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The Mass Spectrum and Strong Decays of Tetraquark States via QCD Sum Rules

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CATALOG PAGE

01 Research background

02 Mass spectrum of tetraquark states with QCD sum rules

03 Strong decays of tetraquark states with QCD sum rules

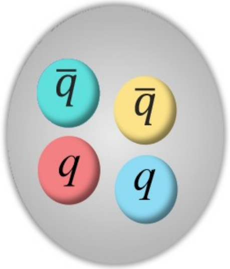
04 Summary

• Exotic hadrons

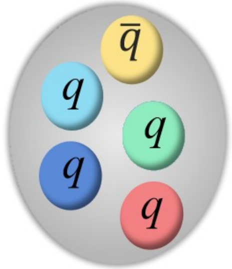
Since the observation of X(3872) in 2003, a number of exotic hadrons states have been observed. There are four types of interpretations.

- Multiquark states
- Multiquark molecular states
- Hybrid states
- Threshold effect, Scattering effect, Cusp effect

- Multiquark states

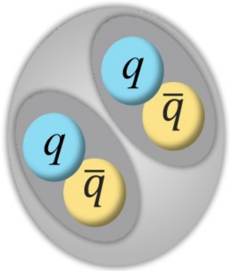


tetraquark states

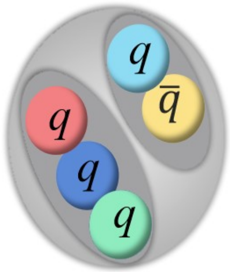


pentaquark states

- Multiquark molecular states

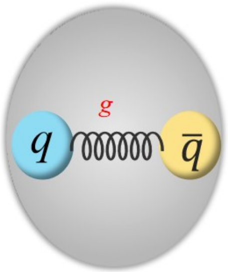


tetraquark molecular states



pentaquark molecular states

- Hybrid states



Research background

Diquark-antiquark type tetraquark currents

Two-point correlation functions for the tetraquark states are,

$$\Pi\left(p^2\right)=i\int d^4x e^{ipx}\left\langle 0\left|T\left\{J(x)J^+(0)\right\}\right|0\right\rangle,$$

The one-gluon-exchange induced attractive interactions favor formation of the diquarks in the color antitriplet, flavor antitriplet and spin singlet.

Using the scalar(S), pseudoscalar(P), vector(V), axialvector(A) and tensor(T) diquarks,

where $C\Gamma=C\gamma_5, C, C\gamma_\mu\gamma_5, C\gamma_\mu$ and $C\sigma_{\mu\nu}\left(C\sigma_{\mu\nu}\gamma_5\right)$,

we can construct diquark-antiquark type tetraquark currents to study the tetraquark states.

The tensor diquarks have both $J^P = 1^+$ and 1^- components, we project out them explicitly, and denote the corresponding $J^P = 1^+$ and 1^- diquarks as $\tilde{A}$ and $\tilde{V}$, respectively.

$$\varepsilon^{ijk}\varepsilon^{imn}q_j^T C\Gamma q_k' \bar{q}_m \Gamma' C \bar{q}_n'^T$$

Research background

- **Color singlet-singlet type currents**
- **P-wave and D-wave**

Using the color singlet operators $q^T(x)\Gamma\overline{c}(x)c(x)\Gamma'\overline{q}'^T(x)$ where $\Gamma=1, i\gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$ and $\sigma_{\mu\nu}$,

we can construct color singlet-singlet type tetraquark currents to study the tetraquark molecular states.

We can introduce the P-wave explicitly in the $C\Gamma$ and Γ to obtain $\varepsilon^{ijk}q_j^T C\Gamma\overleftrightarrow{\partial}_\mu q'_k$ and $q^T(x)\Gamma\overleftrightarrow{\partial}_\mu\overline{q}(x)$

The additional P-wave in the non relativistic quark model can alter the parity by adding a factor $(-)^L = -$, where $L = 1$ is the angular momentum.

We can also adopt the covariant derivative with the simple replacement $\partial_\mu \to D_\mu = \partial_\mu - ig_s G_\mu$,

then the four-quark currents are gauge covariant, however, the covariant derivative D_μ leads to some hybrid components in the hadron states due to the gluon field G_μ.

P-wave and D-wave

P-wave fully-charm tetraquark states

$$0^{++} \quad J(x) = \varepsilon^{ijk}\varepsilon^{imn} c^{Tj}(x) C\gamma_5 \overleftrightarrow{\partial}_\mu c^k(x)\, \bar{c}^m(x) \overleftrightarrow{\partial}_\nu \gamma_5 C \bar{c}^{Tn}(x)\, g^{\mu\nu} ,$$

$$1^{+-} \quad J^1_{\mu\nu}(x) = \varepsilon^{ijk}\varepsilon^{imn}\Big\{ c^{Tj}(x) C\gamma_5 \overleftrightarrow{\partial}_\mu c^k(x)\, \bar{c}^m(x) \overleftrightarrow{\partial}_\nu \gamma_5 C \bar{c}^{Tn}(x) - c^{Tj}(x) C\gamma_5 \overleftrightarrow{\partial}_\nu c^k(x)\, \bar{c}^m(x) \overleftrightarrow{\partial}_\mu \gamma_5 C \bar{c}^{Tn}(x) \Big\} ,$$

$$2^{++} \quad J^2_{\mu\nu}(x) = \varepsilon^{ijk}\varepsilon^{imn}\Big\{ c^{Tj}(x) C\gamma_5 \overleftrightarrow{\partial}_\mu c^k(x)\, \bar{c}^m(x) \overleftrightarrow{\partial}_\nu \gamma_5 C \bar{c}^{Tn}(x) + c^{Tj}(x) C\gamma_5 \overleftrightarrow{\partial}_\nu c^k(x)\, \bar{c}^m(x) \overleftrightarrow{\partial}_\mu \gamma_5 C \bar{c}^{Tn}(x) \Big\} ,$$

