第八届全国重味物理与量子色动力学研讨会

Low-energy $N\phi$ scattering from a pole-enhanced triangle diagram

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Motivation

Theor. predictions

- The non-relativistic Yukawa potential $V_{(Q\bar{Q})A} = -\alpha e^{-\mu r}/r$ is matched to the Pomeron interaction. S. J. Brodsky et al,PRL64(1990); H. Gao et al, PRC63(2001).
- $\{N\phi,\ \Lambda K^*\}$ coupled channel scattering, F. Huang et al,PRC73(2006); J.J Xie et al PLB774(2017);C.S. An et al, PRC98(2018); J. He et al, PRD98(2018).
- Analysis in correlation function：attractive $N\phi$ scattering E. Chizzali et al, PLB848(2024); L. M. Abreu et al, PLB860(2025).

Latt. prediction

- $J = 3/2$ $N\phi$ scattering length $(-1.7,\ -0.9)\,\mathrm{fm}$ fitted to two-pion exchange with $m_\pi = 146.4\,\mathrm{MeV}$ by HAL QCD, Yan Lyu et al, PRD106(2022)

Exp. prediction

- Re. $N\phi$ scattering length around $-1\,\mathrm{fm}$ w/o $\Lambda K^*, \Sigma K^*$ coupled channel scattering by Alice, S. Acharyaet al, PRL127(2021)

$N\phi$ interaction to one loop

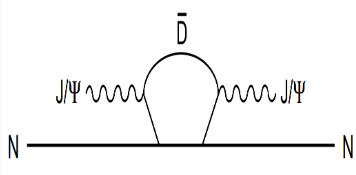
OBE

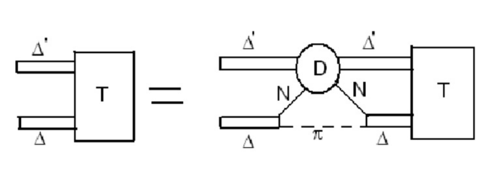
- η-exchange is perturbative.
- σ coupling to strange quark is not clear.
- $\omega - \phi$ mixing is tiny

Two-pion exchange in bubbles

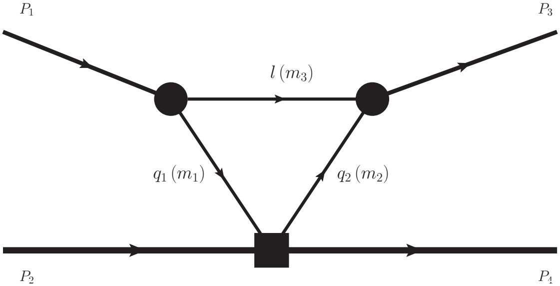
- Two-pion exchange in bubbles is negligible with a vanished WT in $\phi\pi(K)$ scattering.

Dynamics in triangle diagrams S.J. Brodsky et al, PLB 412 (1997); A. Gal et al, PRL111(2013).





Dynamics in the triangle diagram



- $\mathcal{L}_{\phi_{K\bar{K}}} = ig\phi^{\mu}\left(\overline{K}\partial_{\mu}\cdot K - K\partial_{\mu}\cdot\overline{K}\right)$
- $L_1 = \langle \bar{B} i\gamma^{\mu}\frac{1}{4F_{\pi}^2}\left[\left(\Phi\partial_{\mu}\Phi - \partial_{\mu}\Phi\Phi\right)B - B\left(\Phi\partial_{\mu}\Phi - \partial_{\mu}\Phi\Phi\right)\right]\rangle$
- Poles driven by a WT in $NK(\bar{K})$ sacattering
- Spin-independent interaction

Dynamics in the triangle diagram

The S-wave projected loop integral w/o couplings

$$I(l) = \int \frac{d^4 l}{(2\pi)^4} \frac{-g_{\rm eff}\left[4\,\vec{l}^{\,2}+\vec{P}_1\cdot\vec{P}_3\right]\,u(P_2)\,\bar{u}(P_4)}{(l^2-m_1^2)\left[(P_1-l)^2-m_2^2\right]\left[(P_3-l)^2-m_3^2\right]\left[q_\Lambda^2-m_\Lambda^2\right]},$$

