Study of the two-body nonleptonic $B_c(2S)$ weak decays with the QCD factorization approach

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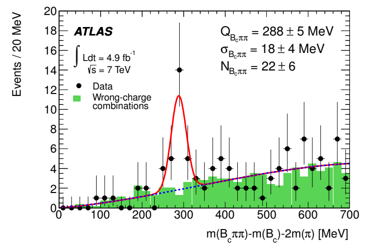
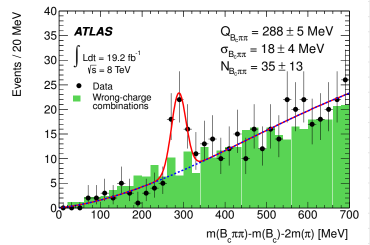
Outline

1. Motivation

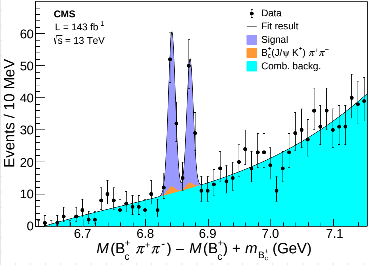
1.1 Observation of $B_c(2S)$

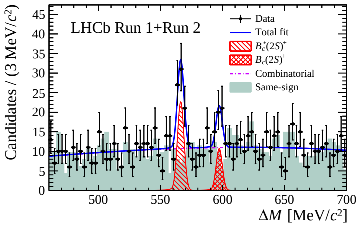




The mass of the observed state is $6842 \pm 4 \pm 5$ MeV. The mass and decay of this state are consistent with expectations for the second S-wave state of the $B_c^{\pm}$ meson, $B_c^{\pm}(2S)$.

Phys.Rev.Lett. 113 (2014) 21, 212004 (ATLAS)

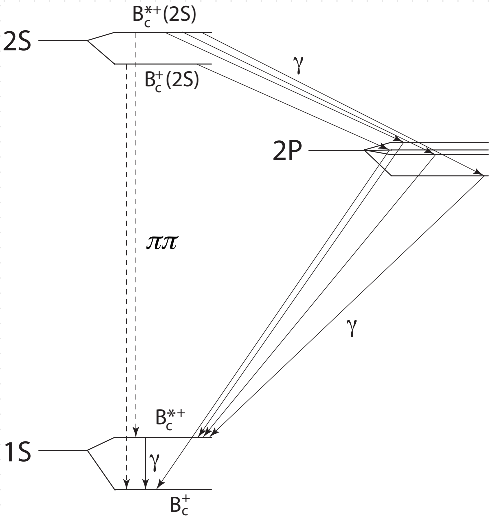




Signals consistent with the $B_c^{\pm}(2S)$ and $B_c^{*\pm}(2S)$ states have been separately observed for the first time by investigating the $B_c^{\pm}\pi^+\pi^-$ invariant mass spectrum measured by CMS. LHCb result confirmed the existence of the two states.

Phys.Rev.Lett. 122 (2019) 13, 132001(CMS)

Phys.Rev.Lett. 122 (2019) 23, 232001 (LHCb)



Transitions between the lightest B_c states, with solid and dashed lines indicating the emission of photons and pion pairs, respectively

1.2 There will be approximately $\mathcal{O}(10^{11})$ $B_c(2S)$ available at HL-LHC experiments

LHC $\sqrt{s} \approx 14$ TeV $\sigma \approx 1\,\mu\mathrm{b}$ $\Longrightarrow$ $\mathcal{O}(10^9)$ $B_c(1S)$ with $1\,\mathrm{fb}^{-1}$

HL-LHC $4000\,\mathrm{fb}^{-1}$ $\Longrightarrow$ $\mathcal{O}(10^{12})$ $B_c(1S)$

CMS group, $R = (3.47{\pm}0.63{\pm}0.33)\%$ $\Longrightarrow$ $\mathcal{O}(10^{11})$ $B_c(2S)$

1.3The $B_c(2S)$ meson is still an immature and up-and-coming particle

- The mass of the $B_c(2S)$ meson is $m = 6871.2 \pm 1.0$ MeV and lies below the open BD threshold.
- The favourite decays, predominantly by hadronic cascades and the electric dipole (E1) or magnetic dipole (M1) transitions, lead to the total width of the $B_c(2S)$ meson are generally less than one hundred keV.
- The weak decays of the $B_c(2S)$ meson are similar to those of the lowest $B_c(1S)$ meson.

