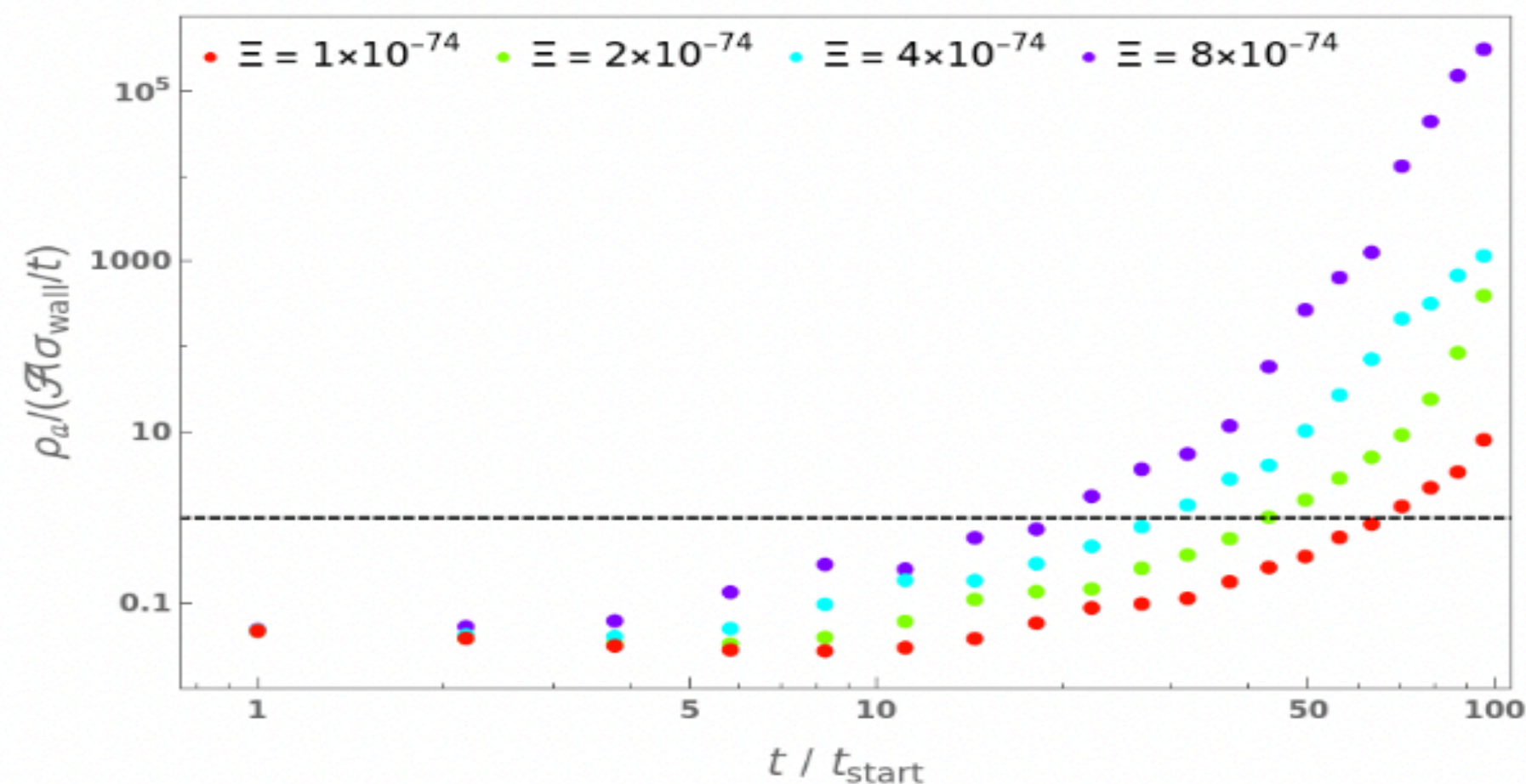
with $p = 1 + \ln C_d / \ln(t_{\text{dec}} / t_{\text{form}})$
 $C_d = \Delta V / (\mathcal{A}\sigma_{\text{wall}}/t) \sim \mathcal{O}(10^2)$

Domain wall decay to free axions



$$\frac{1}{2} \langle \dot{a}_{\text{free}}(\mathbf{k})^* \dot{a}_{\text{free}}(\mathbf{k}') \rangle = (2\pi)^3 \frac{2\pi^2}{k^3} P_{\text{free}}(k) \delta(\mathbf{k} - \mathbf{k}') \quad W(\mathbf{x}) = W_{\text{st}}(\mathbf{x}) \times W_{\text{dw}}(\mathbf{x}), \quad \dot{a}_{\text{free}}(\mathbf{x}) = W(\mathbf{x}) \dot{a}(\mathbf{x}), \quad \dot{a}(\mathbf{x}) = f_a \frac{\dot{\phi}_1 \phi_2 - \phi_1 \dot{\phi}_2}{\phi_1^2 + \phi_2^2}$$

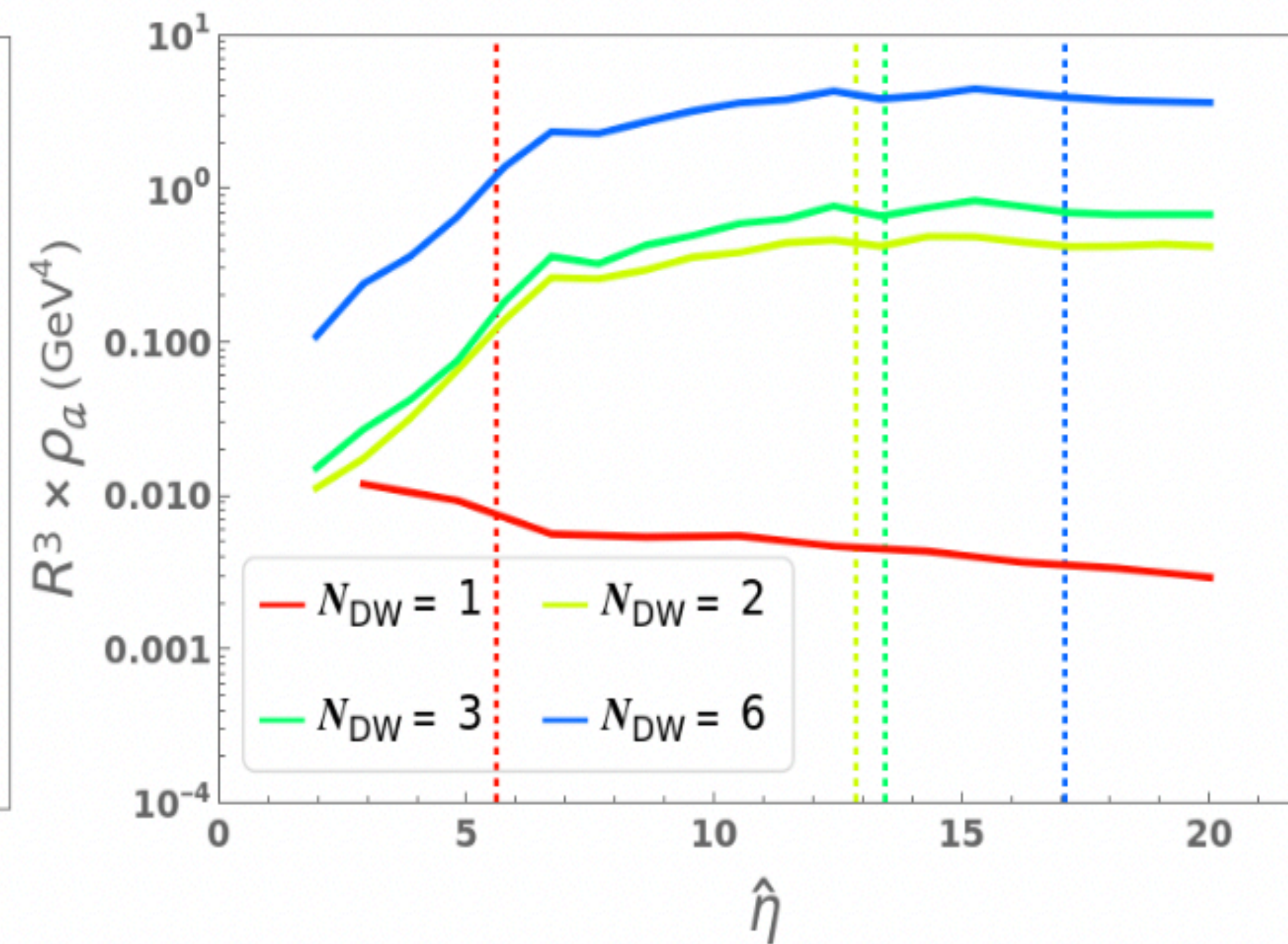
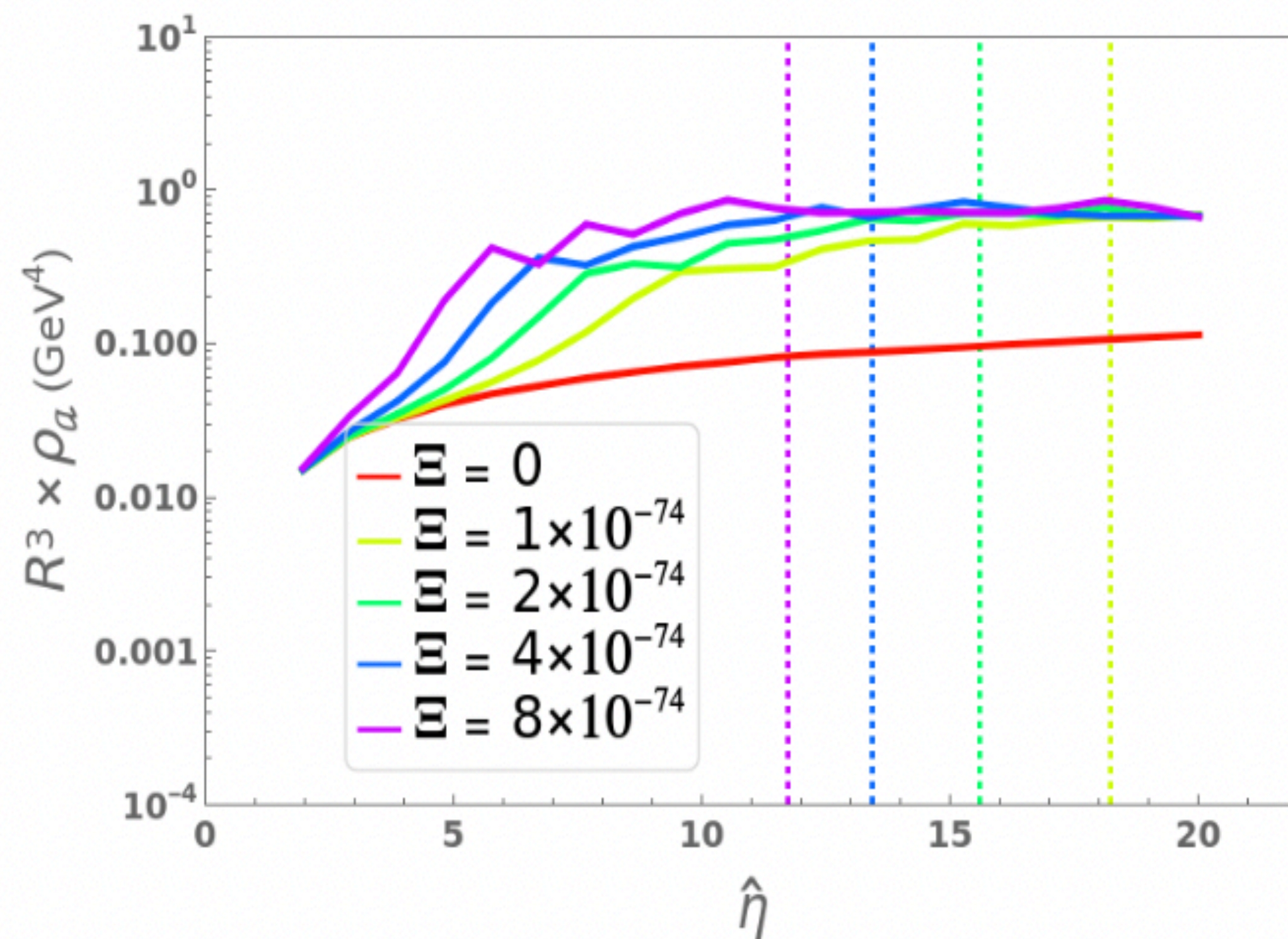
Window function $W_{\text{dw}}(\mathbf{x})$ will take 0 near the DW core and 1 near the true vacuum



$$\rho_a \simeq 2 \times \frac{1}{2} \langle \dot{a}_{\text{free}}^2(\mathbf{x}) \rangle$$

$$\rho_a \sim \mathcal{A} \sigma_{\text{wall}} / t \sim 8 \mathcal{A} m f_a^2 / t_{\text{dec}}$$

► Domain wall decay to free axions



$$\rho_a(t_{\text{dec}}) \simeq \frac{2(2p-1)f_a^4 N_{\text{DW}}^4}{(3-2p)C_d} \Xi \sin^2(\pi/N_{\text{DW}})$$

Axion energy density tends to be a constant at the final moment, and the energy density of the radiated free axion is (almost) proportional to the bias term and N_{DW}

► Domain wall decay to axion Dark matter

$$\Omega_{a,\text{new}}(t_0)h^2 = \Omega_a^{\text{DW}}h^2 + \Omega_{a,0}(t_0)h^2 + \Omega_{a,\text{st}}(t_0)h^2$$

Axion DM from Wall decay:

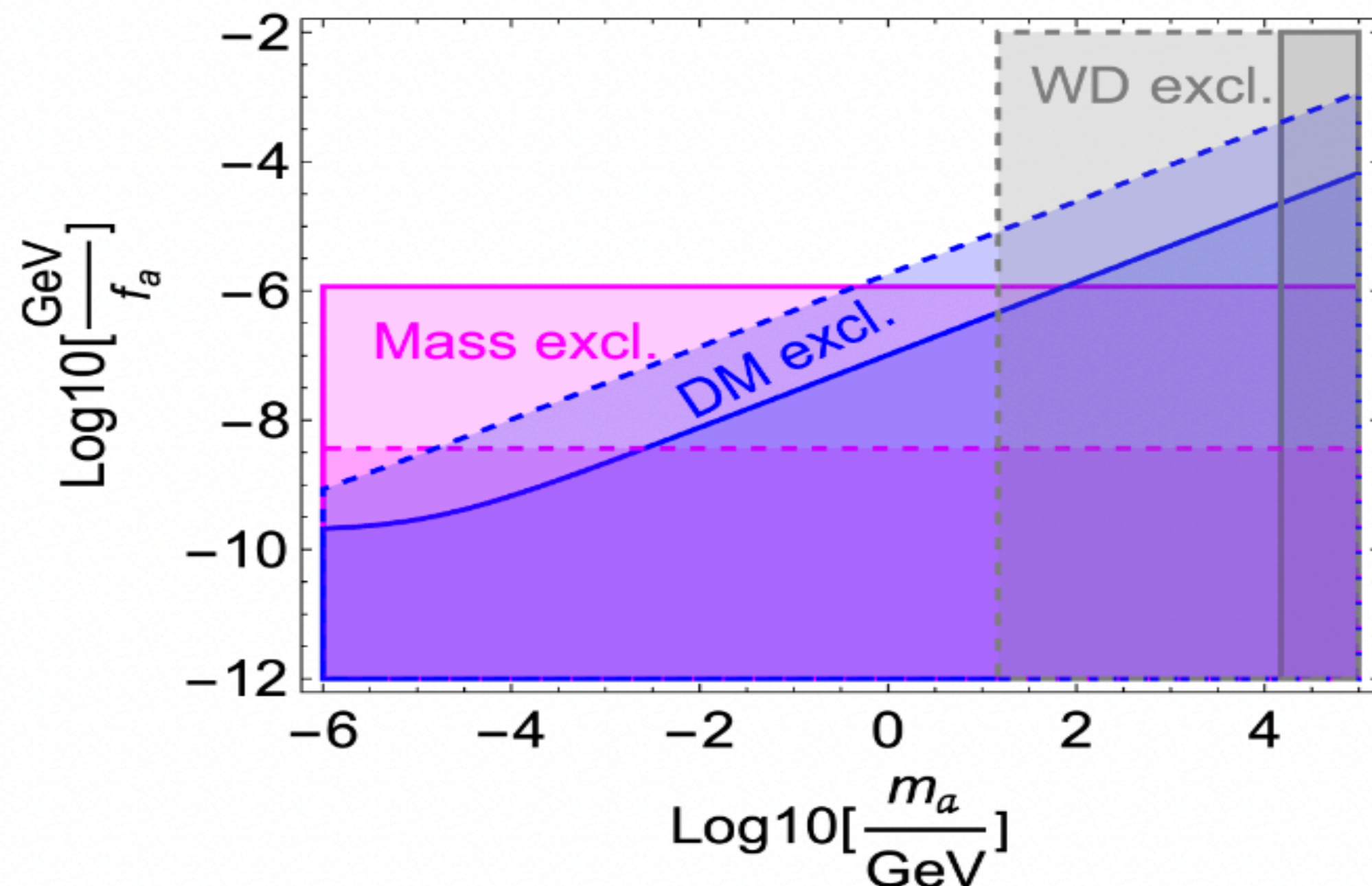
$$\Omega_a h^2 = \frac{\rho_a(t_0)}{\rho_{0,c}/h^2} = 1.024 \times 10^{-19} \left(\frac{8}{3}\right)^{3/(2p)} \left(\frac{2p-1}{3-2p}\right) C_d^{3/(2p)-1} \left(\frac{m}{\text{GeV}}\right)^{3/p-3/2} \left(\frac{f_a}{\text{GeV}}\right)^{4-3/p} \Xi^{1-3/(2p)} \left(\frac{\text{csc}(\pi/N_{\text{DW}})}{N_{\text{DW}}^2}\right)^{3/p-2}$$

Misalignment mechanism:

$$(\Omega_{a,0}(t_0)h^2 \simeq 4.63 \times 10^{-3} (f_{a,\text{new}}/10^{10}\text{GeV})^{1.19})$$

Axion DM from pure axion string:

$$\Omega_{a,\text{st}}(t_0)h^2 = 2.0 \times (f_a / 10^{12}\text{GeV})^{1.19}$$



WD excl.:

$$t_{\text{dec}} < t_{\text{WD}} \quad t_{\text{WD}} = \frac{3}{16\pi G \sigma_{\text{wall}}}$$

Mass excl.:

bias term dominating over the QCD instanton effect in the contributions to axion mass term

$$\Xi < 2 \times 10^{-45} N_{\text{DW}}^{-2} \left(\frac{10^{10}\text{GeV}}{f_a}\right)^4$$

DM excl.

$$\Omega_a(t_0)h^2 \leq 0.12$$

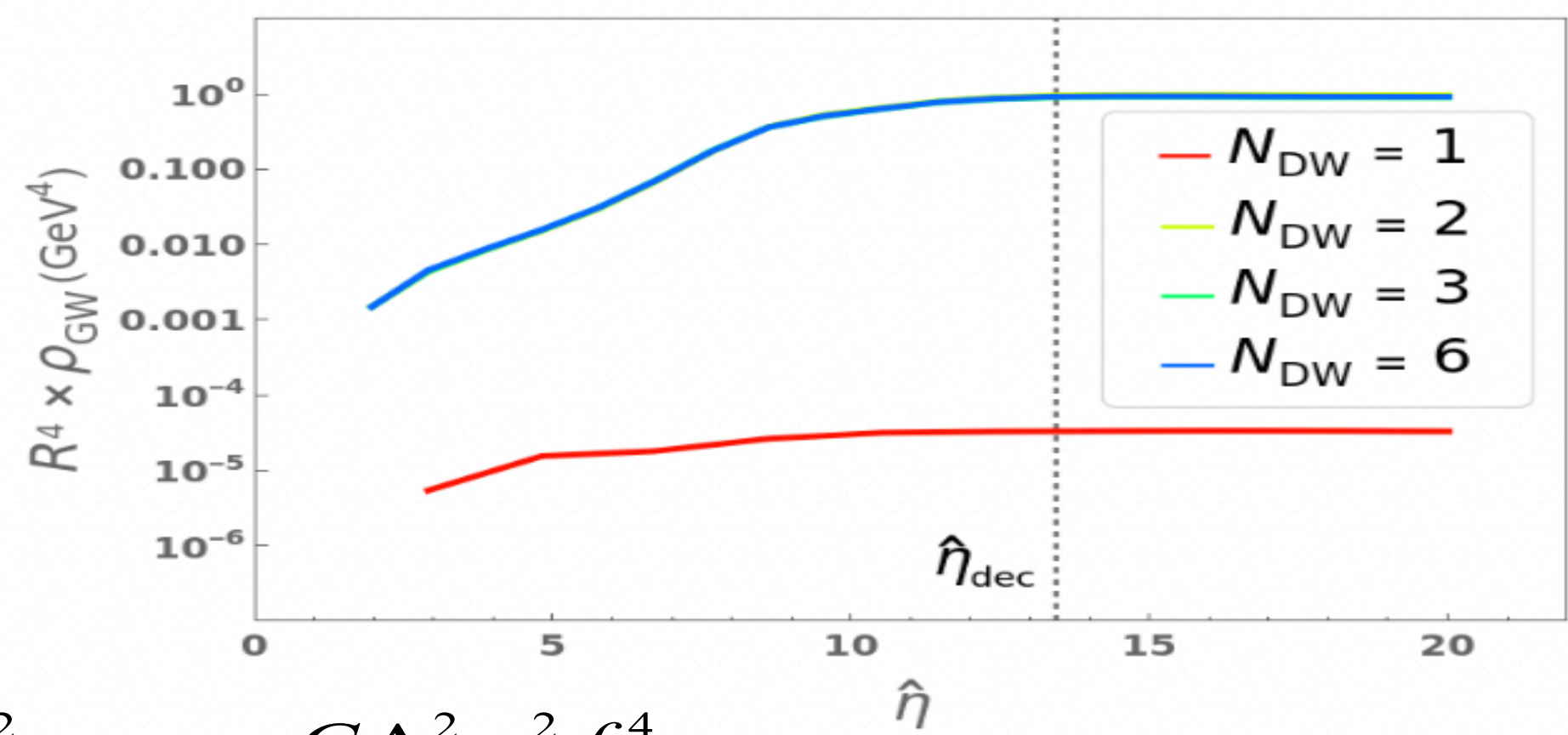
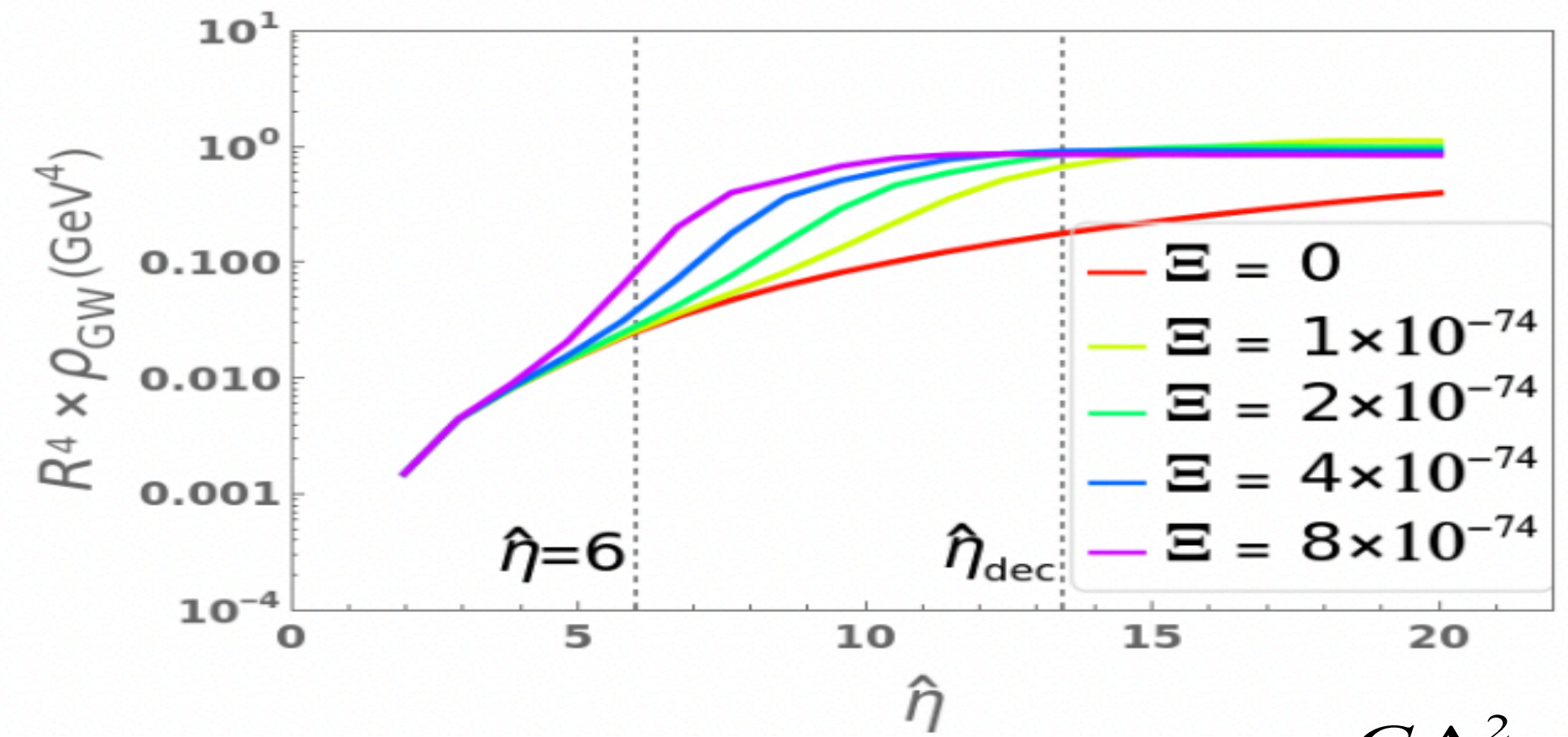
String-wall GW

$$\ddot{h}_{ij} + 3\frac{\dot{R}}{R}\dot{h}_{ij} - \frac{\nabla^2}{R^2}h_{ij} = \frac{16\pi G}{R^2}\Pi_{ij}^{\text{TT}}$$

