第八届全国重味物理与量子色动力学研讨会

V_{cb} determinations from exclusive decays

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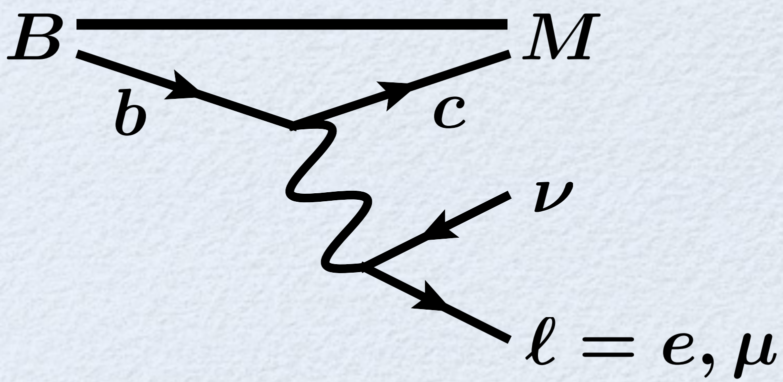
Status

V_{cb} has been measured from semi-leptonic decays

$$\frac{g}{\sqrt{2}} V_{cb}\, \bar{c}_L \gamma^\mu b_L W^+_\mu + \text{h.c.}$$



Exclusive process: $M = D,\ D^*$

Inclusive process: $M = X_c$

Hadronic transitions need to be handled

$$\Gamma_{\text{excl}} \sim m_B^5 G_F^2 |V_{cb}|^2 \times (\text{Form Factor})$$

✔ Today's focus

$$\Gamma_{\text{incl}} \sim m_b^5 G_F^2 |V_{cb}|^2 \times (\text{OPE})$$

PDG average: $|V_{cb}| = \begin{cases} (39.8 \pm 0.6) \times 10^{-3} & (\text{exclusive}) \\ (42.2 \pm 0.5) \times 10^{-3} & (\text{inclusive}) \end{cases}$ ↕ "V_{cb} puzzle"

Form Factors

$$\langle D|\bar{c}\gamma^\nu b|B\rangle = \left[(p_B + p_D)^\nu - \frac{m_B^2 - m_D^2}{q^2} q^\nu\right] f_+(q^2) + \frac{m_B^2 - m_D^2}{q^2} q^\nu f_0(q^2)$$

$$\langle D^*|\bar{c}\gamma^\mu b|B\rangle = i\frac{2V(q^2)}{m_B + m_{D^*}} \varepsilon^{\mu\nu\rho\sigma} \epsilon^*_\nu (p_B)_\rho (p_{D^*})_\sigma$$

$$\langle D^*|\bar{c}\gamma^\mu\gamma^5 b|B\rangle = (m_B + m_{D^*})\epsilon^{*\mu} A_1(q^2) - \frac{(\epsilon^* \cdot q)}{m_B + m_{D^*}}(p_B + p_{D^*})^\mu A_2(q^2)$$
$$- \frac{2m_{D^*}}{q^2}(\epsilon^* \cdot q) q^\mu A_3(q^2)$$

SM contains 6 unknown functions with respect to q^2

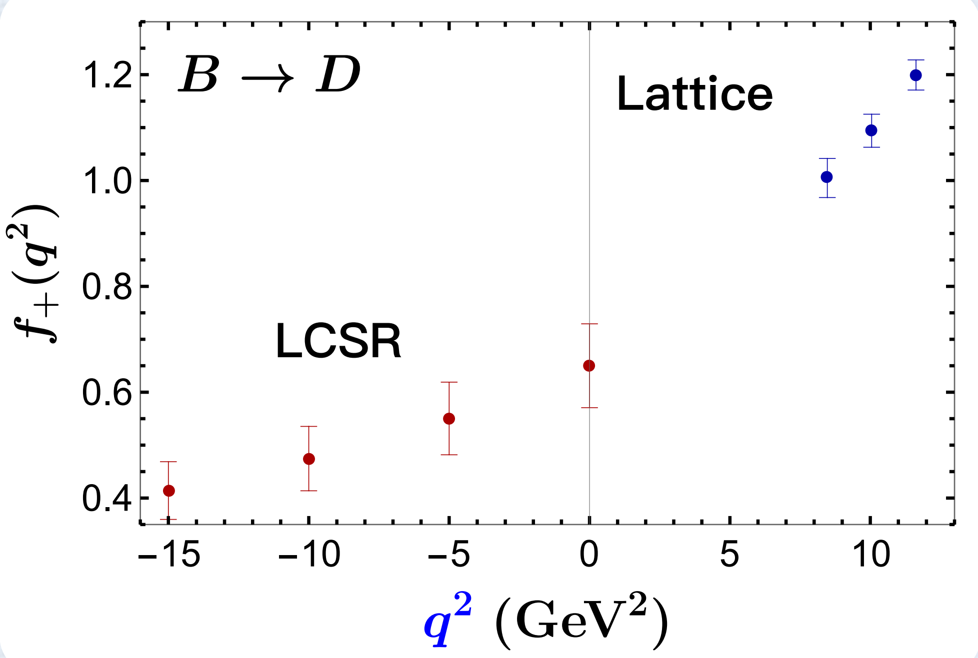
What we need:

Evaluations / Measurements / Parameterizations & Fits

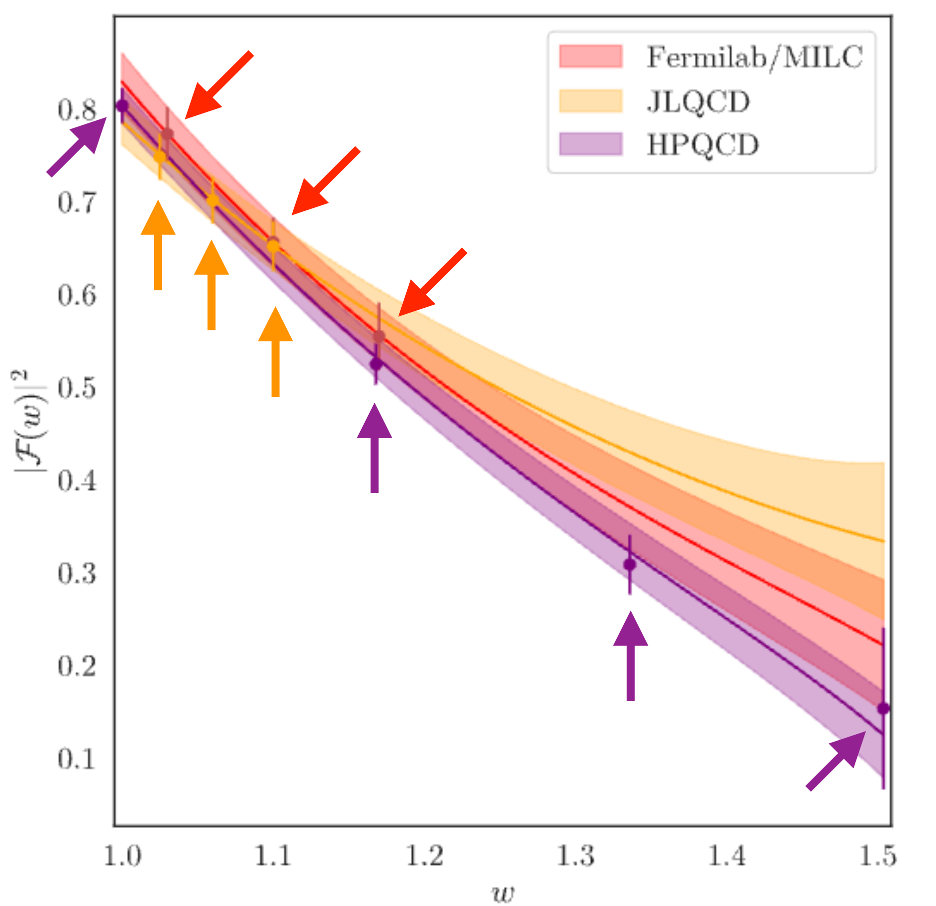
Evaluations

Lattice: $B \to D^{(*)}$ transition (near) zero-recoil $q^2 \sim (m_B - m_{D^{(*)}})^2$

LCSR: $(B, D^{(*)})$ production $q^2 < 0$



Recent evaluations $B \to D^*$



$$q^2 = m_B^2 + m_{D^{(*)}}^2 - 2 m_B m_{D^{(*)}}\, w$$

zero-recoil: $w \sim 1$

Measurements

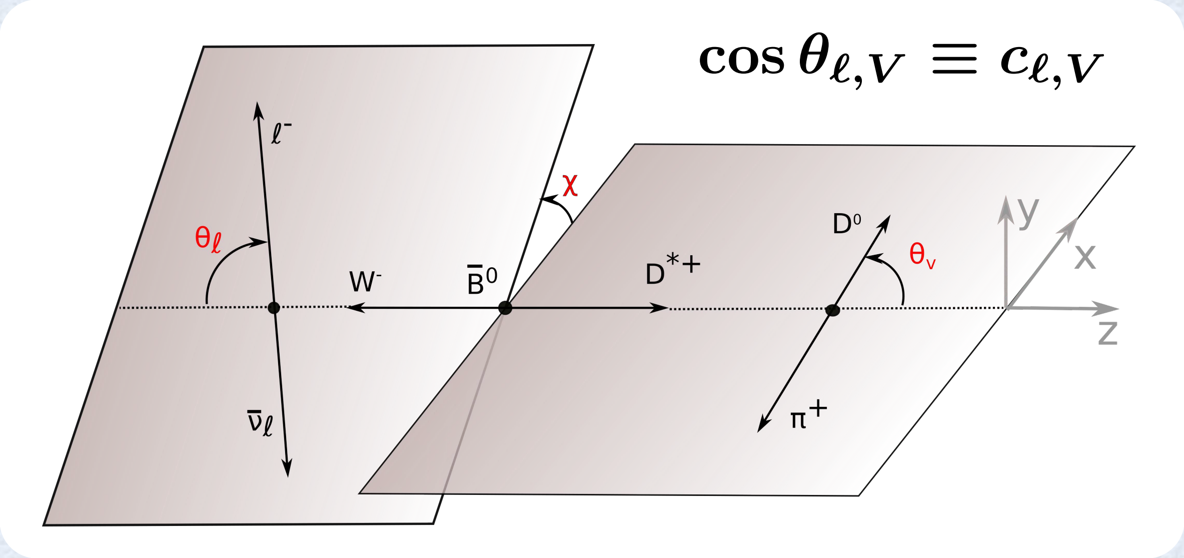
Decay Distribution: $$\frac{d\Gamma(B \to D\ell\nu)}{dq^2} = |V_{cb}|^2 \, \hat{\Gamma}(q^2; f_+(q^2))$$