2.1 The effective Hamiltonian

For the $B_c(2S) \to B\,P$, $B\,V$ decays induced by the c quark decays, the effective weak interaction Hamiltonian can be written as

$$\mathcal{H}^c_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q_1,q_2} V^*_{cq_1} V_{uq_2} \Big[C_1(\mu)\, O_1 + C_2(\mu)\, O_2 \Big] + \text{h.c.}, \tag{1}$$

$$O_1 = [\bar{q}_{1,\alpha}\, \gamma_\mu\, (1-\gamma_5)\, c_\alpha]\; [\bar{u}_\beta\, \gamma^\mu\, (1-\gamma_5)\, q_{2,\beta}], \tag{2}$$

$$O_2 = [\bar{q}_{1,\alpha}\, \gamma_\mu\, (1-\gamma_5)\, c_\beta]\; [\bar{u}_\beta\, \gamma^\mu\, (1-\gamma_5)\, q_{2,\alpha}], \tag{3}$$

both the penguin and annihilation contributions being proportional to the CKM factor $V_{ub}\, V^*_{cb} \sim \mathcal{O}(\lambda^5)$ are highly suppressed relative to those from the operator $O_{1,2}$ being proportional to $V_{ud}\, V^*_{cs} \sim \mathcal{O}(1)$ or $V_{ud}\, V^*_{cd} \sim \mathcal{O}(\lambda)$ or $V_{us}\, V^*_{cs} \sim \mathcal{O}(\lambda)$ or $V_{us}\, V^*_{cd} \sim \mathcal{O}(\lambda^2)$.

For the $B_c(2S) \to \psi(1S, 2S)\, P$, $\eta_c(1S, 2S)\, P$ decays induced by the $b \to c$ quark transition, the expression of the low-energy effective Hamiltonian is written as

$$\mathcal{H}^b_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{cb} V^*_{uq} \Big[C_1(\mu)\, O'_1 + C_2(\mu)\, O'_2 \Big] + {\rm h.c.}, \tag{4}$$

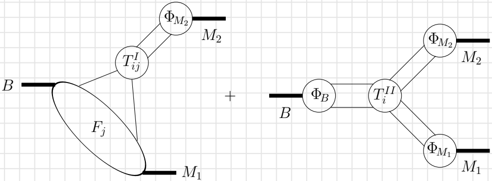
$$O'_1 = [\bar{c}_\alpha\, \gamma_\mu\, (1-\gamma_5)\, b_\alpha]\, [\bar{q}_\beta\, \gamma^\mu\, (1-\gamma_5)\, u_\beta], \tag{5}$$

$$O'_2 = [\bar{c}_\alpha\, \gamma_\mu\, (1-\gamma_5)\, b_\beta]\, [\bar{q}_\beta\, \gamma^\mu\, (1-\gamma_5)\, u_\alpha]. \tag{6}$$

The decay amplitude can be written as,

$$\mathcal{A} = \langle\, \psi\, \pi \,|\, \mathcal{H}_{\rm eff} \,|\, B_c(2S) \,\rangle = \frac{G_F}{\sqrt{2}}\, V_{cb}\, V^*_{ud} \sum_{i=1}^{2} C_i \,\langle\, \psi\, \pi \,|\, O'_i \,|\, B_c(2S) \,\rangle, \tag{7}$$

2.2 Hadronic matrix elements



$$\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle = \sum_j F_j^{B\to M_1}(m_2^2)\int_0^1 du\, T^I_{ij}(u)\,\Phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$
$$+\int_0^1 d\xi du dv\, T^{II}_i(\xi,u,v)\,\Phi_B(\xi)\,\Phi_{M_1}(v)\,\Phi_{M_2}(u) \quad (8)$$

if M_1 and M_2 are both light,

$$\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle = \sum_j F_j^{B\to M_1}(m_2^2)\int_0^1 du\, T^I_{ij}(u)\,\Phi_{M_2}(u) \quad (9)$$

if M_1 is heavy and M_2 is light.