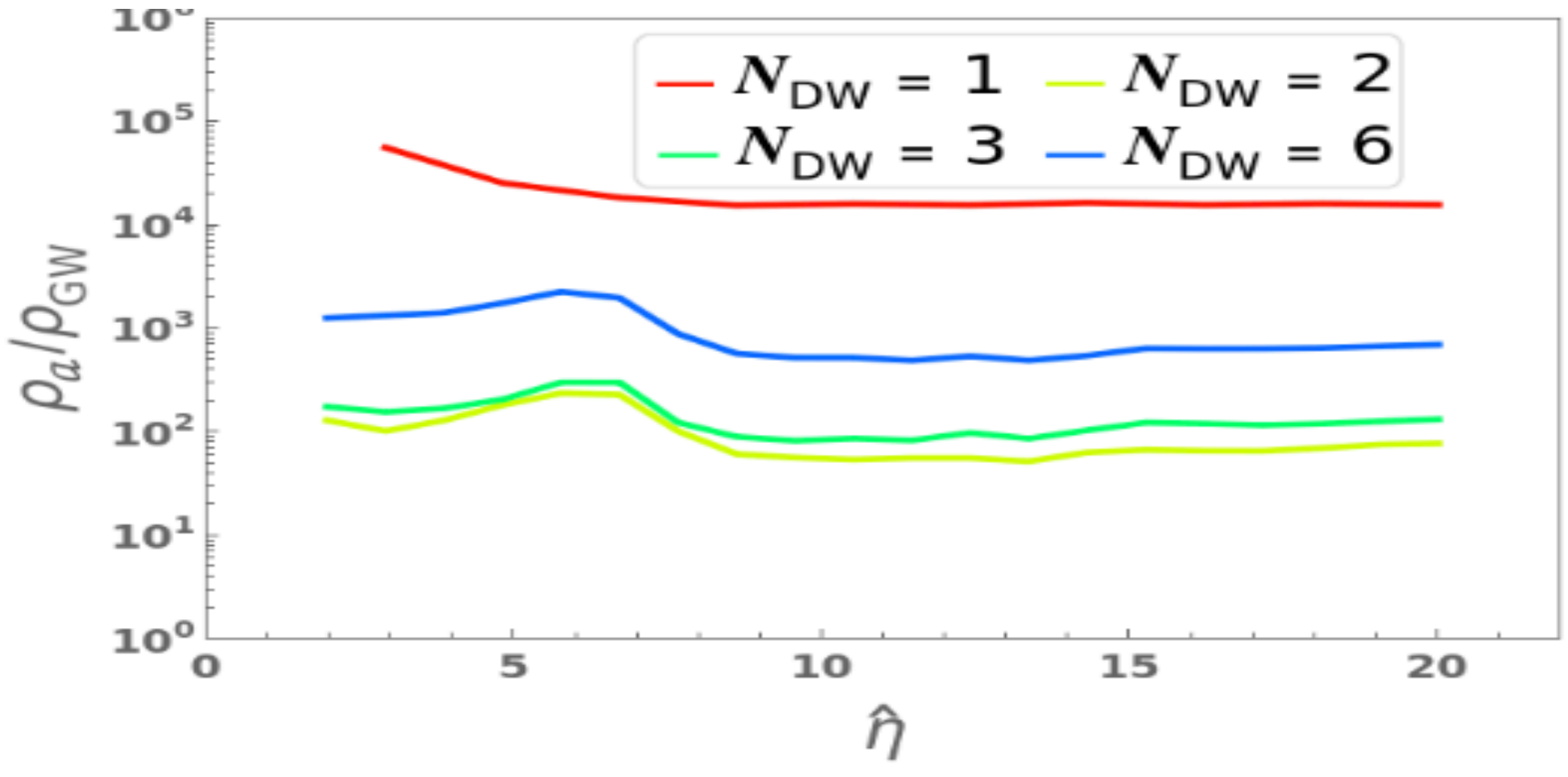
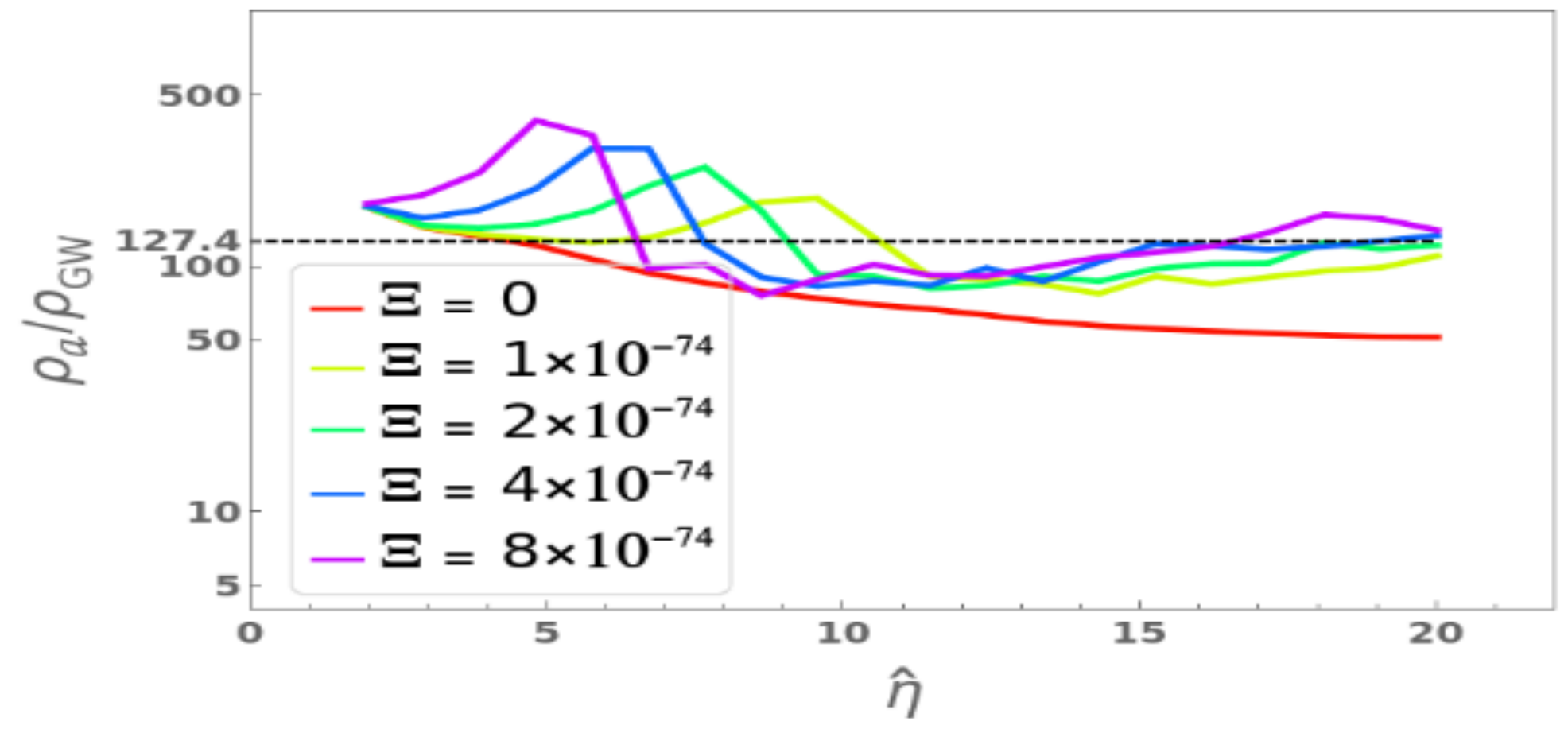
$$\rho_{\text{GW}}(t) = t_{00} = \frac{1}{32\pi G}\langle\partial_\mu h_{ij}^{\text{TT}}(\mathbf{x}, t)\partial_\nu h^{\text{TT},ij}(\mathbf{x}, t)\rangle|_{\mu=\nu=0} = \frac{1}{32\pi G}\langle\dot{h}_{ij}(\mathbf{x}, t)\dot{h}_{ij}(\mathbf{x}, t)\rangle$$

$$\Pi_{ij} \equiv T_{ij} - pg_{ij} = T_{ij} - \frac{\delta_{ij}}{3}\sum_l T_{ll}$$

$$= (\partial_i\phi_1\partial_j\phi_1 + \partial_i\phi_2\partial_j\phi_2) - \frac{\delta_{ij}}{3}\sum_l(\partial_l\phi_1\partial_l\phi_1 + \partial_l\phi_2\partial_l\phi_2)$$

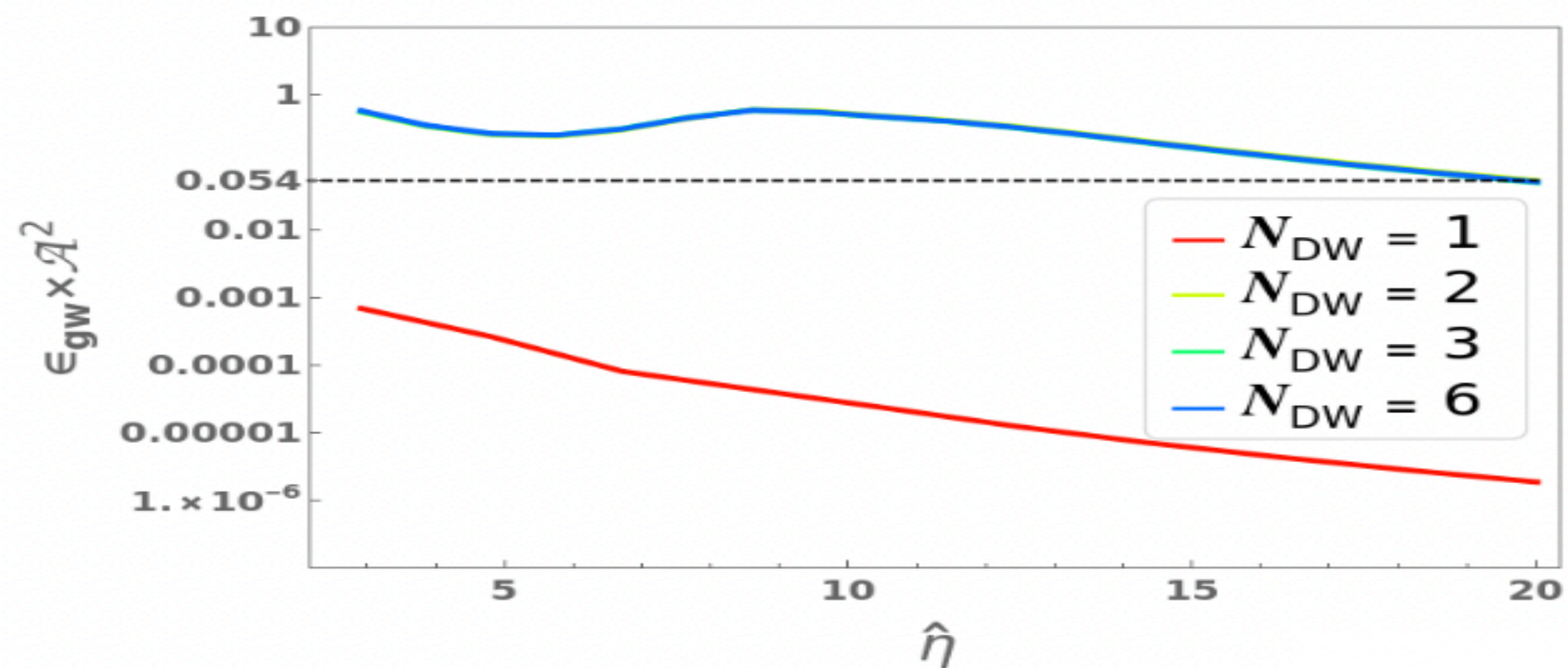


$$\rho_{\text{gw}} = \varepsilon_{\text{gw}} GA^2 \sigma_{\text{wall}}^2 \propto \varepsilon_{\text{gw}} GA^2 m^2 f_a^4$$

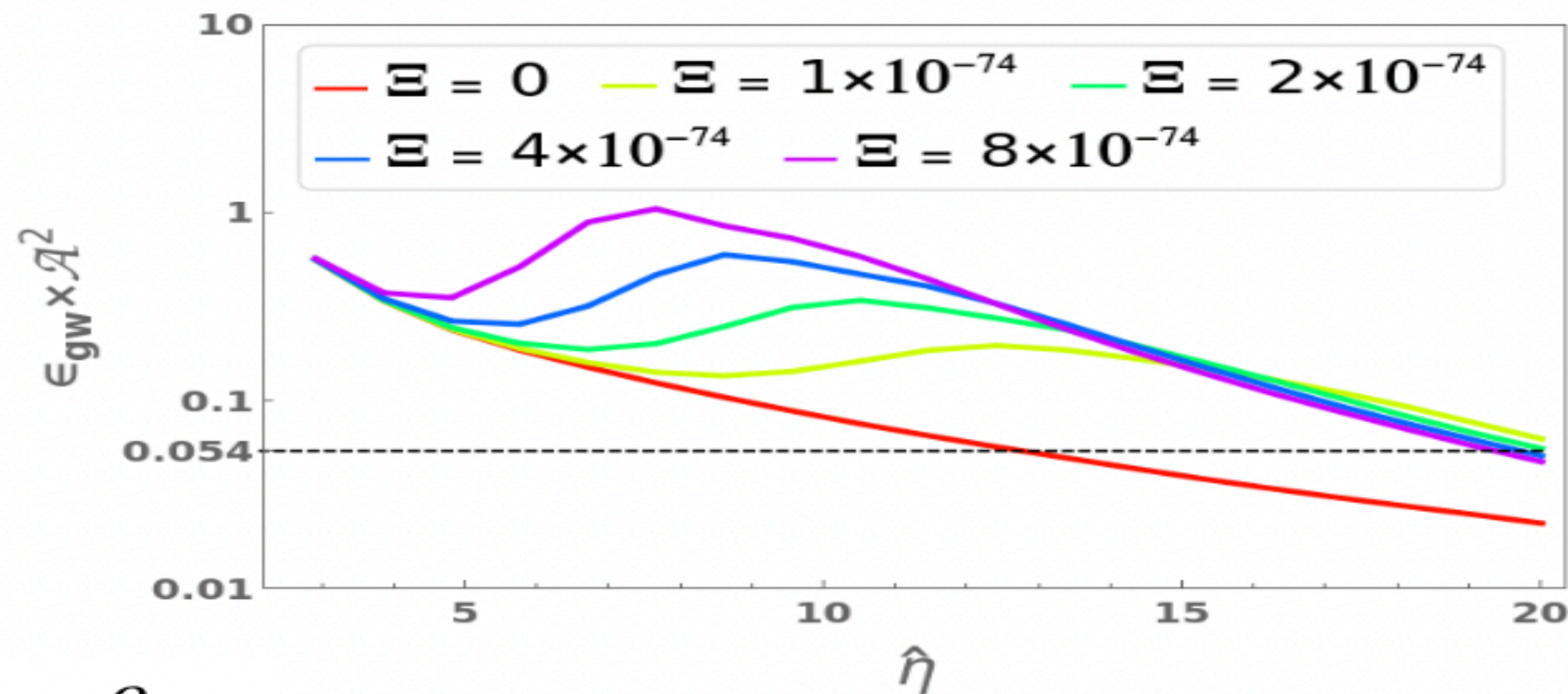


axion string-wall networks mainly decay to axion

String-wall GW

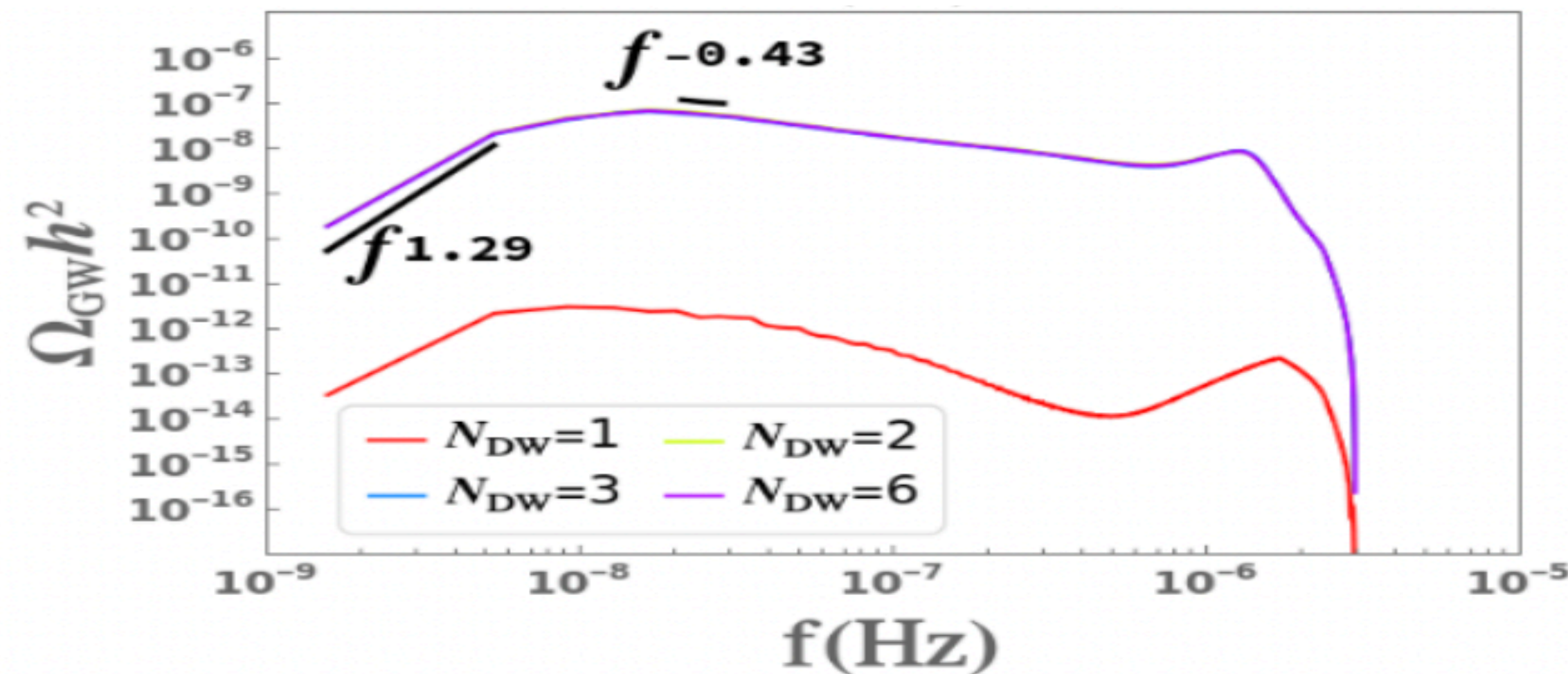
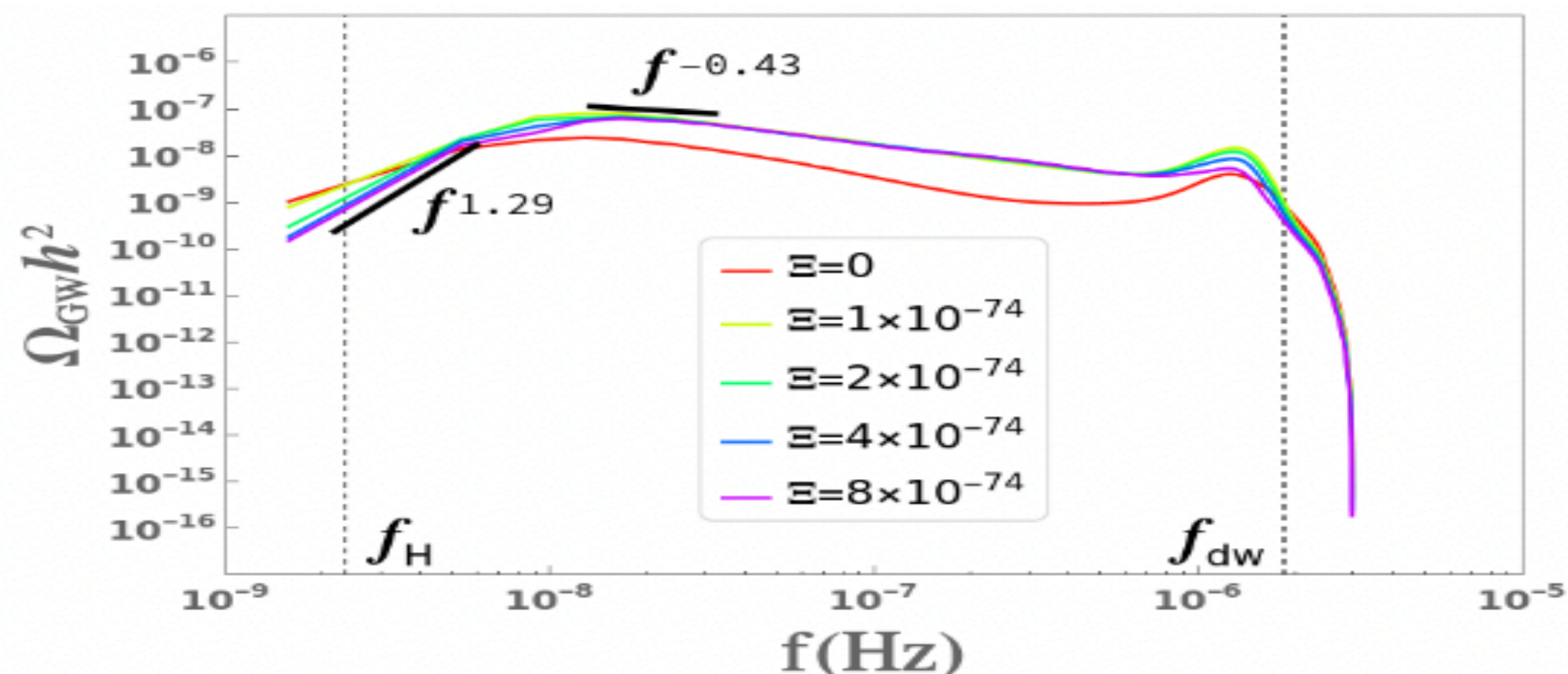


Efficiency parameter



$$\epsilon_{\text{gw}} \mathcal{A}^2 = \frac{\rho_{\text{gw}}}{G\sigma_{\text{wall}}^2} \sim \text{Const.}$$

$$\rho_{\text{gw}} \propto Gm^2 f_a^4$$

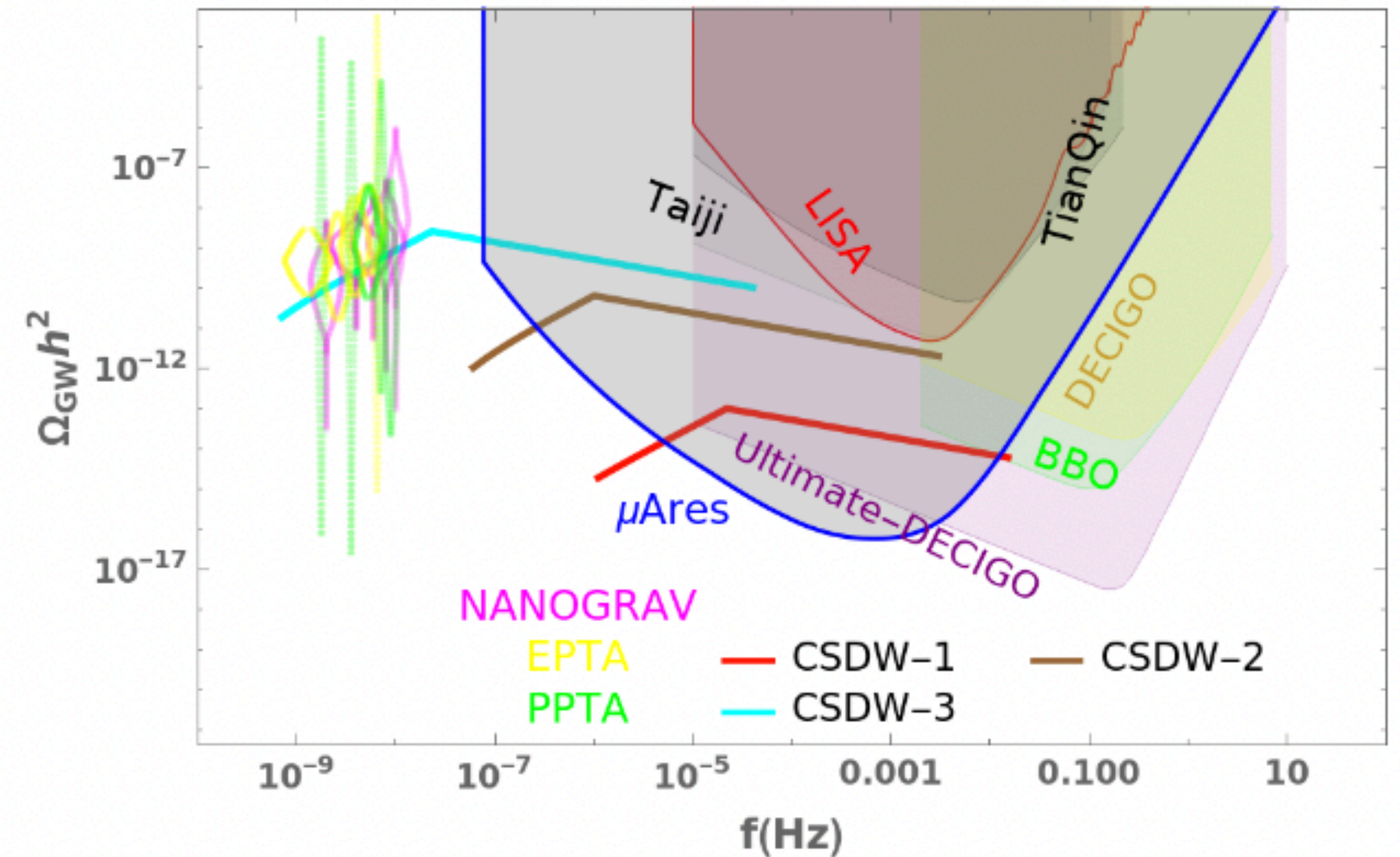
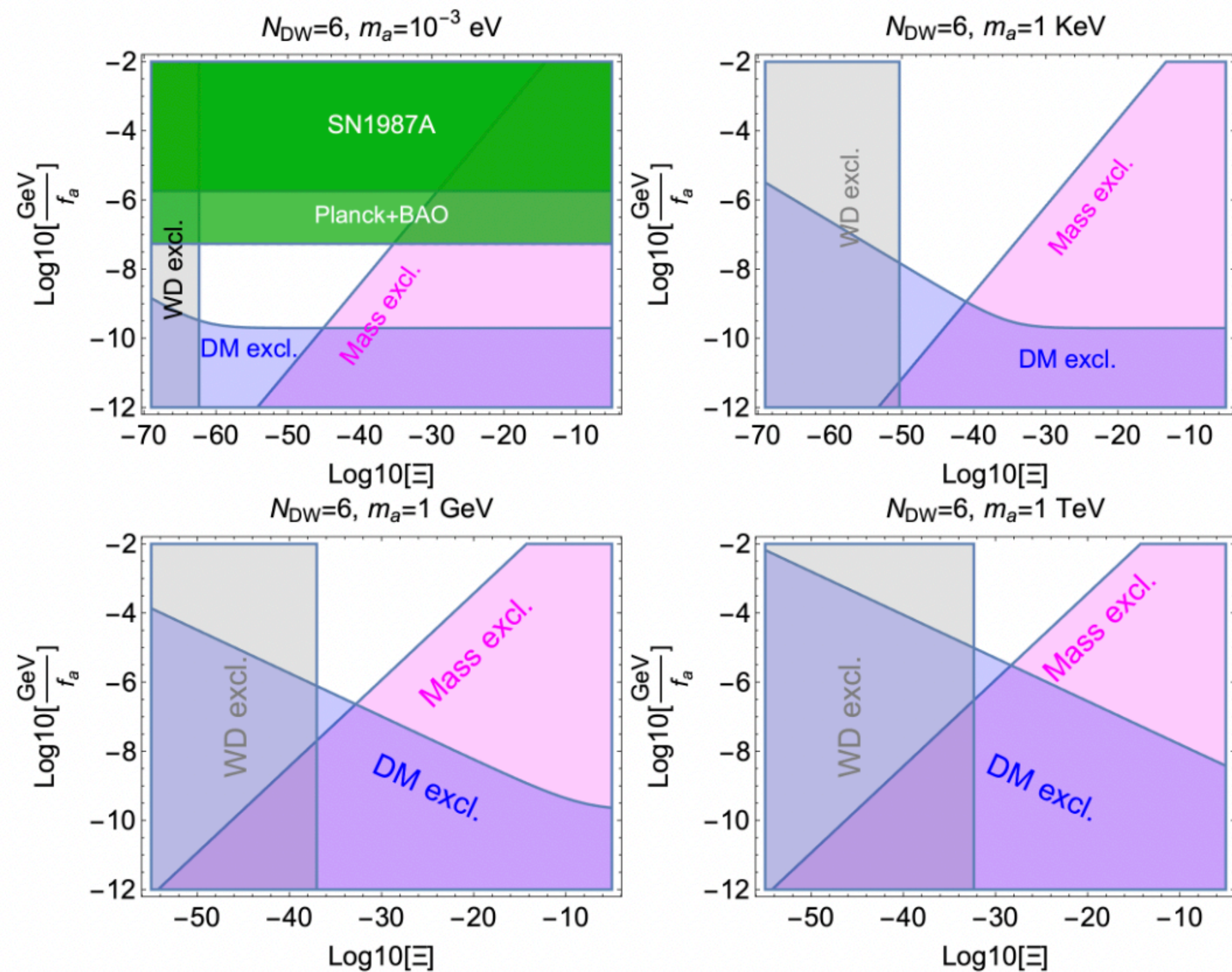


$$\begin{aligned} \Omega_{\text{GW}} &\equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln k} \\ &= \frac{1}{32\pi G} \frac{1}{(2\pi)^3 V} \int d\Omega |\mathbf{k}|^3 |\dot{h}_{ij}(\mathbf{k}, t)|^2 \end{aligned}$$

$$\begin{aligned} \Omega_{\text{GW},0} h^2 &= \frac{h^2}{\rho_{c,0}} \frac{d\rho_{\text{GW},0}}{d \ln k_{p,0}} \\ &= \Omega_{\text{rad},0} h^2 \left(\frac{g_0}{g_e} \right)^{1/3} \left\{ \frac{1}{\rho_{c,e}} \frac{d\rho_{\text{GW},e}}{d \ln k_{co,e}} \right\} \end{aligned}$$

► Axion DM and GW from String-wall

Our research provides the possibility of searching for axion models or constraining model parameters through GW detection experiments



Simultaneously explanation of DM and GWs with axion string-wall networks, **only for $N_{DW} > 1$ of ALP scenarios.**

For $N_{DW} = 1$, the GW energy density appears undetectable for QCD axions and axion-like particles.