Int.J.Mod.Phys.A 36 (2021) 2150014

D-wave charmonium states

$$\psi_1(1^3D_1)1^{--} \quad J_\mu(x) = \bar{c}(x) \overleftrightarrow{D}_\alpha \overleftrightarrow{D}_\beta \gamma_\rho \left(g^{\alpha\beta}g^{\rho\mu} + g^{\alpha\rho}g^{\beta\mu} + g^{\rho\beta}g^{\alpha\mu}\right) c(x) ,$$

$$\psi_2(1^3D_2)2^{--} \quad J^1_{\mu\nu}(x) = \bar{c}(x)\left(\gamma_\mu \gamma\cdot \overleftrightarrow{D}\overleftrightarrow{D}_\nu + \gamma_\mu \overleftrightarrow{D}_\nu \gamma\cdot \overleftrightarrow{D} + \gamma_\nu \gamma\cdot \overleftrightarrow{D}\overleftrightarrow{D}_\mu + \gamma_\nu \overleftrightarrow{D}_\mu \gamma\cdot \overleftrightarrow{D} - g_{\mu\nu} \gamma\cdot \overleftrightarrow{D}\, \gamma\cdot \overleftrightarrow{D}\right) \gamma_5 c(x) ,$$

$$\eta_{c2}(1^1D_2)2^{-+} \quad J^2_{\mu\nu}(x) = \bar{c}(x)\left(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\mu - \frac{1}{2} g_{\mu\nu} \overleftrightarrow{D}\cdot\overleftrightarrow{D}\right)\gamma_5 c(x) ,$$

$$\psi_3(1^3D_2)3^{--} \quad J_{\mu\nu\rho}(x) = \bar{c}(x)\left(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu \gamma_\rho + \overleftrightarrow{D}_\rho \overleftrightarrow{D}_\mu \gamma_\nu + \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\rho \gamma_\mu\right) c(x) ,$$

Int. J. Mod. Phys. A41 (2026) 2650052

- **Two-point correlation functions**

Two-point correlation functions for the tetraquark states are,

$$\Pi\left(p^2\right) = i\int d^4x e^{ipx} \left\langle 0\middle| T\left\{J(x)J^+(0)\right\}\middle|0\right\rangle,$$

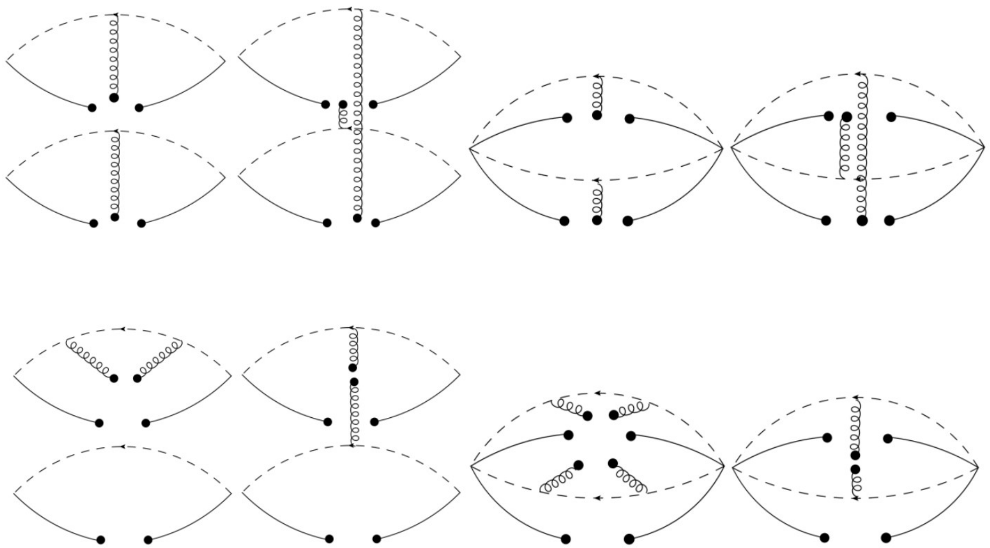
We focus on the quark-gluon degrees of freedom and calculate the correlation functions using Wilson's operator product expansion to separate the physics of short and long distances,

$$\Pi(p^2) = \sum_n C_n(p^2,\mu)\langle\mathcal{O}_n(\mu)\rangle\,,$$

The Wilson coefficients $C_n(p^2,\mu)$ depend only on short-distance dynamics, the vacuum condensates $\langle\mathcal{O}_n(\mu)\rangle$ embody the long-distance effects.

The higher dimensional vacuum condensates

Each heavy quark line emits a gluon and each light quark line contributes a quark-antiquark pair, to reach the higher dimensional vacuum condensates,



The feynman diagrams of 10 dimension vacuum condensate contributions for doubly-charmd tetraquark molecular states

we obtain a quark-gluon operator,

$$\underbrace{G_{\mu\nu}\cdots G_{\alpha\beta}}\,\underbrace{\bar{q}q\,\bar{q}q\cdots\bar{q}q\,\bar{q}q}\,,$$

Research background

- ## Two-point correlation functions

After operator product expansion, the spectral densities at the QCD side are,

$$\Pi\left(p^2\right)=\frac{1}{\pi}\int_{4m_Q^2}^{\infty} ds \frac{\mathrm{Im}\,\Pi_{QCD}(s)}{s-p^2}$$

At the hadron side, the correlation functions can be written as,

$$\Pi\left(p^2\right)=\frac{1}{\pi}\int_{4m_Q^2}^{s_0} ds \frac{\mathrm{Im}\,\Pi_H(s)}{s-p^2}+\frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\,\Pi_H(s)}{s-p^2}$$

$$=\frac{\lambda_X^2}{M_X^2-p^2}+\frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\,\Pi_H(s)}{s-p^2}, \qquad \langle 0|J(0)|X(p)\rangle=\lambda_X,$$

We match the hadron side with the QCD side of the correlation functions according to quark-hadron duality, and obtain the QCD sum rules by performing Borel transform,

$$\lambda_X^2 \exp\left(-\frac{M_X^2}{T^2}\right)=\int_{4m_Q^2}^{s_0} ds \frac{\mathrm{Im}\,\Pi(s)}{\pi}\exp\left(-\frac{s}{T^2}\right),$$

