格点量子色动力学中的多强子散射问题

燕浩波

Based on:

1. **Haobo Yan** et al., **Phys. Rev. D** 111 (2025) 014503 on $D\pi \to D_0^*(2300)$
2. **Haobo Yan** et al., **Phys. Rev. Lett** 133 (2024) 211906 on $\pi\pi\pi \to \omega(782)$ (**Editors' Suggestion**)
3. **Haobo Yan** et al., **Phys. Rev. Lett** 136 (2026) 141901 on $\pi\pi\pi \to \pi(1300)$
4. **Haobo Yan** et al., **JHEP** 10 (2025) 210 on OpTion package

北京大學 PEKING UNIVERSITY

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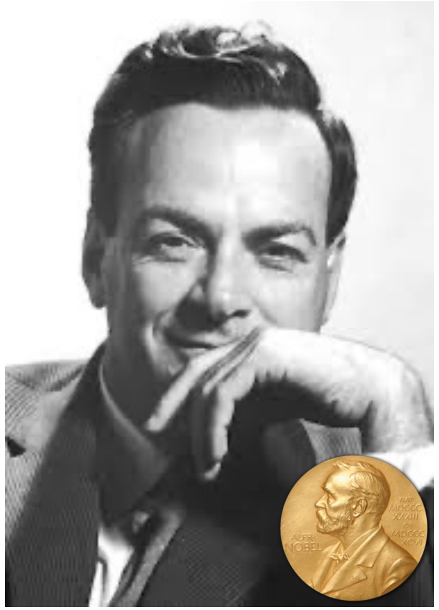
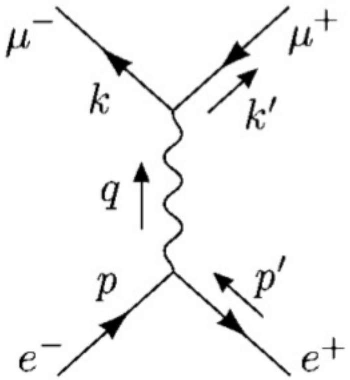
- Short history of multi-hadron scatterings
- Operator constructions
- Two-body systems: $D_0^*(2300) \to D\pi - D\eta - D_s\bar{K} - D^*\pi$
- Two- and three-body systems: $\omega(782) \to \pi\pi\pi$
- Two- and three-body systems: $\pi(1300) \to \pi\pi\pi$

Short introduction to lattice QCD

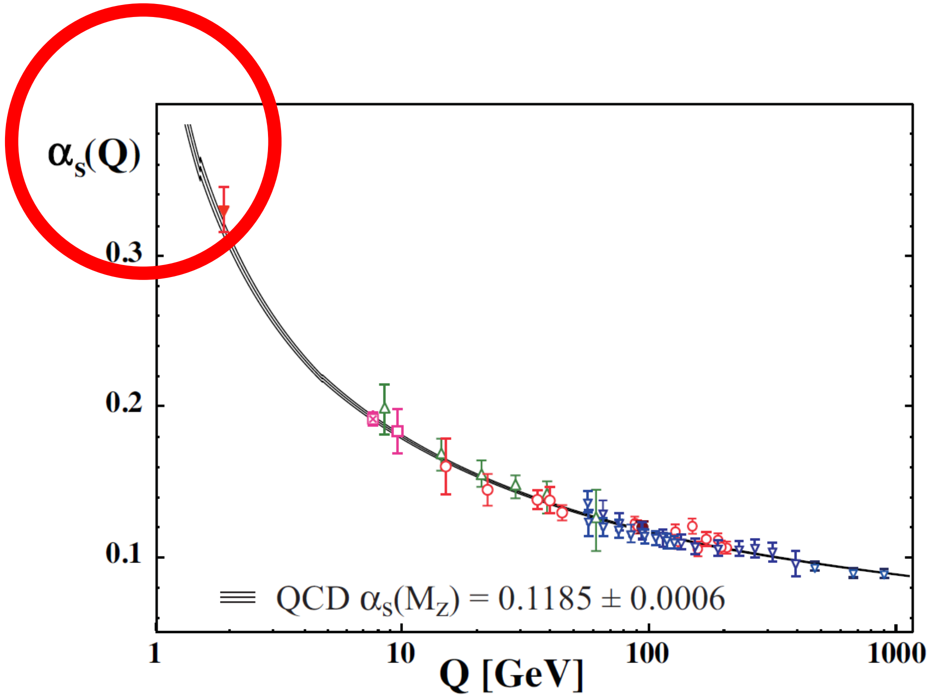


Feynman



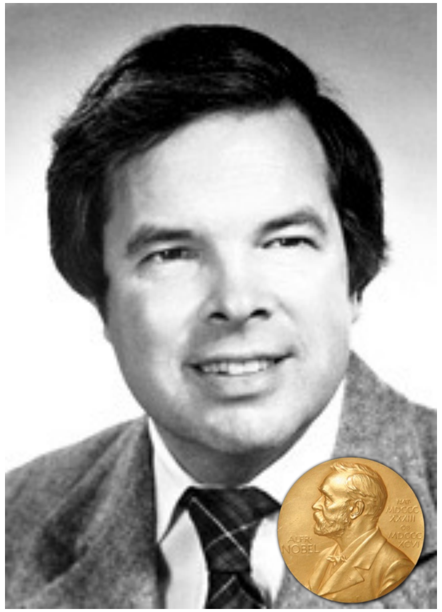
$\mathcal{O}(\alpha)$

$+\mathcal{O}(\alpha^2) + \mathcal{O}(\alpha^3) + \cdots$

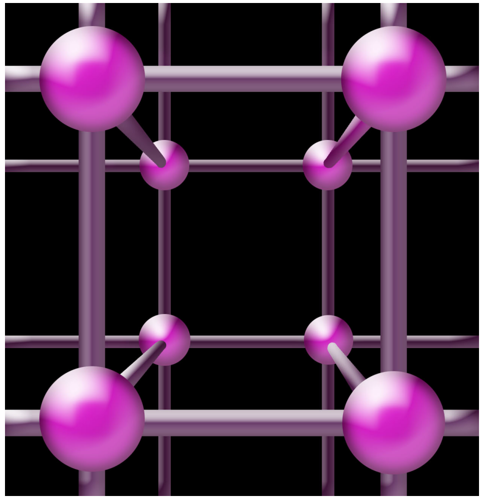


- Works perfectly for small α $\qquad \alpha_{em} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}$
- For strong force when $\alpha \sim 1$, the perturbation theory failed
- Is it possible to calculate all terms non-perturbatively?

Short introduction to lattice QCD



Wilson



- Put QCD on a **finite-volume** discretized **Euclidean** spacetime
- First principle from standard model
- Non-perturbative
- Controlled systematics
- Needs large-scale computing resources

Short introduction to lattice QCD

- **Correlation functions** are calculated by Wick contractions

$$\langle\Omega|O(t)O^\dagger(0)|\Omega\rangle = \frac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi}]\mathcal{D}[U]\mathrm{e}^{-S_F[\psi,\bar{\psi},U]-S_G[U]}\left(O(t)O^\dagger(0)\right)[\psi,\bar{\psi},U]$$

- Impossible to perform the path integral

Dimension of gauge $\sim\ 48^3 \times 96 \times 4 \times 3^2 \sim 10^9$

volume time Lorentz color

- Generate Monte-Carlo ensembles of U according to the gauge action
- Then measure QCD on them

$$\langle O\rangle \approx \frac{1}{N}\sum_{\substack{U_n \text{ with} \\ \text{probability} \\ \propto\, \mathrm{e}^{-S[U_n]}}} O[U_n]$$

- 格点场论既是世界观（非微扰的定义）又是方法论（非微扰的计算）

刘川《格点量子色动力学导论》

Short introduction to lattice QCD

- Correlation function on the other hand

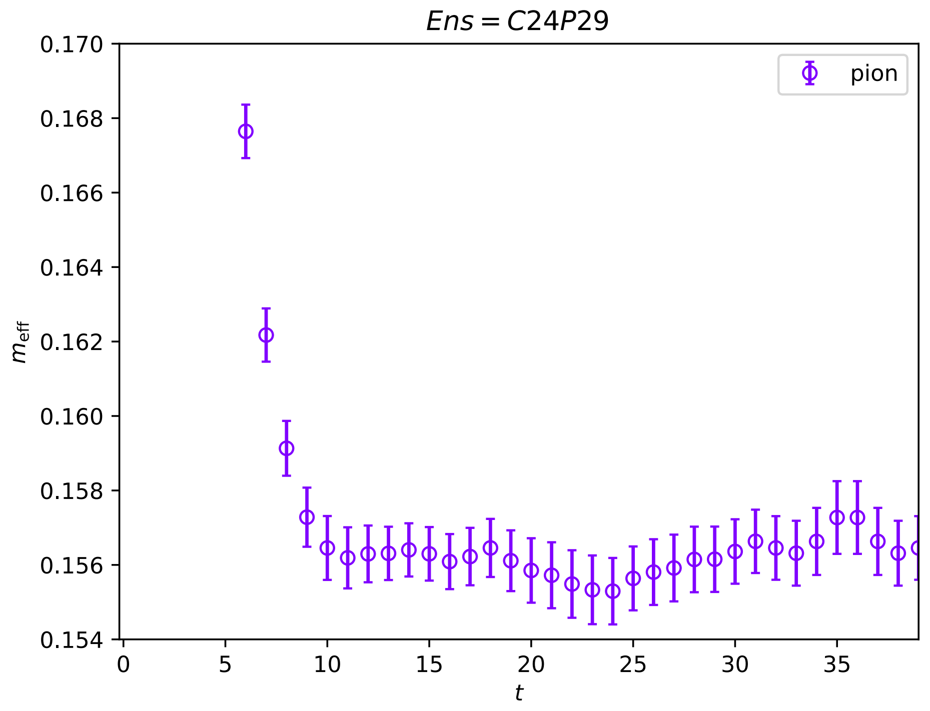
$$\langle\Omega|O(t)O^\dagger(0)|\Omega\rangle = \langle\Omega|O(t)\sum_n \frac{1}{2E_n}|E_n\rangle\langle E_n|O^\dagger(0)|\Omega\rangle$$

$$= \frac{1}{2E_1}|\langle\Omega|O(t)|E_1\rangle|^2 e^{-E_1 t} + \text{exited states}$$