► Axion DM with PMF

- The action and potential are

$$S = \int d^4x \sqrt{-g} \left[-\partial_\mu \phi \partial^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi) - \alpha \frac{\phi}{v} F_{\mu\nu} \tilde{F}^{\mu\nu} \right]$$

$$m_a(T)^2 = \frac{\alpha_a \Lambda^4}{f_a^2 (T/\Lambda)^{6.68}} \quad (T > 100 \text{ MeV}, f_a = \frac{v}{N_{DW}})$$

$$V(\phi) = \frac{m_a(T)^2 v^2}{N_{DW}^2} \left(1 - \cos \left(N_{DW} \frac{\phi}{v} \right) \right)$$

$$V_{eff}(\phi) = V(\phi) + \alpha \frac{\phi}{v} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- The EOMs are

$$\phi'' = \partial_i \partial_i \phi - 2\mathcal{H}\phi' - a^2 \frac{dV}{d\phi} - a^2 \frac{\alpha}{4v} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

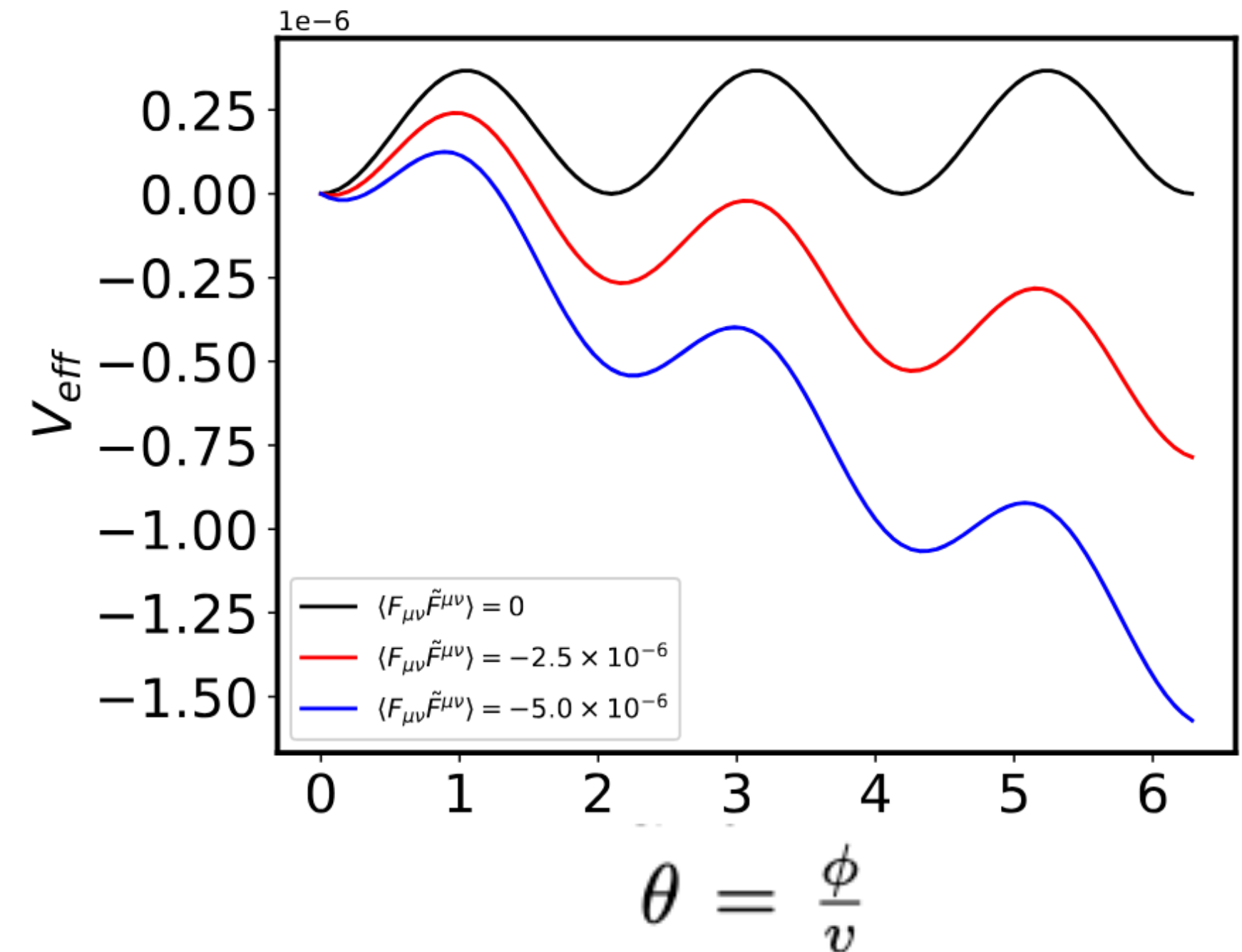
$$E'_i = -\epsilon_{ijk} \partial_j B_k - \frac{\alpha}{v} \phi' B_i + \frac{\alpha}{v} \epsilon_{ijk} \partial_j \phi E_k$$

$$\partial_i A'_i = -\frac{\alpha}{v} \partial_i \phi \epsilon_{ijk} \partial_j A_k$$

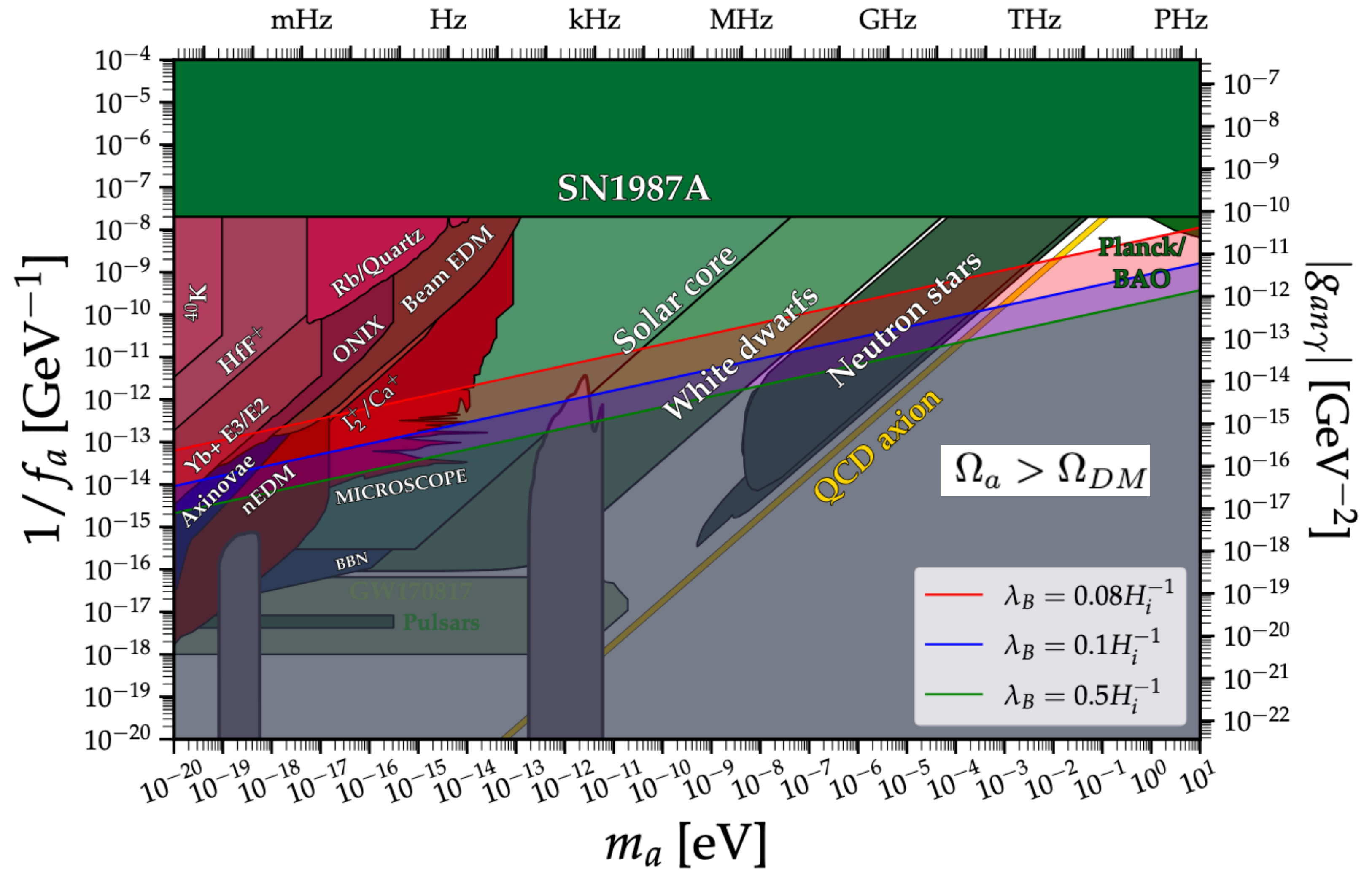
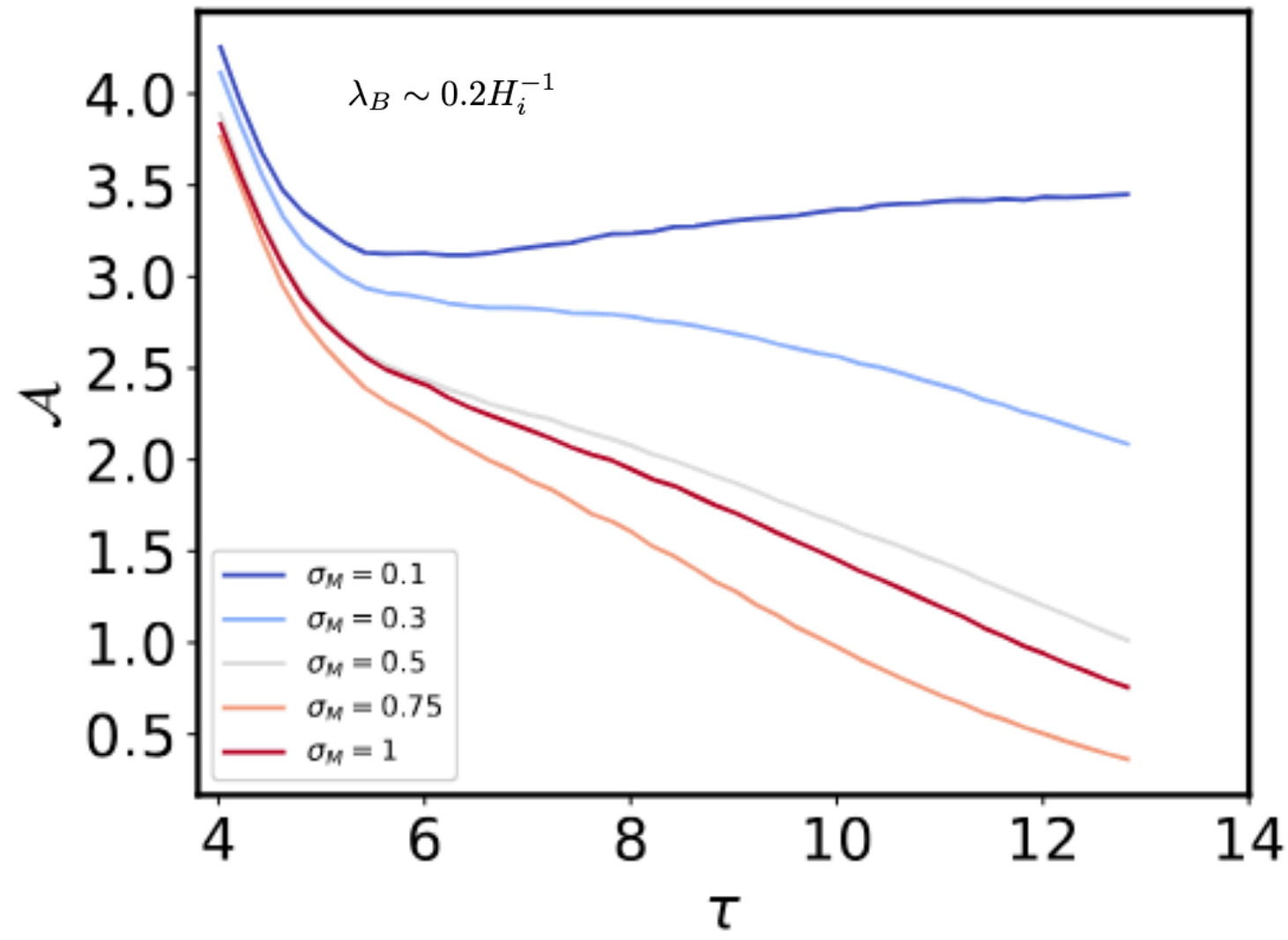
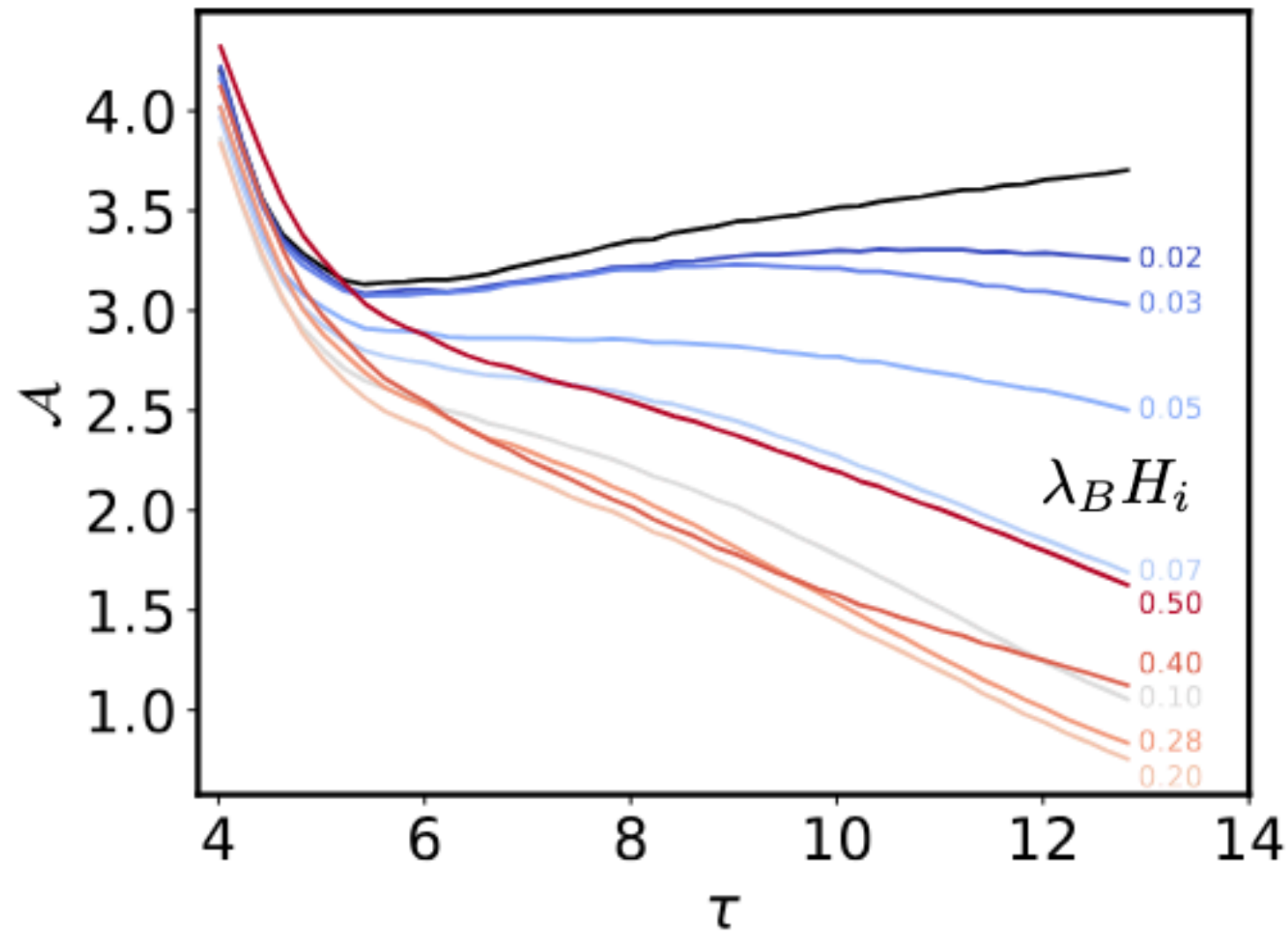
$$E_i = \frac{A'_i}{a^2}, \quad B_i = \frac{1}{a^2} \epsilon_{ijk} \partial_j A_k \quad F_{\mu\nu} \tilde{F}^{\mu\nu} = 4 \sum_i E_i B_i \text{ is Chern-Simons (CS)}$$

magnetic fields $B_i(\vec{k}) = B_{ini} \Theta(k - k_{UV}) (\delta_{ij} - \hat{k}_i \hat{k}_j - i\sigma_M \epsilon_{ijl} \hat{k}_l) g_j(\vec{k}) k^n$

$g_j(\vec{k})$ is Gauss random field and $\sigma_M \in [-1, 1]$ determines the helicity of magnetic field. n is the spectrum index. (2409.16124)