- For the pole z_A : $l^0 = \omega_1 - i\epsilon$

$$2\pi i\,{\rm Res}\,I(z_A) = \int \frac{d^3 l}{(2\pi)^3} \frac{4\vec{l}^2 * g_{\rm eff}/(8\,m_K^3)}{P_1^0-\omega_1-\omega_2} \frac{u(P_2)\,\bar{u}(P_4)}{P_3^0-\omega_1-\omega_3} \frac{1}{2m_\Lambda\left(P_1^0+P_2^0-\omega_1-\omega_\Lambda\right)}$$

$N\phi$ potential w/ $2m_K - m_\phi = \delta$ in HAL QCD

- $\frac{V^s_{LO}}{\mathcal{C}} = \arctan\left(\frac{\Lambda}{a_0}\right)\left[\frac{a_0}{2(b^2-a_0^2)} + \frac{a_0(a_0^2-2b^2)}{(a_0^2-b^2)^2}\right] + \frac{a_0}{2(b^2-a_0^2)}\frac{a_0\Lambda}{a_0^2+\Lambda^2} + \arctan\left(\frac{\Lambda}{b}\right)\frac{b^3}{(b^2-a_0^2)^2}$, w/ $\mathcal{C} = -2m_K^2 g_{\rm eff}/\pi^2$, $a_0 = \sqrt{-m_K\delta}$
  $b^2 = -2m_\Lambda\Delta E$, $\delta = m_\phi - 2m_K$ and $\Delta E = m_\phi + m_p - \omega_1 - m_\Lambda$

Dynamics in the triangle diagram

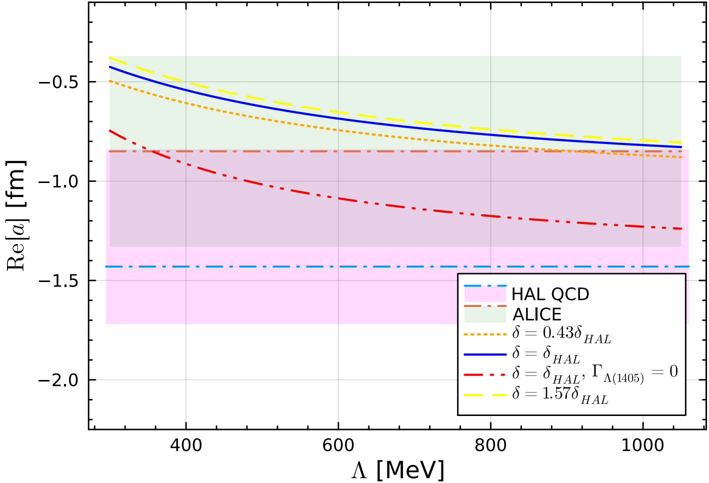
Physical $N\phi$ potential, w/ $k_0 = \sqrt{2m_K\delta}$

$$\frac{V^s_{LO}}{\mathcal{C}} = \frac{k_0}{4(b^2+k_0^2)}\left(\log\frac{\Lambda+k_0}{\Lambda-k_0} - i\pi - \frac{2k_0\Lambda}{\Lambda^2-k_0^2}\right) + \frac{b^3}{(b^2+k_0^2)^2}\arctan\left(\frac{\Lambda}{b}\right) + \frac{k_0(k_0^2+2b^2)}{2(b^2+k_0^2)^2}\left(\log\frac{\Lambda-k_0}{\Lambda+k_0} + i\pi\right),$$