- E_1 measured from

$$\log\frac{C(t)}{C(t+1)} \xrightarrow{t\to\infty} E_1$$

- Higher E_n obtained by diagonalizing the correlation matrices

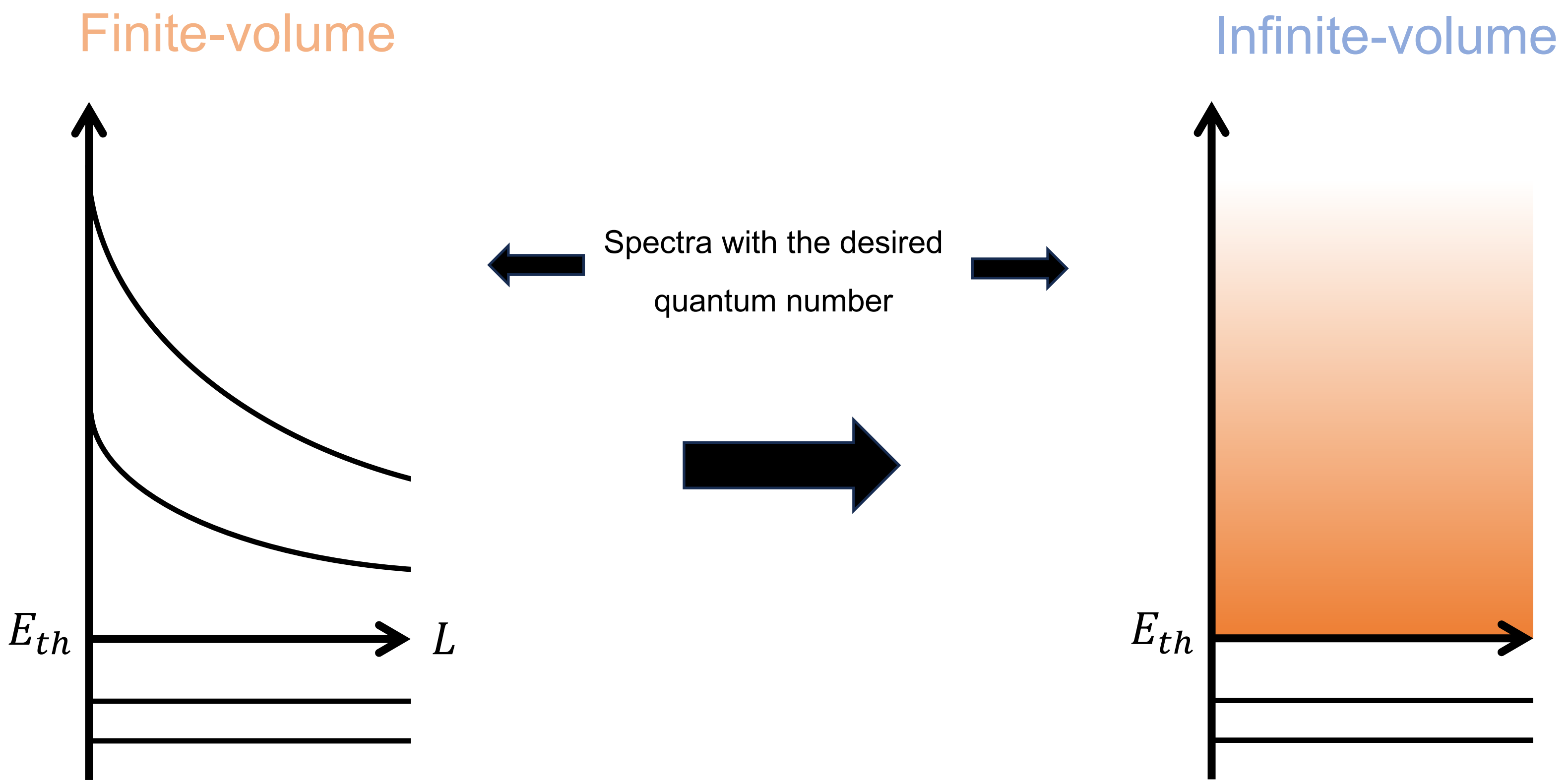


Milestones of hadron scattering

- Two-body
  - 1991: Lüscher – the famous formula
  - 1992-2014 – development of Lüscher formalism
  - 2012: energy-dependent phase shift of $I = 2\ \pi\pi$
  - 2013: $\pi\pi \to \rho$
  - 2012-now: scattering with mesons, baryons…

- Three-body
  - 2014-2017: development of three-particle formalism
  - 2021: energy-dependent $I = 3\ \pi\pi\pi$
  - 2021: $I = 1\ \pi\pi\pi \to a_1(1260)$
  - 2024-now: our work begins

Routine

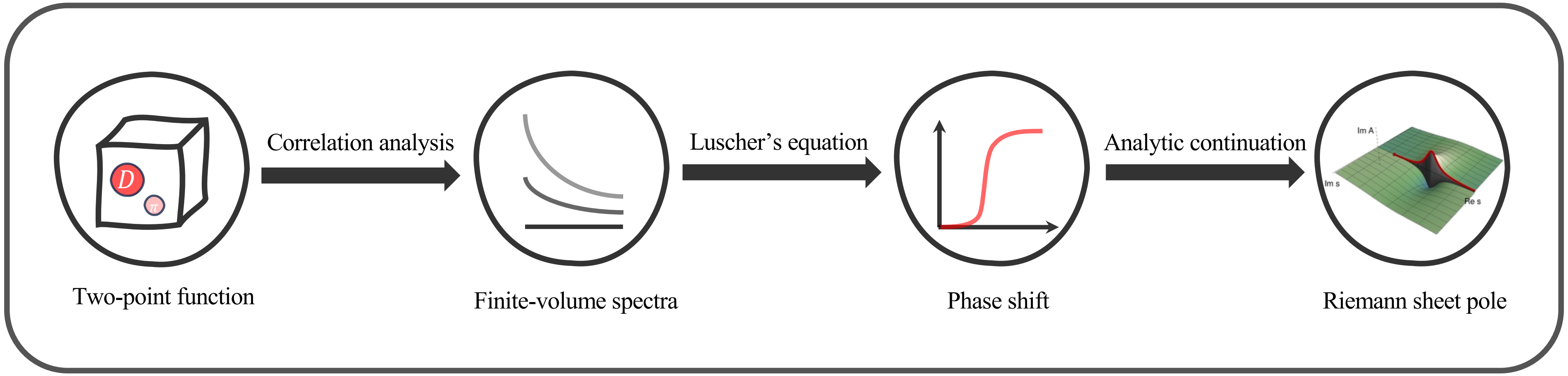


- How to extract the infinite-volume information from finite-volume spectra?

  → Quantization condition: $f\big(\mathcal{M}(E_n),\ \mathcal{F}(E_n, L)\big) = 0$

- More E_n and L give more constraints on $\mathcal{M}(E_n)$

Methodology: two-body



- Operators from OpTion
- Two-body spectra
- Apply the quantization conditions
- Analytic continuate and search for poles

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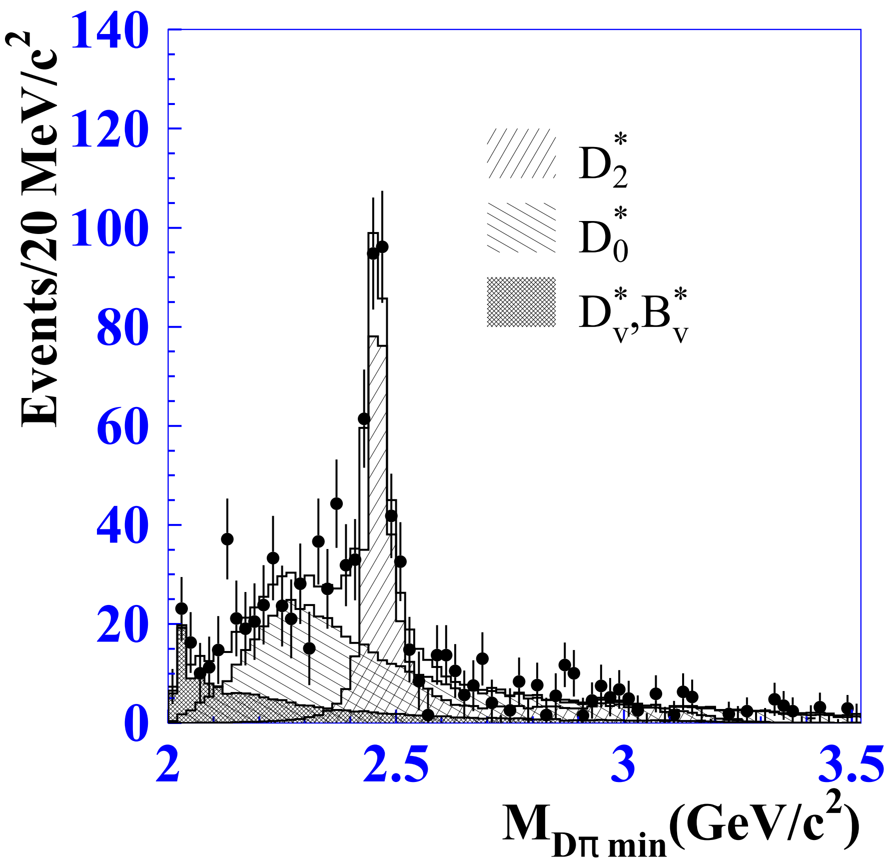
Puzzles on $D_0^*(2300)$

- A broad resonance $D_0^* \to D\pi$ was found by Belle collaboration in 2004, **Breit-Wigner is used**



Satpathy et al, PRB 159 (2003) 553.

