Revising the Mass of Light Hybrid Mesons:

NLO QCD Sum Rules Point to $\phi(2170)$ as a Prime Candidate

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JHEP 03 (2026) 087

Outline

- **Introduction:** history & conflict among different methods
- **NLO Calculation:** renormalization & NLO correction
- **Impact of Vector Meson:** how 1^{--} vector meson affects the prediction
- **Sum Rules Analysis:** Laplace & Gaussian sum rules
- **Conclusion**

History & conflict

$J^{PC} = 1^{--}$ hybrid:

Masses of $\bar{q}qG$ hybrid states (in GeV) and the corresponding values of $\sqrt{s_0}$

J^{PC}	$\bar{b}bG$	$\sqrt{s_0}$	$\bar{c}cG$	$\sqrt{s_0}$	$\bar{s}sG$	$\sqrt{s_0}$
0^{++}	10.9	11.6	5.0	5.4	3.5	3.8
$*0^{--}$	11.4	12.0	5.4	5.8	3.7	4.0
1^{+-}	10.6	11.2	4.1	4.4	2.8	3.2
$*1^{-+}$	10.6	11.5	4.4	5.0	2.5	3.0
0^{-+}	10.5	11.6	4.7	5.4	3.1	3.8
$*0^{+-}$	11.0	12.0	5.1	5.8	3.4	4.0
1^{--}	10.4	11.2	4.1	4.4	2.9	3.2
1^{++}	10.9	11.5	4.7	5.0	2.8	3.0

LO QCD sum rules prediction: $\approx 2.9\text{GeV}\ (s\bar{s}g)$

Nuclear Physics B262 (1985) 575-592

Phys. Rev. D 100, 034012 (2019)

Flux tube model:

$1.8 - 1.9\text{GeV}\ (u\bar{u}g)$;

$2.1 - 2.2\text{GeV}\ (s\bar{s}g)$

Phys. Rev. D 52, 5242 (1995)

Lattice QCD:

$2.2 - 2.3\text{GeV}\ (I = 1)$;

$2.4 - 2.5\text{GeV}\ (I = 0)$

Phys. Rev. D 84, 074023 (2011)

Candidate:

$\phi(2170), I^G(J^{PC}) = 0^-(1^{--})$

Decays into $\phi f_0(980), \phi\eta, \ldots$

different states?

missing something?

QCD (SVZ) Sum Rules of Hybrid

$$J^\mu = g\bar{\Psi}\gamma_\rho\gamma^5\tilde{G}^{\rho\mu}\Psi; \quad \tilde{G}^{\rho\mu} = \frac{1}{2}\varepsilon^{\rho\mu\alpha\beta}G^n_{\alpha\beta}T^n; \quad T^n = \frac{\lambda^n}{2}$$

$$\Pi^{\mu\nu}(q) = i\int d^4x\, e^{-ipx}\langle\Omega|T\, J^\mu(x)\, J^\nu(0)|\Omega\rangle = \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)\Pi_v(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi_s(q^2)$$

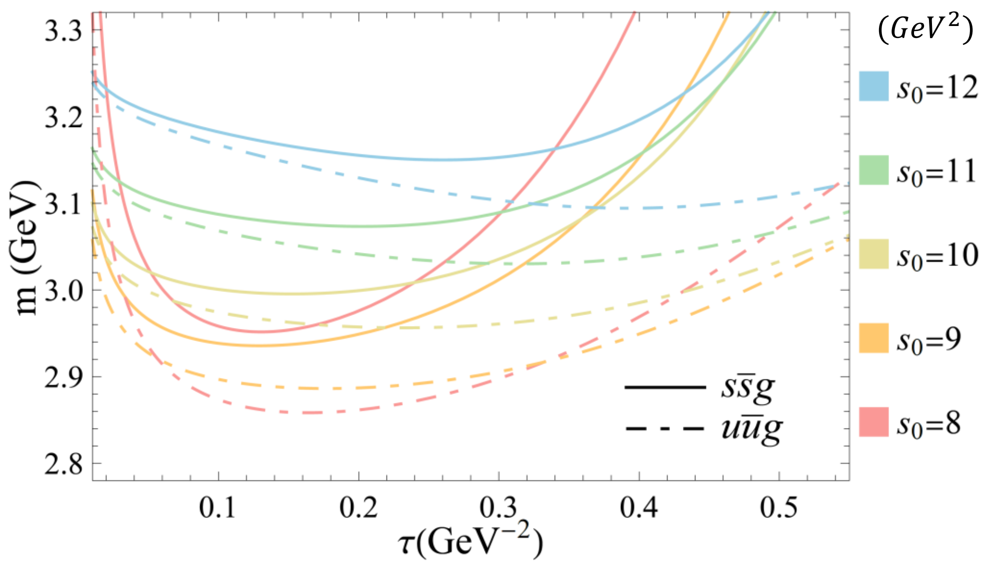
Dispersion relation, "δ + continuum" ansatz

$$\frac{1}{\pi}\mathrm{Im}\Pi(s) \sim f^2\delta(s - m^2) + \theta(s - s_0)\rho(s)$$

Laplace sum rules (Borel transformation)

$$\mathcal{M}^n = \frac{1}{\pi}\int_0^{s_0} ds\, s^n e^{-\tau s}\mathrm{Im}\Pi(s) = f^2 m^{2n} e^{-\tau m^2}$$

$$\mathcal{R}^n = \frac{\mathcal{M}^{n+1}}{\mathcal{M}^n} = m^2$$



LO Laplace sum rules prediction, including dimension-8 condensate

NLO Calculation

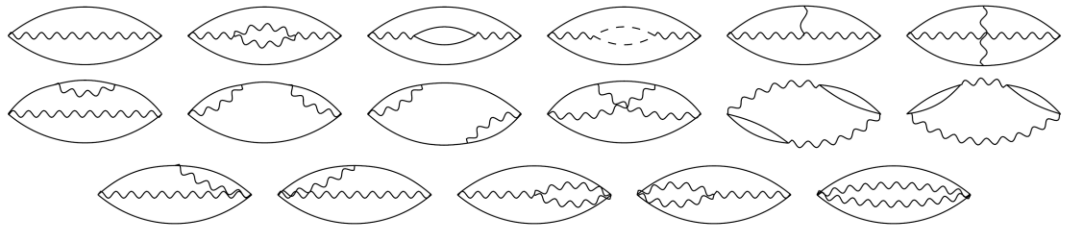
$$\Pi_v(q^2) = \sum_n C^n(q^2)\langle O_n\rangle$$

$$= C^0(q^2) + C_1^4(q^2)\langle GG\rangle + C_2^4(q^2)m\langle\bar{q}q\rangle + C_1^6(q^2)\langle\bar{q}q\rangle^2 + C_2^6(q^2)m\langle\bar{q}Gq\rangle + C^8(q^2)\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle + \cdots$$

