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Lattice Parton
Collaboration

The light-cone structures for light baryons on Lattice

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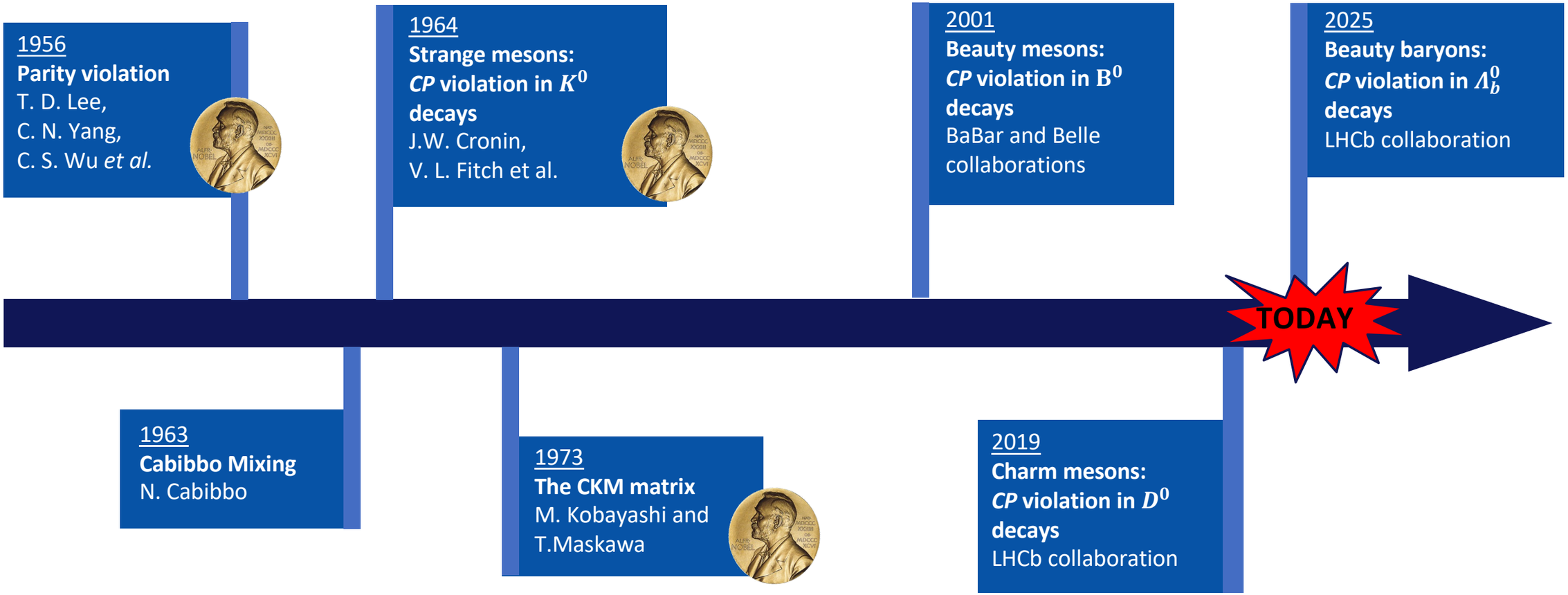
on behalf of Lattice Parton collaboration

2026 04/26 第八届全国重味物理与量子色动力学研讨会 @山城重大

OUTLINE

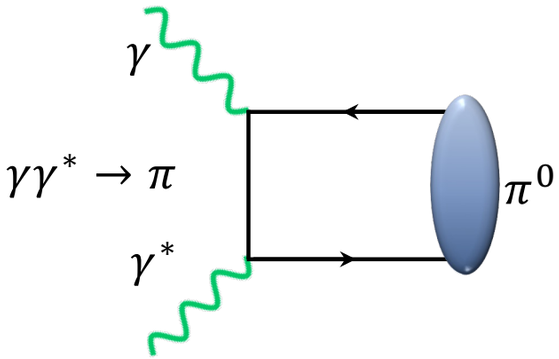
About LCDA

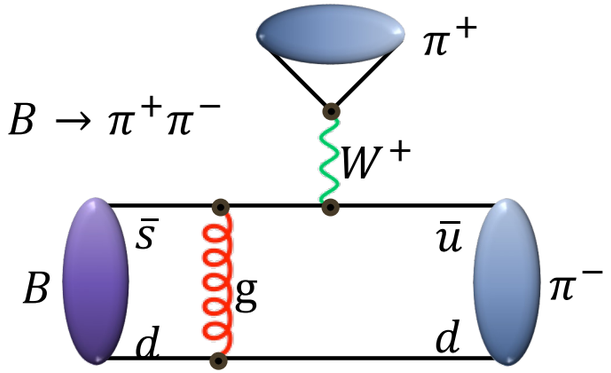


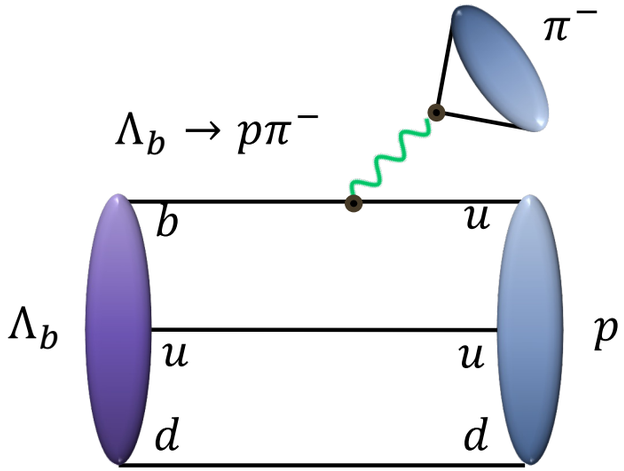
- **Among these, the LCDAs serve as a key theoretical cornerstone for investigating CP violation**

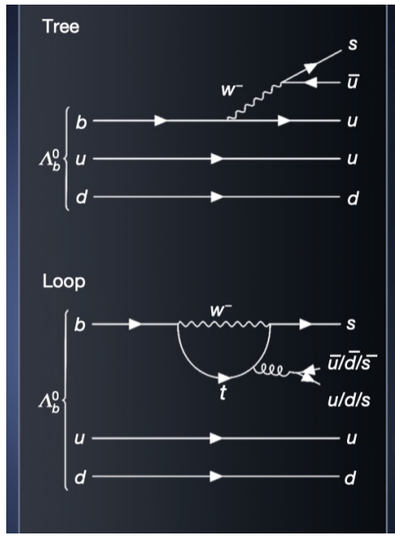
About LCDA

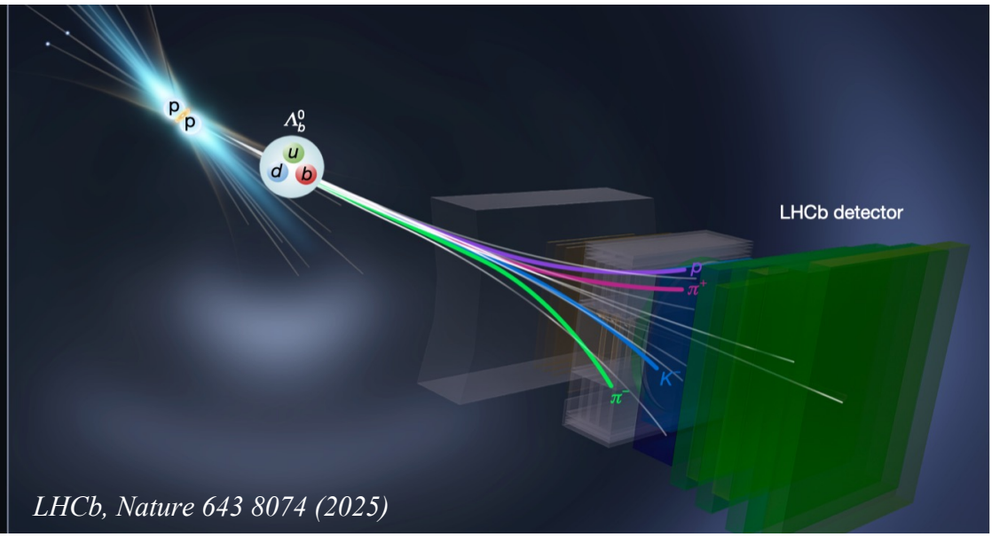
- Richer QCD dynamical information and most important input during describing **Exclusive scattering — LCDAs**











LHCb, Nature 643 8074 (2025)

LCDAs of Meson & Baryon: dynamical progress

Results @ Indirect (moments / priors) reconstruction
Results @ Direct x-dependent distribution

Light Meson

Sum Rule

NPB 201 492 (1982)
ZPC 44 157 (1989)
PRD 54 2182 (1996)
NPB 529 323 (1998)
JHEP 05 004 (2006)
…

Analytic

Model

PRD 66 094016 (2002)
PRD 84 033001 (2011)
PRD 88 034013 (2013)
PRD 104 016021(2021)
…

Global Fits

PRD 62 116002 (2000)
PRD 67 074012 (2003)
PLB 578 91 (2004)
PRD 83 054020 (2011)
PRD 102 074022 (2020)
…

DSE

PRL 110 132001 (2013)
PRL 111 092001 (2013)
PRD 90 014011 (2014)
PPNP 120 103883 (2021)
…

OPE Moments

Lattice OPE

PRD 74 074501 (2006)
PRD 92 014504 (2015)
[RQCD]PLB 774 91 (2017)
[RQCD] JHEP 08 065 (2019)
…

Pseudo

PRD 95 056020 (2017)
PRD 100 116011 (2019)
arXiv:2401.06858
…

Current Current

EPJC 78 217 (2018)
PRD 98 094507 (2018)
…

Full Distribution

LaMET

PRD 95 094514 (2017)
PRD 102 094519 (2020)
[LPC] PRL 127 062002 (2021)
[LPC] PRL 129 132001 (2022)
[BNL/ANL]PRD 106 074505 (2022)
[BNL/ANL] JHEP 07 211 (2024)
…

