Huazhong University of Science and Technology
School of Physics

# Phenomenological study on semi-leptonic inclusive decay of charm meson

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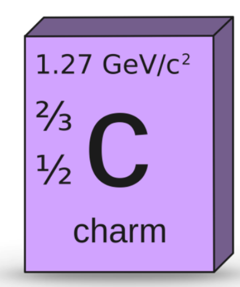


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# Content

1. Why we study semi-leptonic inclusive D meson decays

2. Theoretical Framework

3. Phenomenological Analysis

    - Determine the **HQET basic parameters & CKM matrix elements**
    - Preliminary Extension to $\Lambda_c^+$ **Baryon**

4. Summary

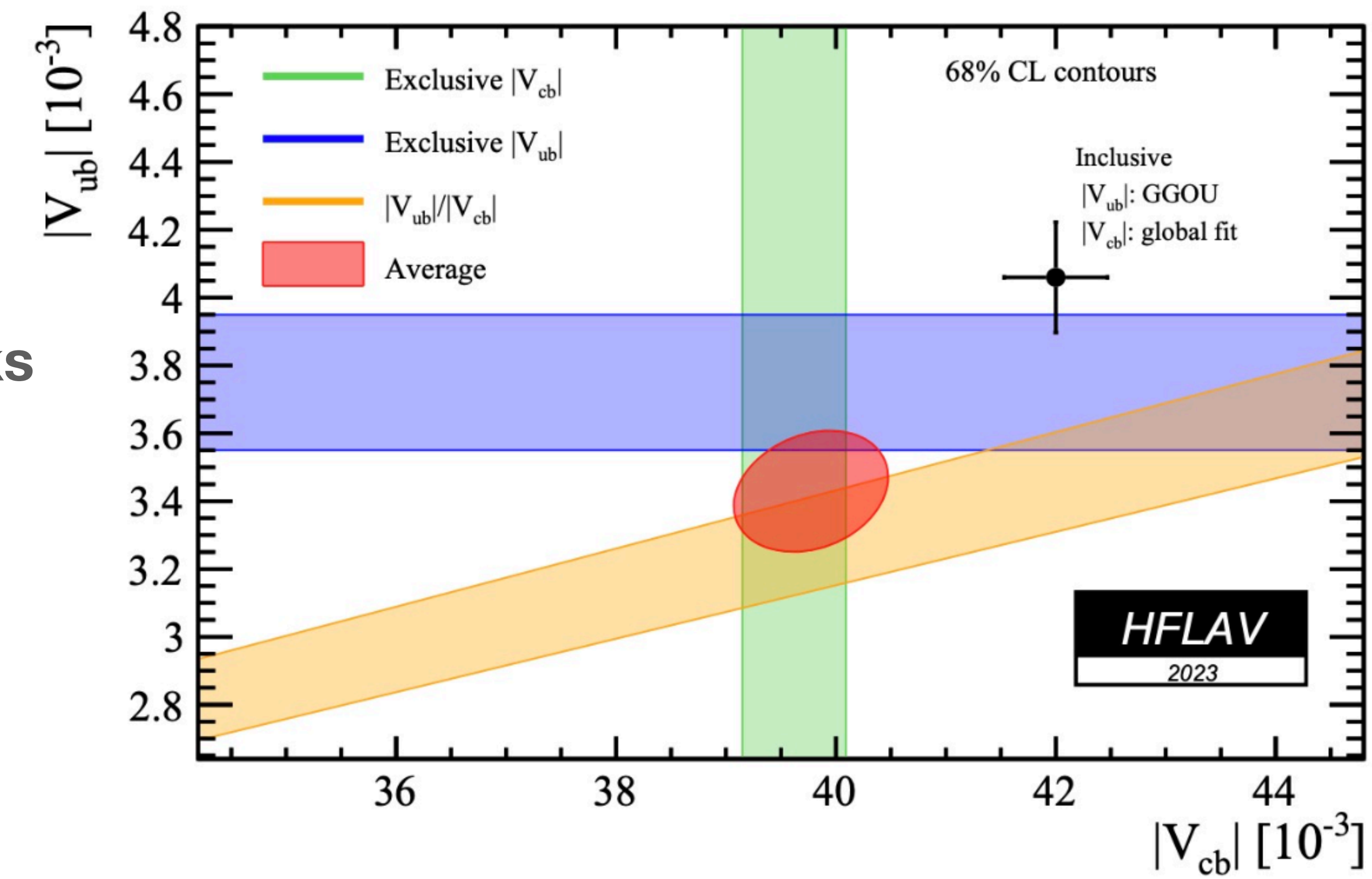
# 1. Why we study semi-leptonic inclusive D meson decays

# What and Why?

**Flavor Puzzle $V_{cb}$ and $V_{ub}$:**
**Inclusive vs Exclusive**

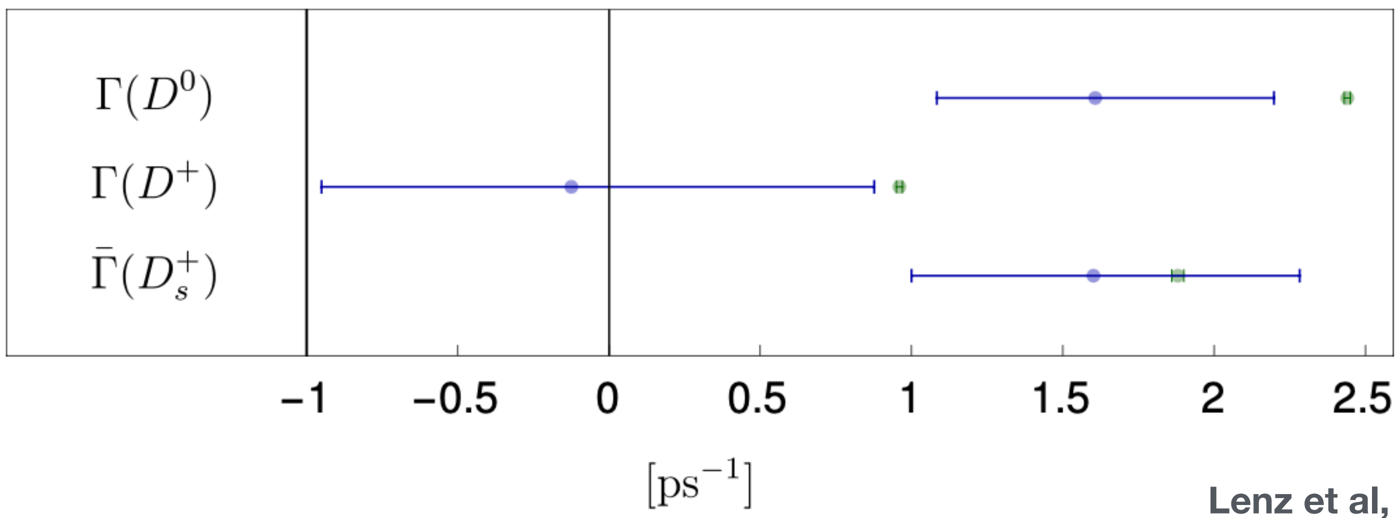
**Key issue:**
**Systematic uncertainties from theoretical incl. and excl. frameworks**

**Hint:**
**Test Vcd and Vcs : incl. VS excl.**
**Await the first inclusive values**



# What and Why?

## Charmed meson lifetimes:
## theory vs experiment




Lenz et al, '22]

Fig 1

**Meson**

Experiment： High precision

Theory： significant uncertainty

**Key issue:**

Theoretical predictions are limited by non-perturbative inputs.

**The extraction of model-independent non-perturbative matrix elements will play an important role**

# 2. Theoretical Framework

# Theoretical Framework and Development

- **Optical theorem**

$$\sum \langle D|H|X\rangle\langle X|H|D\rangle \propto \mathrm{Im}\int \mathrm{d^4x\langle D|T\{H(x)H(0)\}|D\rangle}$$

- **Operator product expansion (OPE)**

  - ⭐ Short distance : $x \sim 1/m_c$
  - ⭐ Fluctuation in D meson $\sim \Lambda_{\mathrm{QCD}}$

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0) \to 1 + \frac{\Lambda_{\mathrm{QCD}}}{m_c} + \frac{\Lambda^2_{\mathrm{QCD}}}{m_c^2} + \ldots$$

**Systematic OPE in HQET.**

- **Heavy Quark Effective Theory**

$$h_v(x) \equiv e^{-im_c v\cdot x}\frac{1+\gamma\cdot v}{2}c(x) \qquad v = (1,0,0,0)$$

$$L \ni \bar{h}_v iv\cdot Dh_v - \bar{h}_v\frac{D_\perp^2}{2m_c}h_v - a(\mu)g\bar{h}_v\frac{\sigma\cdot G}{4m_c}h_v + \ldots$$

**Subtract the big intrinsic momentum,**

**Leave only ~$\Lambda_{\mathrm{QCD}}$ degrees of freedom.**

# Theoretical Framework and Development

- **OPE**

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0)$$

$C_n(x)$

- ☆ LO: $\alpha_s^0(m_c)$
- ☆ NLO: $\alpha_s(m_c)$
- ☆ ...

$O_n(0)$

- ☆ Dim-3: $\bar{h}_v h_v$ $(\bar{c}\gamma^\mu c) \rightarrow$ **partonic decay rate.**
- ☆ Dim-5: $\bar{h}_v D_\perp^2 h_v,\ g\bar{h}_v \sigma \cdot G h_v$ .
- ☆ Dim-6: $\bar{h}_v D_\mu (v \cdot D) D^\mu h_v,\ (\bar{h}_v \Gamma_1 q)(\bar{q}\Gamma_2 h_v), \ldots$
- ☆ ...

- **Contribute to inclusive decay rate and lifetime**

1. Matrix elements of the **same operators** (SL& NL)
2. Only different short-distance coefficients

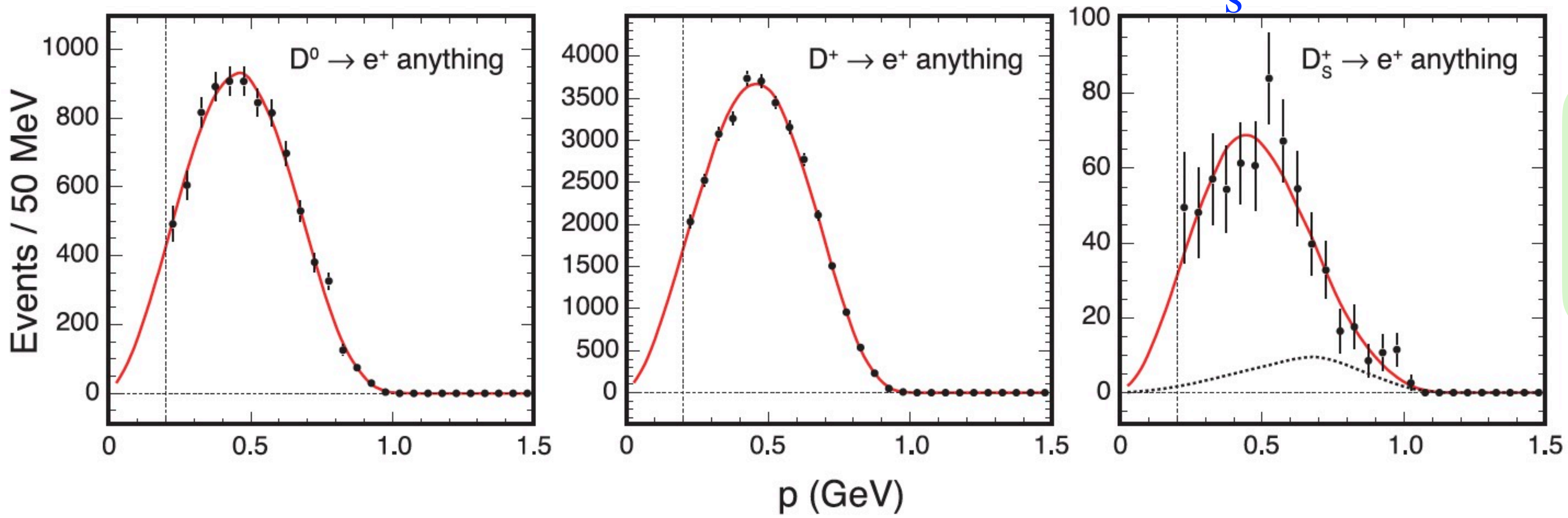
$$\mu_\pi^2 = -\frac{1}{2M_D}\langle D(v)\,|\,\bar{c}_v (iD)^2 c_v\,|\,D(v)\rangle$$
$$\mu_G^2 = \frac{1}{6M_D}\langle D(v)\,|\,\bar{c}_v \frac{g_s}{2} G_{\mu\nu}\sigma^{\mu\nu} c_v\,|\,D(v)\rangle$$

