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Study on the interference mechanism in the multi-body decay process of B mesons

吕　刚

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Research background

CP violation is an important area for testing the Standard Model and searching for new physics signals. In recent years, it has attracted extensive attention. The B meson system, which contains rich physical information, is a good way to explore CP violation. Therefore, the study of B mesons occupies an extremely important position in the field of particle physics.

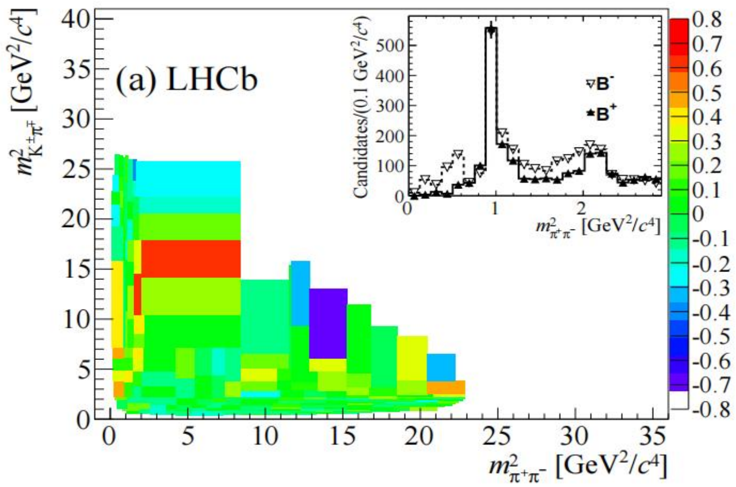
At present, theoretically, with the gradual in-depth study of B mesons, the research on CP violation in the two-body decay process has gradually matured, and the research on CP violation of B mesons is gradually moving towards in-depth study of multi-body decays.

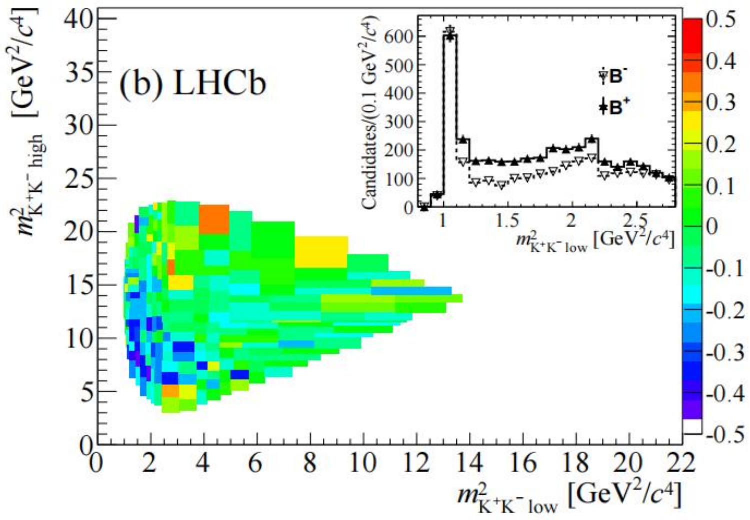
Meanwhile, there is also a large amount of experimental data on CP violation in the multi-body decay processes of B mesons. This not only provides a basis for us to explore the CP violation mechanism in the decay processes of B mesons, but also enables us to detect and search for the CP violation mechanism from the vast amount of data.

The LHCb experiment, the experimental results of the three-body decay of B mesons:

$B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$

Phys. Rev. Lett. 111, 101801(2013)



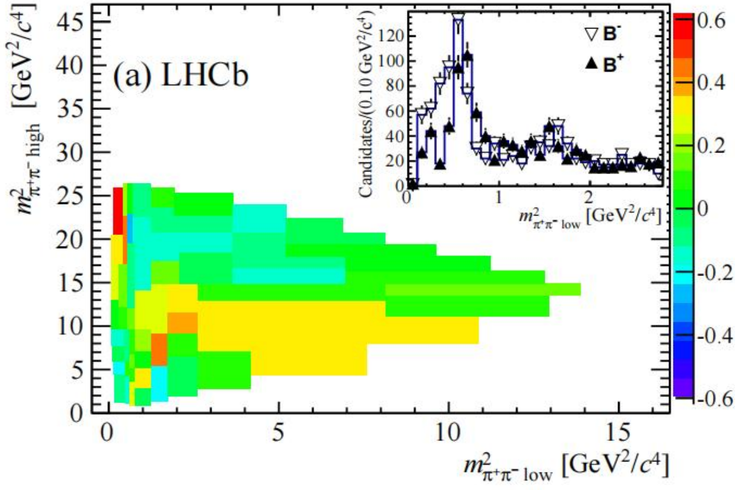


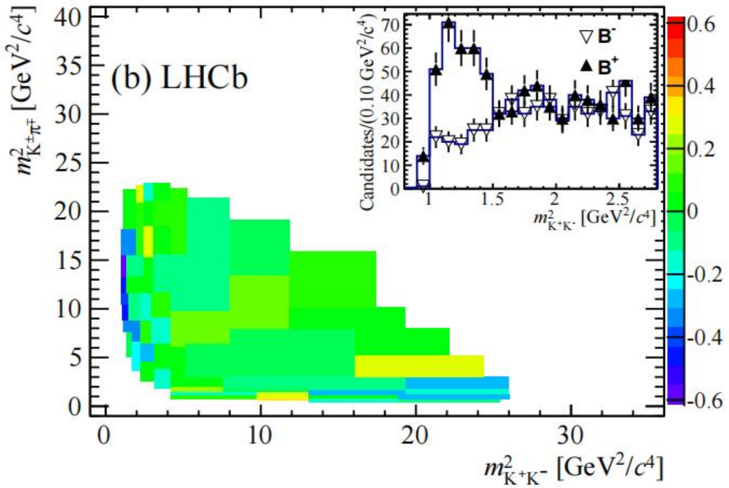
$B^{\pm} \to K^{\pm}K^{+}K^{-}$

Phys. Rev. Lett. 111, 101801(2013)

$B^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$

Phys. Rev. Lett.112, 011801(2014)





$B^{\pm} \to K^{+}K^{-}\pi^{\pm}$

Phys. Rev. Lett.112, 011801(2014)

Research background

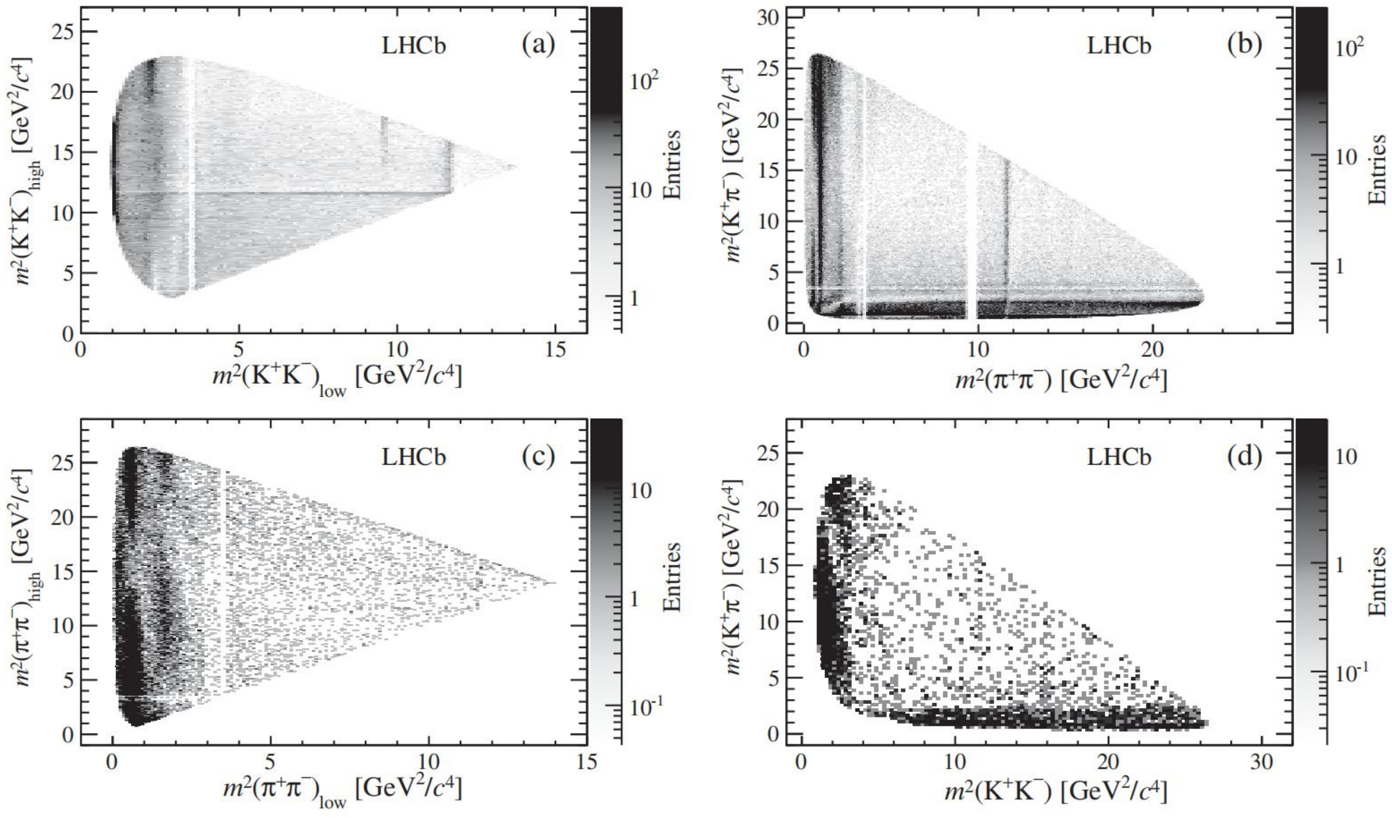


FIG. 2. Dalitz plot distributions of (a) $B^{\pm} \to K^{\pm}K^{+}K^{-}$, (b) $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$, (c) $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and (d) $B^{\pm} \to \pi^{\pm}K^{+}K^{-}$ candidates. The visible gaps correspond to the exclusion of the J/ψ (in the $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ decay) and D^{0} (all plots, except for the $B^{\pm} \to \pi^{\pm}K^{+}K^{-}$ decay) mesons from the samples.

图1 摘自LHCb Collaboration Phys. Rev. D 90, 112004 (2014)

Phys. Rev. Lett. 112, 011801 (2014).

