第八届全国重味物理与
量子色动力学研讨会

NNLO精度的共线碎裂函数

(Collinear Fragmentation Functions at NNLO)

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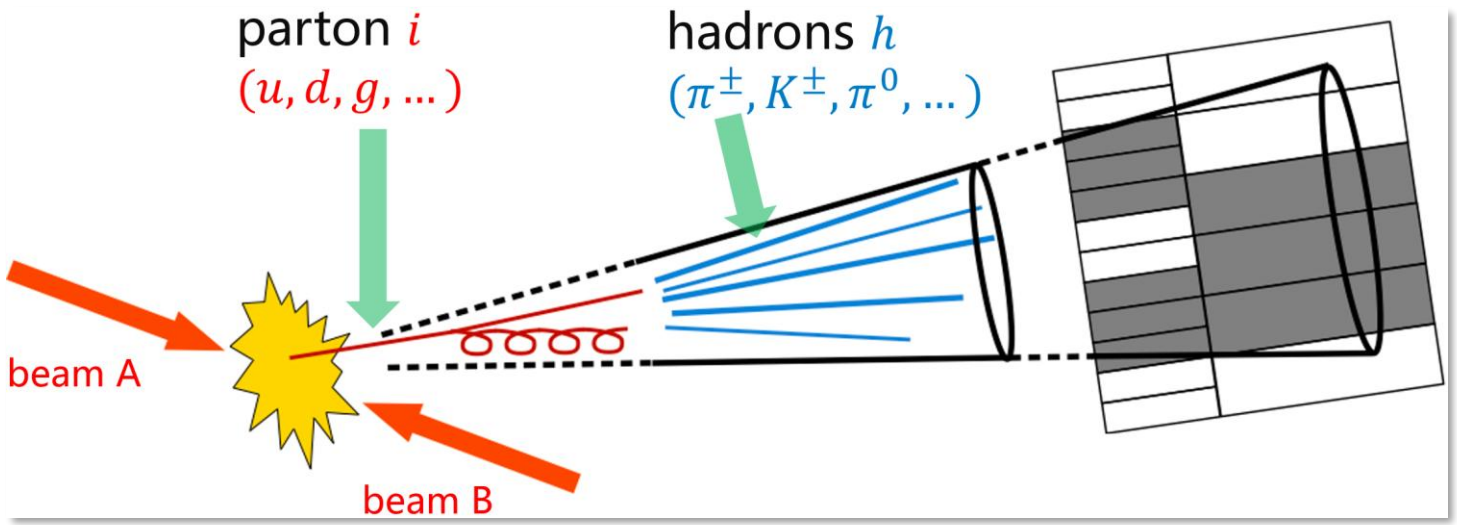
Based on: Jun Gao, **XS**, Hongxi Xing, Yuxiang Zhao, Bin Zhou (NPC Collaboration)
PRL 135, 041902, 2025

Outline

- Introduction
  - Fragmentation Functions (FFs) in parton model and QCD
  - Determination of collinear FFs
- NPC NNLO FFs analysis
  - Determination of NNLO FFs
  - Constraints on PDFs
- Summary

Fragmentation Functions in parton model



FF = number density of finding

- a specific hadron h
- with momentum fraction z "in" parton i

$$D_{h/i}\left(z=\frac{p_h^+}{p_i^+}\right) \longleftrightarrow f_{i/h}\left(x=\frac{p_i^+}{p_h^+}\right)$$

碎裂函数　　部分子分布函数

Fragmentation Functions (FFs) in QCD

Bjorken et al., '71
FFs proposed

Collins, Soper, '81
FFs def. in QCD

1969 **1971** **1977** **1981**

Feynman, '69
Parton model

Field, Feynman, '77
First determination

$$D_{h/q}(z) = \frac{z}{4}\sum_X \int \frac{\mathrm{d}\xi^+}{2\pi} e^{iP_h^-\xi^+/z}\,\mathrm{Tr}\Big[\langle 0\,|\,\mathcal{W}(\infty^+,\xi^+)\;\psi_q(\xi^+,0^-,\vec{0}_T)\,|\,P_h,S_h;X\rangle \times \langle P_h,S_h;X\,|\,\bar{\psi}_q(0^+,0^-,\vec{0}_T)\;\mathcal{W}(0^+,\infty^+)\,|\,0\rangle\;\gamma^-\Big]$$

$D(z,Q_0)$

$$\frac{\mathrm{d}D_{h/i}(z,Q)}{\mathrm{d}\ln^2 Q} = P_{ji}(y)\otimes D_{h/j}\left(\frac{z}{y},Q\right)$$

timelike DGLAP evolution

$D(z,Q)$

PDFs

Hadron

Parton distribution function describes the probability of finding a quark or gluon

Crossing Symmetry

$e^-+h\to e^-+X$

$e^-+e^+\to h+X$

FFs

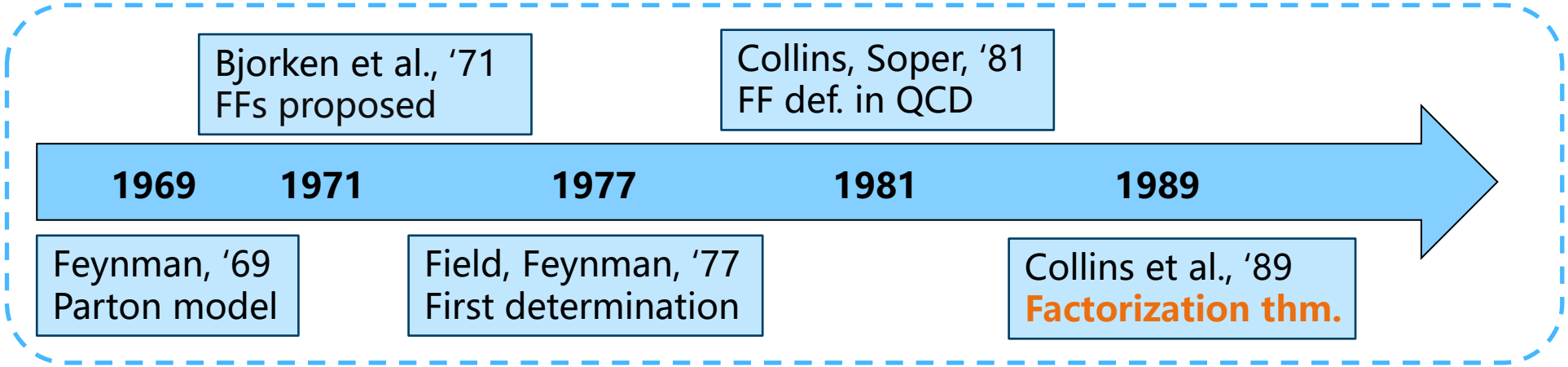
Parton

Fragmentation function describes the probability of producing a specific hadron.

