Determination of B-meson distribution amplitudes from B to π, K, D transition form factors

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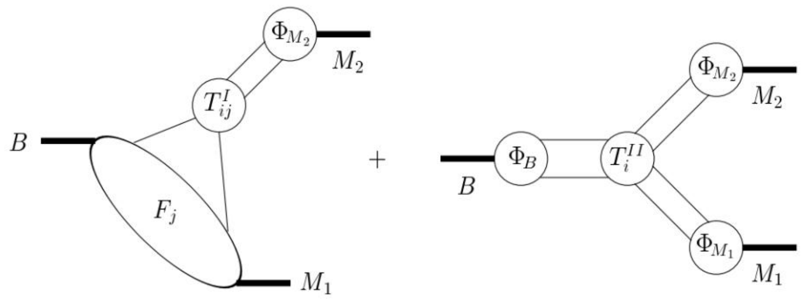
LCDA: encodes the momentum distribution of the light spectator parton at soft-scale

fundamental ingredient for establishing factorization formula in exclusive process

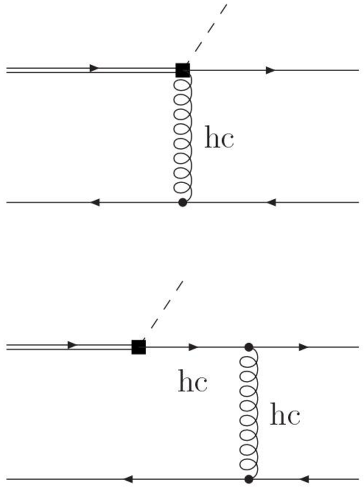
[Lepage and Brodsky, PLB (1979); Efremov and Radyushkin, PLB (1980)]

[Grozin and Neubert, PRD (1997); Beneke, Buchalla, Neubert and Sachrajda, PRL (1999)]

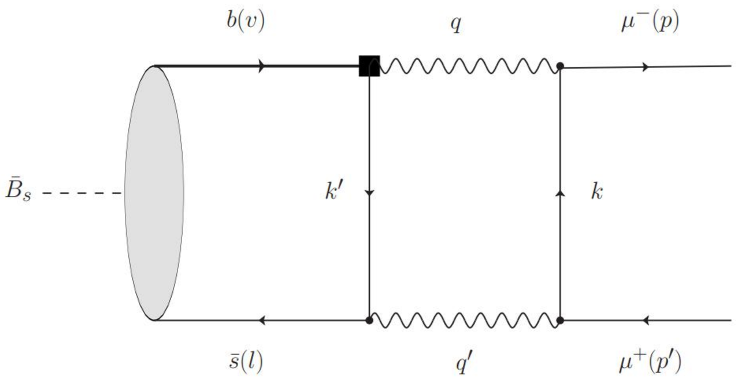
[Keum, H-N Li and Sanda, PRD (2001); Keum, Kurimoto, H-N Li and C-D Lu, PRD (2004)]



$B \to \pi\pi/\pi K$



$B \to \pi/K$ or $B \to D$



$B_s \to \mu^+ \mu^-$

B-meson LCDA and Light-cone Sum Rules

LCDA: renormalization group equation, perturbative QCD constraints, equations of motion, matching function and the model-independent determination of LCDA on the Euclidean lattice

$w \to 0$ and $w \to \infty$ behavior

[Lange, Neubert, PRL 2003; Bell, Feldmann, Wang, Yip, JHEP 2013]

[Braun, Ji, Manashov, JHEP 2017]

[Lee, Neubert, PRD 2005;Feldmann, Lange, Wang, PRD 2014]

[Wang, Wang, Xu, Zhao, PRD 2020; Beneke, Finauri, Vos, Wei, JHEP 2023]

Inverse moment: $\lambda_B = \int \mathrm{dw}\, \phi_B^+(w)/w \sim \Lambda_{\mathrm{QCD}}$

	QCD sum rule	LCSR $B \to \gamma$	LCSR $B \to \pi$	LCSR $B \to K$	LQCD LaMET	Traditional interval
λ_B [MeV]	460(110) 383(153)	310(60)-415(60) >214	303-389 460(160)	338^{+68}_{-9} 472^{+110}_{-41}	389(35)	350(150)
arxiv	hep-ph/0309330 2008.03935	2106.13616 1606.03080	1506.00667 0504091	2308.07033	2403.17492	