Research background

Large CP violation in a local phase space region has been observed in the three-body non-charm decays of B mesons. The result of the integrated CP violation is:

$$A_{CP}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}) = +0.025 \pm 0.004 \pm 0.004 \pm 0.007$$

$$A_{CP}(B^{\pm} \to K^{\pm}K^{+}K^{-}) = -0.036 \pm 0.004 \pm 0.002 \pm 0.007$$

$$A_{CP}(B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}) = +0.058 \pm 0.008 \pm 0.009 \pm 0.007$$

$$A_{CP}(B^{\pm} \to \pi^{\pm}K^{+}K^{-}) = -0.123 \pm 0.017 \pm 0.012 \pm 0.007$$

Research background

The experimental CP violation results in the three-body non-charm decay process of B mesons：

$B^{\pm} \to (\rho^{0}(770) \to \pi^{+}\pi^{-})K^{-}$	$0.150 \pm 0.019 \pm 0.011(\mathrm{LHC})$	Phys. Rev.D 108, 012013 (2023)
	$0.30 \pm 0.11 \pm 0.02(Belle)$	Phys. Rev.Lett. 96, 251803 (2006)
$B^{\pm} \to (\phi(1020) \to K^{+}K^{-})K^{-}$	$0.004 \pm 0.014 \pm 0.007(\mathrm{LHC})$	Phys. Rev. D 108, 012013 (2023)
	$0.128 \pm 0.004 \pm 0.013(BaBar)$	Phys. Rev. D 85,112010 (2012)
$B^{\pm} \to (\rho^{0}(770) \to \pi^{+}\pi^{-})\pi^{-}$	$-0.004 \pm 0.017 \pm 0.009(\mathrm{LHC})$	Phys. Rev. D 108, 012013 (2023)
	$0.007 \pm 0.011 \pm 0.016(LHC)$	Phys.Rev. Lett. 124, 031801 (2020)
$B^{\pm} \to (\phi(1020) \to K^{+}K^{-})\pi^{-}$	$-0.648 \pm 0.070 \pm 0.013 \pm 0.070(\mathrm{LHC})$	Phys.Rev. Lett.112, 011801 (2014)

2

$\rho - \omega - \phi$ mixing mechanism

Introduction to the mixing mechanism

- According to the vector meson dominance (VMD) model, positrons and electrons can annihilate into photons. In a vacuum, photons can be polarized to form vector particles ρ , ω , and ϕ . These vector mesons can then decay into pairs of $\pi^+\pi^-$ or K^+K^- mesons.

- Under this mixing mechanism, through the electromagnetic form factor of the π meson, we can obtain the mixing amplitude parameters of the corresponding two or three particles, and determine their specific values by combining with experimental results.

Introduction to the mixing mechanism

① Carry out the transformation of the physical field

$$\begin{pmatrix}\rho^0\\ \omega\\ \phi\end{pmatrix} = R(s)\begin{pmatrix}\rho_I^0\\ \omega_I\\ \phi_I\end{pmatrix} \quad R = \begin{pmatrix}<\rho_I|\rho> & <\omega_I|\rho> & <\phi_I|\rho>\\ <\rho_I|\omega> & <\omega_I|\omega> & <\phi_I|\omega>\\ <\rho_I|\phi> & <\omega_I|\phi> & <\phi_I|\phi>\end{pmatrix} = \begin{pmatrix}1 & -F_{\rho\omega}(s) & -F_{\rho\phi}(s)\\ F_{\rho\omega}(s) & 1 & -F_{\omega\phi}(s)\\ F_{\rho\phi}(s) & F_{\omega\phi}(s) & 1\end{pmatrix}$$

② Construct the isospin basis vectors $|I, I_3>$, and then according to the orthonormalization relation, we have:

$$\sum_M |M><M| = \sum_{M_I} |M_I><M_I| = I,$$

$$<M|N>=<M_I|N_I>=\delta_{MN}$$

transform

$$|M>=\sum_{N_I} |N_I><N_I|M>$$

$$D(s) = \frac{1}{s - W(s)}$$

According to the diagonalization of the physical state

$$D = \sum_M \frac{|M><M|}{s - Z_M}$$

Introduction to the mixing mechanism

③ From the above transformation, we can obtain the expression form of vector mesons in the description of the physical field.

$$\rho^0 = \rho_I^0 - F_{\rho\omega}(s)\omega_I - F_{\rho\phi}(s)\phi_I, \qquad \omega = F_{\rho\omega}(s)\rho_I^0 + \omega_I - F_{\omega\phi}(s)\phi_I, \qquad \phi = F_{\rho\Phi}(s)\rho_I^0 + F_{\omega\phi}(s)\omega_I + \phi_I.$$

$$W_I = \begin{pmatrix} <\rho_I|W|\rho_I> & <\rho_I|W|\omega_I> & <\rho_I|W|\phi_I> \\ <\omega_I|W|\rho_I> & <\omega_I|W|\omega_I> & <\omega_I|W|\phi_I> \\ <\phi_I|W|\rho_I> & <\phi_I|W|\omega_I> & <\phi_I|W|\phi_I> \end{pmatrix}.$$

Define the square of the complex mass

$$Z_{\rho(\omega,\phi)} = (m_{\rho(\omega,\phi)} - i\,\Gamma_{\rho(\omega,\phi)}/2\,)^2 \simeq m^2_{\rho(\omega,\phi)} - im_{\rho(\omega,\phi)}\Gamma_{\rho(\omega,\phi)},$$

Neglecting the higher-order terms, we can use the matrix R in the physical representation to diagonalize the equation W I.

$$W = RW_IR^{-1} = \begin{pmatrix} Z_\rho & 0 & 0 \\ 0 & Z_\omega & 0 \\ 0 & 0 & Z_\phi \end{pmatrix}.$$

$$F_{\rho\omega} = \frac{\langle\rho_I|\mathrm{W}|\omega_I\rangle}{m_\omega^2 - m_\rho^2 - i(m_\omega\Gamma_\omega - m_\rho\Gamma_\rho)}$$

$$F_{\rho\phi} = \frac{\langle\rho_I|\mathrm{W}|\omega_I\rangle}{m_\phi^2 - m_\rho^2 - i(m_\phi\Gamma_\phi - m_\rho\Gamma_\rho)}$$

$$F_{\omega\phi} = \frac{\langle\rho_I|\mathrm{W}|\omega_I\rangle}{m_\phi^2 - m_\omega^2 - i(m_\phi\Gamma_\phi - m_\omega\Gamma_\omega)}$$

The contribution of the higher-order term $F^{\ 2}$ can be neglected to obtain the following relationship:

$$F_{\rho\omega} = \frac{<\rho_I|W|\omega_I>}{Z_\omega - Z_\rho}, \; F_{\rho\phi} = \frac{<\rho_I|W|\phi_I>}{Z_\phi - Z_\rho}, \quad F_{\omega\phi} = \frac{<\omega_I|W|\phi_I>}{Z_\phi - Z_\omega}.$$

Introduction to the mixing mechanism

④The propagator of the intermediate state particles from vector mesons can be expressed as:

$$D^{\mu\nu}_{V_1V_2}(q^2) = i\int d^4 x e^{iqx} < 0|T(V_1^{\mu}(x)(V_2^{\nu}(0))|0 > .$$

$$D_{\rho\omega} =< 0|T\rho\omega|0 >=< 0|T(\rho_I - F_{\rho\omega}\omega_I - F_{\rho\phi}\phi_I)(F_{\rho\omega}\rho_I + \omega_I - F_{\omega\phi}\phi_I)|0 >$$

$$= F_{\rho\omega}\frac{1}{s_\rho} + \frac{1}{s_\rho}\Pi_{\rho\omega}\frac{1}{s_\omega} - F_{\omega\phi}\frac{1}{s_\rho}\Pi_{\rho\phi}\frac{1}{s_\phi} - F_{\rho\omega}\frac{1}{s_\omega} - F_{\rho\phi}\frac{1}{s_\phi}\Pi_{\phi\omega}\frac{1}{s_\omega} + \mathcal{O}(\varepsilon^2)$$

$$D_{\rho\phi} =< 0|T\rho\phi|0 >=< 0|T(\rho_I - F_{\rho\omega}\omega_I - F_{\rho\phi}\phi_I)(F_{\rho\phi}\rho_I + F_{\omega\phi}\omega_I + \phi_I)|0 >$$

$$= F_{\rho\phi}\frac{1}{s_\rho} + F_{\omega\phi}\frac{1}{s_\rho}\Pi_{\rho\omega}\frac{1}{s_\omega} + \frac{1}{s_\rho}\Pi_{\rho\phi}\frac{1}{s_\phi} - F_{\rho\omega}\frac{1}{s_\omega}\Pi_{\omega\phi}\frac{1}{s_\phi} - F_{\rho\phi}\frac{1}{s_\phi} + \mathcal{O}(\varepsilon^2)$$

$$D_{\omega\phi} =< 0|T\omega\phi|0 \geq< 0|T\big(\omega_I + F_{\rho\omega}\rho_I - F_{\omega\phi}\phi_I\big)\big(F_{\rho\phi}\rho_I + F_{\omega\phi}\omega_I + \phi_I\big)|0 >$$

$$= F_{\rho\omega}\frac{1}{s_\rho}\Pi_{\rho\omega}\frac{1}{s_\phi} + F_{\rho\phi}\frac{1}{s_\omega}\Pi_{\omega\rho}\frac{1}{s_\rho} + F_{\omega\phi}\frac{1}{s_\omega} + \frac{1}{s_\omega}\Pi_{\omega\phi}\frac{1}{s_\phi} - F_{\omega\phi}\frac{1}{s_\phi} + \mathcal{O}(\varepsilon^2)$$

$$\frac{1}{s_\rho}\Pi_{\rho\omega}\frac{1}{s_\omega} - F_{\omega\phi}\frac{1}{s_\rho}\Pi_{\rho\phi}\frac{1}{s_\phi} - F_{\rho\phi}\frac{1}{s_\phi}\Pi_{\phi\omega}\frac{1}{s_\omega} = F_{\rho\omega}(\frac{1}{s_\omega} - \frac{1}{s_\rho})$$

$$F_{\omega\phi}\frac{1}{s_\rho}\Pi_{\rho\omega}\frac{1}{s_\omega} + \frac{1}{s_\rho}\Pi_{\rho\phi}\frac{1}{s_\phi} - F_{\rho\omega}\frac{1}{s_\omega}\Pi_{\omega\phi}\frac{1}{s_\phi} = F_{\rho\phi}\left(\frac{1}{s_\phi} - \frac{1}{s_\rho}\right)$$

$$F_{\rho\omega}\frac{1}{s_\rho}\Pi_{\rho\omega}\frac{1}{s_\phi} + F_{\rho\phi}\frac{1}{s_\omega}\Pi_{\omega\rho}\frac{1}{s_\rho} + \frac{1}{s_\omega}\Pi_{\omega\phi}\frac{1}{s_\phi} = F_{\omega\phi}\left(\frac{1}{s_\phi} - \frac{1}{s_\omega}\right)$$

$$F_{\rho\omega} = \frac{\Pi_{\rho\omega}}{s_\rho - s_\omega} \qquad F_{\rho\phi} = \frac{\Pi_{\rho\phi}}{s_\rho - s_\phi} \qquad F_{\omega\phi} = \frac{\Pi_{\omega\phi}}{s_\omega - s_\phi}$$