Recent measurement: Belle 2015, BelleII 2025

$$\frac{d\Gamma(B \to D^*(\to D\pi)\ell\nu)}{dq^2\, dc_\ell\, dc_V\, d\chi} = |V_{cb}|^2 \, H(q^2; \mathrm{FF}(q^2))\, J(c_\ell, c_V, \chi)$$

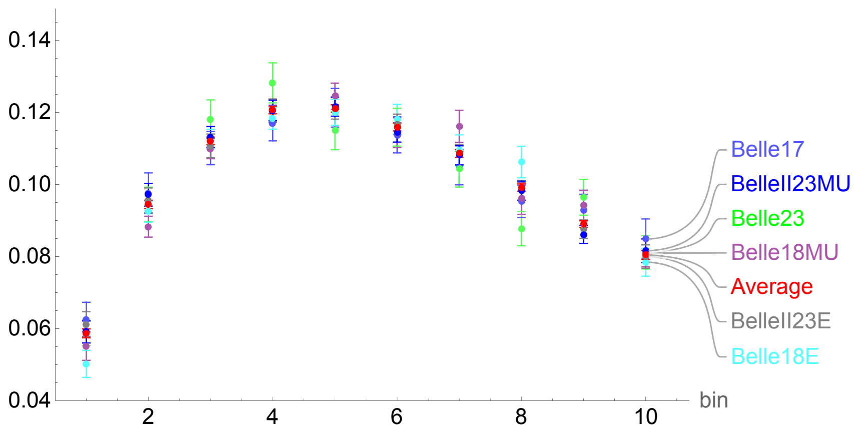
Angular obs:

Recent measurements: Belle 2017, 2018, 2023, BelleII 2023

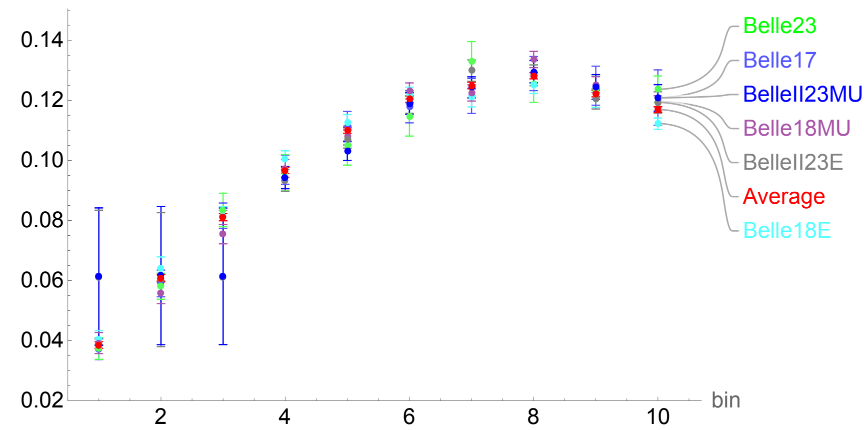


Measurements (combined)

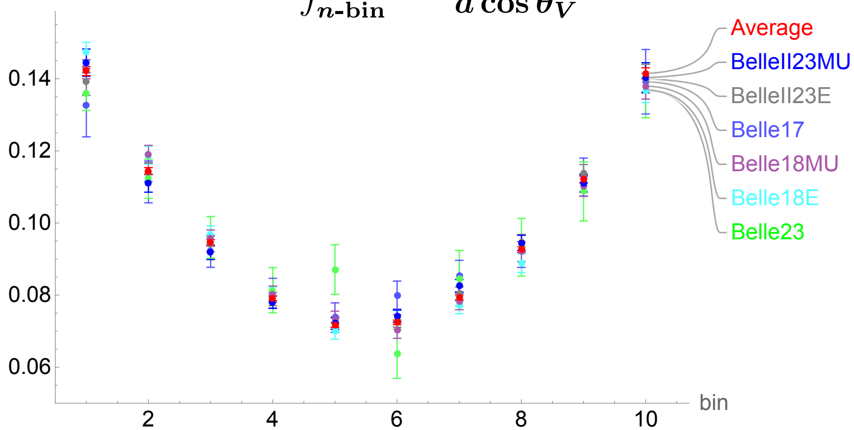
$$\Delta\Gamma_n^w = \int_{n\text{-bin}} \frac{d\Gamma(B \to D^*\ell\nu)}{dw} dw$$



