Charmed Λ_c^+ baryon decays into light scalar mesons in the topological $SU(3)_f$ framework

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Introduction

Light scalar meson (S)

- S with a mass below 1 GeV

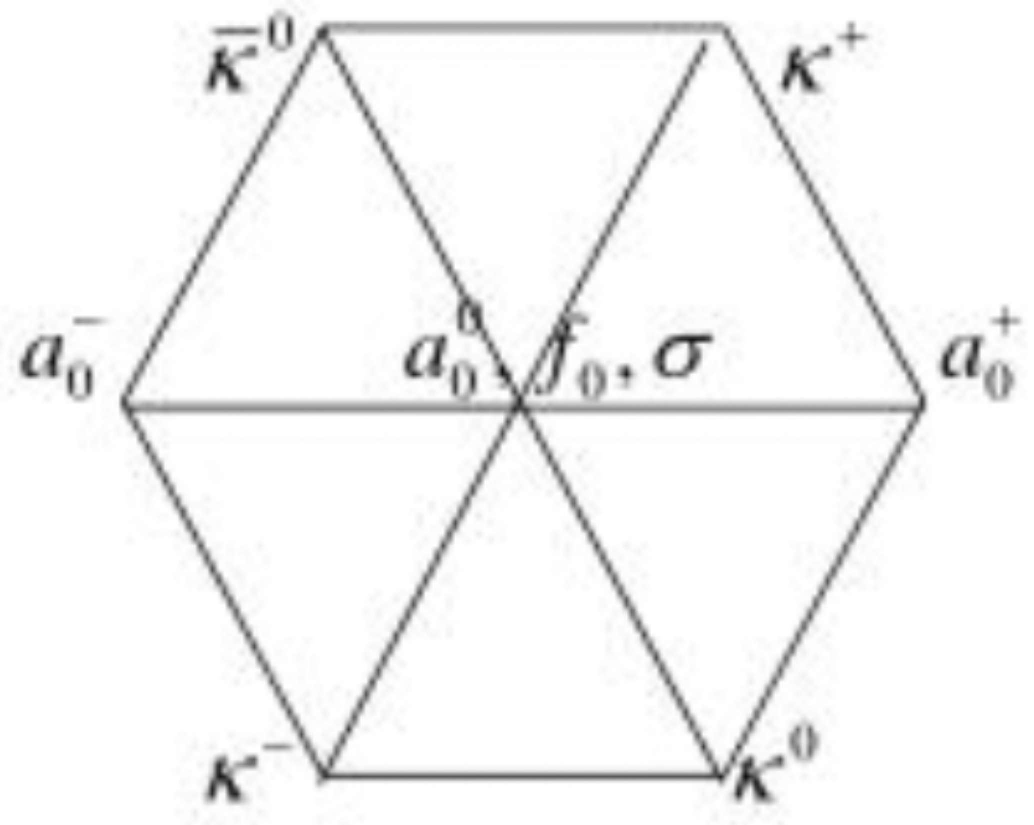
$f_0/f_0(980)$, $\sigma_0/f_0(500)$, $a_0/a_0(980)$, $\kappa_0/K_0^*(700)$

- Regarded as

a normal p-wave meson ($q\bar{q}$), or

an exotic tetraquark [compact $q^2\bar{q}^2$ bound state or

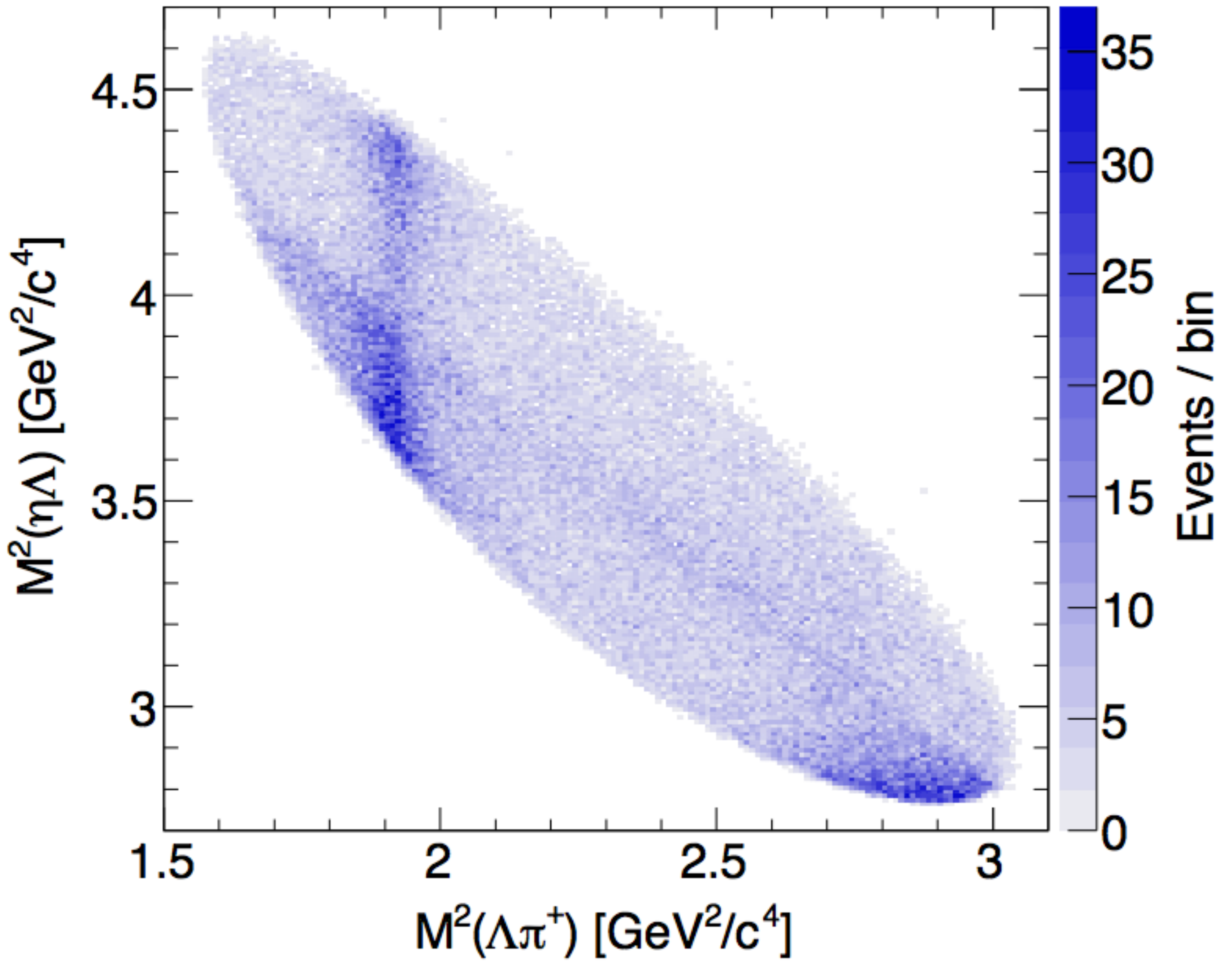
molecular M_1M_2 bound state.]



(From Ying Li)

Introduction (motivation)

The total branching fraction [Belle, PRD103, 052005 (2021)]



$$\mathcal{B}(\Lambda_c^+ \to \Lambda\eta\pi^+) = (18.4 \pm 0.2 \pm 0.9 \pm 0.9) \times 10^{-3},$$

$$\mathcal{B}(\Lambda_c^+ \to \Lambda^*\pi^+, \Lambda^* \to \Lambda\eta) = (3.5 \pm 0.5) \times 10^{-3},$$

$$\mathcal{B}(\Lambda_c^+ \to \Sigma^{*+}\eta, \Sigma^{*+} \to \Lambda\pi^+) = (10.5 \pm 1.2) \times 10^{-3},$$

with $\Lambda^* \equiv \Lambda(1670)$ and $\Sigma^* \equiv \Sigma(1385)$.