$N\phi$ potential and ERE

- $\frac{8\pi th.}{T} = V^{-1} - G = -\frac{1}{a} - ik$, $-\frac{1}{a_{N\phi}} = \frac{1}{V^s_{LO}}$.
- $t = -\lim_{s\to s_R}\frac{g_i^2}{s-s_R}$ w/ $\sqrt{s}_R = 1417^{+4}_{-4} - i24^{+7}_{-4}\,\mathrm{MeV}$ and coupling $g_i = 7.7^{+1.2}_{-0.6}\,\mathrm{GeV}$, D. Jido et al, NPA725(0223); Z.H. Guo and J. A. Oller, PRC87(2013)
- $\frac{g_i^{\mathrm{unphy.}\,2}}{g_i^2} \simeq \left(\frac{m_N^{\mathrm{unphy.}}}{m_N}\right)^2\left(\frac{m_K^{\mathrm{unphy.}}}{m_K}\right)^2 \approx 1.19,.$

$N\phi$ scattering length with unphysical Kaon mass



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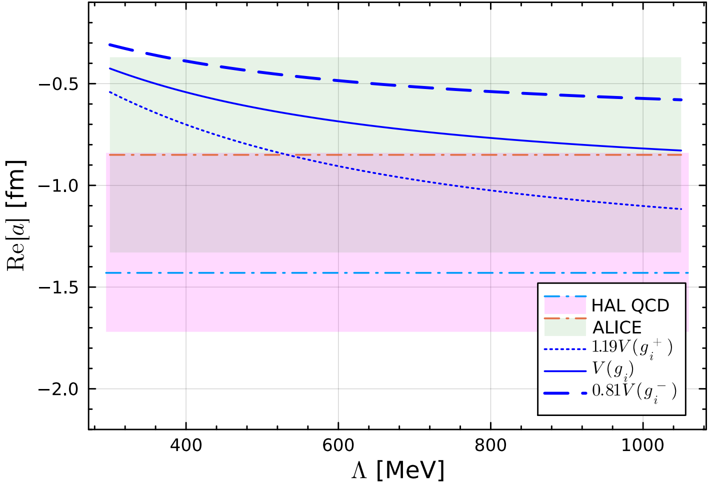

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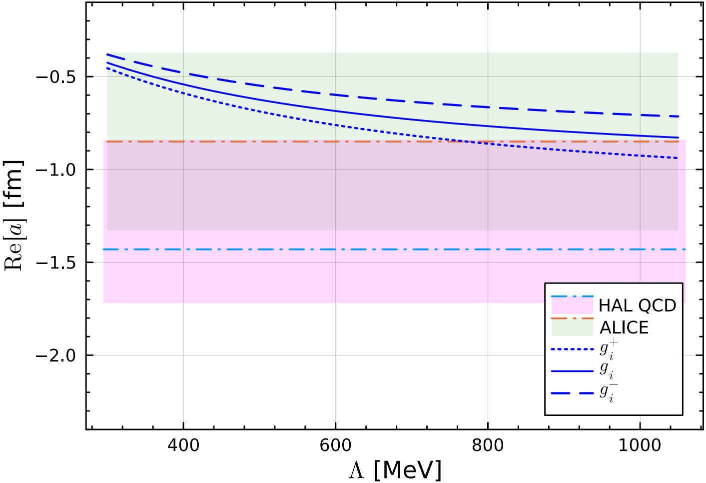
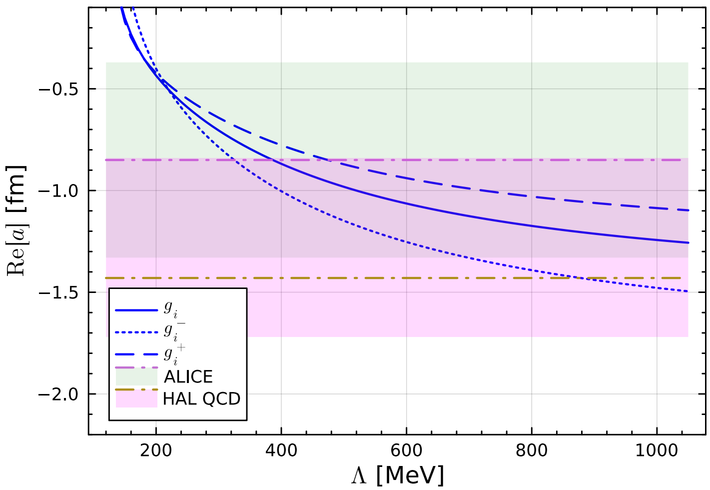
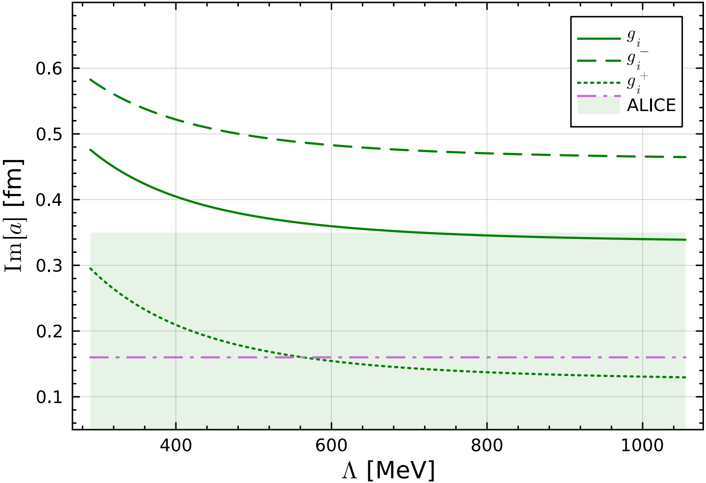