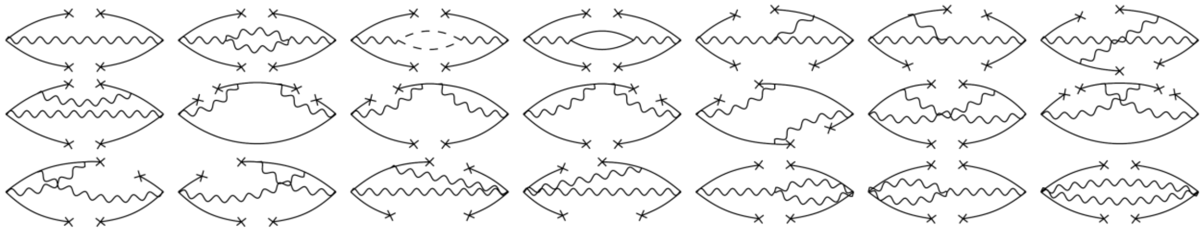
$$+ \alpha_s C^{0'}(q^2) + \alpha_s C_1^{4'}(q^2)\langle GG\rangle + \cdots$$

$$+ \cdots$$

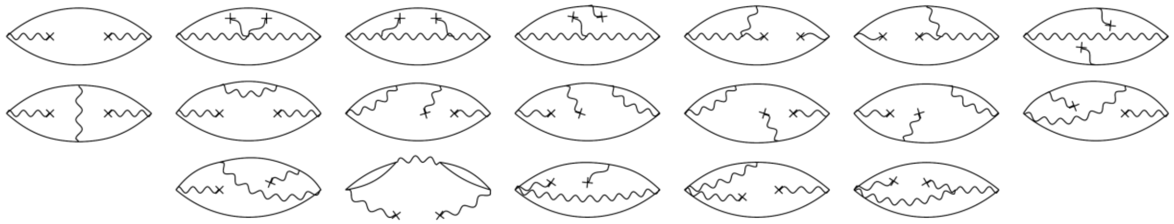
Higher order correction



perturbative

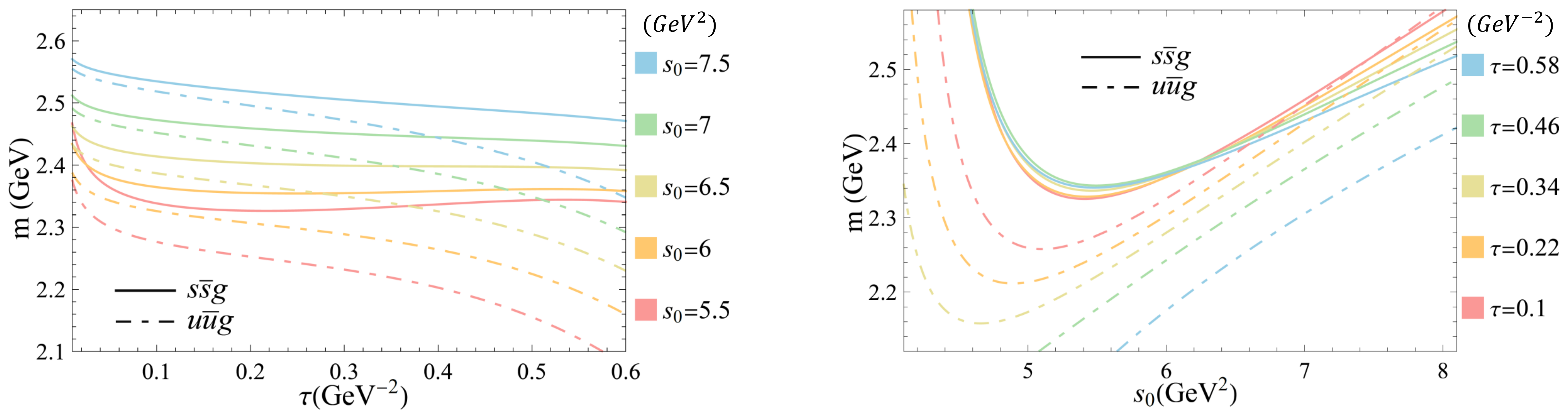


$\langle\bar{q}q\rangle^2$



$\langle GG\rangle$

NLO Result



Lattice QCD:

$2.2 - 2.3\text{GeV}\ (I = 1)$;

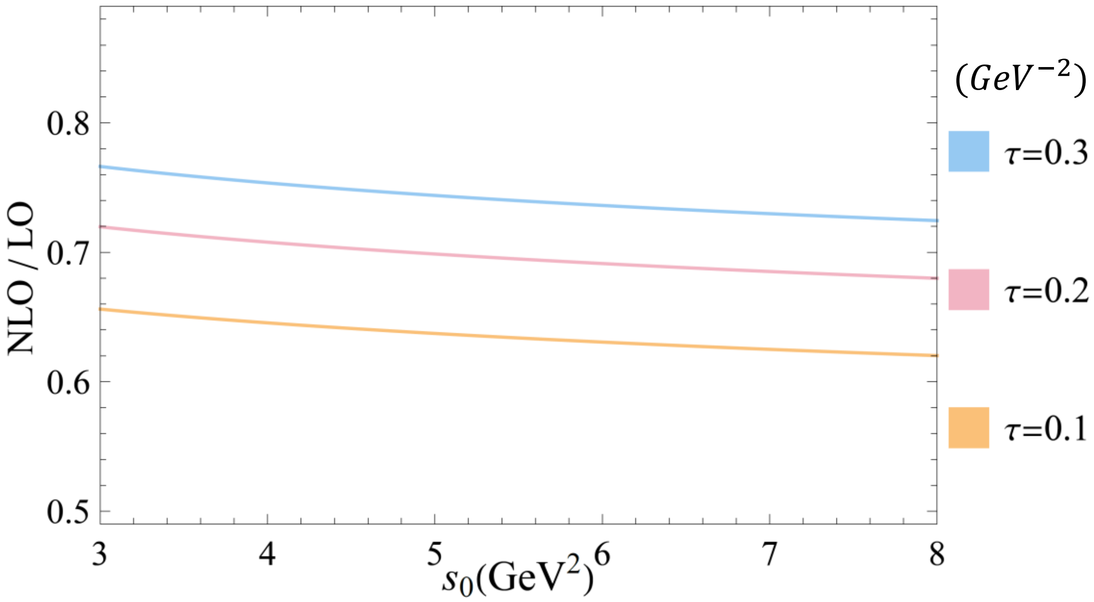
$2.4 - 2.5\text{GeV}\ (I = 0)$

Phys. Rev. D 84, 074023 (2011)

