

B -meson LCDA and power corrections

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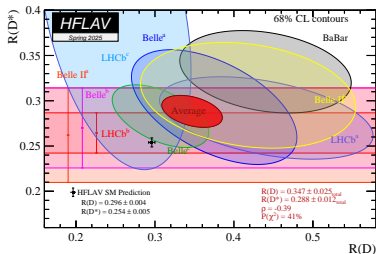
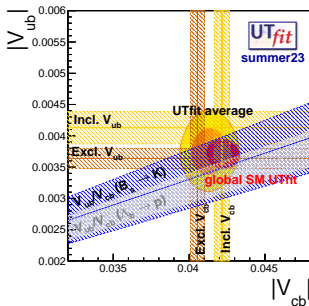
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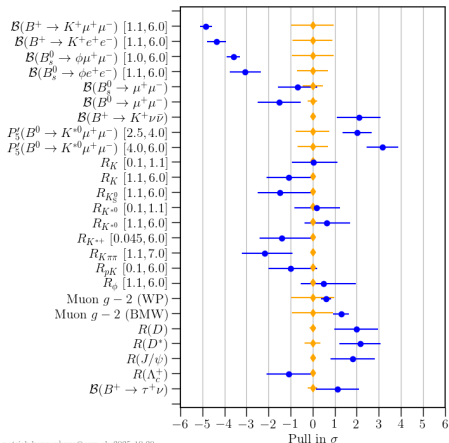
New physics beyond the SM

- Indirect search: flavour physics
- LHC, Belle-II
- HL-LHC



- CPV: $\Lambda_b^0 \rightarrow pK^-\pi^+\pi^-$ [LHCb, 25'], [Wang and Yu, 24']
- $R(D^{(*)})$: [Wang, YBW, Shen and Lü, 17']
- $|V_{ub/cb}|$: [Shen and YBW, 21']





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- Multi-scale problem:

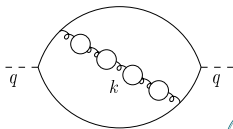


$$B \text{ LCDA} \sim \mathcal{O}(\Lambda)$$

- Power correction:

$$\frac{\Lambda}{m_b} \sim \mathcal{O}(20\%)$$

- IR renormalons [Beneke, '98']



Expansions in B decays

The perturbation expansion and power expansion

$$\alpha_s \sim \Lambda/m_b \sim 20\%$$

\mathcal{A}	LL	NLL	NNLL
LP	LP@LL	LP@NLL	LP@NNLL
NLP	NLP@LL	NLP@NLL	
NNLP	NNLP@LL		

Diagram illustrating the expansion orders for different operators (\mathcal{A}) in B decays. The table shows the expansion orders (LL, NLL, NNLL) for various operators (LP, NLP, NNLP). The expansion orders are indicated by the text in the cells. A red dashed arrow points from the top-right cell (LP@NNLL) to a box labeled 20%. A blue dashed arrow points from the bottom-left cell (NNLP@LL) to a box labeled 4%.

EOM vanishing operators at NNLP.

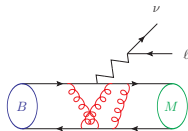


HQET LCDA of heavy meson

The leading-twist heavy-meson HQET LCDA [Grozin and Neubert, 96']

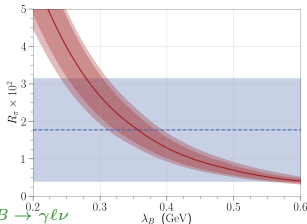
$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 q_s(tn_+) | 0 \rangle = -i \tilde{f}_H(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega)$$

HQET LCDA: appears in exclusive B decays



- $B \rightarrow \pi K \dots$ LQCD: $am_b \gtrsim 1$ [FLAG, 24']
- $B \rightarrow D^{(*)} \ell \nu \dots$ • Effective b quark action
- $B \rightarrow \gamma \ell \nu \dots$ • Quark mass extrapolation

First inverse momentum $\lambda_B \sim \mathcal{O}(\Lambda_{\text{QCD}})$: NOT well determined



