Tomography of proton structures

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J.X.Yu, **SC**, J.J. Han, H.N. Li and F.S. Yu, *EPJC 86 (2026) 300 (Letter)*

J. Chai and **SC**, *PRD 111 (2025) L071902*

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Overview

I Form Factors and LCDAs

II Three-scale Factorization approach

III Scaling behaviors of $Q^4 F_1(Q^2)$

IV Conclusion

”In elementary particle physics and mathematical physics, in particular in effective field theory, a **form factor** is a function that encapsulates the properties of a certain interaction without including all of the underlying physics, but instead, providing the momentum dependence of a suitable matrix elements. Its further measured experimentally in confirmation or specification of a theory.”

Momenta Redistribution

$\Downarrow$ QCD is believed to exhibit confinement

hadron structures $\otimes$ hard scattering

$\Downarrow$ decoupling of LD and SD interactions

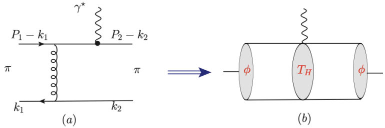
factorisation theorem, EFT; g-2, CKM, B anomalies

PION is the lightest Glodstone boson and the most simplest hadron

- Electromagnetic form factor

$$\langle \pi^-(p_2) \left| J_\mu^{\rm em} \right| \pi^-(p_1) \rangle = e_q \left(p_1 + p_2\right)_\mu F_\pi(Q^2)$$

- Separate the (hard) partonic physics out of the hadronic physics (soft, nonperturbative objects) in exclusive processes **Factorization**



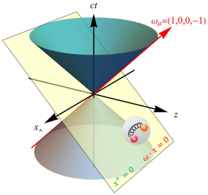
$$\frac{d\sigma}{d\Omega} = \int_0^1 du_i\, \phi(u_1)\, T_H(u_i) \phi(u_2)$$

- The universal nonperturbative objects, LCDAs/GPA, PDF/TMD/GPD see Ding-yu Shao's talk

- studied by QCD-based analytical (QCDSRs, χPT, DSE, instanton) and numerical approaches (Lattice, inverse problem see Fu-sheng Yu's talk)

- more practical by data-driven method (CETQ, CT, MMHT, NNPDF, ABM, JAM, et.al.) need precise QCD calculation of T_H as the inputs

LCDAs Exclusive QCD processes with larger momentum transfers



- The Lorentz and gauge invariant ME

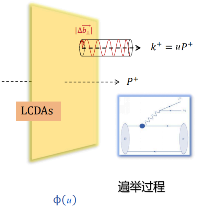
$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(p)\rangle = f_\pi \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[ip_\mu\left(\phi(u,\mu)+\frac{x^2}{4}\phi_1^4(u,\mu)\right) -\right.$$

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\rho^-(p)\rangle = f_\rho m_\rho \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[p_\mu \frac{\epsilon^{(\lambda)}\cdot x}{p\cdot x}\left(\phi_\parallel(u,\mu)-\phi_\perp^3\right.\right.$$

$$\langle 0|\bar{q}(x)\gamma_\mu\gamma_5 q(-x)|f_0(p)\rangle = p_\mu \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[\phi(u,\mu)+\cdots\right]$$

- LCDAs are dimensionless functions of u and renormalization scale μ
- the probability amplitudes to **find the meson** in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$
- the current accuracy up to three-particle ($q\bar{q}g$) current

- **longitudinal** $\otimes$ *transversal* dofs
- the TMD (μ dependence) is governed by the *RGE*
- the LMD is described in terms of irreducible representations of the corresponding symmetry group
  collinear subgroup of conformal group $SL(2,R)\cong SU(1,1)\cong SO(2,1)$
  $\Rightarrow$ collinear twist $\Rightarrow$ **the Gegenbauer polynomials**