NLO Impact

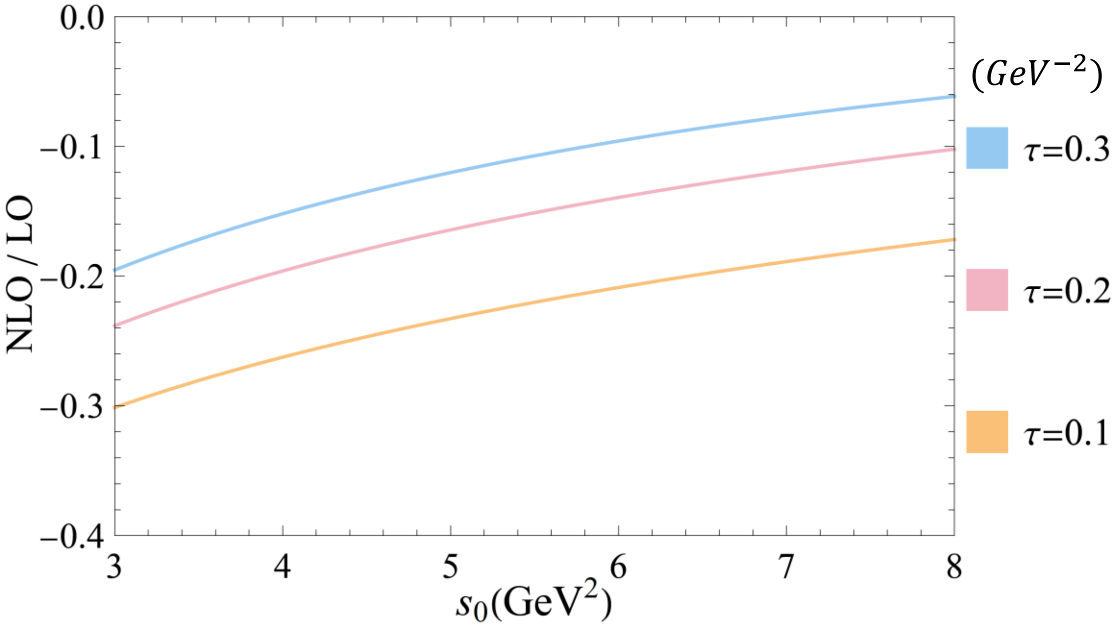
NLO contribution vs LO contribution (after Borel transformation)

NLO perturbative / LO perturbative



$$\frac{\hat{\mathcal{B}}\ \alpha_s C^{0\prime}(q^2)}{\hat{\mathcal{B}}\ C^0(q^2)}$$

NLO $\langle GG \rangle$ / LO $\langle GG \rangle$



$$\frac{\hat{\mathcal{B}}\ \alpha_s C_1^{4\prime}(q^2)\langle GG \rangle}{\hat{\mathcal{B}}\ C_1^4(q^2)\langle GG \rangle}$$

Impact of Vector Meson

$J_H^\mu = g\bar{\Psi}\gamma_\rho\gamma^5\tilde{G}^{\rho\mu}\Psi$ couples to 1^{--} meson as well

$$\frac{1}{\pi}\mathrm{Im}\Pi(s) \sim f_1^2\delta(s-m_1^2) + f_2^2\delta(s-m_2^2) + \theta(s-s_0')\rho(s)$$

$$\langle\Omega|J^\mu|V(q)\rangle = f_1\epsilon^\mu(q), \qquad \langle\Omega|J^\mu|H(q)\rangle = f_2\epsilon^\mu(q)$$

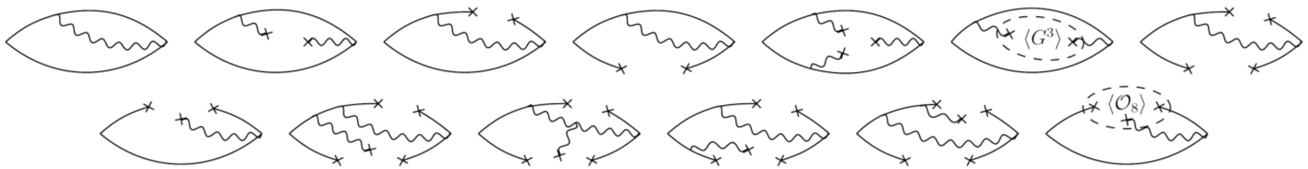
Vector meson $J_V^\mu = \bar{\Psi}\gamma^\mu\Psi$

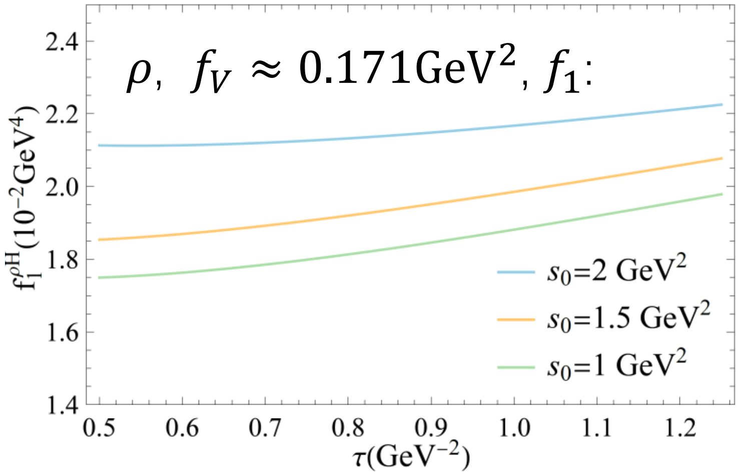
$$\Pi^{\mu\nu}(q) = i\int d^4x\, e^{-ipx}\langle\Omega|T\, J_V^\mu(x)\, J_H^\nu(0)|\Omega\rangle$$

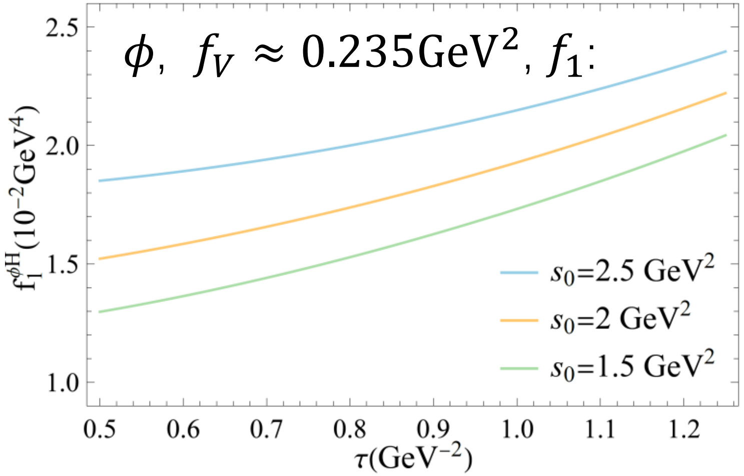
$$= \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)\Pi_v^{\mathrm{VH}}(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi_s^{\mathrm{VH}}(q^2)$$

$$\frac{1}{\pi}\mathrm{Im}\Pi_v^{\mathrm{VH}}(s) = f_V f_1\delta(s-m_1^2) + \theta(s-s_0)\rho(s)$$

$$\langle\Omega|J_V^\mu|V(q)\rangle = f_V\epsilon^\mu(q)$$







$f_1 \sim 0.02\ \mathrm{GeV}^4$
$\to f_1/m_1^3 \sim 40\mathrm{MeV}$
$\sim 20-35\ \mathrm{MeV}$ (P.L.B 175 (1986) 485)

Gaussian Sum Rules

QCD side:

$$G(\tau, s, s_0) = \frac{1}{\sqrt{4\pi\tau}} \int_0^{s_0} dt\ e^{-\frac{(s-t)^2}{4\tau}} \frac{1}{\pi} \mathrm{Im}\Pi_v(s)$$