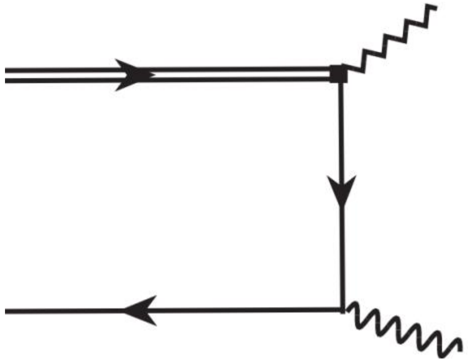
Higher order corrections in B decays: LCSRs with B-LCDA

Leading order results [Khodjamirian, Mannel, Offen, PLB 2005; PRD 2007]

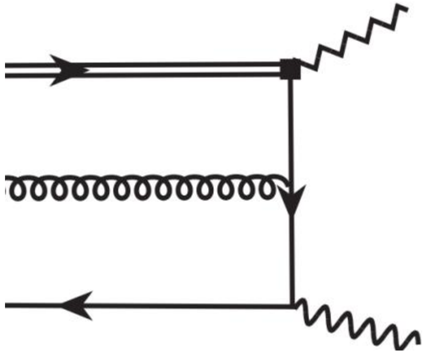
Next-to-leading-logarithmic accuracy [Wang, Shen, NPB 2015; Wang, Wei, Shen, Lü, JHEP 2017]

higher-twist two-particle and three particle contributions [Gubernari, Kokulu, Dyk, JHEP 2019; Lü,Shen,Wang, Wei, JHEP 2019]

Next-to-leading power corrections [Cui, Huang, Shen, Wang, Wang, JHEP 2023; Gao, Huber, Ji, Wang, Wang, Wei, JHEP 2022]



(a)



(b)

The starting point is two-point correlation function (2pt)

$$\Pi_\mu(n\cdot p, \bar{n}\cdot p) = \mathrm{i}\int \mathrm{d}^4\mathrm{x}\, \mathrm{e}^{\mathrm{i}p\cdot x}\langle 0|\mathcal{T}\{\bar{d}(x)\not{n}\gamma_5 q_1(x), \bar{q}_1(0)\Gamma_\mu b(0)\}|\bar{B}(p_B)\rangle, \tag{1}$$

Calculated at the external momenta far below the hadronic thresholds

$$p^2 \ll m_\pi^2 \text{ and } q^2 \ll m_B^2 \qquad n\cdot p = \frac{m_B^2 - q^2}{m_B} \sim \mathcal{O}(m_b), \qquad \bar{n}\cdot p \sim \mathcal{O}(\Lambda_{\mathrm{QCD}}),$$

Assuming the quark-hadron duality above the effective threshold ω_S

And applying the Borel transform $\bar{n}\cdot p \to \omega_M$, the sum rule of $f_{B\pi}^+$ can be obtained,

$$f_{B\pi}^+(q^2) = \frac{f_B m_B}{f_\pi n\cdot p}\,\mathrm{Exp}\left[\frac{m_\pi^2}{n\cdot p\, \omega_M}\right]\int_0^{\omega_s} \mathrm{d}\omega'\,\mathrm{Exp}\left(-\frac{\omega'}{\omega_M}\right)\boxed{\phi_B^-(\omega,\mu)}.$$

B-meson LCDA defined by non-local operator,

$$\langle 0|\bar{d}_\beta(\tau\bar{n})[\tau\bar{n},0]b_\alpha(0)|\bar{B}(p_B)\rangle = -\frac{i f_B m_B}{4}\left\{\frac{1+\not{v}}{2}\left[2\tilde{\phi}_B^+(\tau)+\left(\tilde{\phi}_B^-(\tau)-\tilde{\phi}_B^+(\tau)\right)\not{n}\right]\gamma_5\right\}_{\alpha\beta} \tag{9}$$

$$\phi_B^\pm(\omega',\mu) = \int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}e^{i\omega'\tau}\tilde{\phi}_B^\pm(\tau,\mu), \quad \tilde{\phi}_B^\pm(\tau,\mu) = \int_0^{+\infty} d\omega' e^{-i\omega'\tau}\phi_B^\pm(\omega',\mu), \tag{10}$$

The key parameter in modeling LCDA is the inverse moments of the leading twist LCDA,

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\phi_B^+(\omega,\mu), \quad \frac{\widehat{\sigma}_n(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\ln^n\frac{e^{-\gamma_E}\lambda_B(\mu)}{\omega}\phi_B^+(\omega,\mu). \tag{11}$$

We adopt the three-parameter ansatz [M. Beneke, V. M. Braun, Y. Ji and Y. B. Wei, JHEP (2018)]

$$\phi_B^+(\omega) = \frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha, 3-\alpha, \omega/\omega_0)$$

Determination the Inverse moment using B to π, K, D form factors

$$\phi_B^+(\omega) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} U(\beta-\alpha, 3-\alpha, \omega/\omega_0)$$