- UChPT: **two-pole structure**
- Should be tested on the lattice at a series of m_π
- Input for T_{cc}

Albaladejo et al, PLB 767 (2017) 465

- CLQCD ensembles
- Pion mass = 317 / 305 / 208 MeV to track the chiral behavior

Operator consTruction (OpTion)

- The rotation symmetry **broken** to $O(3) \rightarrow O_h$ or even more
- Software Option has been a routine tool in the CLQCD collaboration

https://github.com/wittscien/OpTion

README MIT license

OpTion

license MIT | Mathematica 10.x 11.x 12.x 13.x

Releases 2

OpTion 1.1.0 Latest
on Jul 24

+ 1 release

Packages

```
(* I=3 πππ or I=1 πππ π(1300) *)
Print /@ NHadronOperatorAll[{0, 0, 0}, "A1-", 1, MaxMom, {"P", "P", "P"}];
打印

P1[0] P2[0] P3[0]
P1[ex] P2[0] P3[-ex] + P1[-ex] P2[0] P3[ex] + P1[ey] P2[0] P3[-ey] + P1[-ey] P2[0] P3[ey] + P1[ez] P2[0] P3[-ez] + P1[-ez] P2[0] P3[ez]
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```
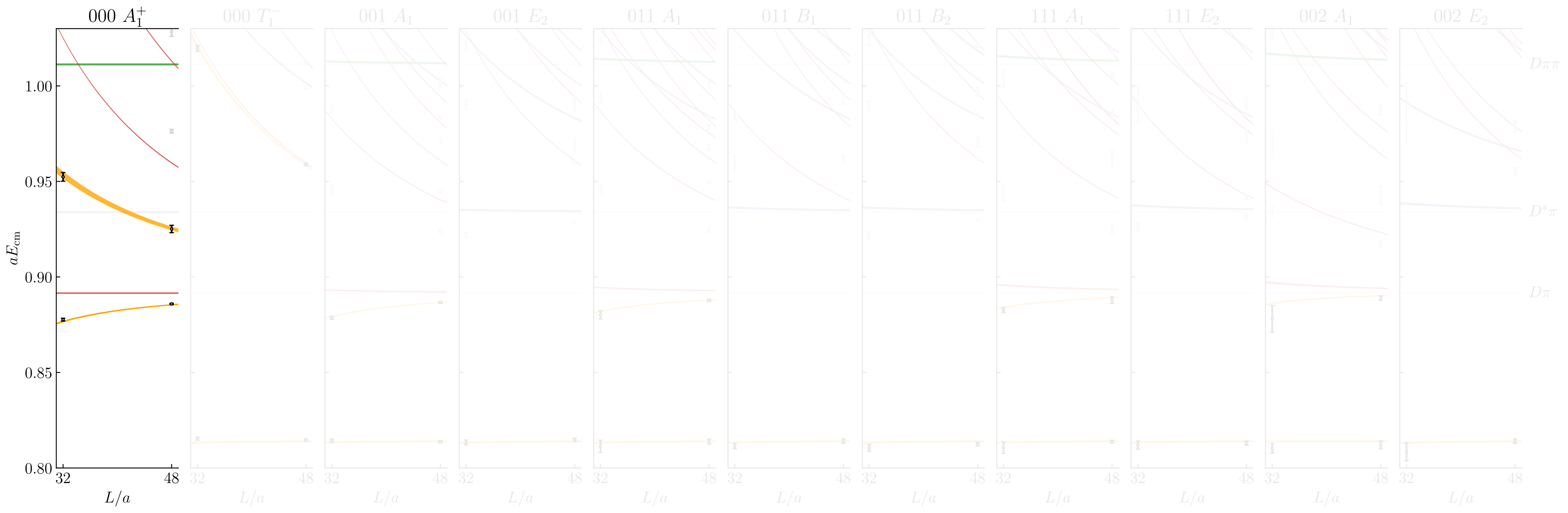

燕浩波（Haobo Yan），刘川（Chuan Liu），刘柳明（Liuming Liu），孟雨（Yu Meng），**JHEP** 10 (2025) 210.