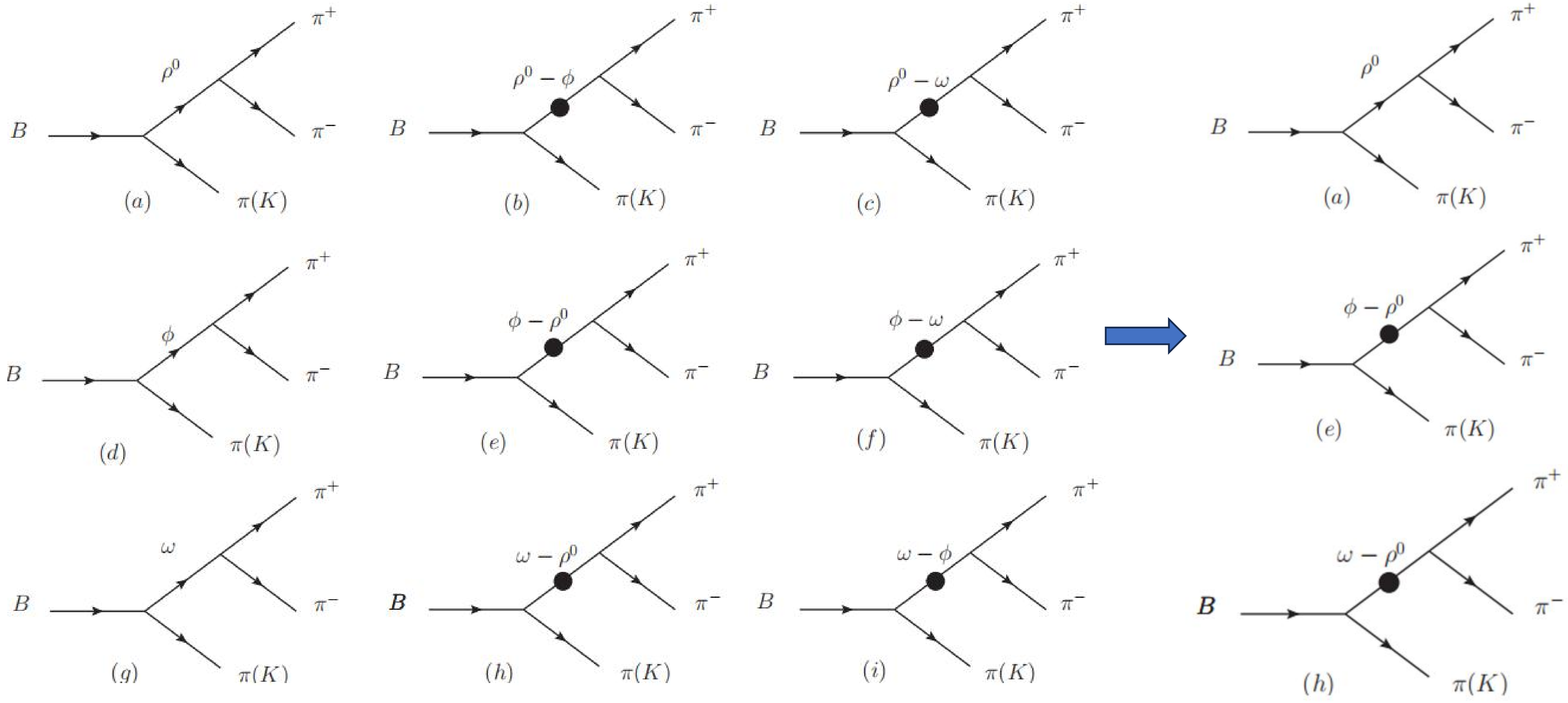
We can define the following mixing parameters:

$$\widetilde{\Pi}_{\rho\omega} = \frac{s_\rho\Pi_{\rho\omega}}{s_\rho - s_\omega} \qquad \widetilde{\Pi}_{\rho\phi} = \frac{s_\rho\Pi_{\rho\phi}}{s_\rho - s_\phi} \qquad \widetilde{\Pi}_{\omega\phi} = \frac{s_\omega\Pi_{\omega\phi}}{s_\omega - s_\phi}$$

3

Study on CP Violation in the Process of Three-Body Decay

Schematic diagram of the B→Vπ(K)→π+π−π(K) decay process



The amplitudes of the B→Vπ→π+π−π decay process in PQCD

$$
\begin{aligned}
A(a) = & A(B_u^- \to (\rho^0 \to \pi^+\pi^-)\pi^-) = \sum_{\lambda=0,\pm1} \frac{G_F P_{B_u^-} \cdot \epsilon^*(\lambda)\; g^{\rho^0\to\pi^+\pi^-}\, \epsilon(\lambda)\cdot(p_{\pi^+} - p_{\pi^-})}{2(s - m_{\rho^0}^2 + i m_{\rho^0}\Gamma_{\rho^0})} \times \Big\{ V_{ub} V_{ud}^* \big\{ a_1 \big[\mathcal{A}_{ab}^{LL}(\pi,\rho) \\
& + \mathcal{A}_{ef}^{LL}(\pi,\rho) - \mathcal{A}_{ef}^{LL}(\rho,\pi)\big] + a_2 \mathcal{A}_{ab}^{LL}(\rho,\pi) + C_2 \big[\mathcal{A}_{cd}^{LL}(\pi,\rho) + \mathcal{A}_{gh}^{LL}(\pi,\rho) - \mathcal{A}_{gh}^{LL}(\rho,\pi)\big] + C_1 \mathcal{A}_{cd}^{LL}(\rho,\pi)\big\} \\
& - V_{tb} V_{td}^* \Big\{ (a_4 + a_{10}) \big[\mathcal{A}_{ab}^{LL}(\pi,\rho) + \mathcal{A}_{ef}^{LL}(\pi,\rho) - \mathcal{A}_{ef}^{LL}(\rho,\pi)\big] + (a_6 + a_8) \big[\mathcal{A}_{ab}^{SP}(\pi,\rho) + \mathcal{A}_{ef}^{SP}(\pi,\rho) - \mathcal{A}_{ef}^{SP}(\rho,\pi)\big] \\
& - \left(a_4 - \frac{3}{2} a_7 - \frac{3}{2} a_9 - \frac{1}{2} a_{10}\right) \mathcal{A}_{ab}^{LL}(\rho,\pi) + (C_3 + C_9) \big[\mathcal{A}_{cd}^{LL}(\pi,\rho) + \mathcal{A}_{gh}^{LL}(\pi,\rho) - \mathcal{A}_{gh}^{LL}(\rho,\pi)\big] + (C_5 + C_7) \big[\mathcal{A}_{cd}^{SP}(\pi,\rho) \\
& + \mathcal{A}_{gh}^{SP}(\pi,\rho) - \mathcal{A}_{gh}^{SP}(\rho,\pi)\big] - \left(C_3 - \frac{3}{2} C_{10} - \frac{1}{2} C_9\right) \mathcal{A}_{cd}^{LL}(\rho,\pi) + \frac{3}{2} C_8 \mathcal{A}_{cd}^{LR}(\rho,\pi) - \left(C_5 - \frac{1}{2} C_7\right) \mathcal{A}_{cd}^{SP}(\rho,\pi) \Big\} \Big\}.
\end{aligned}
$$

$$
\begin{aligned}
A(e) = & A(B_u^- \to (\phi - \rho^0 \to \pi^+\pi^-)\pi^-) = \sum_{\lambda=0,\pm1} \frac{-G_F P_{B_u^-} \cdot \epsilon^*(\lambda)\; g^{\rho^0\to\pi^+\pi^-}\, \epsilon(\lambda)\cdot(p_{\pi^+} - p_{\pi^-})}{2(s - m_{\rho^0}^2 + i m_{\rho^0}\Gamma_{\rho^0})(s - m_\phi^2 + i m_\phi \Gamma_\phi)} \tilde{\Pi}_{\rho^0\phi} \\
& \times \Big\{ V_{tb} V_{td}^* \Big\{ \left(a_3 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9\right) \mathcal{A}_{ab}^{LL}(\phi,\pi) + \left(C_4 - \frac{1}{2} C_{10}\right) \mathcal{A}_{cd}^{LL}(\phi,\pi) + (C_6 - \frac{1}{2} C_8) \mathcal{A}_{cd}^{LR}(\phi,\pi) \Big\}.
\end{aligned}
$$

The amplitude of the B→Vπ→π+π−π decay process in PQCD

$$A(h) = A(B_u^- \to (\omega - \rho^0 \to \pi^+\pi^-)\pi^-) = \sum_{\lambda=0,\pm1} \frac{G_F P_{B_u^-} \cdot \epsilon^*(\lambda)\, g^{\rho^0\to\pi^+\pi^-} \epsilon(\lambda)\cdot(p_{\pi^+} - p_{\pi^-})}{2(s - m_{\rho^0}^2 + \mathrm{i} m_{\rho^0}\Gamma_{\rho^0})(s - m_\omega^2 + \mathrm{i} m_\omega \Gamma_\omega)} \tilde{\Pi}_{\rho^0\omega} \Big\{ V_{ub} V_{ud}^* \big\{ a_1 \big[\mathcal{A}_{ab}^{LL}(\pi,\omega)$$

$$+ \mathcal{A}_{ef}^{LL}(\pi,\omega) + \mathcal{A}_{ef}^{LL}(\omega,\pi)\big] + a_2 \mathcal{A}_{ab}^{LL}(\omega,\pi) + C_2 \big[\mathcal{A}_{cd}^{LL}(\pi,\omega) + \mathcal{A}_{gh}^{LL}(\pi,\omega) + \mathcal{A}_{gh}^{LL}(\omega,\pi)\big] + C_1 \mathcal{A}_{cd}^{LL}(\omega,\pi)\big\}$$

$$- V_{tb} V_{td}^* \Big\{ (a_4 + a_{10}) \big[\mathcal{A}_{ab}^{LL}(\pi,\omega) + \mathcal{A}_{ef}^{LL}(\pi,\omega) + \mathcal{A}_{ef}^{LL}(\omega,\pi)\big] + (a_6 + a_8) \big[\mathcal{A}_{ab}^{SP}(\pi,\omega) + \mathcal{A}_{ef}^{SP}(\pi,\omega) + \mathcal{A}_{ef}^{SP}(\omega,\pi)\big]$$

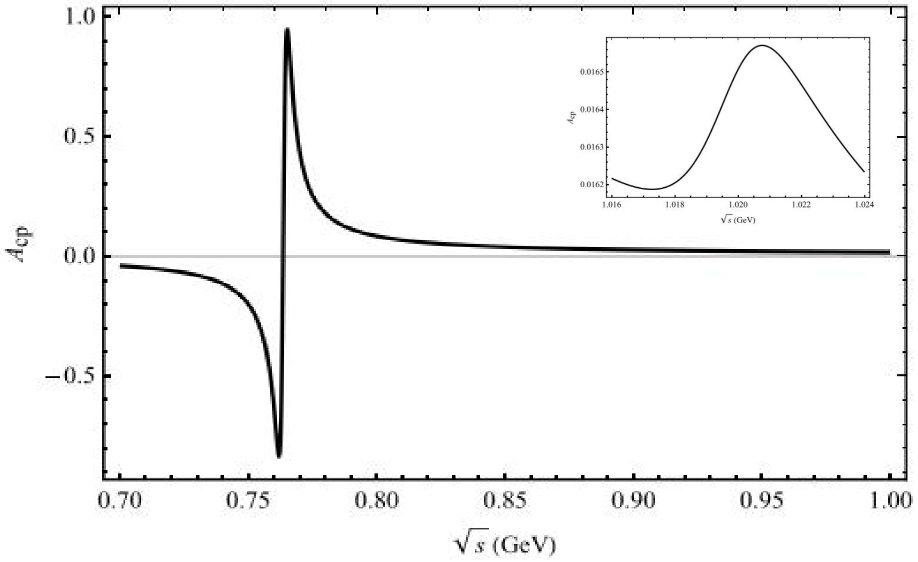
$$+ \left(2a_3 + a_4 + 2a_5 + \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10}\right) \mathcal{A}_{ab}^{LL}(\omega,\pi) + (C_3 + C_9) \big[\mathcal{A}_{cd}^{LL}(\pi,\omega) + \mathcal{A}_{gh}^{LL}(\pi,\omega) + \mathcal{A}_{gh}^{LL}(\omega,\pi)\big]$$