The parameters $\alpha,\ \beta,\ \omega_0$ can be determined via the inverse moments

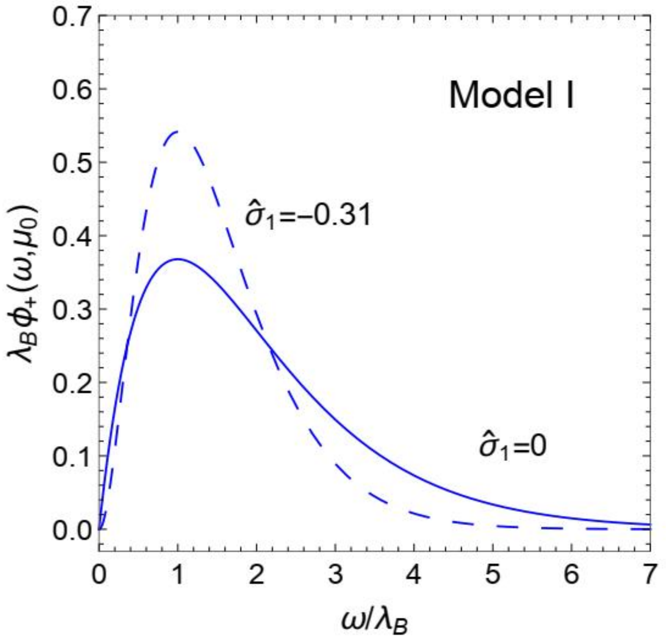
$$\lambda_B(\mu) = \frac{\alpha-1}{\beta-1}\omega_0,$$

$$\widehat{\sigma}_1(\mu) = \psi(\beta-1) - \psi(\alpha-1) + \ln\frac{\alpha-1}{\beta-1},$$

$$\widehat{\sigma}_2(\mu) = \widehat{\sigma}_1^2(\mu) + \psi'(\alpha-1) - \psi'(\beta-1) + \frac{\pi^2}{6},$$

With the choices

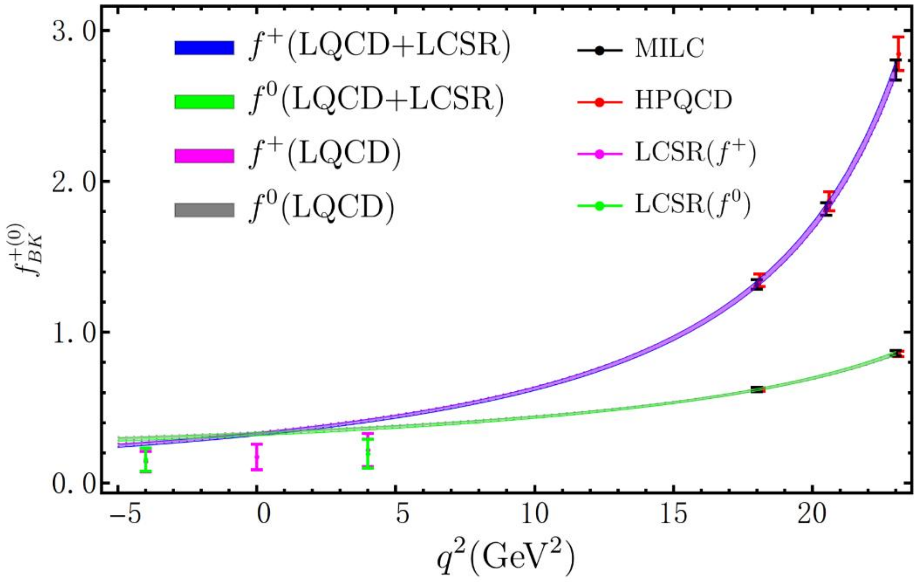
$\lambda_B \in [200,\ 500]$ MeV
$\widehat{\sigma}_1 \in [-0.7,\ 0.7]$
$\widehat{\sigma}_2 \in [-6,\ 6]$ ⇦ Shape parameters, Model dependent