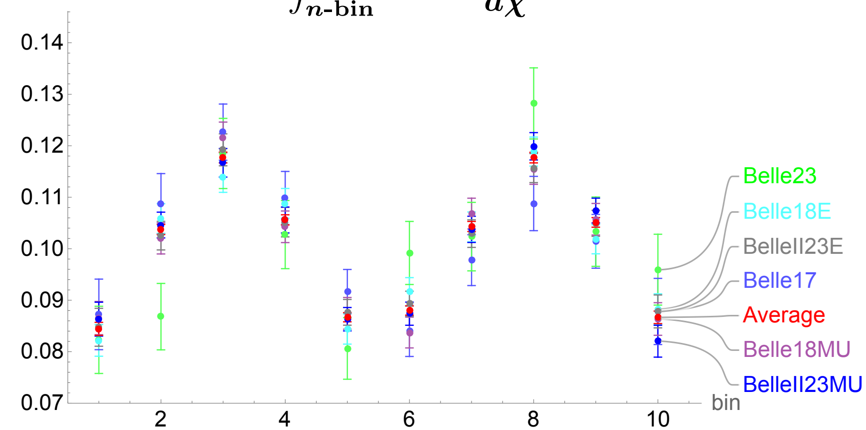
$$\Delta\Gamma_n^{\cos\theta_\ell} = \int_{n\text{-bin}} \frac{d\Gamma(B \to D^*\ell\nu)}{d\cos\theta_\ell} d\cos\theta_\ell$$



$$\Delta\Gamma_n^{\cos\theta_V} = \int_{n\text{-bin}} \frac{d\Gamma(B \to D^*\ell\nu)}{d\cos\theta_V} d\cos\theta_V$$



$$\Delta\Gamma_n^{\chi} = \int_{n\text{-bin}} \frac{d\Gamma(B \to D^*\ell\nu)}{d\chi} d\chi$$



Notice: these 4 distribution data are statistically NOT independent

Parameterizations

Exact q^2 (or w) shape is unknown → Need parameterization

i) All FFs are independent

Boyd, Grinstein, Lebed (1997)

BGL: $$F(w) \equiv \frac{1}{P_F(z)\phi_F(z)} \sum_{n=0}^{N_F} a_n^F z^n$$

— B→D and B→D* are independent subjects

— truncation order N_F is also undetermined → Check dependence

ii) All FFs are related

Jung, Straub (2018), Bordone, Jung, Dyk (2019)

HQET: $$F(w) = F_0(\xi(w)) + \frac{\Lambda}{m_Q} F_1(\chi_1(w), \chi_2(w), \chi_3(w)) + \left(\frac{\Lambda}{m_Q}\right)^2 F_2(\ell_1(w), \cdot$$

— described by "Heavy Quark Expansion"

— leading to common functions. We parameterize them.

$$\xi(w) \equiv \sum_{n=0}^{N_{\rm LO}} b_n^{\xi} (w-1)^n, \quad \chi(w) \equiv \sum_{n=0}^{N_{\rm NLO}} b_n^{\chi} (w-1)^n, \quad \ell(w) \equiv \sum_{n=0}^{N_{\rm NNLO}} b_n^{\ell} (w-1)^n$$

Our Study

$|V_{cb}|$ determination (our study)

1. Show difference in FF parameterization
2. Show difference in fit scenario (explain later)
3. Show New Physics effect (work in progress)

Available data

$|V_{cb}|$ + FF (+NP) fits to available datasets

Lattice/LCSR	
Fermi-MILC 2014 (D^*)	$w = 1$ (zero-recoil)
Fermi-MILC 2015 (D)	$w = 1.00,\ 1.08,\ 1.16$
Fermi-MILC 2021 (D^*)	$w = 1.03,\ 1.10,\ 1.17$
HPQCD 2023 (D^*)	$q^2 = i \times q^2_{max}/3,\ \ i = 0,\ 1,\ 2,\ 3$
JLQCD 2023 (D^*)	$w = 1.025,\ 1.06,\ 1.10$
LCSR 2018 ($D^{(*)}$)	$q^2/\mathrm{GeV}^2 = 0,\ -5,\ -10,\ -15$
LCSR 2023 ($D^{(*)}$)	$q^2/\mathrm{GeV}^2 = 0,\ \pm1,\ \pm2,\ \pm3$

Distributions/Br	
Belle 2015 (D)	w distribution, 10 bins
Belle 2017 (D^*)	$(w, \theta_\ell, \theta_V, \chi)$ distributions, 40 bins
Belle 2018 (D^*)	$(w, \theta_\ell, \theta_V, \chi)$ distributions, 40 / 40 bins
Belle II 2023 (D^*)	$(w, \theta_\ell, \theta_V, \chi)$ distributions, 40 /40 bins
Belle 2023 (D^*)	$(w, \theta_\ell, \theta_V, \chi)$ distributions, 40 bins
Belle II 2025 (D^*)	w distribution, 10 bins
PDG (D, D^*)	

Fit scenarios

(A) fits to raw distribution data with one of $x = \{w, \cos\theta_\ell, \cos\theta_V, \chi\}$

$$\chi^2_{\rm all} = \chi^2(\text{Lattice,LCSR}; \textbf{FF}) + \chi^2(\text{Br}; \textbf{FF}, |V_{cb}|) + \chi^2(\Delta\Gamma^x_n; \textbf{FF}, |V_{cb}|)$$

→ Four fit scenarios: (A, w), $(A, \cos\theta_\ell)$, $(A, \cos\theta_\ell)$, (A, χ)

(B) fits to normalized distribution data with all of $x = \{w, \cos\theta_\ell, \cos\theta_V, \chi\}$

$$\chi^2_{\rm all} = \chi^2(\text{Lattice,LCSR}; \textbf{FF}) + \chi^2(\text{Br}; \textbf{FF}, |V_{cb}|) + \chi^2(\Delta\Gamma^x_n/\Gamma; \textbf{FF})$$