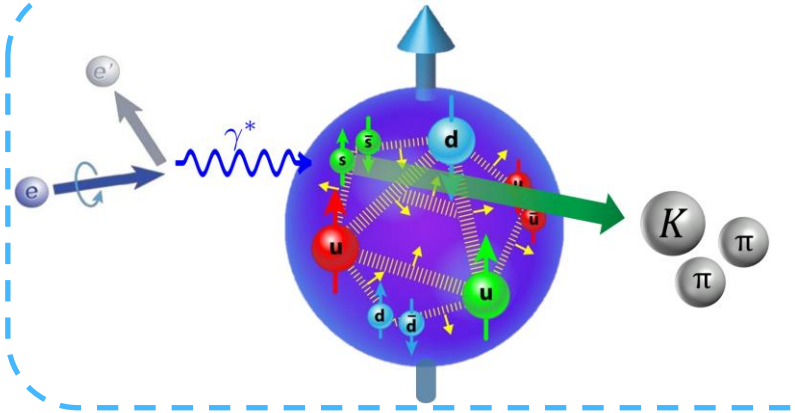
[Figure from PRL.134.111902]

Low Q High

FFs are key ingredients of QCD factorization framework



- Semi-Inclusive DIS (SIDIS) : e + N -> e + **h** + X

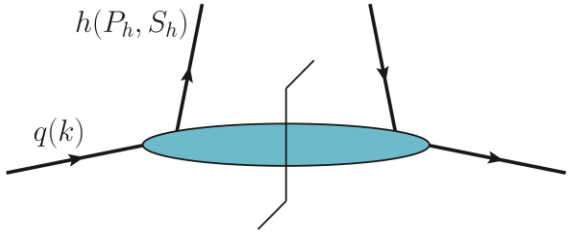


$$\frac{\mathrm{d}^3\sigma_h}{\mathrm{d}x_B\mathrm{d}y\mathrm{d}z_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x,y,z) \otimes D_{h/j}(z)$$

PDF pQCD **FF**

- $e^+e^- \to h + X$ (SIA) => σ = pQCD $\otimes$ **FF**
- $pp \to h + X$ => σ = PDF $\otimes$ PDF $\otimes$ pQCD $\otimes$ **FF**

Determination of FFs



$$D_{h/q}(z) = \frac{z}{4}\sum_X \int \frac{\mathrm{d}\xi^+}{2\pi} e^{iP_h^- \xi^+/z} \mathrm{Tr}\Big[\langle 0 \,|\, \mathcal{W}(\infty^+, \xi^+)\ \psi_q(\xi^+, 0^-, \vec{0}_T) \,|\, P_h, S_h; X\rangle$$
$$\times \langle P_h, S_h; X \,|\, \bar{\psi}_q(0^+, 0^-, \vec{0}_T)\ \mathcal{W}(0^+, \infty^+) \,|\, 0\rangle\ \gamma^-\Big]$$

- Quantum computers

  [Li, Xing, Zhang, 2406.05683]
  [Galvez-Viruet et al. 2510.18869]

- Models

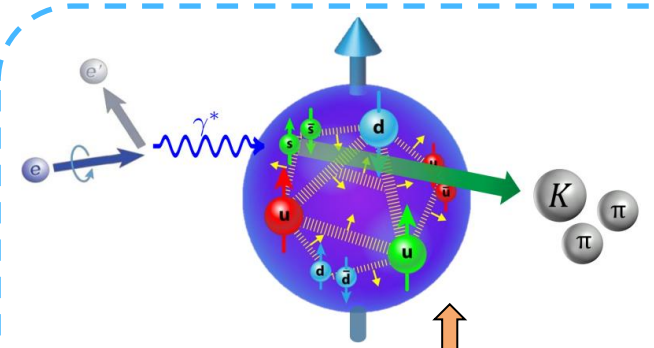
  [Jia, Mo, Xiong, 2310.17640]
  [Xing, Bian, Cui, Roberts 2504.08142]

- Basis light-front quantization?

- Lattice QCD: not yet

Determination of FFs: global data fits

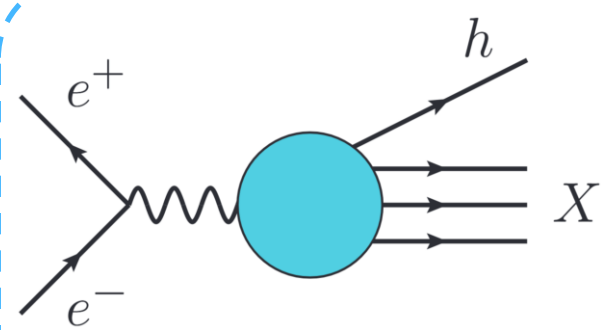
❖ **e p collision:**



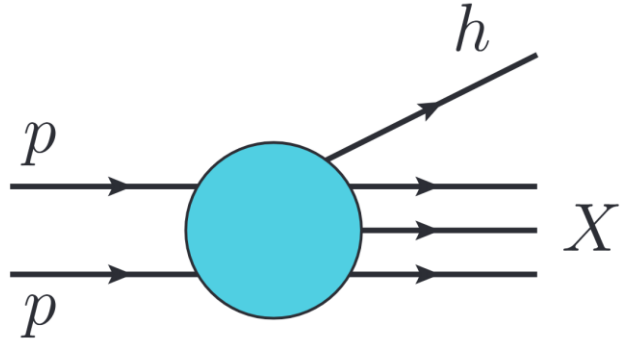
$$= f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x, y, z) \otimes D_{h/j}(z)$$

measurement　input　pQCD　to be determined.　Universal !