A possible $\Lambda_c^+ \to \Lambda a_0^+, a_0^+ \to \eta\pi^+$ process.

Introduction (Motivation)

- Using factorization and pole model,

W_{em} and W_{ex} contributions are estimated:

$\mathcal{B}(\Lambda_c^+ \to \Lambda a_0^+) = 1.9 \times 10^{-4}$.

Sharma, Verma [J. Phys. G **36**, 075005 (2009)]

- By accounting for LD rescattering effects,

we obtain $\mathcal{B}_{\text{th}}(\Lambda_c^+ \to \Lambda a_0^+) = (1.7^{+2.8}_{-1.0} \pm 0.3) \times 10^{-3}$.

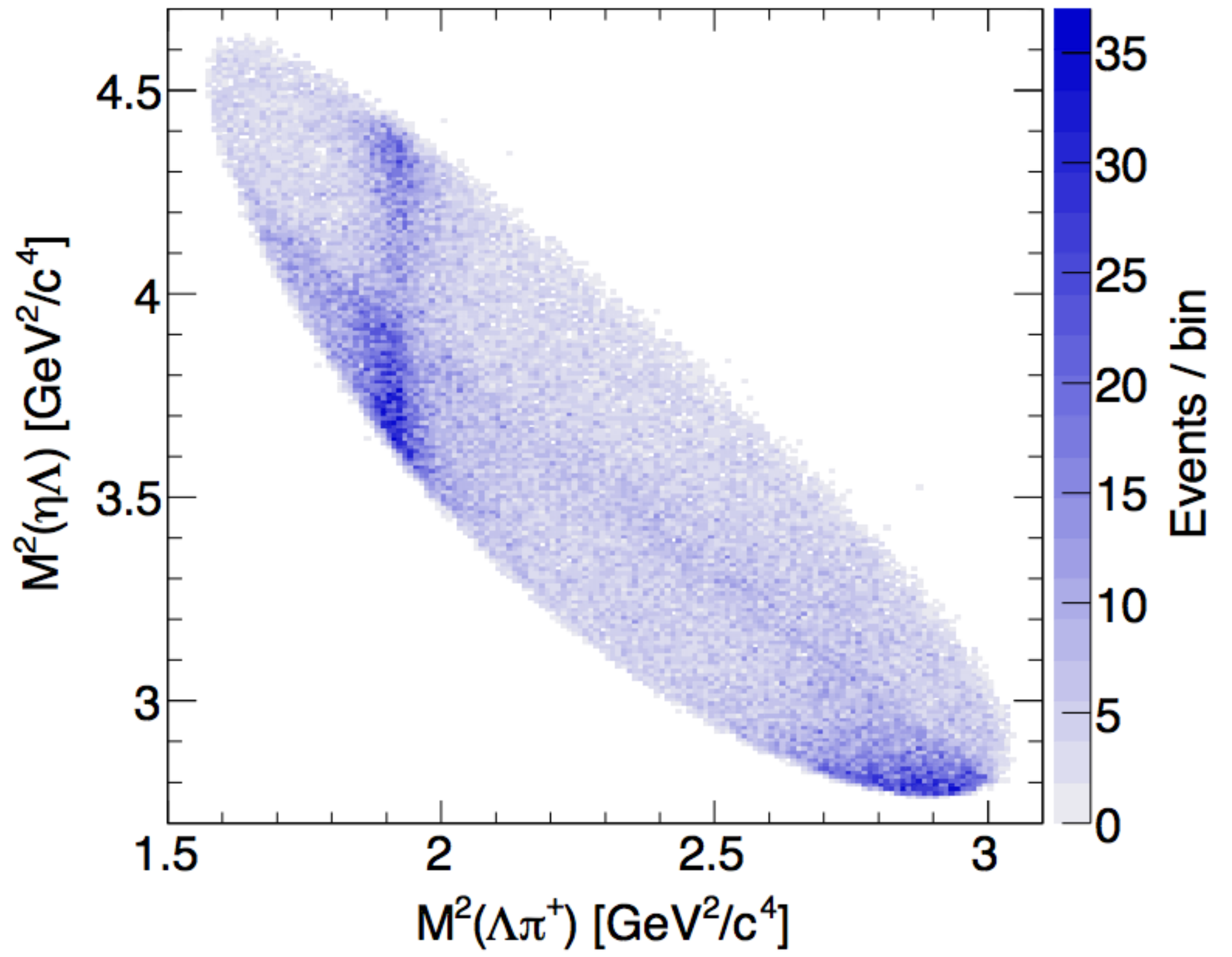
Y. Yu, Hsiao [PLB820, 136586 (2021)]

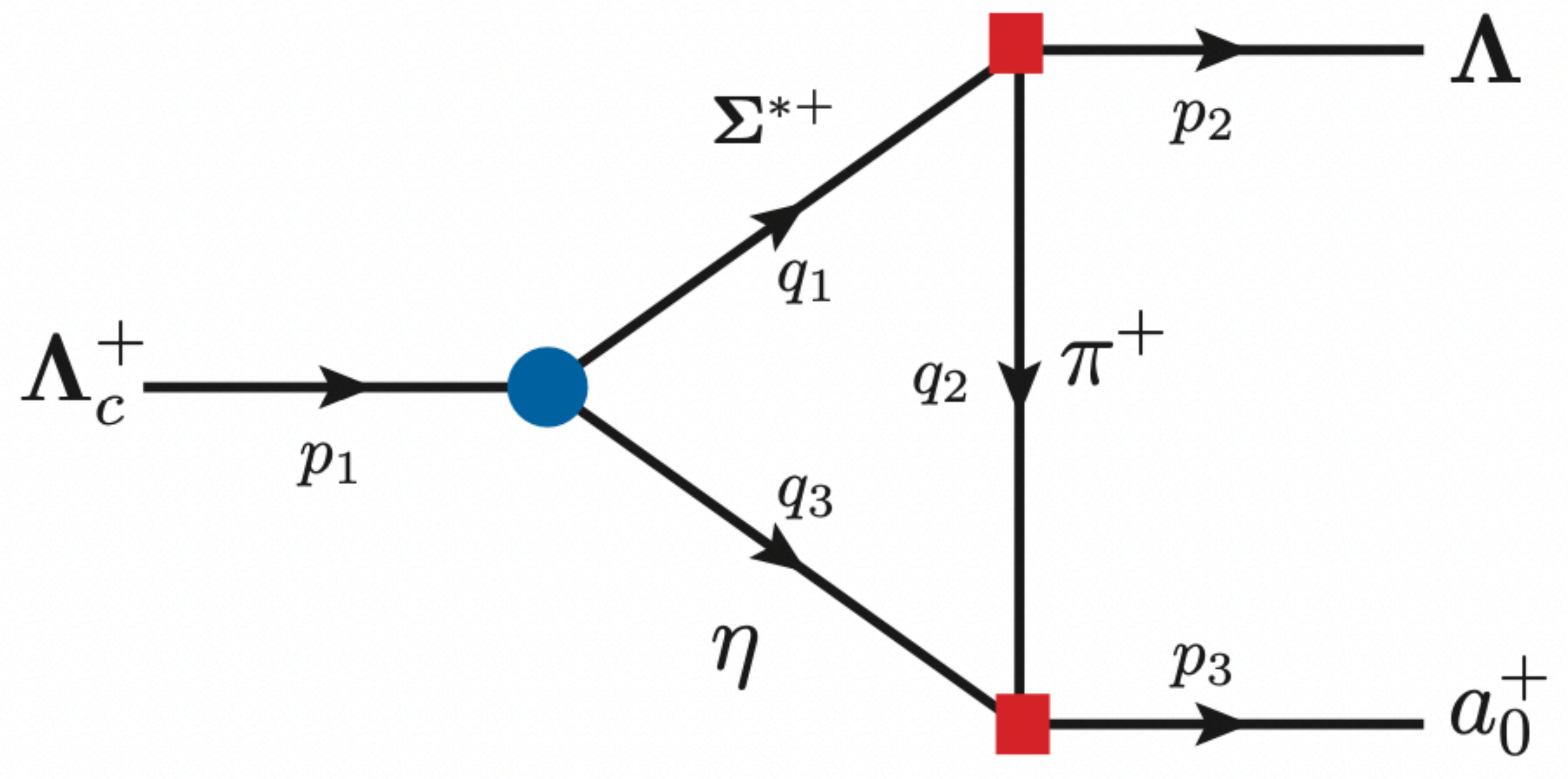
- $\mathcal{B}_{\text{th}} \sim 10^{-3}$ warrants a experimental examination.
- The first discovery is always important.