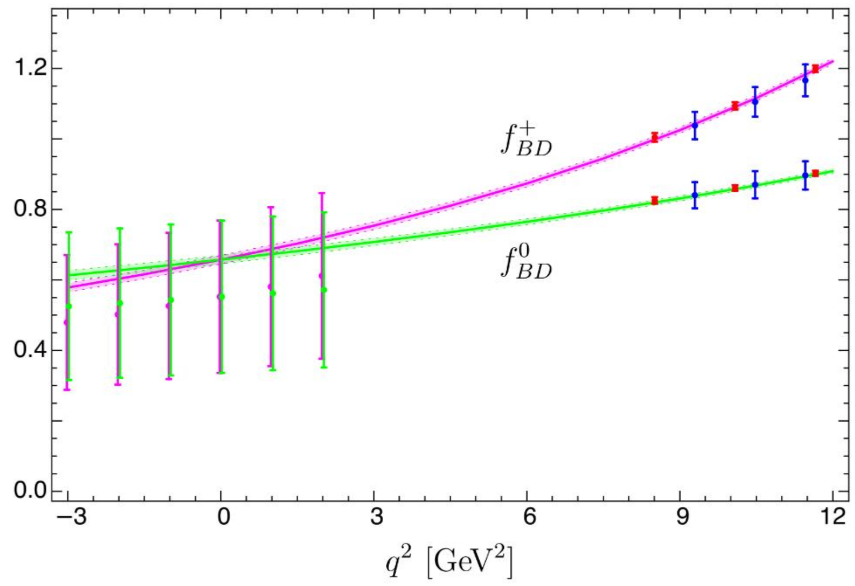
[Beneke, Braun, Ji, Wei, JHEP 2018]

$\widehat{\sigma}_1 = 0,\ \widehat{\sigma}_2 = \pi^2/6$
corresponds to the exponential model
$\phi_B^+(\omega) = \left(\frac{\omega}{\lambda_B^2}\right) \mathrm{Exp}[-\frac{\omega}{\lambda_B}]$

$\lambda_B \in [200,\ 500]$ MeV , $\hat{\sigma}_1 \in [-0.7,\ 0.7]$, $\hat{\sigma}_2 \in [-6,\ 6]$



$B \to K$



$B \to D$

[Cui, Huang, Shen, Wang, Wang, JHEP 2023]

[Gao, Huber, Ji, Wang, Wang, Wei, JHEP 2022]

$\chi^2 = \chi^2_{\mathrm{LQCD}} + \chi^2_{\mathrm{LCSR}} + \chi^2_{\mathrm{exp}}$

Inputs:

1. $B \to \pi,\ B \to K,\ B \to D$ form factors $f^{+,0}_{B\pi}$ $f^{+,0}_{BD}$ $f^{+,0}_{BK}$ f^{T}_{BK} from Lattice QCD
2. experimental measurements on q^2-binned branching fractions of $B \to \pi\ell\nu$

Related by BCL parametrization

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \qquad q^2 > (m_B + m_P)^2 \Rightarrow |z| = 1\,, \quad 0 < q^2 < (m_B - m_P)^2 \Rightarrow z < 1$$

$$f^{+}_{B_{q'}M}(q^2) = \frac{1}{1 - q^2/m^2_{B^*_q}} \sum_{k=0}^{N-1} b_k^+ \left[z(q^2, t_0)^k - (-1)^{k-N} \frac{k}{N} z(q^2, t_0)^N \right] \qquad N = 3$$

22 adjustable parameters : BCL $b_k^i + V_{\mathrm{ub}} + \lambda_B$

40 LQCD data points : $f^{+,0}_{B\pi}$ 6×2, $f^{+,0}_{BK}$ f^{T}_{BK} 8×2, $f^{+,0}_{BD}$ 6×2

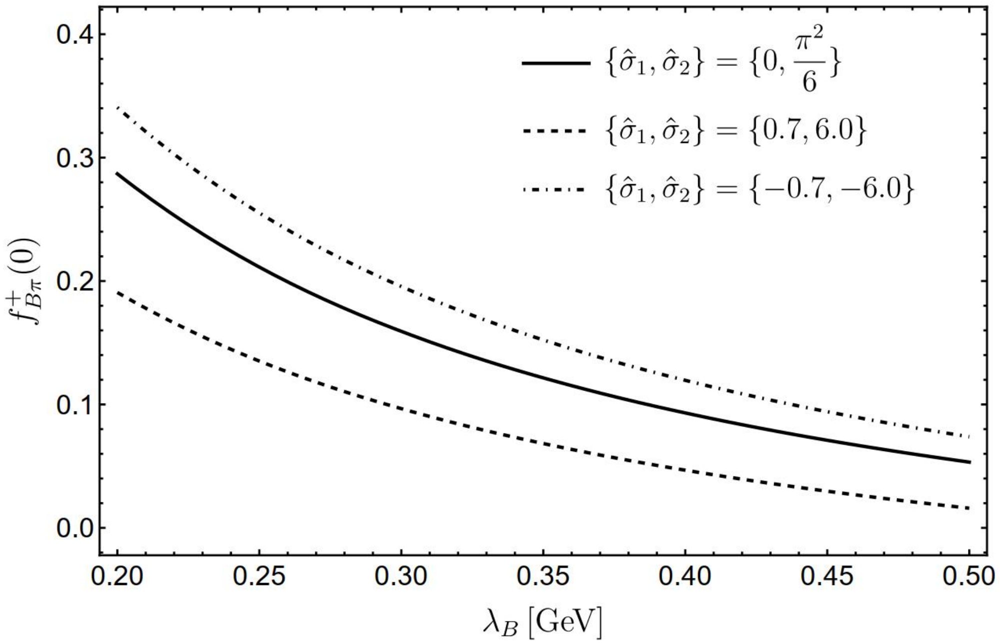
57 data points from experimental measurements