Hadron side:

$$G_h(\tau, s) = \frac{1}{\sqrt{4\pi\tau}} \left(f_1^2 e^{-\frac{(s-m_1^2)^2}{4\tau}} + f_2^2 e^{-\frac{(s-m_2^2)^2}{4\tau}} \right)$$

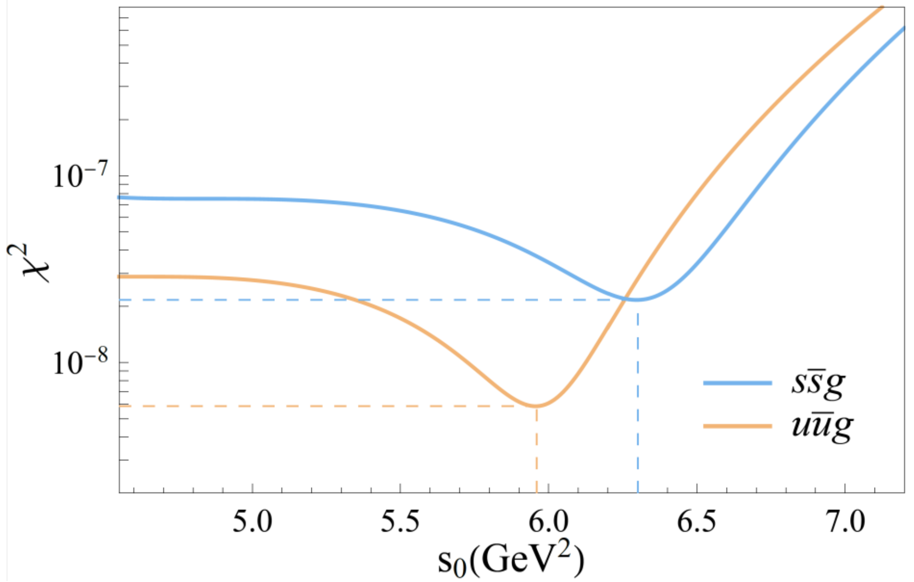
Fitting

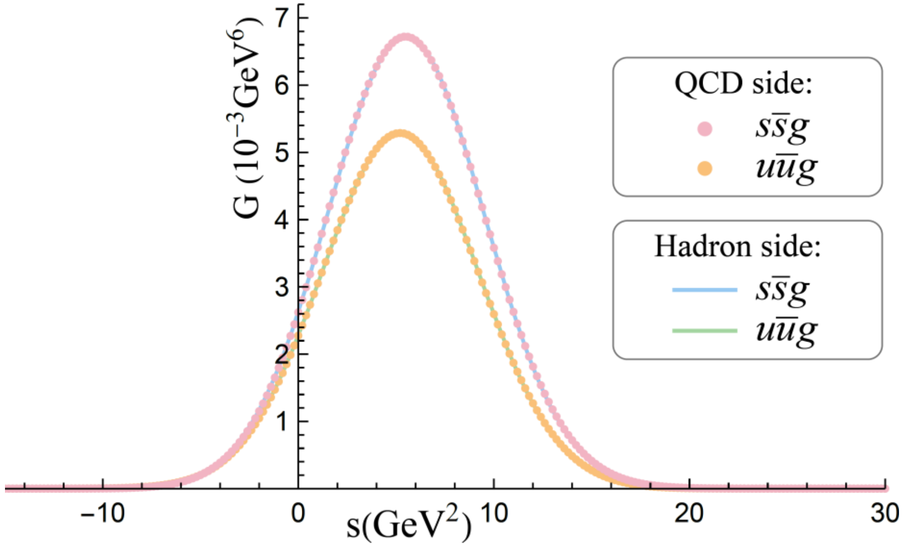
$$\chi^2(\tau, s_0) = \sum_{i=0}^{N} [G(\tau, s_i, s_0) - G_h(\tau, s_i)]^2$$

$$s_i = s_{\min} + i\,\delta s, \qquad \delta s = 0.2 GeV^2$$

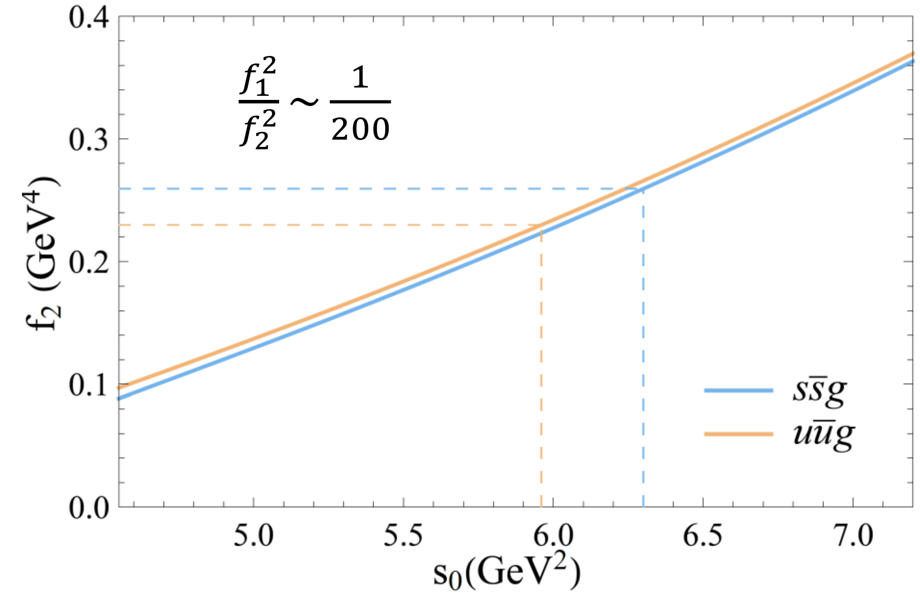
$$s_{\min} = -15 GeV^2, \qquad s_{\max} = 30 GeV^2$$

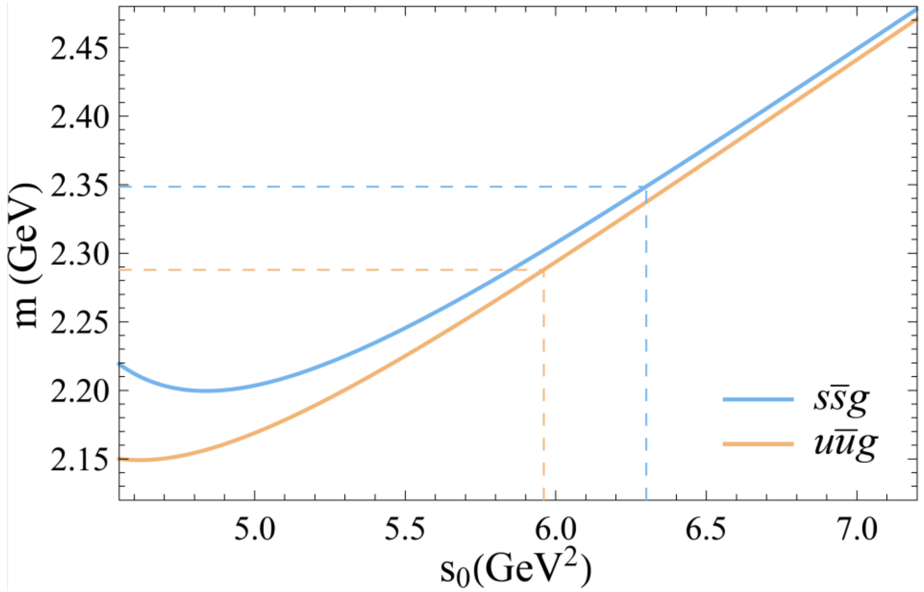
f_1, m_1: fixed according to the vector meson