Heavy Meson

Model

PRD 55 272 (1997)
PRD 69 034014 (2004)
JHEP 07 154 (2018)
…

HQ-LaMET

PRD 102 011502 (2020)
[LPC]PRD 111 L111503 (2025)
[LPC]PRD 111 034503 (2025)
…

Light Baryon

Sum Rule

NPB 279 785 (1987)
PRD 47 R3685 (1993)
…

Model

PRD 65 074011 (2002)
JHEP 11 191 (2013)
…

Lattice OPE

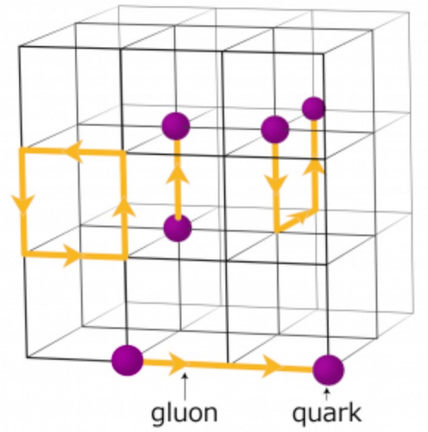
[QCDSF] PRL 101 112002 (2008)
[QCDSF] PRD79 034504 (2009)
[RQCD] EPJA 55 116 (2019)
[RQCD] PRD111 094517 (2025)
…

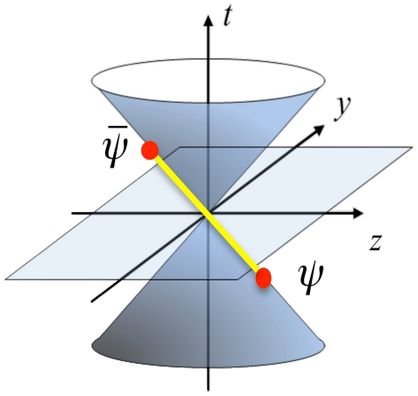
LaMET

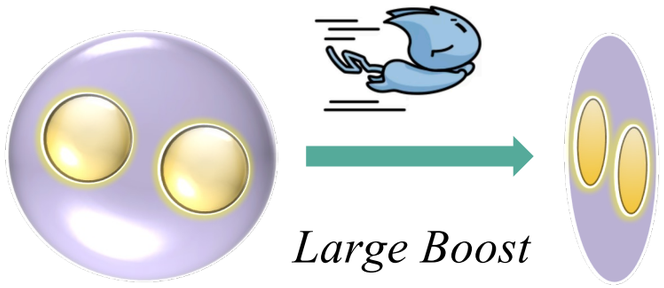
[LPC] PRD 111 034510 (2025)
[LPC] PRD 112 114515 (2025)
…

LQCD for LCDAs OPE VS LaMET

LQCD is formulated as a Feynman path integral on a 4D Euclidean grid, with a Wick rotation from real time to imaginary time.







π LCDA: $\langle 0|\bar{q}_2(\xi^+)\gamma^+\gamma_5 U\left(0,\xi^-\right)q_2\left(\xi^-\right)|\pi\rangle$

- **OPE** ~ in local limit: → moments of DA

$$\langle 0|\bar{q}_2\left(\xi^+\right)\gamma^+\gamma_5 U\left(\xi^+,\xi^-\right)q_2\left(\xi^-\right)|\pi\rangle \overset{\xi\to 0}{\sim} C_m(z)^m \bar{q}_2(0)\left(\gamma^t+\gamma^z\right)\gamma_5(in_z\cdot\overleftrightarrow{D})^m q_1(0)$$

- **LaMET** ~ in large P^z limit: → shape of DA

$$\langle 0|\bar{q}_2\left(\xi^+\right)\gamma^+\gamma_5 U\left(\xi^+,\xi^-\right)q_2\left(\xi^-\right)|\pi\rangle \overset{P^z\to\infty}{\sim} \langle 0|\bar{q}_2\left(z_+\right)(\gamma^t+\gamma^z)\gamma_5 U\left(z_+,z_-\right)q_2\left(z_-\right)|\pi(P^z)\rangle$$

From Meson LCDA

Meson LCDAs by LaMET: (2021~ …)

- π, K LCDA $a \to 0$

  R.Zhang, H.W.Lin et.al. PRD 102,094519

- K^*, ϕ; π, K LCDA $a \to 0$, $\mathrm{m}_\pi \to 130\mathrm{MeV}$

  (LPC) PRL 127 062002(2021); PRL 129 132001(2022)

- Hybrid and Self renormalization

  (LPC) NPB 964 115311(2021); NPB 969 115443(2021)…

- RGR resummation & Renormalon resummation (LRR)

  Y.S.Su et.al. NPB 993 116282(2023)

- Threshold resummation

  Y.S.Su et.al. JHEP 03 045(2025)

- Two loop matching

  F.Yao et.al. arXiv:2504.09367; PRD 113 014505(2026)

- Heavy Meson DA

  X.Y.Han et.al. PRD 111 034504; arXiv: 26xxx

- Not physical π mass, RI/MOM scheme, λ truncation

- Physical π mass with $a \to 0$
- Hybrid scheme, Self renormazaliton

- Towards high precision
- Not yet done with lattice results

- A new frame work
- High precision is challenge

From Meson LCDA

Lattice Advances: (2021 ~ …)

- Kinematically-enhance

  PRD 112 L051502(2025)

- Precision check for meson LCDA

  … JHW et.al.

- Inverse Fourier or Extrapolation

  EPJC 85 12 1409(2025), PRD 113 014509(2026),

  EPJC 86 4 379(2026), arXiv:2601.12189…

- CLQCD ensembles & Pyquda

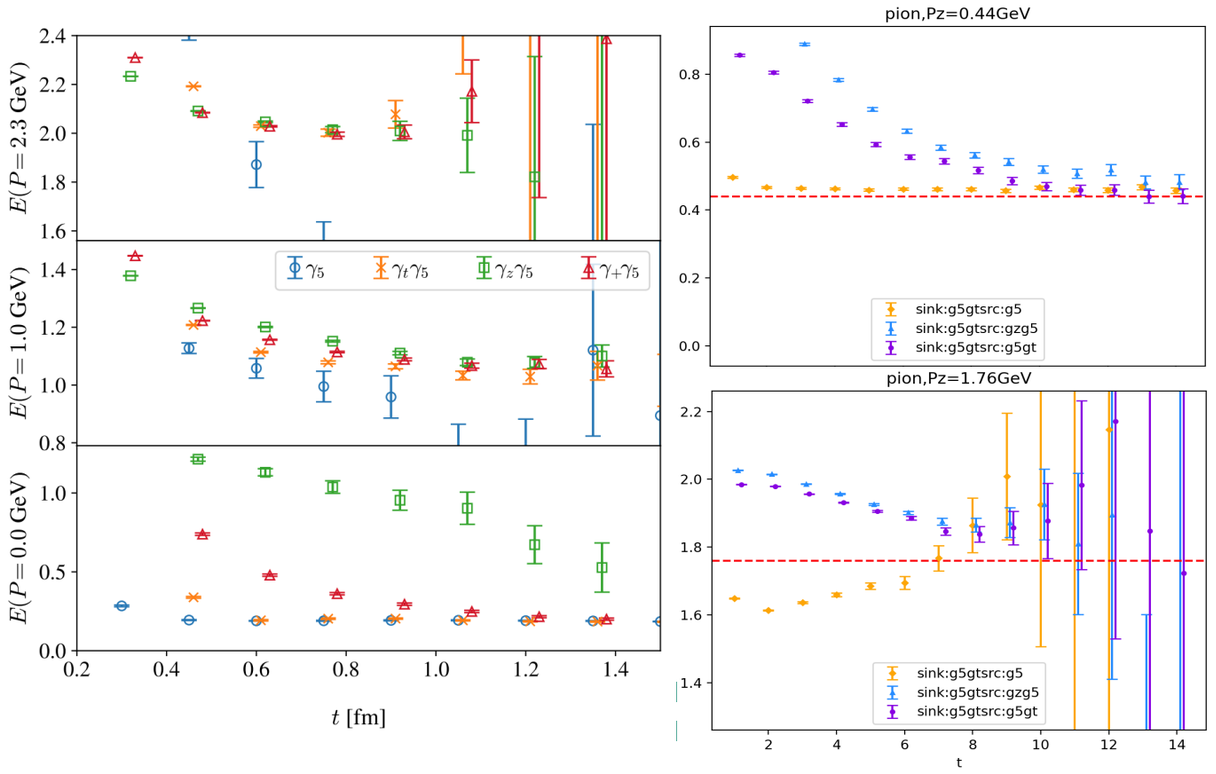
  CLQCD PRD 109(2024) , arXiv: 2411.08461…