Finally, we eliminate the pole residue λ to obtain the QCD sum rules for the ground state mass

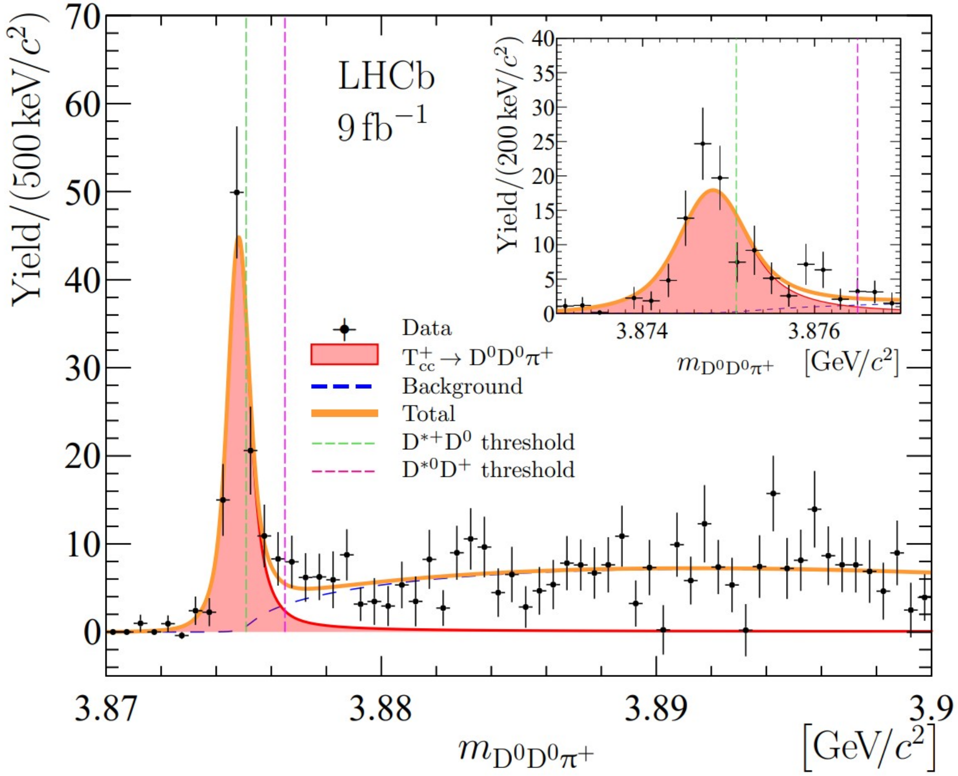
$$M_X^2=-\frac{\frac{d}{d\tau}\int_{4m_Q^2}^{s_0} ds\, \mathrm{Im}\,\Pi(s)\exp(-s\tau)}{\int_{4m_Q^2}^{s_0} ds\, \mathrm{Im}\,\Pi(s)\exp(-s\tau)}\Bigg|_{\tau=\frac{1}{T^2}}$$

Mass spectrum

- *Tcc*(3875)

The exotic state T_{cc}^{+} is consistent with the ground state isoscalar tetraquark state with a valence quark content of $cc\overline{u}\overline{d}$ and spin-parity quantum numbers $J^{P} = 1^{+}$

$$\delta M_{BW} = M_{T_{cc}^{+}} - \left(M_{D^{*+}} + M_{D^{0}}\right)$$
$$= -273 \pm 61 \pm 5_{-14}^{+11}\, KeV$$



Nature Physics, 2022, 18(7): 751

Nature Communications, 2022, 13(1): 3351

Doubly heavy tetraquark states

- Doubly heavy tetraquark states

$$\eta_\mu(x) = \varepsilon^{ijk}\varepsilon^{imn}\, Q_j^T(x)C\gamma_\mu Q_k(x)\, \bar{u}_m(x)\gamma_5 C\bar{d}_n^T(x)\,,$$

	$T^2(\mathrm{GeV}^2)$	$\sqrt{s_0}(\mathrm{GeV})$	$\mu(\mathrm{GeV})$	pole	$M(\mathrm{GeV})$	$\lambda(\mathrm{GeV}^5)$
$cc\bar{u}\bar{d}$	$2.6-3.0$	4.45 ± 0.10	1.3	$(39-63)\%$	3.90 ± 0.09	$(2.64\pm0.42)\times10^{-2}$

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If we perform Fierz rearrangements for the axialvector-diquark-scalar-antidiquark type four quark axialvector currents $\eta_\mu(x)$ both in the color and Dirac spinor spaces, we can obtain special superpositions of the color-singlet-color-singlet type currents,

$$\begin{aligned}
\eta_\mu(x) &= \varepsilon^{ijk}\varepsilon^{imn}Q_j^T(x)C\gamma_\mu Q_k(x)\bar{u}_m(x)\gamma_5 C\bar{d}_n^T(x) \\
&= \frac{i}{2}\left[\bar{u}i\gamma_5 Q\bar{d}\gamma_\mu Q - \bar{d}i\gamma_5 Q\bar{u}\gamma_\mu Q\right] + \frac{1}{2}\left[\bar{u}Q\bar{d}\gamma_\mu\gamma_5 Q - \bar{d}Q\bar{u}\gamma_\mu\gamma_5 Q\right] \\
&\quad - \frac{i}{2}\left[\bar{u}\sigma_{\mu\nu}\gamma_5 Q\bar{d}\gamma^\nu Q - \bar{d}\sigma_{\mu\nu}\gamma_5 Q\bar{u}\gamma^\nu Q\right] + \frac{i}{2}\left[\bar{u}\sigma_{\mu\nu}Q\bar{d}\gamma^\nu\gamma_5 Q - \bar{d}\sigma_{\mu\nu}Q\bar{u}\gamma^\nu\gamma_5 Q\right] \\
&= \frac{i}{2}J_\mu^1(x) + \frac{i}{2}J_\mu^2(x) - \frac{i}{2}J_\mu^3(x) + \frac{i}{2}J_\mu^4(x)
\end{aligned}$$