Gaussian Sum Rules





Normalized single resonance GSR:

$$\tilde{G}(\tau, s, s_0) = \frac{G(\tau, s, s_0)}{\int_{\infty}^{\infty} ds\, G(\tau, s, s_0)}$$

$$\tilde{G}_h(\tau, s) = \frac{1}{\sqrt{4\pi\tau}} e^{-\frac{(s-m^2)^2}{4\tau}}$$

Parameters	NLO	
	$s\bar{s}g$	$u\bar{u}g$
$\overline{\chi^2_{\rm min}}$	1.17×10^{-5}	2.32×10^{-5}
$s_0(\mathrm{GeV}^2)$	6.15 ± 1.21	5.85 ± 1.21
$m(\mathrm{GeV})$	2.31 ± 0.23	2.25 ± 0.23

single resonance fitting with Monte Carlo uncertainty analysis (2000 random samples)

$\langle GG \rangle = 0.07 \pm 0.02\ \mathrm{GeV}^4$
$\langle \bar{q}Gq \rangle = M_0^2 \langle \bar{q}q \rangle, \qquad M_0^2 = 0.8 \pm 0.2\ \mathrm{GeV}^2$
10% error for $\langle \bar{q}q \rangle$

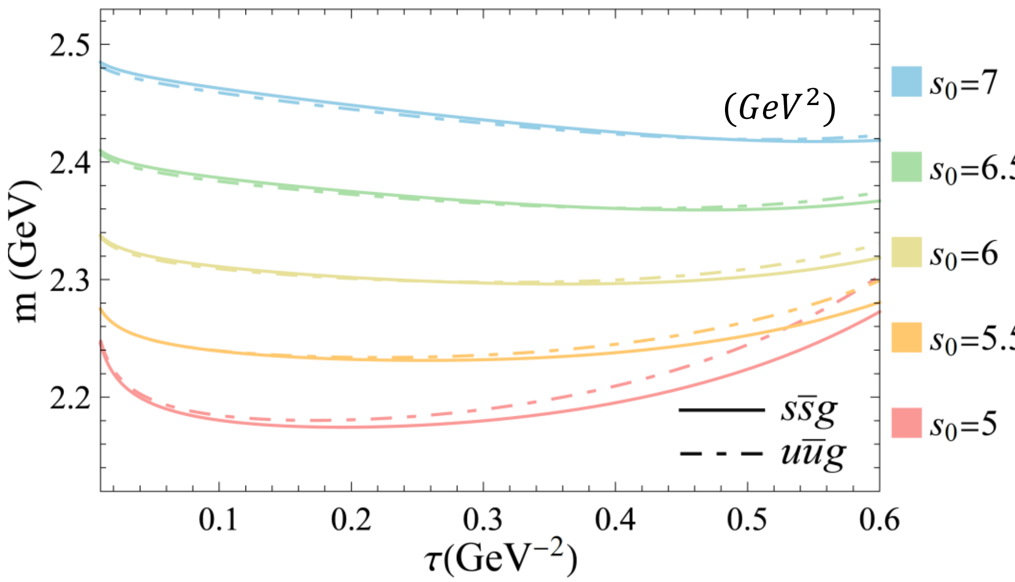
Robustness of Results

Modified LSR:

$$\mathcal{M}(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} ds\, e^{-\tau s}\, \mathrm{Im}\Pi(s) = f_1^2 e^{-\tau m_1^2} + f_2^2 e^{-\tau m_2^2}$$

$$\widetilde{\mathcal{M}}(\tau, s_0) \equiv \mathcal{M}(\tau, s_0)\, e^{\tau m_1^2} = f_1^2 + f_2^2 e^{\tau(m_1^2 - m_2^2)}$$

$$\frac{\partial_\tau^2 \widetilde{\mathcal{M}}(\tau, s_0)}{\partial_\tau \widetilde{\mathcal{M}}(\tau, s_0)} = m_1^2 - m_2^2$$



m_1 is vector meson mass

Factorization deviation

$$\langle \bar{q}q\rangle^2 \to \kappa_6 \langle \bar{q}q\rangle^2$$
$$\langle \bar{q}q\rangle\langle \bar{q}Gq\rangle \to \kappa_8 \langle \bar{q}q\rangle\langle \bar{q}Gq\rangle$$

Parameters	$\kappa_6 = 3,\ \kappa_8 = 2$		$\kappa_6 = 3,\ \kappa_8 = 5$	
	$s\bar{s}g$	$u\bar{u}g$	$s\bar{s}g$	$u\bar{u}g$
$\overline{\chi^2_{\min}}$	2.85×10^{-5}	7.28×10^{-5}	1.74×10^{-4}	4.30×10^{-4}
$s_0(\mathrm{GeV}^2)$	6.03 ± 1.21	6.02 ± 1.31	6.40 ± 1.60	7.03 ± 2.16
$m(\mathrm{GeV})$	2.29 ± 0.23	2.28 ± 0.24	2.34 ± 0.28	2.43 ± 0.37

Conclusion

NLO GSR:

$s\bar{s}g$: 2.31 ± 0.23 GeV
$u\bar{u}g$: 2.25 ± 0.23 GeV

- No conflict between Sum Rules and Lattice QCD
- Necessity of calculation beyond LO

Predicted mass agrees with $\phi(2170)$ in error

Future direction:

- Decay of 1^{--} hybrid

Lattice QCD:

$2.2 - 2.3$ GeV $(I = 1)$;
$2.4 - 2.5$ GeV $(I = 0)$
Phys. Rev. D 84, 074023 (2011)

$2.1 - 2.3$ GeV $(s\bar{s}g)$
Chinese Physics C 45, 013112 (2021)

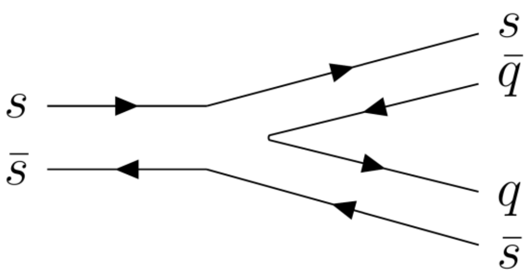
≈ 2.1 GeV $(I = 1)$
Phys. Rev. D 107, 074028 (2023)

$2.2 - 2.3$ GeV (both ρ-like and ϕ-like)
Phys. Rev. D 82, 034508 (2010)