Only an old and less accurate

$\mathcal{B}(\Lambda_c^+ \to p f_0) = (3.5 \pm 2.3) \times 10^{-3}$ exists.

[ACCMOR, Z. Phys. C **48**, 29 (1990)]





- Studies of $\Lambda_c^+ \to \mathbf{B}S$ with LD effects:

1. "The $a_0(980)$ and $\Lambda(1670)$ in the $\Lambda_c^+ \to \pi^+\eta\Lambda$ decay,"
J.J. Xie and L.S. Geng [EPJC76, 496 (2016)].

2. "The scalar $f_0(500)$ and $f_0(980)$ resonances and vector mesons
in the single Cabibbo-suppressed decays $\Lambda_c^+ \to pK^+K^-$ and $p\pi^+\pi^-$,"
Z. Wang, Y.Y. Wang, E. Wang, D.M. Li and J.J. Xie [EPJC80, 842 (2020)].

3. "Roles of $a_0(980)$, $\Lambda(1670)$, and $\Sigma(1385)$ in the $\Lambda_c^+ \to \eta\Lambda\pi^+$ decay,"
G.Y. Wang, N.C. Wei, H.M. Yang, E. Wang, L.S. Geng and J.J. Xie
[PRD106, 056001 (2022)]

4. "The $a_0(980)$ in the single Cabibbo-suppressed process $\Lambda_c^+ \to \pi^0\eta p$,"
X.C. Feng, L.L. Wei, M.Y. Duan, E. Wang and D.M. Li
[PLB846, 138185 (2023)]

- To our surprise, BESIII reported $\mathcal{B}_{\rm ex}(\Lambda_c^+ \to \Lambda a_0^+) = (1.23 \pm 0.21) \times 10^{-2}$,

far above the theoretical expectation [PRL134, 021901 (2025)].

- To interpret this discrepancy, the SD $W_{\rm em}$ and $W_{\rm ex}$ processes

may play a dominant role in $\Lambda_c^+ \to \mathbf{B}S$,

similar to those in $\Lambda_c^+ \to \mathbf{B}M$, but could have been underestimated.

- If this is the case, it is natural to expect $\mathcal{B}(\Lambda_c^+ \to \mathbf{B}S) \simeq \mathcal{B}(\Lambda_c^+ \to \mathbf{B}M)$.

- Currently existing data show that

$\mathcal{B}_{\rm ex}(\Lambda_c^+ \to p f_0) = (3.5 \pm 2.3) \times 10^{-3}$ [ACCMOR, ZPC(1990)],

$\mathcal{B}_{\rm ex}(\Lambda_c^+ \to \Lambda a_0^+) = (1.23 \pm 0.21) \times 10^{-2}$ [BESIII, PRL(2025)],

$\mathcal{B}_{\rm ex}(\Lambda_c^+ \to p \bar{\kappa}^0) = (1.9 \pm 0.6) \times 10^{-3}$ [LHCb, PRD(2023)]

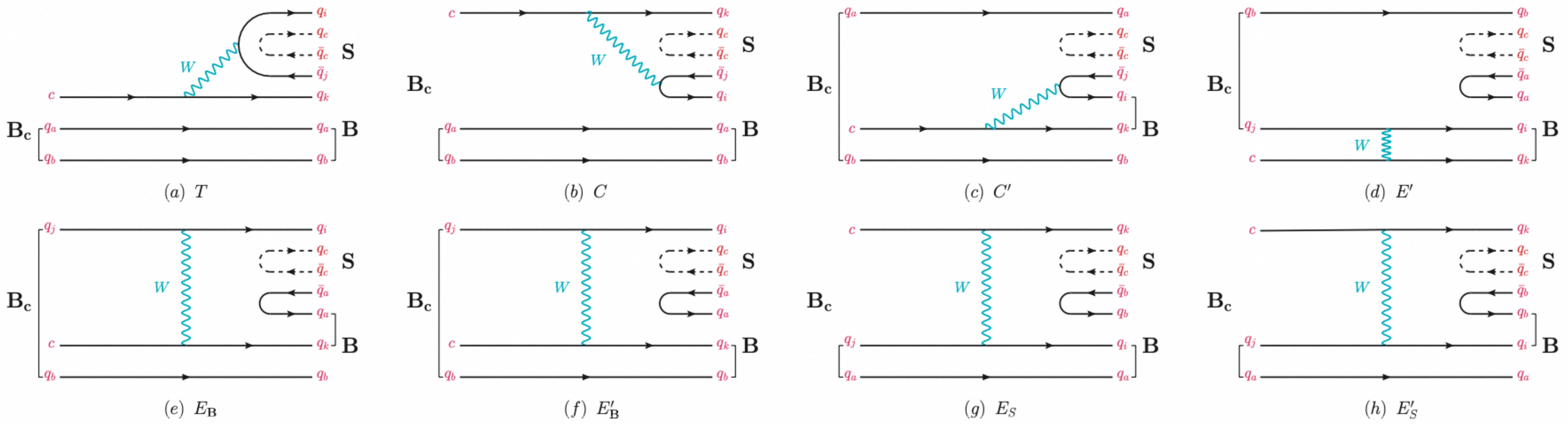
are comparable to $\mathcal{B}(\Lambda_c^+ \to p\eta') = (4.9 \pm 0.9) \times 10^{-4}$,

$\mathcal{B}(\Lambda_c^+ \to \Lambda\pi^+) = (1.31 \pm 0.05) \times 10^{-2}$, and

$\mathcal{B}_{\rm ex}(\Lambda_c^+ \to p K_S^0) = (1.61 \pm 0.07) \times 10^{-2}$, respectively.

Formalism

- The relation $\mathcal{B}_{\rm ex}(\Lambda_c^+ \to \mathbf{B}S) \simeq \mathcal{B}_{\rm ex}(\Lambda_c^+ \to \mathbf{B}M)$ across the measured decay channels supports the dominance of the $W_{\rm EM}$ and $W_{\rm EX}$ processes.
- This observation suggests that the $SU(3)_f$-based topological-diagram approach (TDA), successfully applied to $\Lambda_c^+ \to \mathbf{B}M$, can be consistently extended to $\Lambda_c^+ \to \mathbf{B}S$.
- Topological diagrams for $\mathbf{B}_c \to \mathbf{B}S$



Two $SU(3)_f$ approaches

• Topological diagram approach (intuitive and diagrammatic framework)

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• Irreducible $SU(3)_f$ approach (using the irreducible group-theoretical representations.)