Eur. Phys. J. A58 (2022) 110

Mass spectrum

- **Doubly heavy tetraquark molecular states**

$$J_{DD}(x)=\bar{u}(x)i\gamma_5c(x)\bar{d}(x)i\gamma_5c(x)$$
$$J_{DD_s}(x)=\bar{q}(x)i\gamma_5c(x)\bar{s}(x)i\gamma_5c(x)$$
$$J_{D_sD_s}(x)=\bar{s}(x)i\gamma_5c(x)\bar{s}(x)i\gamma_5c(x)$$
$$J_{D^*D^*}(x)=\bar{u}(x)\gamma_\mu c(x)\bar{d}(x)\gamma^\mu c(x)$$
$$J_{D^*D_s^*}(x)=\bar{q}(x)\gamma_\mu c(x)\bar{s}(x)\gamma^\mu c(x)$$
$$J_{D_s^*D_s^*}(x)=\bar{s}(x)\gamma_\mu c(x)\bar{s}(x)\gamma^\mu c(x)$$

$$J_{D^*D^*,L,\mu\nu}(x)=\frac{1}{\sqrt{2}}\{\bar{u}(x)\gamma_\mu c(x)\bar{d}(x)\gamma_\nu c(x)-\bar{u}(x)\gamma_\nu c(x)\bar{d}(x)\gamma_\mu c(x)\},$$
$$J_{D^*D^*,H,\mu\nu}(x)=\frac{1}{\sqrt{2}}\{\bar{u}(x)\gamma_\mu c(x)\bar{d}(x)\gamma_\nu c(x)+\bar{u}(x)\gamma_\nu c(x)\bar{d}(x)\gamma_\mu c(x)\},$$
$$J_{D^*D_s^*,L,\mu\nu}(x)=\frac{1}{\sqrt{2}}\{\bar{q}(x)\gamma_\mu c(x)\bar{s}(x)\gamma_\nu c(x)-\bar{q}(x)\gamma_\nu c(x)\bar{s}(x)\gamma_\mu c(x)\},$$
$$J_{D^*D_s^*,\mathrm{H},\mu\nu}(x)=\frac{1}{\sqrt{2}}\{\bar{q}(x)\gamma_\mu c(x)\bar{s}(x)\gamma_\nu c(x)+\bar{q}(x)\gamma_\nu c(x)\bar{s}(x)\gamma_\mu c(x)\},$$
$$J_{D_s^*D_s^*,L,\mu\nu}(x)=\frac{1}{\sqrt{2}}\{\bar{s}(x)\gamma_\mu c(x)\bar{s}(x)\gamma_\nu c(x)-\bar{s}(x)\gamma_\nu c(x)\bar{s}(x)\gamma_\mu c(x)\},$$
$$J_{D_s^*D_s^*,H,\mu\nu}(x)=\frac{1}{\sqrt{2}}\{\bar{s}(x)\gamma_\mu c(x)\bar{s}(x)\gamma_\nu c(x)+\bar{s}(x)\gamma_\nu c(x)\bar{s}(x)\gamma_\mu c(x)\},$$

$$J_{DD^*,L,\mu}(x)=\frac{1}{\sqrt{2}}\{\bar{u}(x)i\gamma_5c(x)\bar{d}(x)\gamma_\mu c(x)-\bar{u}(x)\gamma_\mu c(x)\bar{d}(x)i\gamma_5c(x)\},$$
$$J_{DD^*,H,\mu}(x)=\frac{1}{\sqrt{2}}\{\bar{u}(x)i\gamma_5c(x)\bar{d}(x)\gamma_\mu c(x)+\bar{u}(x)\gamma_\mu c(x)\bar{d}(x)i\gamma_5c(x)\},$$
$$J_{DD_s^*,L,\mu}(x)=\frac{1}{\sqrt{2}}\{\bar{q}(x)i\gamma_5c(x)\bar{s}(x)\gamma_\mu c(x)-\bar{q}(x)\gamma_\mu c(x)\bar{s}(x)i\gamma_5c(x)\},$$
$$J_{DD_s^*,H,\mu}(x)=\frac{1}{\sqrt{2}}\{\bar{q}(x)i\gamma_5c(x)\bar{s}(x)\gamma_\mu c(x)+\bar{q}(x)\gamma_\mu c(x)\bar{s}(x)i\gamma_5c(x)\},$$
$$J_{D_sD_s^*,\mu}(x)=\bar{s}(x)i\gamma_5c(x)\bar{s}(x)\gamma_\mu c(x),$$
$$J_{D_1D_0^*,L,\mu}(x)=\frac{1}{\sqrt{2}}\{\bar{u}(x)c(x)\bar{d}(x)\gamma_\mu\gamma_5c(x)+\bar{u}(x)\gamma_\mu\gamma_5c(x)\bar{d}(x)c(x)\},$$
$$J_{D_1D_0^*,H,\mu}(x)=\frac{1}{\sqrt{2}}\{\bar{u}(x)c(x)\bar{d}(x)\gamma_\mu\gamma_5c(x)-\bar{u}(x)\gamma_\mu c(x)\bar{d}(x)c(x)\},$$
$$J_{D_{s1}D_0^*,L,\mu}(x)=\frac{1}{\sqrt{2}}\{\bar{q}(x)c(x)\bar{s}(x)\gamma_\mu\gamma_5c(x)+\bar{q}(x)\gamma_\mu\gamma_5c(x)\bar{s}(x)c(x)\},$$
$$J_{D_{s1}D_0^*,H,\mu}(x)=\frac{1}{\sqrt{2}}\{\bar{q}(x)c(x)\bar{s}(x)\gamma_\mu\gamma_5c(x)-\bar{q}(x)\gamma_\mu\gamma_5c(x)\bar{s}(x)c(x)\},$$
$$J_{D_{s1}D_{s0}^*,\mu}(x)=\bar{s}(x)c(x)\bar{s}(x)\gamma_\mu\gamma_5c(x),$$