[Belle, 18']: $B \rightarrow \gamma \ell \nu$



Study of HQET LCDA

HQET LCDA: non-perturbative physics

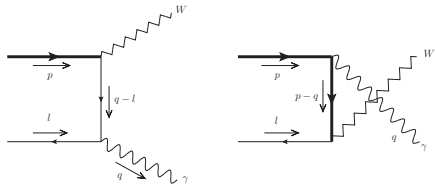
- Inverse moments: [Wang and Shen, 18'], [Beneke, Braun, Ji, and YBW, 18'], [Belle, 18'], [Galda, Neubert and Wang, 22'], [Han, et al., 24']
- EOM [Kawamura, Kodaira, Qiao and Tanaka, 02'], [Lü, Shen, Wang and YBW, 18']
- RG evolution properties: [Braun, Ji and Manashov, 19'], [Galda and Neubert, 20'], [Cui, Shen, Wang and YBW, 23']
- generalized LCDA [Beneke, Böer, Toelstede, and Vos, 22'], [Qin, Shen, Wang and Wang, 22'], [Huang, Ji, Shen, Wang and Wang, 23']

Determination of λ_B

- Extract from the experiments: clean process $B \rightarrow \gamma l \nu$, $W \rightarrow B \gamma$
- Calculate with LQCD [LPC collaboration, 22']



$$B \rightarrow \gamma \ell \nu$$

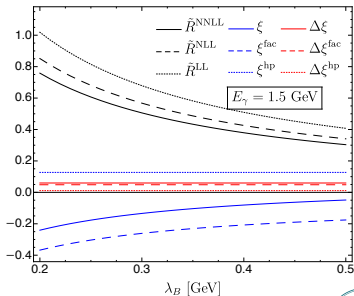


[Wang and Shen, 18'], [Beneke, Braun, Ji, and YBW, 18'], [Galda, Neubert and Wang, 22'], [Cui, Shen, Wang, and YBW, 23']

Decay form factor

$$F_{V/A}(E_\gamma) = R(E_\gamma) + [\xi(E_\gamma) \pm \Delta\xi(E_\gamma)]$$

- Factorizable contributions:
higher-twist B -meson LCDA,
QCD \rightarrow HQET: b quark field
- Non-factorizable contributions:
hadronic structure of the photon



[Cui, Shen, Wang, and YBW, 23']



$$B \rightarrow \gamma \ell \nu$$

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 q_s(tn_+) | 0 \rangle = -i \tilde{f}_H(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega)$$

Exclusive B decays: B -meson HQET decay constant \tilde{f}_B :

$$\tilde{f}_B(\mu) = K^{-1}(\mu) f_B$$

QCD decay constant f_B calculated with LQCD [FLAG].



$$B \rightarrow \gamma \ell \nu$$

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 q_s(tn_+) | 0 \rangle = -i \tilde{f}_B(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega)$$

Exclusive B decays: B -meson **HQET** decay constant \tilde{f}_B :

$$\tilde{f}_B(\mu) = K^{-1}(\mu) f_B$$

QCD decay constant f_B calculated with LQCD [FLAG].

The relation receive

power corrections

[Becher, Neubert and Petrov, 00']

$$\bar{q} \gamma^\alpha Q \cong \sum_{i=1,2} C_i(\mu) J_i + \frac{1}{2m_Q} \sum_{j=1}^{10} B_j(\mu) Q_j$$

$$f_B = K(\mu) \tilde{f}_B(\mu) + \text{NLP}$$

Two loop renormalization of heavy light currents at order $1/m(Q)$ in the heavy quark expansion

Thomas Becher (Cornell U., LNS), Matthias Neubert (Cornell U., LNS), Alexey A. Petrov (Cornell U., LNS)

Dec, 2000

16 pages

Published in: *Nucl.Phys.B* 611 (2001) 367-382

e-Print: [hep-ph/0012183](https://arxiv.org/abs/hep-ph/0012183) [hep-ph]

DOI: [10.1016/S0550-3213\(01\)00341-8](https://doi.org/10.1016/S0550-3213(01)00341-8)

Report number: CLNS-00-1713

View in: [ADS Abstract Service](#)



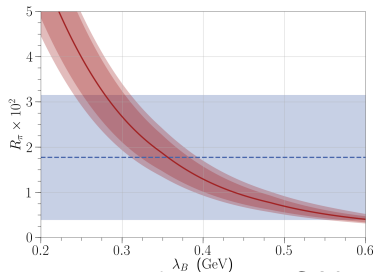
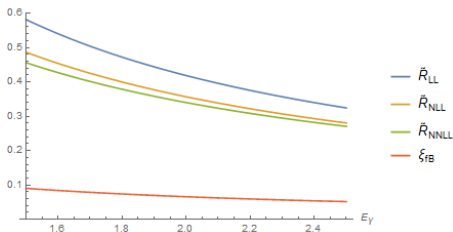
$$B \rightarrow \gamma \ell \nu$$