# 3. Phenomenology

# Experimental status

## CLEO measurements

$D^0 \to e^+X$ $\quad$ $D^+ \to e^+X$ $\quad$ $D_s^+ \to e^+X$

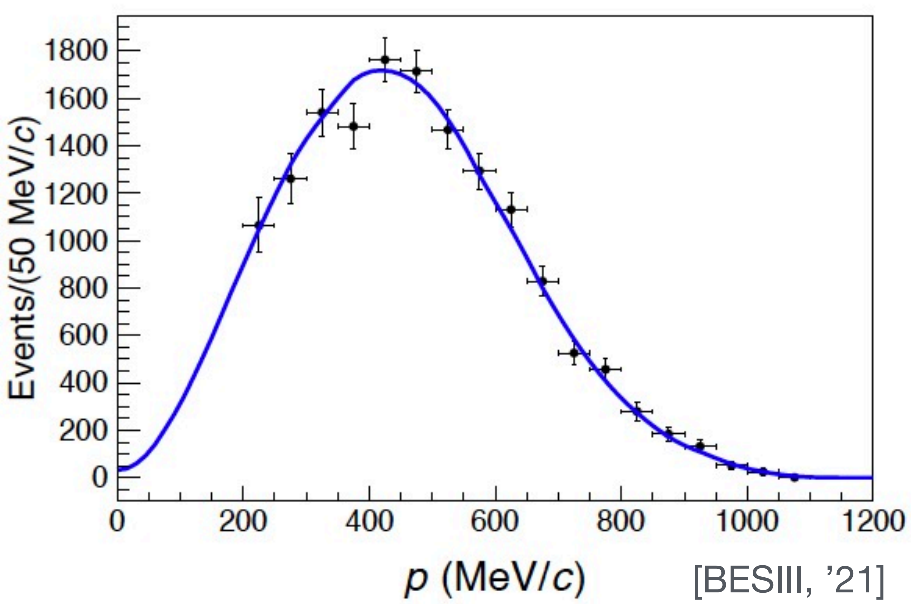



$$B\left(D^0 \to Xe^+\nu_e\right) = (6.46 \pm 0.09 \pm 0.11)\,\%$$
$$B\left(D^+ \to Xe^+\nu_e\right) = (16.13 \pm 0.10 \pm 0.29)\,\%$$
$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.52 \pm 0.39 \pm 0.15)\,\%$$

[CLEO, '09]

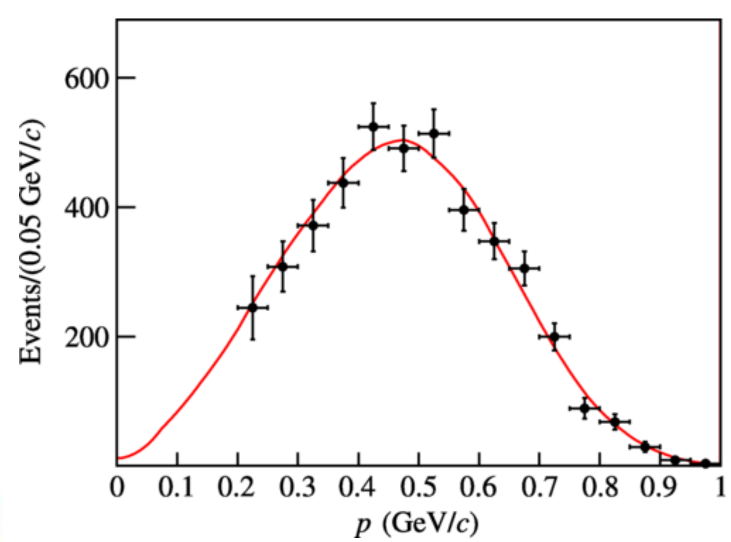
## BESIII measurements

$D_s^+ \to e^+X$




[BESIII, '21]

$\Lambda_c^+ \to e^+X$




[BESIII, '23]

$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.30 \pm 0.13 \pm 0.10)\,\%$$

[BESIII, '21]

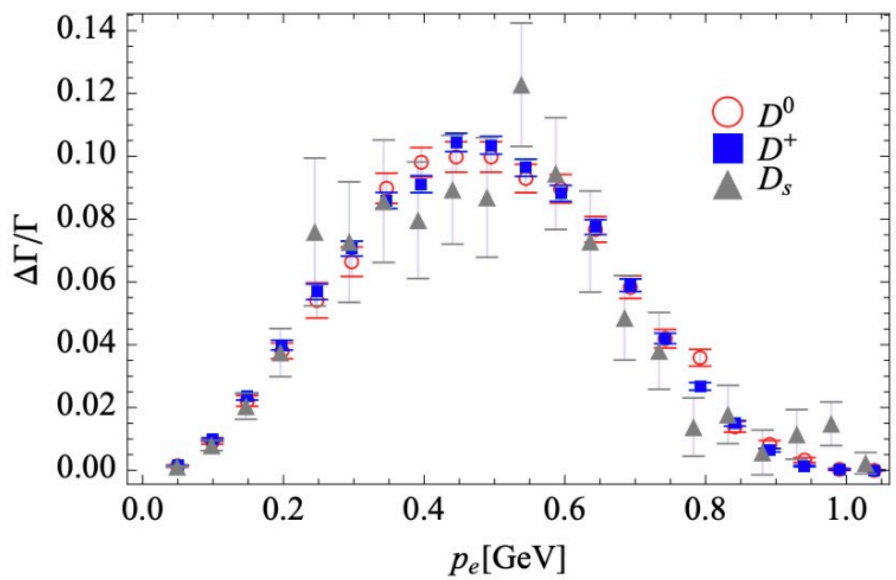
$$B\left(\Lambda_c^+ \to Xe^+\nu_e\right) = (4.06 \pm 0.10 \pm 0.09)\,\%$$

[BESIII, '23]

**2% precision!**

# Phenomenological Progress

- Four quark operators were estimated




$$B^{(0),s}_{\mathrm{WA}}(D^0) = -0.001(3)\mathrm{GeV}^3\,, \qquad B^{(0),s}_{\mathrm{WA}}(D^+) = -0.001(3)\mathrm{GeV}^3\,,$$
$$\bar{B}^{(1),s}_{\mathrm{WA}}(D^0) = -0.0001(6)\mathrm{GeV}^3\,, \qquad \bar{B}^{(1),s}_{\mathrm{WA}}(D^+) = -0.0001(6)\mathrm{GeV}^3\,,$$
$$\bar{B}^{(2),s}_{\mathrm{WA}}(D^0) = -0.0001(10)\mathrm{GeV}^3\,, \qquad \bar{B}^{(2),s}_{\mathrm{WA}}(D^+) = -0.0002(10)\mathrm{GeV}^3\,,$$
$$\bar{B}^{(\sigma),s}_{\mathrm{WA}}(D^0) = -0.0000(7)\mathrm{GeV}^3\,, \qquad \bar{B}^{(\sigma),s}_{\mathrm{WA}}(D^+) = -0.0000(7)\mathrm{GeV}^3\,,$$

**[Gambino, et al ’10]**

- Strong interaction running coupling constant have been estimated

$$\alpha_s = 0.377 \pm 0.008 \pm 0.114$$

**[Wu, et al ’24]**

- Assuming non-perturbative parameters are **identical to those of B meson**
- Global Fitting under **kinetic mass scheme**

?

**Starting from dimension-5!**

# 3. Pheno-1: Determine the HQET parameters

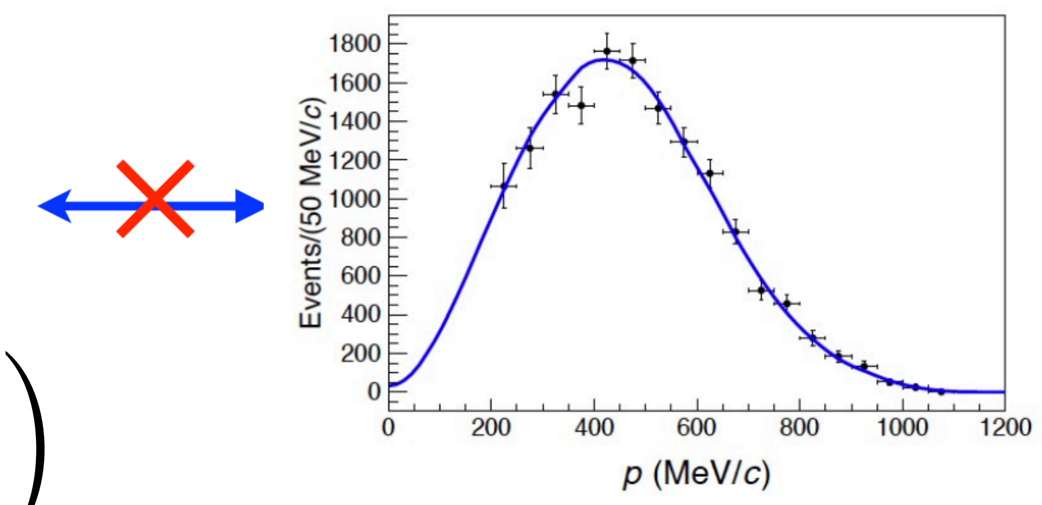
***Eur.Phys.J.C* 85 (2025) 9, 1011**

***Kang Kang Shao*** *Qin Qin*

# Global Fitting Strategy

**Electronic Energy spectrum** $\left(y \equiv 2E_e/m_c\right)$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = 12(1-y)y^2\theta(1-y)$$
$$+\frac{2\mu_\pi^2}{m_c^2}\left[-10y^3\theta(1-y) + 2\delta(1-y)\right.$$
$$-\frac{2\mu_G^2}{3m_c^2}\left[6y^2(6-5y)\theta(1-y)\right] + O\left(\alpha_s, \frac{\Lambda^3}{m_c^3}\right)$$




**[BESIII, ’21]**

- Up to finite power, the obtained differential decay rate is **NOT** the experimental spectrum **[Neubert, 1995]** **[Mannel et al,’94]**
- Observables require integration over final states

$$\Gamma = \int \frac{d\Gamma}{dy}dy, \left\langle E_\ell^n \right\rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell^n dy, n = 1,2,3,4$$

[Gambino,Kamenik, ’10]

# Theoretical Results

## Our main efforts on theory

NLO analytical integration

NNLO numerical results provided by Long Chen

$$\Gamma_{D_i} = \sum_{q=d,s} \hat{\Gamma}_0 |V_{cq}|^2 m_c^5 \Big\{ 1 + \frac{\alpha_s}{\pi}\frac{2}{3}\left(\frac{25}{4} - \pi^2\right) + \frac{\alpha_s^2}{\pi^2}\left[\frac{\beta_0}{4}\frac{2}{3}\left(\frac{25}{4} - \pi^2\right)\log\left(\frac{\mu^2}{m_c^2}\right) + 2.14690 n_l - 29.88311\right]$$

$$- 8\rho\delta_{sq} - \frac{1}{2}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{3}{2}\frac{\mu_G^2(D_i)}{m_c^2} + (6 + 8\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3(D_i)}{m_c^3} + \frac{\tau_0(D_i)}{m_c^3} + \ldots \Big\},$$

[Chen, Chen, Guan, Ma, '23]

Dim-5, $\Lambda_{\mathrm{QCD}}^2/\mathrm{m}_\mathrm{c}^2$

Dim-6, $\Lambda_{\mathrm{QCD}}^3/\mathrm{m}_\mathrm{c}^3$

$D \to e^+ X$ ($D \to e^+ \nu_e X$, only $e^+$ is detected)

Power correction： up to **dim-6** operators contributions

Perturbative correction： up to **NNLO** contributions

# Phenomenological Analysis

The width of semi-leptonic inclusive decays in D mesons is **highly sensitive** to the charm quark mass.