Eg: Finite-volume spectra at $m_\pi \approx 305$ MeV



- Strong **attraction** in the S wave

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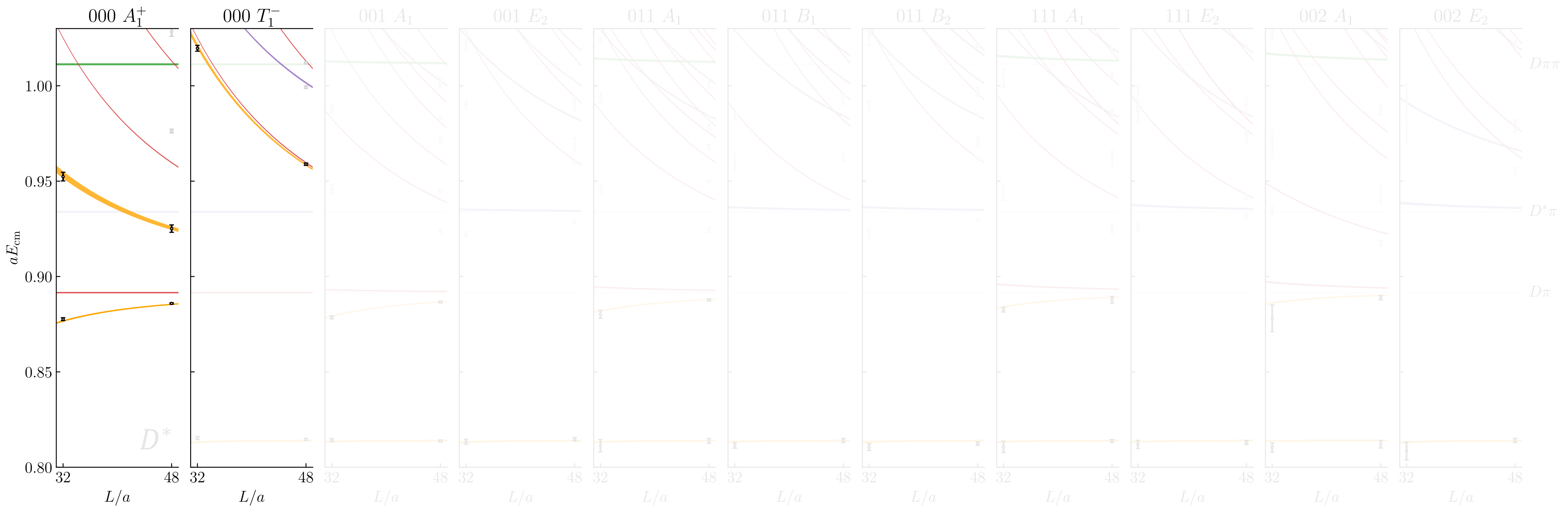

- Strong **attraction** in the S wave
- Negligible interaction in the P wave

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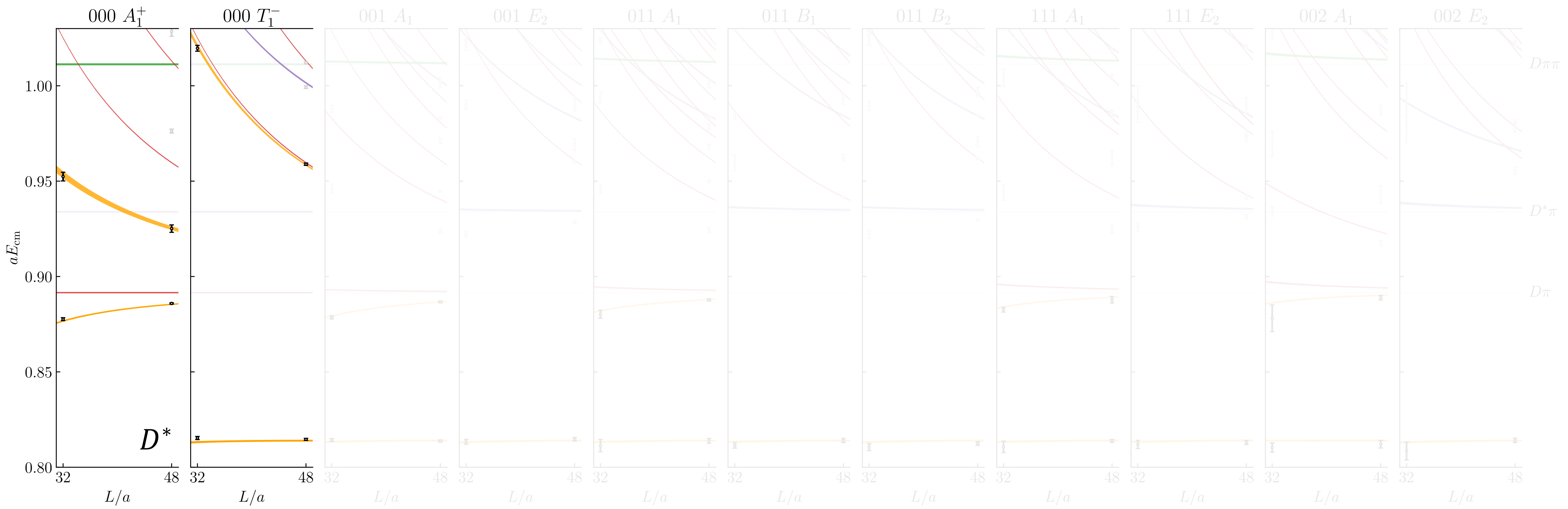

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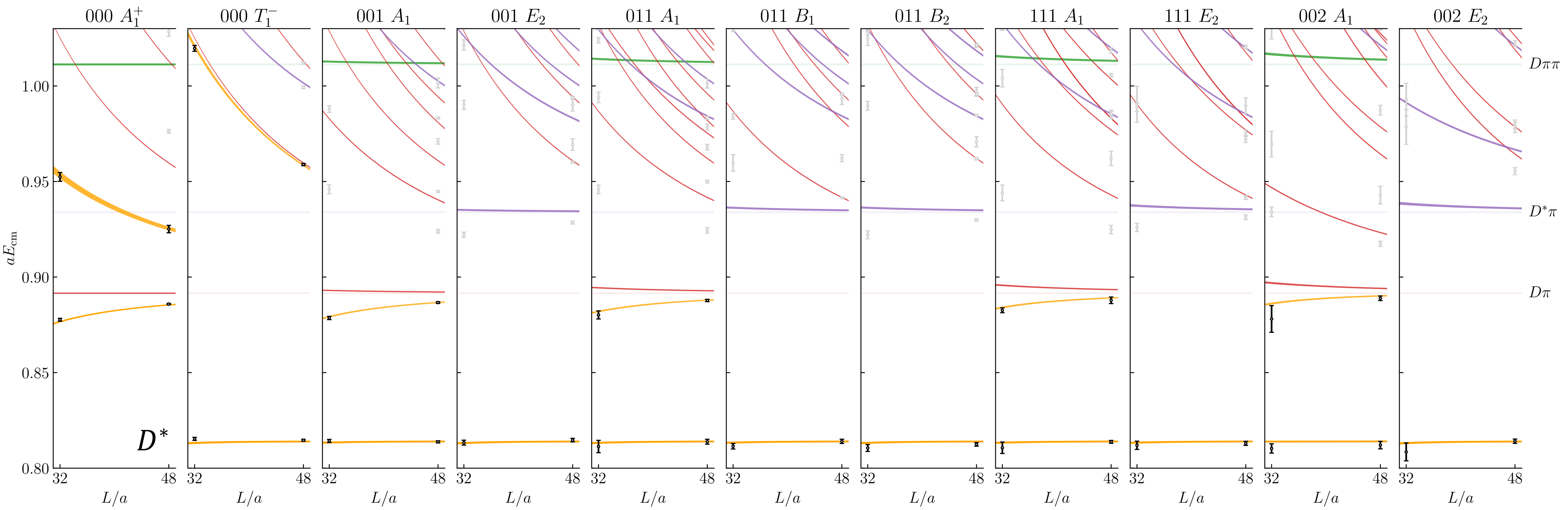

- Strong **attraction** in the S wave
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- Many moving frames to constrain strongly on the amplitude
- Restricted below inelastic thresholds

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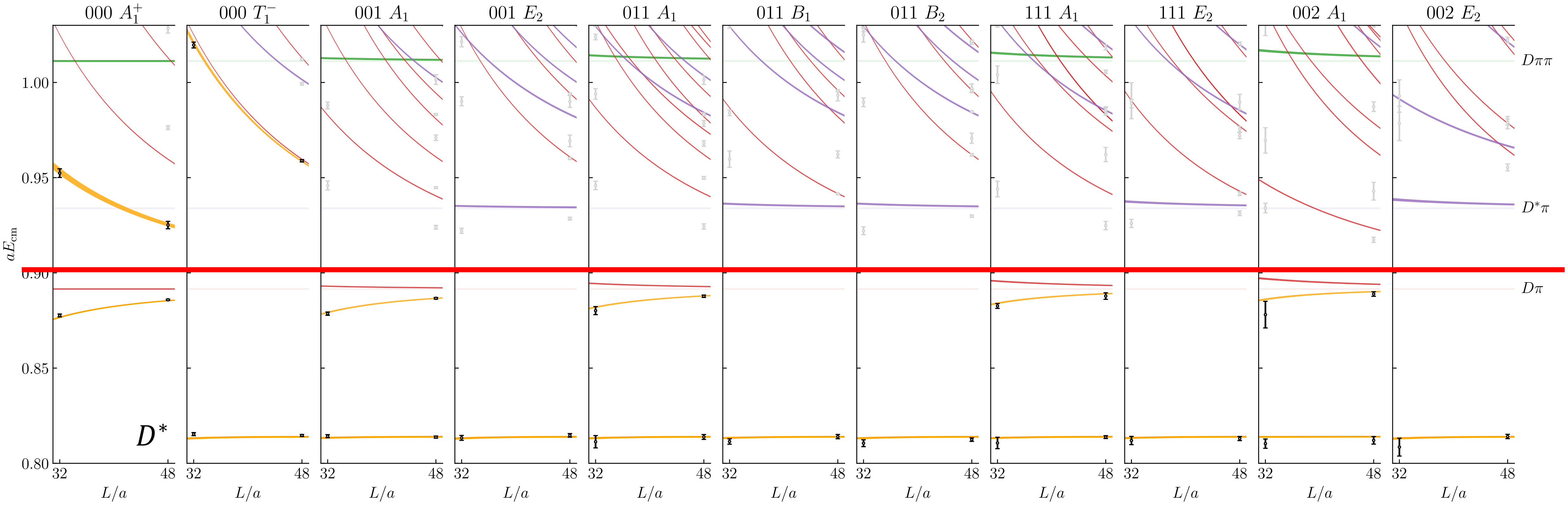

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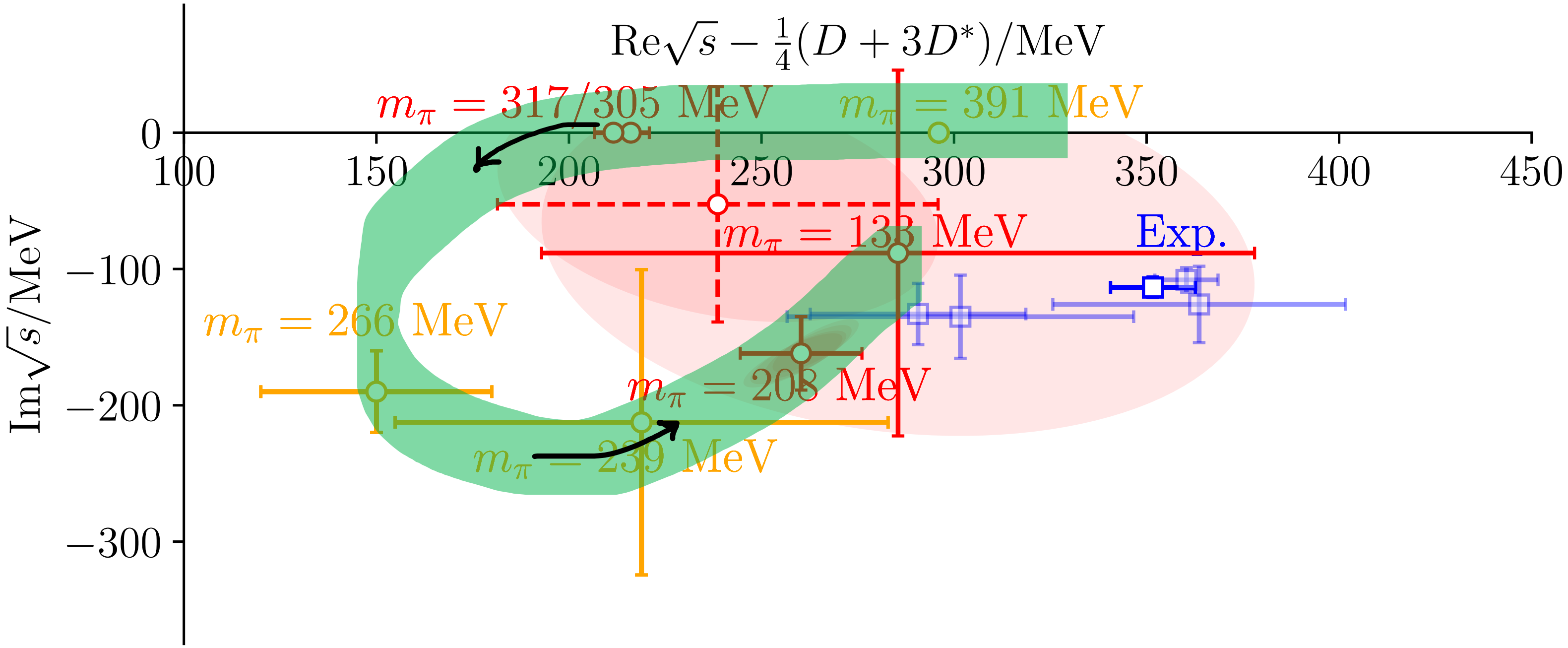
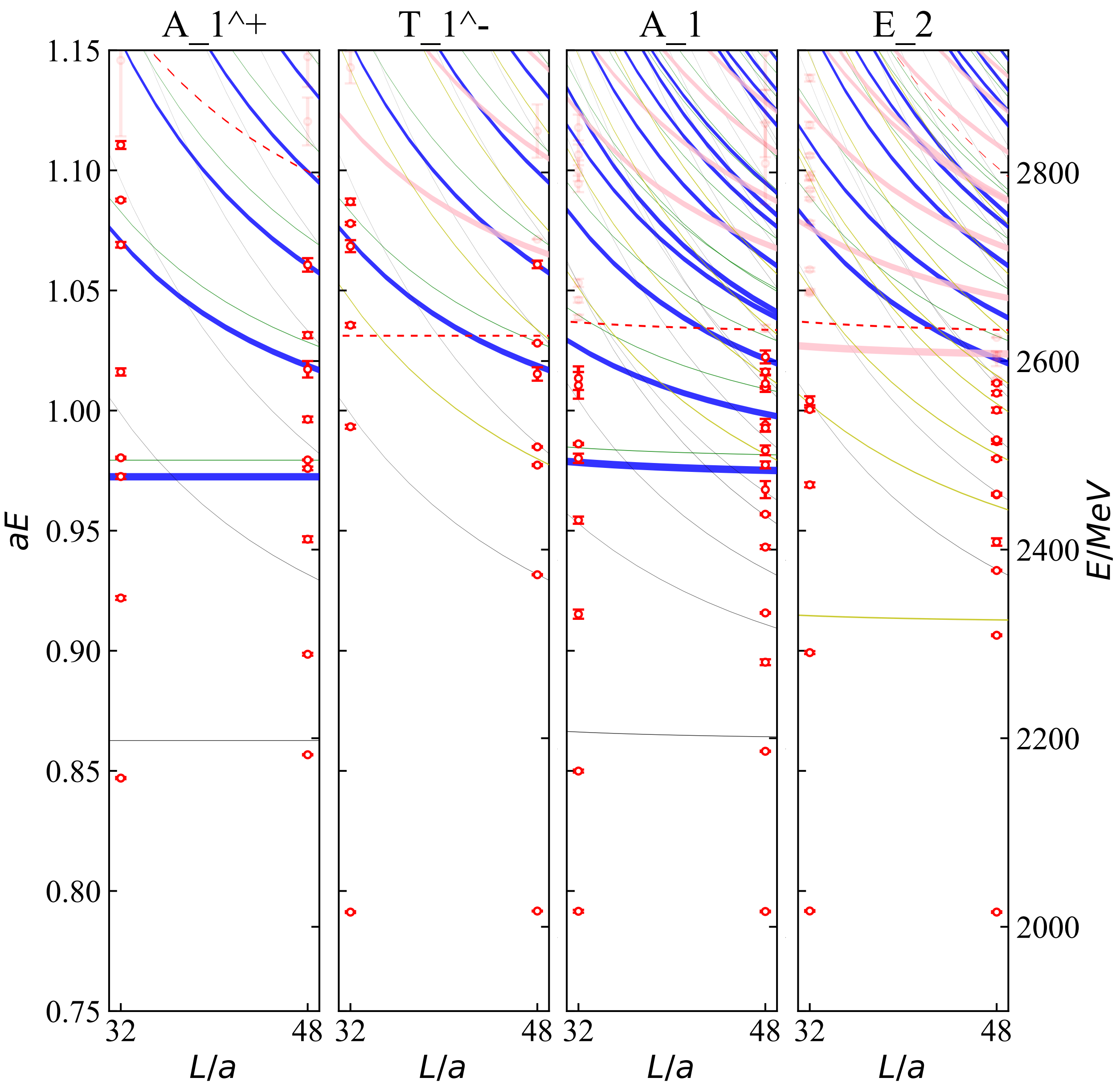


➢ An clear trend for the motion of the D_0^* pole:

bound state → virtual state → resonance

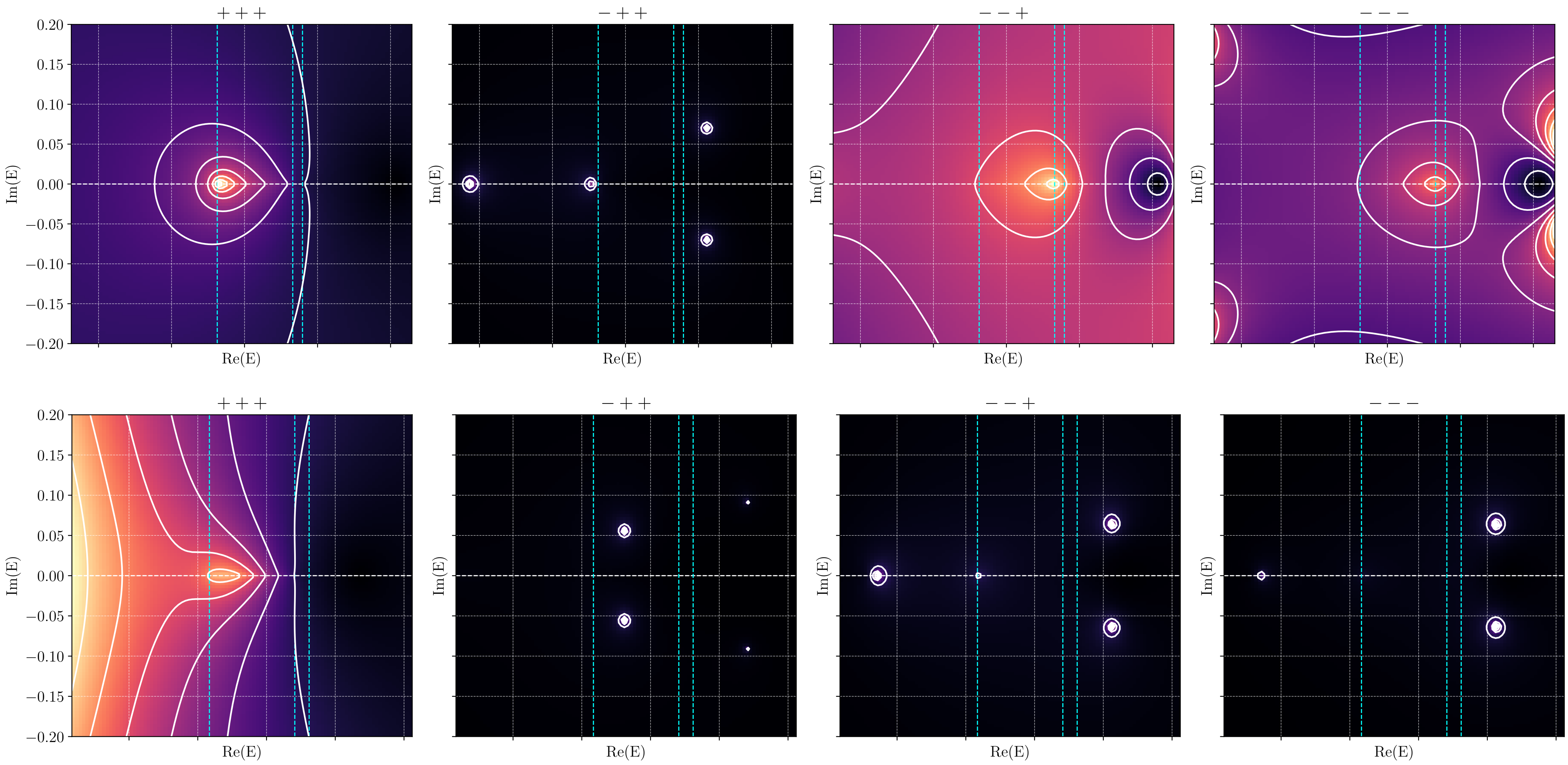