Power correction in f_B

\Rightarrow 20% increase to $F_{V/A}$

\Rightarrow 40% increase to decay width

[Liu, Su and YBW, in progress]



[Belle, 18']: $\lambda_B > 0.24$ GeV

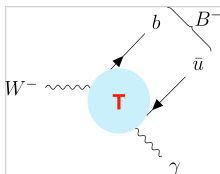
Also affect the exclusive extraction of $|V_{ub}|$ and $|V_{cb}|$.



$$W \rightarrow B\gamma$$

Rare decay $W^- \rightarrow B^-\gamma$: B -meson QCD LCDA

[Grossman, König and Neubert, 15']



Match from QCD LCDA to HQET LCDA [Ishaq, Jia, Xiong and Yang, 19'],

[Zhao, 19'], [Beneke, Finauri, Vos and YBW, 23'], [Deng, Wang, YBW and Zeng, 24']

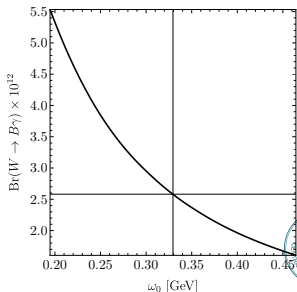
$$\phi_{\text{QCD}}(u) = \mathcal{J}(u, \omega) \otimes \varphi_{\text{HQET}}(\omega)$$

Our result: with $\ln \Lambda_{\text{QCD}}/m_Q$ resummation

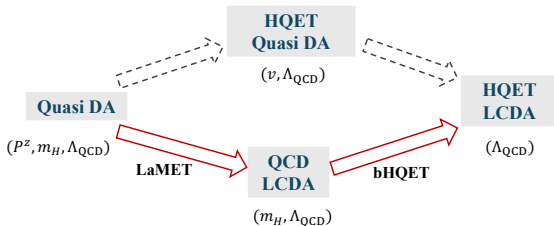
[Beneke, Finauri, Vos and YBW, 23']

$$\text{Br} = (2.58 \pm 0.21_{\text{in}} \begin{matrix} +0.05 \\ -0.08 \end{matrix} \begin{matrix} \mu_h \\ \lambda_B \end{matrix} \begin{matrix} +2.95 \\ -0.98 \end{matrix}) \cdot 10^{-12}$$

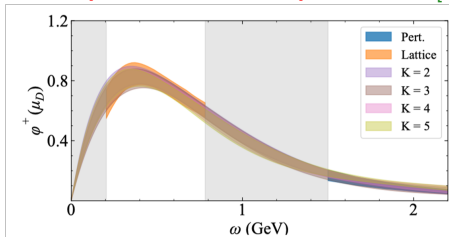
30% increase



LaMET [Ji, 13']



From QCD LCDA to HQET LCDA [Beneke, Finauri, Vos and YBW, 23']



First inverse moment [Qi-An Zhang]
 $\lambda_B = 0.347(21)$ GeV



Indirect study of HQET LCDA

Indirect determination: λ_B

From **QCD LCDA** to **HQET LCDA** [Ishaq, Jia, Xiong and Yang, 19'], [Zhao, 19'],
[Beneke, Finauri, Vos and **YBW**, 23'], [Deng, Wang, **YBW** and Zeng, 24']

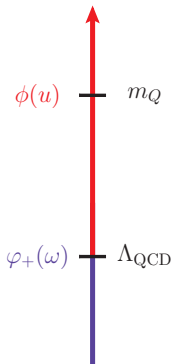
$$\phi(u) = \mathcal{J}(u, \omega) \otimes \varphi_+(\omega)$$

The heavy meson **QCD LCDA** [Braun and Filyanov, 89']

$$\langle H(p_H) | \bar{Q}(0) \not{n}_+ \gamma^5 q(tn_+) | 0 \rangle = -i f_H n_+ \cdot p_H \int_0^1 du e^{iutn_+ \cdot p_H} \phi(u)$$

The leading-twist heavy-meson **HQET LCDA** [Grozin and Neubert, 96']

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 q_s(tn_+) | 0 \rangle = -i \tilde{f}_H(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega)$$