Doubly heavy tetraquark molecular states

The correlation functions $\Pi(p^2)$ do not depend on the energy scale μ,

$$\frac{d}{d\mu}\Pi(p^2) \quad = \quad 0\,,$$

$\frac{\mathrm{Im}\Pi(s)}{\pi} = \rho_{QCD}(s,\mu)$ but which does not mean

$$\frac{d}{d\mu}\int_{4m_Q^2(\mu)}^{s_0} ds \frac{\rho_{QCD}(s,\mu)}{s-p^2} \rightarrow 0\,,$$

- Perturbative corrections are neglected, the higher dimensional vacuum condensates are factorized into lower dimensional ones based on the vacuum saturation.
- Truncations s_0 which are physical quantities determined by the experimental data set in, the correlations between the thresholds $4m_Q^2(\mu)$ and continuum thresholds s_0 are unknown. Quark hadron duality is just an assumption.

Doubly heavy tetraquark molecular states

We use energy scale formula to determine the optimal energy scales,

$$\mu=\sqrt{M_X^2-\left(2\mathrm{M}_Q\right)^2}-k\mathrm{M}_S,$$

and take account of the energy-scale dependence of the quark condensates, mixed quark condensates and MS masses

$$\langle\bar{q}q\rangle(\mu)=\langle\bar{q}q\rangle(1\mathrm{GeV})\left[\frac{\alpha_s(1\mathrm{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}},$$

$$\langle\bar{s}s\rangle(\mu)=\langle\bar{s}s\rangle(1\mathrm{GeV})\left[\frac{\alpha_s(1\mathrm{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}},$$

$$\langle\bar{q}g_s\sigma Gq\rangle(\mu)=\langle\bar{q}g_s\sigma Gq\rangle(1\mathrm{GeV})\left[\frac{\alpha_s(1\mathrm{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}},$$

$$\langle\bar{s}g_s\sigma Gs\rangle(\mu)=\langle\bar{s}g_s\sigma Gs\rangle(1\mathrm{GeV})\left[\frac{\alpha_s(1\mathrm{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}},$$

$$m_c(\mu)=m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}},$$

$$m_s(\mu)=m_s(2\mathrm{GeV})\left[\frac{\alpha_s(\mu)}{\alpha_s(2\mathrm{GeV})}\right]^{\frac{12}{33-2n_f}},$$

$$\alpha_s(\mu)=\frac{1}{b_0t}\left[1-\frac{b_1}{b_0^2}\frac{\log t}{t}+\frac{b_1^2\left(\log^2 t-\log t-1\right)+b_0b_2}{b_0^4t^2}\right],$$

$$t=\log\frac{\mu^2}{\Lambda^2},b_0=\frac{33-2n_f}{12\pi},b_1=\frac{153-19n_f}{24\pi^2},b_2=\frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$$

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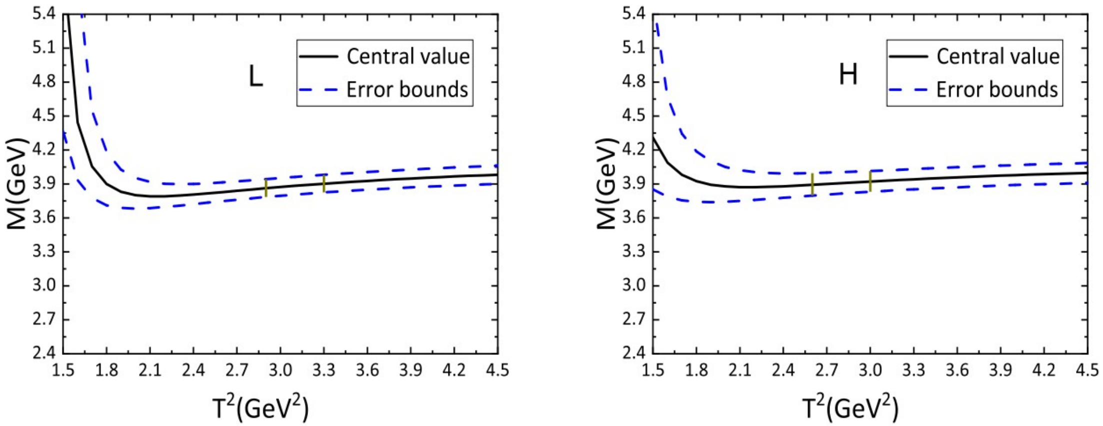
Doubly heavy tetraquark molecular states

Parameters to satisfy the following criteria:

- Appearance of the Borel platforms;
- Pole dominance at the hadron side;
- Convergence of the operator product expansion

$$pole = \frac{\int_{4m_c^2}^{s_0} ds \rho_{QCD}(s) \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{\infty} ds \rho_{QCD}(s) \exp\left(-\frac{s}{T^2}\right)} \approx (40-60)\%$$



The masses with variations of the Borel parameters for the axialvector tetraquark molecular states, where the *L* and *H* denote the lighter and heavier *DD** states, respectively