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• The effective Hamiltonian in the $SU(3)_f$ representation:

1. $\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{uq'} V^*_{cq} [c_1(\bar{u}q')(\bar{q}c) + c_2(\bar{u}_\beta q'_\alpha)(\bar{q}_\alpha c_\beta)]$.

2. $\mathcal{H}_c = \mathcal{H}_{eff}/(G_F/\sqrt{2}) = H^{ki}_j$.

3. The charm quark weak transitions $c \to s\bar{d}u$, $s\bar{s}u$, $d\bar{d}u$, and $d\bar{s}u$

correspond to the non-zero components of H^{ki}_j

$H^{31}_2 = V^*_{cs}V_{ud}$, $H^{31}_3 = V^*_{cs}V_{us}$, $H^{21}_2 = V^*_{cd}V_{ud}$, and $H^{21}_3 = V^*_{cd}V_{us}$, respectively.

• $\mathbf{B}_c$ and $\mathbf{B}$ in the $SU(3)_f$ expression:

$$\mathbf{B}_c = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda^0 + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}.$$

• Light scalar mesons (S) in the $SU(3)_f$ expression:
In the two-quark ($q\bar{q}$) and four-quark ($q^2\bar{q}^2$) structures

$$
\begin{aligned}
f_0 &= \cos\theta_I|s\bar{s}\rangle + \sin\theta_I|\sqrt{1/2}(u\bar{u}+d\bar{d})\rangle\,,\\
\sigma_0 &= -\sin\theta_I|s\bar{s}\rangle + \cos\theta_I|\sqrt{1/2}(u\bar{u}+d\bar{d})\rangle\,,\\
a_0^+ &= |u\bar{d}\rangle\,, a_0^0 = |\sqrt{1/2}(u\bar{u}-d\bar{d})\rangle\,, a_0^- = |d\bar{u}\rangle\,,\\
\kappa^+ &= |u\bar{s}\rangle\,, \kappa^0 = |d\bar{s}\rangle\,, \bar{\kappa}^0 = |s\bar{d}\rangle\,, \kappa^- = |s\bar{u}\rangle\,.
\end{aligned}
$$

$$
\begin{aligned}
f_0 &= \cos\theta_{II}|\sqrt{1/2}(u\bar{u}+d\bar{d})s\bar{s}\rangle + \sin\theta_{II}|u\bar{u}d\bar{d}\rangle\,,\\
\sigma_0 &= -\sin\theta_{II}|\sqrt{1/2}(u\bar{u}+d\bar{d})s\bar{s}\rangle + \cos\theta_{II}|u\bar{u}d\bar{d}\rangle\,,\\
a_0^+ &= |u\bar{d}s\bar{s}\rangle\,,\ a_0^0 = |\sqrt{1/2}(u\bar{u}-d\bar{d})s\bar{s}\rangle\,, a_0^- = |d\bar{u}s\bar{s}\rangle\,,\\
\kappa^+ &= |u\bar{s}d\bar{d}\rangle\,,\ \kappa^0 = |d\bar{s}u\bar{u}\rangle\,,\ \bar{\kappa}^0 = |s\bar{d}u\bar{u}\rangle\,,\ \kappa^- = |s\bar{u}d\bar{d}\rangle\,.
\end{aligned}
$$

$$
S_{q\bar{q}} = \begin{pmatrix}
\frac{1}{\sqrt{2}}(a_0^0 + c\phi_I\sigma_0 + s\phi_I f_0) & a_0^+ & \kappa^+ \\
a_0^- & \frac{-1}{\sqrt{2}}(a_0^0 - c\phi_I\sigma_0 - s\phi_I f_0) & \kappa^0 \\
\kappa^- & \bar{\kappa}^0 & -s\phi_I\sigma_0 + c\phi_I f_0
\end{pmatrix},
$$

$$
S_{q^2\bar{q}^2} = \begin{pmatrix}
\frac{1}{\sqrt{2}}(a_0^0 + c\phi_{II} f_0 - s\phi_{II}\sigma_0) & a_0^+ & \kappa^+ \\
a_0^- & \frac{-1}{\sqrt{2}}(a_0^0 - c\phi_{II} f_0 + s\phi_{II}\sigma_0) & \kappa^0 \\
\kappa^- & \bar{\kappa}^0 & s\phi_{II} f_0 + c\phi_{II}\sigma_0
\end{pmatrix}.
$$

- TDA amplitudes

$$\mathcal{M}_{\rm TDA}(\mathbf{B}_c \to \mathbf{B}S) = T\mathbf{B}_c{}^{ab}H^{ki}_j\mathbf{B}_{abk}S^i_j + C\mathbf{B}_c{}^{ab}H^{ki}_j\mathbf{B}_{abi}S^k_j + C'\mathbf{B}_c{}^{ab}H^{ki}_j\mathbf{B}_{ikb}S^a_j$$
$$+ E_{\mathbf{B}}\mathbf{B}_c{}^{jb}H^{ki}_j\mathbf{B}_{kab}S^i_a + E'_{\mathbf{B}}\mathbf{B}_c{}^{jb}H^{ki}_j\mathbf{B}_{kba}S^i_a$$
$$+ E_S\mathbf{B}_c{}^{jb}H^{ki}_j\mathbf{B}_{iba}S^k_a + E'_S\mathbf{B}_c{}^{jb}H^{ki}_j\mathbf{B}_{iab}S^k_a + E'\mathbf{B}_c{}^{jb}H^{ki}_j\mathbf{B}_{ika}S^b_a\,.$$

The identity $\mathbf{B}_{ijk} = \epsilon_{ijl}\mathbf{B}^l_k$ has been employed.

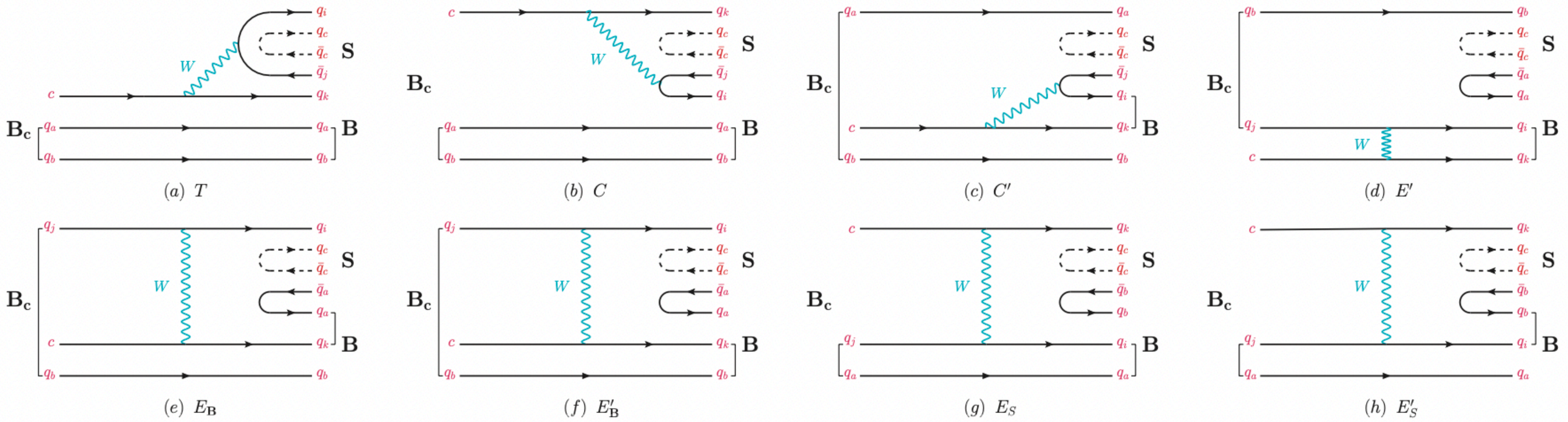


TABLE I. TDA amplitudes of $\Lambda_c^+ \to \mathbf{B} f_0, \mathbf{B}\sigma_0$ in the $f_0 - \sigma_0$ mixing scheme.