- … Gradient flow

  JHEP 06 210 (2024), PRD 112 094504(2025)…

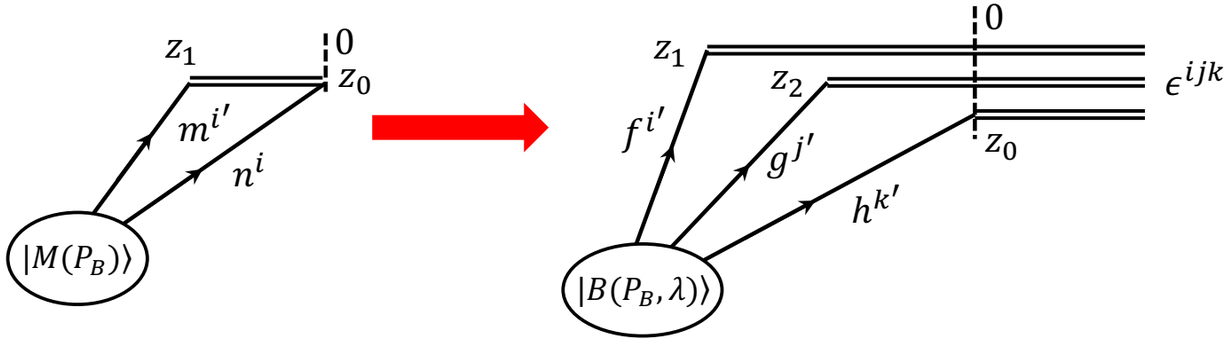
➢ **Precision !**

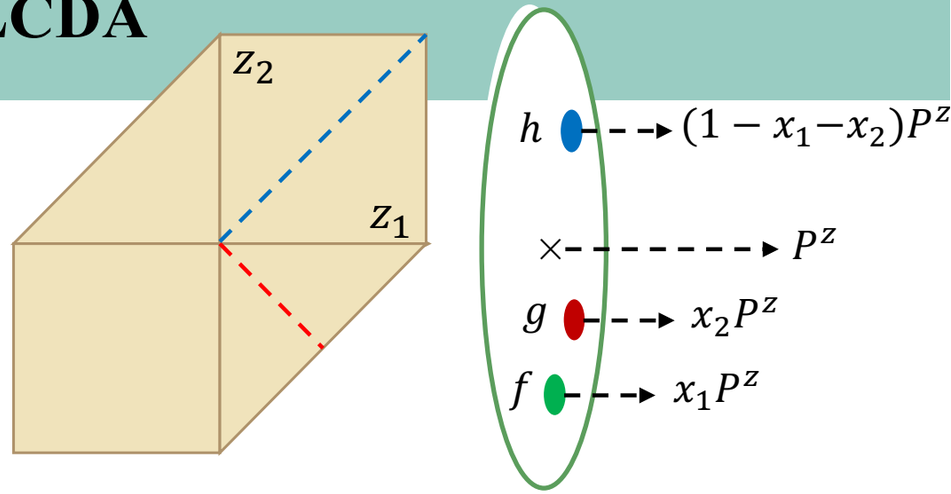
Signal Improve

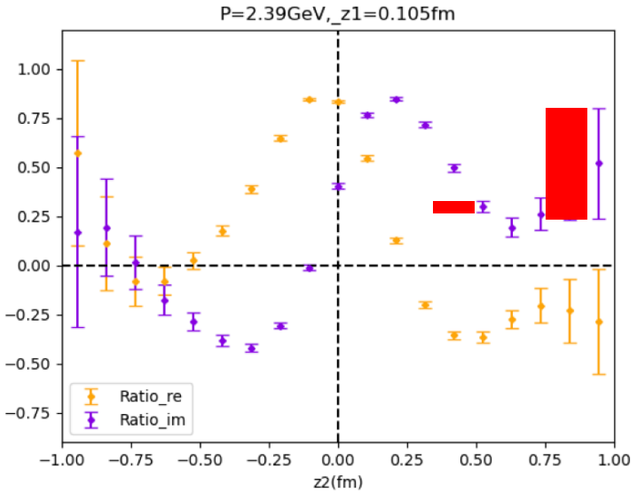


Light Baryon LCDA

Challenge from light meson LCDA to light baryon LCDA







- From effective Wilson length → **Signal to Noise Ratio**
  - Signal to Noise Ratio * 8 -> Statistics * 64
  - Totally 64*10*2
- Dynamic enhance: **SNR 200% improve (eff 400%) !**
- Coding in Pyquda: **Computing efficiency 800% !**

Light Baryon LCDA

- symmetry of z_1, z_2 exchange:

$$V_{(0,0)} = T_{(0,0)} = 0$$

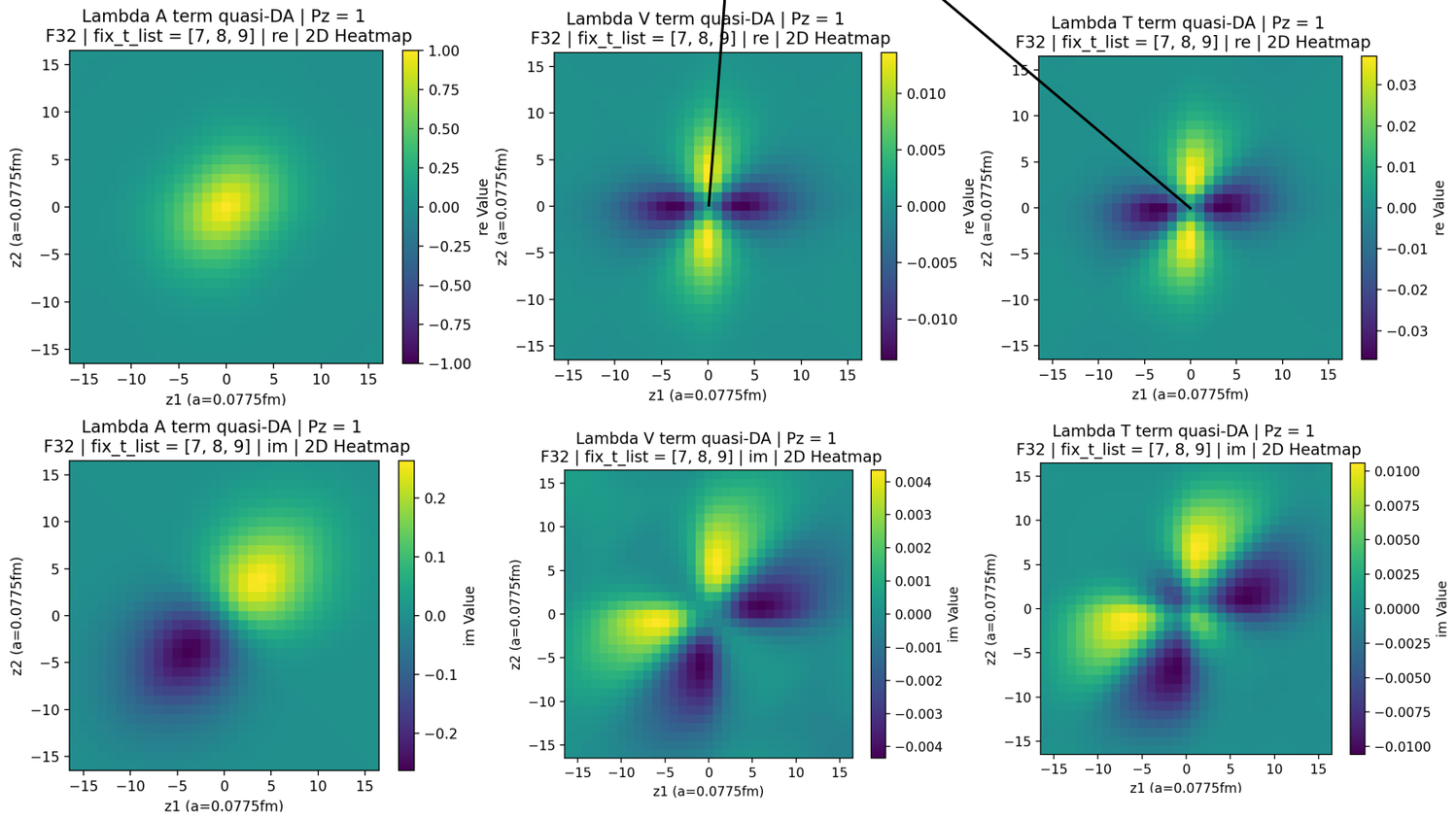
$$\langle 0|f_\alpha(\xi_1 n)g_\beta(\xi_2 n)h_\gamma(\xi_3 n)|B(P_B)\rangle^R$$
$$= \frac{1}{4} f_B \left[(\not{P}_B C)_{\alpha\beta}(\gamma^5 u_B)_\gamma \Phi_B^V(\xi_i n \cdot P_B, \mu) + (\not{P}_B \gamma^5 C)_{\alpha\beta}(u_B)_\gamma \Phi_B^A(\xi_i n \cdot P_B, \mu) \right]$$
$$+ \frac{1}{4} f_B^T (\mathrm{i}\sigma_{\mu\nu} P_B^\nu C)_{\alpha\beta}(\gamma^\mu \gamma^5 u_B)_\gamma \Phi_B^T(\xi_i n \cdot P_B, \mu) \ ,$$

$$\tilde{\Phi}(z_1, z_2) = \tilde{\Phi}^*(-z_1, -z_2)$$

$$\tilde{\Phi}_V^\Lambda(z_1, z_2) = -\tilde{\Phi}_V^\Lambda(z_2, z_1),$$
$$\tilde{\Phi}_A^\Lambda(z_1, z_2) = \tilde{\Phi}_A^\Lambda(z_2, z_1),$$
$$\tilde{\Phi}_T^\Lambda(z_1, z_2) = -\tilde{\Phi}_T^\Lambda(z_2, z_1).$$