燕浩波（Haobo Yan），刘川（Chuan Liu），刘柳明（Liuming Liu），孟雨（Yu Meng），邢瀚洋（Hanyang Xing），**Phys. Rev. D** 111 (2025) 014503.

$D\pi - D\eta - D_s\bar{K} - D^*\pi$ coupled channel scattering

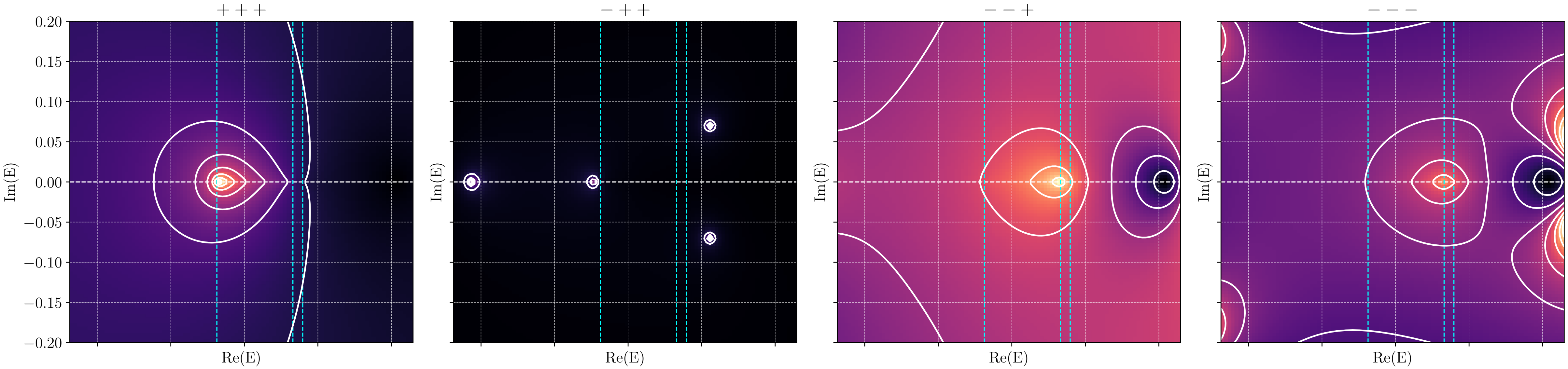


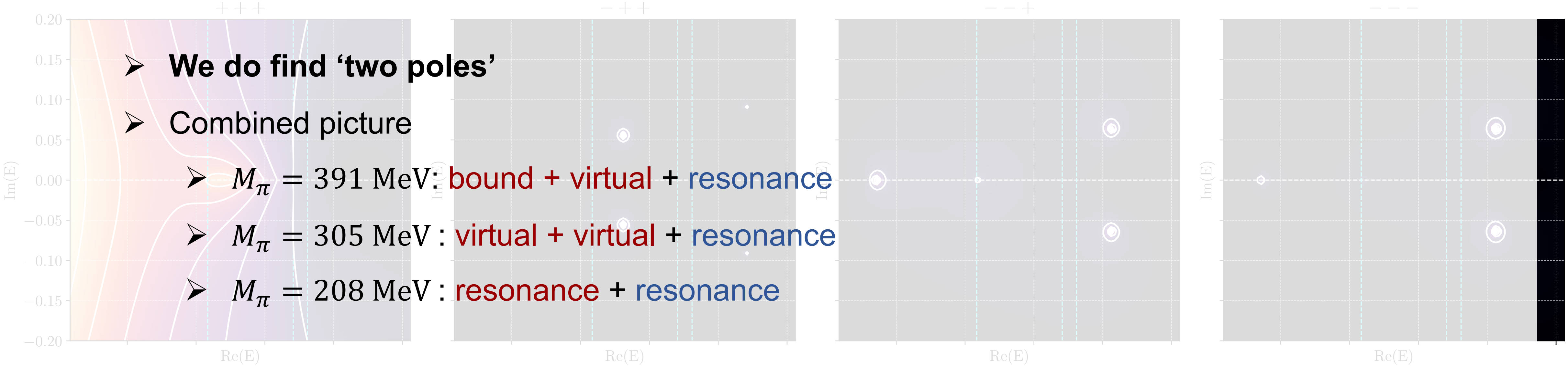
- $\mathcal{O}(1000)$ energy levels

$D\pi - D\eta - D_s\bar{K} - D^*\pi$ coupled channel scattering – pole



$D\pi - D\eta - D_s\bar{K} - D^*\pi$ coupled channel scattering – pole





- **We do find ‘two poles’**
- Combined picture
  - $M_\pi = 391$ MeV: bound + virtual + resonance
  - $M_\pi = 305$ MeV : virtual + virtual + resonance
  - $M_\pi = 208$ MeV : resonance + resonance

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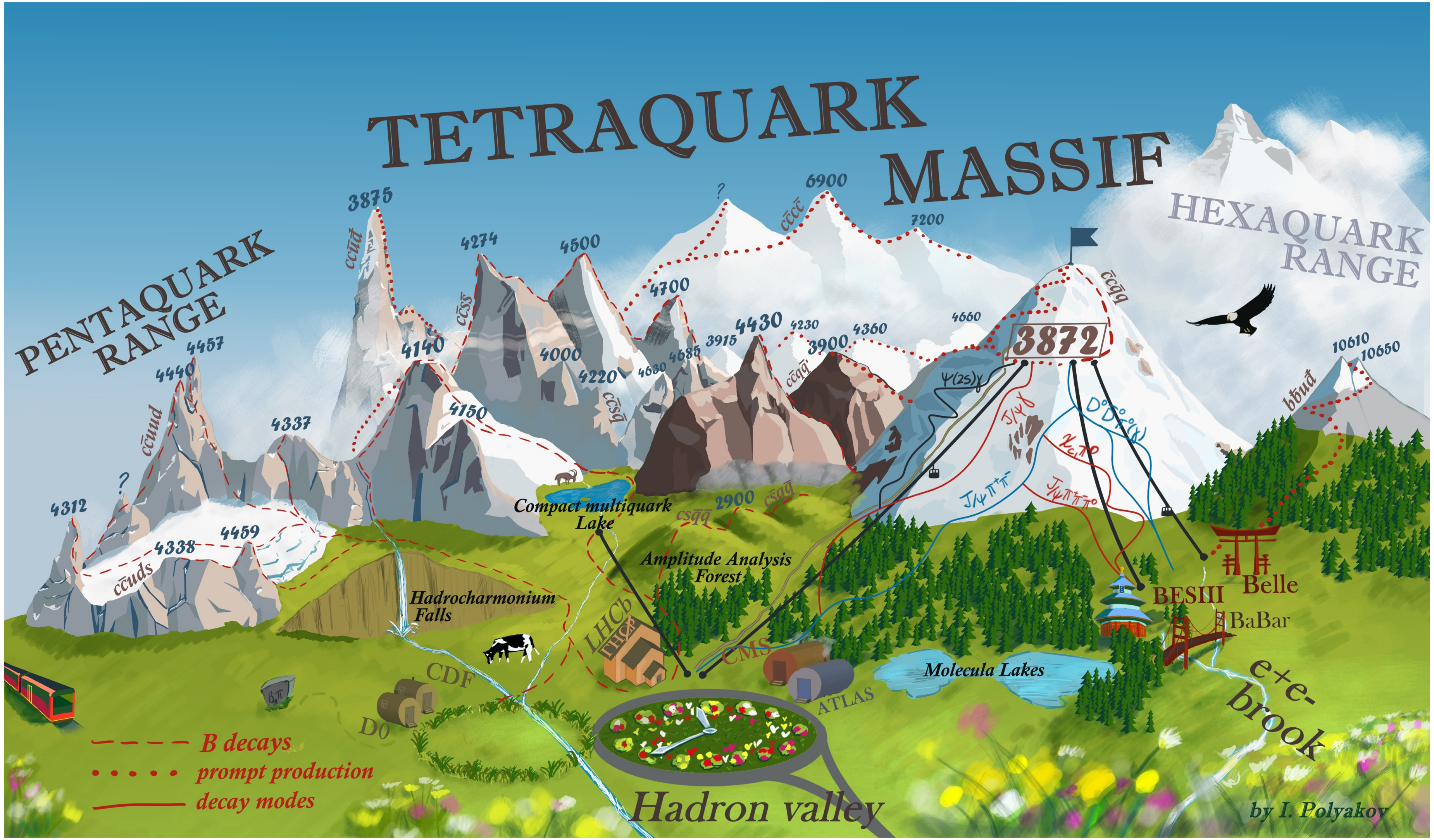
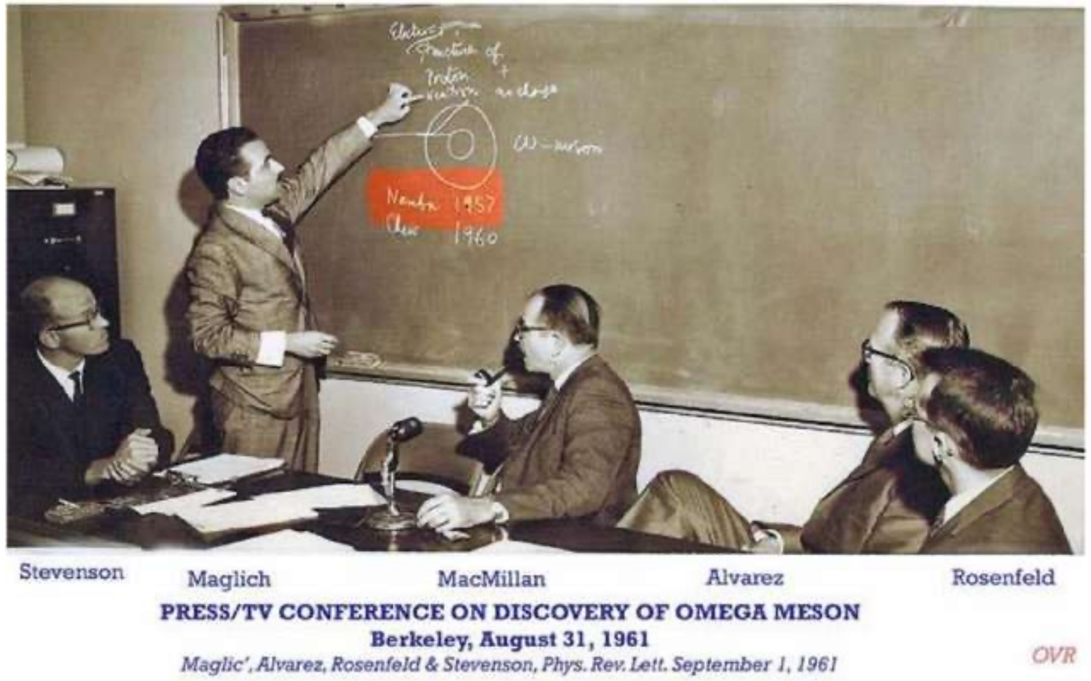
Milestones of hadron scattering

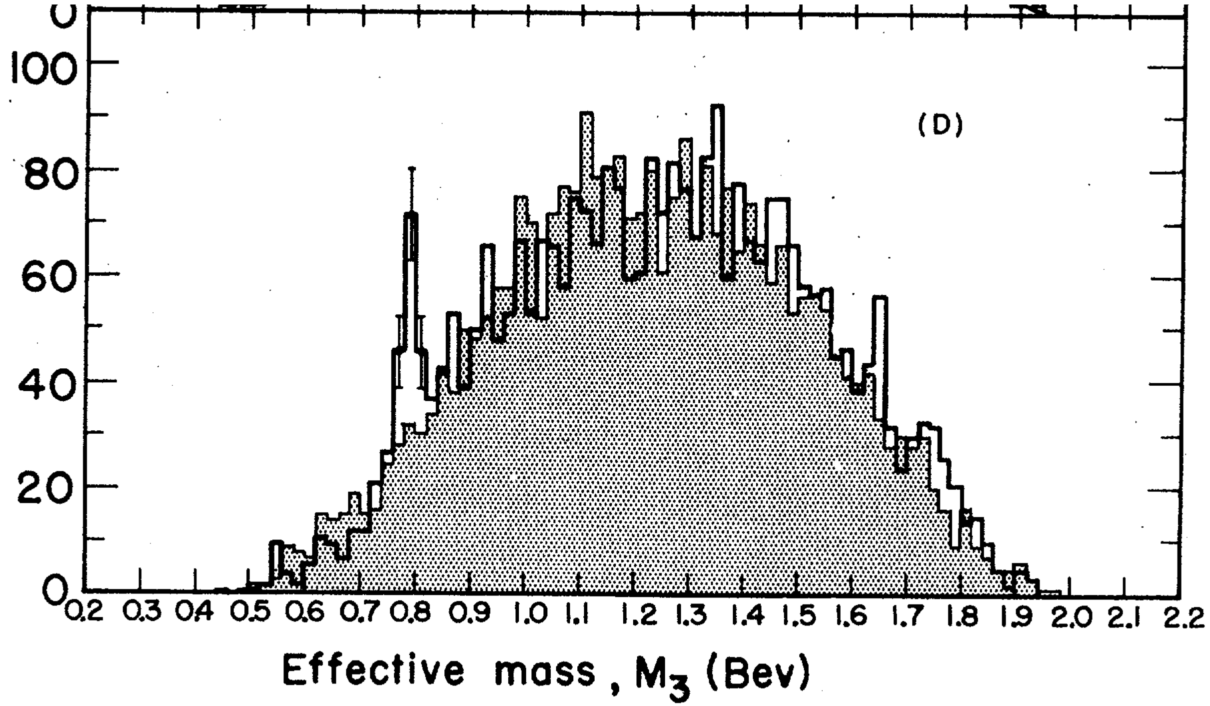


- Many nonperturbative puzzles of strong interactions are related to the **three-body problem**
  - e.g. T_{cc} and Roper
  - But many challenges