- LCDAs of pion achieved **great success** in describing large Q^2 processes

  △ the establishment and development of the pQCD factorization $F_\pi(Q^2)$

$\int_0^1 du_i \phi(u_1, \tilde{Q}_1) T_H(u_i, Q) \phi(u_2, \tilde{Q}_2)$	1980s	[Lepage & Brodsky 1980, Efremov & Radyushkin 1980]
$\Downarrow$		
$\int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/\bar{u}_1} \phi(u_1, k_{1T}) T_H(u_i, Q) \phi(u_2, k_{2T})$	1990s	[Huang, Shen & Kroll 1991, Huang, Shen & Ma 1994]
$\Downarrow$		
$\psi(u_1, \mu_{r_1})\, T_H(u_i, b_i, Q)\, S_t(u_i)\, e^{-S(u_i,b_i,Q)}\, \psi(u_2, \mu_{r_2})$	2000s	[Botts & Sterman 1989, Li & Sterman 1992, Li 1999]
$\Downarrow$		
$\sum_{t_i} \psi^{t_1}(u_1, \mu_{r_1})\, T_H^{t_i,\mathrm{LO+NLO}}(u_i, b_i, Q) S_t(u_i)\, e^{-S(u_i,b,Q)}\, \psi^{t_2}(u_2, \mu_{r_2})$	2010s	[Li, Shen, Wang & Zou 2011, **SC**, Fan & Xiao 2014]
$\sum_{t_i} \psi^{t_1}(u_1, b_1, \mu_{r_1})\, T_H^{t_i,\mathrm{LO+NLO}}(u_i, b_i, Q)\, S_t(u_i)\, e^{-S(u_i,b,Q)}\, \psi^{t_2}(u_2, b_2, \mu_{r_2})$	2020s	[Chai & **SC** 2025] [Chen^2, Feng & Jia 2024], [Ji, Shi, Wang^3 & Yu 2025]

- LCDAs of proton serves as the fundamental input to explain *ep* scattering
  [Chen^2, Feng, Hu, Jia 2025], [Huang, Shi, Wang, Zhao 2025], [Yu, **SC**, Han, Li, Yu 2025]

- The non-perturbative input for HFP theoretical studies that determines the precision and accuracy   predicted the CPVs in the $B \to \pi\pi, K\pi$ decays and et.al.,

- **Claimer**: this talk concentrates on proton EMFFs and LCDAs

Three-scale factorization approach

[Chai and **SC**, 2025]   iTMDs $\otimes$ LCDAs $\otimes$ T

Three-scale Factorization

Exclusive Processes in Perturbative Quantum Chromodynamics #1

G.Peter Lepage (Cornell U., LNS), Stanley J. Brodsky (SLAC) (Mar, 1980)

Published in: *Phys.Rev.D* 22 (1980) 2157

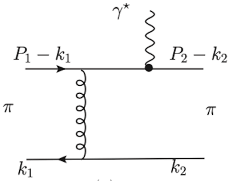
Factorization and Asymptotical Behavior of Pion Form-Factor in QCD #1

A.V. Efremov (Dubna, JINR), A.V. Radyushkin (Dubna, JINR) (Nov, 1979)

Published in: *Phys.Lett.B* 94 (1980) 245-250

- amplitudes are dominated by quark and gluon subprocesses at SDs
- evolution equations for process-independent hadron DAs $\phi(u_i, \tilde{Q}_i)$
- calculations at leading twist DAs and α_s order
  prevents anomalous contributions from the end-point $u_i \sim 1$ integration regions

$$\mathcal{F}_\pi(Q^2) = \int_0^1 du_i\, \phi(u_1, \tilde{Q}_1)\; T_H(u_i, Q)\, \phi(u_2, \tilde{Q}_2)$$



$\phi \propto u(1-u), \quad \phi^{P,\sigma} \propto m_0^\pi$

$$\propto \sum_t \int du_1 du_2 \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{\left(u_1 u_2 Q^2\right)\left(u_2 Q^2\right)}$$

- End-point singularities appear at high twists
- pick up k_T in the internal propagators

$$\mathcal{M} \propto \sum_{t=2,3,4} \int du_1 du_2 dk_{1T} dk_{2T} \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{[u_1 u_2 Q^2 - (\triangle k_T)^2]\,(u_2 Q^2 - k_{2T}^2)}$$

- end-point singularity at leading and subleading powers

$$\mathcal{H} \propto \frac{\alpha_s(\mu)}{u_1 u_2 Q^2 - k_T^2} \sim \frac{\alpha_s(\mu)}{u_1 u_2 Q^2} - \alpha_s(\mu)\frac{k_T^2}{(u_1 u_2 Q^2)^2} + \cdots$$