❖ **ee and pp**



σ = pQCD ⊗ **FF**



σ = PDF ⊗ PDF ⊗ pQCD ⊗ **FF**

Global analyses of collinear FFs to light hadrons

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	**NPC**
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
hadron in jet	✗	✗	✗	✗	✗	✓
FFs (charged h)	$\pi^{\pm},K^{\pm},p$	$\pi^{\pm},K^{\pm}$	$\pi^{\pm},K^{\pm},p$	$\pi^{\pm}$	$\pi^{\pm},K^{\pm}$	$\pi^{\pm},K^{\pm},p$
FFs (neutral h)			η			K^0,η,Λ
pQCD order	**NNLO**	**NLO**	**NLO**	**appr. NNLO**	**appr. NNLO**	**NLO**

- Predictions calculated by **FMNLO** [Gao, Liu, **XS**, Zhou, 2305.14620 (JHEP)]
- **NLO** analyses from Non-perturbative Physics Collaboration (NPC) [See also Jun Gao's talk on Saturday]
  - NPC23 FFs to light **charged** hadrons:

    Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903,* '24
    Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019,* '24 (Editors' suggestion)

  - NPC23 FFs to light **neutral** hadrons:

    Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045, '25*

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  - Determination of NNLO FFs
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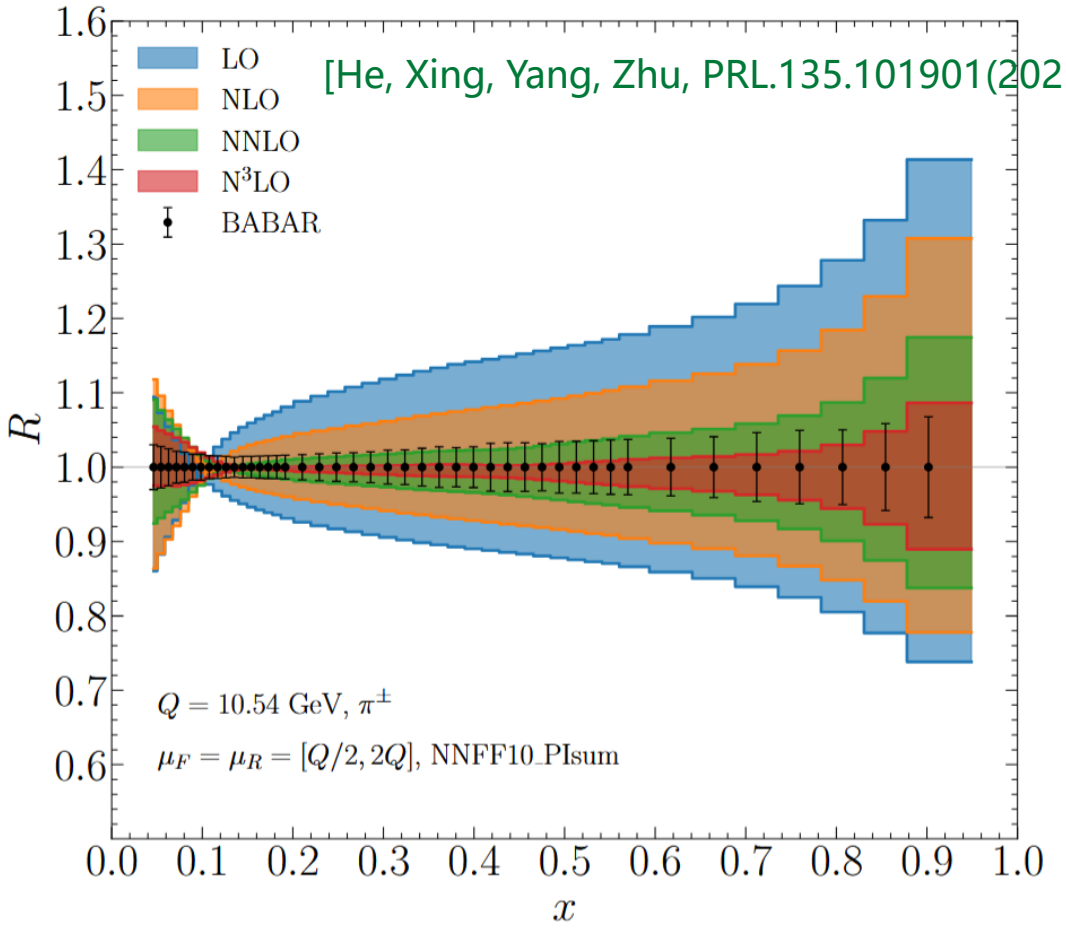
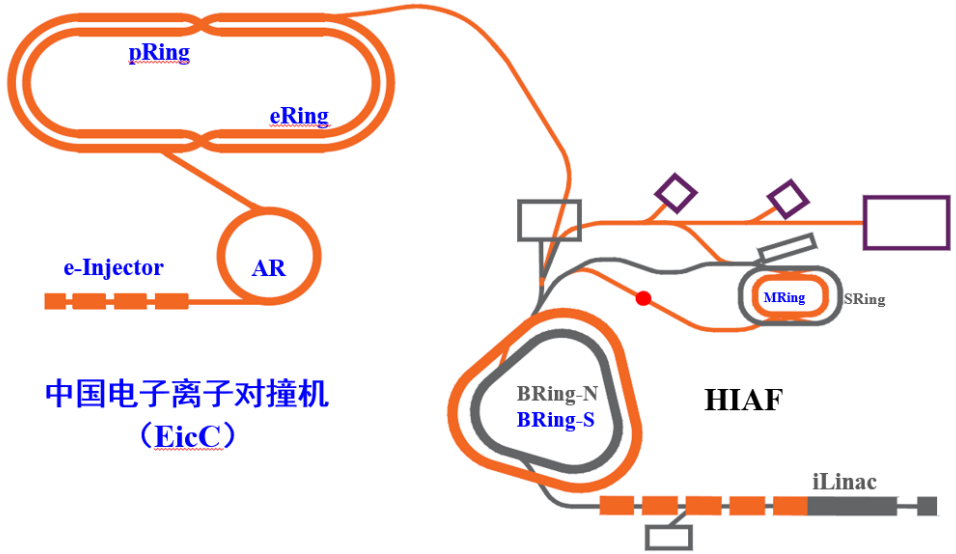
collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✘	✓	✓	✓	✓	✓	✓
pp incl. hadron	✘	✘	✓	✘	✘	✓	✘
hadron in jet	✘	✘	✘	✘	✘	✓	✘
FFs	$\pi^{\pm}$,$K^{\pm}$,p	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ η	$\pi^{\pm}$	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ K^0,η,Λ	$\pi^{\pm}$,$K^{\pm}$
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO	NNLO

[Gao, **XS**, Xing, Zhao, Zhou, ***PRL*** *135, 041902*, 2025]