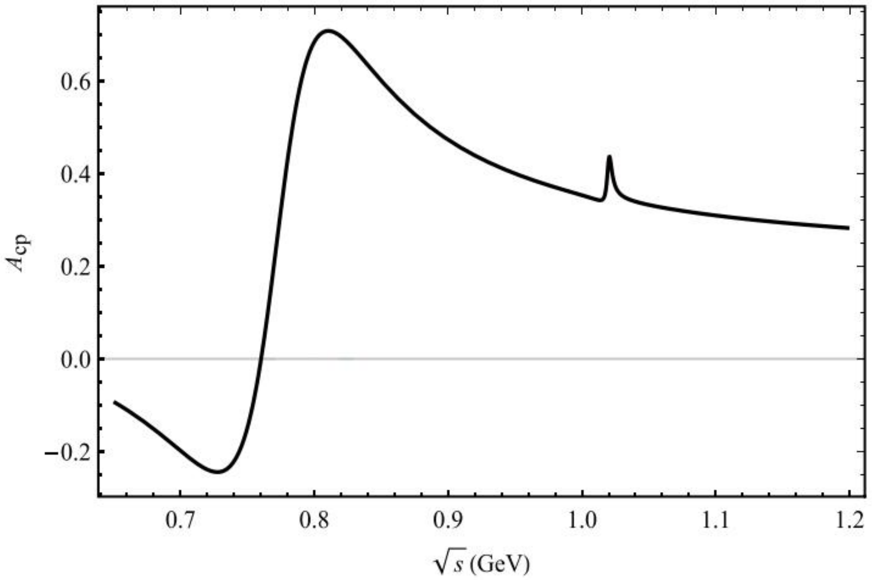
$$+ (C_5 + C_7) \big[\mathcal{A}_{cd}^{SP}(\pi,\omega) + \mathcal{A}_{gh}^{SP}(\pi,\omega) + \mathcal{A}_{gh}^{SP}(\omega,\pi)\big] + \left(C_3 + 2C_4 - \frac{1}{2}C_9 + \frac{1}{2}C_{10}\right) \mathcal{A}_{cd}^{LL}(\omega,\pi)$$

$$+ \left(2C_6 + \frac{1}{2}C_8\right) \mathcal{A}_{cd}^{LR}(\omega,\pi) + \left(C_5 - \frac{1}{2}C_7\right) \mathcal{A}_{cd}^{SP}(\omega,\pi)\Big\}\Big\}.$$