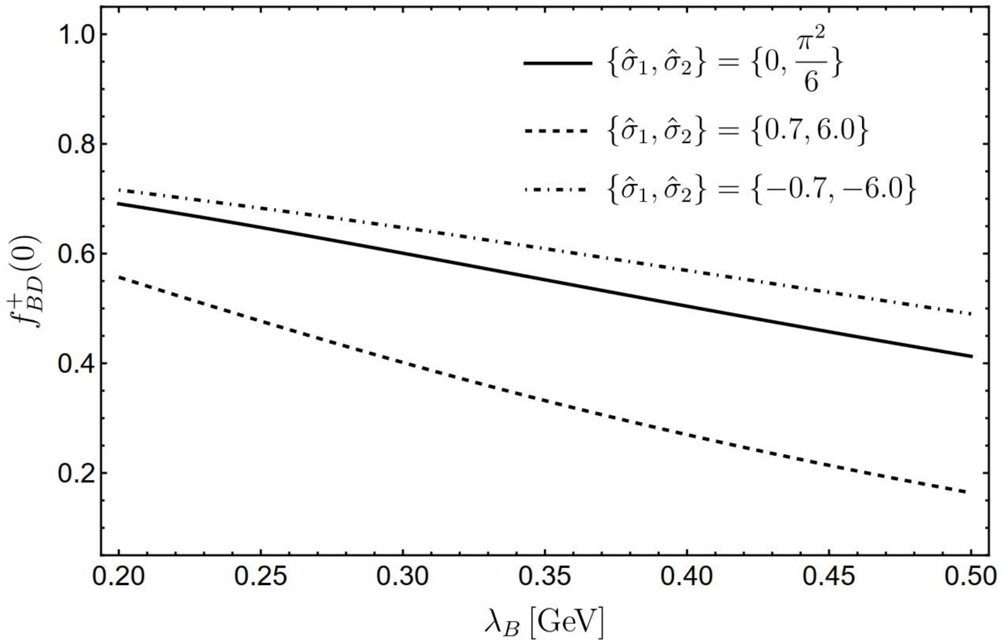
d.o.f = 75

$$\chi^2_{\mathrm{LCSR}} = \sum_{j,k=1}^{4} [f_j(0;b_i) - f_j(0;\lambda_B)] C^{-1}_{jk} [f_k(0;b_i) - f_k(0;\lambda_B)]$$

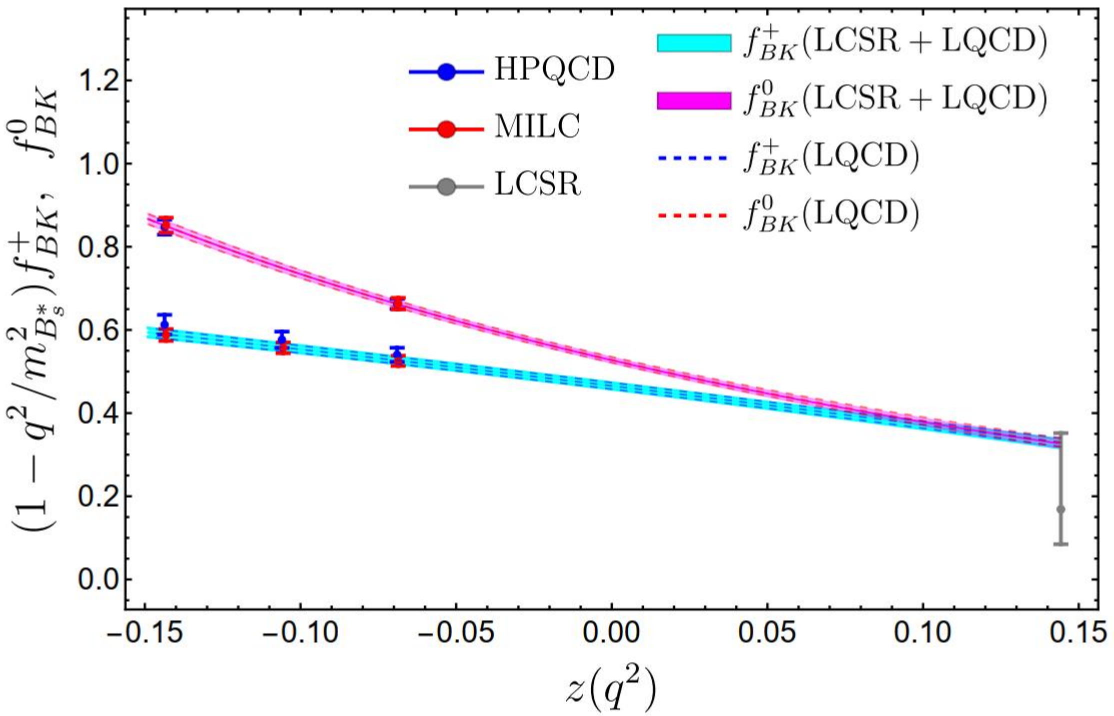
$f^+_{B\pi}$ f^+_{BD} f^+_{BK} f^T_{BK} at $q^2 = 0$