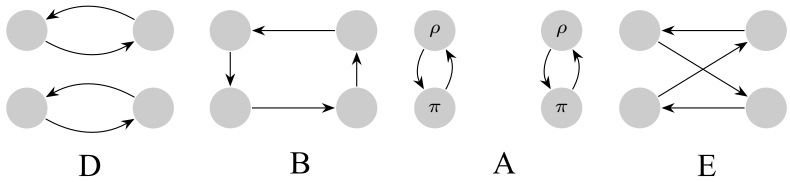
Towards the three-body problem

- We first take a challenge on $\omega \to \pi\pi\pi$
  - dominates the isoscalar response within the VMD picture of the photon-nucleon interactions
  - generates the observed repulsion at < 1 fm in the one-boson-exchange picture of the $N - N$ interaction
  - …

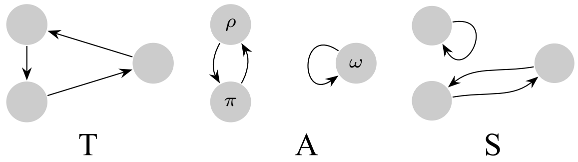




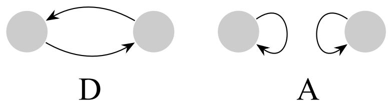
Contractions



$\sim\mathcal{O}(10)$

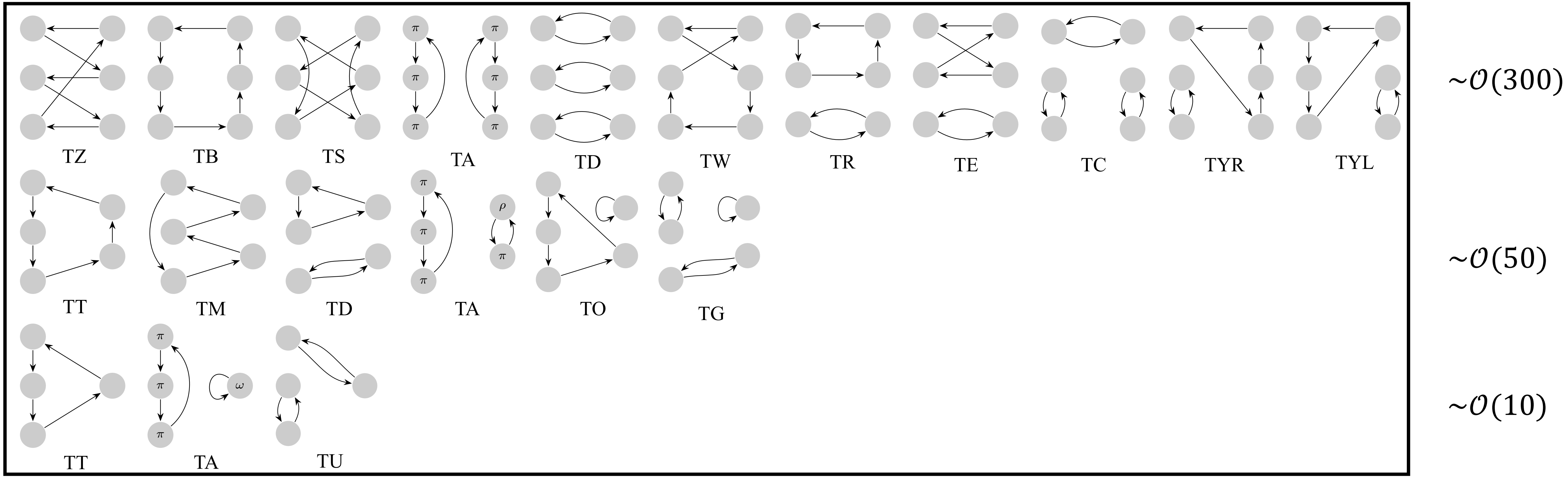


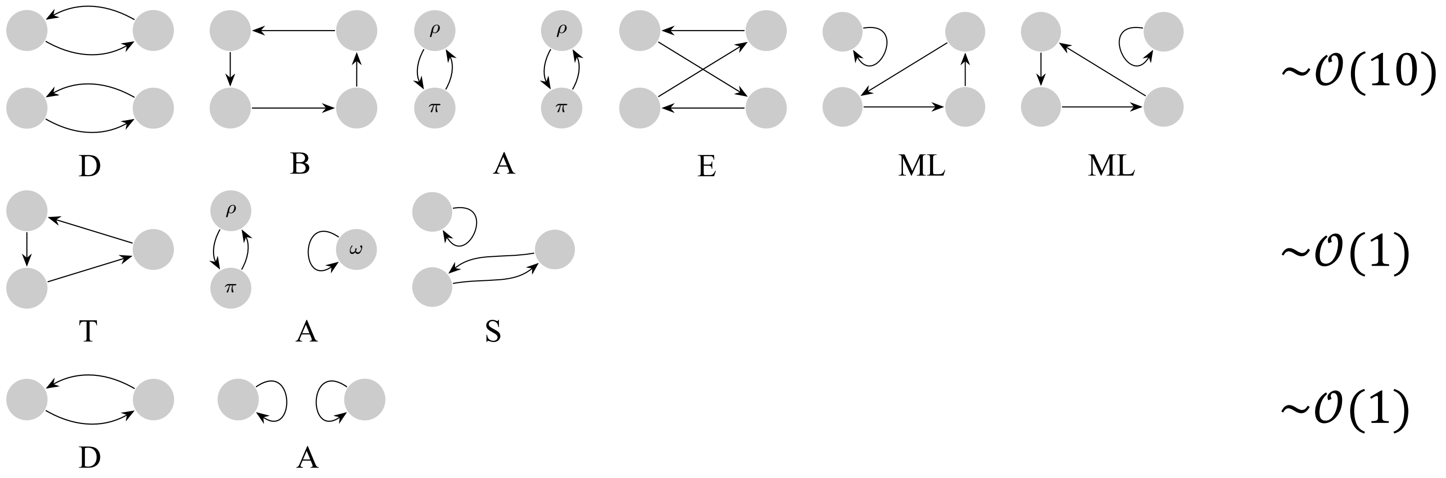
$\sim\mathcal{O}(1)$



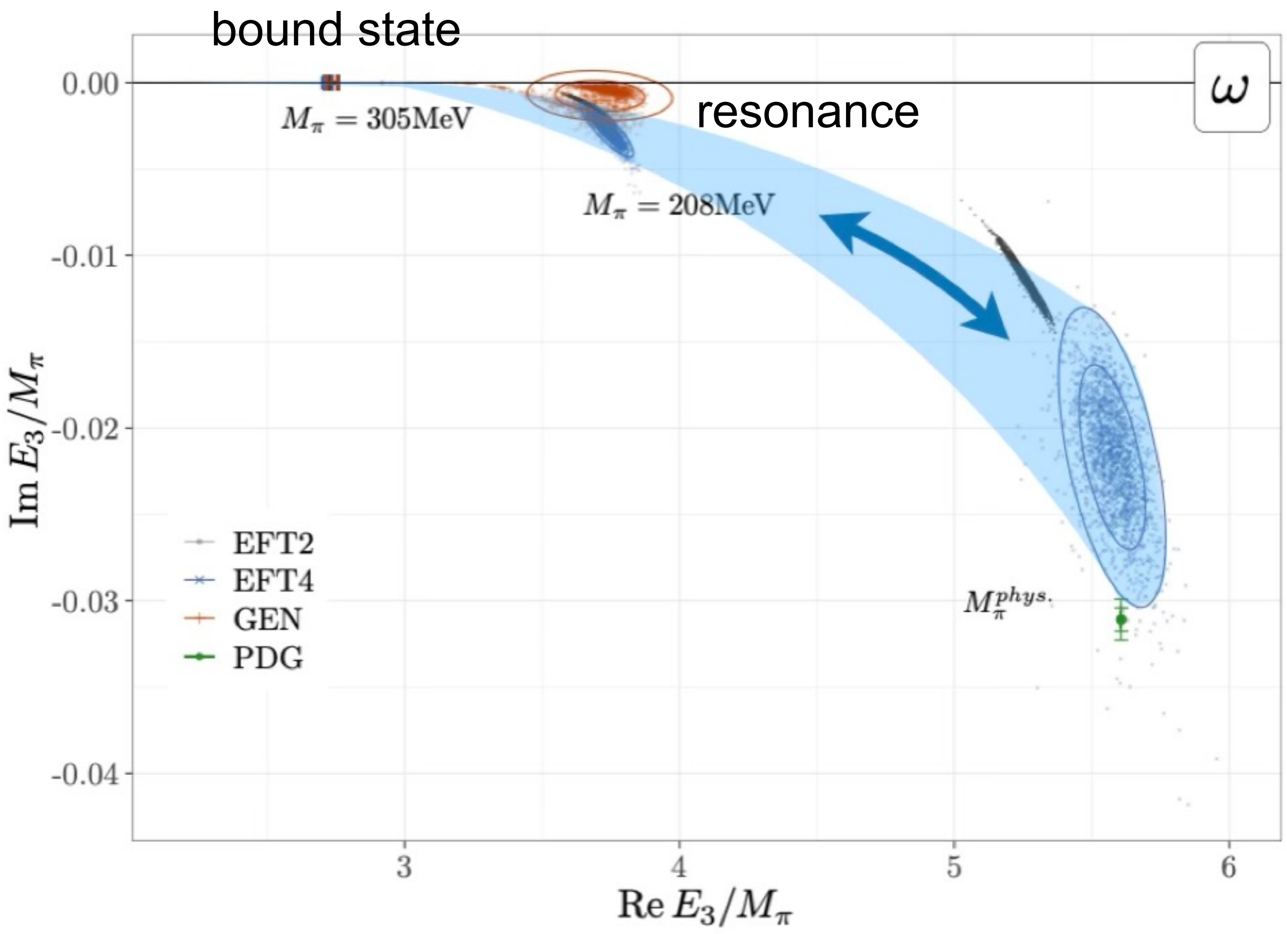
$\sim\mathcal{O}(1)$

Contractions





- A code is written to write code



- ➢ Extrapolate to the physical pion mass
- ➢ Different parametrizations are consistent

$$M_{\omega(782)} = (778.0 \pm 11.2) - i(3.0 \pm 5)\ \text{MeV}$$

燕浩波（Haobo Yan）, Maxim Mai, Marco Garofalo, Ulf-G. Meißner, Chuan Liu, Liuming Liu, Carsten Urbach, **Phys. Rev. Lett** 133 (2024) 211906

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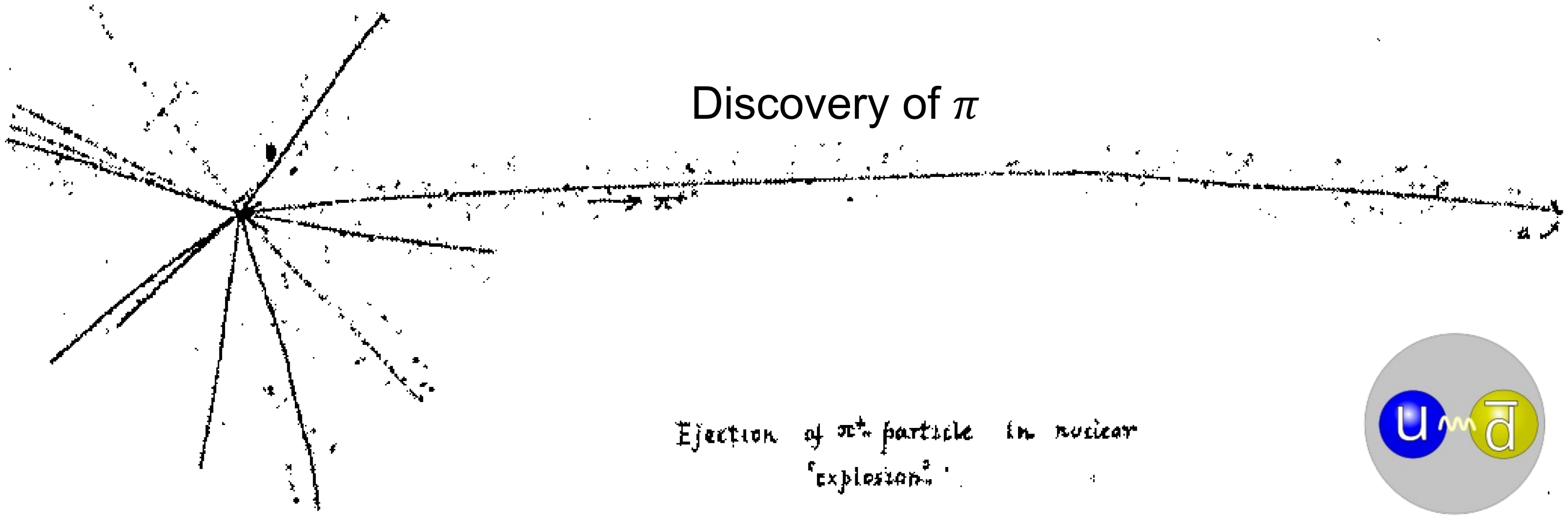