Decay mode	$\mathcal{M}_{q\bar{q}}$	$\mathcal{M}_{q^2\bar{q}^2}$
$\Lambda_c^+ \to \Sigma^+ f_0$	$\frac{1}{\sqrt{2}}(C' - E_{\mathbf{B}} + E')s\phi_I + E_S'^{(s)} c\phi_I$	$\frac{1}{\sqrt{2}}(C' - E_{\mathbf{B}} + E')c\phi_{II} + E_S'^{(s)} s\phi_{II}$
$\Lambda_c^+ \to \Sigma^+ \sigma_0$	$\frac{1}{\sqrt{2}}(C' - E_{\mathbf{B}} + E')c\phi_I - E_S'^{(s)} s\phi_I$	$-\frac{1}{\sqrt{2}}(C' - E_{\mathbf{B}} + E')s\phi_{II} + E_S'^{(s)} c\phi_{II}$
$\Lambda_c^+ \to p f_0$	$[-\frac{1}{\sqrt{2}}(2C - C' - E'_S + E_{\mathbf{B}} - E')s\phi_I + 2Cc\phi_I]s_c$	$[-\frac{1}{\sqrt{2}}(2C - C' - E'_S + E_{\mathbf{B}} - E')c\phi_{II} + 2Cs\phi_{II}]s_c$
$\Lambda_c^+ \to p\sigma_0$	$[-\frac{1}{\sqrt{2}}(2C - C' - E'_S + E_{\mathbf{B}} - E')c\phi_I - 2Cs\phi_I]s_c$	$[\frac{1}{\sqrt{2}}(2C - C' - E'_S + E_{\mathbf{B}} - E')s\phi_{II} + 2Cc\phi_{II}]s_c$

$$\theta_I = (156.7 \pm 0.7)^\circ,\ \theta_{II} = (174.6^{+3.4}_{-3.2})^\circ.$$

- Reduction of the number of parameters

1. The combination of $(E' - E'_{\mathbf{B}})$ appears in the amplitudes, allowing us to set $E'_{\mathbf{B}} = 0$.
2. Using the equivalence relation $\mathcal{M}_{\rm TDA}(\mathbf{B}_c \to \mathbf{B}S) = \mathcal{M}_{\rm IRA}(\mathbf{B}_c \to \mathbf{B}S)$, we idnetify $E'_S = E_{\mathbf{B}} = \hat{a}_4$.
3. For the Factorizable $W_{\rm em}$ terms, $T \sim C \propto f_S \sim 0$.
4. E_S exclusively contributes to $\Xi_c^0 \to \mathbf{B}S$ and therefore is excluded in the this analysis.
5. The set of independent parameters is reduced to $|C'|$, $|E'|e^{\delta_{E'}}$, $|E_{\mathbf{B}}|e^{i\delta_{E_{\mathbf{B}}}}$.

TABLE II. TDA amplitudes of $\Lambda_c^+ \to \mathbf{B}a_0, \mathbf{B}\kappa$.

Decay mode	$\mathcal{M}_{q\bar{q}} = \mathcal{M}_{q^2\bar{q}^2}$
$\Lambda_c^+ \to \Lambda^0 a_0^+$	$-\frac{1}{\sqrt{6}}(4T + C' - E_{\mathbf{B}} - E')$
$\Lambda_c^+ \to \Sigma^0 a_0^+$	$\frac{1}{\sqrt{2}}(C' + E_{\mathbf{B}} - E')$
$\Lambda_c^+ \to \Sigma^+ a_0^0$	$-\frac{1}{\sqrt{2}}(C' + E_{\mathbf{B}} - E')$
$\Lambda_c^+ \to \Xi^0 \kappa^+$	$E'^{(s)}$
$\Lambda_c^+ \to p\bar{\kappa}^0$	$2C - E'_S$
$\Lambda_c^+ \to \Lambda^0 \kappa^+$	$-\frac{1}{\sqrt{6}}(4T + C' - E_{\mathbf{B}}^{(s)} + E'^{(s)})s_c$
$\Lambda_c^+ \to \Sigma^0 \kappa^+$	$\frac{1}{\sqrt{2}}(C' + E_{\mathbf{B}}^{(s)})s_c$
$\Lambda_c^+ \to \Sigma^+ \kappa^0$	$(-E_S'^{(s)} + C')s_c$
$\Lambda_c^+ \to n a_0^+$	$(-2T - C' + E')s_c$
$\Lambda_c^+ \to p a_0^0$	$\frac{1}{\sqrt{2}}(2C - C' - E_{\mathbf{B}} - E'_S + E')s_c$
$\Lambda_c^+ \to p\kappa^0$	$(C' - 2C)s_c^2$
$\Lambda_c^+ \to n\kappa^+$	$-(C' + 2T)s_c^2$

• For a global fit with five parameters,

the currently available three data points are not sufficient.

• Seeking additional inputs for a global fit

1. The following relation can be useful:

$\mathcal{B}_T(\mathbf{B}_c \to \mathbf{B}MM) \simeq \mathcal{B}_N(\mathbf{B}_c \to \mathbf{B}MM)$ $+\mathcal{B}_{\mathbf{B}^*}(\mathbf{B}_c \to M(\mathbf{B}^* \to)\mathbf{B}M)$

$+\mathcal{B}_V(\mathbf{B}_c \to \mathbf{B}(V \to)MM)$$+\mathcal{B}_S(\mathbf{B}_c \to \mathbf{B}(S \to)MM)$.

2. Under the narrow-width approximation (NWA):

a) $\mathcal{B}_V \equiv \mathcal{B}(\mathbf{B}_c \to \mathbf{B}V, V \to MM) \simeq \mathcal{B}(\mathbf{B}_c \to \mathbf{B}V) \times \mathcal{B}(V \to MM)$,

b) $\mathcal{B}_{\mathbf{B}^*} \equiv \mathcal{B}(\mathbf{B}_c \to \mathbf{B}^*M, \mathbf{B}^* \to \mathbf{B}M) \simeq \mathcal{B}(\mathbf{B}_c \to \mathbf{B}^*M) \times \mathcal{B}(\mathbf{B}^* \to \mathbf{B}M)$,

c) Using $\mathcal{B}_T$ from the PDG, together with $\mathcal{B}(\mathbf{B}_c \to \mathbf{B}V)$, $\mathcal{B}(\mathbf{B}_c \to \mathbf{B}^*M)$,

and $\mathcal{B}_N(\mathbf{B}_c \to \mathbf{B}MM)$ calculated within the $SU(3)_f$ framework, we obtain

$\mathcal{B}_{\rm ex(res)}(f_0, \sigma_0) \equiv \mathcal{B}(\Lambda_c^+ \to \Sigma^+[(f_0 + \sigma_0) \to]\pi^+\pi^-) = (11.0 \pm 6.0) \times 10^{-3}$,

$\mathcal{B}_{\rm ex(res)}(f_0, a_0) \equiv \mathcal{B}(\Lambda_c^+ \to \Sigma^+[(f_0 + a_0) \to]K^+K^-) = (10.8 \pm 4.6) \times 10^{-4}$,

$\mathcal{B}_{\rm ex(res)}(a_0^+) \equiv \mathcal{B}(\Lambda_c^+ \to \Sigma^0(a_0^+ \to)\pi^+\eta) = (7.1 \pm 0.7) \times 10^{-3}$,

$\mathcal{B}_{\rm ex(res)}(\kappa^+) \equiv \mathcal{B}(\Lambda_c^+ \to \Xi^0(\kappa^+ \to)K^0\pi^+) = (3.9 \pm 1.9) \times 10^{-3}$.