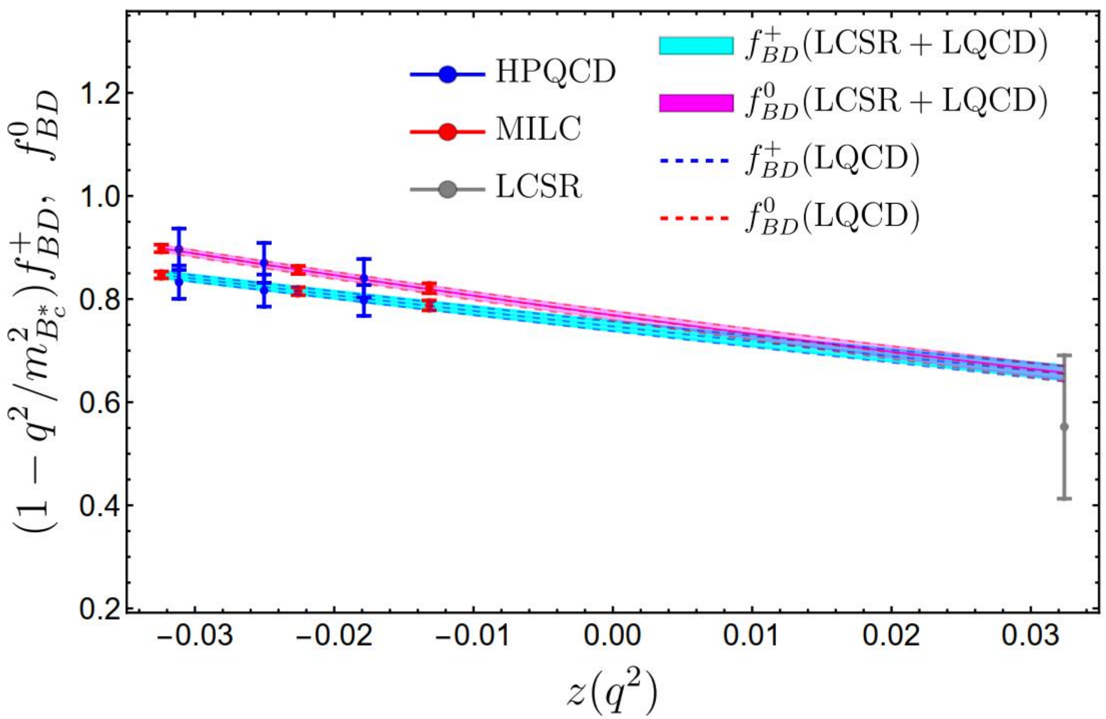
Covariance $C_{jk} = 10\% \times f_j$ uncorrelated + $10\% \times f_j^{\mathrm{NLP}}$ correlated