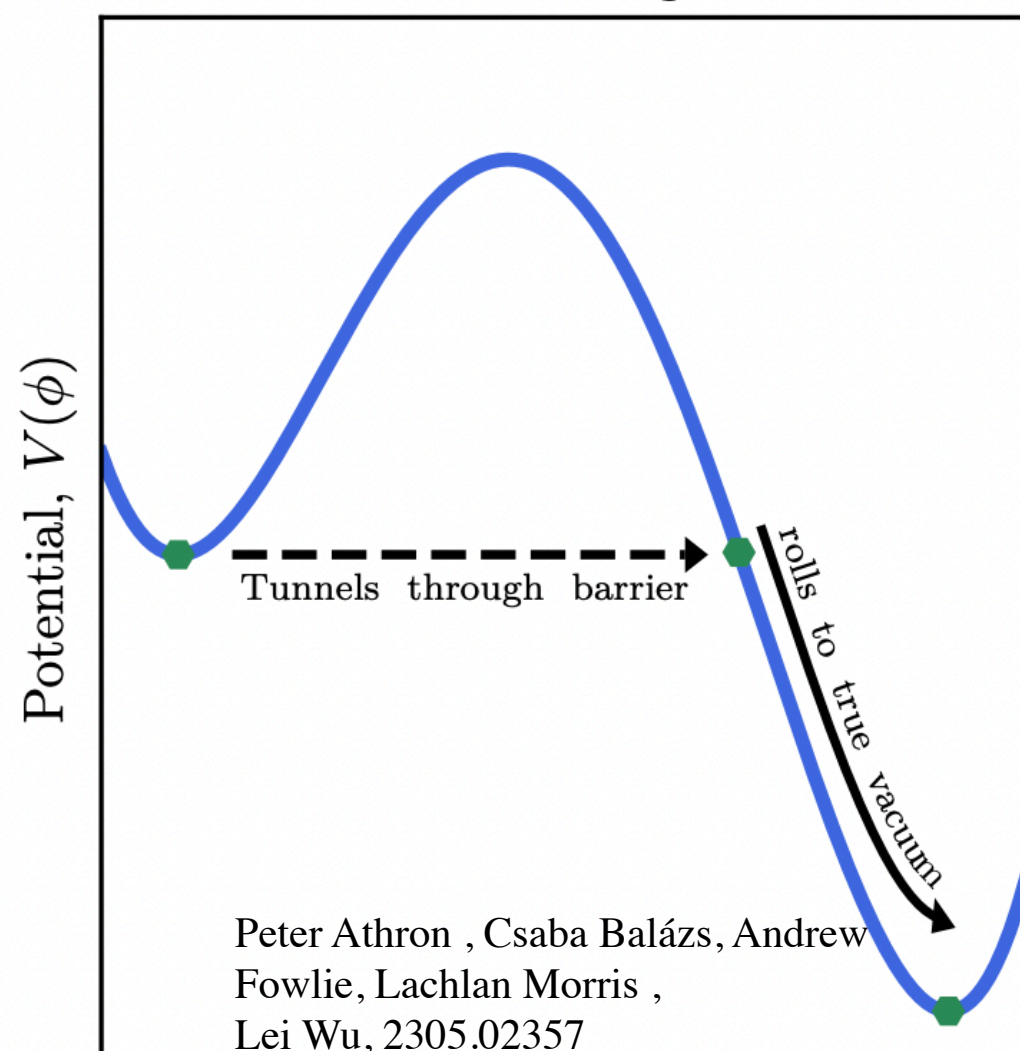


► Axion DM with PMF

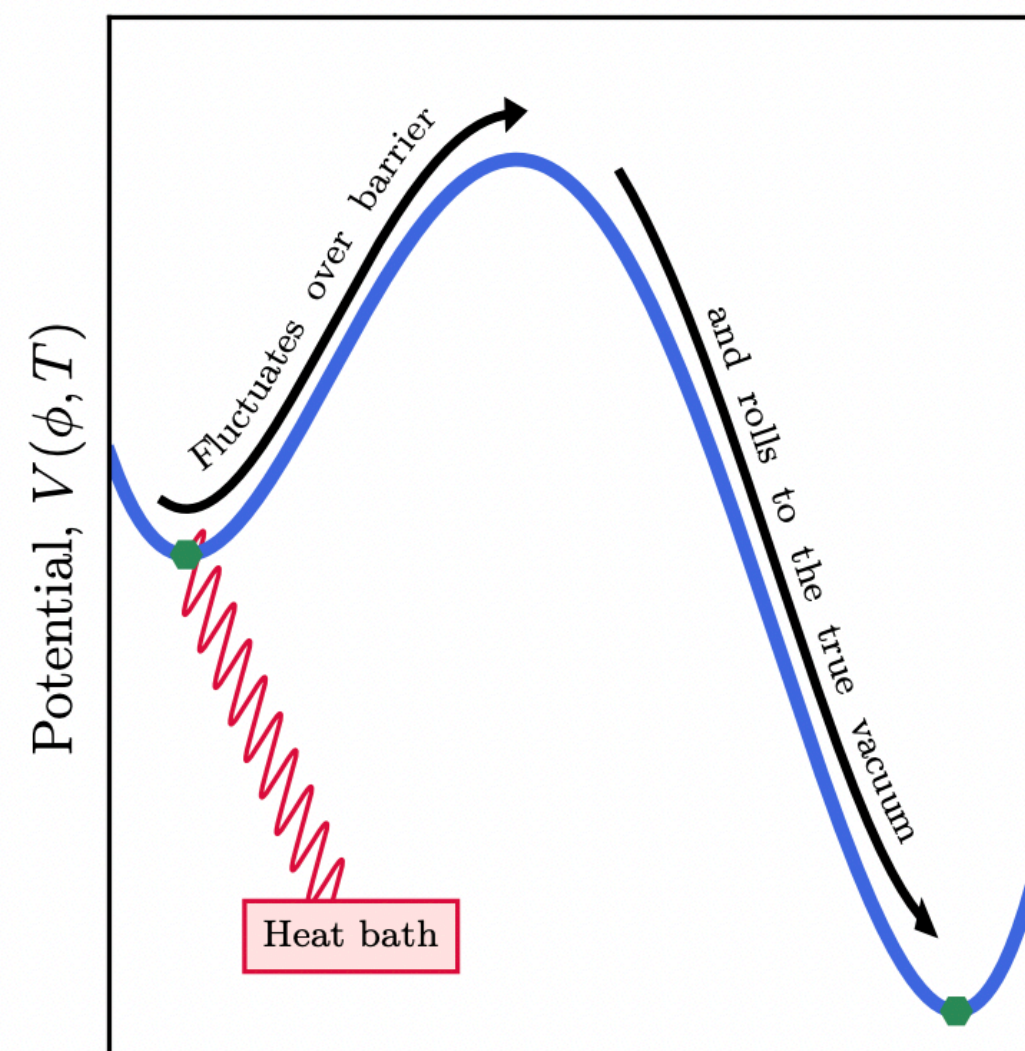


Vacuum decay and FOPT

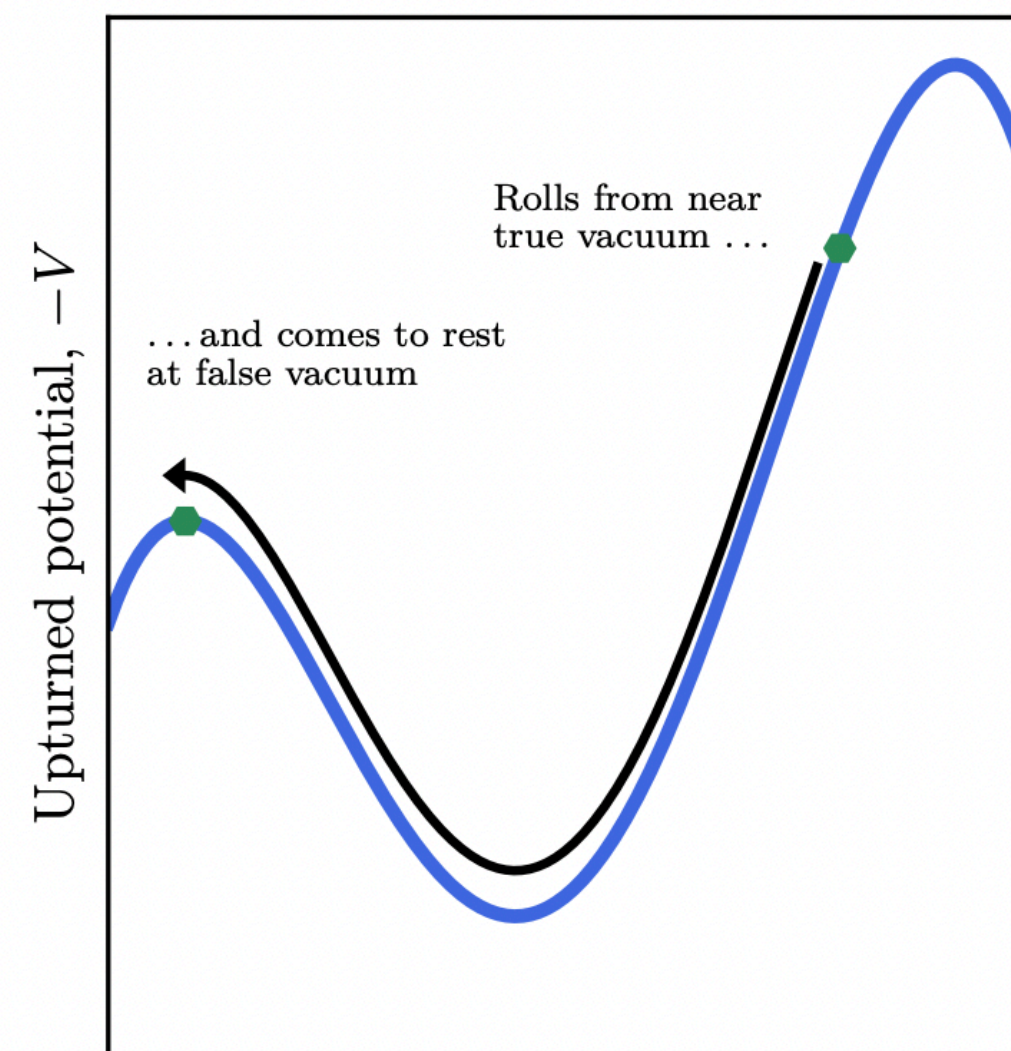
Tunneling



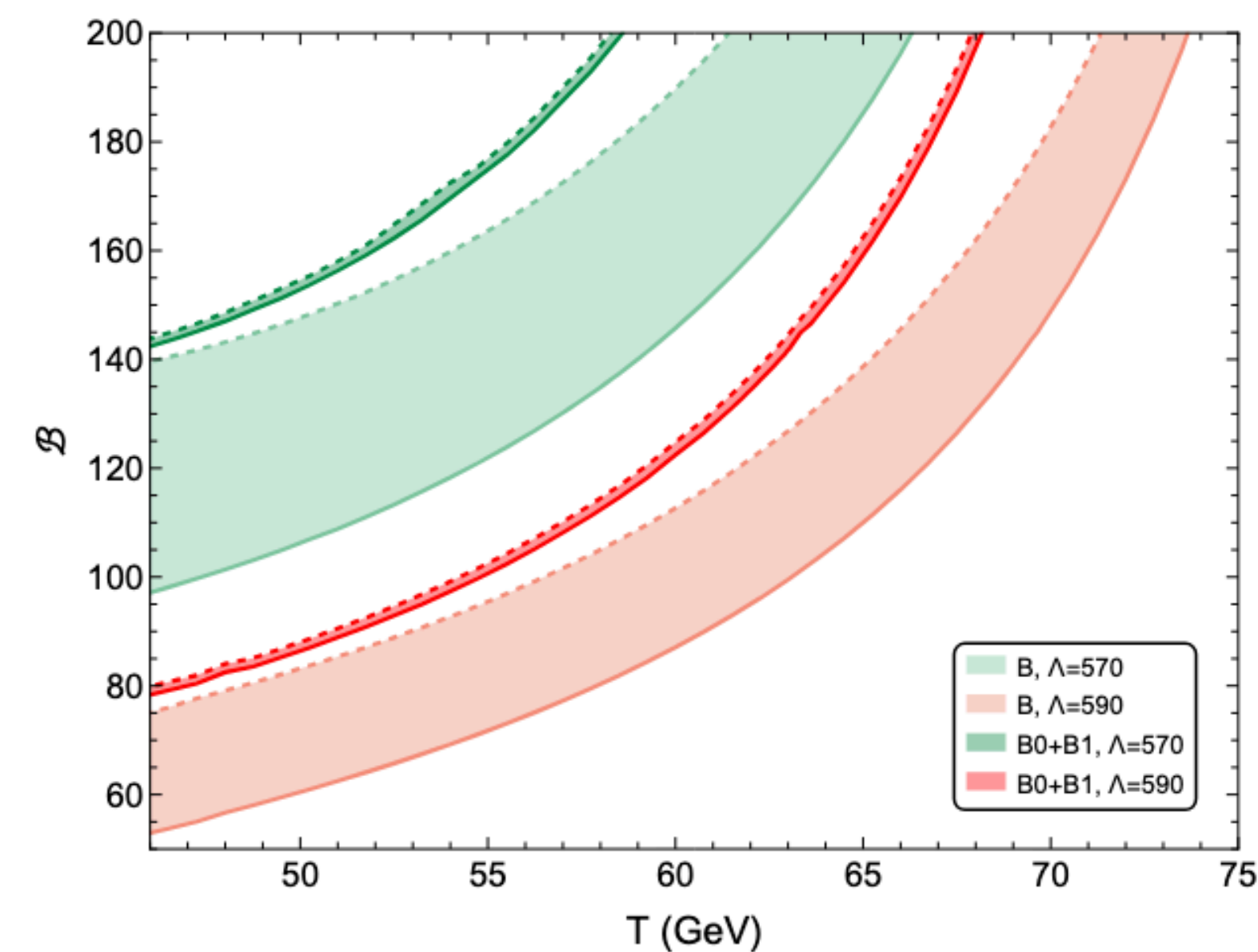
Thermal fluctuations



Bounce solution

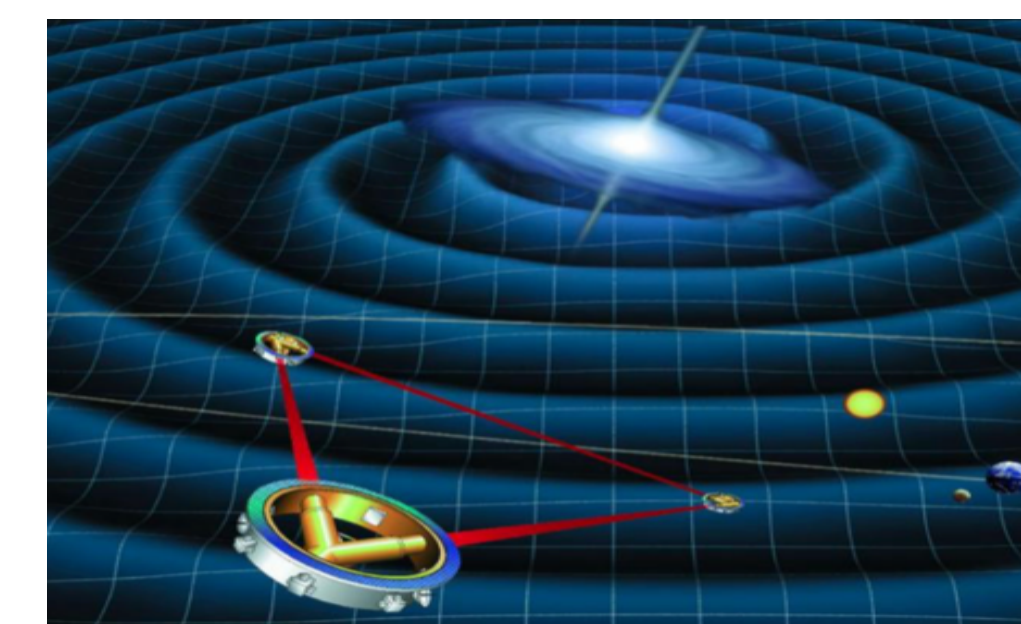
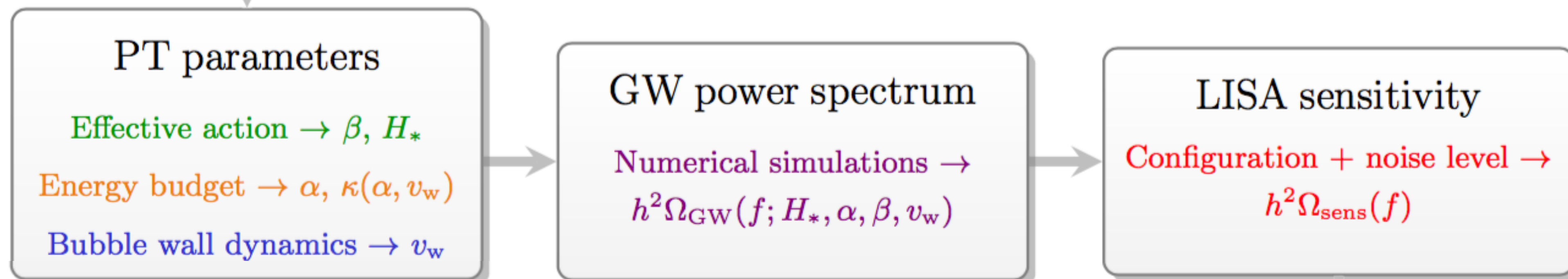


$$p(t; T) \equiv \Gamma/V = |A(T)|e^{-B(T)/T}$$



Thermal EFT+MC lattice simulation

Classical lattice simulation



LISA, TianQin, Taiji, ...

Wigner function in field theory

$$P_R(T) \equiv \int_R dx |\psi(x, T)|^2 \quad \longrightarrow \quad P_R(t) \equiv \int_R \mathcal{D}\phi |\Psi(\phi, t)|^2$$

$$W(q, p; t) \sim |\Psi(x, t)|^2$$

$$\Psi(\phi, t) = \langle \phi(x) | \Psi(t) \rangle = \int \mathcal{D}\phi_i \langle \phi | \hat{U}(t|t_0) | \phi_i \rangle \langle \phi_i(\mathbf{x}) | \Psi(t_0) \rangle$$

$$W(q, p; t) = \int du e^{-\frac{i}{\hbar} pu} \left\langle q + \frac{u}{2} \left| \hat{\rho}(t) \right| q - \frac{u}{2} \right\rangle \quad \longrightarrow \quad W[\phi(x), \Pi(x); t] = \int \mathcal{D}\varphi(x) \exp \left[-\frac{i}{\hbar} \int dx \Pi(x) \varphi(x) \right] \times \left\langle \phi(x) + \frac{\varphi(x)}{2} \left| \hat{\rho}(t) \right| \phi(x) - \frac{\varphi(x)}{2} \right\rangle$$

Wigner function: (ϕ, Π) phase space Quasi-probability distribution

$$\hat{\rho}(0) = \frac{1}{Z} e^{-\beta \hat{H}}$$

$$W[\phi(x), \Pi(x); 0] = \frac{1}{Z} \int \mathcal{D}\varphi(x) \exp \left[-\frac{i}{\hbar} \int dx \Pi(x) \varphi(x) \right] \times \left\langle \phi(x) + \frac{\varphi(x)}{2} \left| e^{-\beta \hat{H}} \right| \phi(x) - \frac{\varphi(x)}{2} \right\rangle$$

$$\approx \frac{1}{Z} \exp \left[-\beta \int dx \left[\frac{1}{2} \Pi^2(x) + \frac{1}{2} (\nabla \phi(x))^2 + V(\phi) \right] \right]$$

$$Z \equiv \text{Tr} \hat{\rho} = \int \mathcal{D}\phi(x) \frac{\mathcal{D}\Pi(x)}{2\pi} W[\phi, \Pi; t]$$

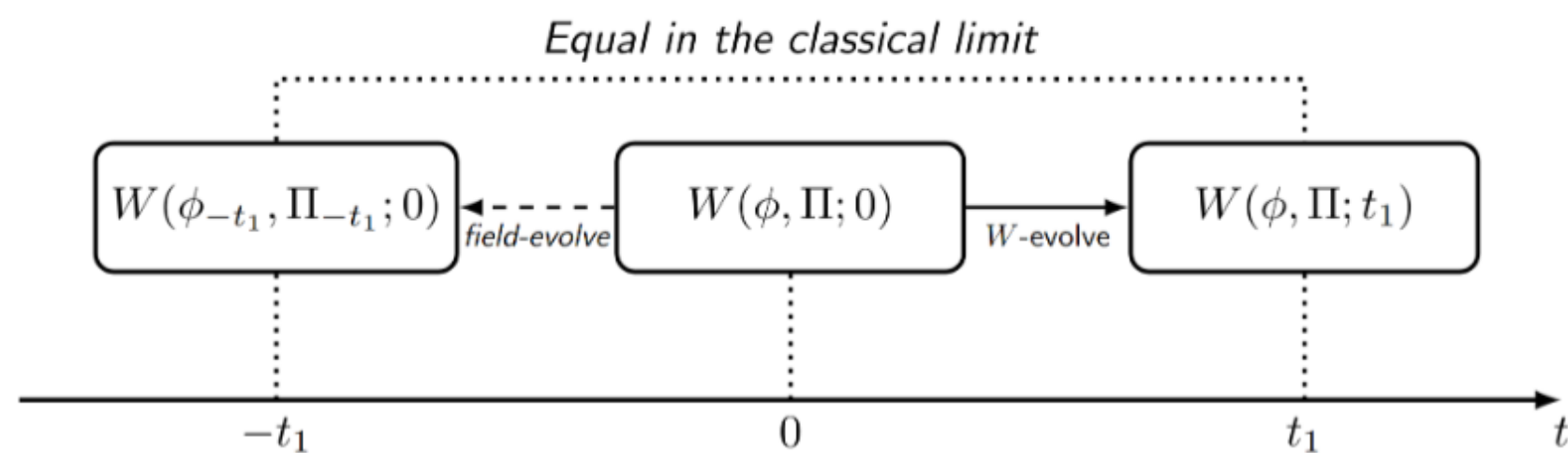
$$W[\phi(x), \Pi(x); t] \gtrsim 0$$

$\phi(x)$ is the real scale field

$$\Pi(x) \equiv \frac{\delta \mathcal{L}}{\delta \dot{\phi}(x)}$$

$$\hat{\rho}(t) = |\Psi(t)\rangle \langle \Psi(t)|$$

Wigner function's EOM



$$\hat{L} = \int d^3x \left(\frac{1}{2} \partial^\mu \hat{\phi} \partial_\mu \hat{\phi} - V(\hat{\phi}) \right) \quad \hat{H} = \int d^3x \hat{\mathcal{H}} = \int d^3x \left[\frac{\hat{\Pi}^2}{2} + \frac{(\nabla \hat{\phi})^2}{2} + V(\hat{\phi}) \right]$$

$$i\hbar \frac{\partial}{\partial t} \hat{\rho}(t) = [\hat{H}, \hat{\rho}(t)] \quad \frac{\partial}{\partial t} W[\phi, \pi; t] = -2H \frac{1}{i\hbar} \sin\left(\frac{i\hbar}{2} \Lambda\right) W[\phi, \pi; t]$$

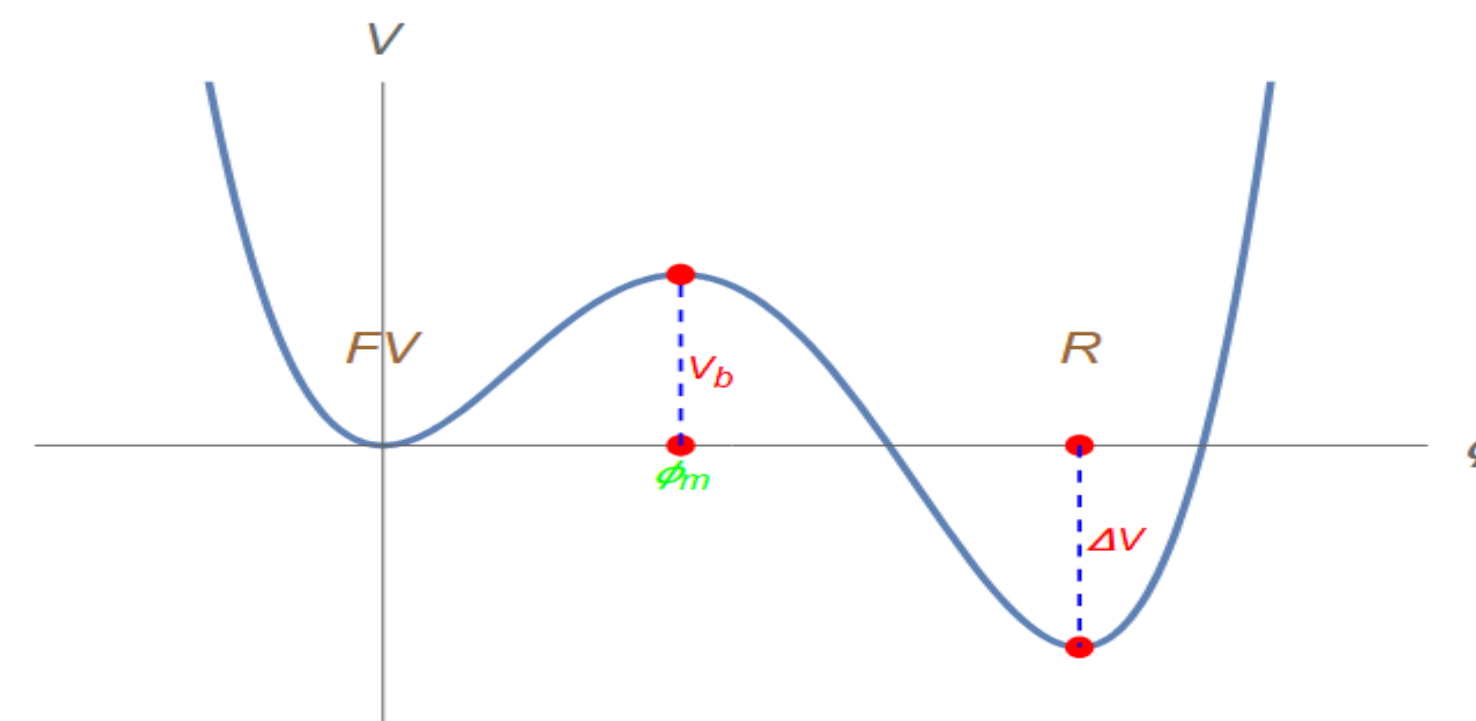
$$\Lambda = \overleftarrow{\frac{\delta}{\delta \Pi}} \overrightarrow{\frac{\delta}{\delta \phi}} - \overleftarrow{\frac{\delta}{\delta \Phi}} \overrightarrow{\frac{\delta}{\delta \Pi}} \text{ is the Poisson bracket operator}$$

Ignore $O(\hbar^2)$

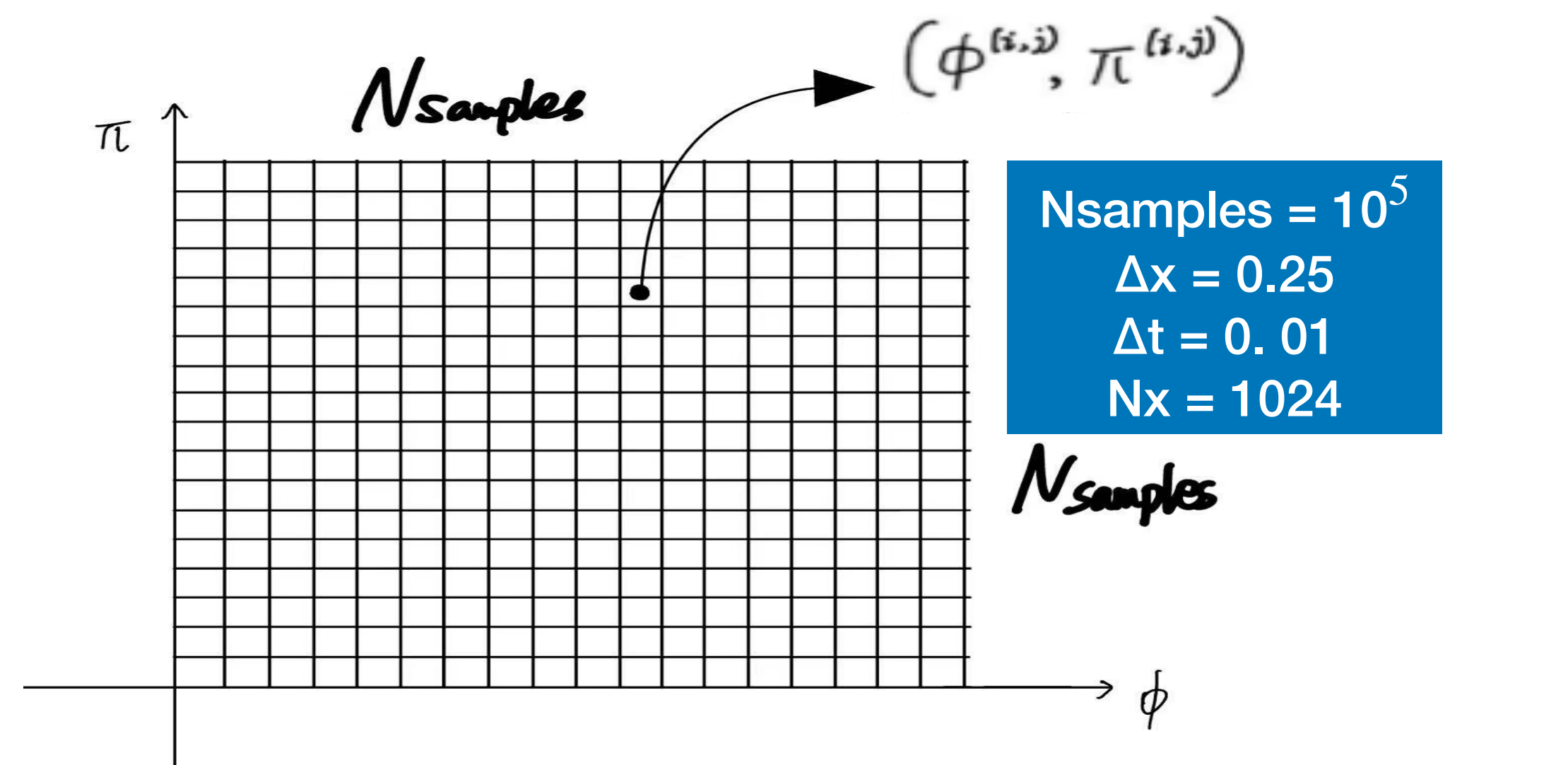
$$\left[\frac{\partial}{\partial t} + \int d^3x \left(\frac{\delta H}{\delta \Pi} \frac{\delta}{\delta \phi} - \frac{\delta H}{\delta \phi} \frac{\delta}{\delta \Pi} \right) \right] W[\phi, \Pi; t] = 0$$

$$W[\phi, \Pi; t] = W[\phi_{-t}, \Pi_{-t}; 0]$$

$$\begin{cases} \frac{\delta H}{\delta \Pi} = \frac{d\phi}{dt} = \Pi \\ -\frac{\delta H}{\delta \phi} = \frac{d\Pi}{dt} = \nabla^2 \phi - \frac{\delta V(\phi)}{\delta \phi} \end{cases}$$