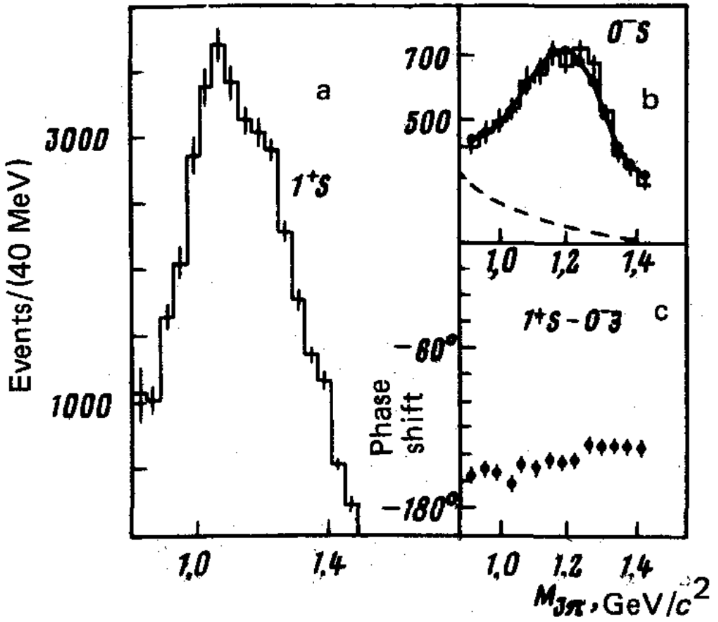
Excitation of pion

Bellini et al (1961)

Discovery of π

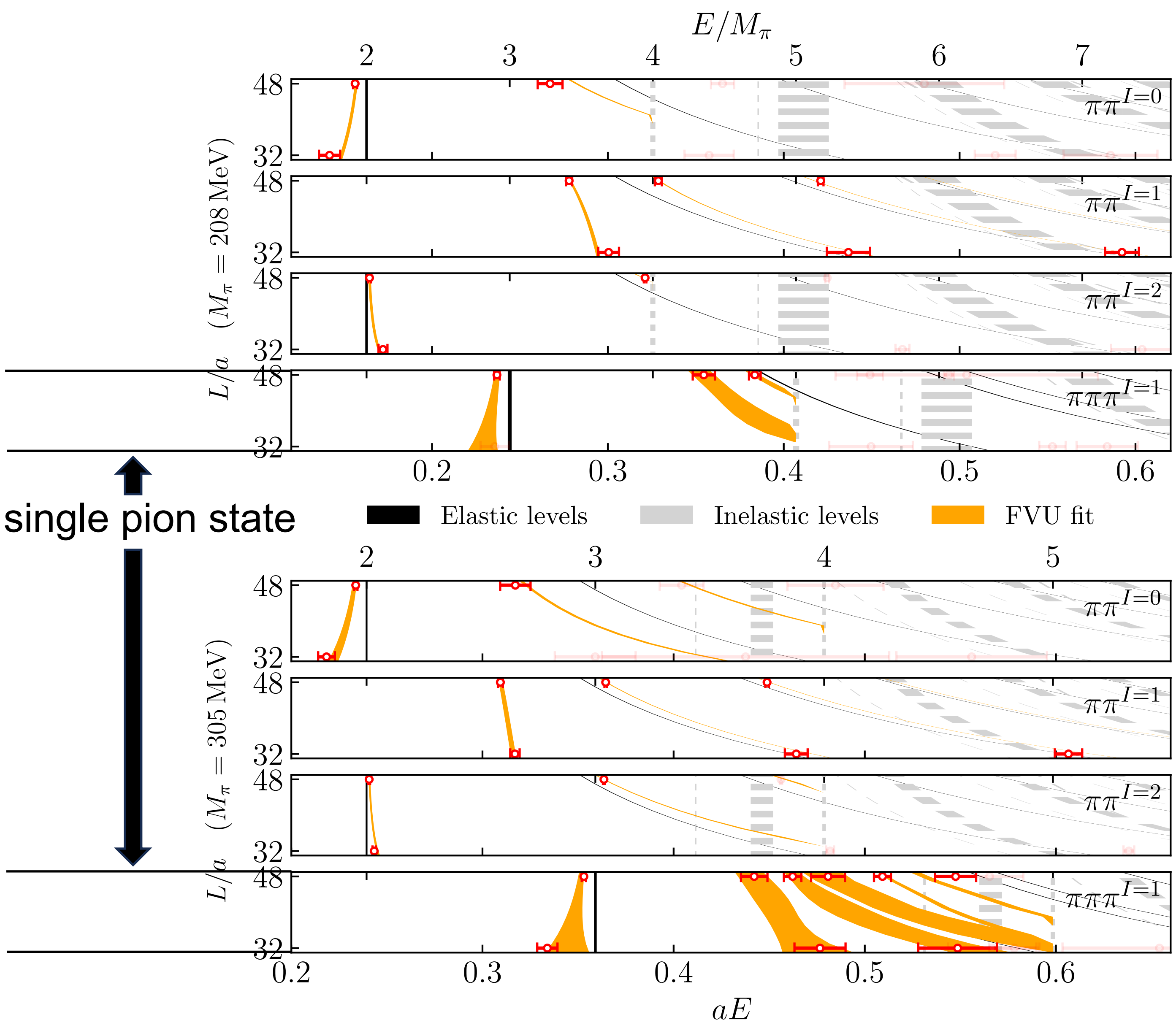


Evidence of $\pi(1300)$



- π: the lightest hadron, Goldstone boson; $\pi(1300)$ is 10x heavier than π, and its existence is not well established
- A direct test of the soft-pion theorem
- Go beyond the post-diction
- Decay to $\pi\pi\pi$ without centrifugal barrier: a second step towards Roper → $N\pi\pi$

Finite-volume spectra and fit

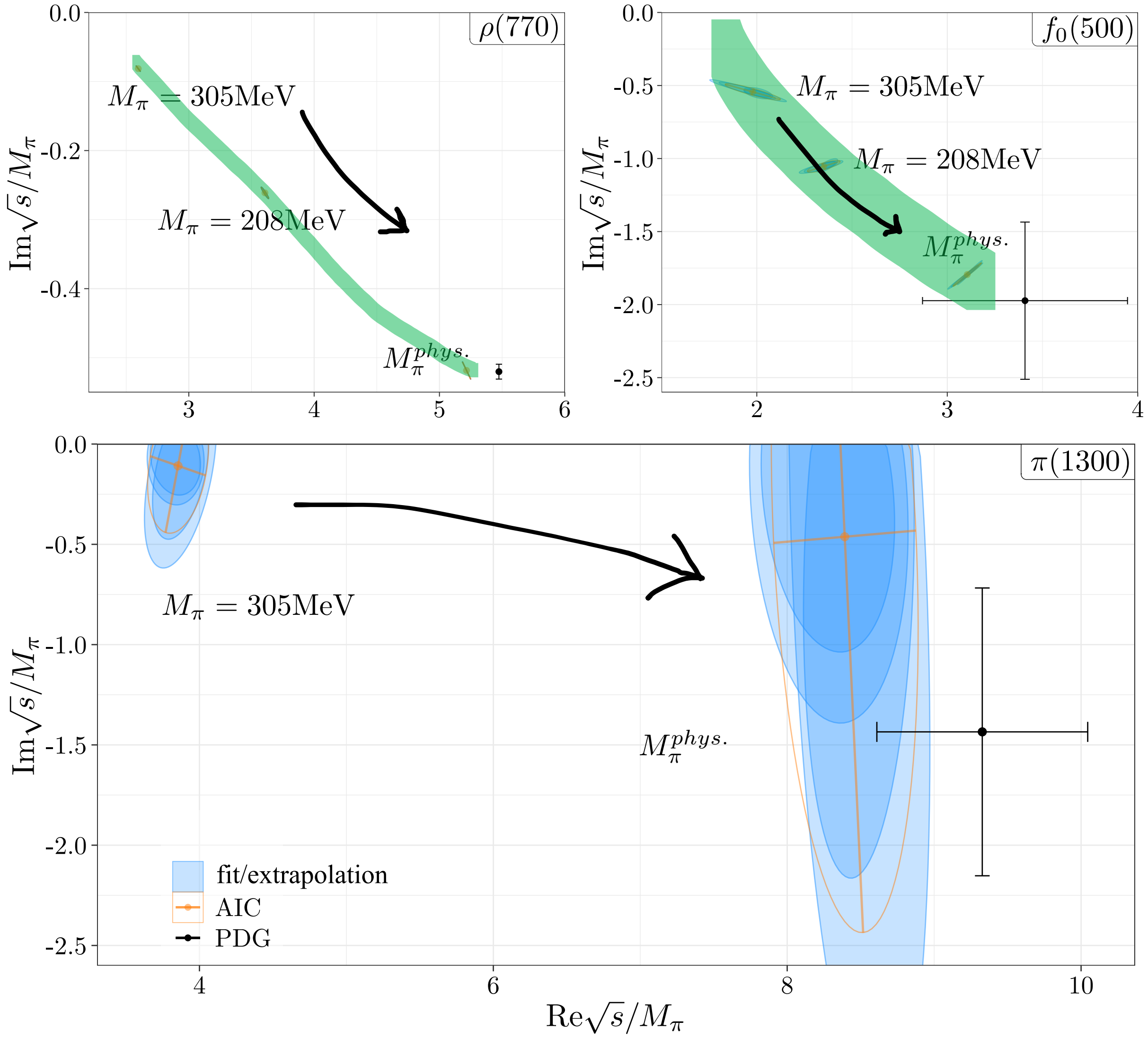


- $I = 0, 1, 2$ $\pi\pi$ levels as expected
- Aggregation of additional states in $\pi\pi\pi$

⬇

- Indication of a resonance in 0^-

Pole positions



- $M_\pi = 305$ MeV: resonance
- $M_\pi = 208$ MeV: only few of the fits lead to poles

$$M_{\pi(1300)} = (1169 \pm 46) - i(62 \pm 169)\ \mathrm{MeV}$$

$$M_{\rho(770)} = (727 \pm 3) - i(72 \pm 1)\ \mathrm{MeV}$$

$$M_{f_0(500)} = (433 \pm 7) - i(250 \pm 7)\ \mathrm{MeV}$$