Motivation to NNLO FF analyses



higher-order calculation is needed for low-energy data

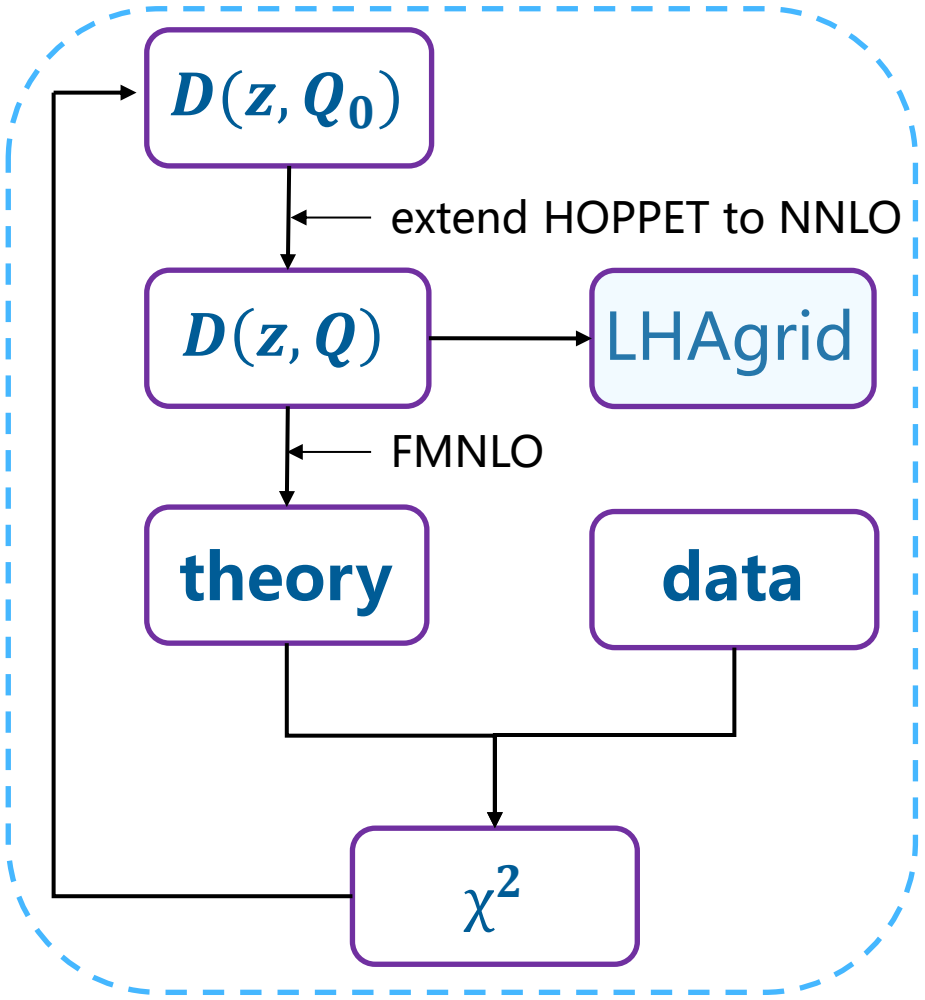


from Jiancheng Yang's talk at EICC2026

	HERA	EIC	EicC
峰值亮度 ($10^{33}cm^{-2}s^{-1}$)	0.05	10	6
积分亮度 (fb^{-1})	0.21	100	**150**
电子极化率	60%	70%	**80%**
质子极化率	无	70%	70%

EIC/EicC require high-precision FFs as input

The framework



- FFs at starting scale $Q_0 = 1.4$ GeV

$$zD_i^h(z, Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

+charge/isospin symmetries

- FFs at arbitrary energy scale Q

3-loop timelike splitting functions

[See Tongzhi's talk for 4-loop]

[Mitov, Moch, Vogt, Almasy]

[Chen, Yang, Zhu, Zhu, '20]

- SIA/SIDIS coefficient functions at NNLO

[Bonino+, '24], [Goyal+, '24]

[See Jun Gao's talk for N3LO]

The **first** global FF fit (ee+SIDIS) at full NNLO accuracy

The datasets

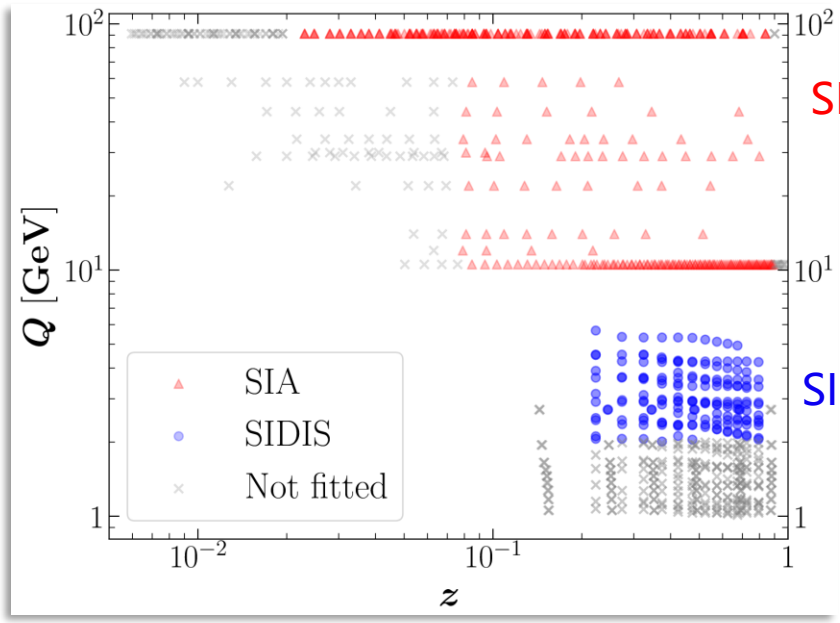
- SIA(e^+e^-) data used in the fit:

exp.	$\sqrt{s}$/GeV	lum.(n_Z)	year	final states	hadrons
DELPHI	189	157.7 pb^{-1}	2002	inc. had.	$\pi^{\pm}$,$K^{\pm}$
OPAL	m_Z	780 000	1994	$Z \to q\bar{q}$	$\pi^{\pm}$,$K^{\pm}$
ALEPH	m_Z	520 000	1995	$Z \to q\bar{q}$	$\pi^{\pm}$, $K^{\pm}$
DELPHI	m_Z	1 400 000	1998	$Z \to q\bar{q}$	$\pi^{\pm}$,$K^{\pm}$
				$Z \to b\bar{b}$	$\pi^{\pm}$,$K^{\pm}$
SLD	m_Z	400 000	2004	$Z \to q\bar{q}$	$\pi^{\pm}$,$K^{\pm}$
				$Z \to b\bar{b}$	$\pi^{\pm}$, $K^{\pm}$
				$Z \to c\bar{c}$	$\pi^{\pm}$,$K^{\pm}$
TASSO	44	34 pb^{-1}	1989	inc. had.	$\pi^{\pm}$, π^0
TASSO	34	77 pb^{-1}	1989	inc. had.	$\pi^{\pm}$,$K^{\pm}$
TPC/2γ	29	70 pb^{-1}	1988	inc. had.	$\pi^{\pm}$,$K^{\pm}$
Belle	10.52	68 fb^{-1}	2013	inc. had.	$\pi^{\pm}$, $K^{\pm}$
BaBar	10.54	0.91 fb^{-1}	2013	inc. had.	$\pi^{\pm}$,$K^{\pm}$
BESIII	2.0-3.671	253 pb^{-1}	2025	inc. had.	$\pi^{\pm}$, $K^{\pm}$

[BESIII, **PRL**135, 151901, 2025]

[See Wenbiao Yan's talk (EICC2026)]

- **separated** kinematic region of e^+e^- and ep data



Datasets used by MAP group

Khalek,Bertone,Nocera PRD.104.034007, '21

- This work uses both ee and ep data at Q~3GeV

The **first** test on **universality of FFs** at Q~3 GeV (using **ee +ep**)

The results

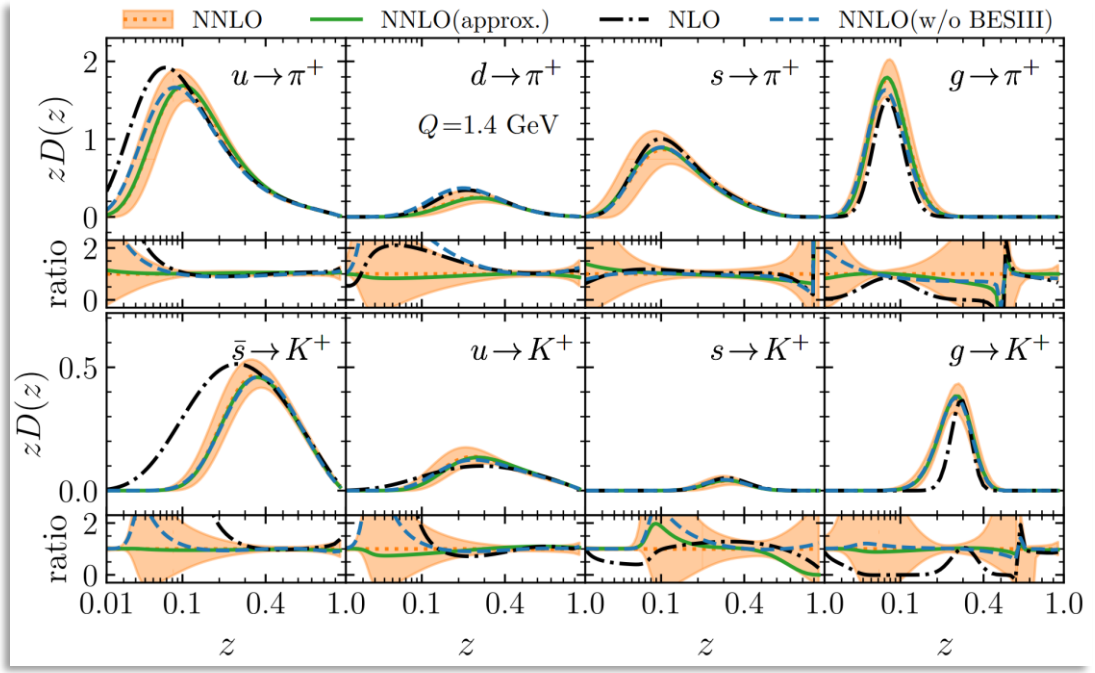
① our FFs describe **both ee and ep** data well

[Gao, **XS**, Xing, Zhao, Zhou, ***PRL 135, 041902***, 2025]

② test of **leading-twist** factorization

	BESIII		COMPASS		B-factories		HE-SIA		global		
$E_{h,\min}$[GeV]	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	χ^2	$\chi^2/N_{\rm pt}$
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77

energy cut of the identified hadron



③ This work: $\chi^2_{\text{full NNLO}} < \chi^2_{\text{NLO}}$

Previous studies: $\chi^2_{\text{appr.NNLO}} > \chi^2_{\text{NLO}}$

[PRL.129.012002, PLB.2022.137456]

LHAgrids of our FFs have been submitted to the LHAPDF repository.