Combined the nonfactorizable contributions with the Wilson coefficients, the scale independent effective coefficients at the order α_s can be obtained as follows:

$$a_1 = C_1^{\rm NLO} + \frac{1}{N_c} C_2^{\rm NLO} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2^{\rm LO} V^{(\prime)}, \tag{10}$$

$$a_2 = C_2^{\rm NLO} + \frac{1}{N_c} C_1^{\rm NLO} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_1^{\rm LO} V^{(\prime)}, \tag{11}$$

c quark decay:

$$V = 6 \log\left(\frac{m_c^2}{\mu^2}\right) - 18 - \left(\frac{1}{2} + i3\pi\right) a_0^M + \left(\frac{11}{2} - i3\pi\right) a_1^M - \frac{21}{20} a_2^M + \cdots, \tag{12}$$

b $\rightarrow$ c decay:

$$V' = 3\ln\left(\frac{m_b^2}{\mu^2}\right) + 3\ln\left(\frac{m_c^2}{\mu^2}\right) - 18 - \int \mathbf{d}z\, \phi(z) H_1(z) \tag{13}$$

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Adv.High Energy Phys. 2015 (2015) 104378

2.3 Form factors

$$\begin{aligned}
&\langle X(p_2) \,|\, \bar{q}_\alpha \, \gamma^\mu \, (1-\gamma_5) \, b_\alpha \,|\, B_c(2S)(p_1) \rangle \\
= \quad & \frac{m^2_{B_c(2S)} - m^2_X}{q^2} \, q^\mu \, F_0^{B_c(2S)\to X}(q^2) \\
+ \quad & \left(p_1^\mu + p_2^\mu - \frac{m^2_{B_c(2S)} - m^2_X}{q^2} \, q^\mu \right) F_1^{B_c(2S)\to X}(q^2),
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\langle \psi(p_2, \epsilon) \,|\, \bar{q}_\alpha \, \gamma^\mu \, (1-\gamma_5) \, b_\alpha \,|\, B_c(2S)(p_1) \rangle \\
= \quad & \epsilon_{\mu\nu\alpha\beta} \, \epsilon_\psi^{*\nu} \, p_1^\alpha \, p_2^\beta \, \frac{2\, V^{B_c(2S)\to\psi}(q^2)}{m_{B_c(2S)} + m_\psi} + i \, \frac{2\, m_\psi \, (\epsilon_\psi^* \cdot q)}{q^2} \, q_\mu \, A_0^{B_c(2S)\to\psi}(q^2) \\
+ \quad & i \, \epsilon^*_{\psi,\mu} \, (m_{B_c(2S)} + m_\psi) \, A_1^{B_c(2S)\to\psi}(q^2) \\
- \quad & i \, \frac{(\epsilon_\psi^* \cdot q)}{m_{B_c(2S)} + m_\psi} \, (p_1 + p_2)_\mu \, A_2^{B_c(2S)\to\psi}(q^2) \\
- \quad & i \, \frac{2\, m_\psi \, (\epsilon^*_{\psi,\mu} \cdot q)}{q^2} \, q_\mu \, A_3^{B_c(2S)\to\psi}(q^2),
\end{aligned} \tag{15}$$

At the large recoil limit, $q^2 = 0$, there is

$$F_0^{B_c(2S)\to X}(0) \;=\; F_1^{B_c(2S)\to X}(0), \tag{16}$$

$$A_0^{B_c(2S)\to \psi}(0) \;=\; A_3^{B_c(2S)\to \psi}(0). \tag{17}$$

Currently, there is no theoretical calculation for these form factors. We will use the Bauer-Stech-Wirbel approach to give a rough estimation of the form factor.