- the power suppressed TMD terms becomes important at the end-points

Three-scale Factorization

A Study of the Applicability of Perturbative {QCD} to the Pion Form-factor #1

Tao Huang (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.), Qi-xing Shen (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.), Peter Kroll (Wuppertal U.) (Aug, 1989)

Published in: *Z.Phys.C* 50 (1991) 139-144 • Contribution to: Quarks 90

DOI cite claim reference search 75 citations

Analysis of the pion wave function in light cone formalism #1

Tao Huang (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.), Bo-Qiang Ma (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.), Qi-Xing Shen (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.) (Mar 8, 1994)

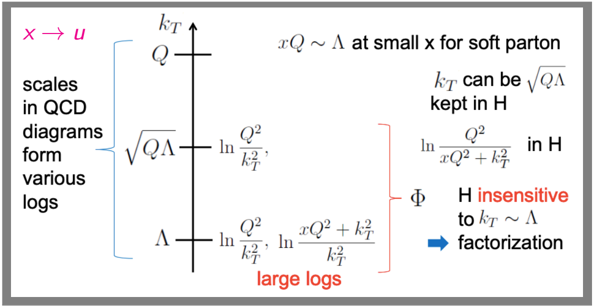
Published in: *Phys.Rev.D* 49 (1994) 1490-1499 • e-Print: hep-ph/9402285 [hep-ph]

pdf DOI cite claim reference search 132 citations

- introduce k_T to regularize the end-point singularity
- constraints on the integration region $b = \left(1 - \sqrt{1-4a}\right)/2, \quad a = \langle k_T^2 \rangle / Q^2, \quad \langle k_T \rangle \sim 300$ MeV
- calculations at leading twist and α_s order within b-dependent models (iTMDs ?)

$$\mathcal{F}_\pi(Q^2) = \int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/\bar{u}_1} du_2 \, \phi(u_1, k_{1T}) T_H(u_i, Q) \phi(u_2, k_{2T})$$

- k_T varies within three scales



- large single and double logarithms from QCD corrections, ie., $\alpha_s(\mu) \ln^2 \frac{k_T^2}{m_B^2}$
- k_T resummation for T to obtain $S(u_i, b_i, Q)$ suppresses the large transversal distances (small k_T) interactions by decreasing $u_i Q^2$ power in denominator
- integrating over k_T, $\ln^2(u_i)$ resides when the internal parton is on shell
- threshold resummation for ϕ to obtain $S_t(u_i, Q)$ suppresses the small u_i regions, repairs the self-consistency between $\alpha_s(t)$ and hard log $\ln(u_1 u_2 Q^2/t^2)$

Three-scale Factorization

Hard Elastic Scattering in QCD: Leading Behavior #1

James Botts (SUNY, Stony Brook), George F. Sterman (SUNY, Stony Brook) (Mar 20, 1989)

Published in: *Nucl.Phys.B* 325 (1989) 62-100

DOI cite claim reference search 598 citations

The Perturbative pion form-factor with Sudakov suppression #1

Hsiang-nan Li (SUNY, Stony Brook), George F. Sterman (SUNY, Stony Brook) (Mar, 1992)

Published in: *Nucl.Phys.B* 381 (1992) 129-140

DOI cite claim reference search 527 citations

- threshold-suppressed hard amplitude $T_H S_t$
- sudokov-multiplied light-cone distribution amplitudes ψe^{-S}
- leading twist & QCD leading order & resolution of endpoint singularities

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$$\mathcal{F}_\pi(Q^2) = \phi(u_1, \mu_{r_1})\, T_H(u_i, b_i, Q)\, S_t(u_i)\, e^{-S(u,b,Q)}\, \phi(u_2, \mu_{r_2})$$

Three-scale Factorization

- twist 2@NLO+twist 3@LO [H.N. Li, Y.L. Shen, Y.M. Wang and H. Zou, PRD 83 (2011) 054029]
- twist 2@NLO+twist 3@NLO [**SC**, Y.Y. Fan and Z.J. Xiao, PRD 89 (2014) 054015]
- twist 2@NLO+twist 3@NLO+twist 4, scale revolutions [**SC**, PRD 100 (2019) 013007]

- joint resummation for $\ln(x_1 x_2 Q^2 b^2)$ for large b and small $x_1 x_2 Q^2$
  [H.N. Li, Y.L. Shen and Y.M. Wang, JHEP 01(2014)004]
- rapidity singularity and pinch singularity, non-dipolar Wilson links for TMDWFs
  [H.N. Li and Y.M. Wang, JHEP 06(2015)013]