→ V_{cb} is determined solely from $\text{Br}(B \to D\ell\nu)$ and $\text{Br}(B \to D^*\ell\nu)$

→ Statistically unfair, but adopted for PDG average

FF parameterizations (reminder)

$$F = \begin{cases} f_+,\ f_0 \quad (B \to D) \\ f,\ \mathcal{F}_1,\ \mathcal{F}_2,\ g \quad (B \to D^*) \end{cases}$$

↕ related (HQET) / independent (BGL)

↔ related (HQET) / independent (BGL)

$|V_{cb}|$ fit results

	Fit scenario	$\lvert V_{cb}\rvert$ fit result ($\times 10^3$) BGL ($N_F = 1$)	($N_F = 2$)	HQET (2/1/0)	(3/2/1)
(A)	w	40.2 ± 0.4	39.8 ± 0.4	40.3 ± 0.5	42.1 ± 0.6
	$\cos\theta_\ell$	40.4 ± 0.4	39.9 ± 0.4	42.0 ± 0.8	39.5 ± 0.7
	$\cos\theta_V$	40.5 ± 0.4	40.1 ± 0.4	42.1 ± 0.8	39.9 ± 0.7
	χ	41.3 ± 0.4	40.8 ± 0.5	42.4 ± 0.8	40.6 ± 0.8
(B)	D^0-mode	39.4 ± 0.8	39.3 ± 0.8	38.6 ± 1.0	39.2 ± 1.0
	D^{*0}-mode	40.6 ± 0.5	39.9 ± 0.6	41.3 ± 0.8	43.0 ± 1.0
	Combined	40.2 ± 0.4	$\mathbf{39.7 \pm 0.5}$	40.3 ± 0.6	41.2 ± 0.7

$|V_{cb}|$ fit results

	Fit scenario	$\lvert V_{cb}\rvert$ fit result ($\times 10^3$) BGL ($N_F=1$)	($N_F=2$)	HQET (2/1/0)	(3/2/1)
(A)	w	40.2 ± 0.4	39.8 ± 0.4	40.3 ± 0.5	42.1 ± 0.6
	$\cos\theta_\ell$	40.4 ± 0.4	39.9 ± 0.4	42.0 ± 0.8	39.5 ± 0.7
	$\cos\theta_V$	40.5 ± 0.4	40.1 ± 0.4	42.1 ± 0.8	39.9 ± 0.7
	χ	41.3 ± 0.4	40.8 ± 0.5	42.4 ± 0.8	40.6 ± 0.8
(B)	D^0-mode	39.4 ± 0.8	39.3 ± 0.8	38.6 ± 1.0	39.2 ± 1.0
	D^{*0}-mode	40.6 ± 0.5	39.9 ± 0.6	41.3 ± 0.8	43.0 ± 1.0
	Combined	40.2 ± 0.4	$\mathbf{39.7 \pm 0.5}$	40.3 ± 0.6	41.2 ± 0.7

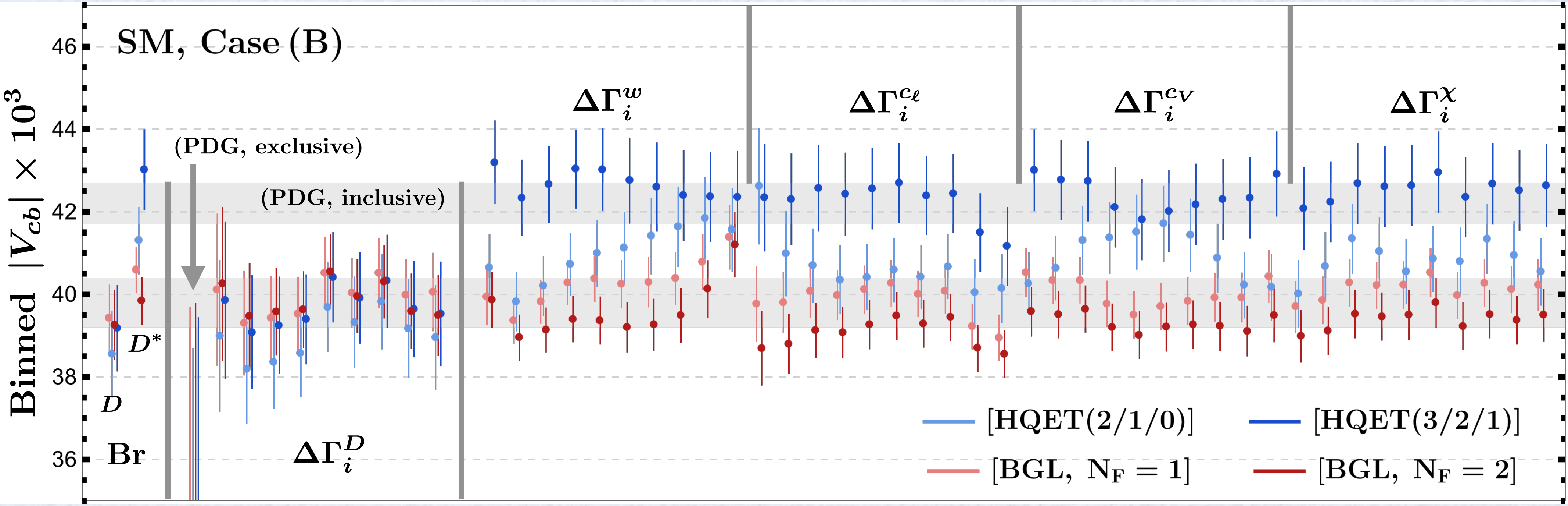
- BGL (N_F=2) reproduces the PDG average in fit scenario (B)
- BGL gives consistent results among (A, w), (A, $\cos\theta_l$), (A, $\cos\theta_V$), and (B)
- HQET shows large difference between D and D* modes in (B)

→ indicates that HQET does not properly describe B→D and B→D* (?)