$$F_0^{B_c(2S)\to X}(0) \;=\; \int d\vec{k}_\perp \int_0^1 dx\, \Phi_X(\vec{k}_\perp, x, 0, 0)\, \Phi_{B_c(2S)}(\vec{k}_\perp, x, 0, 0), \tag{18}$$

$$A_0^{B_c(2S)\to \psi}(0) \;=\; \int d\vec{k}_\perp \int_0^1 dx\, \Phi_\psi(\vec{k}_\perp, x, 1, 0)\, \sigma_z\, \Phi_{B_c(2S)}(\vec{k}_\perp, x, 0, 0), \tag{19}$$

$$\Phi_{B_c(2S)}(\vec{k}_\perp, x) \;=\; A\left\{\frac{\vec{k}_\perp^2 + \bar{x}\,m_c^2 + x\,m_b^2}{6\,\alpha_1^2\,x\,\bar{x}} - 1\right\}\exp\left\{-\frac{\vec{k}_\perp^2 + \bar{x}\,m_c^2 + x\,m_b^2}{8\,\alpha_1^2\,x\,\bar{x}}\right\}, \tag{20}$$

$$\Phi_{\eta_c(1S)}(\vec{k}_\perp, x) \;=\; \Phi_{\psi(1S)}(\vec{k}_\perp, x) \;=\; B\exp\left\{-\frac{\vec{k}_\perp^2 + m_c^2}{8\,\alpha_2^2\,x\,\bar{x}}\right\}, \tag{21}$$

$$\Phi_{\eta_c(2S)}(\vec{k}_\perp, x) \;=\; \Phi_{\psi(2S)}(\vec{k}_\perp, x) \;=\; C\left\{\frac{\vec{k}_\perp^2 + m_c^2}{6\,\alpha_2^2\,x\,\bar{x}} - 1\right\}\exp\left\{-\frac{\vec{k}_\perp^2 + m_c^2}{8\,\alpha_2^2\,x\,\bar{x}}\right\}, \tag{22}$$

the mean value of square of transverse momentum is $\alpha_1^2 = \mu\,\omega$ with the reduced mass $\mu = \frac{m_b\,m_c}{m_b + m_c}$ and the characteristic frequency $\omega \approx 0.50 \pm 0.05$ GeV; and $\alpha_2 = m_c\,\alpha_s$ for the charmonium.

$$\Phi_{B_{u,d,s}}(\vec{k}_\perp, x) = D\, x^2\, \bar{x}^2 \exp\Big\{ -\frac{\vec{k}_\perp^2 + x^2\, m_B^2}{2\, \alpha_3^2} \Big\}, \qquad (23)$$

$\alpha_3 = 0.45 \pm 0.05$ GeV for $B_{u,d}$ meson and 0.55 ± 0.05 GeV for B_s meson

$$F_0^{B_c(2S)\to B_{u,d}}(0) = 0.397, \qquad (24)$$

$$F_0^{B_c(2S)\to B_s}(0) = 0.426, \qquad (25)$$

$$F_0^{B_c(2S)\to \eta_c(1S)}(0) = A_0^{B_c(2S)\to \psi(1S)}(0) = 0.270, \qquad (26)$$

$$F_0^{B_c(2S)\to \eta_c(2S)}(0) = A_0^{B_c(2S)\to \psi(2S)}(0) = 0.260, \qquad (27)$$

3.Numerical results

$$\mathcal{B}r = \frac{p_{\rm cm}}{8\,\pi\, m^2_{B_c(2S)}\,\Gamma_{B_c(2S)}}\,|\mathcal{A}|^2, \tag{28}$$

Table: The partial widths (Γ_i in unit of keV) and branching ratios ($\mathcal{B}r_i$ in unit of %) of the $B_c(2S)$ meson decay.

decay mode	final states	E. Eichten[1]		L. Fulcher[2]		S. Godfrey[3]		I. Asghar[4]		B. Martín-González [5]		X. Li[6]	
		Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$
hadronic	$B_c(1S)\,\pi\,\pi$	50	90.4	50	71.9	57	88.1	10.68	51.42	42	54.4	25	76.9
E1	$B_c(1P)\,\gamma$	0.0	0.0	6.4	9.2	1.3	2.0	7.11	34.23	0.0	0.0	2.8	8.6
E1	$B_c(1P')\,\gamma$	5.2	9.4	13.1	18.8	6.1	9.4	2.91	14.01	35	45.3	4.4	13.5
M1	$B_c^*(1S)\,\gamma$	0.1	0.2	0.06	0.1	0.3	0.5	0.07	0.34	0.25	0.3	0.3	1.0
total		55.3	100	69.6	100	64.7	100	20.77	100	77.3	100	33	100