Mass spectrum

Doubly heavy tetraquark molecular states

T_{cc}	Isospin	$T^2(\text{GeV}^2)$	$\sqrt{s_0}(\text{GeV})$	$\mu(\text{GeV})$	pole	$\|D(10)\|$	$M_T(\text{GeV})$	$\lambda_T(\text{GeV}^5)$
DD	1	$2.6-3.0$	4.30 ± 0.10	1.4	$(40-64)\%$	$\ll 1\%$	3.75 ± 0.09	$(1.48\pm0.23)\times10^{-2}$
DD_s	$\frac{1}{2}$	$2.7-3.1$	4.40 ± 0.10	1.4	$(41-64)\%$	$\ll 1\%$	3.85 ± 0.09	$(1.69\pm0.26)\times10^{-2}$
D_sD_s	0	$2.8-3.2$	4.50 ± 0.10	1.4	$(41-63)\%$	$\ll 1\%$	3.95 ± 0.09	$(2.00\pm0.32)\times10^{-2}$
D^*D^*	1	$2.8-3.2$	4.55 ± 0.10	1.7	$(41-61)\%$	$< 1\%$	4.04 ± 0.11	$(4.99\pm0.66)\times10^{-2}$
$D_s^*D^*$	$\frac{1}{2}$	$2.9-3.3$	4.65 ± 0.10	1.7	$(42-62)\%$	$\ll 1\%$	4.12 ± 0.10	$(5.74\pm0.78)\times10^{-2}$
$D_s^*D_s^*$	0	$3.2-3.5$	4.80 ± 0.10	1.8	$(42-61)\%$	$\ll 1\%$	4.22 ± 0.10	$(7.46\pm0.89)\times10^{-2}$
D^*D-DD^*	0	$2.9-3.3$	4.45 ± 0.10	1.4	$(42-62)\%$	$\ll 1\%$	3.88 ± 0.11	$(1.92\pm0.29)\times10^{-2}$
D^*D+DD^*	1	$2.6-3.0$	4.40 ± 0.10	1.4	$(42-63)\%$	$\ll 1\%$	3.90 ± 0.11	$(1.50\pm0.22)\times10^{-2}$
$D_s^*D-D_sD^*$	$\frac{1}{2}$	$3.0-3.4$	4.50 ± 0.10	1.5	$(40-62)\%$	$< 1\%$	3.97 ± 0.10	$(2.40\pm0.41)\times10^{-2}$
$D_s^*D+D_sD^*$	$\frac{1}{2}$	$2.9-3.3$	4.50 ± 0.10	1.5	$(40-60)\%$	$\ll 1\%$	3.98 ± 0.11	$(2.06\pm0.30)\times10^{-2}$
$D_s^*D_s$	0	$3.0-3.4$	4.60 ± 0.10	1.5	$(41-63)\%$	$\ll 1\%$	4.10 ± 0.12	$(2.31\pm0.45)\times10^{-2}$
$D_0^*D_1-D_1D_0^*$	0	$5.6-7.0$	6.35 ± 0.10	4.6	$(41-60)\%$	$\ll 1\%$	5.79 ± 0.15	$(2.13\pm0.19)\times10^{-1}$
$D_0^*D_1+D_1D_0^*$	1	$4.7-6.1$	5.90 ± 0.10	4.0	$(42-61)\%$	$\ll 1\%$	5.37 ± 0.13	$(1.30\pm0.11)\times10^{-1}$
$D_0^*D_{s1}-D_{s0}^*D_1$	$\frac{1}{2}$	$5.8-7.2$	6.50 ± 0.10	4.6	$(43-60)\%$	$< 1\%$	5.93 ± 0.27	$(2.80\pm0.33)\times10^{-1}$
$D_0^*D_{s1}+D_{s0}^*D_1$	$\frac{1}{2}$	$4.7-6.1$	6.05 ± 0.10	4.0	$(42-62)\%$	$< 1\%$	5.54 ± 0.20	$(1.51\pm0.16)\times10^{-1}$
$D_{s1}D_{s0}^*$	0	$4.9-6.3$	6.20 ± 0.10	4.0	$(43-61)\%$	$< 1\%$	5.67 ± 0.27	$(1.77\pm0.27)\times10^{-1}$
$D^*D^*-D^*D^*$	0	$3.2-3.6$	4.55 ± 0.10	1.7	$(42-61)\%$	$< 1\%$	4.00 ± 0.11	$(2.47\pm0.32)\times10^{-2}$
$D^*D^*+D^*D^*$	1	$3.0-3.4$	4.55 ± 0.10	1.7	$(41-60)\%$	$< 1\%$	4.02 ± 0.11	$(2.83\pm0.30)\times10^{-2}$
$D_s^*D^*-D_s^*D^*$	$\frac{1}{2}$	$3.3-3.7$	4.65 ± 0.10	1.7	$(40-59)\%$	$\ll 1\%$	4.08 ± 0.10	$(2.81\pm0.40)\times10^{-2}$
$D_s^*D^*+D_s^*D^*$	$\frac{1}{2}$	$3.1-3.5$	4.65 ± 0.10	1.7	$(42-61)\%$	$< 1\%$	4.10 ± 0.11	$(3.19\pm0.44)\times10^{-2}$
$D_s^*D_s^*-D_s^*D_s^*$	0	$3.6-4.0$	4.80 ± 0.10	1.8	$(40-60)\%$	$\ll 1\%$	4.19 ± 0.09	$(3.49\pm0.49)\times10^{-2}$
$D_s^*D_s^*+D_s^*D_s^*$	0	$3.4-3.9$	4.80 ± 0.10	1.8	$(41-61)\%$	$< 1\%$	4.20 ± 0.10	$(4.00\pm0.53)\times10^{-2}$

Eur. Phys. J. A58 (2022) 110
AAPPS Bull. 32 (2022) 37

Mass spectrum

Hidden-charm tetraquark molecular states

If one or both of the color-neutral clusters contain P-waves, the mass of the tetraquark molecular states calculated using QCD sum rules greater than or significantly greater than the threshold of the corresponding two mesons.

	$T^2(\mathrm{GeV}^2)$	$\sqrt{s_0}(\mathrm{GeV})$	pole	$\mu(\mathrm{GeV})$	$M_Y(\mathrm{GeV})$	$\lambda_Y(10^{-2}\mathrm{GeV}^5)$
$D\bar{D}_1\ (1^{--})$	$3.2-3.6$	4.9 ± 0.1	$(45-65)\%$	2.3	4.36 ± 0.08	3.97 ± 0.54
$D\bar{D}_1\ (1^{-+})$	$3.5-3.9$	5.1 ± 0.1	$(44-63)\%$	2.7	4.60 ± 0.08	5.26 ± 0.65
$D^*\bar{D}_0^*\ (1^{--})$	$4.0-4.4$	5.3 ± 0.1	$(44-61)\%$	3.0	4.78 ± 0.07	7.56 ± 0.84
$D^*\bar{D}_0^*\ (1^{-+})$	$3.8-4.2$	5.2 ± 0.1	$(44-61)\%$	2.9	4.73 ± 0.07	6.83 ± 0.84

Do not support interpreting Y(4260) as a $D\bar{D}_1$ molecular state.