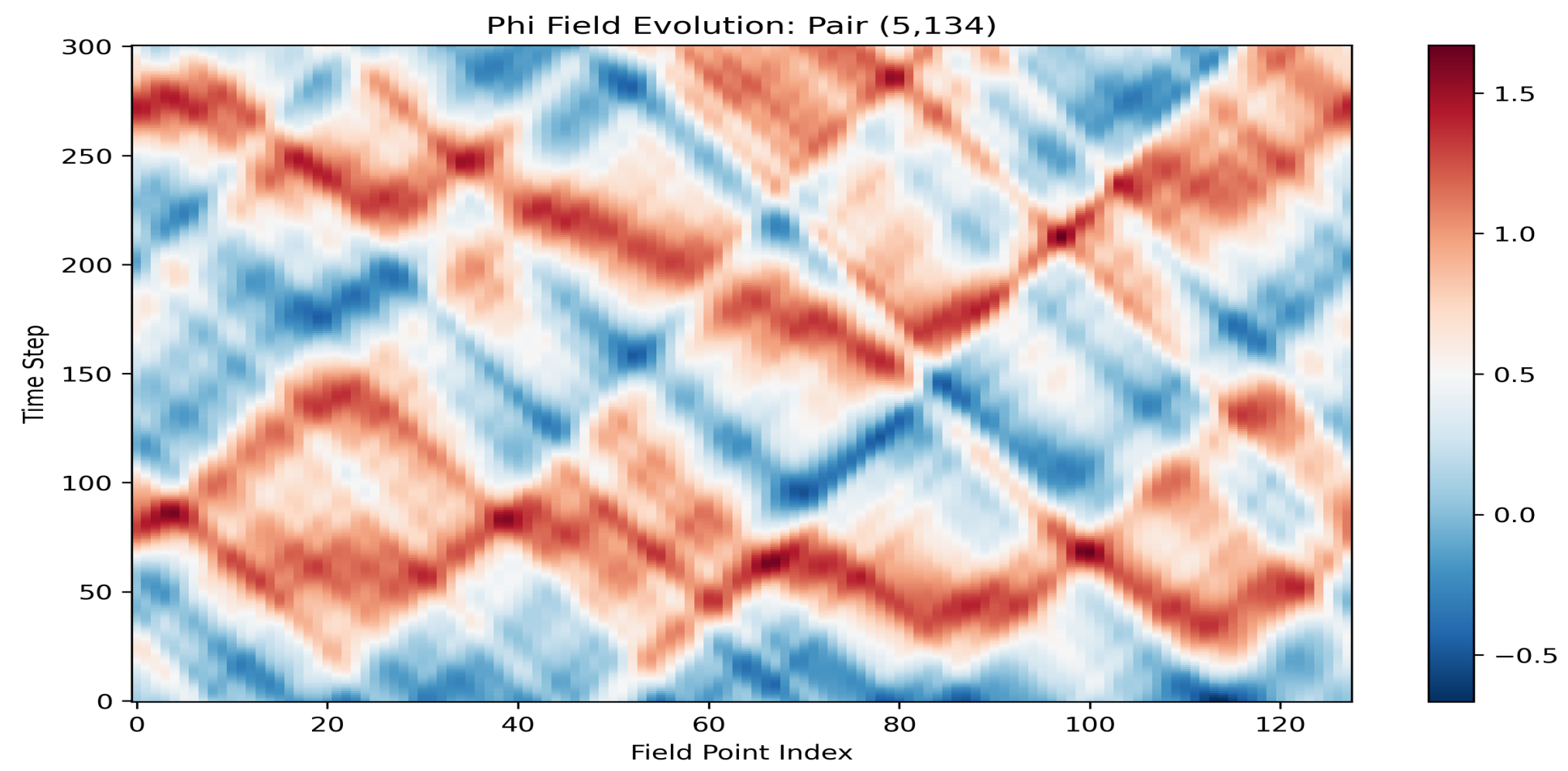


$$P_{\text{FV}}(T) \sim e^{-\Gamma T} \implies \Gamma = -\frac{1}{P_{\text{FV}}} \frac{d}{dT} P_{\text{FV}}$$

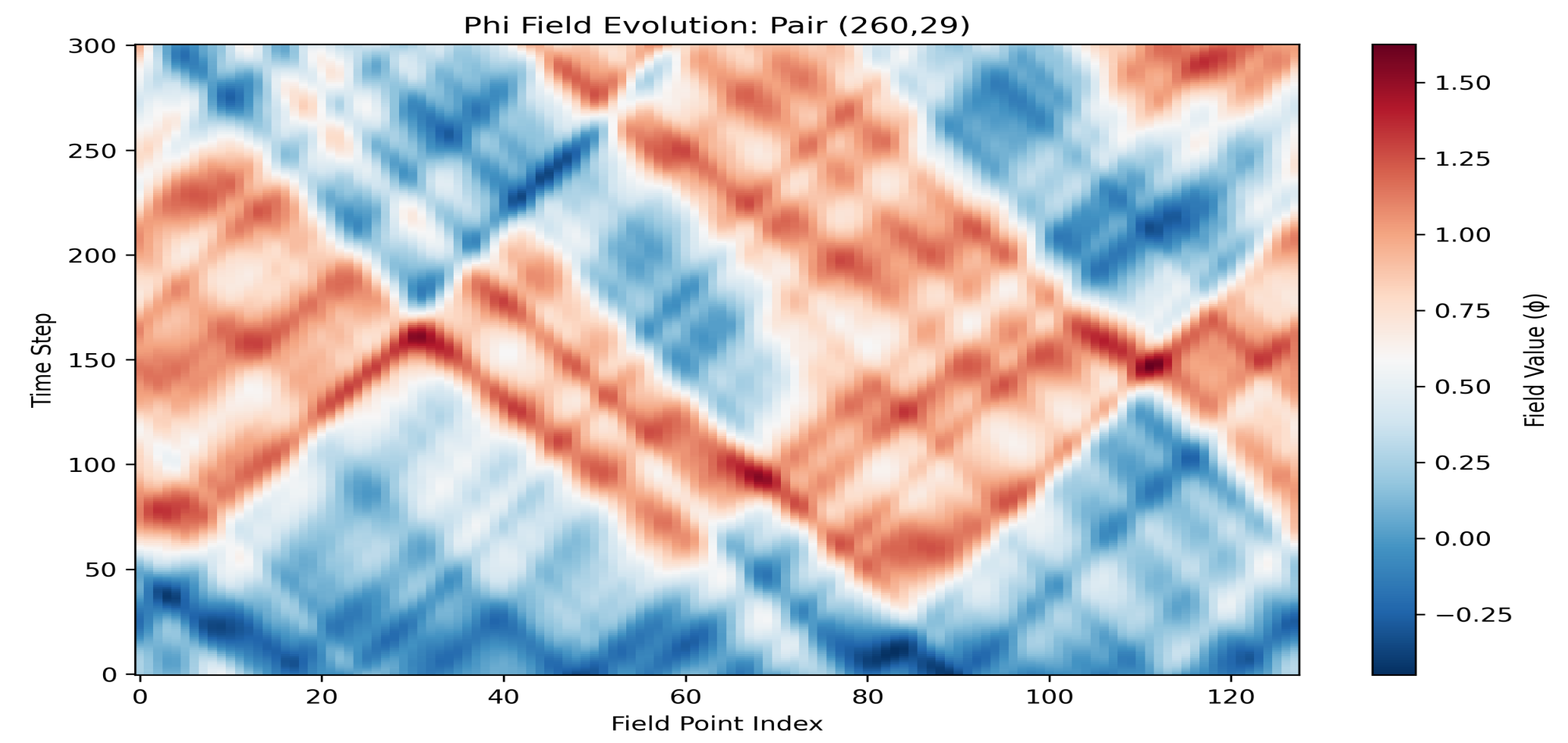


$N_{\text{samples}} * N_{\text{samples}}$ for (ϕ, π) phase space
Summation range: $i, j = 1, 2, 3, \dots, N_{\text{samples}}$

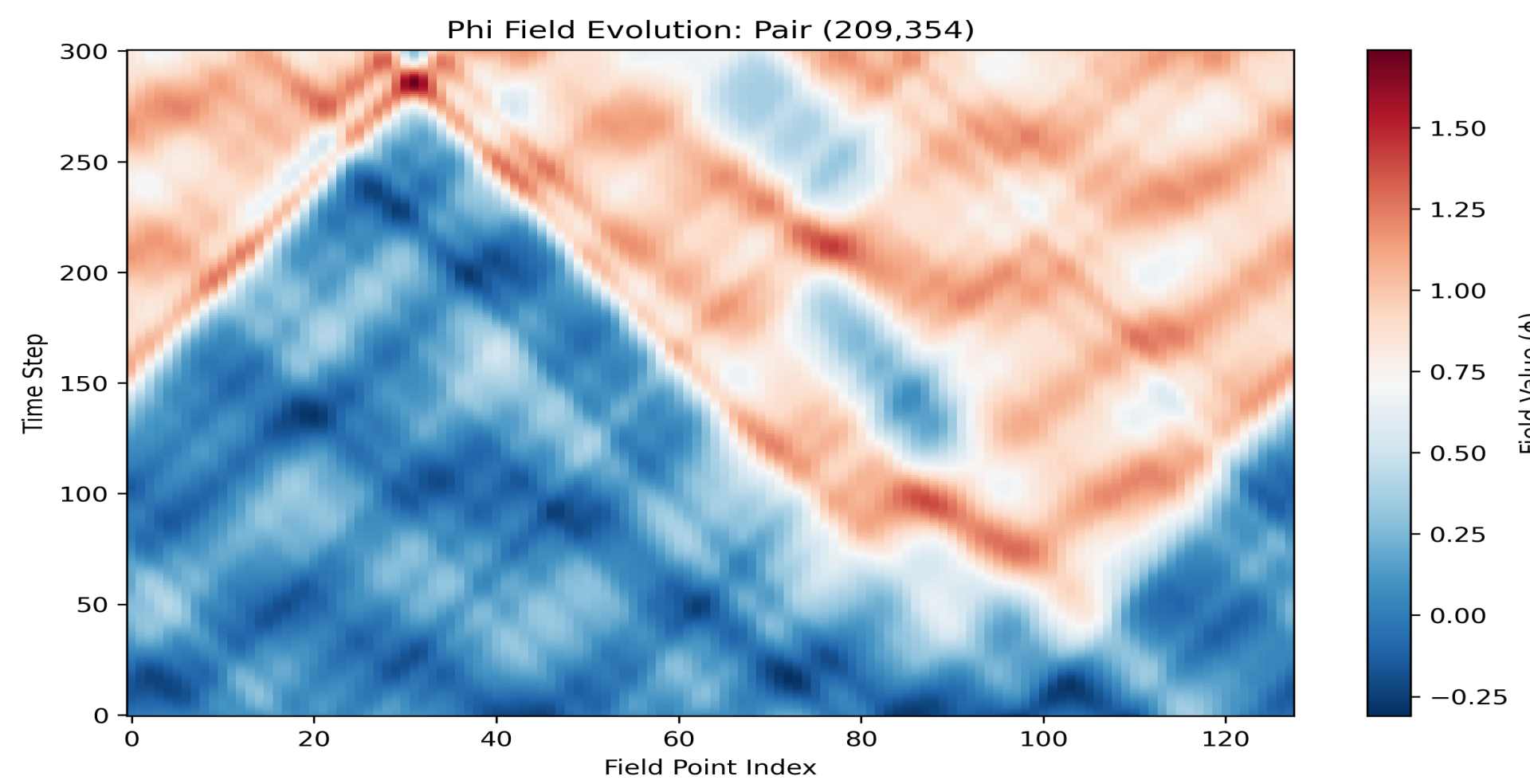
► Simulation results



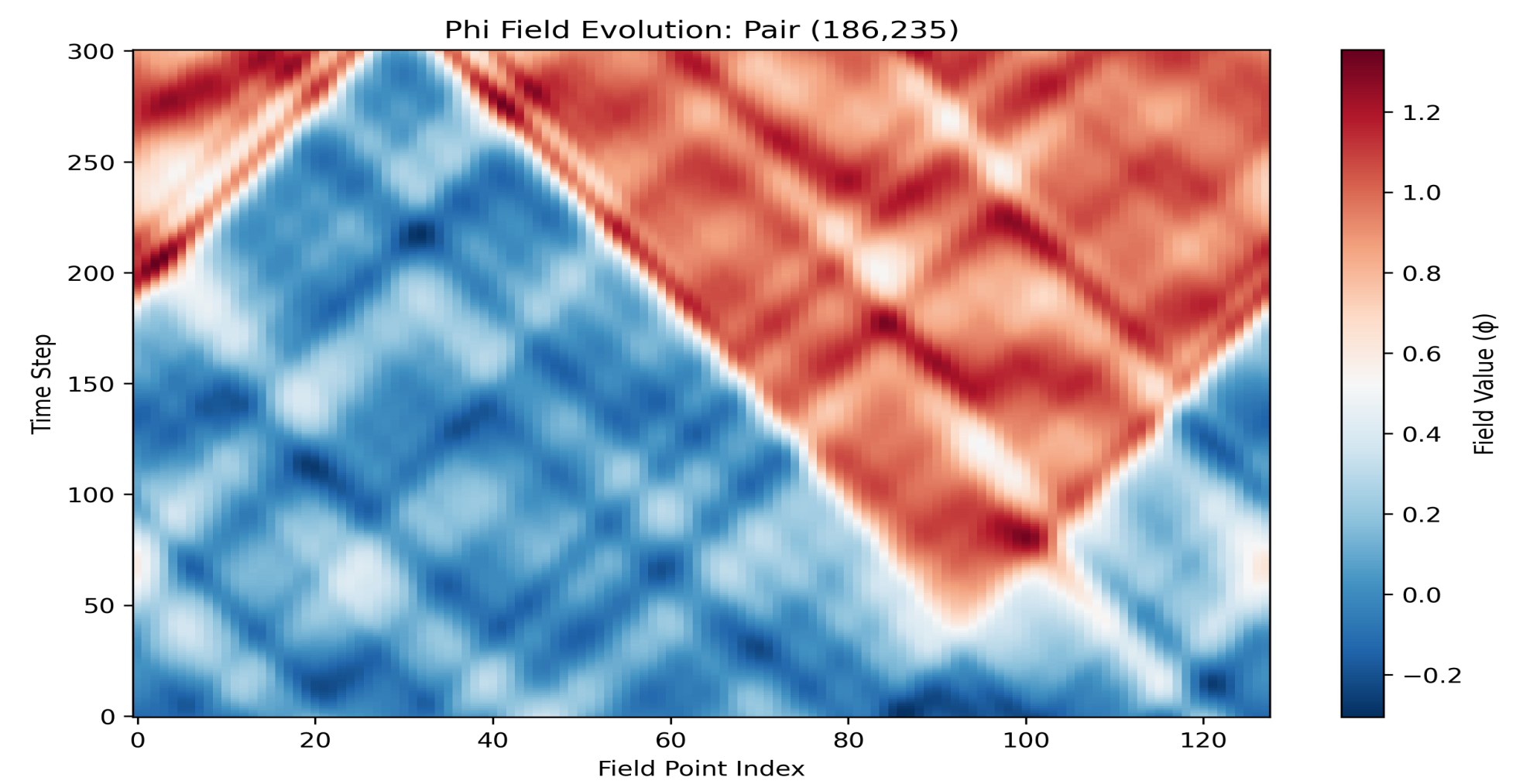
$\beta=0.8$



$\beta=1.0$

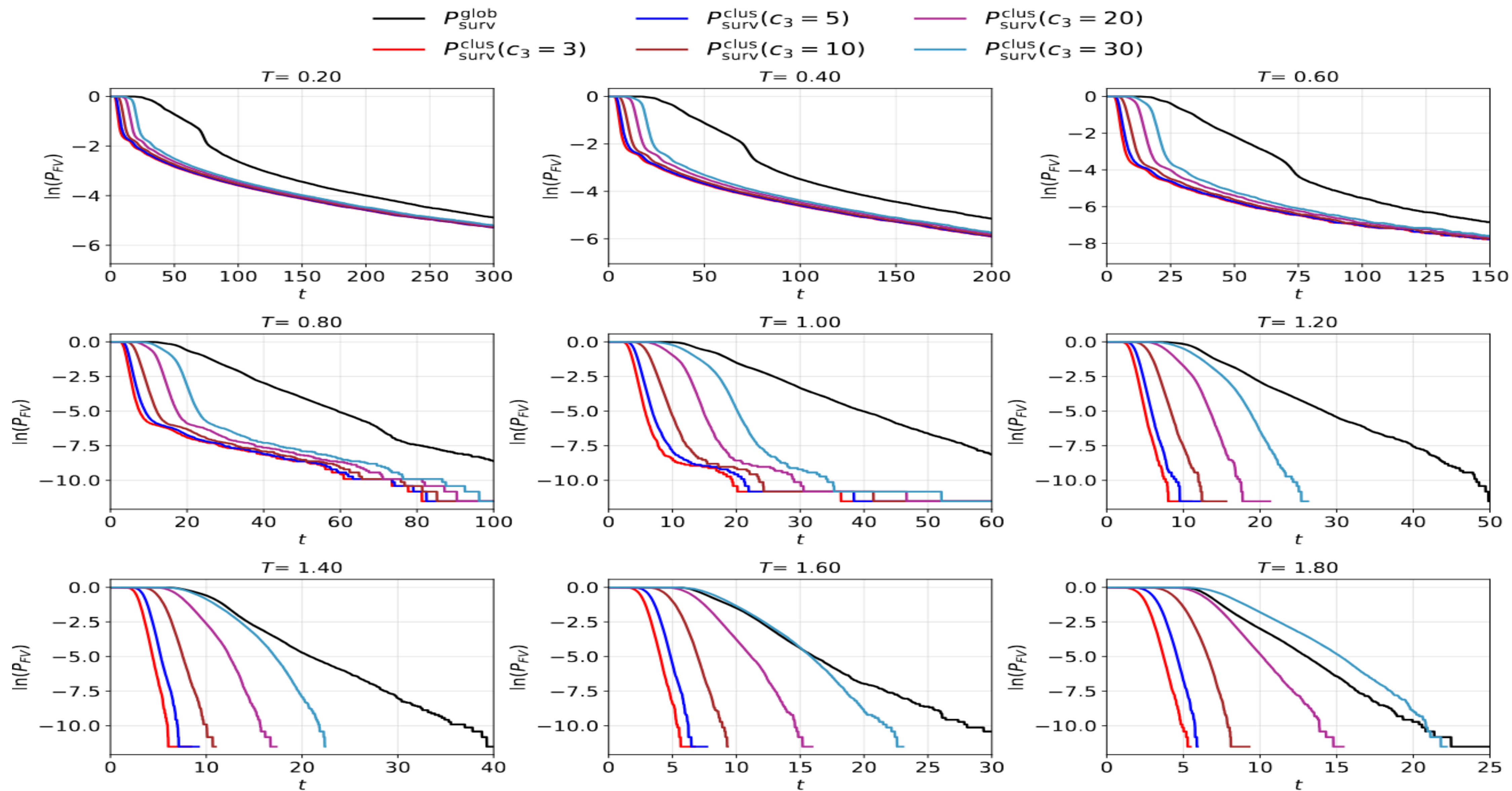


$\beta=1.2$

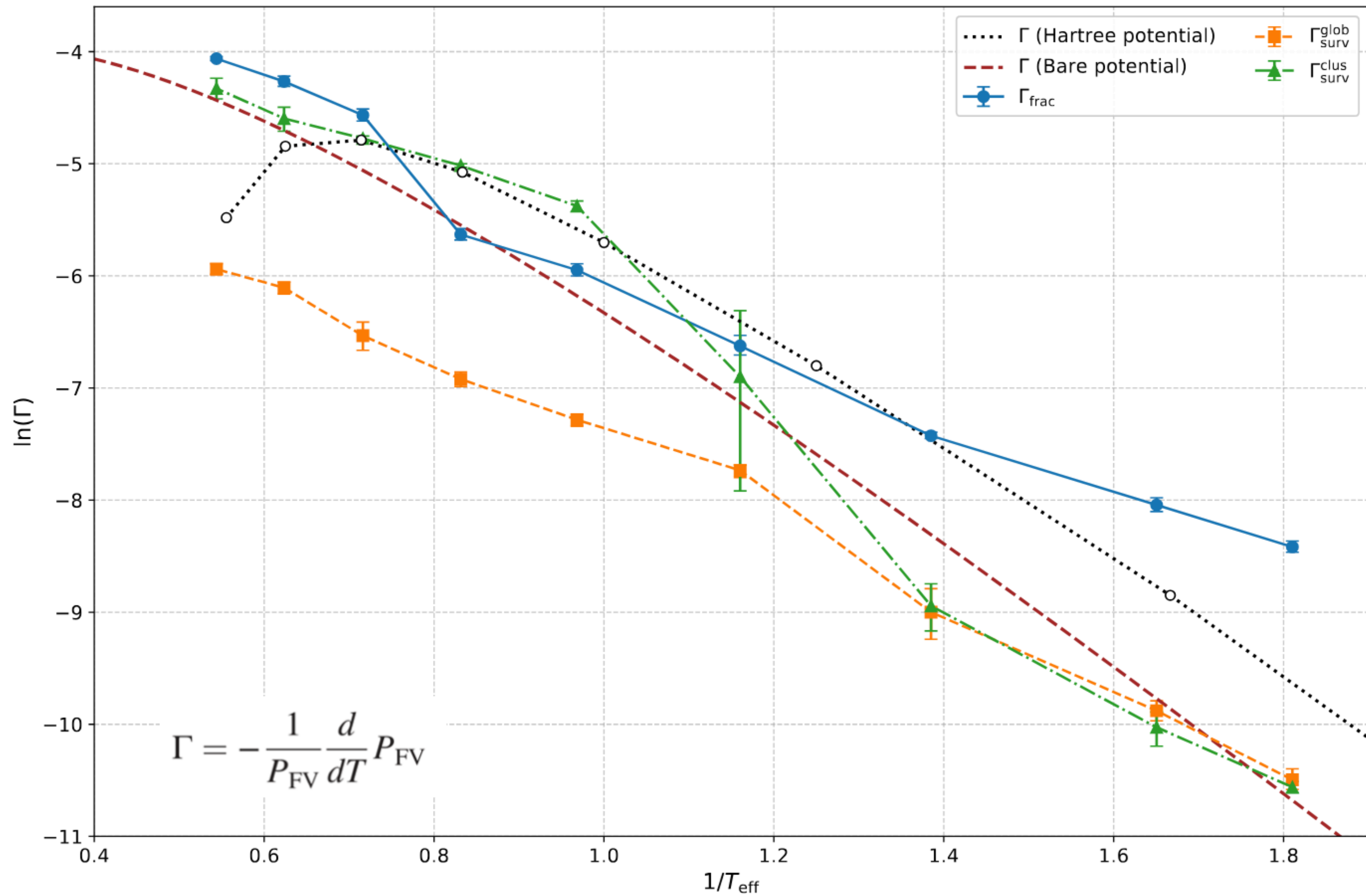


$\beta=1.4$

Simulation results



Simulation results



► Lattice electroweak theory-classical simulation

$\Phi(t, \mathbf{x})$: Higgs field doublet defined on sites;

$U_i(t, \mathbf{x})$ and $V_i(t, \mathbf{x})$: SU(2) and U(1) link fields, defined on the link between the neighboring sites \mathbf{x} and $\mathbf{x} + \mathbf{i}$, $\Phi(t, \mathbf{x})$, $U_i(t, \mathbf{x})$ and $V_i(t, \mathbf{x})$ are defined at time steps $t + \Delta t, t + 2\Delta t, \dots$;

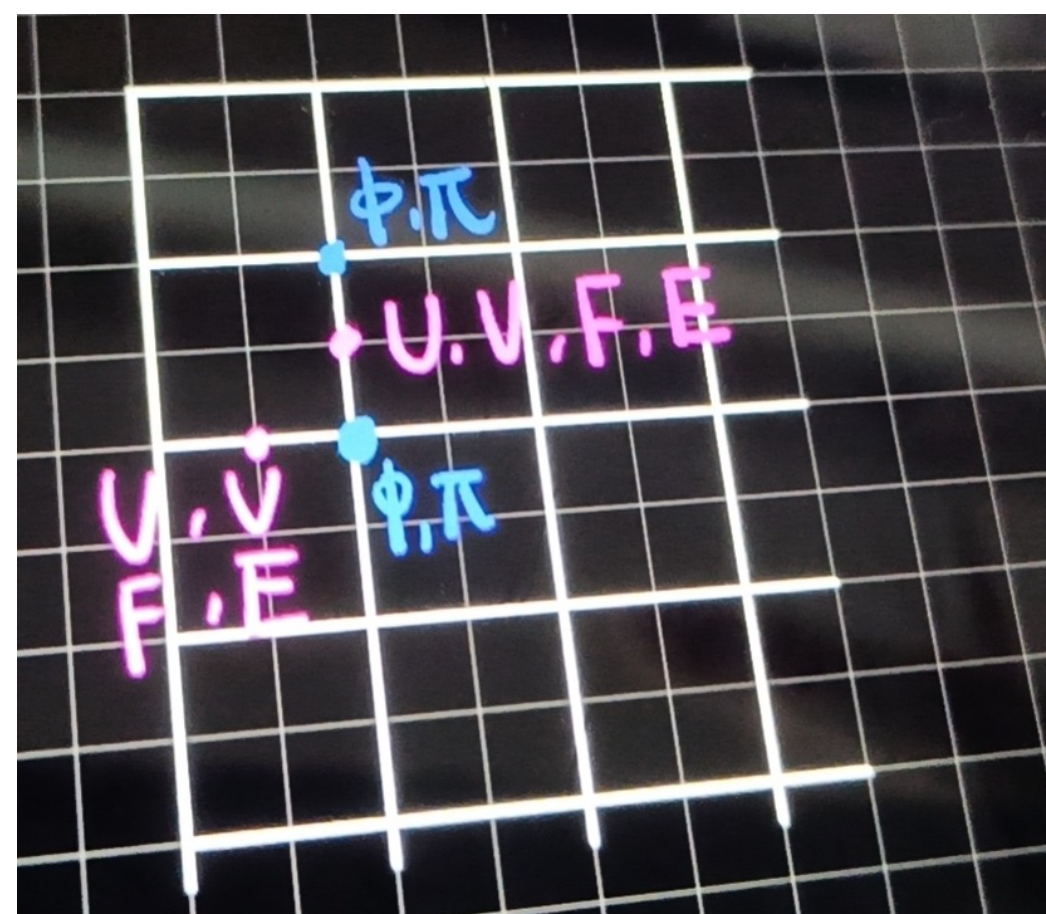
Conjugate momentum fields: $\Pi(t+\Delta t/2, \mathbf{x})$, $F(t+\Delta t/2, \mathbf{x})$ and $E(t+\Delta t/2, \mathbf{x})$, are defined at time steps $t + \Delta t/2, t + 3\Delta t/2$.

$$U_i(t, \mathbf{x}) = \exp\left(-\frac{i}{2}g\Delta x\sigma^a W_i^a\right)$$

$$U_0(t, \mathbf{x}) = \exp\left(-\frac{i}{2}g\Delta t\sigma^a W_0^a\right)$$

$$V_i(t, \mathbf{x}) = \exp\left(-\frac{i}{2}g\Delta x B_i\right)$$

$$V_0(t, \mathbf{x}) = \exp\left(-\frac{i}{2}g\Delta t B_0\right).$$



Temporal gauge
 $U_0(t, \mathbf{x}) = \mathbf{I}_2, V_0(t, \mathbf{x}) = 1$

leapfrog

$$D_i\Phi = \frac{1}{\Delta x} [U_i(t, \mathbf{x})V_i(t, \mathbf{x})\Phi(t, \mathbf{x} + \mathbf{i}) - \Phi(t, \mathbf{x})]$$

$$D_0\Phi = \frac{1}{\Delta t} [U_0(t, \mathbf{x})V_0(t, \mathbf{x})\Phi(t + \Delta t, \mathbf{x}) - \Phi(t, \mathbf{x})].$$

$$\Phi(t + \Delta t, \mathbf{x}) = \Phi(t, \mathbf{x}) + \Delta t\Pi(t + \Delta t/2, \mathbf{x})$$

$$V_i(t + \Delta t, \mathbf{x}) = \frac{1}{2}g'\Delta x\Delta t E_i(t + \Delta t/2, \mathbf{x})V_i(t, \mathbf{x})$$

$$U_i(t + \Delta t, \mathbf{x}) = g\Delta x\Delta t F_i(t + \Delta t/2, \mathbf{x})U_i(t, \mathbf{x}),$$