• Relating $\mathcal{B}(\Lambda_c^+ \to \mathbf{B}(S \to)MM)$ with $\mathcal{B}(\Lambda_c^+ \to \mathbf{B}S)$

1. The resonant amplitude can be written as

$\mathcal{M}(\Lambda_c^+ \to \mathbf{B}(S \to)MM) = \mathcal{M}(S \to MM)D_S^{-1}\mathcal{M}(\Lambda_c^+ \to \mathbf{B}S)$,

where D_S^{-1} denotes the resonance propagator.

2. For $f_0 \to (\pi\pi, K\bar{K})$ and $a_0 \to (\eta\pi, K\bar{K})$, multiple coupled channels are involved.

In addition, the broad decay widths $\Gamma_{\sigma_0} \sim m_{\sigma_0}$ and $\Gamma_\kappa \sim m_\kappa$

invalidate both the conventional Breit–Wigner form $1/[(p^2 - m^2) + im\Gamma]$.

3. We therefore adopt the Flatte parameterization for f_0 and a_0,

and energy-dependent-width Breit-Wigner form for σ_0 and κ.

4. Accordingly, we obtain $\mathcal{B}_{\text{th(res)}}(f_0, \sigma_0) = \bar{\alpha}_1\mathcal{B}_{\text{th}}(\Lambda_c^+ \to \Sigma^+ f_0) + \bar{\alpha}_2\mathcal{B}_{\text{th}}(\Lambda_c^+ \to \Sigma^+\sigma_0)$,

$\mathcal{B}_{\text{th(res)}}(f_0, a_0) = \bar{\alpha}_3\mathcal{B}_{\text{th}}(\Lambda_c^+ \to \Sigma^+ f_0) + \bar{\alpha}_4\mathcal{B}_{\text{th}}(\Lambda_c^+ \to \Sigma^+ a_0^0)$,

$\mathcal{B}_{\text{th(res)}}(a_0^+) = \bar{\alpha}_5\mathcal{B}_{\text{th}}(\Lambda_c^+ \to \Sigma^0 a_0^+)$, $\mathcal{B}_{\text{th(res)}}(\kappa^+) = \bar{\alpha}_6\mathcal{B}_{\text{th}}(\Lambda_c^+ \to \Xi^0\kappa^+)$,

where the coefficients $\bar{\alpha}_i$ encode the resonant effects of $S \to MM$.

Specifically, $(\bar{\alpha}_1, \bar{\alpha}_2)$ correspond to $(f_0, \sigma_0) \to \pi^+\pi^-$, $(\bar{\alpha}_3, \bar{\alpha}_4)$ to $(f_0, a_0^0) \to K^+K^-$,

$\bar{\alpha}_5$ to $a_0^+ \to \pi^+\eta$, $\bar{\alpha}_6$ to $\kappa^+ \to K^0\pi^+$.

Numerically, we obtain $(\alpha_1, \alpha_2) = (0.17 \pm 0.09, 0.33 \pm 0.03)$,

$(\alpha_3, \alpha_4) = (0.02 \pm 0.01, 0.04 \pm 0.01)$, $(\alpha_5, \alpha_6) = (0.59 \pm 0.06, 0.26 \pm 0.03)$.

- The parameters are determined from a χ^2 fit defined as

$$\chi^2 = \Sigma_i \left(\frac{\mathcal{B}^i_{\rm th} - \mathcal{B}^i_{\rm ex}}{\sigma^i_{\rm ex}} \right)^2 + \Sigma_j \left(\frac{\bar{\alpha}_j - \alpha_j}{\sigma_{\alpha_j}} \right)^2 .$$

$$(\alpha_1, \alpha_2) = (0.17 \pm 0.09, 0.33 \pm 0.03) ,$$

$$(\alpha_3, \alpha_4) = (0.02 \pm 0.01, 0.04 \pm 0.01) ,$$

$$(\alpha_5, \alpha_6) = (0.59 \pm 0.06, 0.26 \pm 0.03) .$$

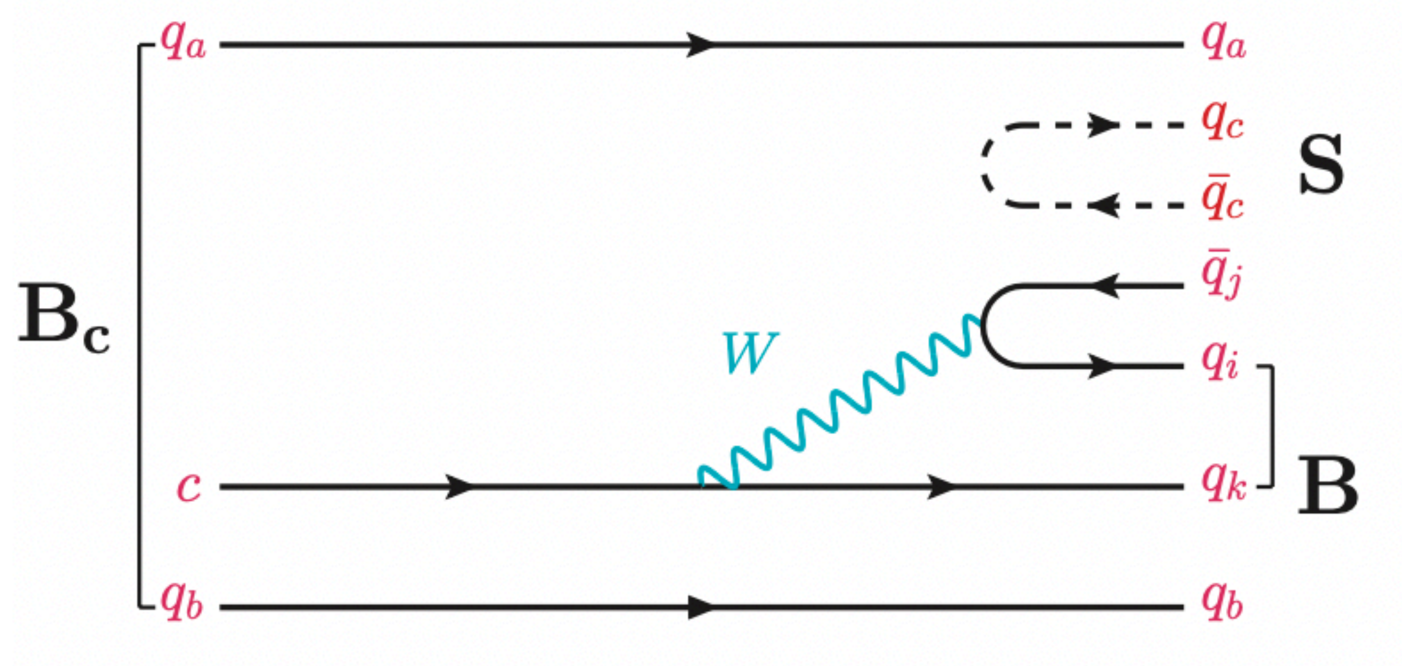
- We thus obtain

$$\text{S1} : (|C'|, |E'|, |E_{\bf B}|) = (0.67 \pm 0.16, 0.53 \pm 0.13, 0.15 \pm 0.02)\ \text{GeV}^3 ,$$