Lambda

P=0.5 GeV



Light Baryon LCDA

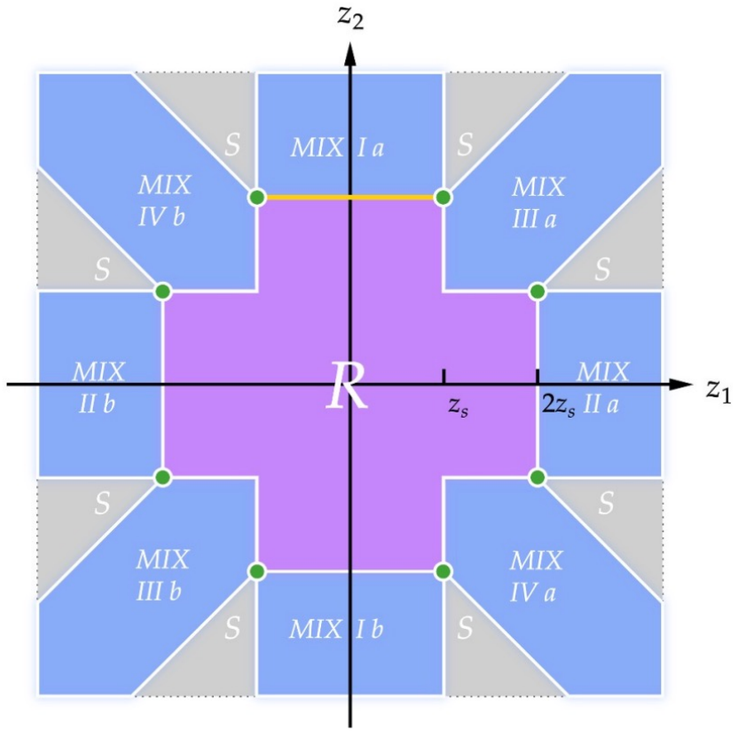
CLQCD Ensembles —— fully exploiting available

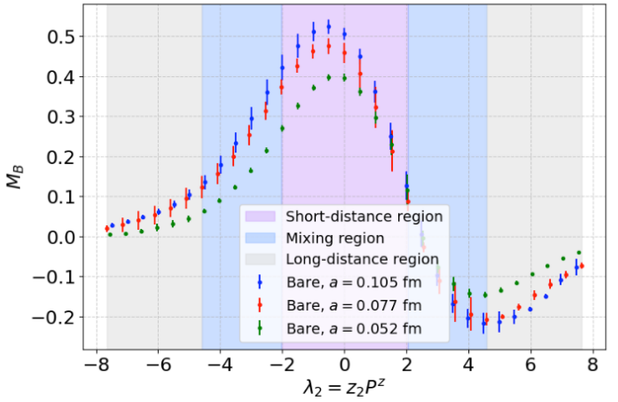
Ensembles	a (fm)	m_π (MeV)	Volume	n_{cfg}	n_{src}	P^z (GeV)
C24P29	0.1052	292.3	24×72	864	4×9	0, 1.96, 2.45, 2.94
C32P23	0.1052	227.9	32×64	954	4×8	0, 1.84, 2.21, 2.57, 2.94
C48P14	0.1052	136.4	48×96	302	4×16	0, 1.96, 2.45, 2.94
F32P30	0.0775	300.4	32×96	777	4×8	0, 2.00, 2.49, 2.99
F32P21	0.0775	210.3	32×64	459	4×16	0, 2.00, 2.49, 2.99
G36P29	0.0689	297.2	36×108	656	6×8	0, 2.00, 2.50, 3.00
H48P32	0.0520	316.6	48×144	550	6×9	0, 1.98, 2.48, 2.98

Table 2: Ensembles

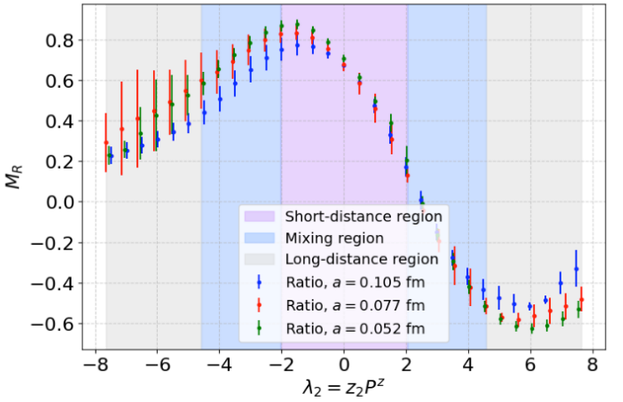
Light Baryon LCDA

Hybrid renormalization —— Done

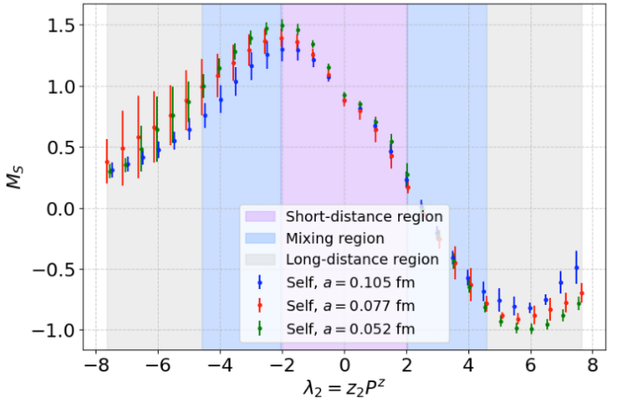




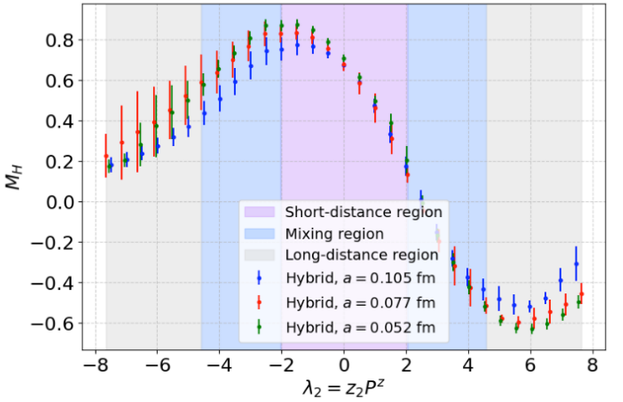
(a) Bare result of Λ at $P = 2.0$ GeV



(b) Ratio scheme result of Λ at $P = 2.0$ GeV



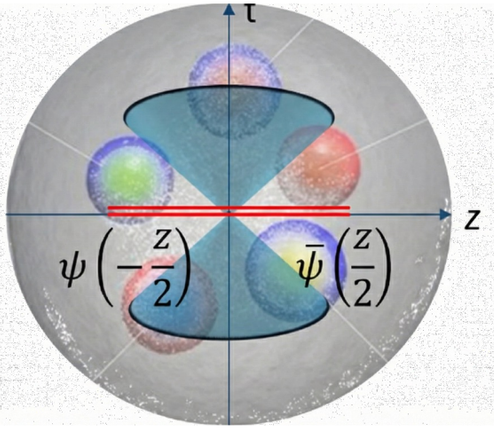
(c) Self renormalized result of Λ at $P = 2.0$ GeV



(d) Hybrid scheme result of Λ at $P = 2.0$ GeV

Light Baryon LCDA

New λ-Extrapolation —— Done



$$z \to +\infty \quad \underset{\sim}{} \quad \frac{e^{-\Lambda z} e^{i\phi(z)}}{z^{\alpha}}$$

- HQET reduction: represent gauge link with heavy quark

$$\tilde{h}_{\Gamma_1\Gamma_2}(P,x) = \left\langle P \left| \bar{\psi}(x)\Gamma_1 U(x,0) \frac{ix\!\!\!/ + \sqrt{-x^2}}{2\sqrt{-x^2}} \Gamma_2 \psi(0) \right| P \right\rangle_c$$

$$M_{\Gamma_1\Gamma_2}(P,x) = \langle P | \bar{\psi}(x)\Gamma_1 Q(x)\ \bar{Q}(0)\Gamma_2\psi(0) | P \rangle_c$$

$$M_{\Gamma_1\Gamma_2}(P,x) = H_{\Gamma_1\Gamma_2}\left(\frac{m_Q^2}{\mu^2}\right) \frac{m_Q^3}{2\sqrt{2}\pi^{3/2}} \frac{e^{-m_Q\sqrt{-x^2}}}{\left(m_Q\sqrt{-x^2}\right)^{3/2}}\ \tilde{h}_{\Gamma_1\Gamma_2}(P,x) \left[1 + O\left(\frac{1}{m_Q\sqrt{-x^2}}\right)\right]$$

- Dispersive analysis: Insert a complete basis for $x = (t,0,0,0)$ with $t > 0$

$$M_{\gamma^t,I}(P,x) = \sum_X \int d\Gamma_X(k)\ e^{i(P-k)\cdot x} \left\langle P | \bar{\psi}\gamma^z Q | X(k) \right\rangle \left\langle X(k) | \bar{Q}\psi | P \right\rangle \Big|_c$$

- Lorentz symmetry: transfer timelike to spacelike

LA NLA

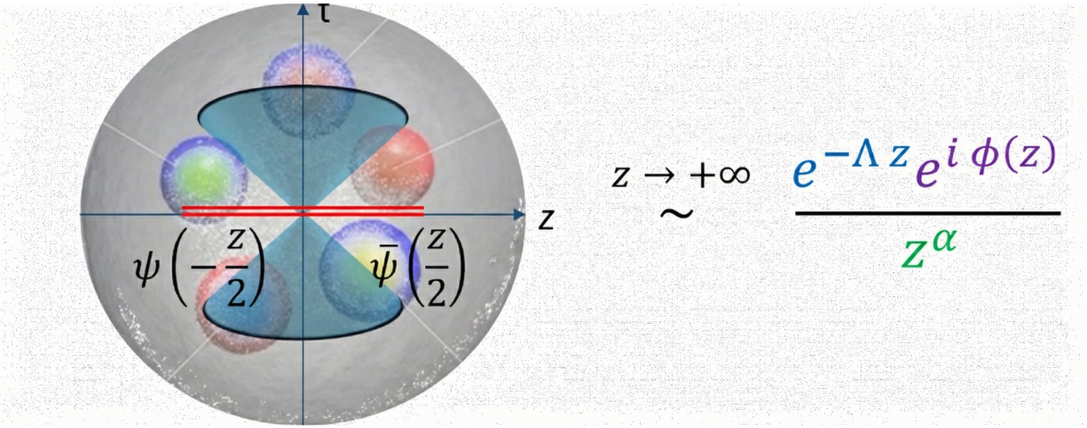
$$\langle P | \bar{\psi}(z)\gamma^t U(z,0)\psi(0) | P \rangle_{|z|\to+\infty} = e^{-\Lambda_0|z|}\left(a_{0,0} + \frac{a_{0,1}}{|z|} + \frac{a_{0,2}}{|z|^2} + \cdots\right)$$