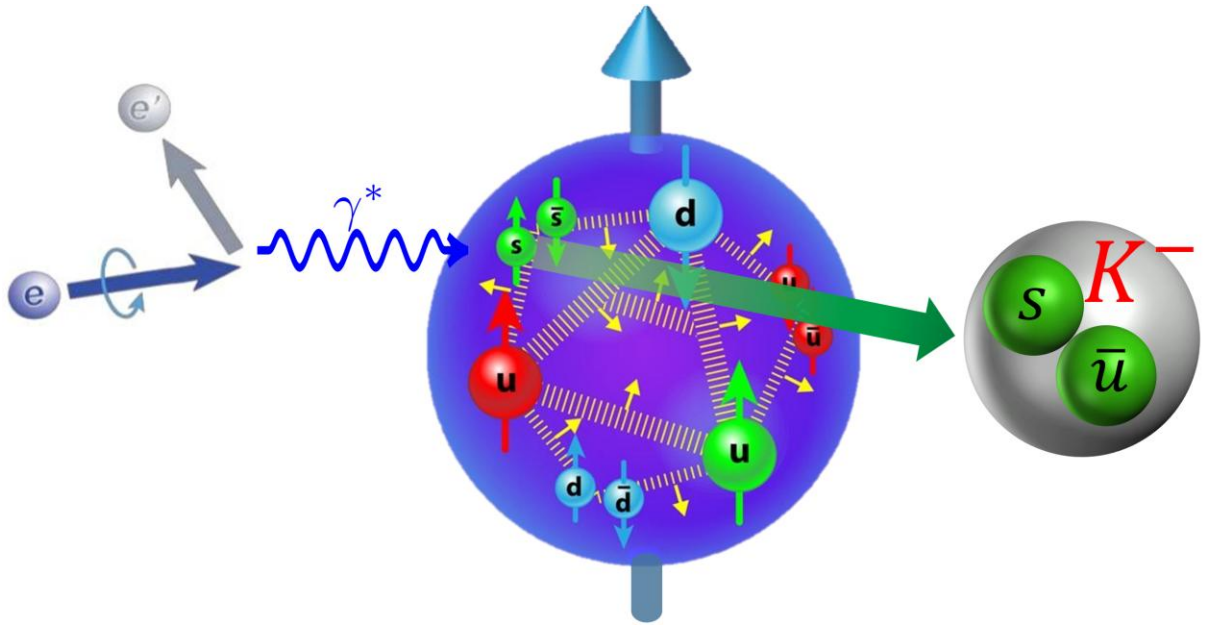
https://www.lhapdf.org/pdfsets.html

Application: constraining proton PDFs at NNLO

SIDIS may also constrain PDFs:

$$\frac{\mathrm{d}^3\sigma_h}{\mathrm{d}x_B\mathrm{d}y\mathrm{d}z_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x,y,z) \otimes D_{h/j}(z)$$

unpolarized PDF　　FF



- LO xsec of SIDIS off an isoscalar target (COMPASS)

$$\frac{\mathrm{d}\sigma^{K^+}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} - \frac{\mathrm{d}\sigma^{K^-}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z}$$

$$\sim 2\Big(u_v(x)+d_v(x)\Big)\Big(D_u^{K^+}(z) - D_{\bar{u}}^{K^+}(z)\Big)$$

$$+\Big(s(x)-\bar{s}(x)\Big)\Big(D_s^{K^+}(z) - D_{\bar{s}}^{K^+}(z)\Big) + \cdots$$