→ some of fits are close to inclusive one (~42.2) but not reliable

|V_cb| from the distribution data

$$\Delta\Gamma_n^x(\text{theory}) = \int_{x_n}^{x_{n+1}} \frac{d\Gamma}{dx} dx = |V_{cb}|^2 \times \hat{\Gamma}_n^x(\mathbf{FF}) \quad \Rightarrow \quad |V_{cb}|_n^x = \sqrt{\frac{\Delta\Gamma_n^x(\exp)}{\hat{\Gamma}_n^x(\mathbf{FF})}}$$



- BGL gives consistent Vcb between Branching ratios and Distribution data

  → "V_{cb} puzzle" remains (exclusive ~40 vs inclusive ~42)

- HQET shows large difference between D and D* modes also for distribution

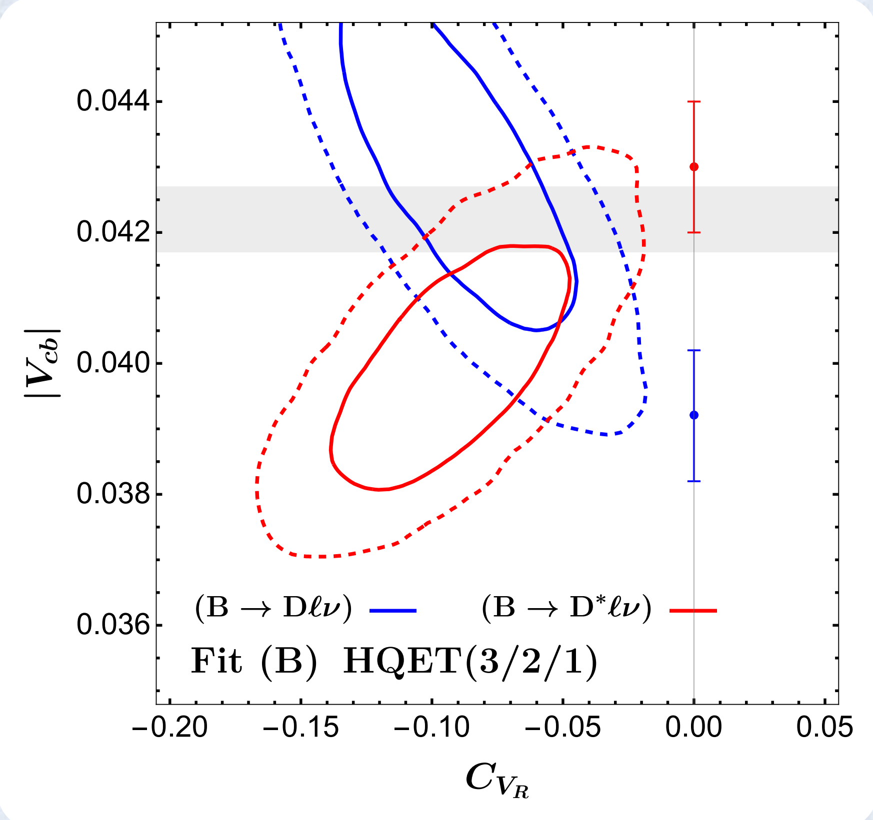
  → V_{cb} from D* mode is in agreement with inclusive while D mode is not.

New Physics effect

Setup

$$\mathcal{H}_{\rm eff}^{b\to c\ell\nu} = \frac{4G_F}{\sqrt{2}} V_{cb}\Big[(\bar{c}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell) + C_{V_R}(\bar{c}\gamma^\mu P_R b)(\bar{\ell}\gamma_\mu P_L \nu_\ell) + C_{S_L}(\bar{c}P_L b)(\bar{\ell}P_L\nu_\ell) + C_{S_R}(\bar{c}P_R b)(\bar{\ell}P_L\nu_\ell) + C_T(\bar{c}\sigma^{\mu\nu}P_L b)(\bar{\ell}\sigma_{\mu\nu}P_L\nu_\ell)\Big]$$

Work in progress...



HQET(3/2/1)

- Large difference between D & D* in SM
- NP contribution can reduce this issue!
- NP can also resolve V_{cb} puzzle? (~0.042)

Preliminary!

- Wait for arXiv:2605.xxxxx

Summary

What we saw in $|V_{cb}|$ determination

1. Show difference in FF parameterization

2. Show difference in fit scenario

3. Show New Physics effect (work in progress)

What we got

- BGL (NF=2) reproduces the PDG average

- BGL gives consistent results among different fit scenarios

- HQET shows large difference between D and D* modes

  → indicates that HQET does not properly describe B→D and B→D* (?)

  → NP contribution can reduce this large difference (!)