The CP violation plots of the B→Vπ(K)→π+π−π(K) decay process

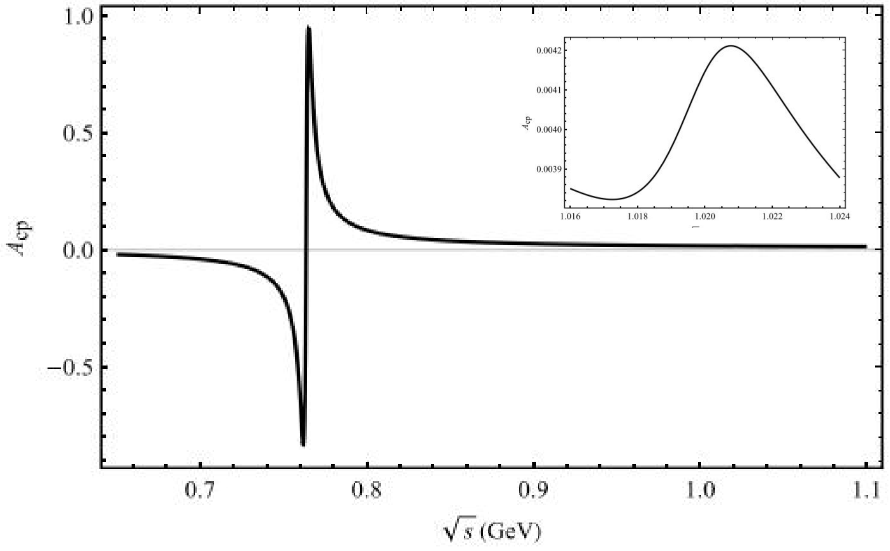
$B^- \to \pi^+\pi^-\pi^-$

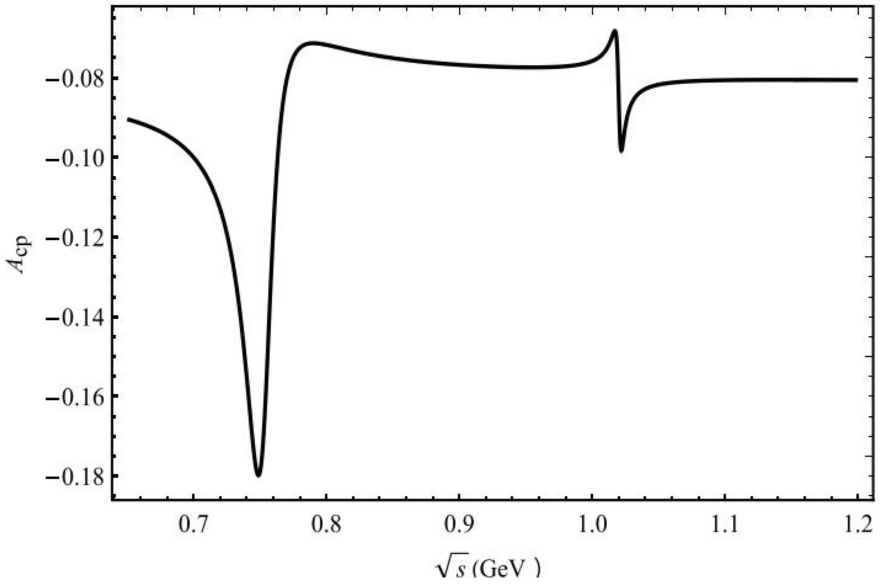




$B_u^- \to \pi^+\pi^- K^-$

$\bar{B}_d^0 \to \pi^+\pi^-\pi^0$





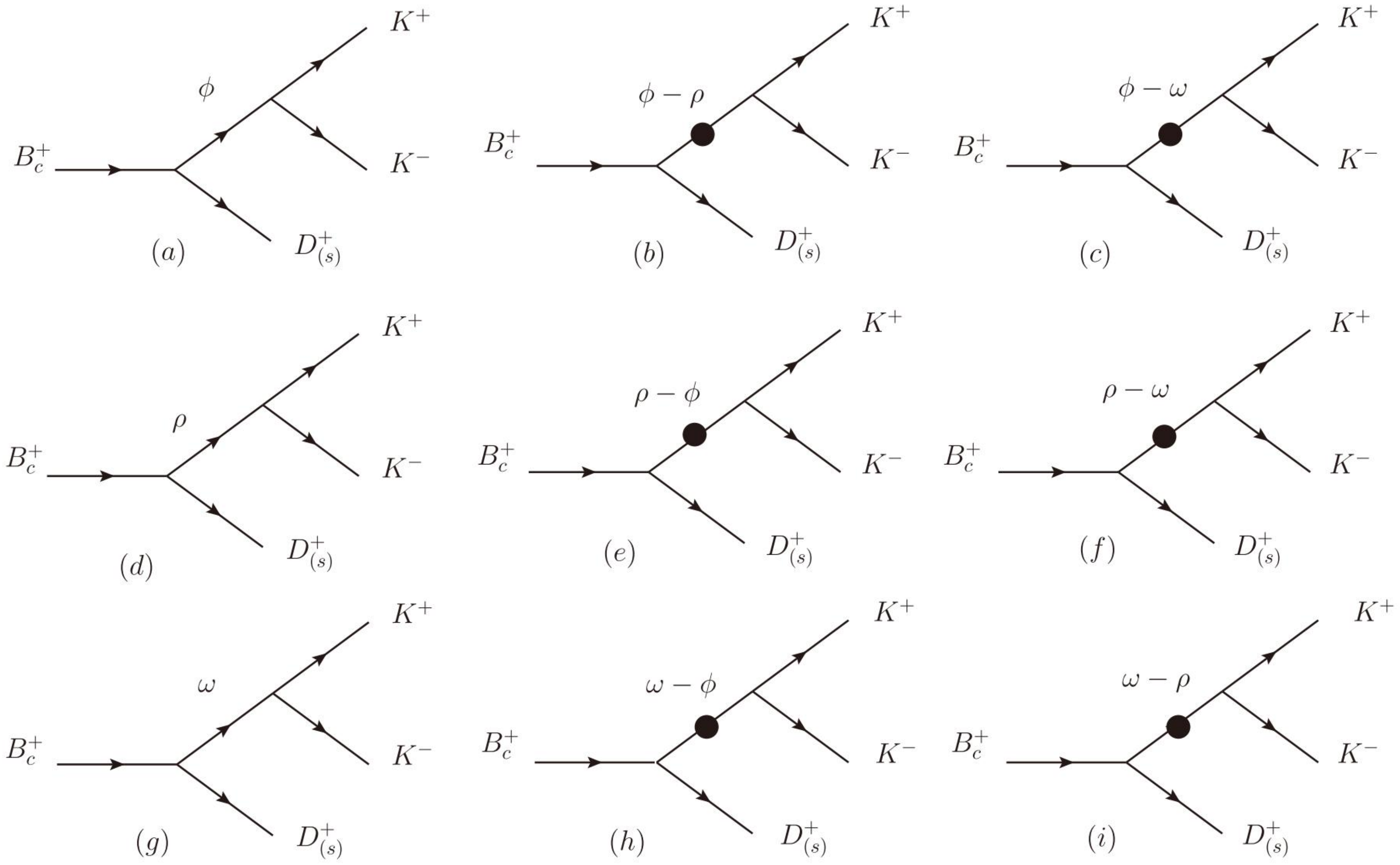
$\bar{B}_d^0 \to \pi^+\pi^- \bar{K}^0$

The results of CP violation for the B→Vπ(K)→π+π−π(K) decay process

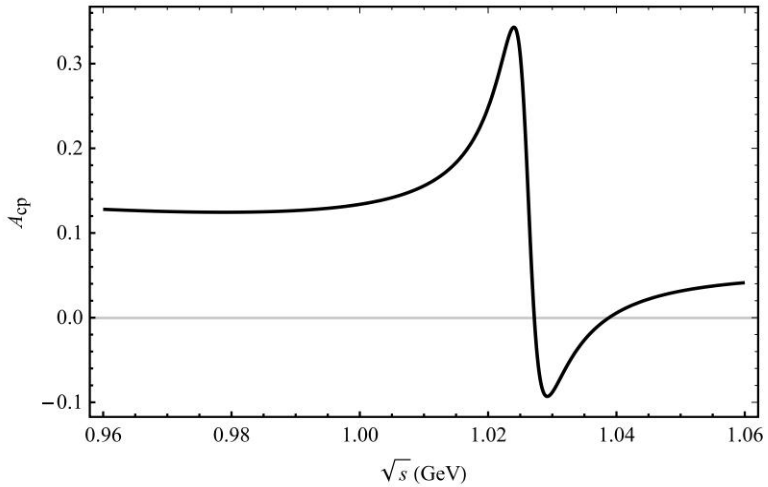
Decay channel	This work	Previous measurements(no mixing)
$\bar{B}_d^0 \to \pi^+\pi^-\bar{K}^0$	$-0.1683 \pm 0.0013 \pm 0.0000$ (ρ^0)	
	$-0.0847 \pm 0.0000 \pm 0.0084$ ($\rho^0 - \omega$ mixing)	----------------------------
	$-0.0813 \pm 0.0064 \pm 0.0052$ ($\rho^0 - \phi$ mixing)	
	$-0.0847 \pm 0.0056 \pm 0.0089$ ($\phi - \rho^0 - \omega$ mixing)	
$\bar{B}_d^0 \to \pi^+\pi^-\pi^0$	$-0.0054 \pm 0.0004 \pm 0.0011$ (ρ^0)	
	$0.0173 \pm 0.0109 \pm 0.0015$ ($\rho^0 - \omega$ mixing)	----------------------------
	$-0.0055 \pm 0.0006 \pm 0.0011$ ($\rho^0 - \phi$ mixing)	
	$0.0147 \pm 0.0014 \pm 0.0086$ ($\phi - \rho^0 - \omega$ mixing)	
$B_u^- \to \pi^+\pi^-K^-$	$0.2093 \pm 0.0206 \pm 0.0044$ (ρ^0)	0.150 ± 0.019 ± 0.011 LHCb [8]
	$0.1771 \pm 0.0084 \pm 0.0061$ ($\rho^0 - \omega$ mixing)	0.44 ± 0.10 ± 0.04 BaBar [42]
	$0.2109 \pm 0.0207 \pm 0.0023$ ($\rho^0 - \phi$ mixing)	0.30 ± 0.11 ± 0.02 Belle [43]
	$0.3470 \pm 0.0310 \pm 0.0709$ ($\phi - \rho^0 - \omega$ mixing)	
$B_u^- \to \pi^+\pi^-\pi^-$	$0.0065 \pm 0.0014 \pm 0.0031$ (ρ^0)	-0.004 ± 0.017 ± 0.009 LHCb [8]
	$0.0256 \pm 0.0013 \pm 0.0016$ ($\rho^0 - \omega$ mixing)	0.30 ± 0.11 ± 0.02 Belle [43]
	$0.0076 \pm 0.0006 \pm 0.0023$ ($\rho^0 - \phi$ mixing)	
	$0.0260 \pm 0.0034 \pm 0.0047$ ($\phi - \rho^0 - \omega$ mixing)	

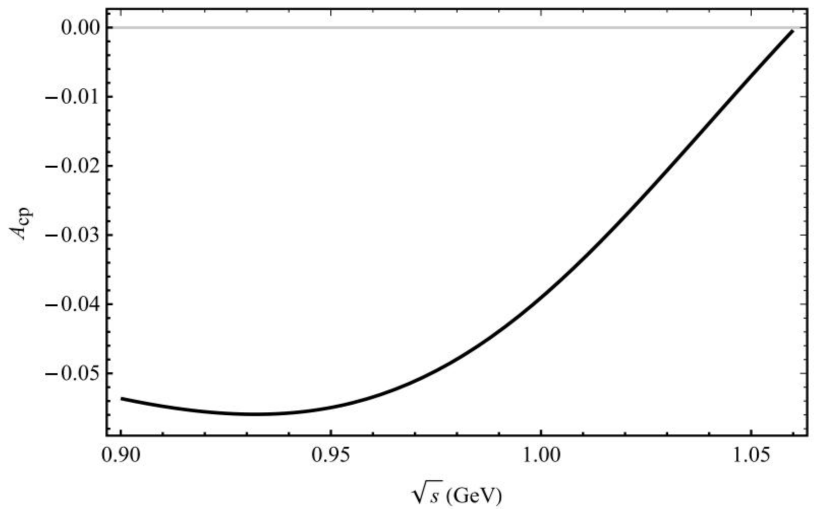
Phys.Rev. D **108**, 012013 (2023);Phys. Rev. Lett. 96, 251803(2006) ;Phys.Rev.D 78, 012004 (2008)

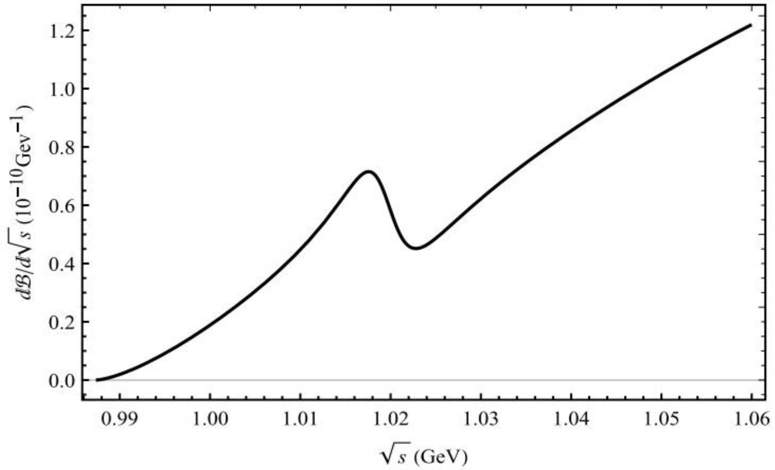
The CP violation for the Bc→K+K−($\pi^+\pi^-$)D(s)+ decay process

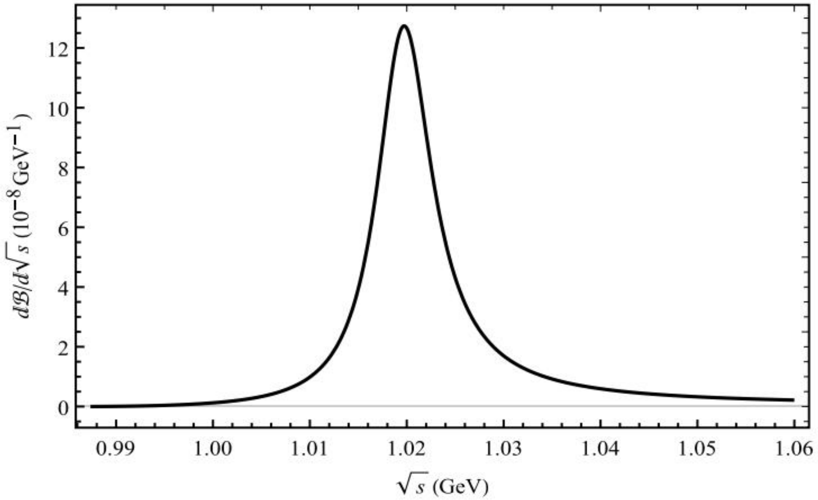


The CP violation and branching ratio for the Bc→K+K−D(s)+decay process









$B_c^+ \to D^+ V \to D^+ K^+ K^-$

$B_c^+ \to D_s^+ V \to D_s^+ K^+ K^-$

Table 1. Peak local integral ($0.65\ \mathrm{GeV} \le \sqrt{s} \le 1.06\ \mathrm{GeV}$ for the $\pi^+\pi^-$ final states and $0.98\ \mathrm{GeV} \le \sqrt{s} \le 1.06\ \mathrm{GeV}$ for the K^+K^- final states) of A_{CP}^{Ω} from different resonance ranges for $B_c^+ \to D_{(s)}^+\pi^+\pi^-$ and $B_c^+ \to D_{(s)}^+K^+K^-$ decay processes.

Decay channel	$B_c^+ \to D^+\pi^+\pi^-$	$B_c^+ \to D_s^+\pi^+\pi^-$	$B_c^+ \to D^+K^+K^-$	$B_c^+ \to D_s^+K^+K^-$
$\phi-\rho-\omega$ mixing	$0.0706^{+0.002-0.063}_{-0.002-0.091}$	$-0.0134^{+0.0005-0.0016}_{-0.0005+0.0006}$	$0.1636^{+0.006+0.046}_{-0.006-0.067}$	$-0.0278^{+0.001-0.0004}_{-0.001+0.0012}$
$\rho-\omega$ mixing	$0.0706^{+0.002-0.064}_{-0.002-0.091}$	$-0.0133^{+0.0005-0.0013}_{-0.0005+0.0012}$	$0.0906^{+0.006+0.074}_{-0.006-0.072}$	$-0.0267^{+0.001+0.014}_{-0.001-0.004}$
$\phi-\rho$ mixing	$-0.0176^{+0.0006+0}_{-0.0006-0}$	$-0.0156^{+0.0006-0.0005}_{-0.0006+0.00001}$	$0.0290^{+0.0005-0.045}_{-0.0005-0.041}$	$-0.0258^{+0.0009+0.0012}_{-0.0009+0.0008}$
$\phi-\omega$ mixing	–	–	$-0.0381^{+0.0006+0.012}_{-0.0006-0.0096}$	$-0.0264^{+0.00003-0.002}_{-0.00003-0.001}$
no mixing	$-0.0176^{+0.0006+0}_{-0.0006-0}$	$-0.0155^{+0.0006+0}_{-0.0006-0}$	$0.0784^{+0.001+0.091}_{-0.001+0.050}$	$-0.0252^{+0.0009+0.0015}_{-0.0009+0.0005}$

The branching ratio for the Bc→K+K−D(s)+decay process

Table 2. PQCD predictions of the branching ratios for quasi-two-body decays $B_c^+ \to D^+ V \to D^+ K^+ K^-$ and $B_c^+ \to D_s^+ V \to D_s^+ K^+ K^-$.

Decay channel	Branching ratio	Direct three-body decay/Mixed three-body decay
$B_c^+ \to D^+ \rho^0 \to D^+ K^+ K^-$	$1.66^{+0.15}_{-0.31} \times 10^{-11}$	35.37%
$B_c^+ \to D^+ \omega \to D^+ K^+ K^-$	$2.36^{+0.37}_{-0.37} \times 10^{-11}$	50.21%
$B_c^+ \to D^+ \phi \to D^+ K^+ K^-$	$4.37^{+0.20}_{-0.21} \times 10^{-11}$	92.98%
$B_c^+ \to D^+ \phi(\rho^0, \omega) \to D^+ K^+ K^-$	$4.70^{+0.53}_{-0.51} \times 10^{-11}$	1
$B_c^+ \to D_s^+ \rho^0 \to D_s^+ K^+ K^-$	$1.10^{+0.15}_{-0.14} \times 10^{-11}$	0.0083%
$B_c^+ \to D_s^+ \omega \to D_s^+ K^+ K^-$	$3.76^{+0.48}_{-0.47} \times 10^{-12}$	0.0028%
$B_c^+ \to D_s^+ \phi \to D_s^+ K^+ K^-$	$1.44^{+0.95}_{-0.24} \times 10^{-7}$	109.09%
$B_c^+ \to D_s^+ \phi(\rho^0, \omega) \to D_s^+ K^+ K^-$	$1.32^{+0.12}_{-0.12} \times 10^{-7}$	1

The branching ratio for the Bc→K+K−D(s)+decay process

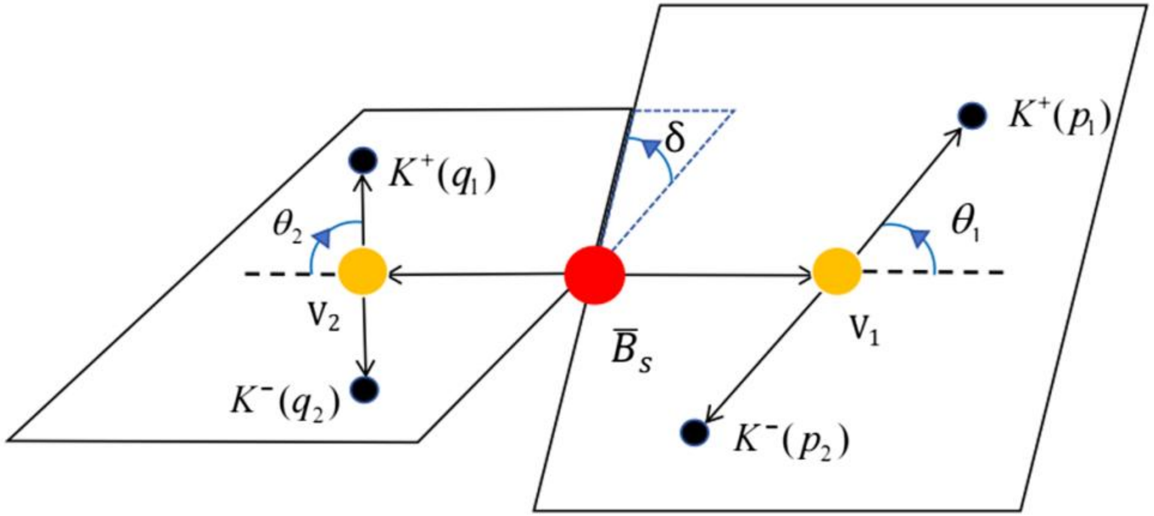
Table 3. Branching ratio results of $B_c^+ \to D^+ V \to D^+\pi^+\pi^-$ and $B_c^+ \to D_s^+ V \to D_s^+\pi^+\pi^-$. We also present the outcomes of quasi-two-body decay for comparison.