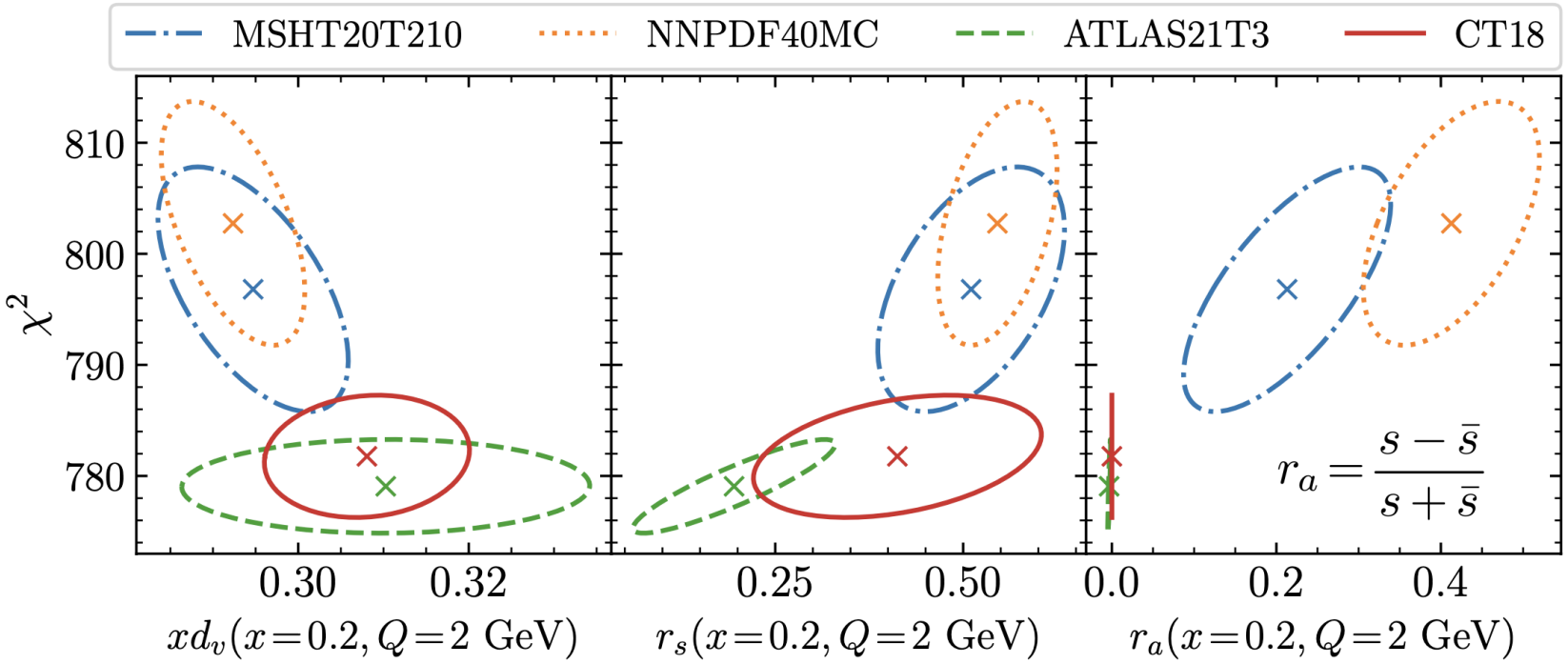
PDF　　FF

is sensitive to **strangeness asymmetry**

$$r_a = \frac{s-\bar{s}}{s+\bar{s}}$$

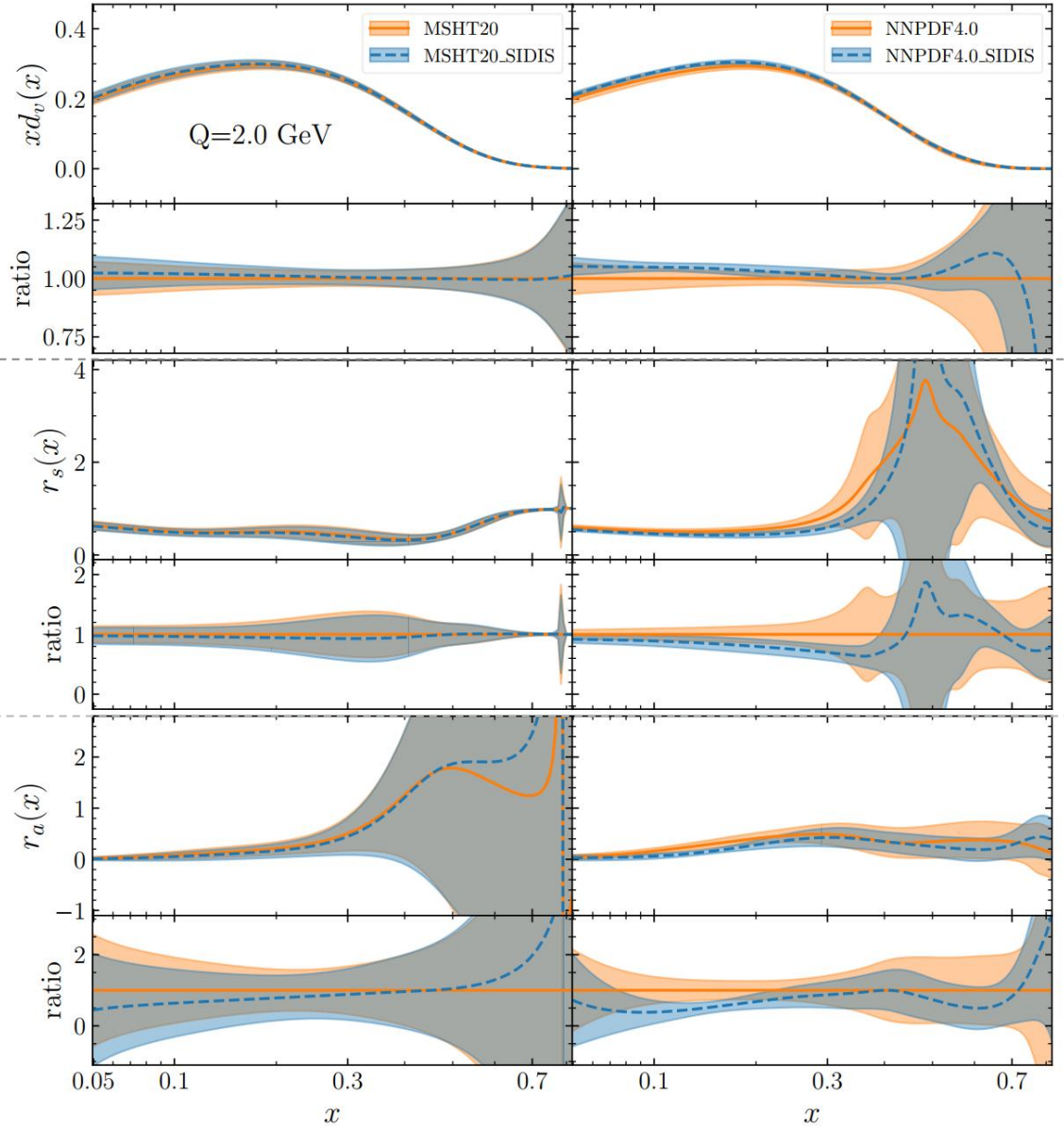
Application: constraining proton PDFs at NNLO

- Correlation between χ^2 and PDFs



- We generate **modified PDFs** which reflect the impact of SIDIS data
  - Reweighting of the NNPDF4.0 PDF set
  - Profiling of the MSHT20 PDF set

Application: PDF sets before and after reweighting/profiling



[Gao, **XS**, Xing, Zhao, Zhou, ***PRL 135, 041902***, 2025]

$$d_v := d(x,Q) - \bar{d}(x,Q)$$

$$r_s := \frac{s(x,Q)+\bar{s}(x,Q)}{\bar{u}(x,Q)+\bar{d}(x,Q)}$$

$$r_a := \frac{s(x,Q)-\bar{s}(x,Q)}{s(x,Q)+\bar{s}(x,Q)}$$

Summary

- FFs are key inputs for calculations of hadron production rate from first principles.
- We present the first global (ee+SIDIS) FFs determination at full **NNLO**.
- Our FFs are in good agreement with low-Q data from **both ee and SIDIS**.
- We find a preference for a reduced asymmetry in the strange (anti-)quark PDFs.

NPC FF sets		
SIA (ee)	✓	✓
SIDIS (ep)	✓	✓
pp incl. hadron	✓	
hadron in jet	✓	
hadrons	$\pi^{\pm}, K^{\pm}, p$ K^0, η, Λ	$\pi^{\pm}, K^{\pm}$
pQCD order	**NLO**	**NNLO**
$Q_{\min}$	4GeV	1.3GeV

NPC FF sets are available at https://www.lhapdf.org/pdfsets.html

NLO charged hadron:
Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903*, 2024
Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019*, 2024
NLO neutral hadron:
Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045*, 2025
NNLO:
Gao, **XS**, Xing, Zhao, Zhou, ***PRL*** *135, 041902*, 2025