$N\phi$ scattering length with physical Kaon mass



The momentum carried by the outgoning K is $118.17\,\mathrm{MeV}$ in $\phi \to K\bar{K}$.

$N\phi$ scattering length with physical Kaon mass



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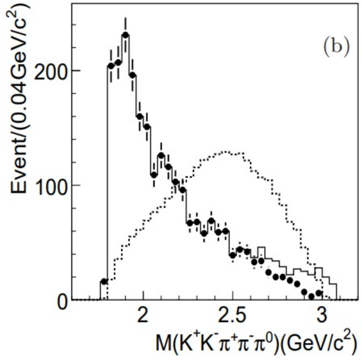
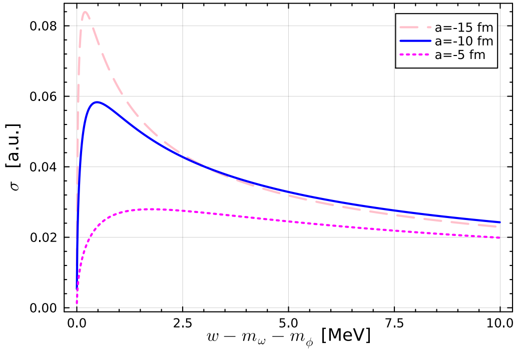
$\omega\phi$ threshold enhancement





M. Ablikim et al. (BESIII), PRD87, 032008 (2013)

A pole from $\omega\phi$ scattering around the threshold in addition to $f_0(1710)$?

Summary

1. The $N\phi$ scattering length evaluated in the triangle diagram with including $\Lambda(1405)$ matches to the one from HAL QCD, where the open channel effect in $\Sigma\pi$, $N\bar{K}$ scattering is ignored.
2. The scattering length is a power law of a_0 and b and differs from two-pion exchange and the Van der Waals force.
3. When the 3-B pole closes to the 2-B threshold, the 3B pole contributes to the scattering length.
4. This dynamic driven from the triangle diagram is adaptable to $\omega\phi$ threshold enhancement.
5. $P_c(4312)\pi$ couple to $I = 3/2$ sector.
6. $w_{c1}\pi$, $X(3872)\pi$ couple to $Z_c(4020)$.

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Thanks !

Backup

The $\mathrm{SU}(3)$ matrices for the mesons and the baryons are the following

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}.$$

Table 1: Inputs of the isospin-averaged hadron masses

Hadron	Lattice [MeV]	Expt. [MeV]
K	524.7	495.6
ϕ	1048.0	1019.5
N	954.0	938.9