Decay channel	The three-particle admixture in this paper	Quasi-two-body
$B_c^+ \to D^+\rho^0 \to D^+\pi^+\pi^-$	$9.037^{+0.30}_{-0.21} \times 10^{-7}$	$6.61^{+1.64}_{-0.99} \times 10^{-7}$
$B_c^+ \to D_s^+\rho^0 \to D^+\pi^+\pi^-$	$4.55^{+0.33}_{-0.31} \times 10^{-8}$	$2.63^{+0.35}_{-0.35} \times 10^{-7}$

4

Study on CP Violation in Four-Body Decay Processes

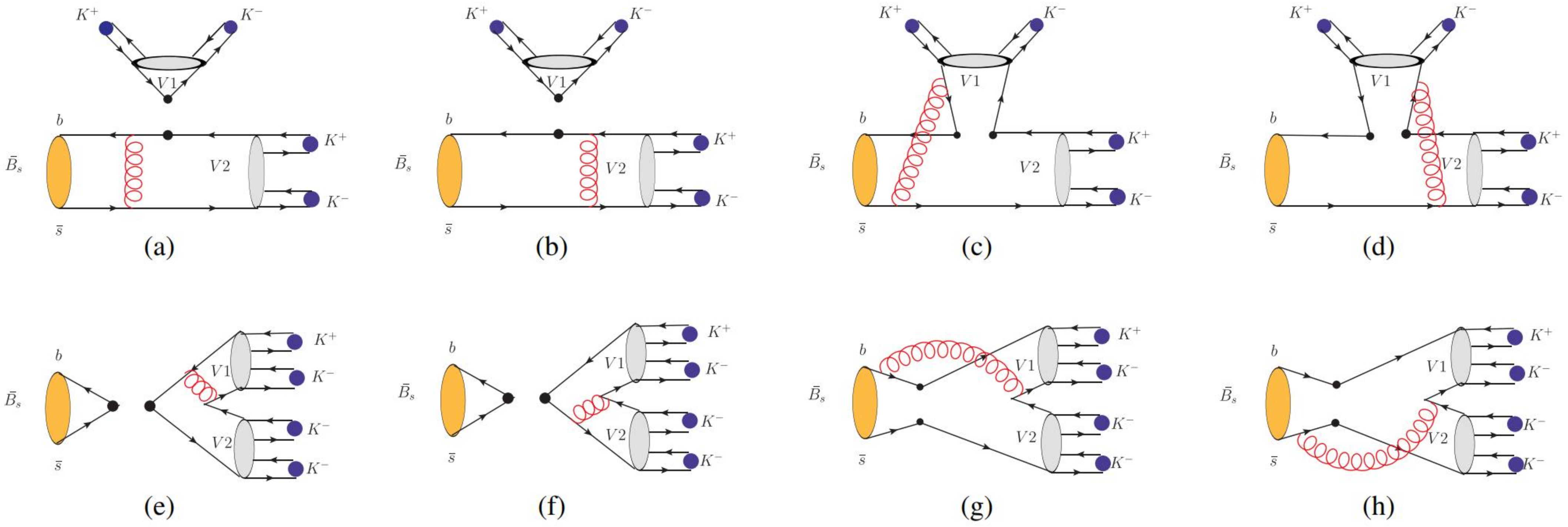
The kinematic diagram of the Bs→VV→K+K−K+K- decay process



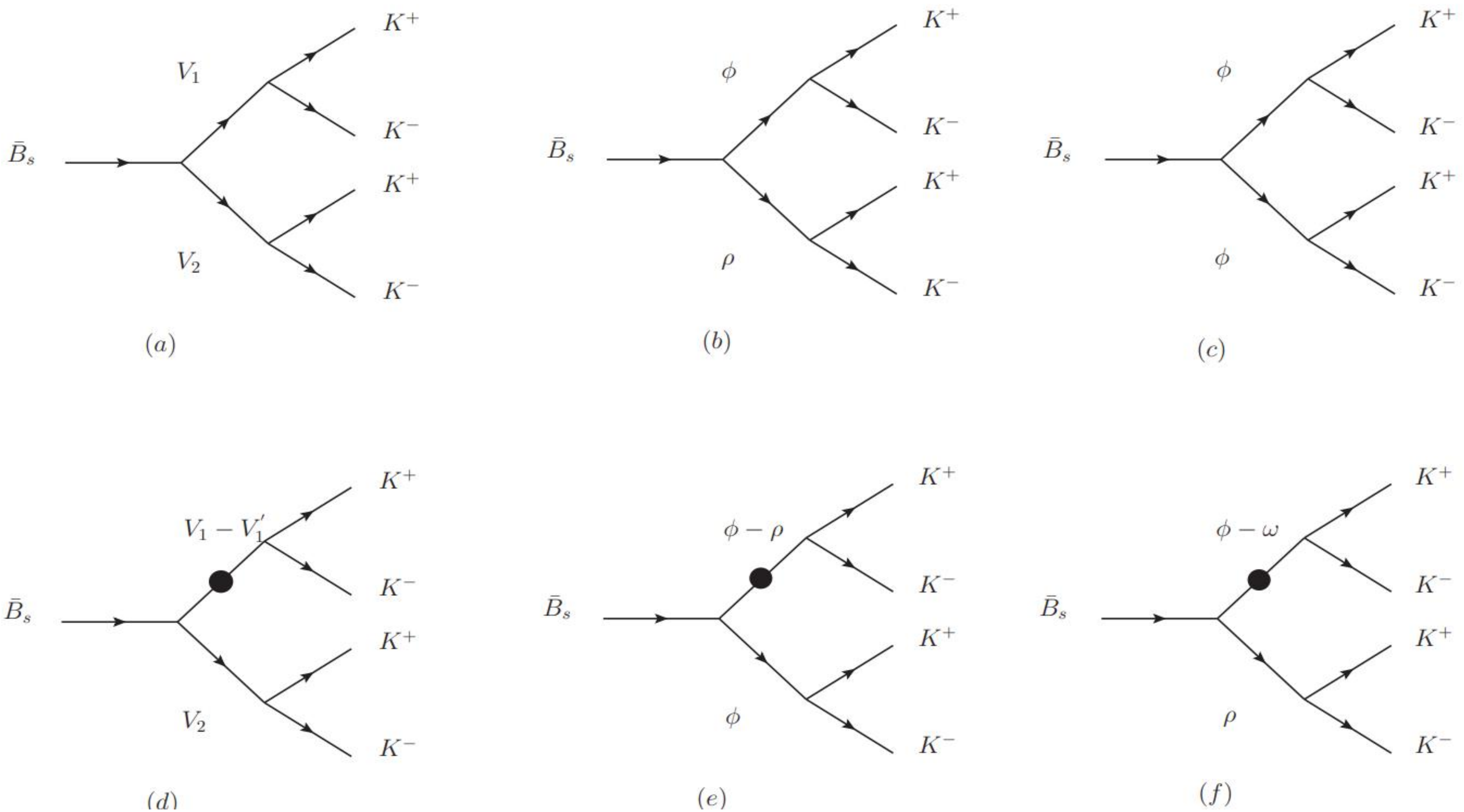
$$d\Gamma = \frac{|\mathcal{M}|^2}{4(4\pi)^6 m_B^3} X\beta_1\beta_2 ds_1 ds_2 d\cos\theta_1 d\cos\theta_2 d\delta$$

The expression $X = [(m_{\bar{B}_s}^2 - s_1 - s_2)^2/4 - s_1 s_2]^{1/2}$ is introduced, where $\beta_1 = \lambda^{1/2}(s_1, m_{p_1}^2, m_{p_2}^2)/s_1$ and $\beta_2 = \lambda^{1/2}(s_2, m_{q_1}^2, m_{q_2}^2)/s_2$, with $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$. Here, $|\mathcal{M}|^2$ represents the squared amplitude

Fenman diagram of the Bs→VV→K+K−K+K- decay process in PQCD



Schematic diagram of the Bs→VV→K+K−K+K- decay process



The form total amplitude of the B→VV→K+K−K+K- decay process

$$
\begin{aligned}
\langle K^+K^-K^+K^-|\mathcal{A}|\bar{B}_s\rangle = {} & \frac{g_\phi^2}{s_\phi^2}A_{\phi\phi} + \frac{2g_\rho g_\phi}{s_\rho s_\phi^2}\widetilde{\Pi}_{\rho\phi}A_{\phi\phi} + \frac{2g_\omega g_\phi}{s_\omega s_\phi^2}\widetilde{\Pi}_{\omega\phi}A_{\phi\phi} \\
& + \frac{g_\rho^2}{s_\rho^2}A_{\rho\rho} + \frac{2g_\phi g_\rho}{s_\phi s_\rho^2}\widetilde{\Pi}_{\phi\rho}A_{\rho\rho} + \frac{2g_\omega g_\rho}{s_\omega s_\rho^2}\widetilde{\Pi}_{\omega\rho}A_{\rho\rho} \\
& + \frac{g_\omega^2}{s_\omega^2}A_{\omega\omega} + \frac{2g_\phi g_\omega}{s_\phi s_\omega^2}\widetilde{\Pi}_{\phi\omega}A_{\omega\omega} + \frac{2g_\rho g_\omega}{s_\rho s_\omega^2}\widetilde{\Pi}_{\rho\omega}A_{\omega\omega} \\
& + \frac{g_\phi g_\rho}{s_\phi s_\rho}A_{\phi\rho} + \frac{g_\phi^2}{s_\phi^2 s_\rho}\widetilde{\Pi}_{\phi\rho}A_{\phi\rho} + \frac{g_\phi g_\omega}{s_\phi s_\omega s_\rho}\widetilde{\Pi}_{\omega\rho}A_{\phi\rho} + \frac{g_\rho^2}{s_\phi s_\rho^2}\widetilde{\Pi}_{\rho\phi}A_{\phi\rho} + \frac{g_\rho g_\omega}{s_\phi s_\rho s_\omega}\widetilde{\Pi}_{\omega\phi}A_{\phi\rho} \\
& + \frac{g_\phi g_\omega}{s_\phi s_\omega}A_{\phi\omega} + \frac{g_\phi^2}{s_\phi^2 s_\omega}\widetilde{\Pi}_{\phi\omega}A_{\phi\omega} + \frac{g_\phi g_\rho}{s_\phi s_\rho s_\omega}\widetilde{\Pi}_{\rho\omega}A_{\phi\omega} + \frac{g_\omega^2}{s_\phi s_\omega^2}\widetilde{\Pi}_{\omega\phi}A_{\phi\omega} + \frac{g_\omega g_\rho}{s_\phi s_\omega s_\rho}\widetilde{\Pi}_{\rho\phi}A_{\phi\omega} \\
& + \frac{g_\rho g_\omega}{s_\rho s_\omega}A_{\rho\omega} + \frac{g_\rho^2}{s_\rho^2 s_\omega}\widetilde{\Pi}_{\rho\omega}A_{\rho\omega} + \frac{g_\rho g_\phi}{s_\rho s_\omega s_\phi}\widetilde{\Pi}_{\phi\omega}A_{\rho\omega} + \frac{g_\omega^2}{s_\rho s_\omega^2}\widetilde{\Pi}_{\omega\rho}A_{\rho\omega} + \frac{g_\omega g_\phi}{s_\rho s_\omega s_\phi}\widetilde{\Pi}_{\phi\rho}A_{\rho\omega}
\end{aligned}
$$