Summary

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- Our FFs are in good agreement with low-Q data from **both ee and SIDIS**.
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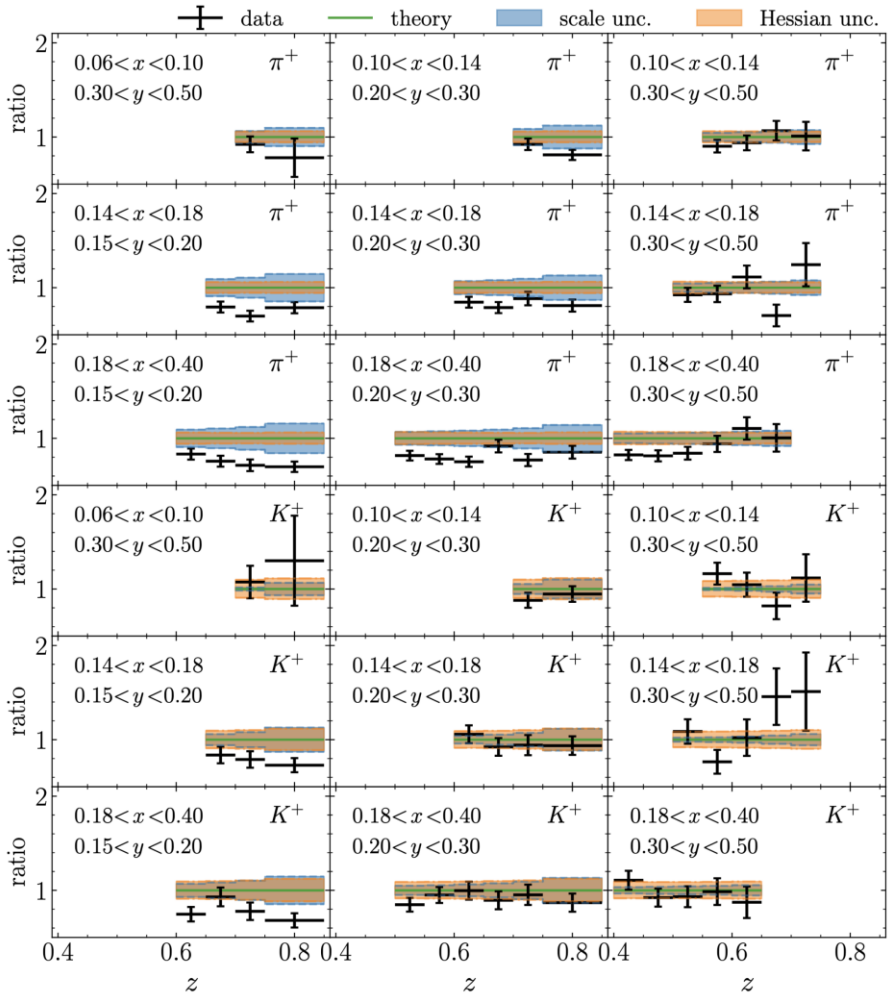
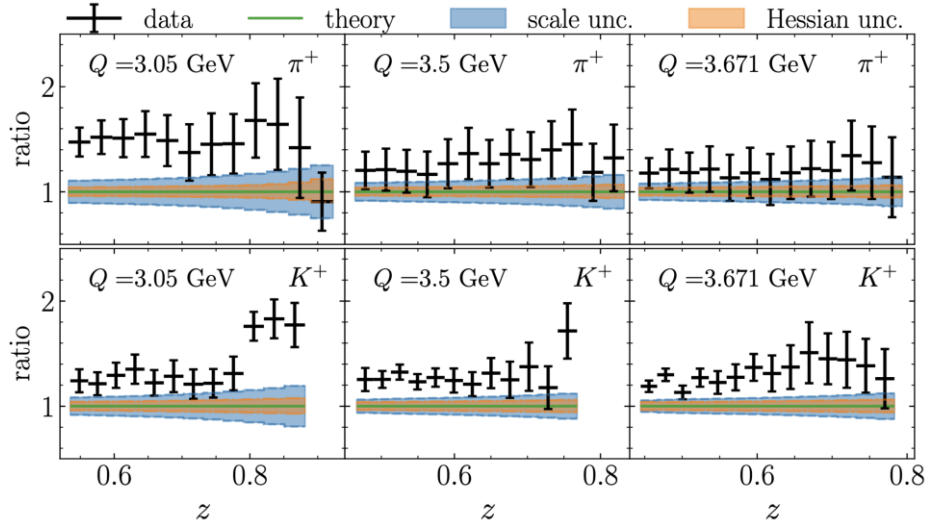
Thank you!

Backup

Theory v.s. data for COMPASS06 (SIDIS: 2~5GeV)

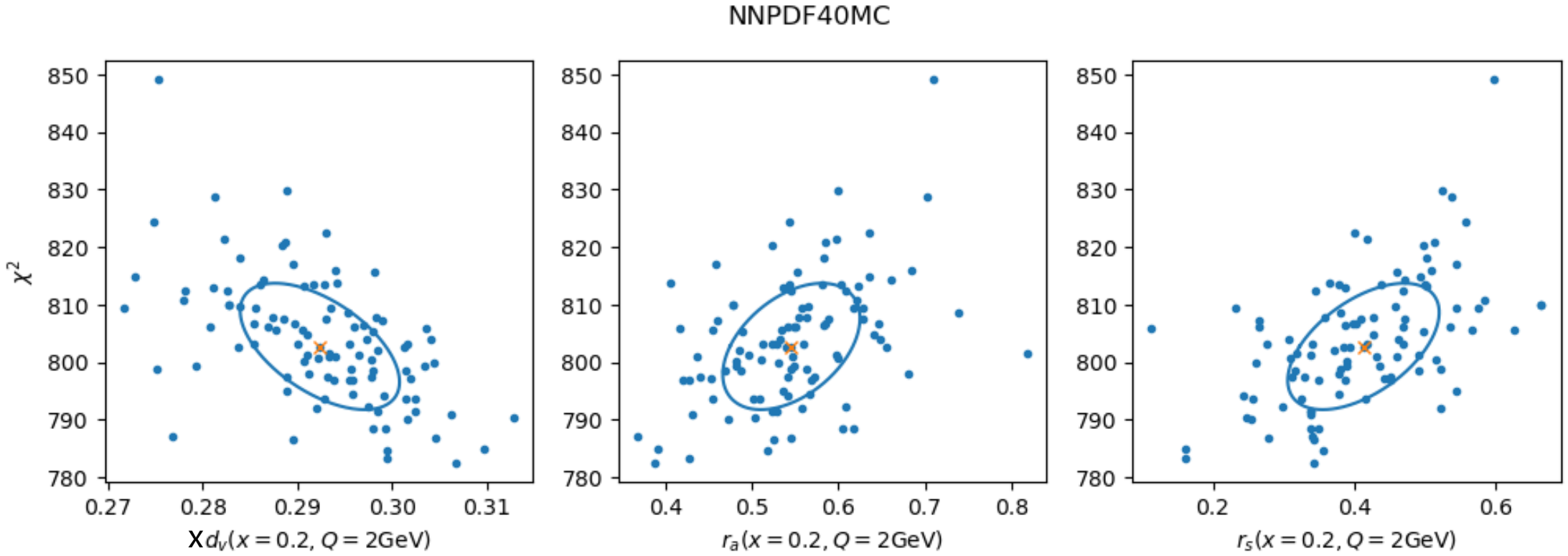


Theory v.s. data for BESIII (SIA: ~3GeV)



- Kinematic cuts in our analyses:
  - $Q > 3$ GeV (SIA)
  - $Q > 2$ GeV(SIDIS)
  - $z > 0.01, E_h > 0.8$ GeV

Backup


NNPDF40MC
χ^2
850
840
830
820
810
800
790
780
0.27
0.28
0.29
0.30
0.31
$\mathrm{X}d_v(x=0.2, Q=2\mathrm{GeV})$
0.4
0.5
0.6
0.7
0.8
$r_a(x=0.2, Q=2\mathrm{GeV})$
0.2
0.4
0.6
$r_s(x=0.2, Q=2\mathrm{GeV})$