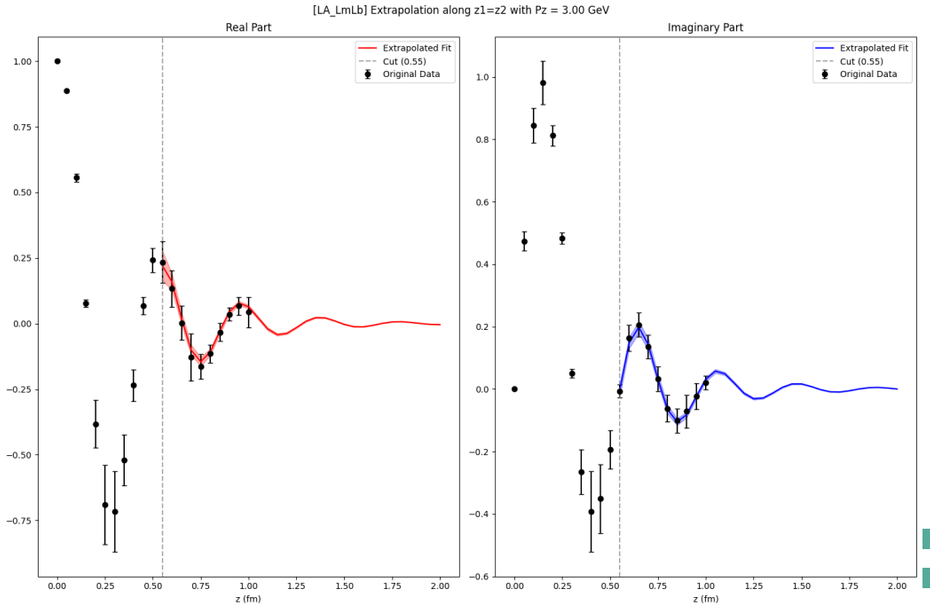
$$+ e^{-\Lambda_1|z|}\left(a_{1,0} + \frac{a_{1,1}}{|z|} + \frac{a_{1,2}}{|z|^2} + \cdots\right) + e^{-\Lambda_2|z|}\left(a_{2,0} + \frac{a_{2,1}}{|z|} + \frac{a_{2,1}}{|z|^2} \cdots\right) + \cdots$$

where the mass gaps $\{\Lambda_0 < \Lambda_1 < \Lambda_2 < \Lambda_3 < \cdots\}$ are related to the binding energies of inserted intermediate heavy-quark systems

Light Baryon LCDA

New λ-Extrapolation —— Done





Asymptotic Long-Distance Expansion of Euclidean Correlators in Lattice Parton Applications

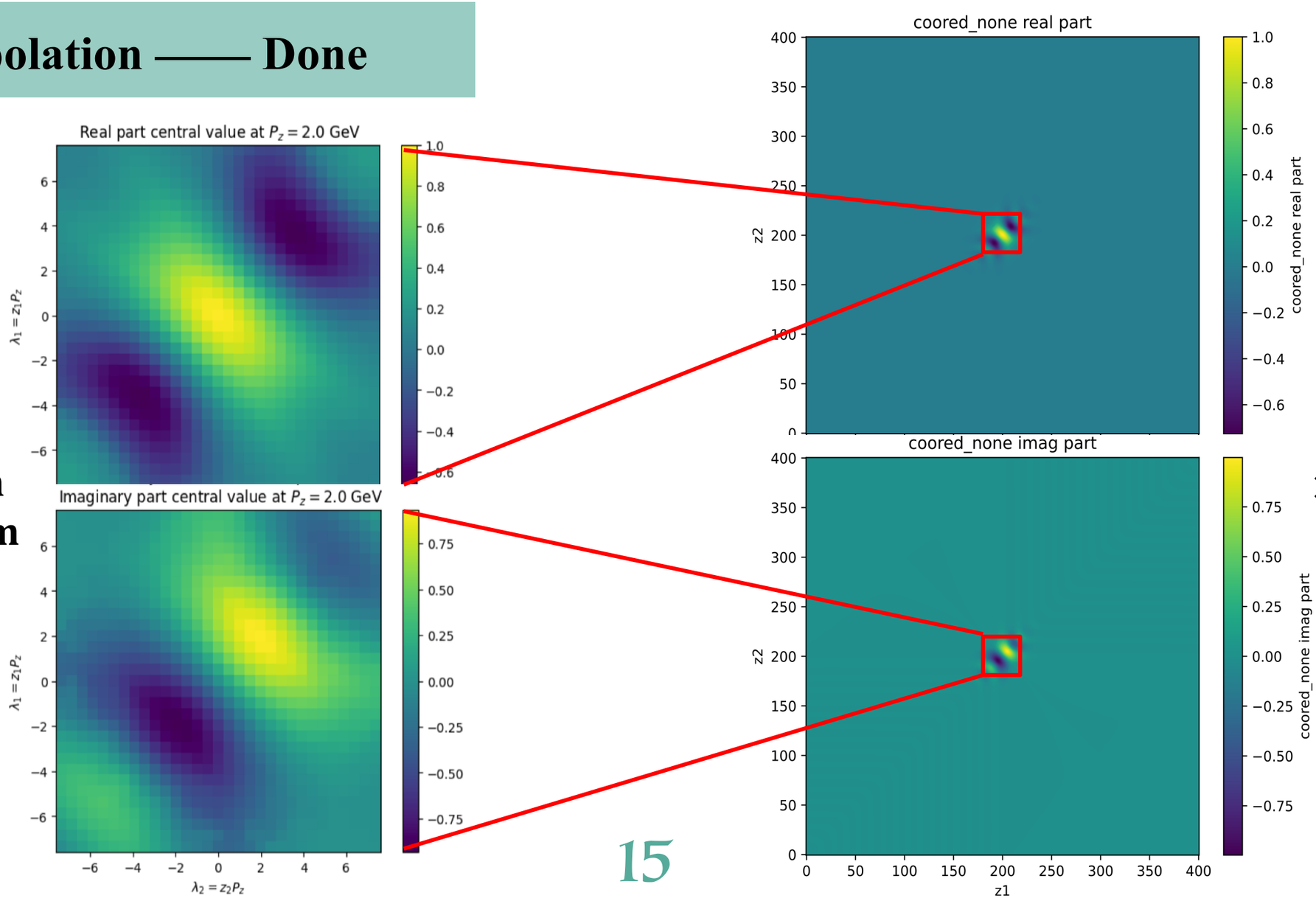
X. Ji, Y. Liu, Y. Su, arXiv:2601.12189

LA NLA

$$A(z_1,z_2) = e^{iz_1P^z}e^{-\Lambda^{0^-}|z_2|}\left[P_1(iz_1,i\hat{z}_2,P^z) + \frac{P_1'(iz_1,i\hat{z}_2,P^z)}{|z_1|} + \cdots\right]$$
$$+e^{-\Lambda^{0^-}|z_2|}\left[P_2(iz_1,i\hat{z}_2,P^z) + \frac{P_2'(iz_1,i\hat{z}_2,P^z)}{|z_1|} + \cdots\right]$$
$$+e^{iz_2P^z}e^{-\Lambda^{1/2^-}|z_2|}\left[P_3(iz_1,i\hat{z}_2,P^z) + \frac{P_3'(iz_1,i\hat{z}_2,P^z)}{|z_1|} + \cdots\right]$$
$$+\cdots$$