The amplitude of the B→VV→K+K−K+K- decay process

$$\sqrt{2}A^i(\overline{B}_s^0 \to \phi(\phi \to K^+K^-)\phi(\phi \to K^+K^-)) =- \frac{2G_F g_\phi \epsilon(\lambda)\cdot(p_1 - p_2) g_\phi \epsilon(\gamma)\cdot(q_1 - q_2)}{\sqrt{2} s_\phi s_\phi}$$

$$\times V_{tb}V_{ts}^* \Big\{ f_\phi F_{B_s\to\phi}^{LL,i}\left[a_3 + a_4 + a_5 - \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10}\right] - f_{B_s} F_{ann}^{SP,i}\left[a_6 - \frac{1}{2}a_8\right]$$

$$+ M_{B_s\to\phi}^{LL,i}\left[C_3 + C_4 - \frac{1}{2}C_9 - \frac{1}{2}C_{10}\right] - M_{B_s\to\phi}^{LR,i}\left[C_5 - \frac{1}{2}C_7\right]$$

$$+ f_{B_s} F_{ann}^{LL,i}\left[a_3 + a_4 - \frac{1}{2}a_9 - \frac{1}{2}a_{10}\right] - M_{B_s\to\phi}^{SP,i}\left[C_6 - \frac{1}{2}C_8\right]$$

$$+ f_{B_s} F_{ann}^{LR,i}\left[a_5 - \frac{1}{2}a_7\right]$$

$$- M_{ann}^{LR,i}\left[C_5 - \frac{1}{2}C_7\right] + M_{ann}^{LL,i}\left[C_3 + C_4 - \frac{1}{2}C_9 - \frac{1}{2}C_{10}\right] + M_{ann}^{SP,i}\left[C_6 - \frac{1}{2}C_8\right]\Big\}$$

CP violation of the Bs→VV→K+K−K+K- decay process

TABLE I. The peak integrated values of A_{CP}^{Ω} for process $\bar{B}_s \to K^+K^-K^+K^-$ in different resonance ranges.

$\sqrt{s}$ (GeV)	0.99–1.05
$\bar{B}_s \to V_1(\rho-\phi-\omega)V_2(\rho-\phi-\omega) \to K^+K^-K^+K^-$	$0.0 \pm 0.0\%$
$\bar{B}_s \to V_1(\rho-\phi)V_2(\rho-\phi) \to K^+K^-K^+K^-$	$0.0 \pm 0.0\%$
$\bar{B}_s \to V_1(\omega-\phi)V_2(\omega-\phi) \to K^+K^-K^+K^-$	$0.0 \pm 0.0\%$
$\bar{B}_s \to V_1(\rho-\omega)V_2(\rho-\omega) \to K^+K^-K^+K^-$	$1.57 \pm 0.01 \pm 0.90\%$
$\bar{B}_s \to V_1V_2(no\,\phi\phi) \to K^+K^-K^+K^-$	$-21.47 \pm 0.61 \pm 5.70\%$
$\bar{B}_s \to V_1(\phi)V_2(\rho-\omega-\phi) \to K^+K^-K^+K^-$	$0.0 \pm 0.0\%$
$\bar{B}_s \to V_1(\phi)V_2(\rho-\omega) \to K^+K^-K^+K^-$	$-76.59 \pm 3.26 \pm 9.01\%$
$\bar{B}_s \to V_1(\phi)V_2(\rho-\phi) \to K^+K^-K^+K^-$	$0.0 \pm 0.0\%$
$\bar{B}_s \to V_1(\phi)V_2(\omega-\phi) \to K^+K^-K^+K^-$	$0.0 \pm 0.0\%$

CP violation of the Bs→VV→K+K−(π + π-)K+π- decay process

TABLE II: Direct A_{CP} values (in units of %) of $\bar{B}_s \to (K^+K^-)(K^+\pi^-)$ and $\bar{B}_s \to (\pi^+\pi^-)(K^+\pi^-)$ decay processes in different intermediate states. The errors of these items arise from the CKM matrix elements and the uncertainties of the input hadronic quantities.

Decay Modes	$\mathcal{A}_0^{\rm CP}$	$\mathcal{A}_{\parallel}^{\rm CP}$	$\mathcal{A}_{\perp}^{\rm CP}$	$\mathcal{A}^{\rm CP}$
$\bar{B}_s^0 \to \rho^0\,(\rho^0 \to K^+K^-)\,K^{*0}(\,K^{*0} \to K^+\pi^-)$	$63.57^{\pm 1.46}_{\pm 8.45}$	$71.01^{\pm 0.73}_{\pm 6.10}$	$25.15^{\pm 0.14}_{\pm 5.34}$	$53.24^{\pm 0.57}_{\pm 15.73}$
$\bar{B}_s^0 \to \omega\,(\omega \to K^+K^-)\,K^{*0}(\,K^{*0} \to K^+\pi^-)$	$-77.38^{\pm 1.97}_{\pm 7.92}$	$53.44^{\pm 0.92}_{\pm 5.23}$	$78.43^{\pm 1.34}_{\pm 8.11}$	$-60.31^{\pm 1.82}_{\pm 13.12}$
$\bar{B}_s^0 \to \phi\,(\phi \to K^+K^-)\,K^{*0}(\,K^{*0} \to K^+\pi^-)$	$\cdots$	$\cdots$	$\cdots$	0
$\bar{B}_s^0 \to \rho^0\,(\rho^0 \to \pi^+\pi^-)\,K^{*0}(\,K^{*0} \to K^+\pi^-)$	$63.58^{\pm 1.44}_{\pm 8.45}$	$71.01^{\pm 0.72}_{\pm 6.11}$	$25.14^{\pm 0.14}_{\pm 5.34}$	$53.24^{\pm 0.57}_{\pm 15.73}$
$\bar{B}_s^0 \to \omega\,(\omega \to \pi^+\pi^-)\,K^{*0}(\,K^{*0} \to K^+\pi^-)$	$-77.39^{\pm 1.98}_{\pm 7.92}$	$53.44^{\pm 0.92}_{\pm 5.23}$	$78.44^{\pm 1.35}_{\pm 8.11}$	$-60.31^{\pm 1.82}_{\pm 13.12}$
$\bar{B}_s^0 \to \phi\,(\phi \to \pi^+\pi^-)\,K^{*0}(\,K^{*0} \to K^+\pi^-)$	$\cdots$	$\cdots$	$\cdots$	0

CP violation of the Bs→VV→K+K−(π + π-)K+π- decay process

TABLE III: The peak regional integral of $A_{\rm CP}^{\Omega}$ from different resonance rangs for $\bar{B}_s \to (K^+K^-)(K^+\pi^-)$ decay processes. For the K^+K^- final state,the integration range is set to 0.99 - 1.20GeV. The source of the theoretical error is the same as that shown in Table II.

Decay channel	Different resonance effect	$\sqrt{s_1} = 0.99 - 1.20(GeV)$
$\bar{B}_s \to (K^+K^-)(K^+\pi^-)$	$\phi - \rho - \omega$	$-0.291 \pm 0.078 \pm 0.198$
	$\phi - \rho$	$-0.006 \pm 0.001 \pm 0.012$
	$\phi - \omega$	$-0.021 \pm 0.009 \pm 0.034$
	$\rho - \omega$	$0.016 \pm 0.029 \pm 0.154$

CP violation of the Bs→VV→π + π-K+π- decay process

TABLE IV: The peak regional integral of $A_{\rm CP}^{\Omega}$ from different resonance rangs for $\bar{B}_s \to (\pi^+\pi^-)(K^+\pi^-)$ decay processes. For the $\pi^+\pi^-$ final state,the integration range is set to 0.99 - 1.20GeV.

Decay channel	Different resonance effect	$\sqrt{s_1} = 0.70 - 1.10(GeV)$
$\bar{B}_s \to (\pi^+\pi^-)(K^+\pi^-)$	$\rho - \omega - \phi$	$-0.537 \pm 0.020 \pm 0.135$
	$\rho - \phi$	$-0.532 \pm 0.006 \pm 0.085$
	$\rho - \omega$	$-0.537 \pm 0.021 \pm 0.134$
	$\omega - \phi$	$-0.302 \pm 0.012 \pm 0.192$

The differential branching ratio of the Bs→VV→K+K−K+π- decay process

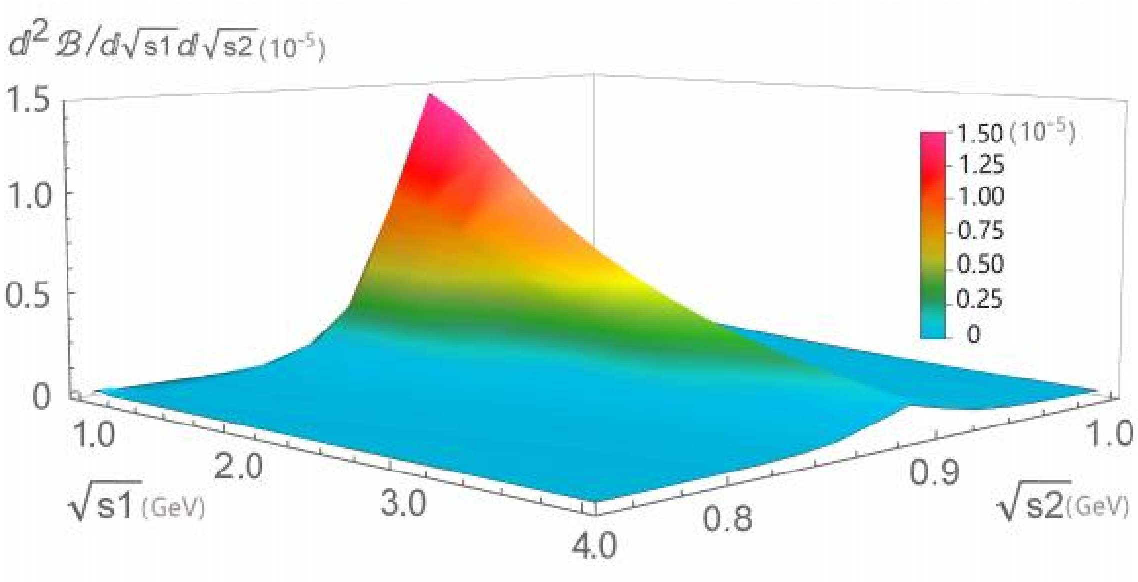


FIG. 1: The differential branching ratio plot for the quasi-two-body decay process $\bar{B}_s \to [\phi(\rho^0,\omega) \to K^+K^-][K^{*0} \to K^+\pi^-]$ .The numerical range corresponding to the blue-to-pink gradient area (0-1.50×10^{-5}) reflects the upward trend of these values.