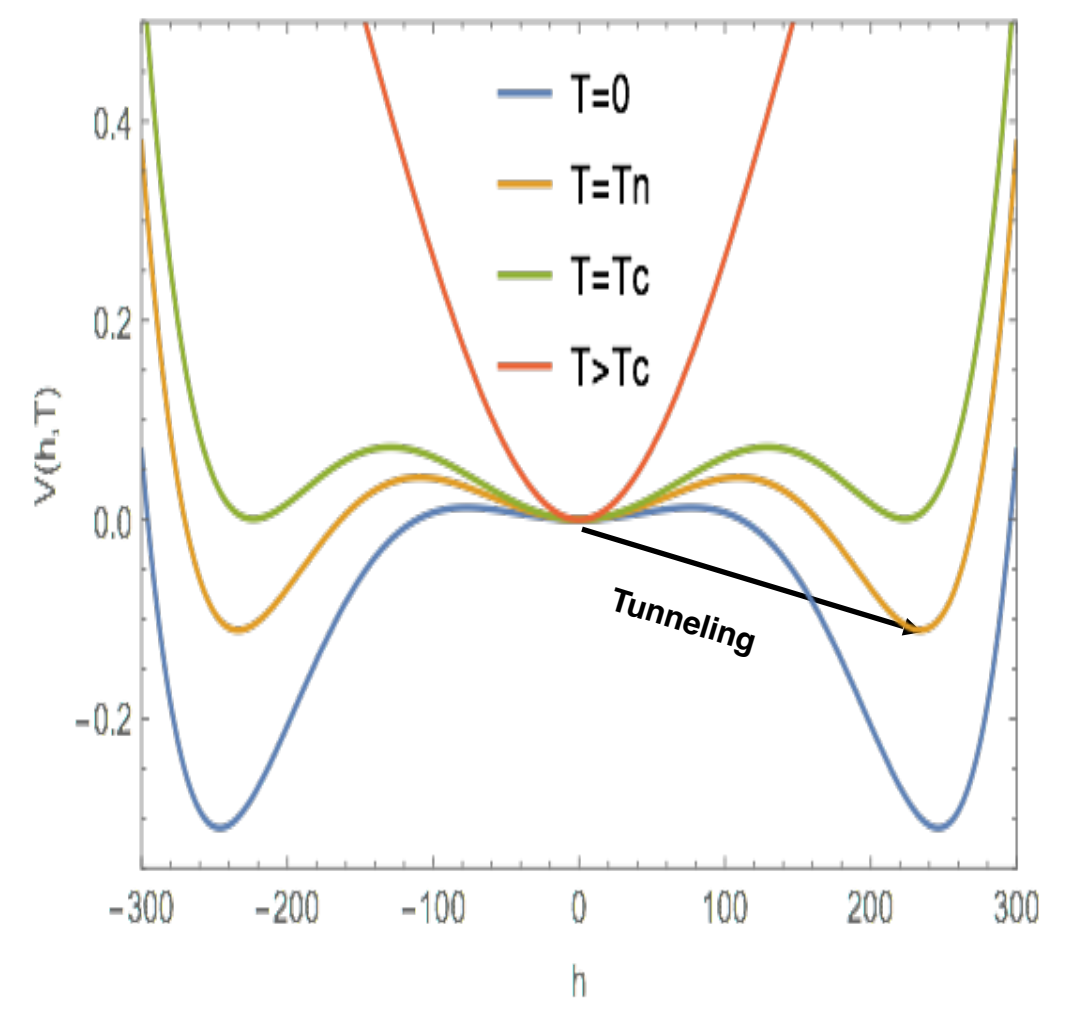
Field basis equation of motion

$$\begin{aligned} \partial_0^2 \Phi &= D_i D_i \Phi - \frac{dV(\Phi)}{d\Phi}, \\ \partial_0^2 B_i &= -\partial_j B_{ij} + g' \text{Im}[\Phi^\dagger D_i \Phi], \\ \partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g \epsilon^{abc} W_k^b W_{ik}^c + g \text{Im}[\Phi^\dagger \sigma^a D_i \Phi], \\ \partial_0 \partial_j B_j - g' \text{Im}[\Phi^\dagger \partial_0 \Phi] &= 0, \\ \partial_0 \partial_j W_j^a + g \epsilon^{abc} W_j^b \partial_0 W_j^c - g \text{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] &= 0. \end{aligned}$$

Lattice implementation

$$\begin{aligned} \Pi(t + \Delta t/2, x) &= \Pi(t - \Delta t/2, x) + \Delta t \left\{ \frac{1}{\Delta x^2} \sum_i [U_i(t, x) V_i(t, x) \Phi(t, x + i) \right. \\ &\quad \left. - 2\Phi(t, x) + U_i^\dagger(t, x - i) V_i^\dagger(t, x - i) \Phi(t, x - i)] - \frac{\partial U}{\partial \Phi^\dagger} \right\} \\ \text{Im}[E_k(t + \Delta t/2, x)] &= \text{Im}[E_k(t - \Delta t/2, x)] + \Delta t \left\{ \frac{g'}{\Delta x} \text{Im}[\Phi^\dagger(t, x + k) U_k^\dagger(t, x) V_k^\dagger(t, x) \Phi(t, x)] \right. \\ &\quad \left. - \frac{2}{g' \Delta x^3} \sum_i \text{Im}[V_k(t, x) V_i(t, x + k) V_k^\dagger(t, x + i) V_i^\dagger(t, x) \right. \\ &\quad \left. + V_i(t, x - i) V_k(t, x) V_i^\dagger(t, x + k - i) V_k^\dagger(t, x - i)] \right\} \\ \text{Tr}[i\sigma^m F_k(t + \Delta t/2, x)] &= \text{Tr}[i\sigma^m F_k(t - \Delta t/2, x)] + \Delta t \left\{ \frac{g}{\Delta x} \text{Re}[\Phi^\dagger(t, x + k) U_k^\dagger(t, x) V_k^\dagger(t, x) i\sigma^m \Phi(t, x)] \right. \\ &\quad \left. - \frac{1}{g \Delta x^3} \sum_i \text{Tr}[i\sigma^m U_k(t, x) U_i(t, x + k) U_k^\dagger(t, x + i) U_i^\dagger(t, x) \right. \\ &\quad \left. + i\sigma^m U_k(t, x) U_i^\dagger(t, x + k - i) U_k^\dagger(t, x - i) U_i(t, x - i)] \right\}, \end{aligned}$$

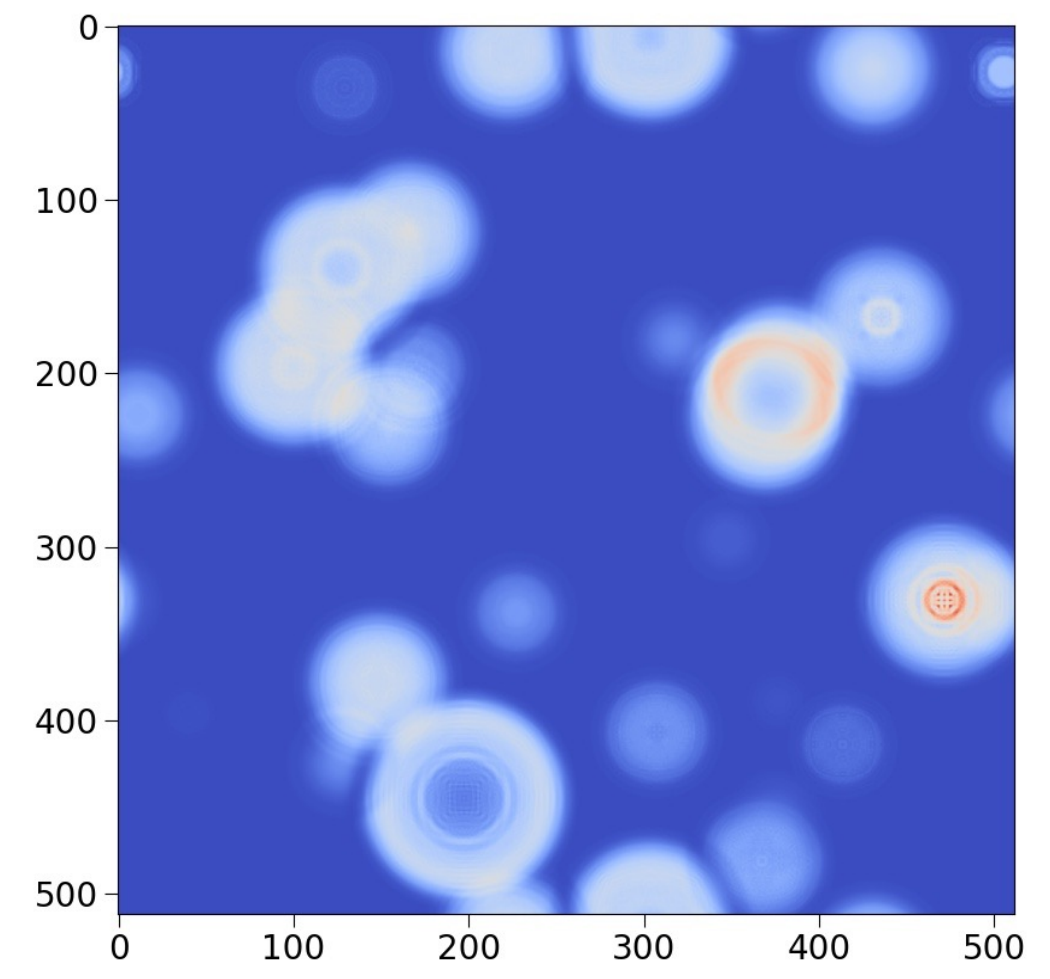
Finite-T Veff



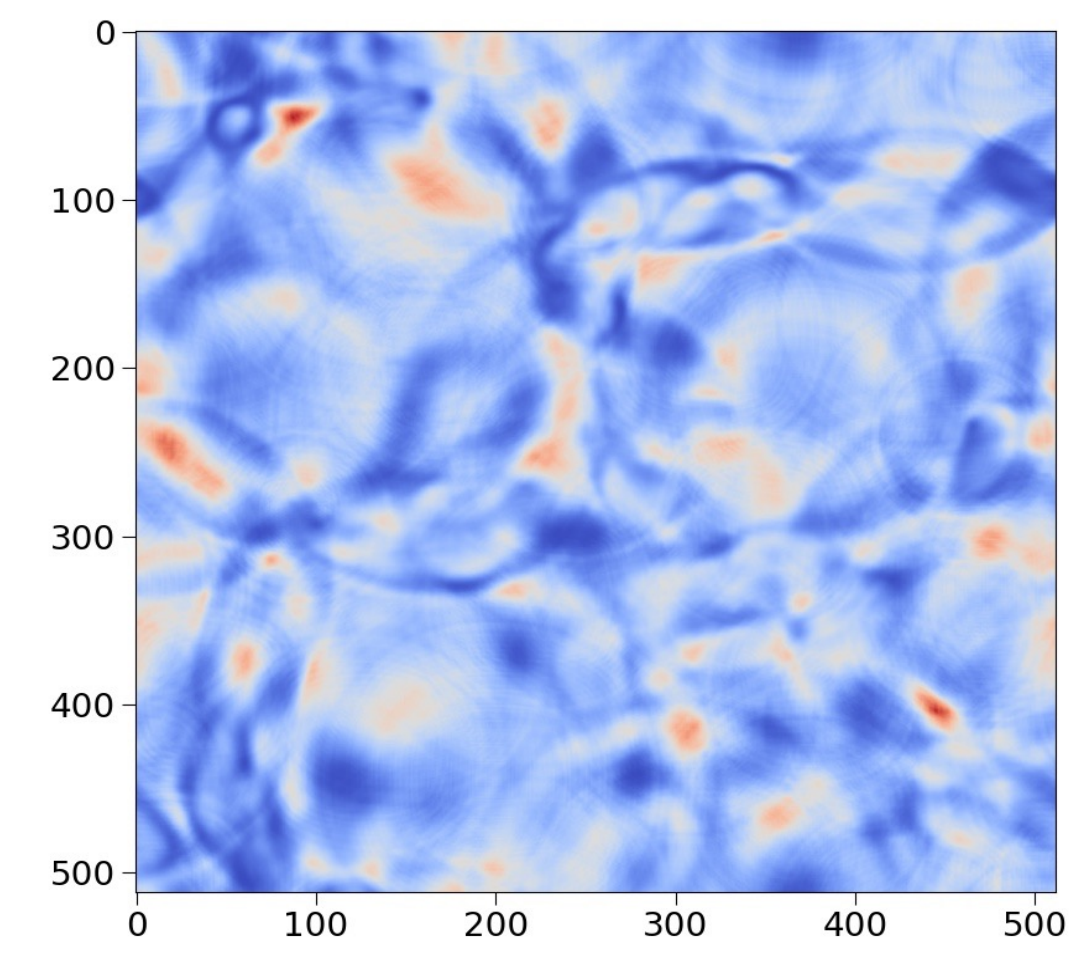
Finite-T calculation

temperature (T_*)
duration (β)
strength (α)

Nucleation



Expansion&Percolation



Bubble collision and PMF

1st Phase transition + Magnetic field = Matter-antimatter asymmetry ?

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} - \frac{1}{2} Y_{\mu\nu}^{\text{ex}} Y^{\mu\nu} + \mathcal{V}(\Phi)$$

Higgs doublet U(1) gauge field strength
SU(2) gauge field strength External U(1) gauge field strength
Dose not change with time

$$D_\mu = \partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a - ig' \frac{1}{2} (Y_\mu + Y_\mu^{\text{ex}}) \quad \text{is the covariant derivative.}$$

$$\mathcal{V}(\Phi) = -\mu^2 (\Phi^\dagger \Phi - \eta^2) + A (\Phi^\dagger \Phi - \eta^2)^{3/2} + \lambda (\Phi^\dagger \Phi - \eta^2)^2$$

$$\partial_\mu j_B^\mu = N_g \left[\frac{g^2}{16\pi^2} \text{Tr} (W_{\mu\nu} \tilde{W}^{\mu\nu}) - \frac{g'^2}{32\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right]$$

Adler, PR 177, 2426 (1969)
 Bell, Jackiw, NCA 60, 47 (1969)
 't Hooft, PRL 39, 8 (1976)
 't Hooft, PRD 14, 3432 (1976)

$$\Delta n_B = n_B(t) - n_B(0) = N_g \frac{\Delta N_{\text{CS}}(t)}{V} = N_g \frac{N_{\text{CS}}(t) - N_{\text{CS}}(0)}{V}$$

Number of families of fermions $N_g = 3$

Chern-Simons Number density:

$$\frac{N_{\text{CS}}(t)}{V} = \frac{1}{V} \frac{1}{32\pi^2} \varepsilon^{ijk} \int d^3x \left[-g'^2 (Y_i + Y_i^{\text{ex}})(Y_{jk} + Y_{jk}^{\text{ex}}) + g^2 (W_i^a W_{jk}^a - \frac{g}{3} \varepsilon^{abc} W_i^a W_j^b W_k^c) \right]$$

Helicity density:

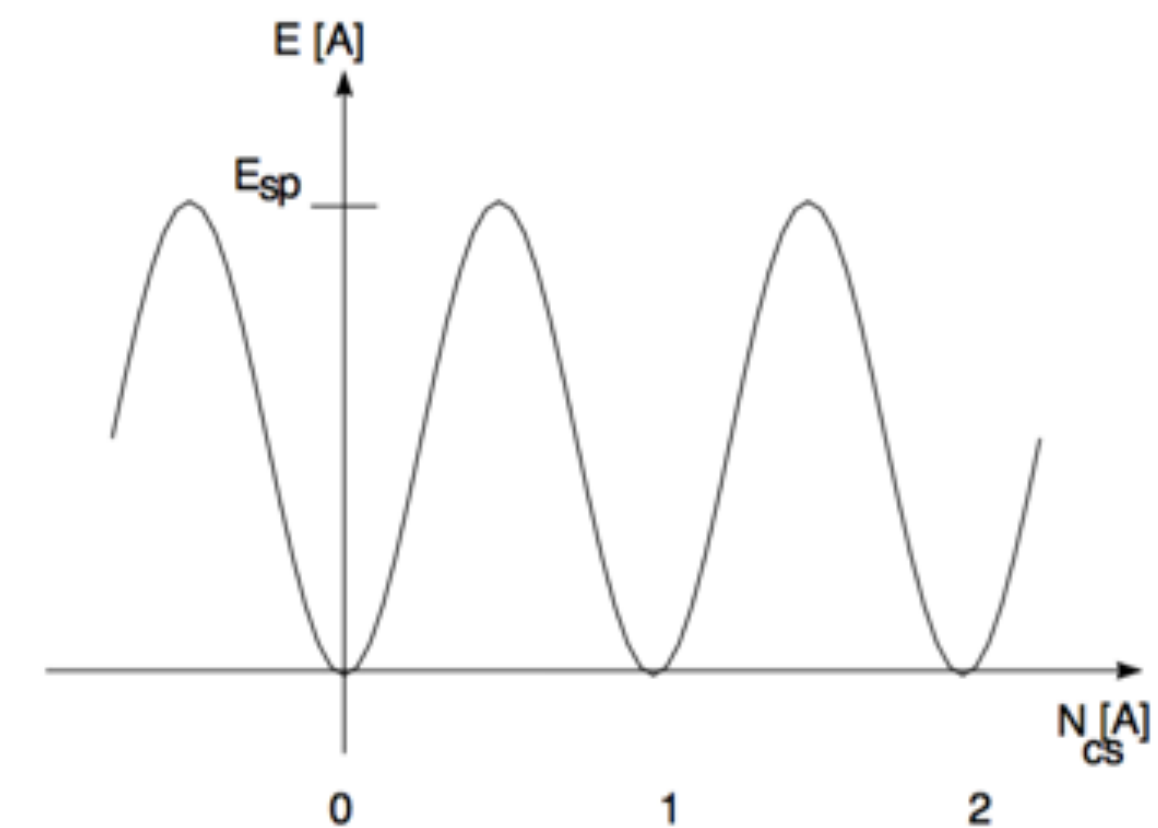
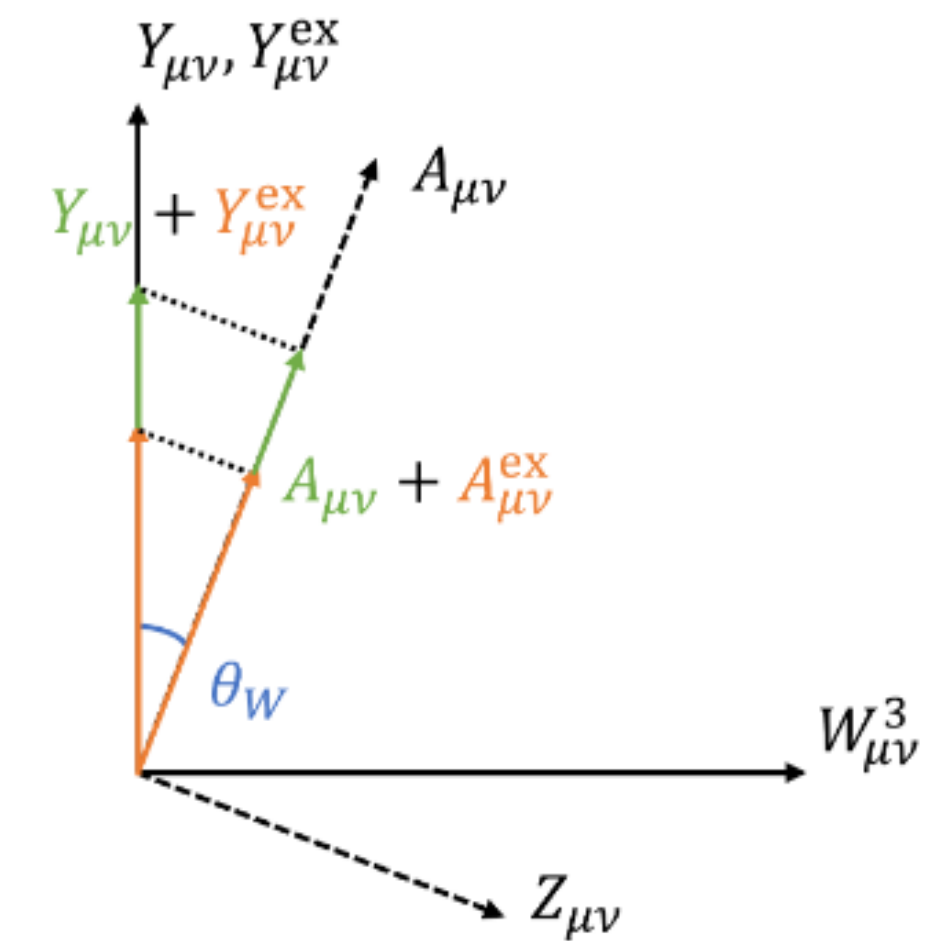
$$h_Y = \frac{H_Y}{V} = \frac{1}{V} \varepsilon^{ijk} \int dx^3 (Y_i + Y_i^{\text{ex}})(Y_{jk} + Y_{jk}^{\text{ex}}) = \frac{1}{V} \int dx^3 (\mathbf{Y} + \mathbf{Y}^{\text{ex}}) \cdot (\mathbf{B}_Y + \mathbf{B}_Y^{\text{ex}})$$

Net Baryon density:

$$\eta_B(t) = \frac{n_B}{s} = \frac{45 N_g}{2\pi^2 g_{*S} T^3} \frac{\Delta N_{\text{CS}}(t)}{V}$$

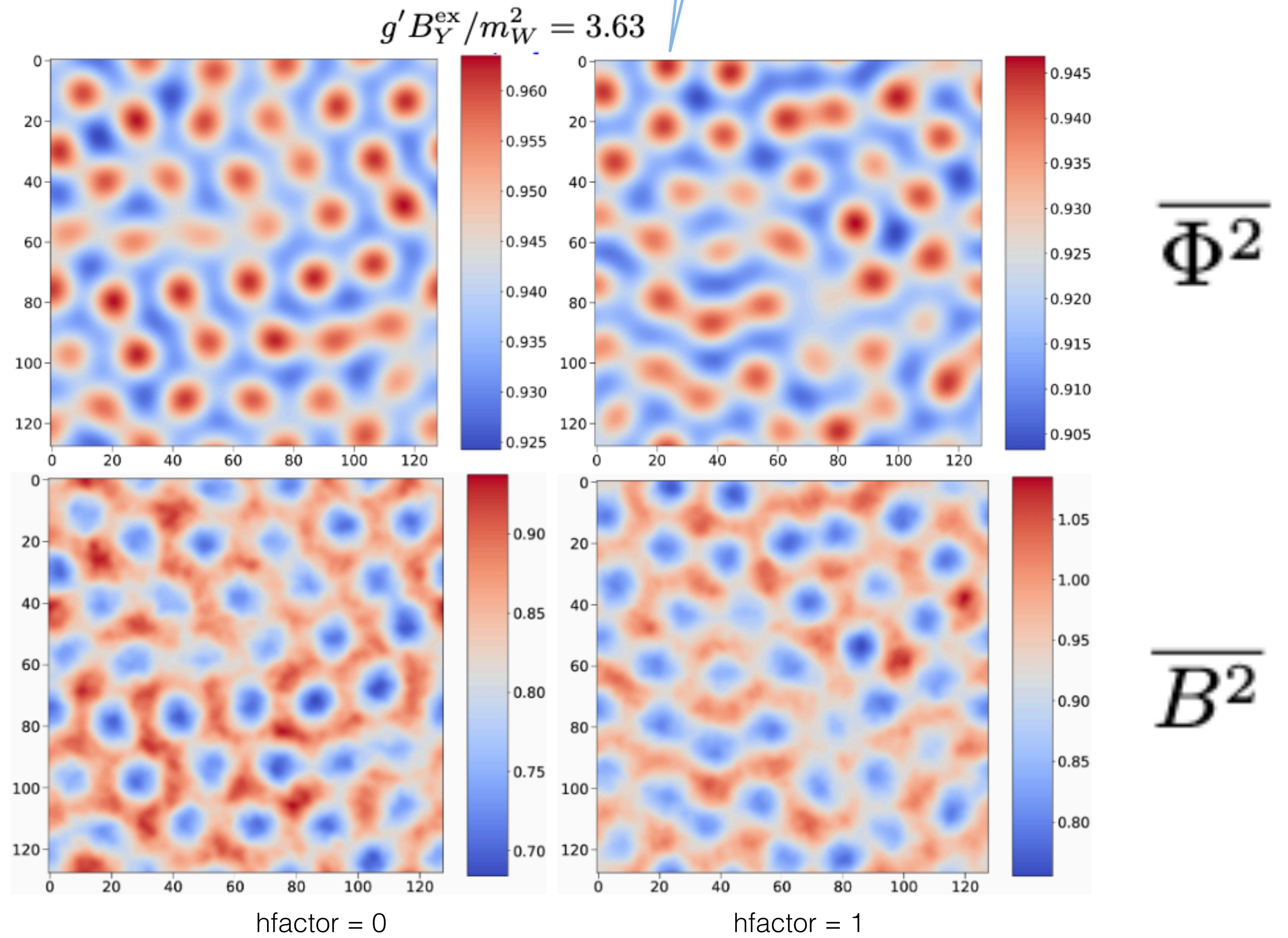
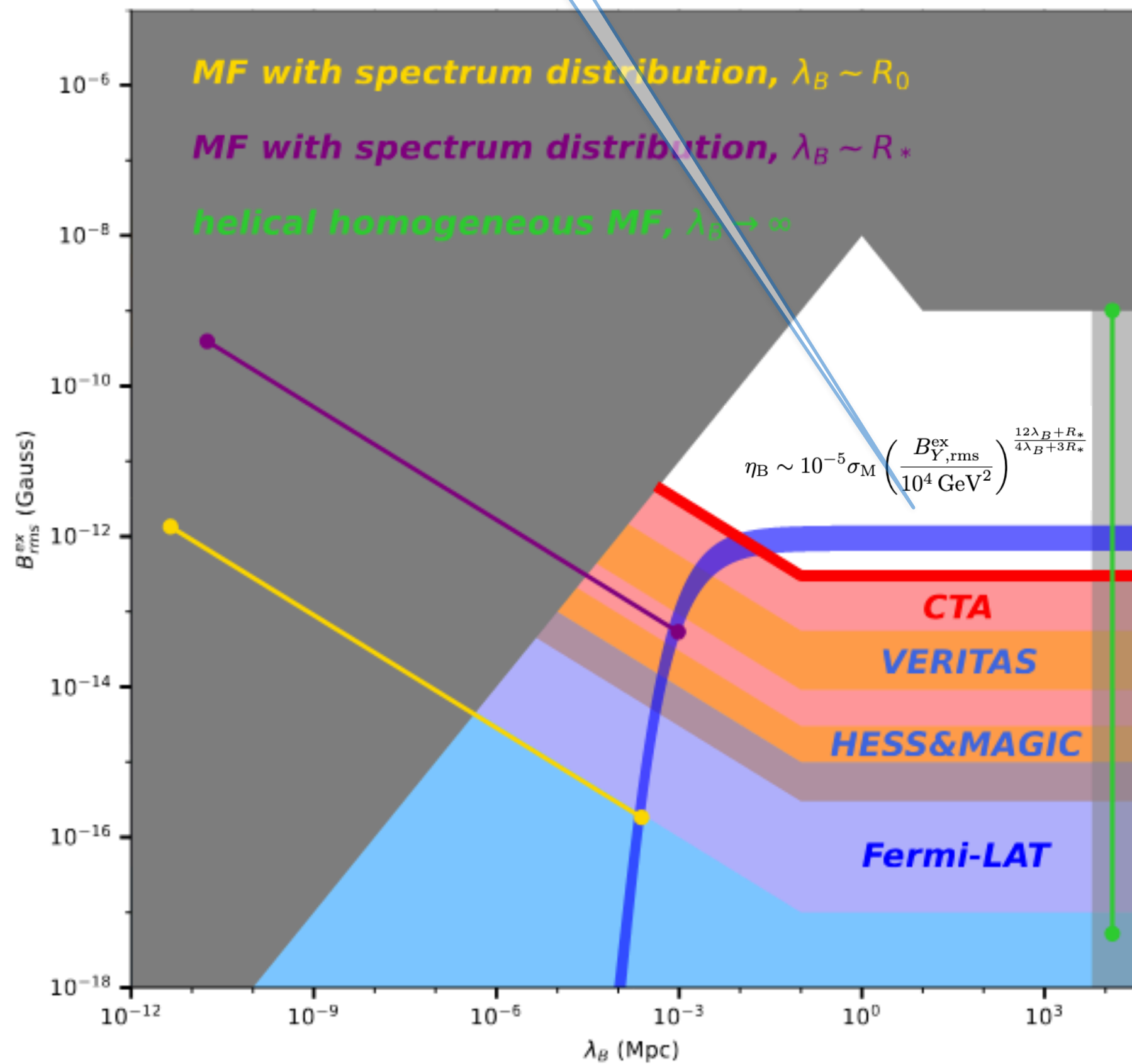
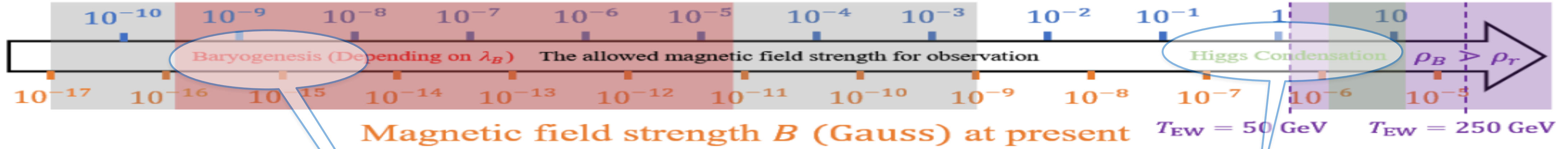
$$\text{SU}(2)_L \times \text{U}(1)_Y \xrightarrow{\text{Spontaneous symmetry breaking}} \text{U}(1)_{\text{em}}$$