- high twist contributions, more fruitful hadron structures
- NLO QCD corrections in hard kernel and TMDWFs

- hard scale choice [Majaza, Brodsky and Wu, PRL 109 (2012) 042002, 110 (2013) 192001]
- $\mathrm{N^2LO}$ from QCD collinear factorization leading twist
  [Chen^2, Feng and Jia, PRL 132 (2024) 201901], [Ji, Shi, Wang^3 and Yu, PRL 134 (2025) 221901]

$$\mathcal{F}_\pi(Q^2) = \sum_{t_i} \varphi^{t_1}(u_1, \mu_{r_1})\, T_H^{t_i,\mathrm{NLO}}(u_i, b_i, Q)\, S_t(u_i)\, e^{-S(u_i,b,Q)}\, \varphi^{t_2}(u_2, \mu_{r_2})$$

Three-scale Factorization $\mathcal{F}_\pi(Q^2)$

$$\int_0^1 du_i \phi(u_1, \tilde{Q}_1)\, T_H(u_i, Q) \phi(u_2, \tilde{Q}_2) \quad \boxed{\text{1980s}}$$

$$\Downarrow$$

$$\int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/\bar{u}_1} \phi(u_1, k_{1T})\, T_H(u_i, Q) \phi(u_2, k_{2T}) \quad \boxed{\text{1990s}}$$

$$\Downarrow$$

$$\phi(u_1, \mu_{r_1})\ T_H(u_i, b_i, Q)\ S_t(u_i)\ e^{-S(u_i, b_i, Q)}\ \phi(u_2, \mu_{r_2}) \quad \boxed{\text{2000s}}$$

$$\Downarrow$$

$$\sum_{t_i} \varphi^{t_1}(u_1, \mu_{r_1})\ T_H^{t_i, \text{LO+NLO}}(u_i, b_i, Q) S_t(u_i)\ e^{-S(u_i, b, Q)}\ \varphi^{t_2}(u_2, \mu_{r_2}) \quad \boxed{\text{2010s}}$$

- $T_H(u_i, b_i, Q) S_t(x_i, Q)$ threshold-suppressed hard scattering amplitude
  including both the longitudinal and transversal dynamics
- $e^{-S(u_i, b_i, Q)} \psi(u_i, \mu_r)$ sudokov-multiplied LCDAs wave functions at zero transversal separations $b_i \sim 0$, only the soft longitudinal dynamics, oversight of the soft transversal dynamics (intrinsic transverse momentum distributions/iTMDs) $\Downarrow$ [Chai, **SC**, PRD 111 (2025) L071902]

$$\sum_{t_i} \psi^{t_1}(u_1, b_1, \mu_{r_1})\ T_H^{t_i, \text{LO+NLO}}(u_i, b_i, Q)\ S_t(u_i)\ e^{-S(u_i, b, Q)}\ \psi^{t_2}(u_2, b_2, \mu_{r_2}) \quad \boxed{\text{2020s}}$$

Scaling behaviors of proton FFs

[Yu, **SC**, Han, Li and Yu 2026]

Proton LCDAs

- The proton emergence challenges naive quark-model expectations (spin crisis and mass decomposition)   see Jian Zhou's talk

- $\left\langle 0 \left| \varepsilon^{ijk} u^i_\alpha \left(a_1 z\right) u^j_\beta \left(a_2 z\right) d^k_\gamma \left(a_3 z\right) \right| p \right\rangle$ define 24 LCDAs, in 8 parameters

	Twist-3 $\sim (p^+)^{3/2}$	**Twist-4** $\sim (p^+)^{1/2}$
LCDAs	V_1, A_1, T_1	$V_2, V_3, A_2, A_3, T_2, T_3, T_7, S_1, P_1$
Parameters	f_p, V_1^d, A_1^u	$f_p, V_1^d, A_1^u, \lambda_1, \lambda_2, f_1^d, f_1^u, f_2^d$
	Twist-5 $\sim (p^+)^{-1/2}$	**Twist-6** $\sim (p^+)^{-3/2}$
LCDAs	$V_4, V_5, A_4, A_5, T_4, T_5, T_8, S_2, P_2$	V_6, A_6, T_6
Parameters	$f_p, V_1^d, A_1^u, \lambda_1, \lambda_2, f_1^d, f_1^u, f_2^d$	$f_p, V_1^d, A_1^u, \lambda_1, f_1^d, f_1^u$