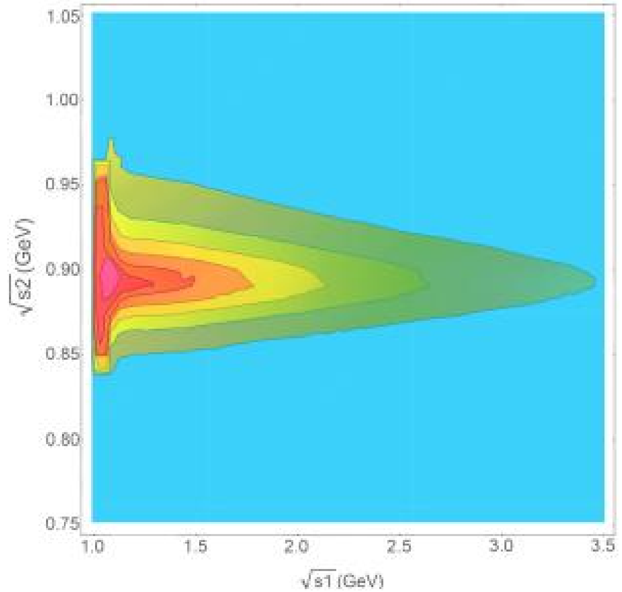


FIG. 2 : The plane mapping plot of the differential branching ratios for the decay process $\bar{B}_s \to [\phi(\rho^0,\omega) \to K^+K^-][K^{*0} \to K^+\pi^-]$. That is, the planar projection diagram shown in Fig. 1, and the color representation method is the same as that in Fig. 1.

The differential branching ratio of the Bs→VV→$\pi+\pi$-K+π- decay process

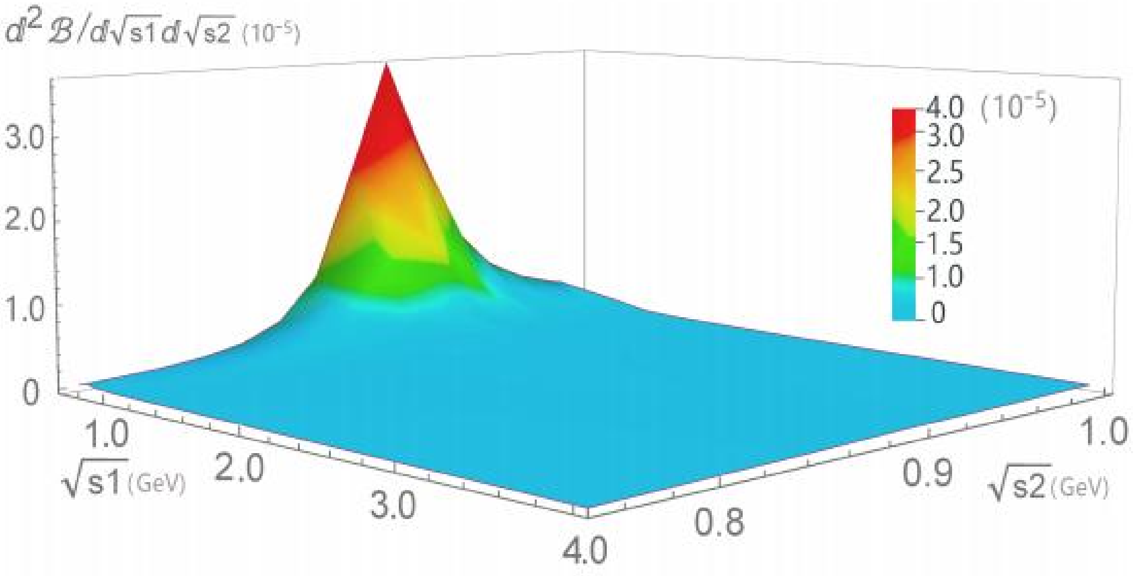


FIG. 3 : The differential branching ratio plot for the quasi-two-body decay process $\bar{B}_s \to [\rho^0(\omega,\phi) \to (\pi^+\pi^-)][K^{*0} \to K^+\pi^-]$ .The numerical range corresponding to the blue-to-pink gradient area (0-4.0×10^{-5}) reflects the upward trend of these values.

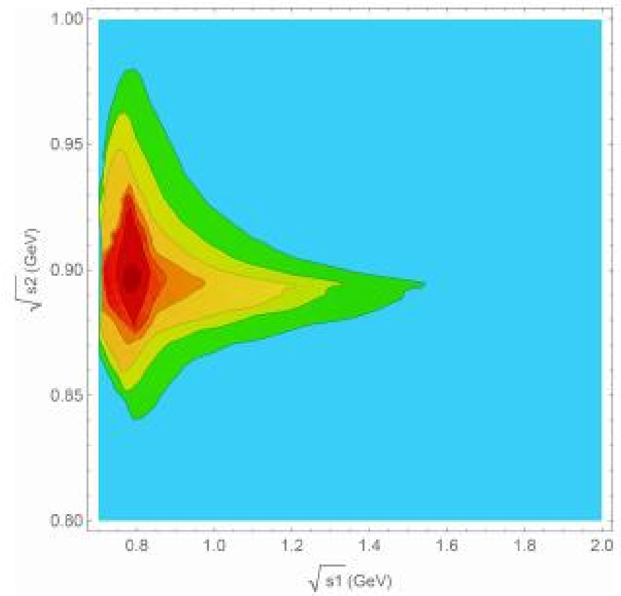


FIG. 4 : The plane mapping plot of the differential branching ratios for the decay process $\bar{B}_s \to [\rho^0(\omega,\phi) \to (\pi^+\pi^-)][K^{*0} \to K^+\pi^-]$. That is, the planar projection diagram shown in Fig. 3 , and the color representation method is the same as that in Fig. 3 .

Polarization fractions and branching ratio of the Bs→VV→K+K−(π + π-)K+π- decay process

TABLE V: Polarization fractions and branching ratio ($\times 10^{-6}$) of $\bar{B}_s \to (K^+K^-)(K^+\pi^-)$ and $\bar{B}_s \to (\pi^+\pi^-)(K^+\pi^-)$ decay processes. The source of the theoretical error is the same as that shown in Table 2.

Decay Channel	Modes	$f_0(\%)$	$f_\parallel(\%)$	$f_\perp(\%)$	$\mathcal{B}(10^{-6})$
$\bar{B}_s \to (K^+K^-)(K^+\pi^-)$	$\rho^0 K^{*0}$	$46.65^{\pm 0.32}_{\pm 9.31}$	$27.50^{\pm 0.02}_{\pm 2.65}$	$25.84^{\pm 0.30}_{\pm 2.35}$	$0.063^{\pm 0.006}_{\pm 0.039}$
	ωK^{*0}	$50.12^{\pm 2.99}_{\pm 8.66}$	$25.78^{\pm 1.57}_{\pm 3.84}$	$24.10^{\pm 1.43}_{\pm 6.82}$	$0.014^{\pm 0.002}_{\pm 0.015}$
	ϕK^{*0}	$78.22^{\pm 0.53}_{\pm 12.76}$	$11.54^{\pm 0.32}_{\pm 3.25}$	$10.23^{\pm 0.18}_{\pm 1.56}$	$1.49^{\pm 0.10}_{\pm 0.56}$
	$(\phi - \rho^0)K^{*0}$	$77.43^{\pm 0.02}_{\pm 4.53}$	$11.29^{\pm 0.01}_{\pm 1.45}$	$11.28^{\pm 0.01}_{\pm 1.16}$	$1.51^{\pm 0.07}_{\pm 0.52}$
	$(\phi - \omega)K^{*0}$	$76.54^{\pm 0.08}_{\pm 6.26}$	$12.18^{\pm 0.03}_{\pm 2.01}$	$11.28^{\pm 0.05}_{\pm 1.12}$	$1.28^{\pm 0.07}_{\pm 0.57}$
	$(\rho^0 - \omega)K^{*0}$	$59.25^{\pm 2.36}_{\pm 8.89}$	$25.27^{\pm 1.42}_{\pm 5.84}$	$15.48^{\pm 0.94}_{\pm 3.94}$	$0.092^{\pm 0.017}_{\pm 0.043}$
	$(\phi - \rho^0 - \omega)K^{*0}$	$75.90^{\pm 0.07}_{\pm 9.02}$	$12.68^{\pm 0.03}_{\pm 6.57}$	$11.41^{\pm 0.05}_{\pm 3.56}$	$1.28^{\pm 0.07}_{\pm 0.57}$
$\bar{B}_s \to (\pi^+\pi^-)(K^+\pi^-)$	$\rho^0 K^{*0}$	$46.64^{\pm 0.31}_{\pm 9.19}$	$27.51^{\pm 0.01}_{\pm 2.43}$	$28.84^{\pm 0.12}_{\pm 2.24}$	$0.25^{\pm 0.03}_{\pm 0.07}$
	$(\rho^0 - \omega)K^{*0}$	$39.65^{\pm 1.96}_{\pm 12.80}$	$36.03^{\pm 1.31}_{\pm 10.92}$	$24.32^{\pm 0.65}_{\pm 8.54}$	$0.38^{\pm 0.11}_{\pm 0.19}$
	$(\rho^0 - \phi)K^{*0}$	$46.65^{\pm 0.32}_{\pm 9.18}$	$27.50^{\pm 0.02}_{\pm 2.42}$	$25.85^{\pm 0.29}_{\pm 2.25}$	$0.25^{\pm 0.02}_{\pm 0.06}$
	$(\rho^0 - \omega - \phi)K^{*0}$	$39.66^{\pm 0.023}_{\pm 12.82}$	$36.03^{\pm 1.31}_{\pm 10.93}$	$24.31^{\pm 0.36}_{\pm 8.52}$	$0.58^{\pm 0.01}_{\pm 0.11}$

5

Summary

Summary

1、Within the PQCD framework, we investigated CP violation and decay branching ratios in the decay processes of $B \to \pi\pi\pi(K)$ and $Bc \to KK(\pi\pi)D_{(s)}$ induced by the $\rho - \omega - \phi$ mixing mechanism. Our results indicate that this mixing mechanism can significantly influence both CP violation and branching ratios.

2、Furthermore, within the same PQCD framework, we examined CP violation in the $Bs \to KKKK$ and $Bs \to K^+K^-(\pi^+\pi^-)K^+\pi^-$ decay process. We observed that larger CP violation effects can be generated under different combinations of vector mesons.