- 1S mass scheme: **well perturbative behaviors** 1/2 $J/\psi$ mass

$$\Gamma/\Gamma_{\rm LO} \approx 1 - 13.1\% - 4.8\% + 1.8\,\%$$

[Hoang,Ligeti,Manohar, ’98; Hoang,Teubner, ’99]

- $\overline{\rm MS}$ mass scheme: **slow** convergence

$$\Gamma/\Gamma_{\rm LO} = 1 + 1.35\alpha_s + 3.02\alpha_s^2 + 7.69\alpha_s^3 \approx 1 + 52\% + 46\% + 44\,\%$$

[Melnikov,van Ritbergen, ’99]

- Pole mass scheme**: no** convergence

Naive parameters of HQET

$$\Gamma/\Gamma_{\rm LO} = 1 - 0.77\alpha_s - 2.38\alpha_s^2 - 10.73\alpha_s^3 \approx 1 - 30\% - 36\% - 62\,\%$$

- Kinetic mass scheme: $\left(\alpha_s/\pi\right)\mu^n/m_c^n$ cut-off scale **somewhat subtle**

[Fael,Schönwald, Steinhauser, ’20]

# Theoretical Results

## Our main efforts on theory

$$\langle E_e \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 m_c^6 \left[ \frac{3}{10} + \frac{\alpha_s}{\pi} a_1^{(1)} + \frac{\alpha_s^2}{\pi^2} a_1^{(2)} - 3\rho\delta_{sq} - \frac{1}{2}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{139}{30} + 4\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3(D_i)}{m_c^3} + \frac{3}{10}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \frac{\tau_0}{2m_c^3} + \ldots \right],$$

$$\langle E_e^2 \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 m_c^7 \left[ \frac{1}{10} + \frac{\alpha_s}{\pi} a_2^{(1)} + \frac{\alpha_s^2}{\pi^2} a_2^{(2)} - \frac{6}{5}\rho\delta_{sq} + \frac{1}{12}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{11}{60}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{17}{6} + 2\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3(D_i)}{m_c^3} + \frac{7}{30}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \frac{\tau_0}{4m_c^3} + \ldots \right],$$

$$\langle E_e^3 \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 m_c^8 \left[ \frac{1}{28} + \frac{\alpha_s}{\pi} a_3^{(1)} + \frac{\alpha_s^2}{\pi^2} a_3^{(2)} - \frac{1}{2}\rho\delta_{sq} + \frac{1}{14}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{1}{14}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{223}{140} + \log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3(D_i)}{m_c^3} + \frac{1}{7}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \frac{\tau_0}{8m_c^3} + \ldots \right],$$

$$\langle E_e^4 \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 m_c^9 \left[ \frac{3}{224} + \frac{\alpha_s}{\pi} a_4^{(1)} + \frac{\alpha_s^2}{\pi^2} a_4^{(2)} - \frac{3}{14}\rho\delta_{sq} + \frac{3}{64}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{13}{448}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{481}{560} + \frac{1}{2}\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3(D_i)}{m_c^3} + \frac{9}{112}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \frac{\tau_0}{16m_c^3} + \ldots \right],$$

$$\left\langle E_e^2 \right\rangle_{\text{center}} \equiv \left\langle (E_e - \langle E_e \rangle)^2 \right\rangle$$

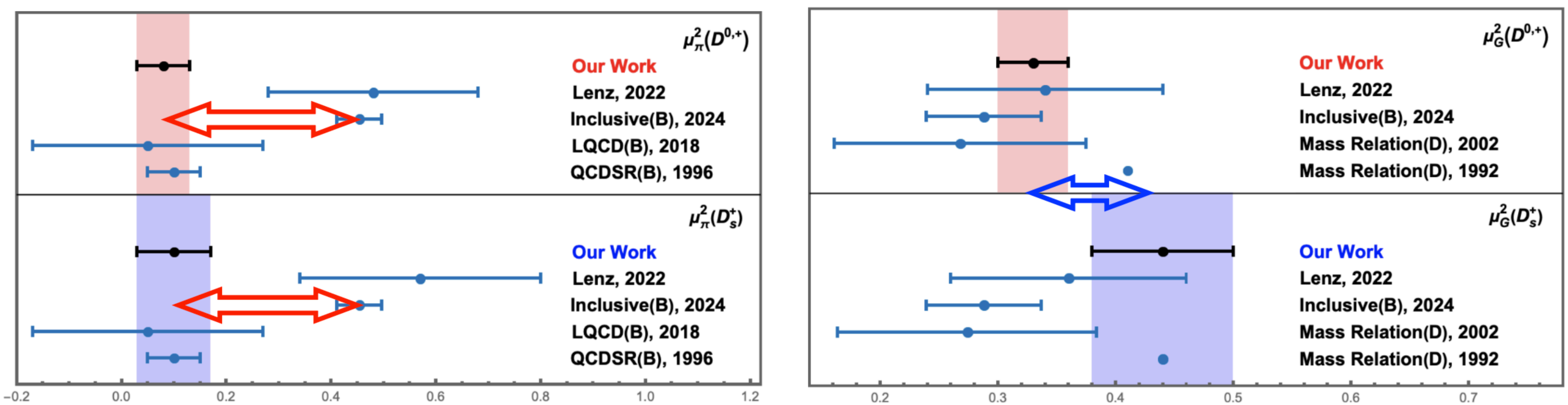
# Fitting Results

❖ Extracted nonperturbative HQE parameters

$$\mu_\pi^2(D^{0,+}) = (0.08 \pm 0.05)\mathrm{GeV}^2, \qquad \mu_\pi^2(D_s^+) = (0.10 \pm 0.07)\mathrm{GeV}^2,$$
$$\mu_G^2(D^{0,+}) = (0.33 \pm 0.03)\mathrm{GeV}^2, \qquad \mu_G^2(D_s^+) = (0.44 \pm 0.06)\mathrm{GeV}^2,$$
$$\rho_D^3(D^{0,+}) = (-0.003 \pm 0.002)\mathrm{GeV}^3, \qquad \rho_D^3(D_s^+) = (-0.004 \pm 0.002)\mathrm{GeV}^3,$$
$$\rho_{LS}^3(D^{0,+}) = (0.004 \pm 0.002)\mathrm{GeV}^3, \qquad \rho_{LS}^3(D_s^+) = (0.005 \pm 0.002)\mathrm{GeV}^3,$$

**Sizable** breaking effects of flavor SU(3) symmetry and heavy quark symmetry.

$\mu_\pi^2$

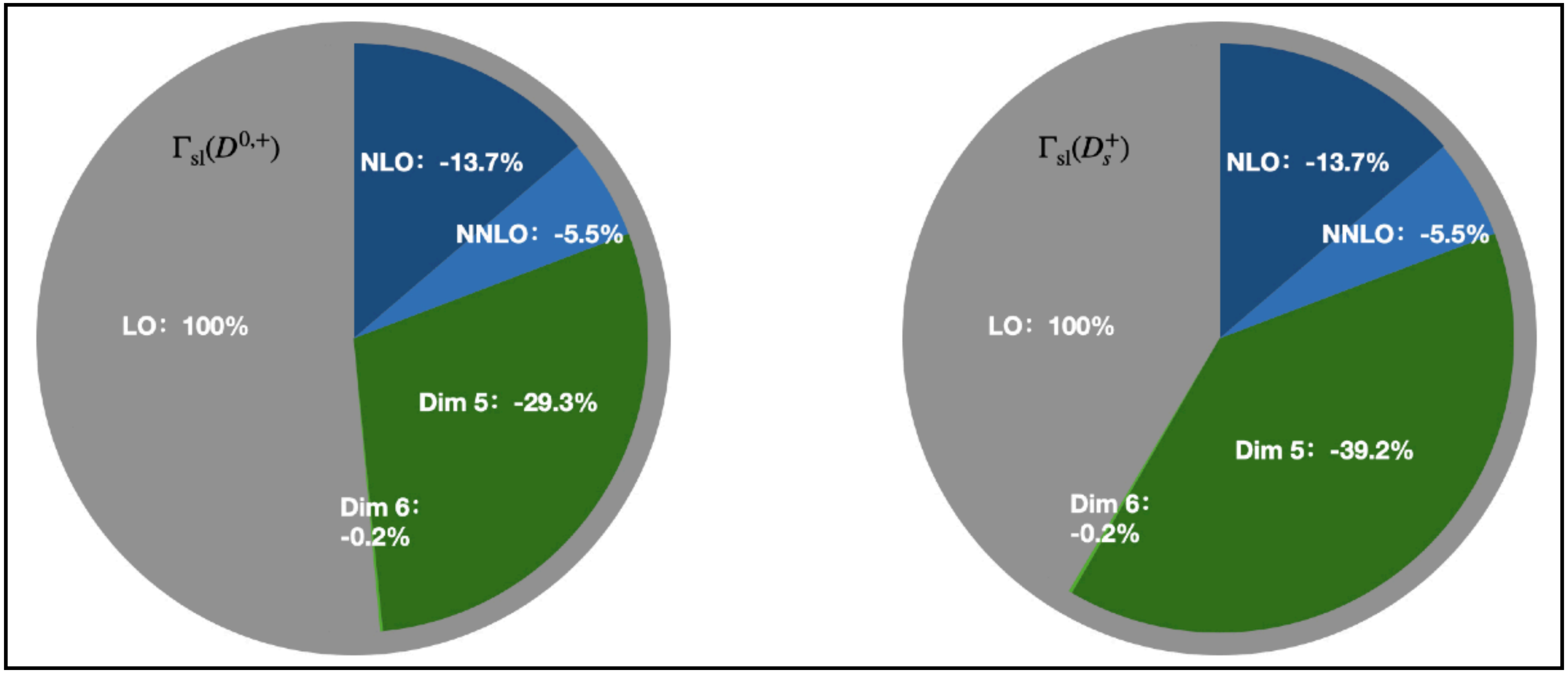



$\mu_G^2$

**Inappropriate to use beauty parameters for charm!**

# Convergence

The non-perturbative series exhibits **good** convergence behavior in D-meson semi-leptonic inclusive decays under the **1S mass** scheme




**We propose the 1S mass scheme as the optimal choice for calculating charmed meson lifetimes**

# Weak Annihilation Uncertainty

❖ Redo the fits, adopting the HQET SR calculation for the weak annihilation contributions

$$\tau_0(D_d \to X_d) = \tau_0(D_s \to X_s) = \tau_{\rm val} = (-0.18 \pm 0.65)\ \text{GeV}^3$$

[King, Lenz, Piscopo, Rauh,'19]

$$\tau_0(D_u \to X_{d,s}) = \tau_0(D_d \to X_s) = \tau_0(D_s \to X_d) = \tau_{\rm nonval} = (0.45 \pm 2.10)\ \text{GeV}^3$$

$$\chi^2(\boldsymbol{\theta}) = \sum_{l=u,d,s} \sum_{i=1}^{5} \sum_{j=1}^{5} (y_{i,l} - \eta_{i,l})\, V_{ij,l}^{-1}\, (y_{j,l} - \eta_{j,l}) + \left(\frac{\tau_{\rm nonval} - (-0.18)}{0.65}\right)^2 + \left(\frac{\tau_{\rm val} - 0.45}{2.10}\right)^2$$

$$\mu_\pi^2(D^{0,+}) = 0.08\text{GeV}^2\,,\ \mu_G^2(D^{0,+}) = 0.33\text{GeV}^2\,,\ \rho_D^3(D^{0,+}) = -0.003\text{GeV}^3\,,\ \rho_{LS}^3(D^{0,+}) = 0.004\text{GeV}^3$$
$$\mu_\pi^2(D_s) = 0.15\text{GeV}^2\,,\ \mu_G^2(D_s) = 0.38\text{GeV}^2\,,\ \rho_D^3(D_s) = -0.005\text{GeV}^3\,,\ \rho_{LS}^3(D_s) = 0.006\text{GeV}^3\,,$$
$$\tau_{\rm val} = -0.11\text{GeV}^3,\ \tau_{\rm nonval} = 0.002\text{GeV}^3\,.$$

**The best-fit values only slightly change.**

# 3. Pheno-2: Determine the CKM matrix elements

***https://arxiv.org/abs/2509.11404***

***Kang Kang Shao**, Hai-Long Feng, Xue-Yin Han, Qin Qin, Fu-sheng Yu, Liang Sun*

# Phenomenological Analysis

1. Treat Vcs and Vcd as free parameters rather than fixed inputs.
2. Difficult for incl. measurements to Xs and Xd → **Sum of excl. channels**

| $D^+$ decays | | $D^0$ decays | | $D_s$ decays | |
|---|---|---|---|---|---|
| Mode | BR(%) | Mode | BR(%) | Mode | BR(%) |
| $D^+ \to \bar{K}^0 e^+\nu_e$ | $8.72 \pm 0.09$ | $D^0 \to K^- e^+\nu_e$ | $3.549 \pm 0.026$ | $D_s \to \phi e^+\nu_e$ | $2.34 \pm 0.12$ |
| $(K^-\pi^+)_{\bar{K}^*(892)^0} e^+\nu_e$ | $3.54 \pm 0.09$ [23] | $(\bar{K}^0\pi^-)_{\text{S-wave}} e^+\nu_e$ | $0.079 \pm 0.017$ | $D_s \to \eta e^+\nu_e$ | $2.27 \pm 0.06$ |
| $(K^-\pi^+)_{\text{S-wave}} e^+\nu_e$ | $0.228 \pm 0.011$ | $D^0 \to K^*(892)^- e^+\nu_e$ | $2.04 \pm 0.047$ [24] | $D_s \to \eta' e^+\nu_e$ | $0.81 \pm 0.04$ |
| $D^+ \to \bar{K}_1(1270)^0 e^+\nu_e$ | $0.230 \pm 0.026$ | $D^0 \to \bar{K}_1(1270)^- e^+\nu_e$ | $0.101 \pm 0.018$ | $D_s \to f_0(980) e' \nu_e$ | $0.164 \pm 0.013$ |
| $D^+ \to \eta e^+\nu_e$ | $0.111 \pm 0.007$ | $D^0 \to \pi^- e^+\nu_e$ | $0.291 \pm 0.004$ | $D_s \to K^0 e^+\nu_e$ | $0.288 \pm 0.026$ |
| $D^+ \to \pi^0 e^+\nu_e$ | $0.372 \pm 0.017$ | $D^0 \to \rho(770)^- e^+\nu_e$ | $0.145 \pm 0.007$ | $D_s \to K^*(892)^0 e^+\nu_e$ | $0.205 \pm 0.020$ |
| $D^+ \to \pi^+\pi^- e^+\nu_e$ | $0.245 \pm 0.008$ | $D^0 \to a(980)^- e^+\nu_e$ | $0.0133^{+0.0034}_{-0.0030}$ | | |
| $D^+ \to \pi^0\pi^0 e^+\nu_e$ | $0.0315 \pm 0.0027$ [25] | | | | |
| $D^+ \to \omega e^+\nu_e$ | $0.169 \pm 0.011$ | | | | |
| $D^+ \to \eta' e^+\nu_e$ | $0.020 \pm 0.004$ | | | | |
| $D^+ \to a(980)^0 e^+\nu_e$ | $0.017 \pm 0.008$ | | | | |

TABLE III. Branching ratios of selected exclusive decays of the $D^+$, $D^0$, and $D_s$ mesons in units of percentage. All the values without references are taken from the PDG [21].