燕浩波（Haobo Yan）, Maxim Mai, Marco Garofalo, Yuchuan Feng, Micheal Döring, Chuan Liu, Liuming Liu, Ulf-G. Meißner, Carsten Urbach, **Phys. Rev. Lett** 136 (2026) 141901

Summary

- OpTion for the community
- $D_0^*(2300)$ pole movement as a function pion mass
- Two-pole structure of $D_0^*(2300)$ identified, results expected soon
- Discovery of $\omega(782)$ from QCD
- Discovery of $\pi(1300)$ from QCD
- More measurements coming up soon

Thank you!

Quantization condition

- Using FVU (**F**inite-**V**olume **U**nitarity)

$$\begin{cases} \tilde{K}^{-1}(\sigma) - \Sigma^{FV}(\sigma) = 0 \\ \det[(\tilde{K}^{-1}(s) - \Sigma^{FV}(s))E_L - (\tilde{B}(s) + \tilde{C}(s))] = 0 \end{cases}$$

B fixed by unitarity | C generic pole / EFT form | Σ fixed by unitarity | $\tilde{K}^{-1}$ EFT form

State-of-the-art formalisms

- FVU [Mai and Döring, 2017]
- RFT [Hansen and Sharpe, 2014]
- NREFT [Hammer, Pang, and Rusetsky, 2017]

For review of the EFT used, see Meißner (1988).

See Mai PRL (2019) for quantitative tests of smooth and hard cutoffs