Light Baryon LCDA

New λ-Extrapolation —— Done



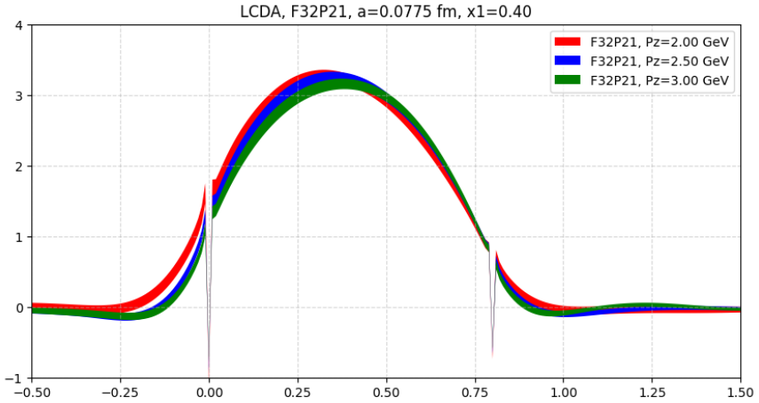
1.2 fm ×1.2 fm

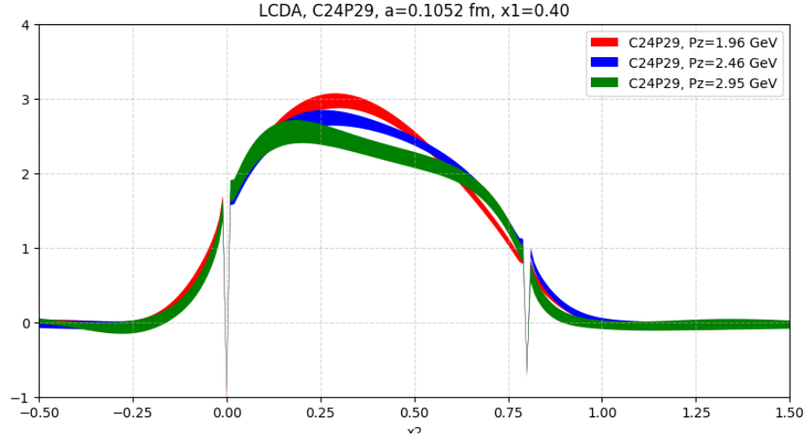
10 fm ×10 fm

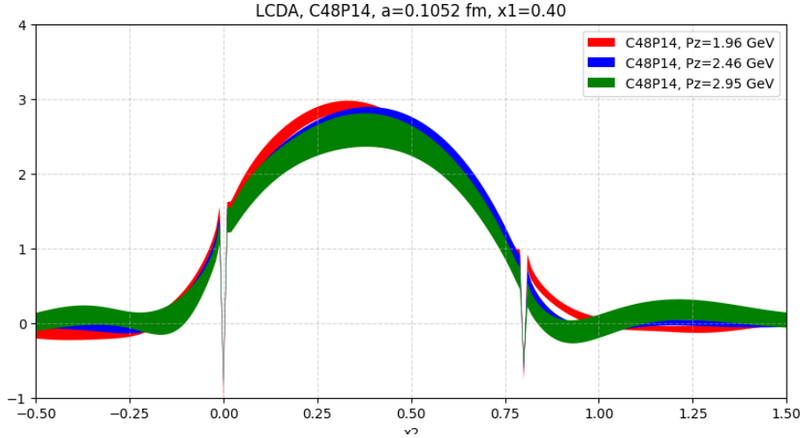
Matching $\phi(x_1, x_2, P^z, \mu) = \int dy_1 dy_2\, C(x_1, x_2; y_1, y_2; P^z, \mu)\, \tilde{\varphi}(y_1, y_2, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(x_1 P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(x_2 P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{[(1-x_1-x_2)P^z]^2}\right)$