→

| | $D^+$ | $D^0$ | $D_s$ |
|---|---|---|---|
| $X_s e^+\nu_e$ | $14.60 \pm 0.16\%$ | $5.81 \pm 0.06\%$ | $5.58 \pm 0.14\%$ |
| $X_d e^+\nu_e$ | $0.96 \pm 0.03\%$ | $0.45 \pm 0.01\%$ | $0.49 \pm 0.03\%$ |

TABLE I. The branching ratios of semi-leptonic inclusive $D$ decays, derived from the sum of the experimentally well-measured exclusive decay modes.

$$B(D^+ \to X_{d+s} e^+\nu_e) = 0.1602(32),$$
$$B(D^0 \to X_{d+s} e^+\nu_e) = 0.0636(15),$$
$$B(D_s \to X_{d+s} e^+\nu_e) = 0.0631(14),$$

**In agreement with 1～2$\sigma$**

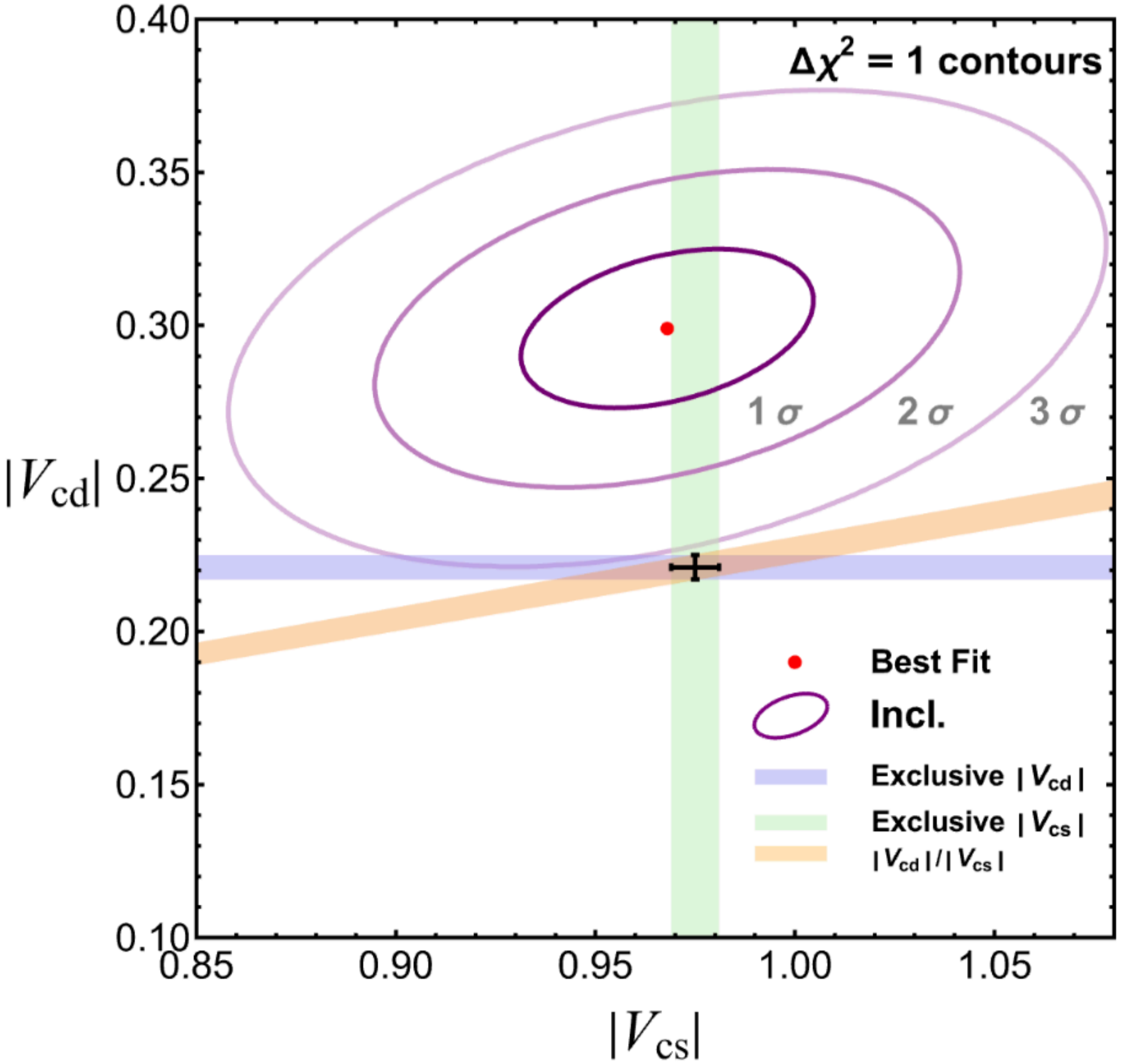
# Phenomenological Analysis

- Data choice strategy: all inclusive $X_{s+d}$ data are used
  - **S1**: Sum-of-exclusive $X_s$ data, plus sum-of-exclusive $X_d$ data,
  - **S2**: Sum-of-exclusive $X_s$ data
  - **S3**: Sum-of-exclusive $X_d$ data
- For each strategy, two scenarios for weak-annihilation contributions
  - **Scenario 1**: VIA, $\tau_0 = 0$
  - **Scenario 2**: HQET SR, input the previous best-fit values $\tau_{\rm val} = -0.11 {\rm GeV}^3, \ \tau_{\rm nonval} = 0.002 {\rm GeV}^3$ .

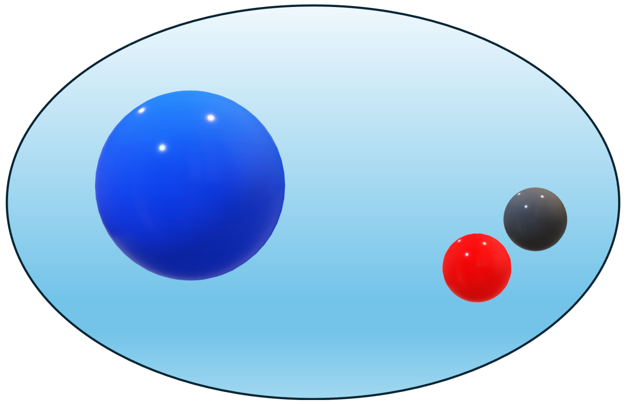
# Phenomenological Analysis

| **S1** $(X_{s,d})$ | $\chi^2$/d.o.f. | $\|V_{cs}\|$ | $\|V_{cd}\|$ |
|---|---|---|---|
| Scenario 1 | 3.24 | $0.977 \pm 0.022 \pm 0.026$ | $0.252 \pm 0.006 \pm 0.006$ |
| Scenario 2 | 1.12 | $0.969 \pm 0.022 \pm 0.026$ | $0.253 \pm 0.006 \pm 0.006$ |
| **S2** $(X_s)$ | $\chi^2$/d.o.f. | $\|V_{cs}\|$ | $\|V_{cd}\|$ |
| Scenario 1 | 0.43 | $0.974 \pm 0.022 \pm 0.026$ | $0.279 \pm 0.031 \pm 0.007$ |
| Scenario 2 | 0.29 | $0.960 \pm 0.021 \pm 0.026$ | $0.278 \pm 0.031 \pm 0.006$ |
| **S3** $(X_d)$ | $\chi^2$/d.o.f. | $\|V_{cs}\|$ | $\|V_{cd}\|$ |
| Scenario 1 | 4.08 | $0.982 \pm 0.023 \pm 0.027$ | $0.253 \pm 0.006 \pm 0.005$ |
| Scenario 2 | 1.24 | $0.974 \pm 0.022 \pm 0.026$ | $0.254 \pm 0.006 \pm 0.006$ |

$$|V_{cs}| = 0.968 \pm 0.022 \pm 0.026 \pm 0.014$$
$$|V_{cd}| = 0.299 \pm 0.025 \pm 0.007 \pm 0.002$$




**Inclusive vs exclusive: 3$\sigma$ tension!**



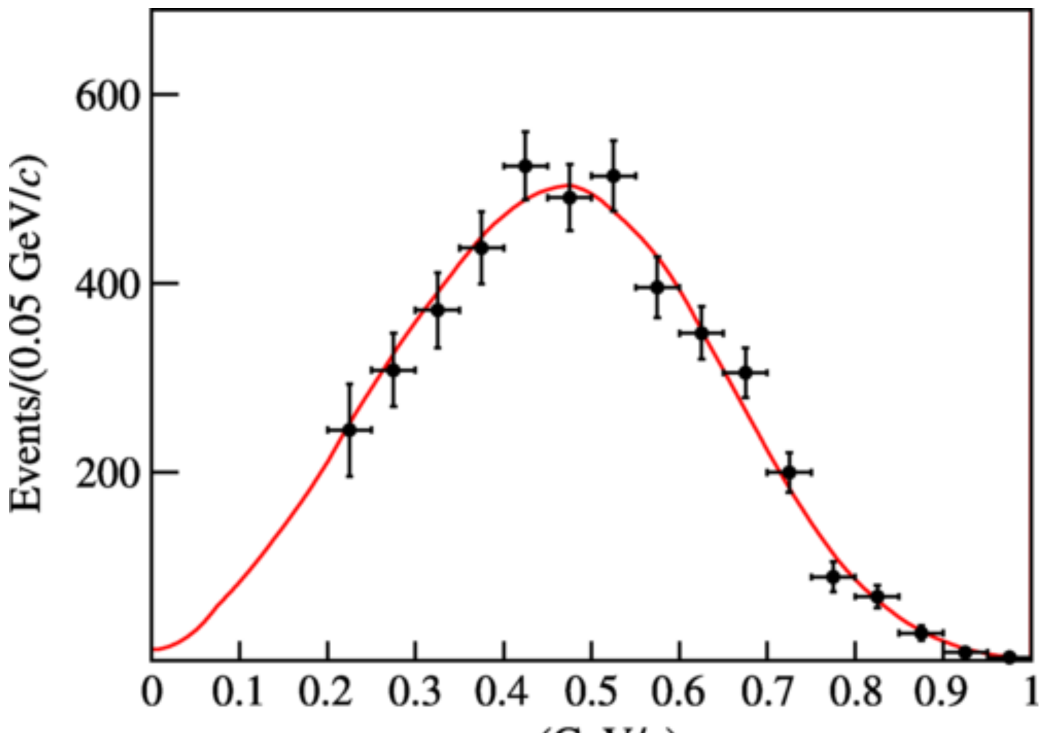
# 3. Pheno-3: Preliminary Extension to $\Lambda_c^+$ Baryon