[1] nonrelativistic quarkonium quantum mechanics. PhysRevD.49.5845
[2] potential models. PhysRevD.60.074006
[3] the relativized quark model. PhysRevD.70.054017
[4] non-relativistic quark potential model. PhysRevD.100.096002
[5] non-relativistic constituent quark model. PhysRevD.106.054009
[6] modified Godfrey-Isgur model. EPJC.83.1080

Table: The numerical results on CP-averaged branching ratios for the $B_c(2S) \to BP$, BV decays.

final states	decay amplitude ($\mathcal{A}$)		case	branching ratio ($\mathcal{B}r$)
	coefficient	CKM factor		
$B_s^0\,\pi^+$	a_1	$V_{cs}^*\,V_{ud} \sim \mathcal{O}(1)$	I-a	$(1.93^{+0.00+0.18+0.01}_{-0.00-0.10-0.01}) \times 10^{-9}$
$B_s^0\,\rho^+$	a_1	$V_{cs}^*\,V_{ud} \sim \mathcal{O}(1)$	I-a	$(3.34^{+0.00+0.31+0.10}_{-0.00-0.17-0.10}) \times 10^{-9}$
$B_s^0\,K^+$	a_1	$V_{cs}^*\,V_{us} \sim \mathcal{O}(\lambda)$	I-b	$(1.40^{+0.01+0.13+0.02}_{-0.01-0.07-0.02}) \times 10^{-10}$
$B_s^0\,K^{*+}$	a_1	$V_{cs}^*\,V_{us} \sim \mathcal{O}(\lambda)$	I-b	$(1.53^{+0.01+0.14+0.07}_{-0.01-0.08-0.07}) \times 10^{-10}$
$B_d^0\,\pi^+$	a_1	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	I-b	$(1.04^{+0.01+0.10+0.01}_{-0.01-0.05-0.01}) \times 10^{-10}$
$B_d^0\,\rho^+$	a_1	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	I-b	$(1.90^{+0.01+0.18+0.06}_{-0.01-0.10-0.05}) \times 10^{-10}$
$B_d^0\,K^+$	a_1	$V_{cd}^*\,V_{us} \sim \mathcal{O}(\lambda^2)$	I-c	$(7.60^{+0.09+0.72+0.10}_{-0.09-0.40-0.10}) \times 10^{-12}$
$B_d^0\,K^{*+}$	a_1	$V_{cd}^*\,V_{us} \sim \mathcal{O}(\lambda^2)$	I-c	$(8.97^{+0.11+0.84+0.43}_{-0.11-0.47-0.42}) \times 10^{-12}$
$B_u^+\,\overline{K}^0$	a_2	$V_{cs}^*\,V_{ud} \sim \mathcal{O}(1)$	II-a	$(2.85^{+0.00+1.60+0.07}_{-0.00-0.79-0.07}) \times 10^{-10}$
$B_u^+\,\overline{K}^{*0}$	a_2	$V_{cs}^*\,V_{ud} \sim \mathcal{O}(1)$	II-a	$(3.33^{+0.00+1.87+0.19}_{-0.00-0.92-0.18}) \times 10^{-10}$
$B_u^+\,\pi^0$	a_2	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	II-b	$(5.61^{+0.03+3.15+0.09}_{-0.03-1.55-0.09}) \times 10^{-12}$
$B_u^+\,\rho^0$	a_2	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	II-b	$(1.02^{+0.01+0.57+0.04}_{-0.01-0.28-0.03}) \times 10^{-11}$
$B_u^+\,\omega$	a_2	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	II-b	$(7.55^{+0.04+4.24+0.46}_{-0.04-2.09-0.44}) \times 10^{-12}$
$B_u^+\,\eta$	a_2	$V_{cs}^*\,V_{us}$, $V_{cd}^*\,V_{ud}$	II-b	$(2.17^{+0.01+1.22+0.19}_{-0.01-0.60-0.18}) \times 10^{-11}$
$B_u^+\,\eta'$	a_2	$V_{cs}^*\,V_{us}$, $V_{cd}^*\,V_{ud}$	II-b	$(2.79^{+0.02+1.56+0.64}_{-0.02-0.77-0.56}) \times 10^{-12}$
$B_u^+\,\phi$	a_2	$V_{cs}^*\,V_{us} \sim \mathcal{O}(\lambda)$	II-b	$(1.37^{+0.01+0.77+0.00}_{-0.01-0.38-0.00}) \times 10^{-11}$
$B_u^+\,K^0$	a_2	$V_{cd}^*\,V_{us} \sim \mathcal{O}(\lambda^2)$	II-c	$(8.21^{+0.10+4.61+0.21}_{-0.10-2.27-0.21}) \times 10^{-13}$
$B_u^+\,K^{*0}$	a_2	$V_{cd}^*\,V_{us} \sim \mathcal{O}(\lambda^2)$	II-c	$(9.53^{+0.12+5.35+0.54}_{-0.12-2.63-0.52}) \times 10^{-13}$