  → NP can also resolve V_{cb} puzzle (!!), rising the exclusive V_{cb} value close to inclusive

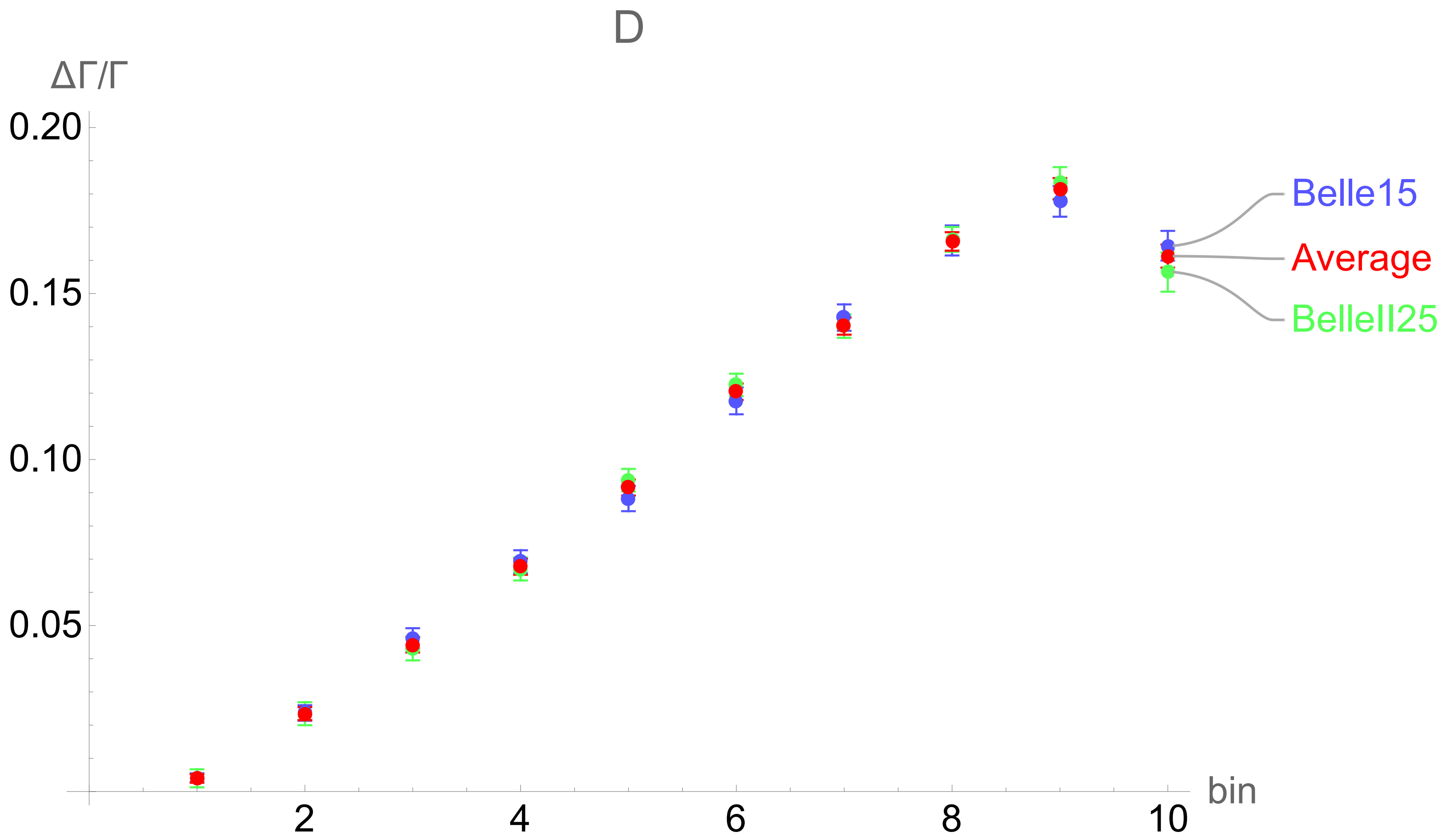
Backup

Fit details

	χ^2_{best} regarding **Distribution (Combined)** and **BrRatio**.					
Fit scenario	**BSZ** ($N_F = 1$)	($N_F = 2$)	**BGL** ($N_F = 1$)	($N_F = 2$)	**HQET (2/1/0)**	**(3/2/1)**
(A, w)	14.4	20.6	14.7	19.7	33.5	40.0
$(A, \cos\theta_\ell)$	12.5	12.0	11.3	16.6	42.3	105
$(A, \cos\theta_V)$	14.9	14.4	13.2	17.4	50.5	42.6
(A, χ)	22.7	30.6	46.7	40.1	32.0	47.6
(B)	16.5	14.6	20.0	17.8	31.4	22.6

	χ^2_{best} for MILC15 (6) / MILC21 (12) / JLQCD23 (12) / HPQCD23 (20)					
Fit scenario	**BSZ** ($N_F = 1$)	($N_F = 2$)	**BGL** ($N_F = 1$)	($N_F = 2$)	**HQET (2/1/0)**	**(3/2/1)**
(A, w)	26/29/5/5	8/31/6/5	4/25/5/5	5/23/9/5	16/42/20/15	7/12/20/6
$(A, \cos\theta_\ell)$	21/35/7/5	9/39/9/6	7/28/8/4	4/40/11/8	13/43/23/14	11/24/10/5
$(A, \cos\theta_V)$	23/31/5/5	12/34/6/6	4/29/7/4	3/39/13/5	14/36/21/14	13/18/13/5
(A, χ)	23/28/7/5	13/27/5/5	6/18/11/4	2/30/18/6	17/41/28/19	11/18/10/4
(B)	22/28/5/5	6/38/6/6	3/24/5/4	5/30/10/4	18/42/17/14	6/23/6/4