***https://arxiv.org/abs/2512.08559***

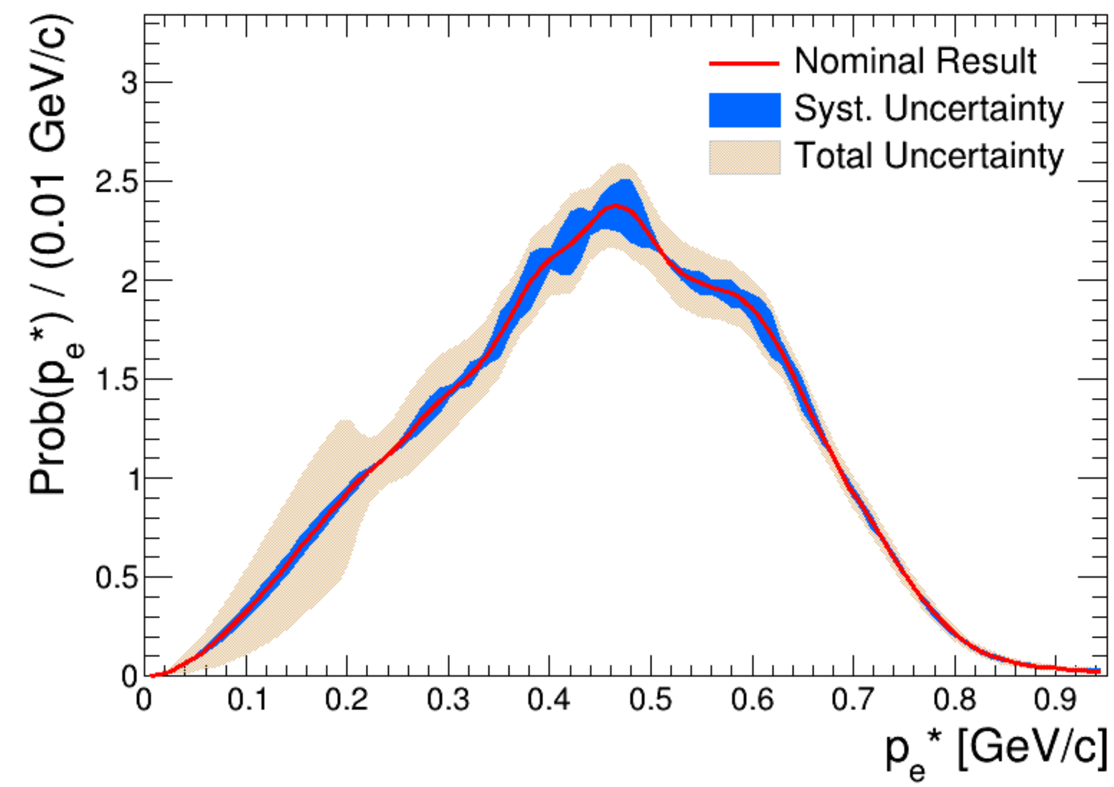
***Dong Xiao, Kang Kang shao***

https://arxiv.org/abs/2512.08559

# Phenomenological Analysis




[BESIII, '23]




| $\mu_\pi^2\,[10^{-1}\,\mathrm{GeV}^2]$ | $\rho_D^3\,[10^{-4}\,\mathrm{GeV}^3]$ | $\tau_{\mathrm{WA}}\,[10^{-1}\,\mathrm{GeV}^3]$ | $\chi^2/\mathrm{d.o.f.}$ | $N_{data}$ v.s. $N_{param}$ |
|---|---|---|---|---|
| $1.33 \pm 0.18 \pm 0.02 \pm 0.40$ | $-2.95 \pm 2.59 \pm 0.36 \pm 1.18$ | $-2.69 \pm 0.33 \pm 0.12 \pm 0.81$ | 0.53 | 5 v.s. 3 |

- **Agree with ones in D mesons**
- **Differ from bag-model and wave-functions approachs**

| Energy Moment | Value |
|---|---|
| $\langle E_e\rangle[\mathrm{GeV}]$ | $(4.55 \pm 0.16 \pm 0.03) \times 10^{-1}$ |
| $\langle E_e^2\rangle_{\mathrm{center}}[\mathrm{GeV}^2]$ | $(2.75 \pm 0.37 \pm 0.07) \times 10^{-2}$ |
| $\langle E_e^3\rangle_{\mathrm{center}}[\mathrm{GeV}^3]$ | $(-3.67 \pm 4.39 \pm 0.91) \times 10^{-4}$ |
| $\langle E_e^4\rangle_{\mathrm{center}}[\mathrm{GeV}^4]$ | $(1.89 \pm 0.45 \pm 0.08) \times 10^{-3}$ |

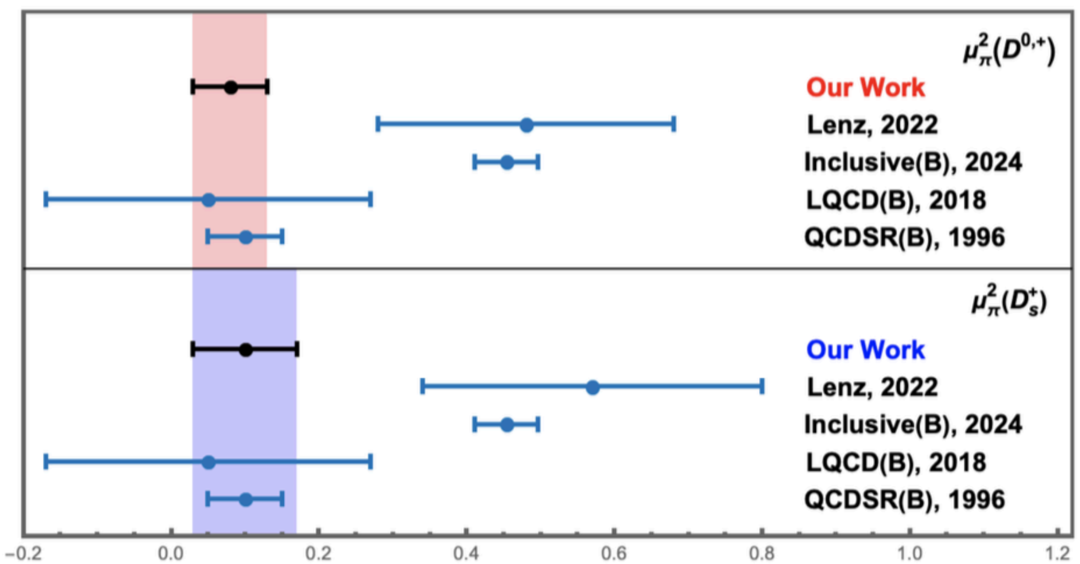
# Summary

## Phenomenology of inclusive D decays

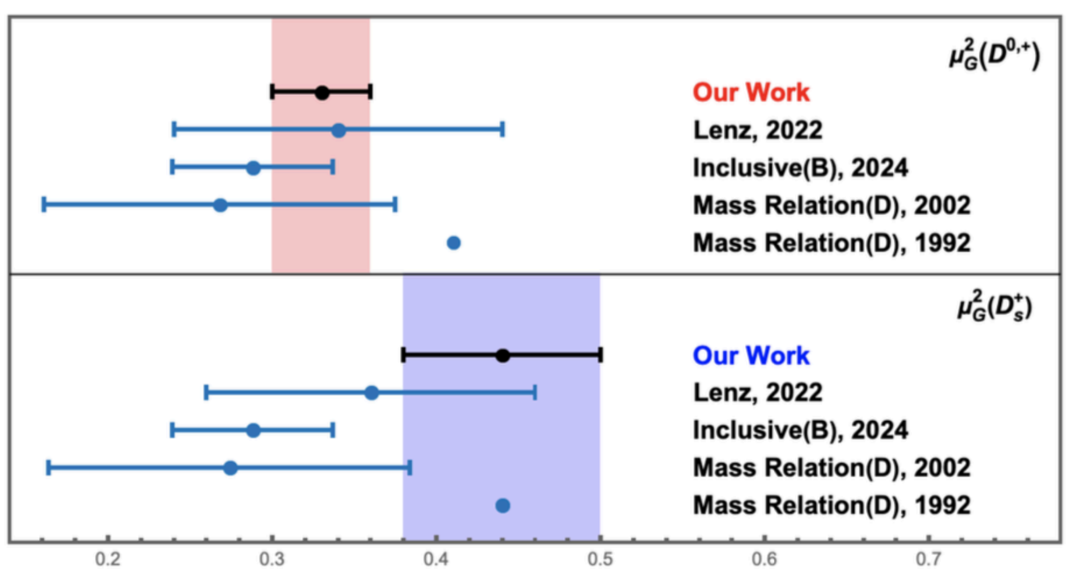
❖ By fitting the HQET formulas for inclusive D decays to data, we determine for the first time

➡ The **HQE parameters** for D mesons

$\mu_\pi^2$




$\mu_G^2$

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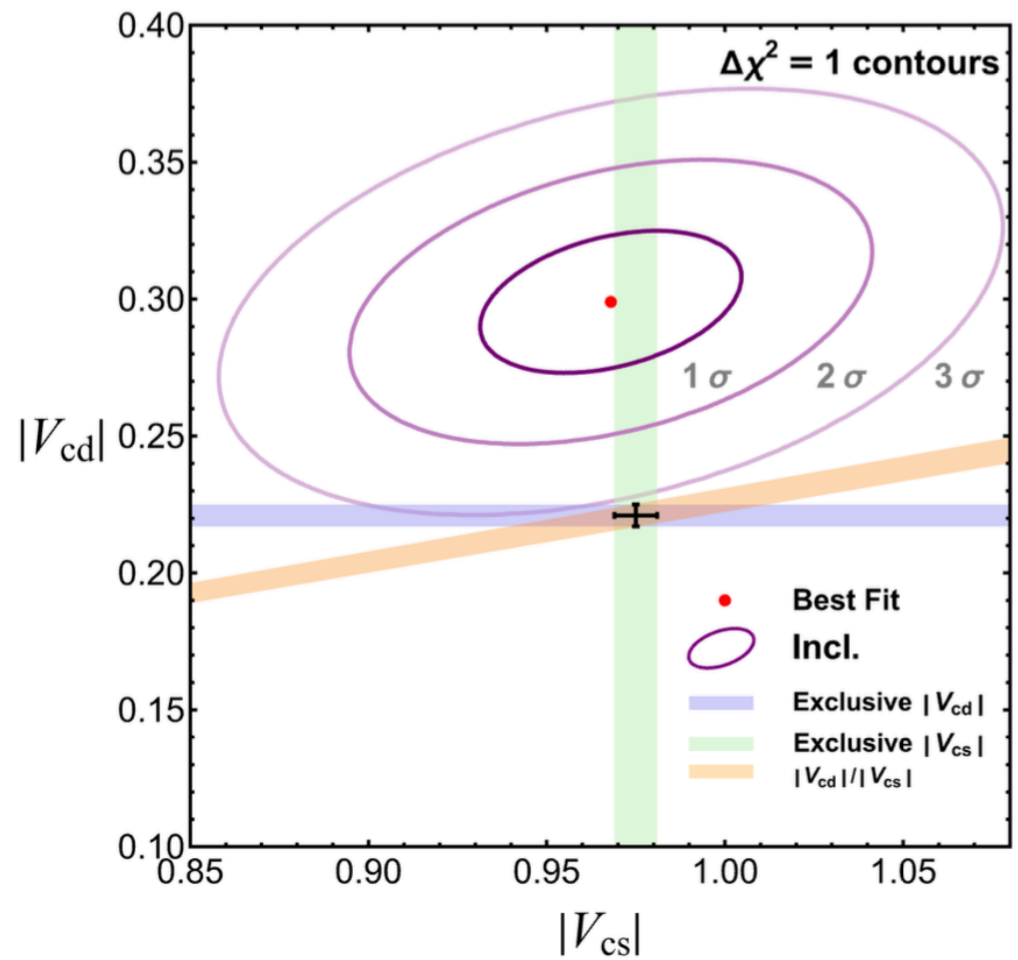
➡ The **CKM matrix elements** $V_{cs,cd}$ (inclusive)




[Shao,Huang,**QQ**,2502.05901]

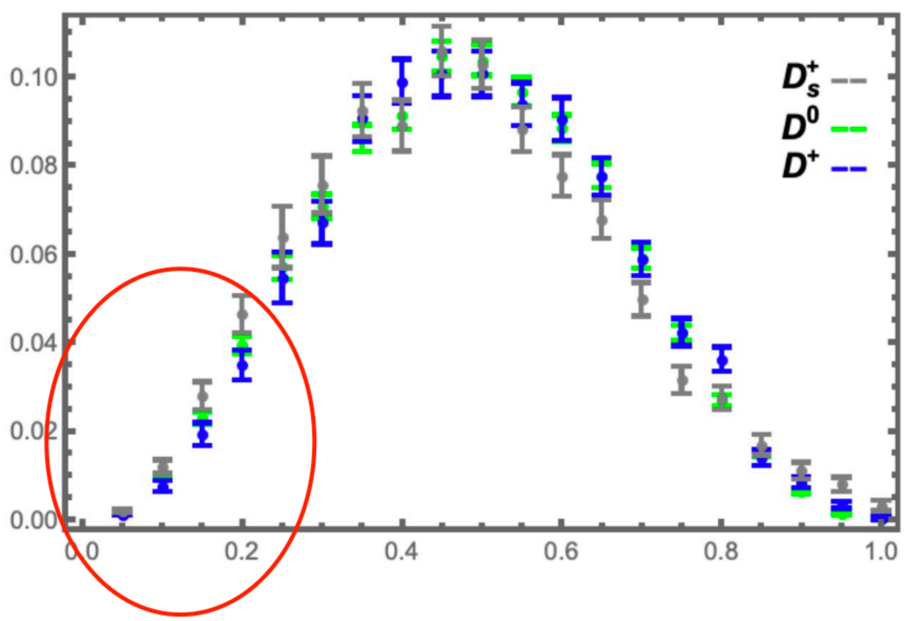
[Shao,Feng,Liu,**QQ**,Sun,Yu,2509.11404]