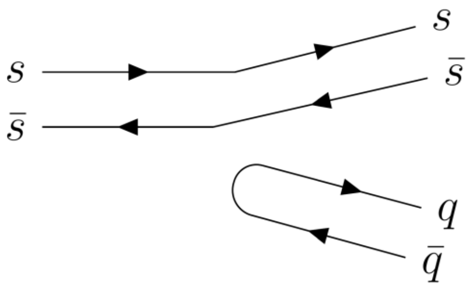
Thank You

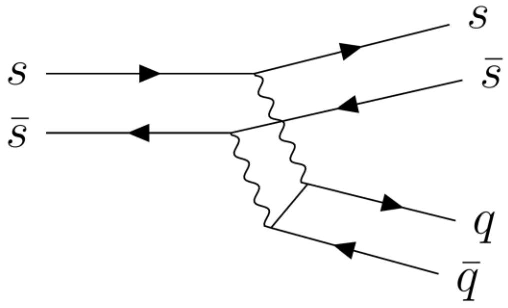
Backup: Decay

Favored



Suppressed



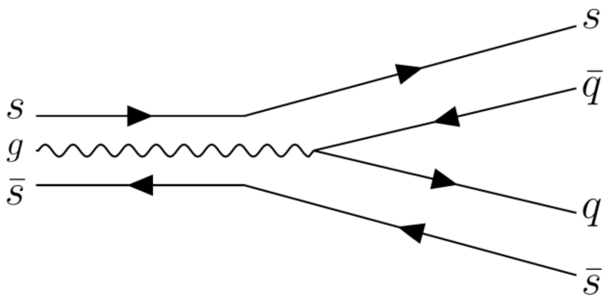


disturb the vacuum → $q\bar{q}$ or two virtual gluons → $q\bar{q}$
transfers momentum $|p| \sim m_{q\bar{q}}$

1^{--} strangeonium state ($2D$ or $3S$)

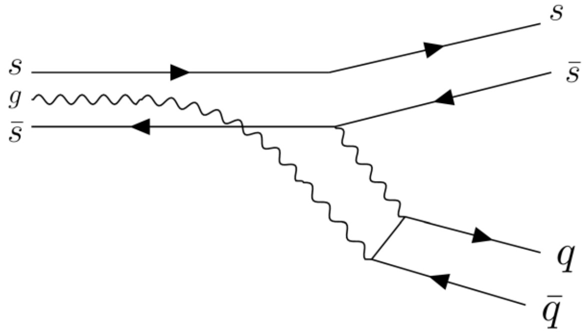
- decay into open strangeness states ($K\bar{K}^*$...)
- branching ratio to $\phi f_0(980)$ should be small
- broad width (large phase space)

Suppressed



(1^{--} $c\bar{c}g$ → $D^{(*)}\bar{D}^{(*)}$ is forbidden by symmetry
Physics Letters B 631 (2005) 164–169)

Allowed



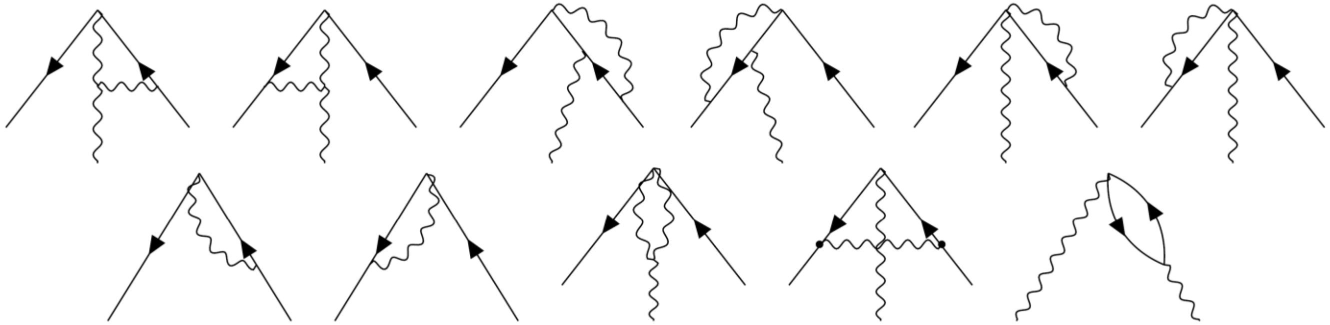
$\phi(2170)$

- decay into closed strangeness states ($\phi f_0(980)$ ...)
- Narrower width ~ 100 MeV (selection rules)

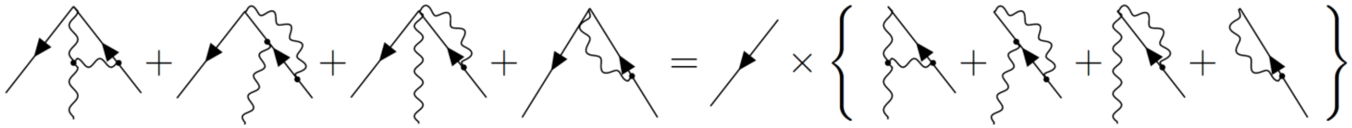
Backup: Renormalization of Generic Hybrid Current

$J = g\bar{\Psi}\Gamma G\Psi$

$\langle T\, J(0)\, \Psi(q_1)\, A(q_2)\, \bar{\Psi}(q_3)\rangle$
$\langle T\, J(0)\, \Psi(q_1)\, \bar{\Psi}(q_2)\rangle$
$\langle T\, J(0)\, A(q_1)\, A(q_2)\rangle$



Observation:



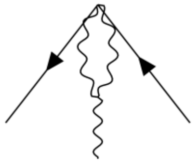
Corresponding counter terms ~ $\frac{1}{\varepsilon}\bar{\Psi}\Gamma \times \{\text{opeators acting on } \Psi\}$

Counterterm bases:

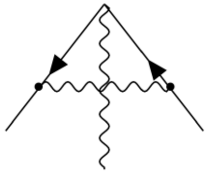
$\frac{1}{\varepsilon}\bar{\Psi}\Gamma.\Gamma' G\Psi$, $\frac{1}{\varepsilon}\bar{\Psi}\Gamma.\Gamma'\nabla\nabla\Psi$, $\frac{1}{\varepsilon}\bar{\Psi}\Gamma.\Gamma' A(\gamma_\mu\nabla^\mu + i\, m)\,\Psi$, $\frac{1}{\varepsilon}\bar{\Psi}\Gamma.\Gamma' m^2\Psi$

non-gauge-invariant counterterms must vanish under equation of motion

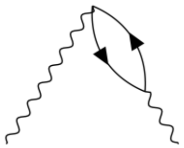
$\Gamma' G \sim G^{\mu\nu}$, $\sigma^{\mu\rho}G_\rho{}^\nu$, $i\,\gamma^5\epsilon^{\mu\nu\rho\eta}G_{\rho\eta}$; $\Gamma'\nabla\nabla \sim \sigma^\nu{}_\rho\,\nabla^\mu\nabla^\rho$, $\sigma^{\mu\nu}\nabla^2$; $\cdots$