- Result from QCDSR and LQCD exhibit magnificent discrepancies

  [**QCDSR** V.M. Braun, A. Lenz and M. Wittmann, PRD 73 (2006) 094019]
  [**LQCD** V.M. Braun, R.J Fries, N. Mahnke and E. Stein, NPB 589 (2000) 381-409]

	f_N (10^{-3})	λ_1 (10^{-2})	λ_2 (10^{-2})	V_1^d	A_1^u	f_1^d	f_2^d
QCDSR	5.0 ± 0.5	-2.7 ± 0.9	5.4 ± 1.9	0.23 ± 0.03	0.38 ± 0.15	0.4 ± 0.05	0.22 ± 0
LQCD	3.7 ± 0.1	-4.0 ± 0.4	8.4 ± 0.4	0.29 ± 0.01	0.10 ± 0.01	...	...

- data-driven analysis of the FFs are worthwhile

Proton EMFFs

$$\mathcal{A}_\mu \equiv \langle P'|j_\mu(0)|P\rangle = \bar{u}(P')\left[\gamma_\mu F_1(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2m_p}F_2(Q^2)\right]u(P)$$

$$= \sum_{n=1}^{42}\sum_{t,t'=3}^{6}\int[dx^{(\prime)}][d^2\mathbf{b}^{(\prime)}]\,\psi^{t'}(x_i',b_i',Q,\mu_f)\mathcal{H}_{n,\mu}^{t,t'}(x_i^{(\prime)},b_i^{(\prime)},Q,\mu_f)\psi^t(x_i,b_i,Q,\mu_f).$$

- Dirac FF and Pauli FF are normalized to the proton charge and magnetic moments $F_1(0) = 1,\ F_1(0) + F_2(0) = 2.793$
- 42 Feynman dias at LO, Sudakov-multiplied twist 3-6 proton LCDAs
- $\psi = e^{-S(x_i,b_i,Q)}\phi$ absorbes the soft and hard-collinear k_T logarithms
- pQCD is applicable at intermediate energies (**DATA available**)
- ignore the intrinsic b dependence and the μ_f in φ for simplicity $\Rightarrow$ the parton transversal distributions are completely described by $S(x_i, b_i, Q)$

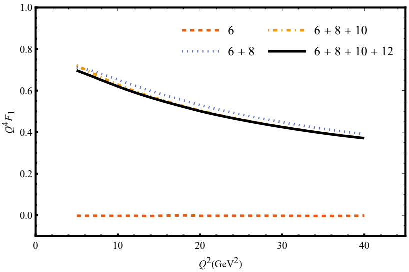
Subleading twist dominanted $Q^4 F_1(Q^2)$

- Typical integrands of $F_1(Q^2)$ from each twist combination of LCDAs

Twists	3	4	5	6
3	$\frac{[x_i]^3[x_i']^3 f_p^2}{\tilde{Q}^4}$	$\frac{[x_i]^3[x_i']^2 f_p\lambda_j}{\tilde{Q}^5}$	$\frac{[x_i]^3[x_i'] f_p\lambda_j}{\tilde{Q}^6}$	$\frac{[x_i]^3 f_p^2}{\tilde{Q}^7}$
4	$\frac{[x_i]^2[x_i']^3 \lambda_j f_p}{\tilde{Q}^5}$	$\frac{[x_i]^2[x_i']^2[\lambda_j]^2}{\tilde{Q}^6}$	$\frac{[x_i]^2[x_i'][\lambda_j]^2}{\tilde{Q}^7}$	$\frac{[x_i]^2 \lambda_j f_p}{\tilde{Q}^8}$
5	$\frac{[x_i][x_i']^3 \lambda_j f_p}{\tilde{Q}^6}$	$\frac{[x_i][x_i']^2[\lambda_j]^2}{\tilde{Q}^7}$	$\frac{[x_i][x_i'][\lambda_j]^2}{\tilde{Q}^8}$	$\frac{[x_i]\lambda_j f_p}{\tilde{Q}^9}$
6	$\frac{[x_i']^3 f_p^2}{\tilde{Q}^7}$	$\frac{[x_i']^2 f_p\lambda_j}{\tilde{Q}^8}$	$\frac{[x_i'] f_p\lambda_j}{\tilde{Q}^9}$	$\frac{f_p^2}{\tilde{Q}^{10}}$