Power corrections

At NLP in Λ/m_b , we have [Lu and YBW, in progress]

$$\tilde{O}_1 = \bar{h}_v(0) \frac{i\overleftarrow{\not{D}}}{2m_Q} \not{p}_+ \gamma_5 q(nt_+)$$

From EOM, need three-particle B -meson LCDA

$$\begin{aligned} \partial_x^\rho \bar{q}(x) \Gamma[x, 0] h_v(0) &= \partial^\rho [\bar{q}(x) \Gamma[x, 0] h_v(0)] - \bar{q}(x) \Gamma[x, 0] D^\rho h_v(0) \\ &+ i \int_0^1 du \bar{u} \bar{q}(x) [x, ux] g_s x_\sigma G^{\sigma\rho}(ux) [ux, 0] \Gamma h_v(0) \end{aligned}$$

Higher power HQET Lagrangian O_{kin} [Neubert, 93']

$$\tilde{O}_{\text{kin}} = \frac{1}{2m_Q} \int d^4x T \{ \tilde{O}_{\text{LP}}, O_{\text{kin}} \}$$

Nonperturbative matrix elements of the operators.



Systematic power counting

QCD LCDA \Rightarrow B -meson production: **highly boosted**

※① **Systematic power counting**: boosted frame

The LP operators

$$\hat{\mathcal{O}}_k = \frac{1}{n_+ v} \sqrt{\frac{n_+ v}{2}} \bar{h}_n \not{v}_+ \left(n_+ v \frac{i \not{D}_\perp}{i n_+ D} \right)^k \gamma^5 \xi_{sc}$$

Only operators $\hat{\mathcal{O}}_0$ and $\hat{\mathcal{O}}_1$ will appear at LP [Beneke, Finauri, Vos and **YBW**, 23']

For heavy particle with large energy

- **Top-quark** production [Fleming, Hoang, Mantry and Stewart, 07']
- B -meson leptonic decay with τ **lepton** [Huang, Shen, Zhao and Zhou, 23'], [Shen, Wang, **YBW** and Zhou, in progress]



Factorization of the QCD LCDA

※② **Cut δ** to QCD LCDA: avoid end-point singularity

The factorization formula

$$\phi(u) = \begin{cases} u \sim \delta: & \mathcal{J}_p(u, \omega) \otimes \varphi_+(\omega), \\ u \sim 1: & \mathcal{J}_{\text{tail}}(u), \end{cases} \quad \begin{aligned} \mathcal{O}_{\text{QCD}} &= \mathcal{J}_p \otimes \mathcal{O}_0, \\ \mathcal{O}_{\text{QCD}} &= \mathcal{J}_0 \mathcal{O}_0 + \mathcal{J}_1 \mathcal{O}_1, \end{aligned}$$

End-point singularity: refactorization-based subtraction (RBS)

- NLP in $h \rightarrow \gamma\gamma$: [Liu and Neubert, 19']
- Off-diagonal parton scattering, NLP in gluon thrust: [Beneke et. al., 20', 22']
- QED correction to $B \rightarrow \mu^+ \nu_\mu$: [Cornella, König and Neubert, 22', 26']



Summary

- ✧ **Power correction** in f_B matching and application to $B \rightarrow \gamma \ell \nu$
 - 40% increase of decay width

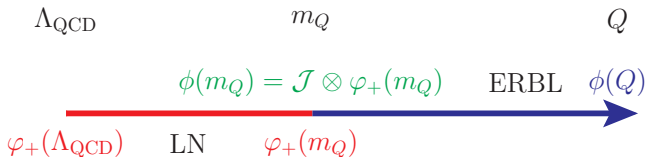
- ✧ Match QCD LCDA to the HQET LCDA
 - Application in $W \rightarrow B\gamma$ decay
 - LaMET quasi DA: $\lambda_B = 0.347(21)$ GeV
 - **Power correction** at tree level

Thank you!



Evolution of the LCDAs: φ_+ and ϕ

Two-step evolutions: $\Lambda_{\text{QCD}} \rightarrow m_Q \rightarrow Q$



$\phi(m_Q) \rightarrow \phi(Q)$:
evolve of the Gegenbauer moments

$$\frac{a_n(\mu_h)}{a_n(\mu)} = \left(\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)} \right)^{\frac{\gamma_n}{2\beta_0}}$$