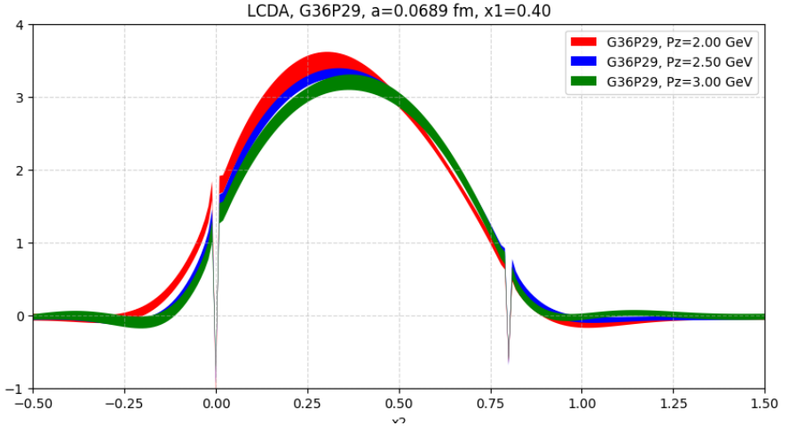
The Best Results of Baryon LCDA

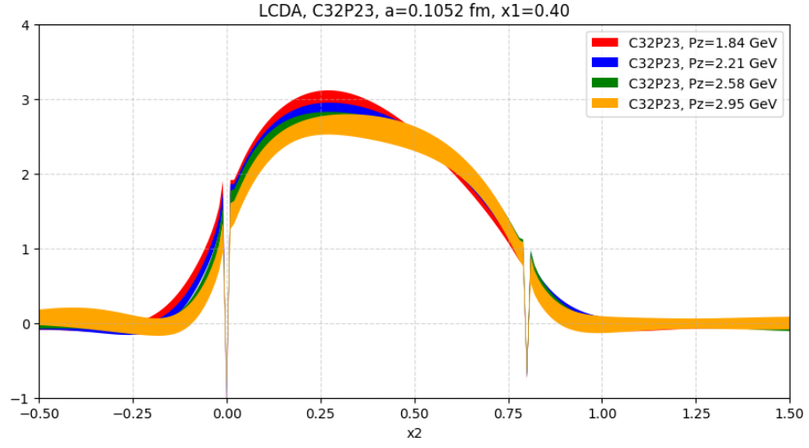
A Terms for each ensemble

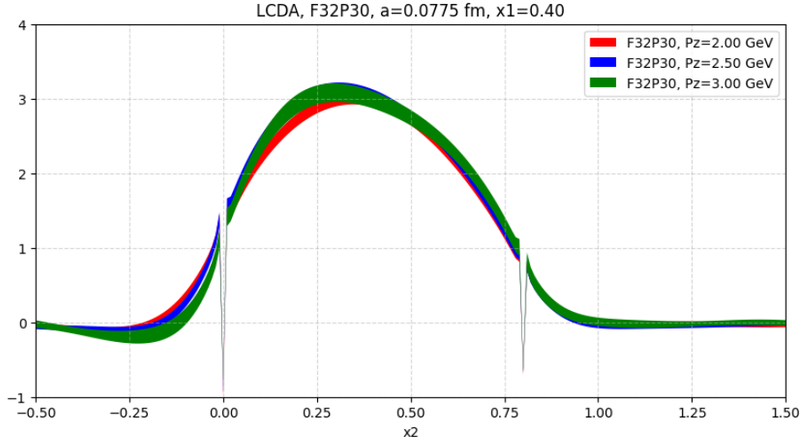


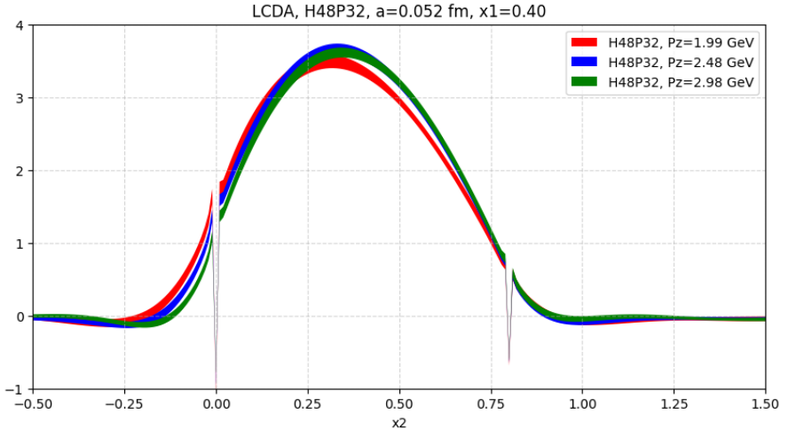






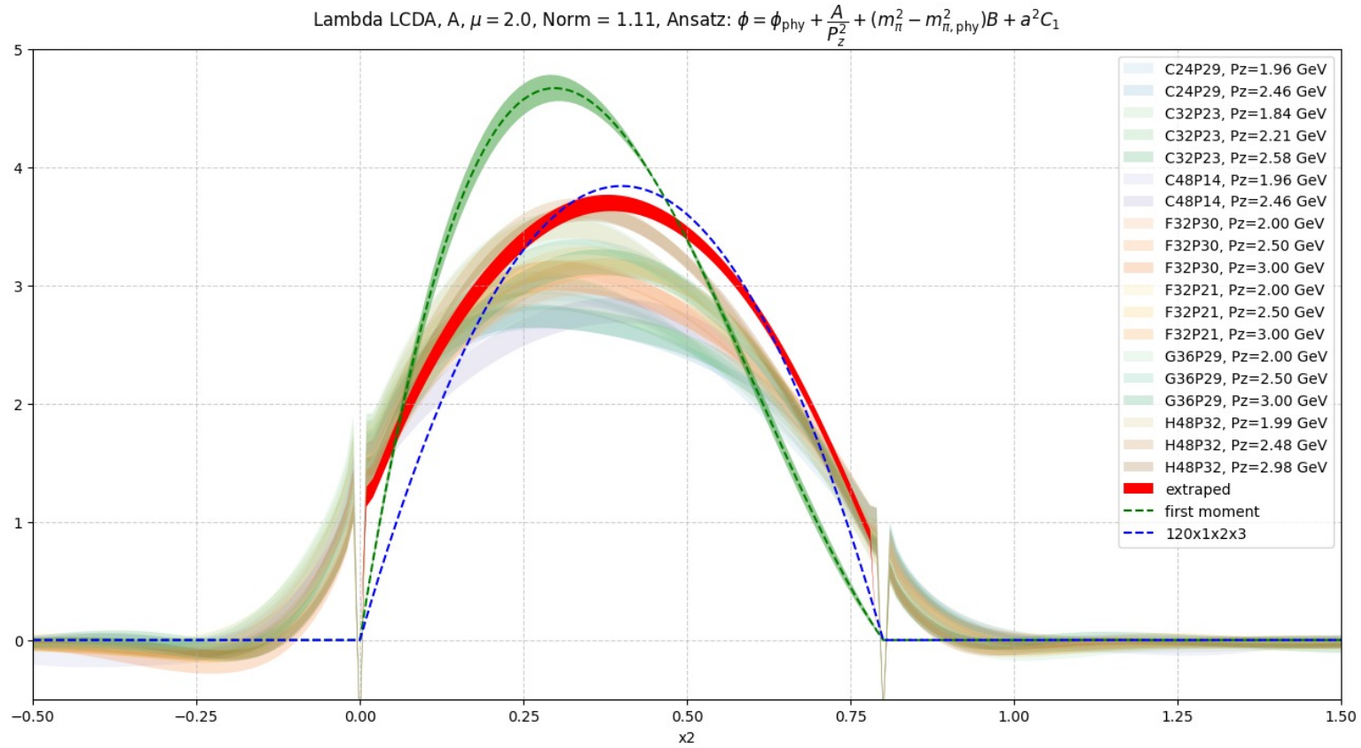


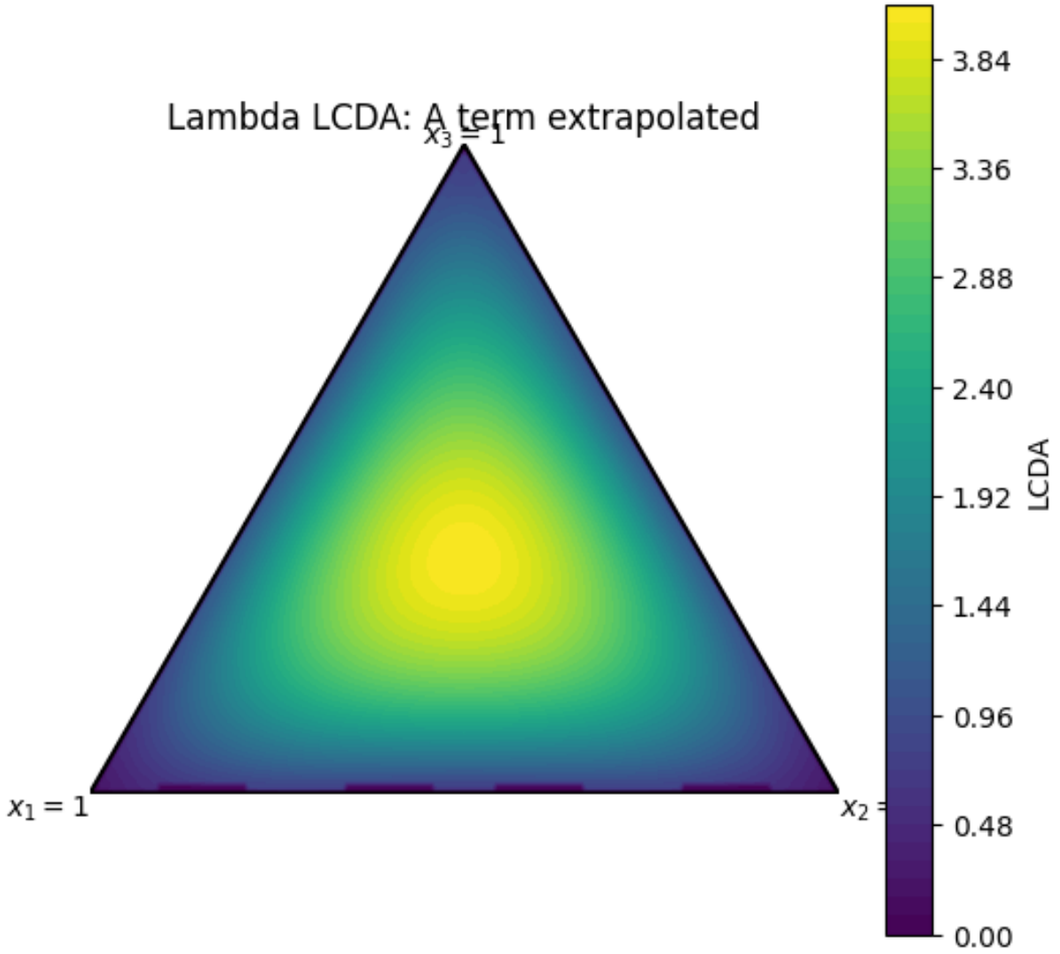




The Best Results of Baryon LCDA

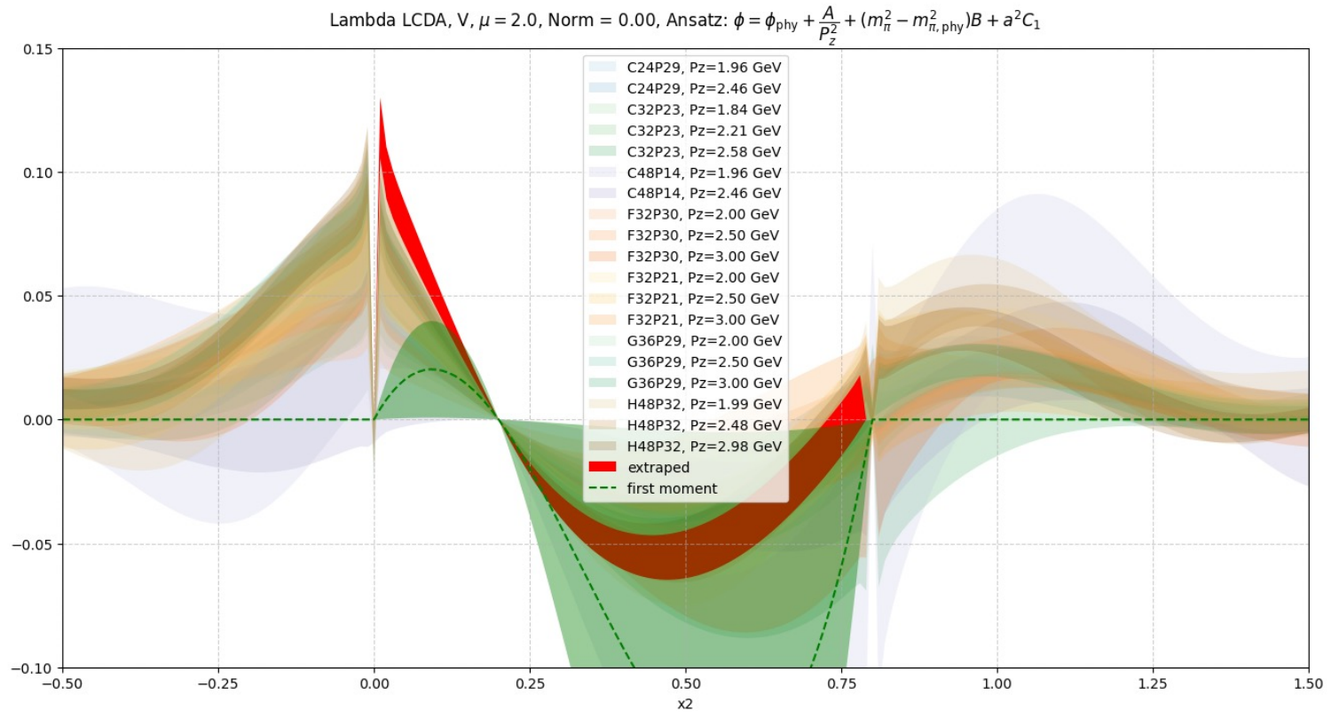
A Term:

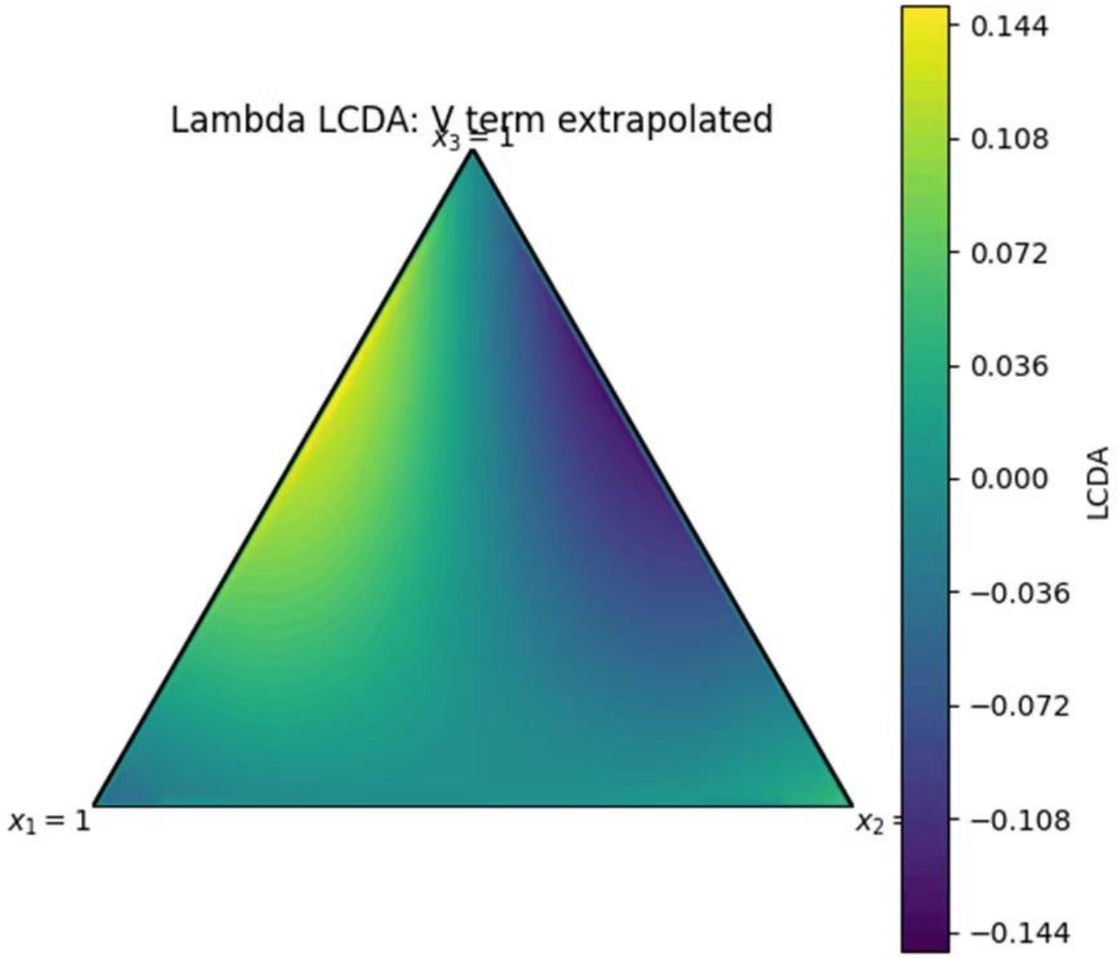




The Best Results of Baryon LCDA

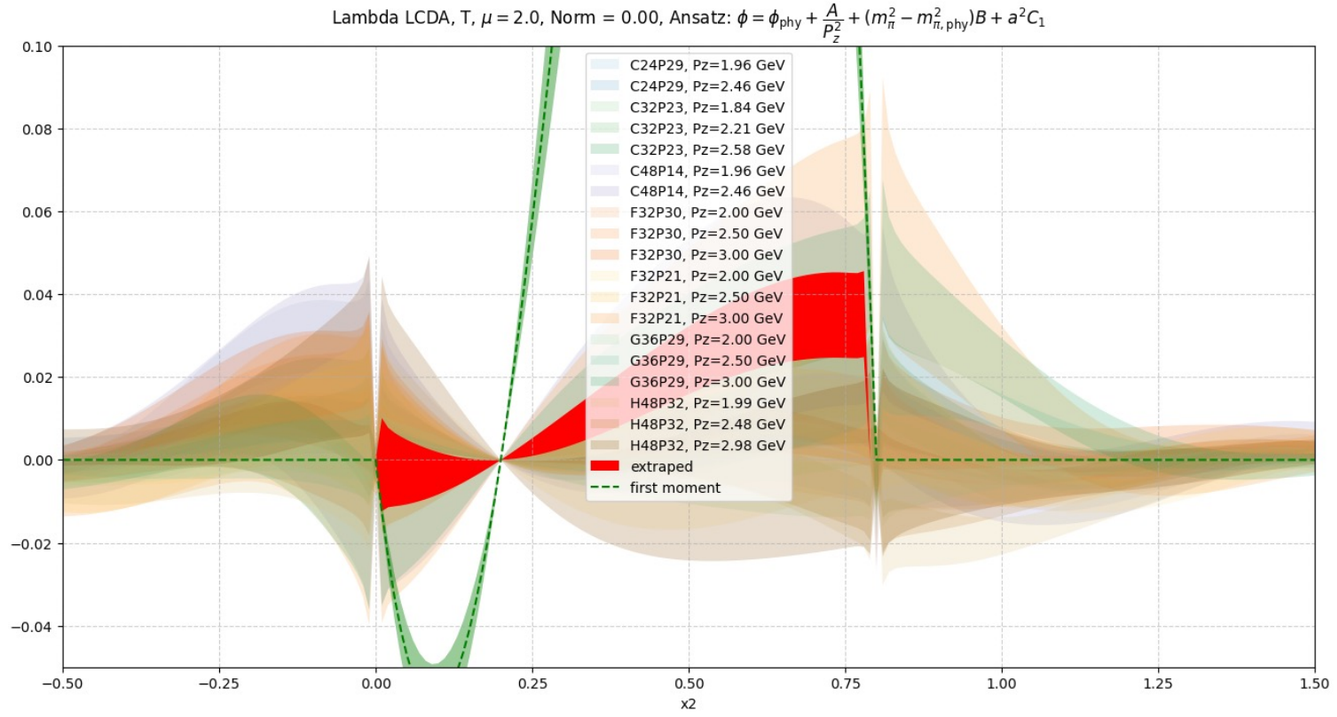
V Term:

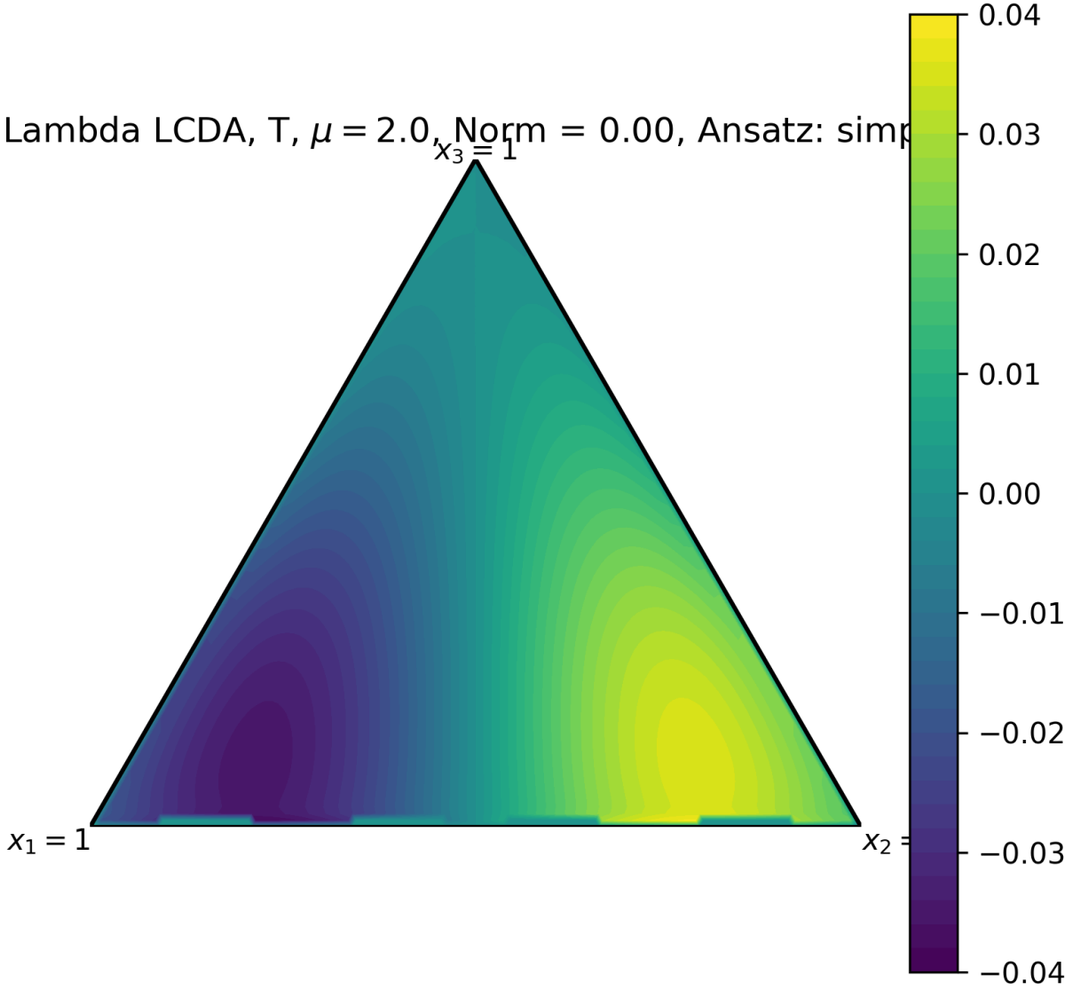




The Best Results of Baryon LCDA

T Term:





A benchmark with lattice OPE

Our results:

	A (NLA_lm)	V (LA_lm)	T (LA_lm)
Norm	1.10891(46)	2.0(6.5)e-05	1.4(5.8)e-05
$\langle \tilde{x}_1 \rangle$	0.36035(17)	0.003448(28)	-0.001885(26)
$\langle \tilde{x}_2 \rangle$	0.36035(17)	-0.003434(28)	0.001895(26)
$\langle \tilde{x}_3 \rangle$	0.38821(22)	0.6(2.5)e-05	0.4(2.1)e-05
$\frac{\varphi_{10}^{\Lambda}}{f^{\Lambda}}[V-A]$	0.11639(68)	0.11639(68)	
$\frac{\varphi_{11}^{\Lambda}}{f^{\Lambda}}\Big\vert_1 [V-A]$	0.04568(25)	0.04568(25)	
$\frac{\varphi_{11}^{\Lambda}}{f^{\Lambda}}\Big\vert_2 [V-A]$	-0.11766(51)	-0.11766(51)	

RQCD (PRD 111 094517)

$$f^{\Lambda} = 4.75^{+6}_{-4}(2)_r(5)_a(16)_m$$
$$\varphi_{10}^{\Lambda} = 0.563^{+33}_{-27}(6)_r(27)_a(43)_m \ ,$$
$$\varphi_{11}^{\Lambda} = 0.242^{+8}_{-6}(9)_r(46)_a(8)_m \ ,$$
$$\pi_{10}^{\Lambda} = 0.237^{+30}_{-35}(4)_r(12)_a(19)_m$$

$$\frac{\varphi_{10}^{\Lambda}}{f^{\Lambda}} = 0.119(14)$$
$$\frac{\varphi_{11}^{\Lambda}}{f^{\Lambda}} = 0.051(10)$$

Summary & Outlook

Light meson

Light baryon

Nucleon GPD…

Lattice scheme results

Renormalization

A well-defined scheme

LaMET Matching

(Calculating the LaMET matching kernel under self scheme)

Done!

Light cone results Λ

Few weeks …

Proton

Ensembles	a (fm)	m_π (MeV)	
C24P29	0.1052	292.3	
C32P23	0.1052	227.9	New
C48P14	0.1052	136.4	
F32P30	0.0775	300.4	
F32P21	0.0775	210.3	
G36P29	0.0689	297.2	New
H48P32	0.0520	316.6	

Ratio scheme

Self renormalization New

Hybrid scheme New

LPC, PRD 111, 034510 (2025)

LPC, PRD 112, 114515 (2025)

Thanks for Your Attention !