B → D l ν distribution data (combined)



New Physics effect

Setup

$$\mathcal{H}_{\rm eff}^{b\to c\ell\nu} = \frac{4G_F}{\sqrt{2}} V_{cb}\Big[(\bar{c}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell) + C_{V_R}(\bar{c}\gamma^\mu P_R b)(\bar{\ell}\gamma_\mu P_L \nu_\ell)$$
$$+ C_{S_L}(\bar{c}P_L b)(\bar{\ell}P_L\nu_\ell) + C_{S_R}(\bar{c}P_R b)(\bar{\ell}P_L\nu_\ell) + C_T(\bar{c}\sigma^{\mu\nu}P_L b)(\bar{\ell}\sigma_{\mu\nu}P_L\nu_\ell)\Big]$$

Form Factors

SM scalar FFs

Scalar: $\langle D|\bar{c}b|B\rangle = \frac{m_B^2 - m_D^2}{m_b - m_c} f_0(q^2)\,,\quad \langle D^*|\bar{c}\gamma^5 b|B\rangle = \frac{2m_{D^*}}{m_b + m_c}(\epsilon^*\cdot q) A_0(q^2)\,,$

Tensor: $\langle D|\bar{c}\sigma^{\mu\nu}b|B\rangle = \frac{-2i}{m_B + m_D}(p_B^\mu p_D^\nu - p_B^\nu p_D^\mu) f_T(q^2)\,,$

$\langle D^*|\bar{c}\sigma^{\mu\nu}b|B\rangle \supset T_1(q^2)\,, T_2(q^2)\,, T_2(q^2)$

Parameterization

BGL: additional independent FFs (HQPCD23, LCSR18, dist. available)

HQET: common IW functions with SM

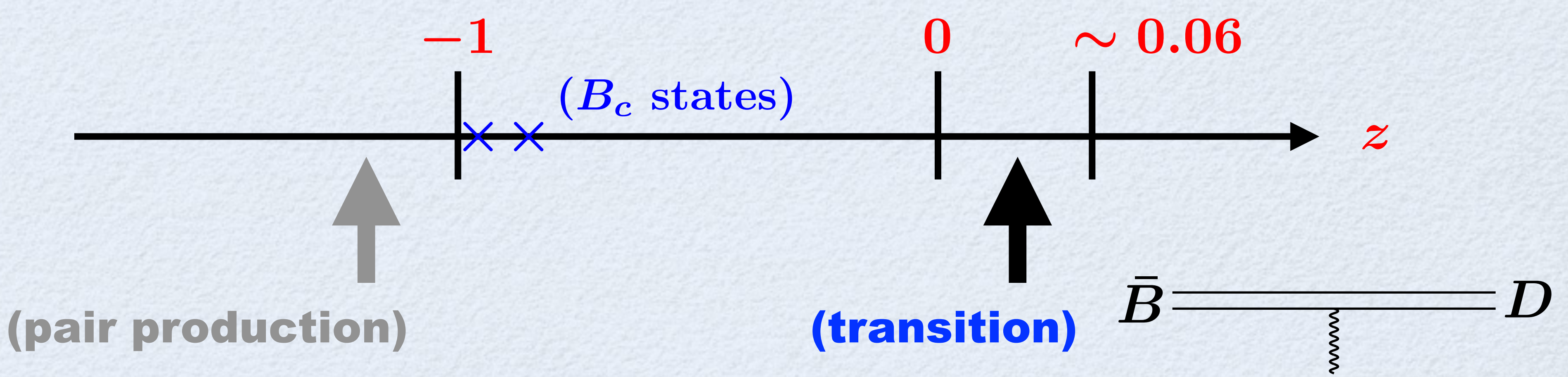
(FF properties)

Boyd, Grinstein, Lebed (1997)

→ **Analyticity + Dispersion relation requires**

$$\oint_C \frac{dz}{z} \left| \text{something}(z) \times \mathbf{FF}(z) \right|^2 \leq \text{constant}$$

- **complex integral:** $C \in \text{unit circle}$
- **z variable:** $z \equiv \left[(m_B - m_D)^2 - q^2\right] \Big/ \left[\sqrt{(m_B + m_D)^2 - q^2} + \sqrt{4 m_B m_D}\right]^2$
- **something(z) = QCD calculation + Bc poles = obtainable**
- **z region of interest:**



(FF parameterization)

$\rightarrow$ **Analyticity + Dispersion relation + Unitarity bound**

$$\oint_C \frac{dz}{z} \left| \text{something}(z) \times \mathbf{FF}(z) \right|^2 \leq \text{constant}$$

$\rightarrow$ **leads to the general form of the parameterization**

$$\text{something}(z) \times \mathbf{FF}(z) \equiv \sum_{n=0}^{\infty} a_n \, z^n$$

- a_n **are free parameters to be fitted**
- **In general, infinite number sum. In practice, needs truncation.**