- naive power counting in the longitudinal virtuality $\tilde{Q}^2 \sim x_i x_i' Q^2, x_i Q^2, x_i' Q^2$
- scales like $\mathcal{O}(\frac{1}{Q^4})$ asymptotically, tamed by k_T in the intermediate regions
- corrected by higher-twist contributions $\mathcal{O}(1/\tilde{Q}^{n-2}), n = t + t'$
- t3 LCDAs $\sim [x_i]^3$, t4 and t5 LCDAs $\sim [x_i]^2$, t6 LCDAs $\sim [x_i]^0$ $\Downarrow$
  the increasing importance of the endpoint regions with twists
- t3, t6 LCDAs $\propto f_P$, t4, t5 LCDAs $\propto \lambda_i \sim \mathcal{O}(10) f_P$ $\Downarrow$
  high-twist contributions may not be suppressed at the intermediate Q^2

Scaling behavior of $Q^4 F_1(Q^2)$



take the QCDSR's parameters

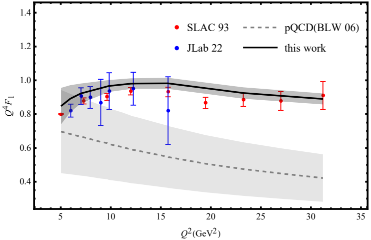
[V.M. Braun, A. Lenz and M. Wittmann, PRD 73 (2006) 094019]

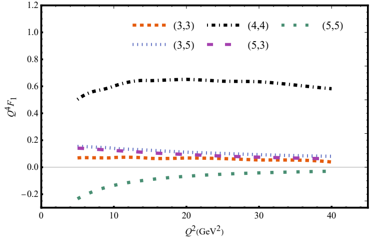
- the piece from twist $n = 8$ dominate
- the power expansion converges above $n = 9$

- the Q^2 dependence decreases dramatically within $[5, 30]$ GeV^2, while the data is roughly constant $Q^4 F_1(Q^2) \sim 1.0$
- this deviation can not be understood in collinear factorization: $\sim \mathcal{O}(1/Q^6)$ contribution to $F_1(Q^2)$ leads to descent

  [Chen^2, Feng, Hu, Jia PRL 135 (2025) 131903, Huang, Shi, Wang, Zhao, PRL 135 (2025) 061901]
- the additional hard-collinear scale k_T may modify the power-law behaviors associated with different twist: $\mathcal{O}(1/\tilde{Q}^6) \sim \mathcal{O}(1/Q^4)$
- the endpoint enhancement, tamed by the k_T resummation effect, is crucial for accommodating the approximating scaling behavior of the $Q^4 F_1(Q^2)$

Scaling behavior of $Q^4 F_1(Q^2)$





- the enhancement effect arises from subleading twist LCDAs, higher-twist contributions exhibit good convergence
- the extracted parameters show significant shifts from QCDSRs and LQCD
- NLO correction would solidify/resolve this discrepancies
- amplify more the endpoint contributions (iTMDs ?)

	f_N (10^{-3})	λ_1 (10^{-2})	λ_2 (10^{-2})	V_1^d	A_1^u	f_1^d	f_2^d	f_1^u
QCDSR	5.0 ± 0.5	-2.7 ± 0.9	5.4 ± 1.9	0.23 ± 0.03	0.38 ± 0.15	0.4 ± 0.05	0.22 ± 0.05	0.07 ± 0.05
LQCD	3.67 ± 0.06	-4.02 ± 0.38	8.37 ± 0.43	0.288 ± 0.007	0.096 ± 0.010	...	...	...
this work	3.67 ± 0.06	-3.98 ± 0.37	8.32 ± 0.43	1.15 ± 0.44	1.27 ± 0.72	0.85 ± 0.12	0.59 ± 0.04	-0.23 ± 0.03

Conclusion

- The subleading twist contributions, tamed by the k_T resummation effect, can produce the scaling behavior of FFs at the intermediate Q^2
- NLO correction would solidify/resolve the discrepancies of proton LCDAs between LQCD, QCDSR and our fit

Thank you for your patience.