Φ, W_μ^a, Y_μ $A_\mu, H, Z_\mu, W_\mu^\pm$



Bubble collision and PMF

Hypermagnetic field strength $g' B_Y^{\text{ex}} / m_W^2$ at electroweak scale



❖ Summary

- **GW of ALP particles in the DFSZ case can be probed by GW detectors**
- **GW of Axion/ALP dark matter scenario in KSVZ cannot be probed**
- **Direct measurement of the vacuum decay rate at finite temperature**
- **Bubble dynamics prefer PMF and GW production from EW lattice**

❖ Future

- **Dark matter and PT**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls**
- **Magnetic fields, baryogenesis, and Axion dynamics**

Thanks

谢谢!

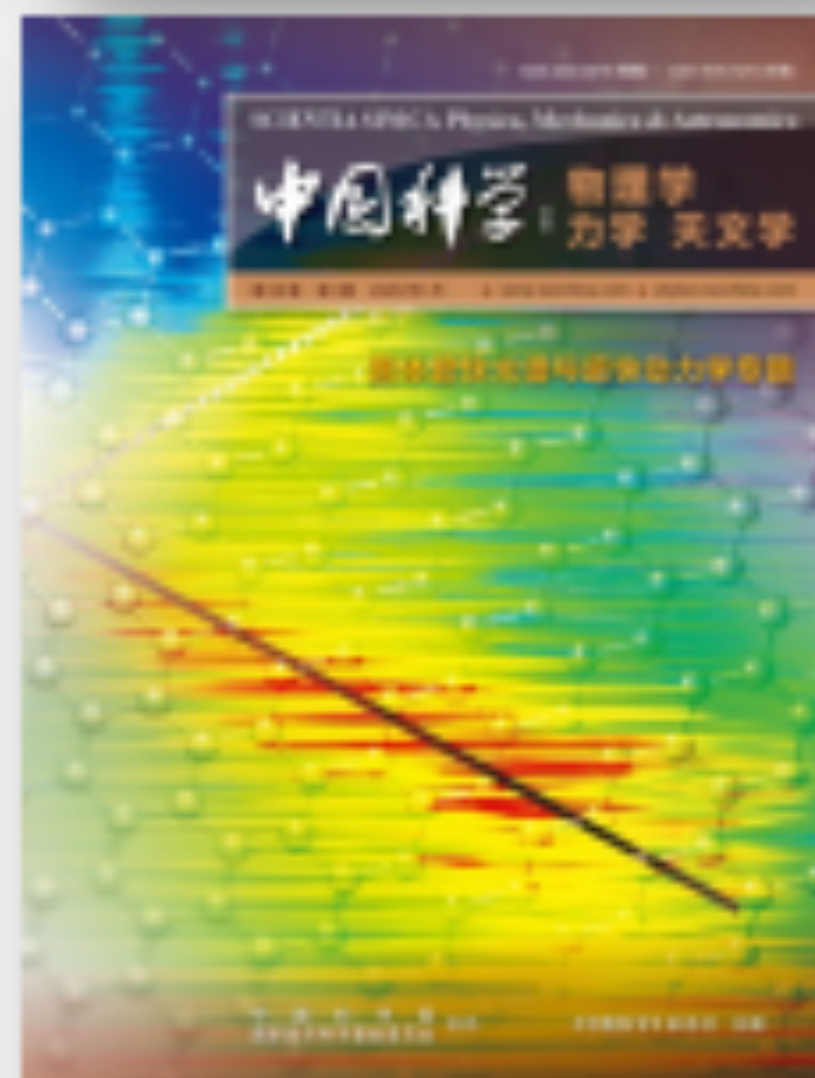
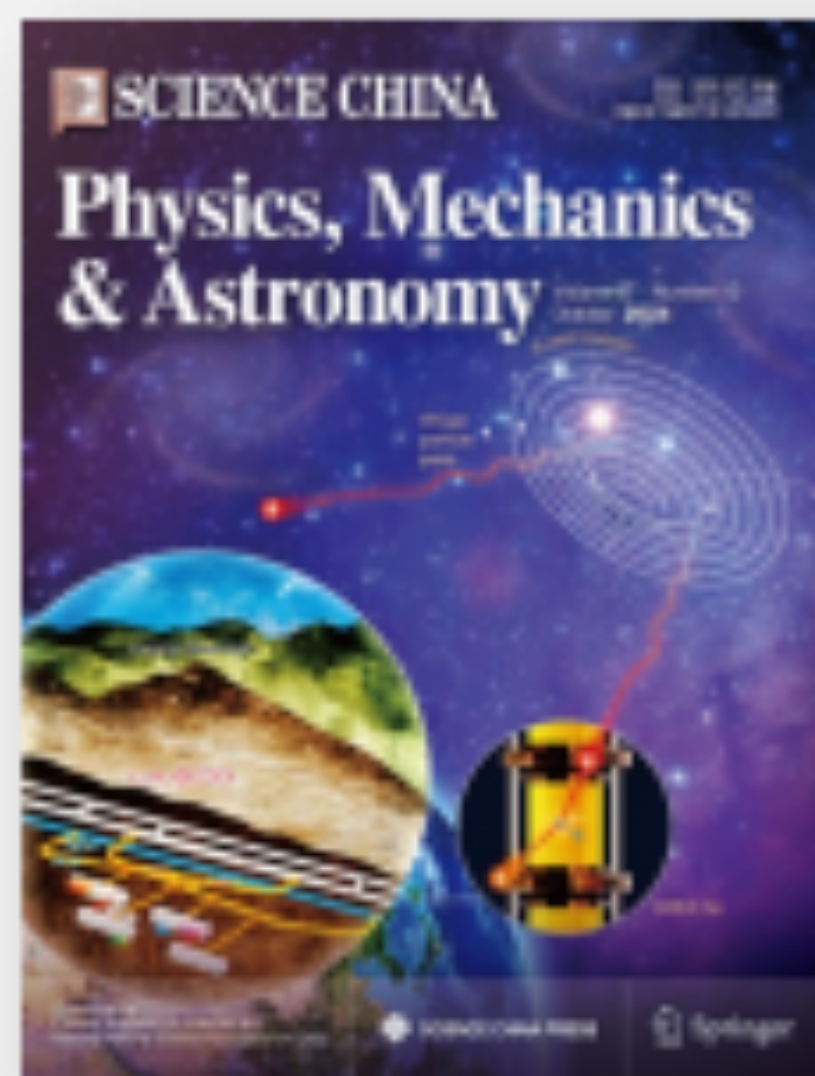
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7.5

IMPACT FACTOR | Q1



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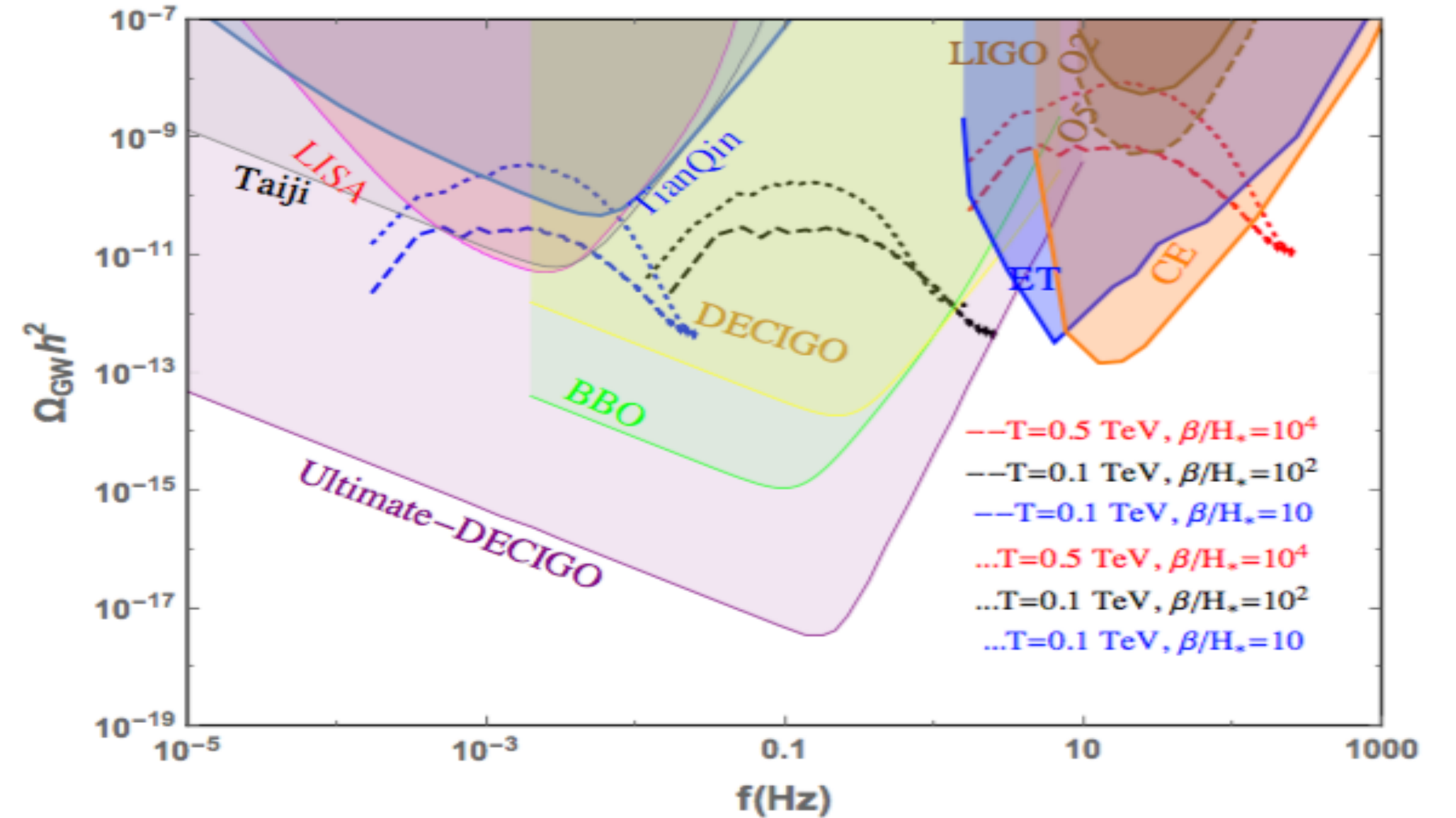
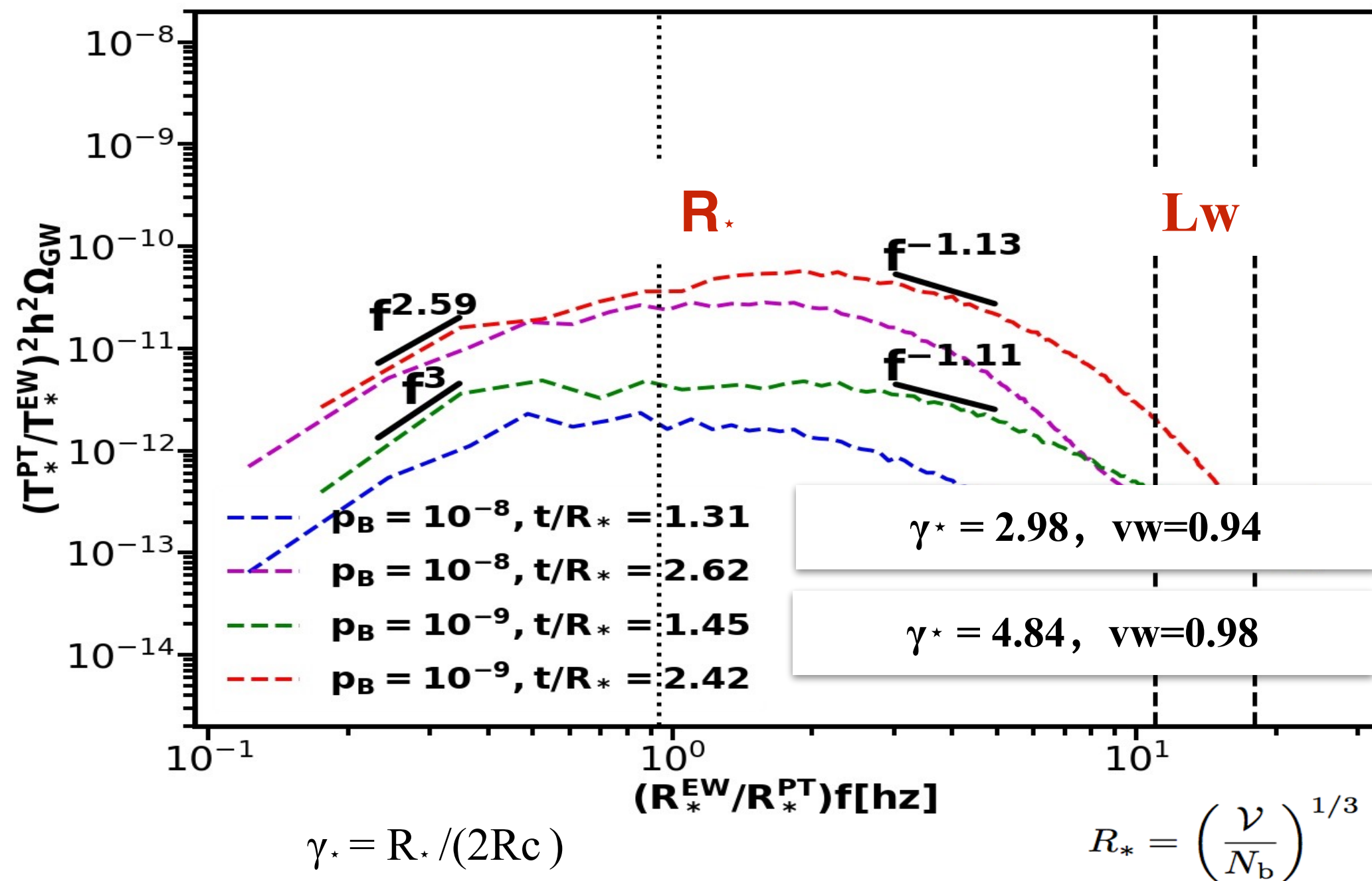
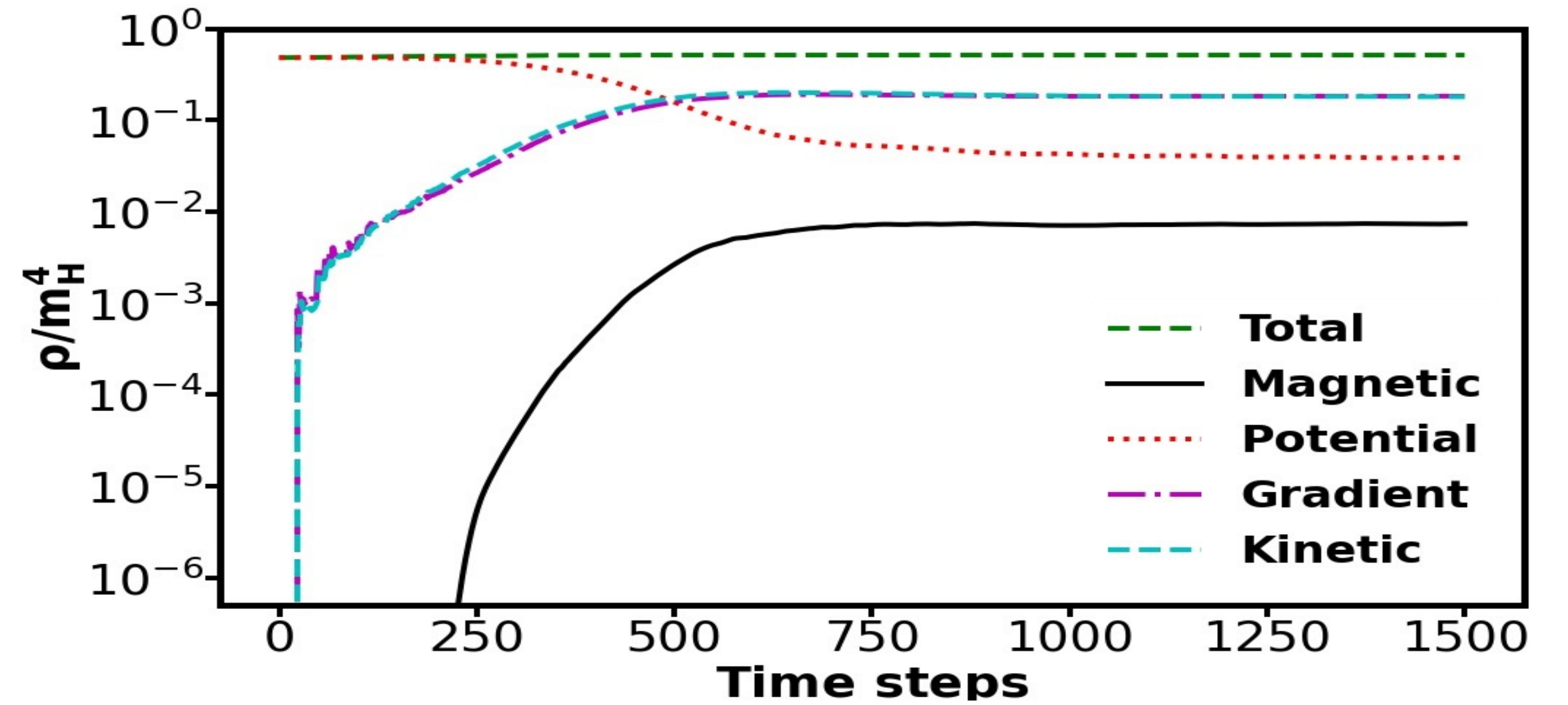
Bubble collision and GWs generation

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

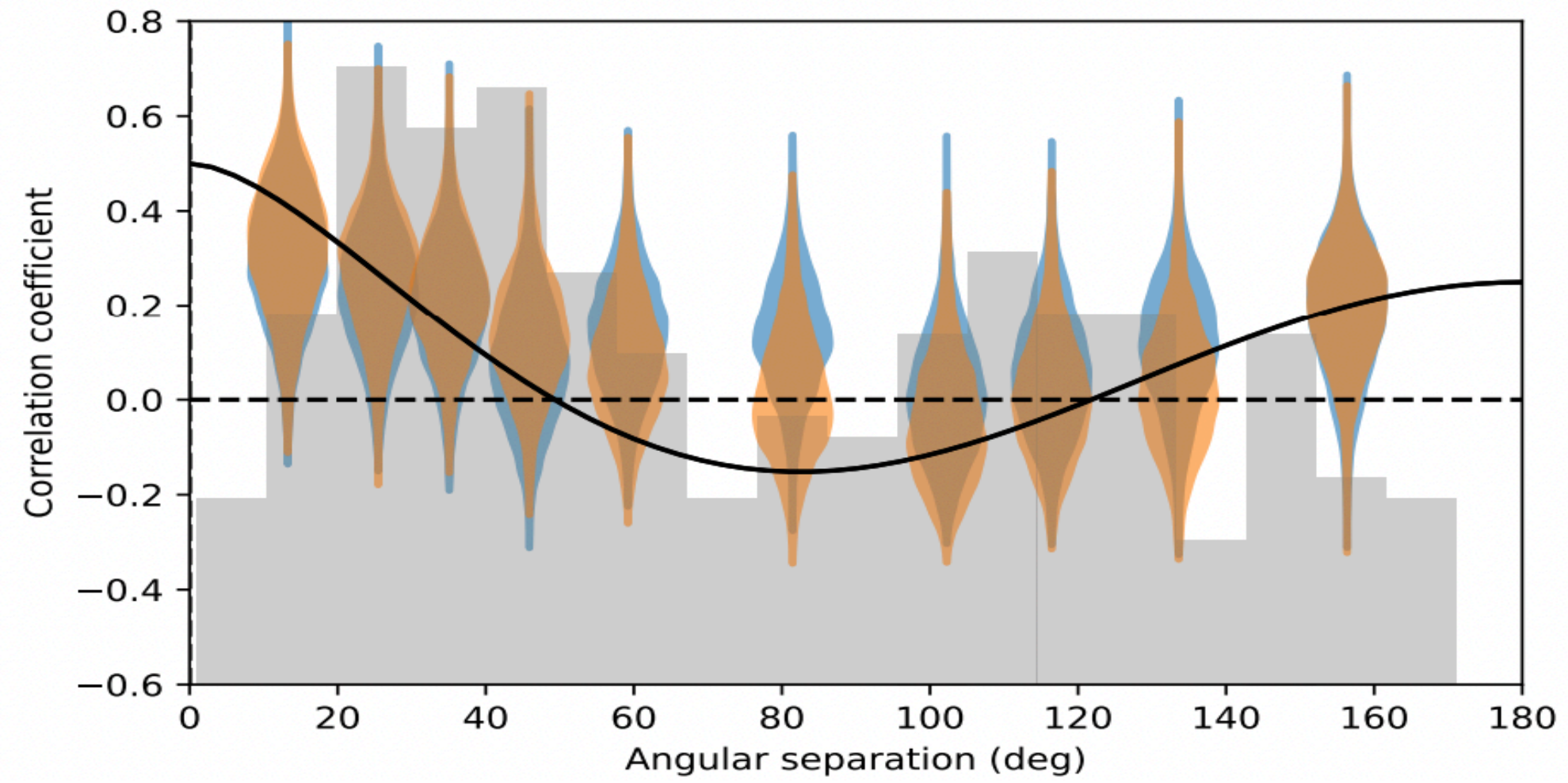
$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu} \frac{1}{2} \text{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k}, t) \dot{h}_{ij}^{TT}(\mathbf{k}', t) \rangle = P_h(\mathbf{k}, t) (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