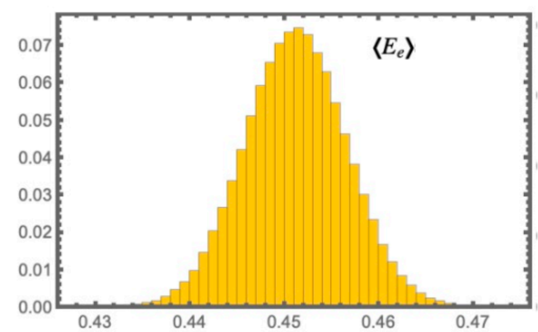
# Appendix

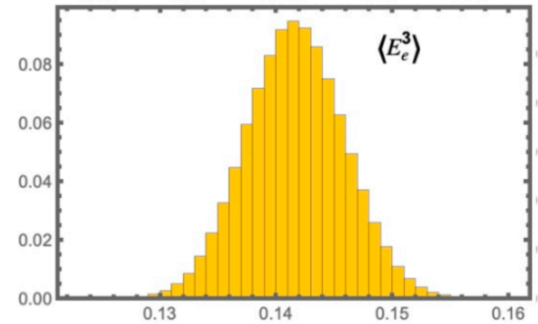
# Data Aspect

$$\frac{d\Gamma}{dy} = ay^2(1+by)(1-y)$$

**Original data** → **Extrapolation** [Gambino,Kamenik, '10] → **All-bin distribution** → **Lorentz boost** → **Rest frame** → **MC Simulation** → **Electron energy moments**




$$\langle E_e \rangle_{exp}^{D_s} = 0.437(6)\text{GeV}, \quad \langle E_e^2 \rangle_{exp}^{D_s} = 0.220(5)\text{GeV}^2$$
$$\langle E_e \rangle_{exp}^{D^0} = 0.462(5)\text{GeV}, \quad \langle E_e^2 \rangle_{exp}^{D^0} = 0.242(5)\text{GeV}^2$$
$$\langle E_e \rangle_{exp}^{D^+} = 0.455(4)\text{GeV}, \quad \langle E_e^2 \rangle_{exp}^{D^+} = 0.236(4)\text{GeV}^2$$

$$\langle E_e^3 \rangle_{exp}^{D_s} = 0.121(4)\text{GeV}^3, \quad \langle E_e^4 \rangle_{exp}^{D_s} = 0.072(3)\text{GeV}^4$$
$$\langle E_e^3 \rangle_{exp}^{D^0} = 0.138(4)\text{GeV}^3, \quad \langle E_e^4 \rangle_{exp}^{D^0} = 0.084(3)\text{GeV}^4$$
$$\langle E_e^3 \rangle_{exp}^{D^+} = 0.134(3)\text{GeV}^3, \quad \langle E_e^4 \rangle_{exp}^{D^+} = 0.081(3)\text{GeV}^4$$

Statistical uncertainties uncorrelated,
Systematic uncertainties fully correlated.

# Fitting Results

❖ Global fit in two mass schemes, each with **Scenario 1** ( $\Lambda_{\rm QCD}^2/m_c^2$ ) and **Scenario 2** ( $\Lambda_{\rm QCD}^3/m_c^3$ )

4-q operator contributions vanish under VIA, $\tau_0 = 0$.

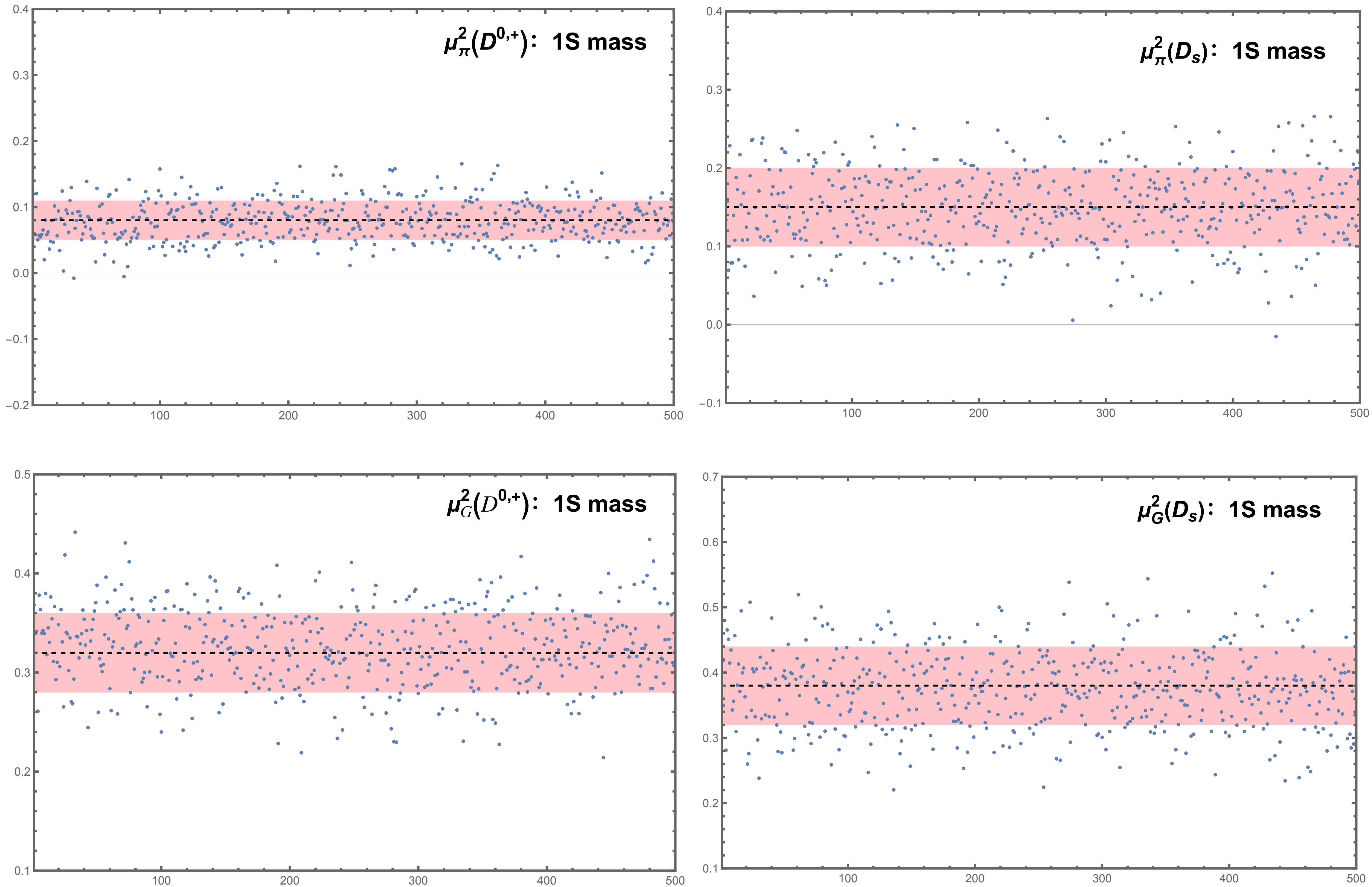
| $\overline{\rm MS}$ scheme | $\chi^2$/d.o.f. | $D_i$ | $\mu_\pi^2/{\rm GeV}^2$ | $\mu_G^2/{\rm GeV}^2$ | $\rho_{\rm D}^3/{\rm GeV}^3$ | $\rho_{LS}^3/{\rm GeV}^3$ |
|---|---|---|---|---|---|---|
| Scenario 1 | 4.5 | $D^{0,+}$ | $0.09 \pm 0.01$ | $0.27 \pm 0.14$ | - | - |
| | | $D_s$ | $0.09 \pm 0.02$ | $0.39 \pm 0.12$ | - | - |
| Scenario 2 | 2.1 | $D^{0,+}$ | $0.11 \pm 0.02$ | $0.26 \pm 0.14$ | $-0.002 \pm 0.002$ | $0.003 \pm 0.002$ |
| | | $D_s$ | $0.12 \pm 0.02$ | $0.38 \pm 0.13$ | $-0.003 \pm 0.002$ | $0.005 \pm 0.002$ |

| 1S scheme | $\chi^2$/d.o.f. | $D_i$ | $\mu_\pi^2/{\rm GeV}^2$ | $\mu_G^2/{\rm GeV}^2$ | $\rho_{\rm D}^3/{\rm GeV}^3$ | $\rho_{LS}^3/{\rm GeV}^3$ |
|---|---|---|---|---|---|---|
| Scenario 1 | 4.9 | $D^{0,+}$ | $0.04 \pm 0.01$ | $0.33 \pm 0.02$ | - | - |
| | | $D_s$ | $0.06 \pm 0.02$ | $0.44 \pm 0.02$ | - | - |
| Scenario 2 | 0.33 | $D^{0,+}$ | $0.09 \pm 0.02$ | $0.32 \pm 0.02$ | $-0.003 \pm 0.002$ | $0.004 \pm 0.002$ |
| | | $D_s$ | $0.11 \pm 0.02$ | $0.43 \pm 0.02$ | $-0.004 \pm 0.002$ | $0.005 \pm 0.002$ |

**Reliable perturbative calculation ensures a good fit!**

Differences between Scenarios 1 and 2 as systematic uncertainties.