Analysis and Results

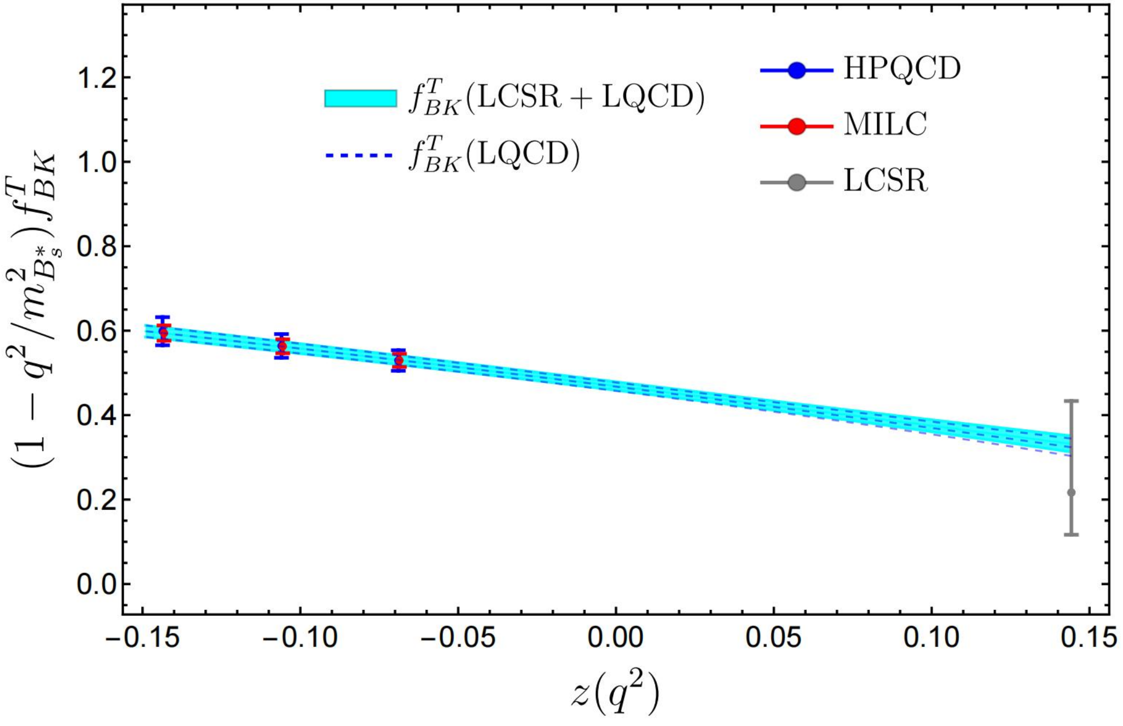


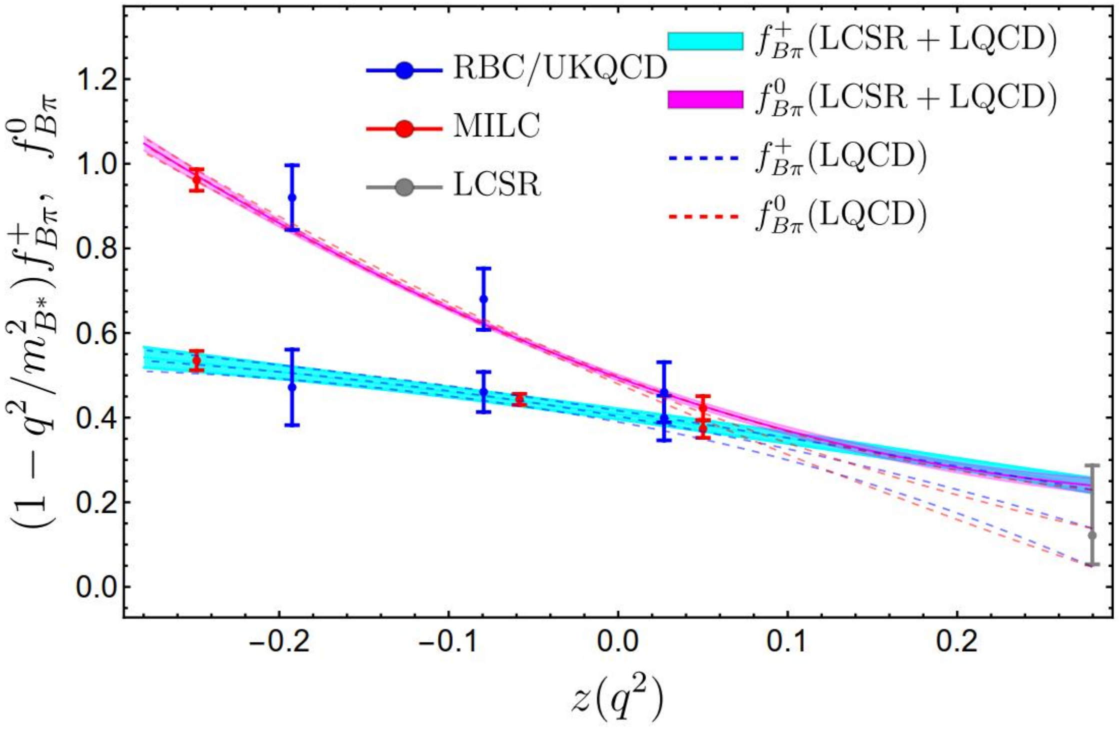


Gray line represents the LCSR calculated form factors with the interval $\lambda_B \in [200,\ 500]$ MeV and $\widehat{\sigma}_1 = 0, \widehat{\sigma}_2 = \pi^2/6$