Chin.Phys. C41 (2017) 083103

- **Hidden-charm tetraquark states**

Y_c	J^{PC}	M_Y(GeV)	Assignments
$[uc]_P[\overline{dc}]_A-[uc]_A[\overline{dc}]_P$	1^{--}	4.66 ± 0.07	? $Y(4660)$
$[uc]_P[\overline{dc}]_A+[uc]_A[\overline{dc}]_P$	1^{-+}	4.61 ± 0.07	
$[uc]_S[\overline{dc}]_V+[uc]_V[\overline{dc}]_S$	1^{--}	4.35 ± 0.08	? $Y(4360/4390)$
$[uc]_S[\overline{dc}]_V-[uc]_V[\overline{dc}]_S$	1^{-+}	4.66 ± 0.09	
$[uc]_{\tilde{V}}[\overline{dc}]_A-[uc]_A[\overline{dc}]_{\tilde{V}}$	1^{--}	4.53 ± 0.07	? $Y(4500)$
$[uc]_{\tilde{V}}[\overline{dc}]_A+[uc]_A[\overline{dc}]_{\tilde{V}}$	1^{-+}	4.65 ± 0.08	
$[uc]_{\tilde{A}}[\overline{dc}]_V+[uc]_V[\overline{dc}]_{\tilde{A}}$	1^{--}	4.48 ± 0.08	? $Y(4500)$
$[uc]_{\tilde{A}}[\overline{dc}]_V-[uc]_V[\overline{dc}]_{\tilde{A}}$	1^{-+}	4.55 ± 0.07	
$[uc]_S[\overline{dc}]_{\tilde{V}}-[uc]_{\tilde{V}}[\overline{dc}]_S$	1^{--}	4.50 ± 0.09	? $Y(4500)$
$[uc]_S[\overline{dc}]_{\tilde{V}}+[uc]_{\tilde{V}}[\overline{dc}]_S$	1^{-+}	4.50 ± 0.09	
$[uc]_P[\overline{dc}]_{\tilde{A}}-[uc]_{\tilde{A}}[\overline{dc}]_P$	1^{--}	4.60 ± 0.07	
$[uc]_P[\overline{dc}]_{\tilde{A}}+[uc]_{\tilde{A}}[\overline{dc}]_P$	1^{-+}	4.61 ± 0.08	
$[uc]_A[\overline{dc}]_A$	1^{--}	4.69 ± 0.08	? $Y(4660)$

Eur. Phys. J. C78 (2018) 518
Front. Phys. 21 (2026) 016300

Three-point correlation functions,

$$\Pi(p,q)=i^2\int d^4x d^4y e^{ipx}e^{iqy}\langle 0|T\left\{J_B(x)J_C(y)J_A^+(0)\right\}|0\rangle,$$

where the currents $J_A(0)$ interpolate the initial tetraquark state, the $J_B(x)$ and $J_C(y)$ interpolate the final conventional mesons B and C,

$$\langle 0|J_A(0)|A(p')\rangle=\lambda_A,$$
$$\langle 0|J_B(0)|B(p)\rangle=\lambda_B,$$
$$\langle 0|J_C(0)|C(q)\rangle=\lambda_C,$$

the λ_A, λ_B and λ_C are pole residues or decay constants. Then we define the hadronic coupling constants,

$$\langle B(p)C(q)|A(p')\rangle=iG_{ABC}.$$

- **Hidden-charm tetraquark states**

At the hadron side, the correlation functions can be expressed as,

$$\Pi_H\left(p'^2,p^2,q^2\right)=\int_{(m_B+m_C)^2}^{s_A^0}ds'\int_{\Delta_s^2}^{s_B^0}ds\int_{\Delta_u^2}^{s_C^0}du\frac{\rho_H\left(s',s,u\right)}{\left(s'-p'^2\right)\left(s-p^2\right)\left(u-q^2\right)}$$

$$+\int_{s_A^0}^{\infty}ds'\int_{\Delta_s^2}^{s_B^0}ds\int_{\Delta_u^2}^{s_C^0}du\frac{\rho_H\left(s',s,u\right)}{\left(s'-p'^2\right)\left(s-p^2\right)\left(u-q^2\right)},$$

At the QCD side, we carry out operator product expansion and the correlation functions can be written as,

$$\Pi_{QCD}\left(p'^2,p^2,q^2\right)=\int_{\Delta_s^2}^{s_B^0}ds\int_{\Delta_u^2}^{s_C^0}du\frac{\rho_{QCD}\left(p'^2,s,u\right)}{\left(s-p^2\right)\left(u-q^2\right)}+\dots,$$

The triple dispersion relation at the hadron side cannot match with the double dispersion relation at the QCD side,

$$\int_{\Delta_s^2}^{s_B^0}ds\int_{\Delta_u^2}^{s_C^0}du\frac{\rho_{QCD}\left(s,u\right)}{\left(s-p^2\right)\left(u-q^2\right)}=\int_{\Delta_s^2}^{s_B^0}ds\int_{\Delta_u^2}^{s_C^0}du\int_{(m_B+m_C)^2}^{\infty}ds'\frac{\rho_H\left(s',s,u\right)}{\left(s'-p'^2\right)\left(s-p^2\right)\left(u-q^2\right)}$$

$$=\frac{\lambda_A\lambda_B\lambda_C G_{ABC}}{\left(m_A^2-p'^2\right)\left(m_B^2-p^2\right)\left(m_C^2-q^2\right)}+\frac{C_{A'BC}}{\left(m_B^2-p^2\right)\left(m_C^2-q^2\right)}$$