# 1S mass scheme robust Testing： scenario 3



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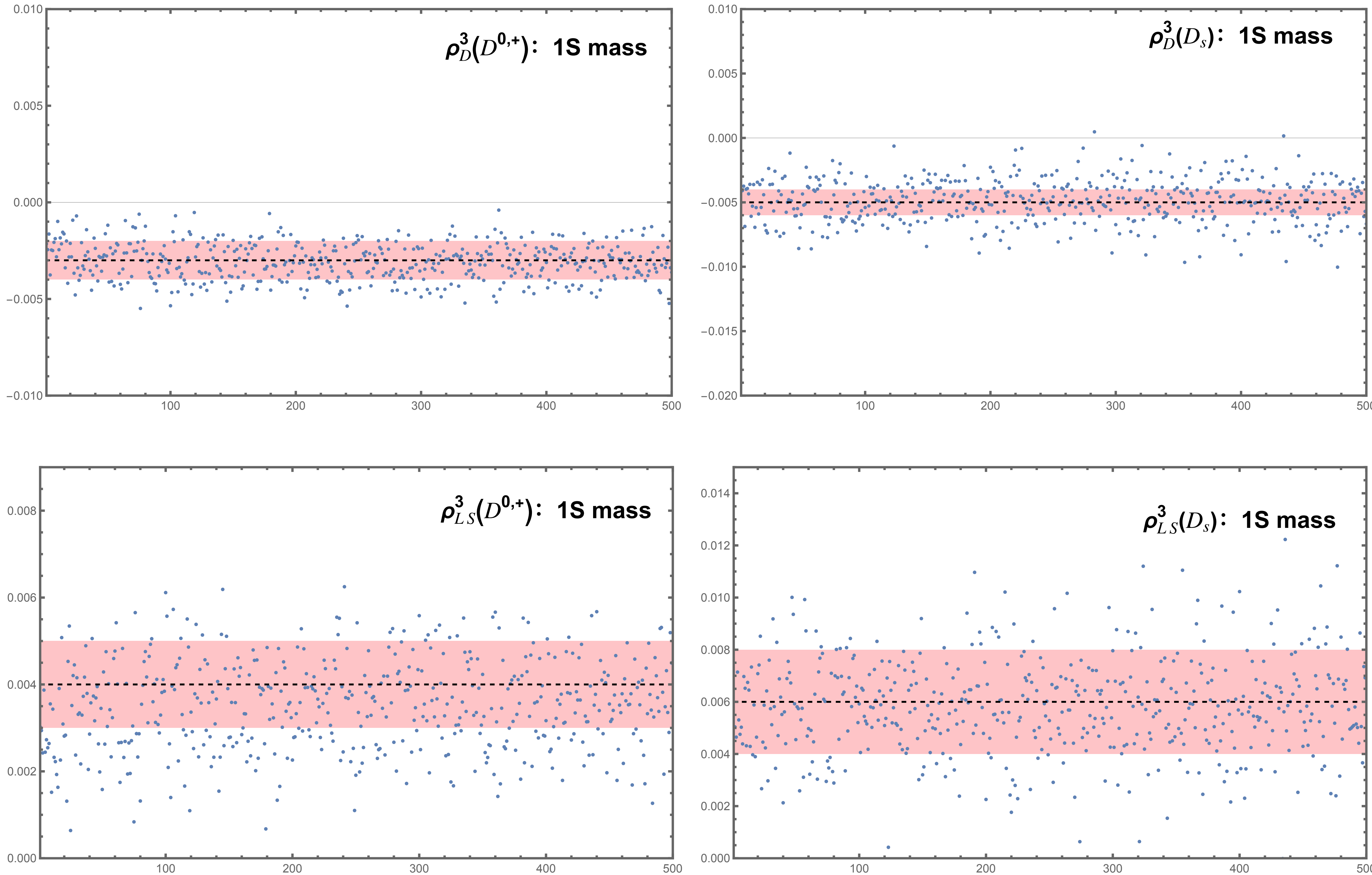
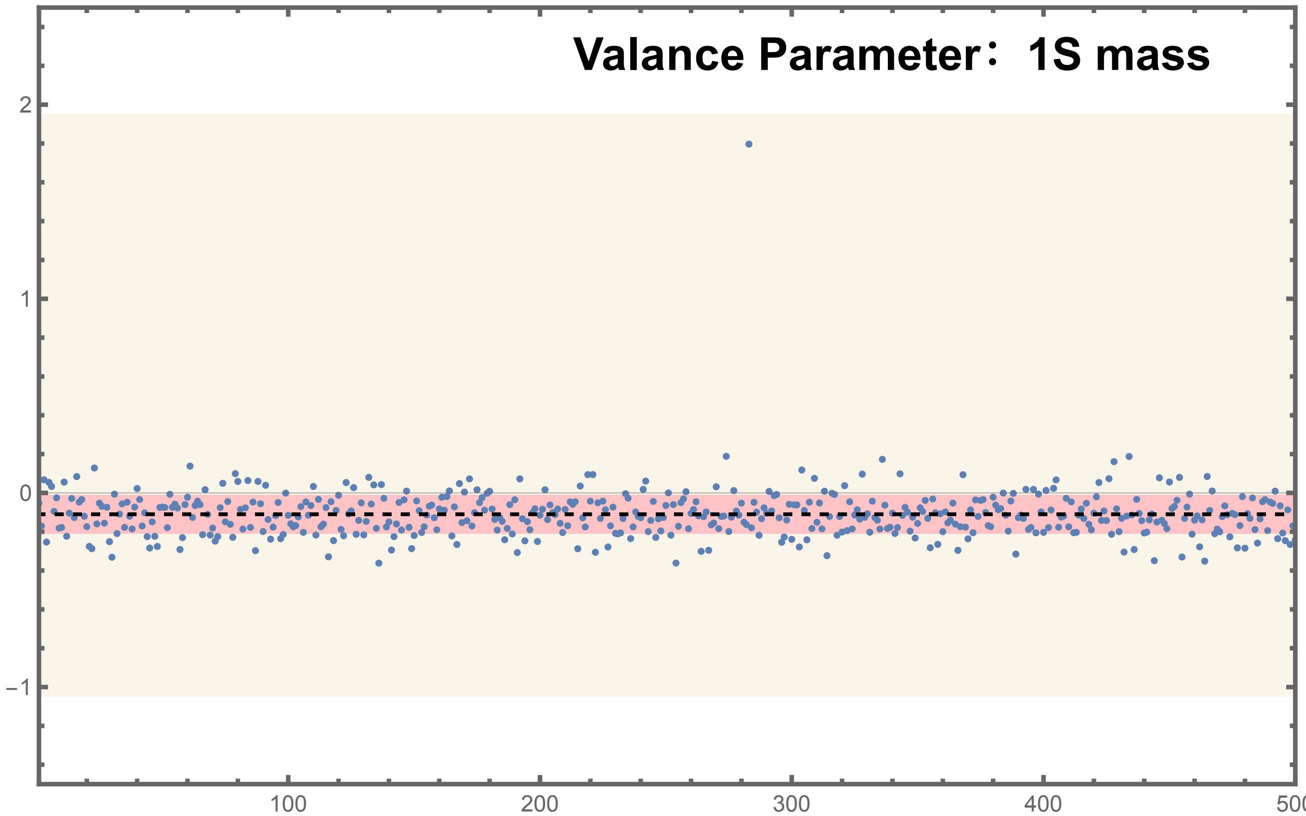
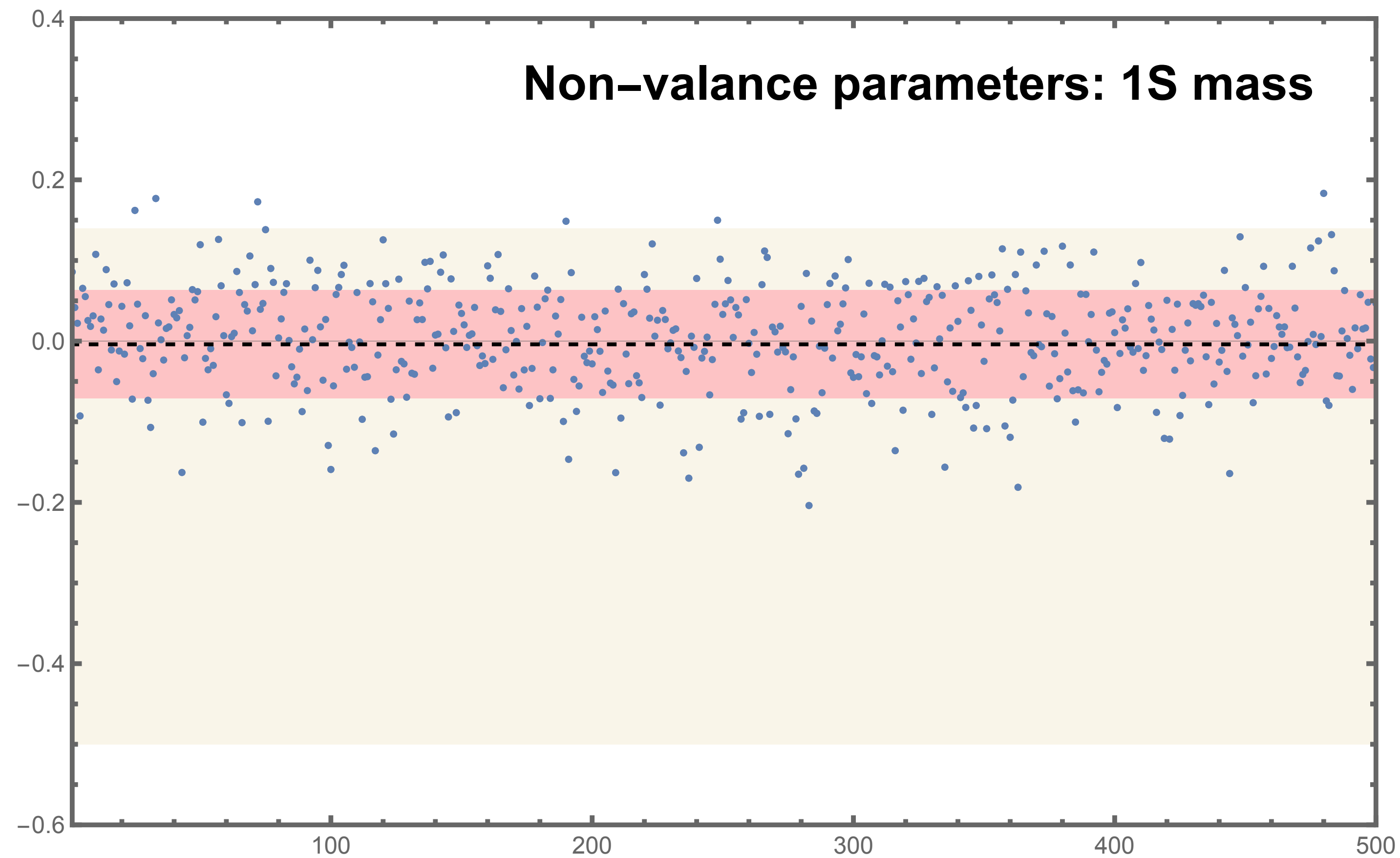


1S mass scheme robust Testing：  scenario 3



# 形状函数

b to u 跃迁电子能谱

$$\frac{1}{2\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = yF(y)\Theta(1-y) - \frac{\lambda_1 + 33\lambda_2}{6m_b^2}\delta(1-y) - \frac{\lambda_1}{6m_b^2}\delta'(1-y)$$

$$\frac{1}{2\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = yF(y)\frac{1}{N}\sum_{i=1}^{N}\Theta\left(1-y+\varepsilon_i\right)$$

$$\delta y = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i = -\frac{\lambda_1+33\lambda_2}{6m_b^2}, \quad \sigma_y^2 = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i^2 = -\frac{\lambda_1}{3m_b^2}$$

$$\frac{1}{N}\sum_{i=1}^{N}\Theta\left(1-y+\varepsilon_i\right) \xrightarrow{N\to\infty} \vartheta(y) = \Theta(1-y) + S(y)F(1)/F(y)$$

$$S(y) = \left\langle \Theta\left[1-y+\frac{2}{m_b}(v-\hat{p})\cdot iD\right] - \Theta(1-y)\right\rangle + \text{ less singular terms}$$

P: 轻子动量

D：b夸克动量

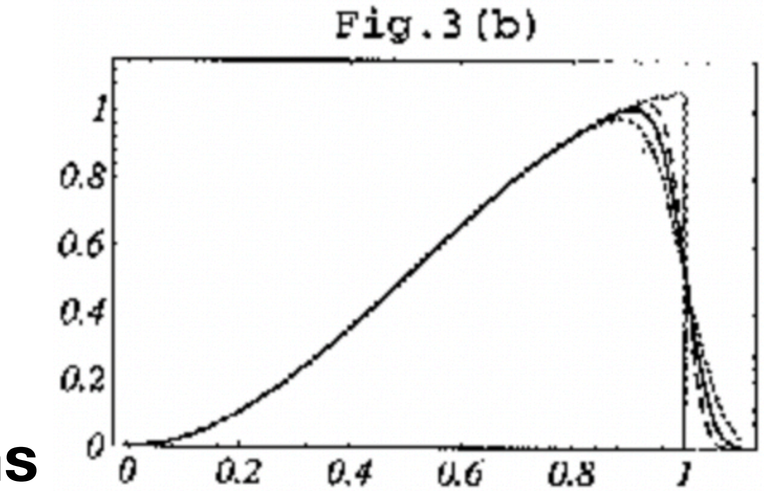
部分子模型

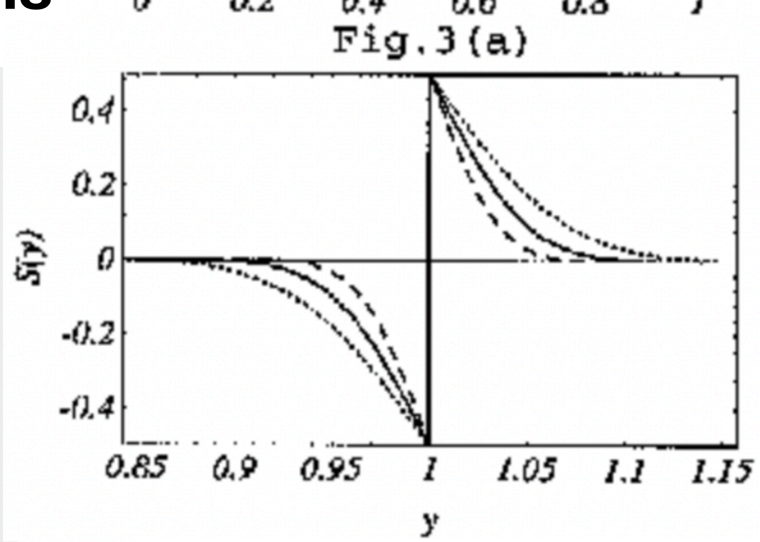
束缚态效应 $\phi\left(\left|\vec{p}_b\right|\right)$

忽略胶子场　A.C.M. 模型

$$\phi\left(\left|\vec{p}_b\right|\right) = \frac{4}{\sqrt{\pi}p_F^3}\exp\left(-\frac{\left|\vec{p}_b\right|^2}{p_F^2}\right)$$

$$S_{\mathrm{ACM}}(y) = \int \cdots\phi\left(\left|\vec{p}_b\right|\right)\cdots \mathrm{d}\vec{p}_b = \left[\frac{1}{2} - \Theta(1-y)\right]\Phi\left(\frac{m_b}{p_F}|y-1|\right)$$