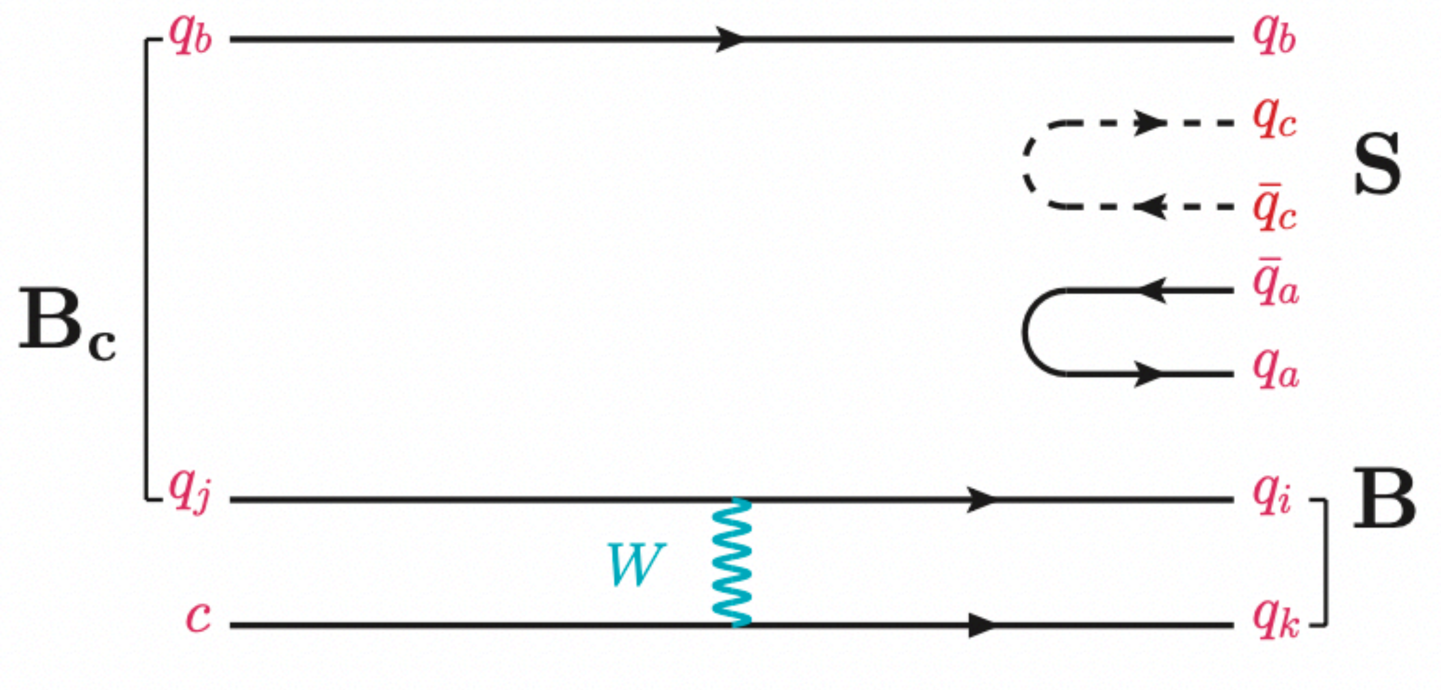
$$(\delta_{E'}, \delta_{E_{\bf B}}) = (-90.3 \pm 15.0, -144.4 \pm 40.7)^\circ ,$$

$$\text{S2} : (|C'|, |E'|, |E_{\bf B}|) = (0.97 \pm 0.20, 0.54 \pm 0.13, 0.15 \pm 0.02)\ \text{GeV}^3 ,$$

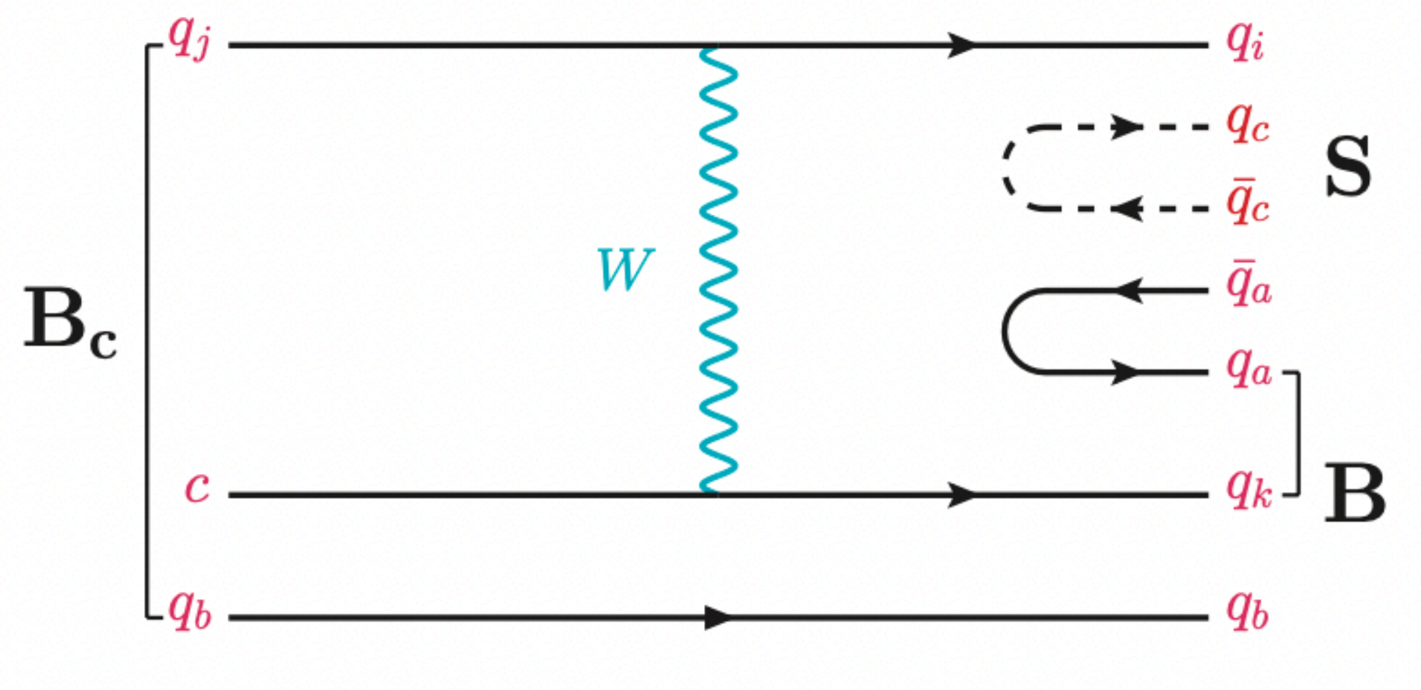
$$(\delta_{E'}, \delta_{E_{\bf B}}) = (-61.0 \pm 17.7, -144.2 \pm 39.6)^\circ .$$



(c) C'



(d) E'