Table: The numerical results on CP-averaged branching ratios for the $B_c(2S) \to \eta_c(1S, 2S)\, P$, $\psi(1S, 2S)\, P$ decays, where the theoretical uncertainties come from the CKM parameters, the renormalization scale $\mu = (1{\pm}0.2)\, m_b$, decay constants, and Gegenbauer moments, respectively.

final states	decay amplitude ($\mathcal{A}$) coefficient	CKM factor	case	branching ratio ($\mathcal{B}r$)
$\eta_c(1S)\, \pi^+$	a_1	$V_{cb}\, V_{ud}^{\ast} \sim \mathcal{O}(\lambda^2)$	I-c	$(9.90^{+0.48+0.14+0.03}_{-0.47-0.11-0.03}) \times 10^{-12}$
$\eta_c(2S)\, \pi^+$	a_1	$V_{cb}\, V_{ud}^{\ast} \sim \mathcal{O}(\lambda^2)$	I-c	$(6.40^{+0.31+0.10+0.02}_{-0.30-0.07-0.02}) \times 10^{-12}$
$\psi(1S)\, \pi^+$	a_1	$V_{cb}\, V_{ud}^{\ast} \sim \mathcal{O}(\lambda^2)$	I-c	$(9.36^{+0.46+0.14+0.03}_{-0.44-0.11-0.03}) \times 10^{-12}$
$\psi(2S)\, \pi^+$	a_1	$V_{cb}\, V_{ud}^{\ast} \sim \mathcal{O}(\lambda^2)$	I-c	$(6.19^{+0.30+0.09+0.02}_{-0.29-0.07-0.02}) \times 10^{-12}$
$\eta_c(1S)\, K^+$	a_1	$V_{cb}\, V_{us}^{\ast} \sim \mathcal{O}(\lambda^3)$	I-d	$(7.49^{+0.42+0.11+0.07}_{-0.40-0.09-0.07}) \times 10^{-13}$
$\eta_c(2S)\, K^+$	a_1	$V_{cb}\, V_{us}^{\ast} \sim \mathcal{O}(\lambda^3)$	I-d	$(4.83^{+0.27+0.07+0.04}_{-0.26-0.06-0.04}) \times 10^{-13}$
$\psi(1S)\, K^+$	a_1	$V_{cb}\, V_{us}^{\ast} \sim \mathcal{O}(\lambda^3)$	I-d	$(6.95^{+0.39+0.11+0.06}_{-0.37-0.08-0.06}) \times 10^{-13}$
$\psi(2S)\, K^+$	a_1	$V_{cb}\, V_{us}^{\ast} \sim \mathcal{O}(\lambda^3)$	I-d	$(4.56^{+0.25+0.07+0.04}_{-0.24-0.05-0.04}) \times 10^{-13}$

4.Summary

- It is expected to have hundreds or dozens of the $B_c(2S)$ weak decay events belonging to the I-a, I-b and II-a cases with the potential $\mathcal{O}(10^{11})$ $B_c(2S)$ data in the coming HL-LHC experiment, which might be observed and used to identify the $B_c(2S)$ mesons.
- The $B_c(2S)$ weak decays into final states containing one charmonium are severely suppressed by the CKM factors, and have significantly small branching ratios, $\mathcal{O}(10^{-12})$ and less, which might be outside the future experimental detectability.

感谢各位专家、老师
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