# 粲介子寿命与半轻单举衰变宽度

- 同位旋对称性: $D^0, D^+$

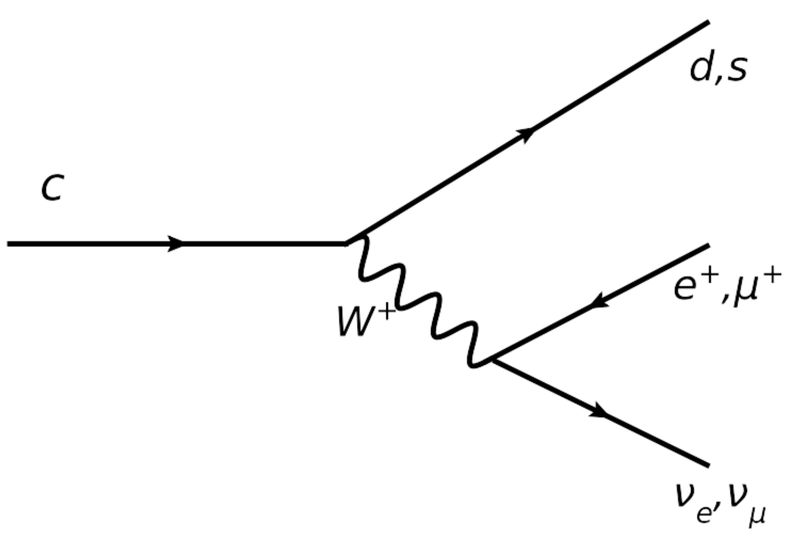
强子化过程中

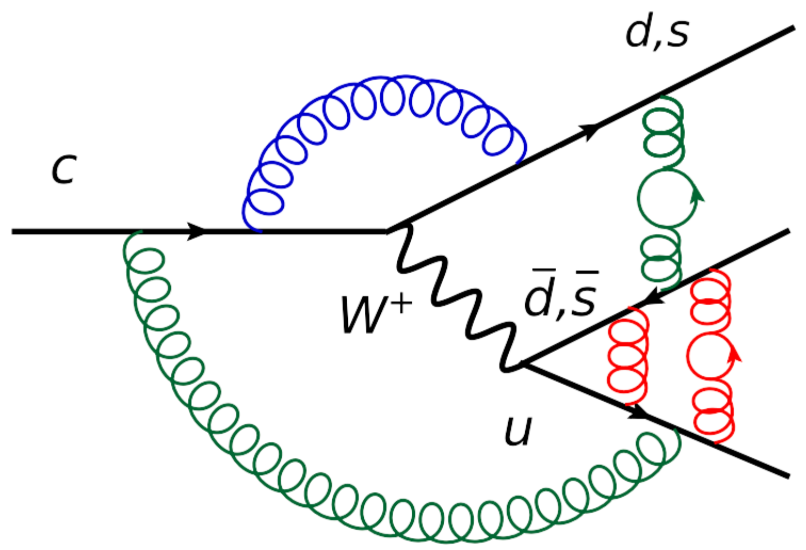
| Quantity | $D^0$ | $D^+$ | $D_s^+$ |
|---|---|---|---|
| $\tau\,[ps]$ | $0.4101 \pm 0.0015$ | $1.040 \pm 0.007$ | $0.504 \pm 0.004$ |
| $\Gamma\,[ps^{-1}]$ | $2.438 \pm 0.009$ | $0.962 \pm 0.006$ | $1.984 \pm 0.0016$ |
| $BR(D_i \to Xe\nu)\,[\%]$ | $6.49 \pm 0.16$ | $16.07 \pm 0.30$ | $6.30 \pm 0.16$ |
| $\Gamma(D_i \to Xe\nu)\,[ps^{-1}]$ | $0.158 \pm 0.004$ | $0.155 \pm 0.003$ | $0.125 \pm 0.003$ |

旁观者效应贡献很小

HQETSR, VIA

- 粲介子寿命差异: 旁观者夸克效应：弱湮灭、弱交换和泡利干涉项



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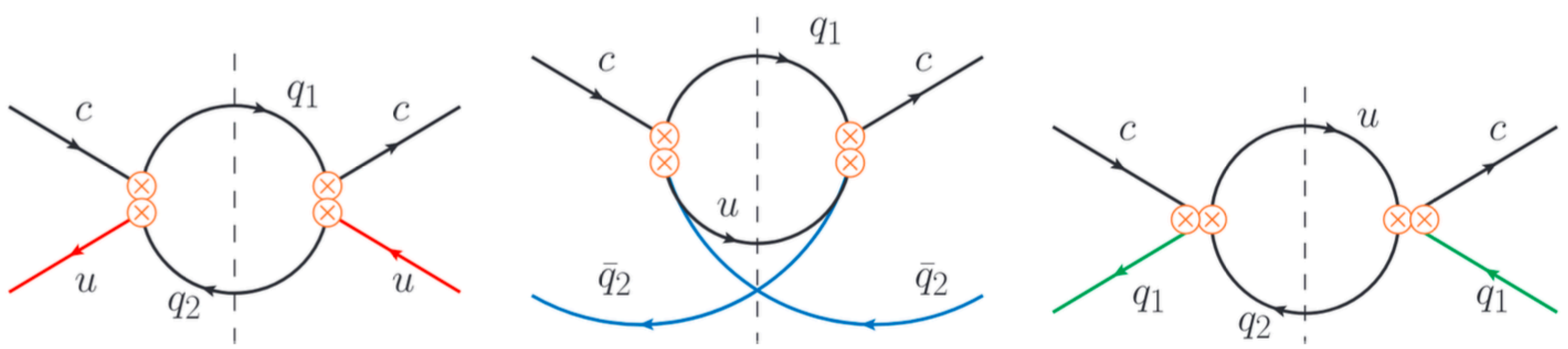



弱交换　　泡利干涉　　弱湮灭

半轻衰变过程仅包含弱湮灭过程

# 粲介子寿命与半轻单举衰变宽度

- 动力学质量方案下粲介子衰变宽度对非微扰矩阵元依赖形式为（$\mu = 0.5\text{GeV}$）：

$$\Gamma\left(\mathrm{D}^0\right) = \Gamma_0[\underbrace{6.15}_{c_3^{\text{Lo}}} + \underbrace{2.95}_{\Delta c_3^{\text{NLO}}} - 1.66\frac{\mu_\pi^2(D)}{\text{GeV}^2} + 0.13\frac{\mu_G^2(D)}{\text{GeV}^2} + 23.6\frac{\rho_D^3(D)}{\text{GeV}^3}$$

$$-1.60\tilde{B}_1^q + 1.53\tilde{B}_2^q - 21.0\tilde{\epsilon}_1^q + 19.2\tilde{\epsilon}_2^q + \underbrace{0.00}_{\text{dim}-7,\text{VIA}}$$

$$-10.7\tilde{\delta}_1^{qq} + 1.53\tilde{\delta}_2^{qq} + 54.6\tilde{\delta}_3^{qq} + 0.13\tilde{\delta}_4^{qq} - 29.2\tilde{\delta}_1^{sq} + 28.8\tilde{\delta}_2^{sq} + 0.56\tilde{\delta}_3^{sq} + 2.36\tilde{\delta}_4^{sq}]$$

$$= 6.15\Gamma_0\left[1 + 0.48 - 0.13\frac{\mu_\pi^2(D)}{0.48\text{GeV}^2} + 0.01\frac{\mu_G^2(D)}{0.34\text{GeV}^2} + 0.31\frac{\rho_D^3(D)}{0.082\text{GeV}^3}\right.$$

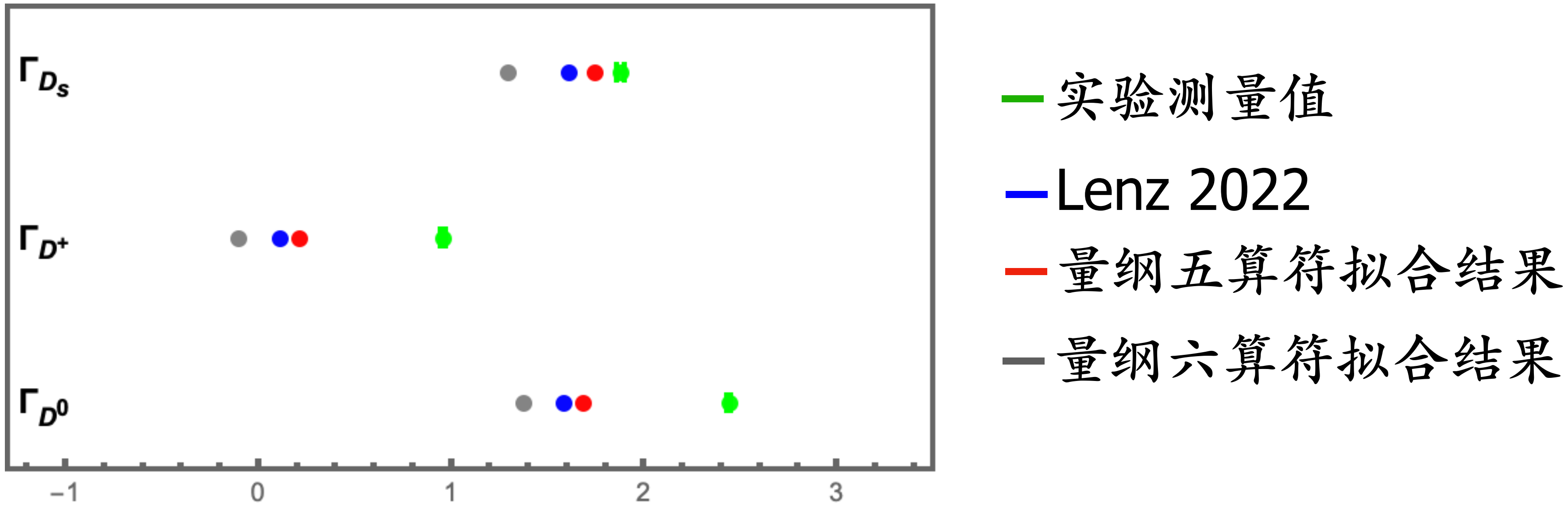
$$-\underbrace{0.01}_{\text{dim-6,VIA}} - 0.005\frac{\delta\tilde{B}_1^q}{0.02} + 0.005\frac{\delta\tilde{B}_2^q}{0.02} + 0.137\frac{\tilde{\epsilon}_1^q}{-0.04} - 0.125\frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{0.00}_{\text{dim}-7,\text{VIA}}$$

$$-0.0045r_1^{qq} - 0.0004r_2^{qq} - 0.0035r_3^{qq} + 0.0000r_4^{qq}$$

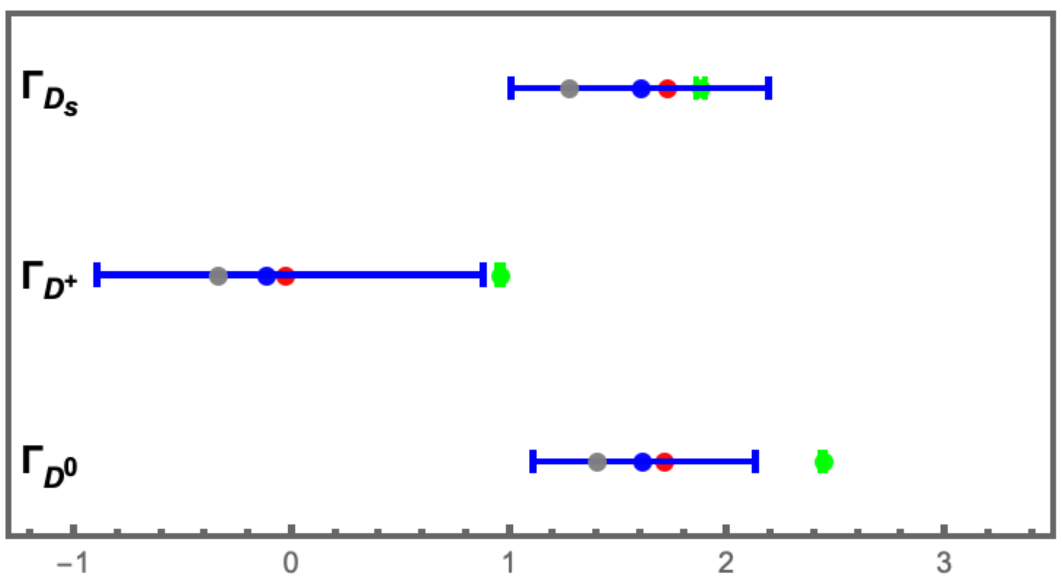
$$-0.0109r_1^{sq} - 0.0079r_2^{sq} - 0.0000r_3^{sq} + 0.0001r_4^{sq}\Big].$$

# 粲介子寿命的初步定性讨论

动力学质量方案下基于VIA对粲介子衰变宽度的理论预言




动力学质量方案下基于HQETSR对粲介子衰变宽度的理论预言




- 实现了量纲五算符非微扰矩阵元模型无关的高精度全局拟合估计
- 该估计值使得衰变宽度更靠近实验测量值
- 该现象没有模型依赖