Strong decays of tetraquark states with QCD sum rules

Hidden-charm tetraquark states

We set $p'^2=\alpha p^2$, where the α is a constant, and perform double Borel transform to get the QCD sum rules,

$$\frac{\lambda_A\lambda_B\lambda_C G_{ABC}}{\left(\tilde m_A^2-m_B^2\right)}\left[\exp\left(-\frac{m_B^2}{T_1^2}\right)-\exp\left(-\frac{\tilde m_A^2}{T_1^2}\right)\right]\exp\left(-\frac{m_C^2}{T_2^2}\right)+C_{A'BC}\exp\left(-\frac{m_B^2}{T_1^2}-\frac{m_C^2}{T_2^2}\right)$$
$$=\int_{\Delta_s^2}^{s_B^0}ds\int_{\Delta_u^2}^{s_C^0}du\,\rho_{QCD}(s,u)\exp\left(-\frac{s}{T_1^2}-\frac{u}{T_2^2}\right),$$

Channels	Γ(MeV)
$Y_{\tilde A V}\to \bar D^0 D^0,\ \bar D^- D^+$	22.5 ± 2.1
$Y_{\tilde A V}\to \frac{\bar D^{0*}D^0-\bar D^0 D^{*0}}{\sqrt2},\ \frac{\bar D^{-*}D^+-\bar D^- D^{*+}}{\sqrt2}$	0.08 ± 0.01
$Y_{\tilde A V}\to \bar D^{*0} D^{*0},\ \bar D^{*-} D^{*+}$	9.93 ± 0.84
$Y_{\tilde A V}\to \frac{\bar D_0^0 D^{*0}-\bar D^{*0} D_0^0}{\sqrt2},\ \frac{\bar D_0^- D^{*+}-\bar D^{*-} D_0^+}{\sqrt2}$	1.92 ± 0.13
$Y_{\tilde A V}\to \frac{\bar D_1^0 D^0-\bar D^0 D_1^0}{\sqrt2},\ \frac{\bar D_1^- D^+-\bar D^- D_1^+}{\sqrt2}$	59.7 ± 5.5
$Y_{\tilde A V}\to \eta_c\omega$	3.83 ± 0.66
$Y_{\tilde A V}\to J/\psi\omega$	0.0
$Y_{\tilde A V}\to \chi_{c0}\omega$	11.3 ± 1.7
$Y_{\tilde A V}\to \chi_{c1}\omega$	24.4 ± 1.9
$Y_{\tilde A V}\to J/\psi f_0(500)$	13.9 ± 1.7

$Y_{\tilde V A}\to \bar D^0 D^0,\ \bar D^- D^+$	0.009 ± 0.001
$Y_{\tilde V A}\to \frac{\bar D^{0*}D^0-\bar D^0 D^{*0}}{\sqrt2},\ \frac{\bar D^{-*}D^+-\bar D^- D^{*+}}{\sqrt2}$	3.88 ± 0.26
$Y_{\tilde V A}\to \bar D^{*0} D^{*0},\ \bar D^{*-} D^{*+}$	0.047 ± 0.003
$Y_{\tilde V A}\to \frac{\bar D_0^0 D^{*0}-\bar D^{*0} D_0^0}{\sqrt2},\ \frac{\bar D_0^- D^{*+}-\bar D^{*-} D_0^+}{\sqrt2}$	66.3 ± 6.1
$Y_{\tilde V A}\to \frac{\bar D_1^0 D^0-\bar D^0 D_1^0}{\sqrt2},\ \frac{\bar D_1^- D^+-\bar D^- D_1^+}{\sqrt2}$	4.10 ± 0.27
$Y_{\tilde V A}\to \eta_c\omega$	5.65 ± 0.85
$Y_{\tilde V A}\to J/\psi\omega$	0.0
$Y_{\tilde V A}\to \chi_{c0}\omega$	11.1 ± 1.6
$Y_{\tilde V A}\to \chi_{c1}\omega$	29.9 ± 2.5
$Y_{\tilde V A}\to J/\psi f_0(500)$	15.2 ± 1.8

$Y_{S\tilde V}\to \bar D^0 D^0,\ \bar D^- D^+$	1.88 ± 0.38
$Y_{S\tilde V}\to \frac{\bar D^{0*}D^0-\bar D^0 D^{*0}}{\sqrt2},\ \frac{\bar D^{-*}D^+-\bar D^- D^{*+}}{\sqrt2}$	0.20 ± 0.01
$Y_{S\tilde V}\to \frac{\bar D^{0*}D^0-\bar D^0 D^{*0}}{\sqrt2},\ \frac{\bar D^{-*}D^+-\bar D^- D^{*+}}{\sqrt2}$	0.38 ± 0.04
$Y_{S\tilde V}\to \frac{\bar D_0^0 D^{*0}-\bar D^{*0} D_0^0}{\sqrt2},\ \frac{\bar D_0^- D^{*+}-\bar D^{*-} D_0^+}{\sqrt2}$	4.51 ± 0.97
$Y_{S\tilde V}\to \frac{\bar D_1^0 D^0-\bar D^0 D_1^0}{\sqrt2},\ \frac{\bar D_1^- D^+-\bar D^- D_1^+}{\sqrt2}$	24.4 ± 4.3
$Y_{S\tilde V}\to \eta_c\omega$	96.0 ± 34.8
$Y_{S\tilde V}\to J/\psi\omega$	18.0 ± 6.3
$Y_{S\tilde V}\to \chi_{c0}\omega$	49.4 ± 17.2
$Y_{S\tilde V}\to \chi_{c1}\omega$	2.76 ± 0.76
$Y_{S\tilde V}\to J/\psi f_0(500)$	0.49 ± 0.29

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Summary

- Using QCD sum rules to calculate the mass spectra have achieved promising and satisfactory results, further systematic calculations on the strong decays of multiquark states are still required for in-depth research.
- The experimental values that can be reproduced for tetraquark states are larger than those for molecular states.
- For molecular states containing P-wave components, the current calculations need improvement.
- More support from high-energy physics experimental data is required.

Thanks for your attention!