(e) $E_{\bf B}$

Branching fraction	This work: $(S1, S2)$	Data
$10^2\mathcal{B}(\Lambda_c^+ \to \Lambda^0 a_0^+)$	$(1.1 \pm 0.4, 1.1 \pm 0.6)$	1.23 ± 0.21 [36]
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^0 a_0^+)$	$(1.3 \pm 0.8, 1.2 \pm 1.0)$	$\mathcal{B}_{\rm ex(res)}(a_0^+)$
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+ a_0^0)$	$(1.3 \pm 0.8, 1.2 \pm 1.0)$	$\mathcal{B}_{\rm ex(res)}(f_0, a_0)$
$10^2\mathcal{B}(\Lambda_c^+ \to \Xi^0 \kappa^+)$	$(1.4 \pm 0.8, 1.5 \pm 0.8)$	$\mathcal{B}_{\rm ex(res)}(\kappa^+)$
$10^2\mathcal{B}(\Lambda_c^+ \to p\bar{\kappa}^0)$	$(0.21 \pm 0.06, 0.21 \pm 0.06)$	0.19 ± 0.06 [31, 44]
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+ f_0)$	$(0.5 \pm 0.4, 4.9 \pm 1.9)$	$\mathcal{B}_{\rm ex(res)}(f_0, a_0)$, $\mathcal{B}_{\rm ex(res)}(f_0, \sigma_0)$
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \sigma_0)$	$(3.4 \pm 1.7, 0.1 \pm 0.2)$	$\mathcal{B}_{\rm ex(res)}(f_0, \sigma_0)$
$10^3\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \kappa^+)$	$(1.2 \pm 0.6, 2.5 \pm 1.0)$	
$10^3\mathcal{B}(\Lambda_c^+ \to \Sigma^0 \kappa^+)$	$(0.6 \pm 0.5, 1.4 \pm 0.8)$	
$10^3\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \kappa^0)$	$(2.5 \pm 1.2, 4.7 \pm 2.0)$	
$10^3\mathcal{B}(\Lambda_c^+ \to n a_0^+)$	$(3.2 \pm 1.5, 3.1 \pm 1.9)$	
$10^3\mathcal{B}(\Lambda_c^+ \to p a_0^0)$	$(0.7 \pm 0.6, 0.7 \pm 0.7)$	
$10^3\mathcal{B}(\Lambda_c^+ \to p f_0)$	$(0.2 \pm 0.1, 3.6 \pm 1.4)$	3.4 ± 2.3 [31, 32]
$10^3\mathcal{B}(\Lambda_c^+ \to p \sigma_0)$	$(2.0 \pm 0.9, 0.05 \pm 0.02)$	
$10^4\mathcal{B}(\Lambda_c^+ \to p \kappa^0)$	$(1.2 \pm 0.7, 2.6 \pm 1.2)$	
$10^4\mathcal{B}(\Lambda_c^+ \to n \kappa^+)$	$(1.2 \pm 0.7, 2.6 \pm 1.2)$	

- Goodness of the fit

$(S1, S2)$: $\chi^2/\text{n.d.f.} = (2.6.0.6)$,

with n.d.f.=2 in both cases.

Tetraquark scenario is slightly favored.

- The SD contributions dominate the branching fractions, with non-factorizable $W_{\rm em,ex}$ terms playing a key role.
- Similar to $\Lambda_c^+ \to \mathbf{B}M$, the CA and SCS modes of $\Lambda_c^+ \to \mathbf{B}S$ have branching fractions at the levels of 10^{-2} and 10^{-3}, respectively, making them accessible to experiments.
- In most decay channels: $\mathcal{B}_{q\bar{q}} \simeq \mathcal{B}_{q^2\bar{q}^2}$, implying limited sensitivity to the internal $q\bar{q}$ versus $q^2\bar{q}^2$ structures.

$$\mathcal{B}_{\rm ex(res)}(f_0, \sigma_0) \equiv \mathcal{B}(\Lambda_c^+ \to \Sigma^+[(f_0 + \sigma_0) \to]\pi^+\pi^-) = (11.0 \pm 6.0) \times 10^{-3},$$

$$\mathcal{B}_{\rm ex(res)}(f_0, a_0) \equiv \mathcal{B}(\Lambda_c^+ \to \Sigma^+[(f_0 + a_0) \to]K^+K^-) = (10.8 \pm 4.6) \times 10^{-4},$$

$$\mathcal{B}_{\rm ex(res)}(a_0^+) \equiv \mathcal{B}(\Lambda_c^+ \to \Sigma^0(a_0^+ \to)\pi^+\eta) = (7.1 \pm 0.7) \times 10^{-3},$$

$$\mathcal{B}_{\rm ex(res)}(\kappa^+) \equiv \mathcal{B}(\Lambda_c^+ \to \Xi^0(\kappa^+ \to)K^0\pi^+) = (3.9 \pm 1.9) \times 10^{-3}.$$

- The $f_0 - \sigma_0$ mixing in S1 and S2 exhibits notable sensitivity.

$[\theta_I = (156.7 \pm 0.7)^\circ,\ \theta_{II} = (174.6^{+3.4}_{-3.2})^\circ]$.

1. CA decay channels:

$$\mathcal{B}_{q\bar{q}}(\Lambda_c^+ \to \Sigma^+ f_0, \Sigma^+ \sigma_0) = (0.5 \pm 0.4, 3.4 \pm 1.7) \times 10^{-2},$$

$$\mathcal{B}_{q^2\bar{q}^2}(\Lambda_c^+ \to \Sigma^+ f_0, \Sigma^+ \sigma_0) = (4.9 \pm 1.9, 0.1 \pm 0.2) \times 10^{-2},$$

$$\mathcal{M}_{q\bar{q}}(\Lambda_c^+ \to \Sigma^+ f_0, \Sigma^+ \sigma_0) \simeq E_{\mathbf{B}} c\phi_I,\ \frac{1}{\sqrt{2}}(C' + E')c\phi_I$$

$$\mathcal{M}_{q^2\bar{q}^2}(\Lambda_c^+ \to \Sigma^+ f_0, \Sigma^+ \sigma_0) \simeq \frac{1}{\sqrt{2}}(C' + E')c\phi_{II},\ E_{\mathbf{B}} c\phi_{II}.$$

($E_{\mathbf{B}}$ is relatively small.)

2. SCS decay channels:

$$\mathcal{B}_{q\bar{q}}(\Lambda_c^+ \to p f_0, p\sigma_0) = (0.2 \pm 0.1, 2.0 \pm 0.9) \times 10^{-3},$$

$$\mathcal{B}_{q^2\bar{q}^2}(\Lambda_c^+ \to p f_0, p\sigma_0) = (3.6 \pm 1.4, 0.05 \pm 0.02) \times 10^{-3},$$

$$\mathcal{M}_{q\bar{q}}(\Lambda_c^+ \to p f_0, p\sigma_0) \simeq \frac{1}{\sqrt{2}}(C' + E')s\phi_I,\ \frac{1}{\sqrt{2}}(C' + E')c\phi_I$$

$$\mathcal{M}_{q^2\bar{q}^2}(\Lambda_c^+ \to p f_0, p\sigma_0) \simeq \frac{1}{\sqrt{2}}(C' + E')c\phi_{II},\ \frac{1}{\sqrt{2}}(C' + E')s\phi_{II}.$$

Summary

• Using $SU(3)_f$-based TDA, we demonstrate that

the SD $W_{\rm em,ex}$ processes provide the dominant contributions to $\Lambda_c^+ \to \mathbf{B}S$.

• The branching fractions of the CA and SCS decay modes

are predicted at the levels of 10^{-2} and 10^{-3}, respectively,

making them accessible for experimental investigation.

• In the tetraquark scenario, we obtain

$\mathcal{B}(\Lambda_c^+ \to \Sigma^+ f_0) = (4.9 \pm 1.9) \times 10^{-2}$,

$\mathcal{B}(\Lambda_c^+ \to p f_0) = (3.6 \pm 1.4) \times 10^{-3}$,

which are highly sensitive to the internal structure of light scalar mesons.

Thank You