$\bar{\Psi}\Gamma T^n\Psi \times \frac{1}{\varepsilon} G^n_{\mu\nu}$



$\frac{1}{\varepsilon}\bar{\Psi}\Gamma_1.\Gamma.\Gamma_2\, T^n\Psi \times G^n_{\mu\nu}$



$\frac{1}{\varepsilon} mGG$, $\frac{1}{\varepsilon} GDG$

Backup: Renormalization of Generic Hybrid Current

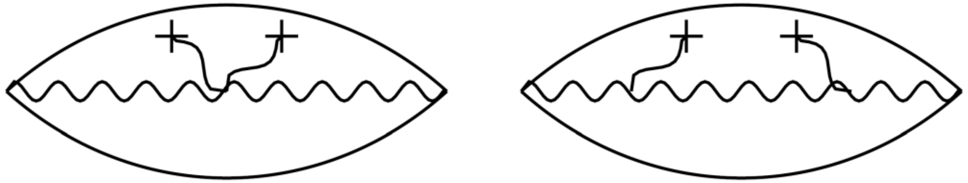
$J = g\overline{\Psi}\Gamma G^{\mu\nu}\Psi$

$$
\begin{aligned}
J_r = &(1 + C_0 + C_4) Z_2^{-1} Z_3^{-\frac{1}{2}}\, \overline{\Psi}_{f_a}\, g\Gamma G^{\mu\nu}\Psi_{f_b} \\
&+ C_1 \left(\overline{\Psi}_{f_a}\big[(\overleftarrow{\nabla}^{\mu}\overleftarrow{\nabla}^{\rho} + \overleftarrow{\nabla}^{\rho}\overleftarrow{\nabla}^{\mu})\sigma_{\rho}{}^{\nu}.\Gamma + \Gamma.\sigma^{\nu}{}_{\rho}(\nabla^{\mu}\nabla^{\rho} + \nabla^{\rho}\nabla^{\mu})\big]\Psi_{f_b} - \{\mu \leftrightarrow \nu\}\right) \\
&+ C_2 \left(\overline{\Psi}_{f_a}\big[(\overleftarrow{\not{\nabla}} - i\, m_{f_a})\gamma^{\nu}.\Gamma A^{\mu} g + g A^{\mu}\Gamma.\gamma^{\nu}.(\not{\nabla} + i\, m_{f_b})\big]\Psi_{f_b} - \{\mu \leftrightarrow \nu\}\right) \\
&+ C_3\, \overline{\Psi}_{f_a}\big(\overleftarrow{\nabla}^{2}\sigma^{\mu\nu}.\Gamma + \Gamma.\sigma^{\mu\nu}\nabla^{2}\big)\Psi_{f_b} \\
&+ C_5\, \overline{\Psi}_{f_a}\big(\gamma^{5}.\Gamma + \Gamma.\gamma^{5}\big) i\, \varepsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}\Psi_{f_b} \\
&+ C_6 \left(\overline{\Psi}_{f_a}\big[i\sigma^{\rho\mu}.\Gamma + i\Gamma.\sigma^{\mu\rho}\big] G_{\rho}{}^{\nu}\Psi_{f_b} - \{\mu \leftrightarrow \nu\}\right) \\
&+ C_7 \left(\overline{\Psi}_{f_a}\big[\overleftarrow{\nabla}^{\mu}\gamma^{\nu}.\Gamma m_{f_a} + m_{f_b}\Gamma.\gamma^{\nu}\nabla^{\mu}\big]\Psi_{f_b} - \{\mu \leftrightarrow \nu\}\right) \\
&+ C_8\, \overline{\Psi}_{f_a}\big(\sigma^{\mu\nu}.\Gamma m_{f_a}^{2} + m_{f_b}^{2}\Gamma.\sigma^{\mu\nu}\big)\Psi_{f_b} \\
&+ C_9\, \overline{\Psi}_{f_a}\gamma^{\rho}.\gamma^{\sigma}.\Gamma.\gamma_{\sigma}.\gamma_{\rho} G_{\mu\nu}\Psi_{f_b} \\
&+ C_{10}\, \delta_{f_a f_b}\left(\mathrm{Tr}\big[\Gamma.\sigma^{\alpha\beta}\big] m_{f_a}\, G^{n\,\mu\nu} G^{n}_{\alpha\beta} + \frac{2}{3}\mathrm{Tr}\big[\Gamma.\gamma_{\alpha}\big] G^{n\,\mu\nu} D^{nm}_{\beta} G^{m\,\alpha\beta}\right).
\end{aligned}
$$

C_0	C_1	C_2	C_3	C_4	C_5
$\frac{3C_A g}{32\pi^2\varepsilon}$	$\frac{C_F g^2}{192\pi^2\varepsilon}$	$\frac{C_A g^2}{128\pi^2\varepsilon}$	$\frac{C_F g^2}{96\pi^2\varepsilon}$	$-\frac{C_A g}{16\pi^2\varepsilon}$	$\frac{C_A g^2}{128\pi^2\varepsilon}$
C_6	C_7	C_8	C_9	C_{10}	
$\frac{(3C_A^2+1)g^2}{128 C_A\pi^2\varepsilon}$	$\frac{C_F g^2}{32\pi^2\varepsilon}$	$\frac{C_F g^2}{32\pi^2\varepsilon}$	$\frac{g^2}{128 C_A\pi^2\varepsilon}$	$\frac{g^2}{64\pi^2\varepsilon}$	

In Feynman gauge

Backup: Infrared divergence



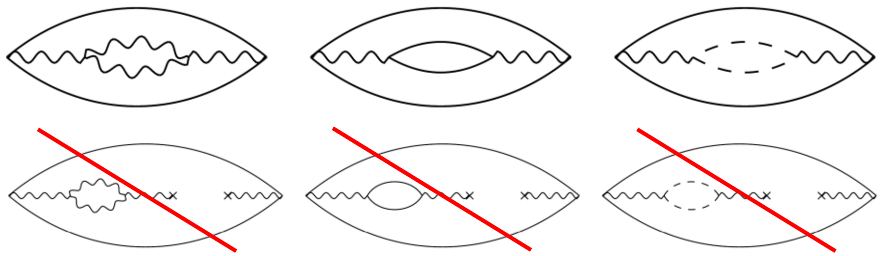
$$\langle T\, G^{\mu\nu}(k) G^{\alpha\beta}(k)\rangle \sim \frac{\langle GG\rangle}{k^4}\left(g^{\mu\alpha}g^{\nu\beta}+\cdots\right)$$

$$\int d^d k\, \frac{1}{k^4}, \qquad \mathrm{d}=4-2\epsilon, \qquad \Rightarrow \sim \frac{1}{\epsilon} \text{ for } k\approx 0$$

$$J_r = (1 + C_0 + C_4) Z_2^{-1} Z_3^{-1/2} g \bar{\Psi}\Gamma G^{\mu\nu}\Psi + \cdots$$