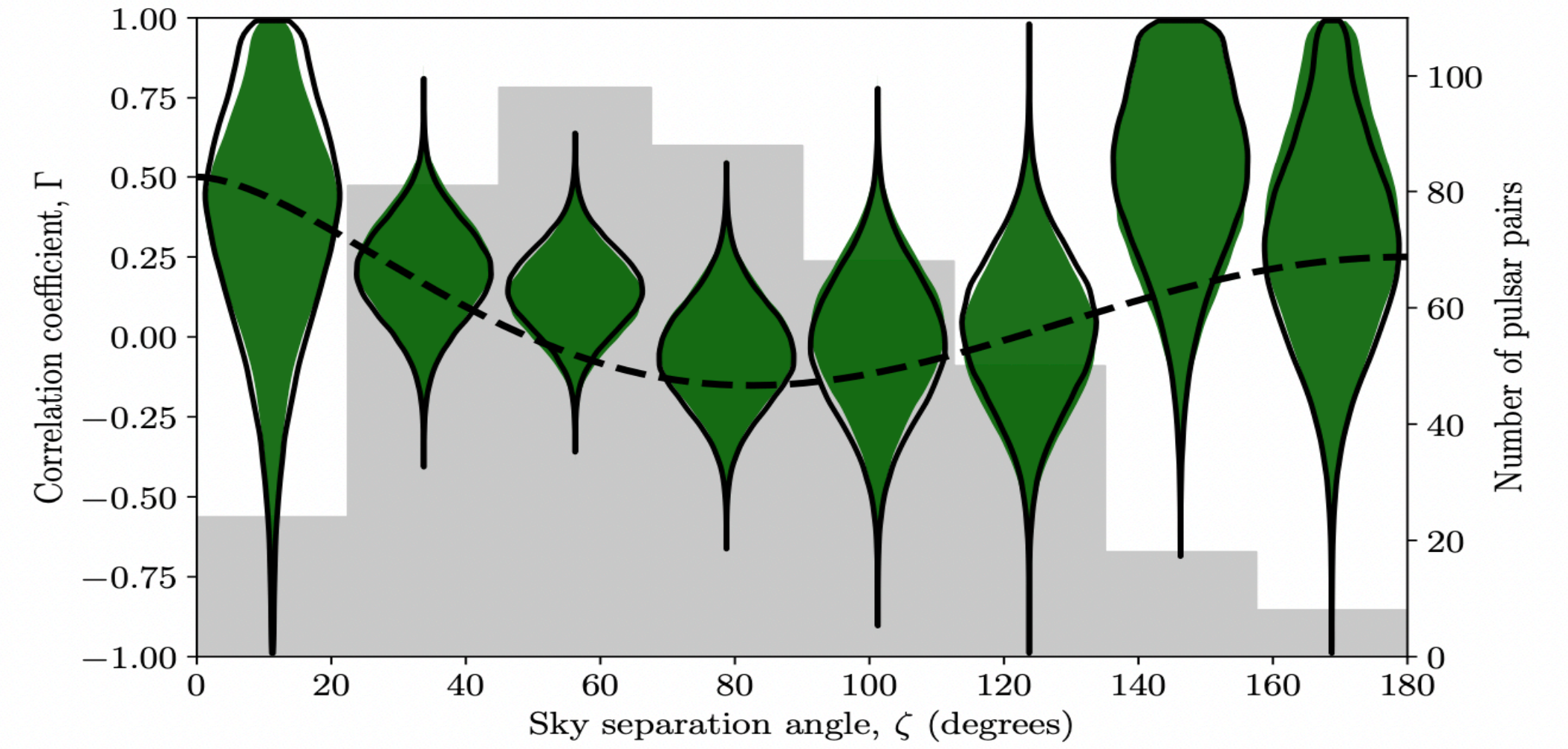
$$\frac{d\Omega_{\text{gw}}}{d\ln(k)} = \frac{1}{32\pi G \rho_c} \frac{k^3}{2\pi^2} P_h(\mathbf{k}, t)$$



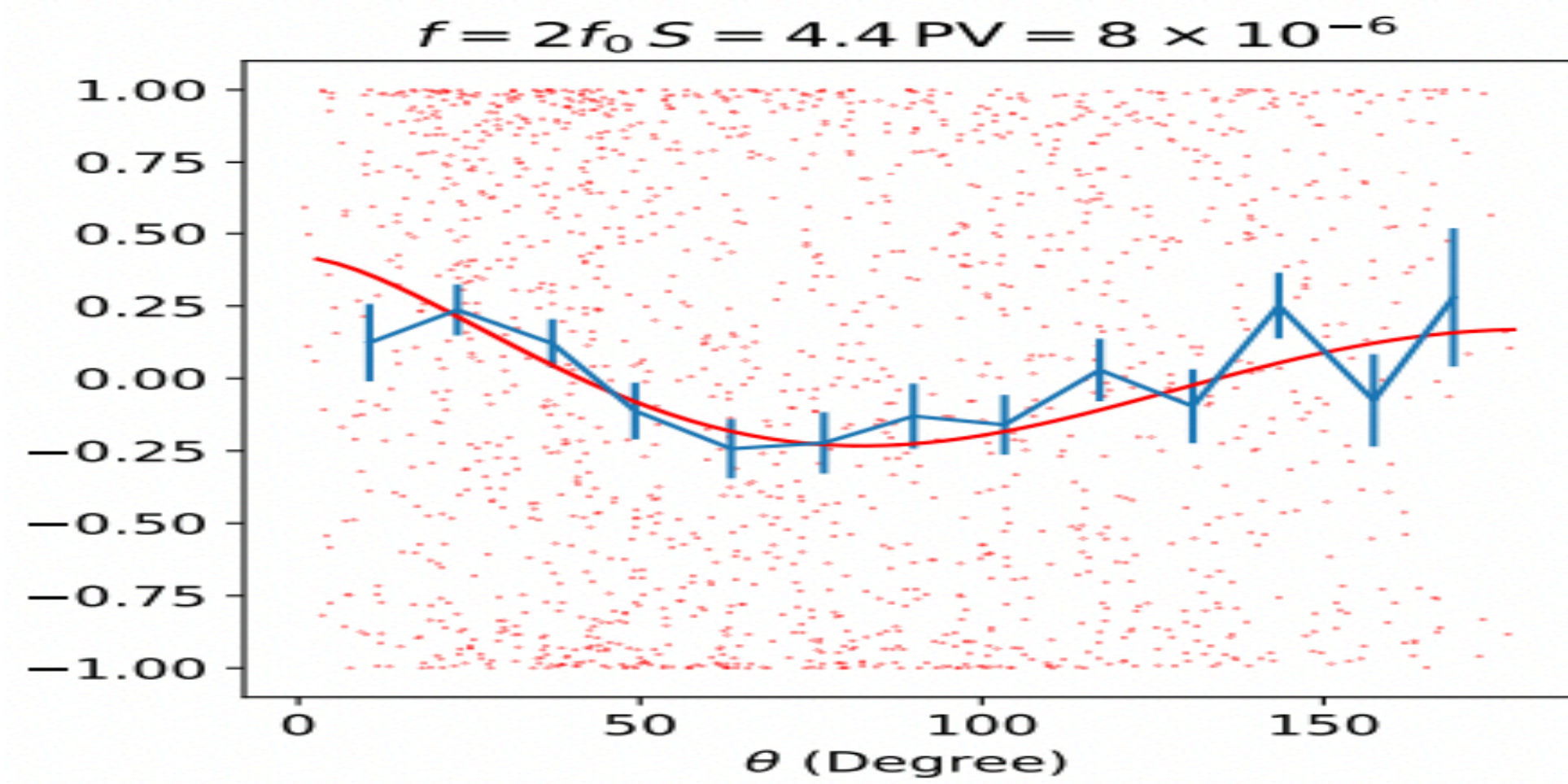
New dataset from PTAs



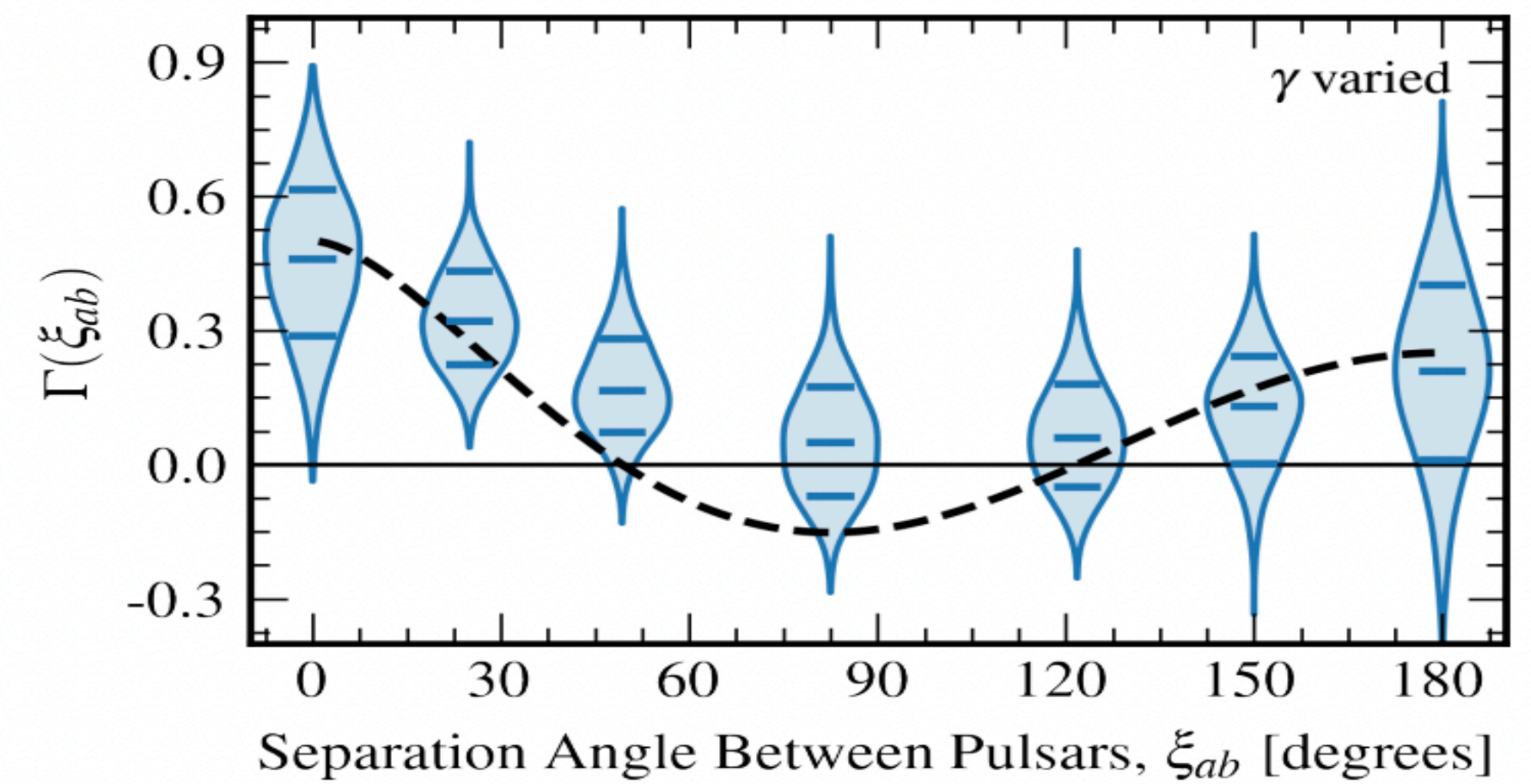
EPTA,2306.16214



PPTA,2306.16215

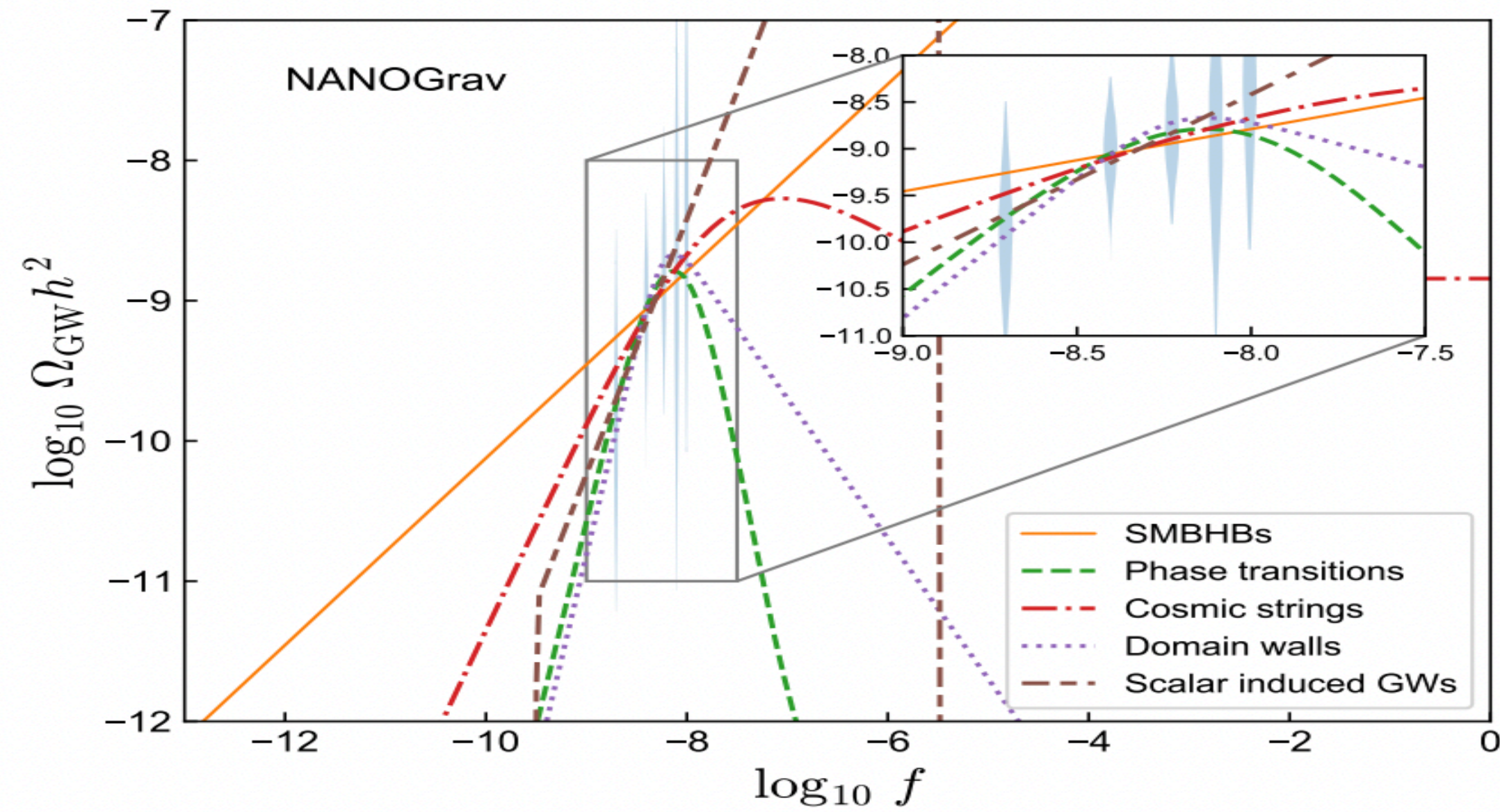
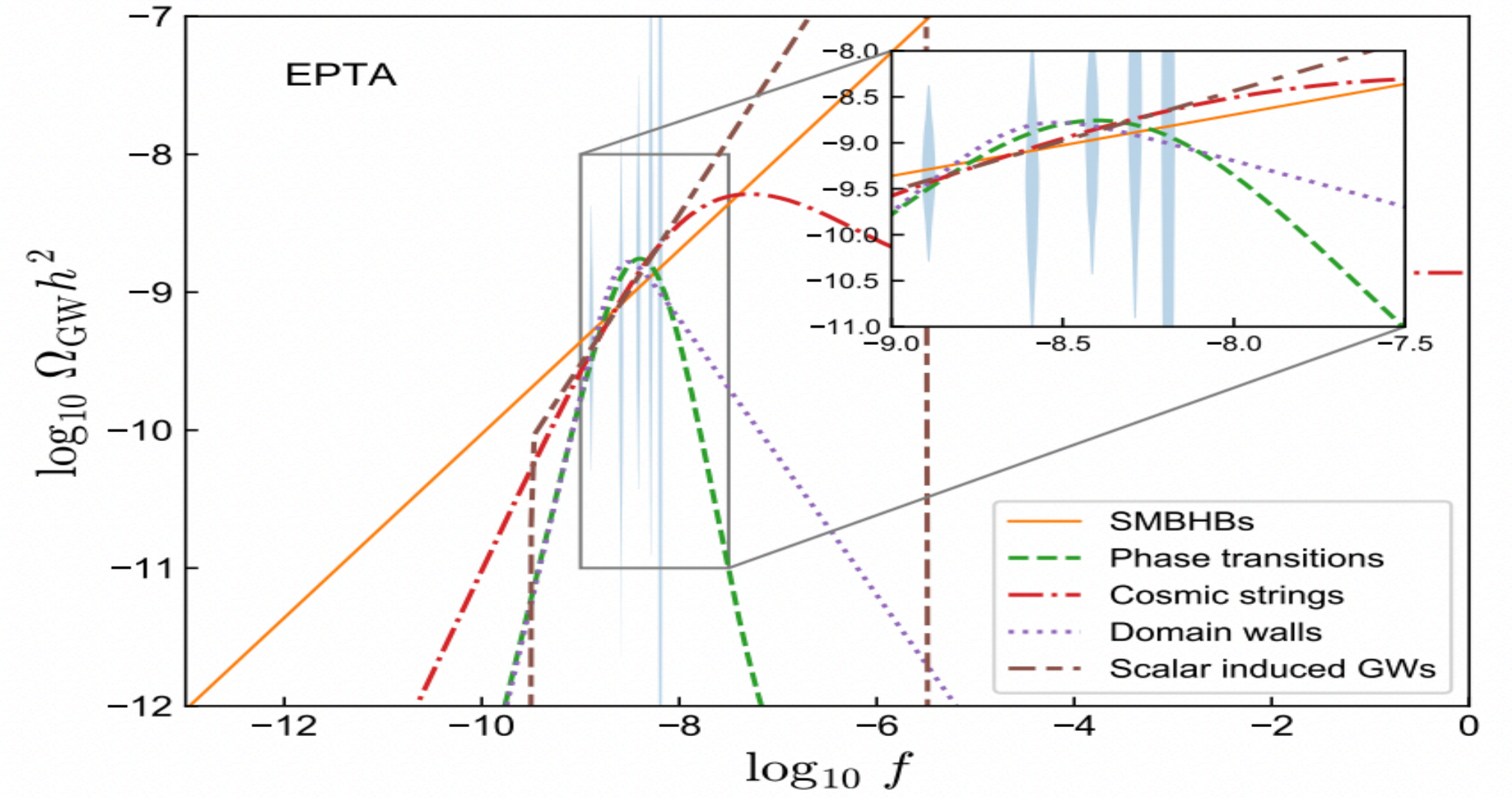
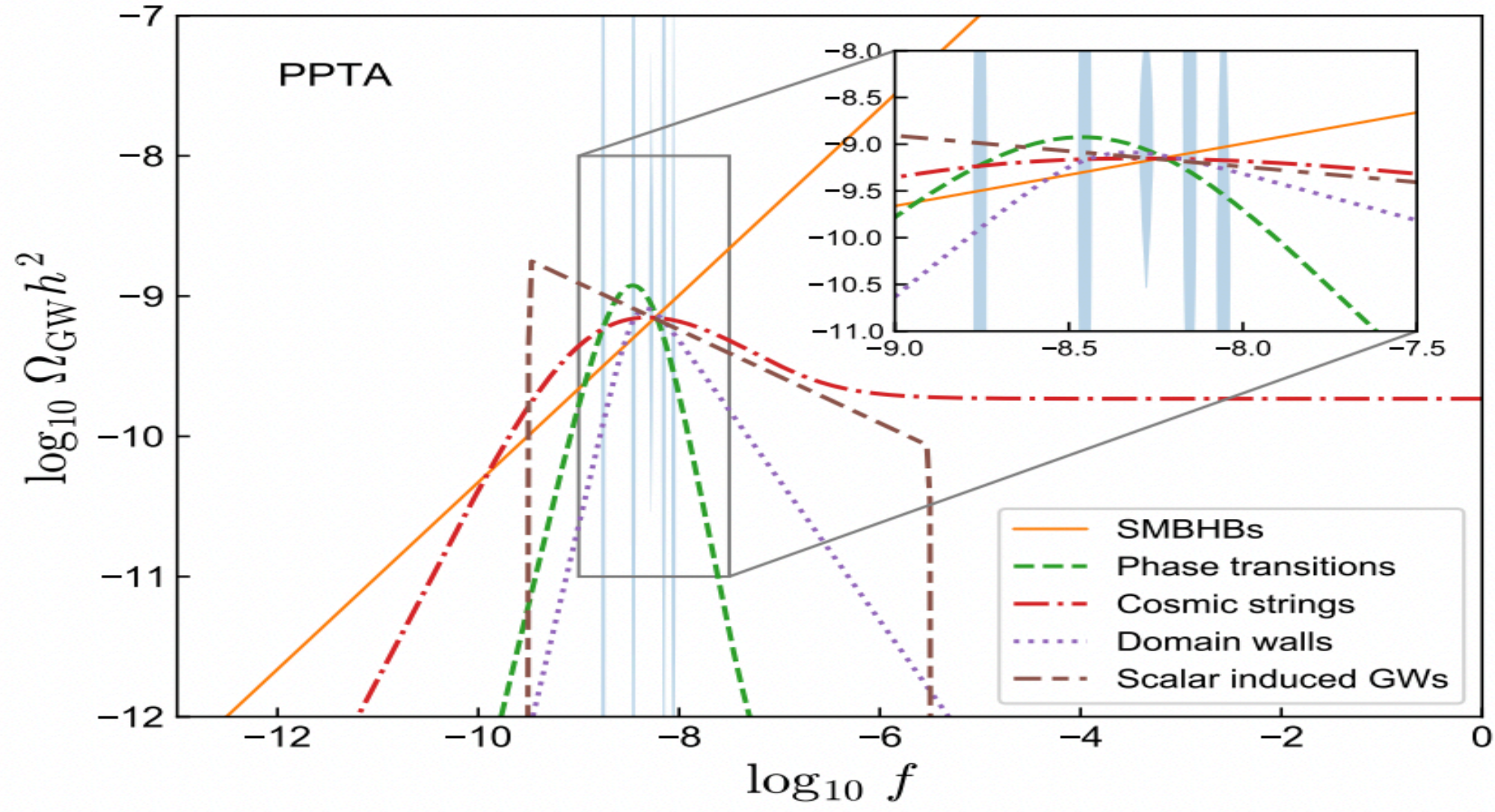


CPTA,2306.16216



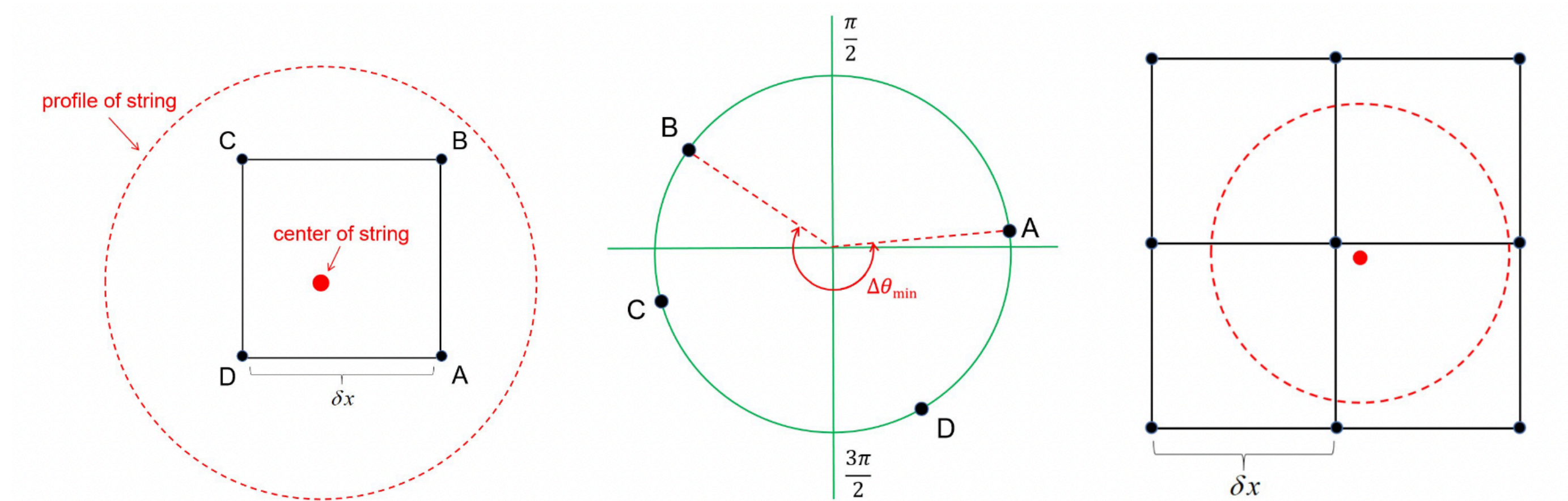
NANOGrav,2306.16213

Gravitational wave sources for Pulsar Timing Arrays



► Identification of Cosmic string

String penetrates the square loop if the minimum phase range which contains the four points is greater than π and the phase changes continuously



For a specific square loop, assuming that the minimum phase at four points is θ_{\min}

- (1) $\theta_{\min} < \pi$.
- (2) There exists at least one phase at another point minus θ_{\min} is greater than π .
- (3) There exists at least one phase at another point minus θ_{\min} is smaller than π .
- (4) Denote the phase closest to π in all phases greater than π as θ_a , and denote the phase closest to π in all phases smaller than π as θ_b , it is required to meet $\theta_a - \theta_b < \pi$.
- (5) Calculate the difference between the phases at each of two adjacent points in a counterclockwise direction, the multiplication of the four differences is required to be negative.

Bubble dynamics of FOPT

From QFT

$$p(t; T) \equiv \Gamma/V = |A(T)|e^{-B(T)/T}$$

Tunneling

$$A(T) = T \left(\frac{S_3[\bar{\phi}(T)]}{2\pi T} \right)^{\frac{3}{2}} \left(\frac{\det'[-\nabla^2 + V''(\bar{\phi})]}{\det[-\nabla^2 + V''(\phi_f)]} \right)^{-\frac{1}{2}}$$

Fluctuation

$$\begin{aligned} A(T) &= \frac{\sqrt{-\lambda_-}}{2\pi} \left(\frac{S_3[\bar{\phi}(T)]}{2\pi T} \right)^{\frac{3}{2}} \left(\frac{\det'[-\nabla^2 + V''(\bar{\phi})]}{\det[-\nabla^2 + V''(\phi_f)]} \right)^{-\frac{1}{2}} \\ &= \frac{1}{2\pi} \left(\frac{S_3[\bar{\phi}(T)]}{2\pi T} \right)^{\frac{3}{2}} \left(\frac{\det^+[-\nabla^2 + V''(\bar{\phi})]}{\det[-\nabla^2 + V''(\phi_f)]} \right)^{-\frac{1}{2}}, \end{aligned}$$

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False vacuum fraction considering bubbles expansion

Bubble Volume

$$h(t) = \exp \left[- \int_{t_c}^t dt' \frac{4\pi}{3} v_w^3 (t-t')^3 \frac{\Gamma(t')}{\mathcal{V}} \right]$$

$$\frac{\Gamma}{\mathcal{V}} = \frac{\Gamma_f}{\mathcal{V}} e^{\beta(t-t_f)} \quad \beta \equiv \left. \frac{d}{dt} \log \left(\frac{\Gamma(t)}{\mathcal{V}} \right) \right|_{t=t_f}$$

$$\begin{aligned} -\log h(t) &\simeq \int_{t_c}^t dt' \frac{4\pi}{3} v_w^3 (t-t')^3 \frac{\Gamma_f}{\mathcal{V}} e^{\beta(t'-t_f)} \\ &= \frac{4\pi}{3} v_w^3 \frac{\Gamma_f}{\mathcal{V}} \frac{3!}{\beta^4} e^{\beta(t-t_f)}. \end{aligned}$$

$$h(t_f) = 1/e$$

$$8\pi \frac{v_w^3}{\beta^4} \frac{\Gamma_f}{\mathcal{V}} = 1$$

$$h(t) = \exp \left[- e^{\beta(t-t_f)} \right]$$

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