$Z_3 \leftrightarrow$

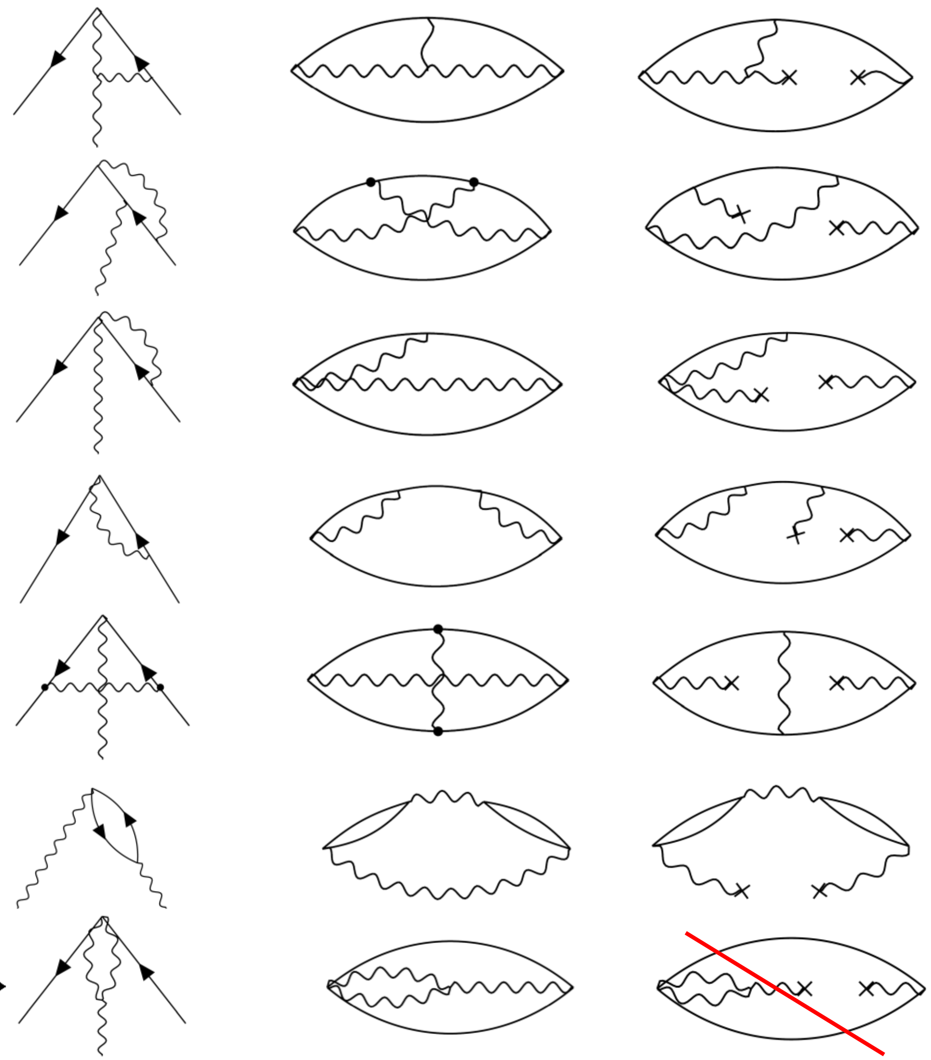
$A_0^\mu = Z_3 A^\mu$



$$\left(C_0 + Z_3^{-1/2} Z_g^{-1} - 1\right) g_0 \bar{\Psi}\Gamma G^{\mu\nu}\Psi$$

$$Z_3 = 1 - \frac{g^2}{16\pi^2\varepsilon}\left[-\frac{5}{3}C_A + \frac{2}{3}n_f\right], \qquad Z_g = 1 + \frac{g^2}{16\pi^2\varepsilon}\left(\frac{11}{6}C_A - \frac{1}{3}n_f\right)$$

Perturbative $\langle GG\rangle$



$C_0 g\bar{\Psi}\Gamma G\Psi \leftrightarrow$

Backup: Error Estimation

$$\chi^2(\tau, s_0) = \sum_{i=0}^{N} \left[\tilde{G}(\tau, s_i, s_0) - \tilde{G}_h(\tau, s_i)\right]^2$$

$$s_i = s_{\min} + i\,\delta s, \qquad \delta s = 0.2\ \mathrm{GeV}^2$$

$$s_{\min} = -15\ \mathrm{GeV}^2, \qquad s_{\max} = 30\ \mathrm{GeV}^2$$

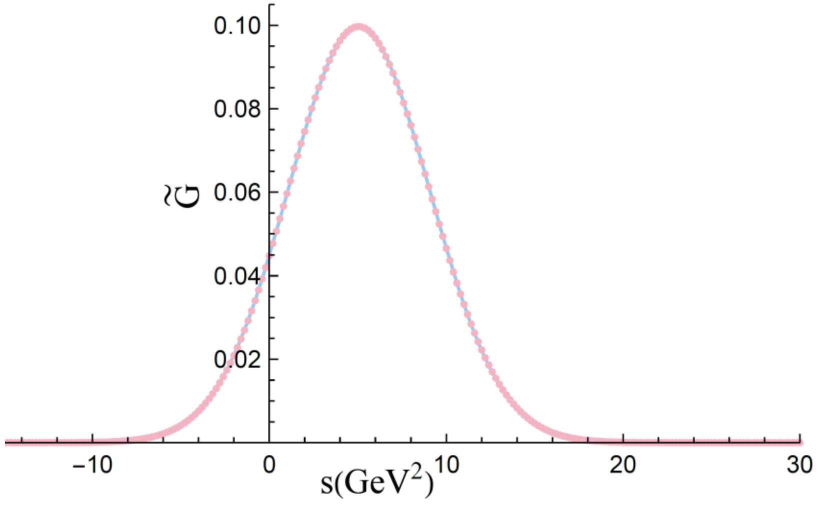
Error is defined by:

$$\sigma = \sqrt{\hat{\sigma}^2 + \overline{\Delta^2}}$$

$\hat{\sigma}^2$: variance of 2000 fitted values ~ statistic error

$$\Delta = \sqrt{2\chi^2_{\min} H^{-1}(\chi^2_{\min})} \qquad \sim \text{systemtic error}$$

$$H(\chi^2) = \begin{bmatrix} \frac{\partial^2 \chi^2}{\partial s_0^2} & \frac{\partial^2 \chi^2}{\partial s_0 \partial m} \\ \frac{\partial^2 \chi^2}{\partial s_0 \partial m} & \frac{\partial^2 \chi^2}{\partial m^2} \end{bmatrix}$$



for small δs

$$\chi^2 \sim \frac{1}{\delta s}, \qquad H^{-1} \sim \delta s \quad \rightarrow \quad \chi^2 H^{-1} \sim 1$$

for $s_{\min}$ and $s_{\max}$ away from the peak
$\tilde{G}(\tau, s_i, s_0)$, $\tilde{G}_h(\tau, s_i) \rightarrow 0$, irrelevant to the specific $s_{\min}, s_{\max}$

(if average the Δ as $\Delta/(s_{\max} - s_{\min})$, choose large $s_{\max} - s_{\min}$ manually reduce the error)

$\Rightarrow$

Δ in the left is irrelevant to the choice of $\delta s, s_{\min}, s_{\max}$
reflect the intrinsic error of the fitting procedure