The change of $\widehat{\sigma}_1, \widehat{\sigma}_2$ resulting an overall shift of the gray lines $\widehat{\sigma}_1 \in [-0.7,\ 0.7]$ $\widehat{\sigma}_2 \in [-6,\ 6]$

Analysis and Results





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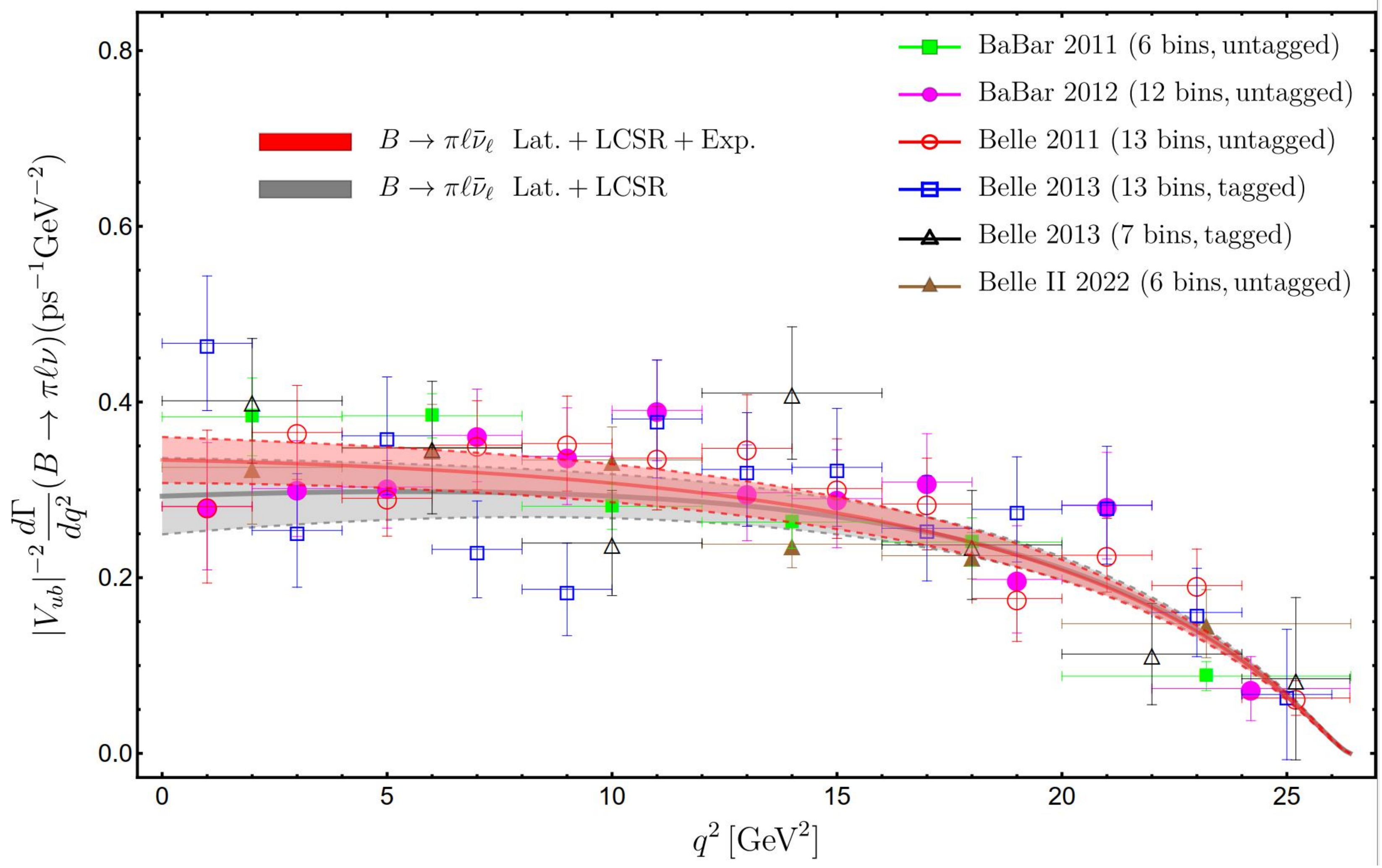


Results:

$\chi^2/\text{dof} = 1.1$

$\lambda_B = 217(19)^{+82}_{-17}$ MeV

$V_{\text{ub}} = 3.68(13)^{+1}_{-0} \times 10^{-3}$

$f^+_{B\pi}(0) = 0.258(10)^{+1}_{-1}$

$\hat{\sigma}_1 < 0.35$

Constraint:

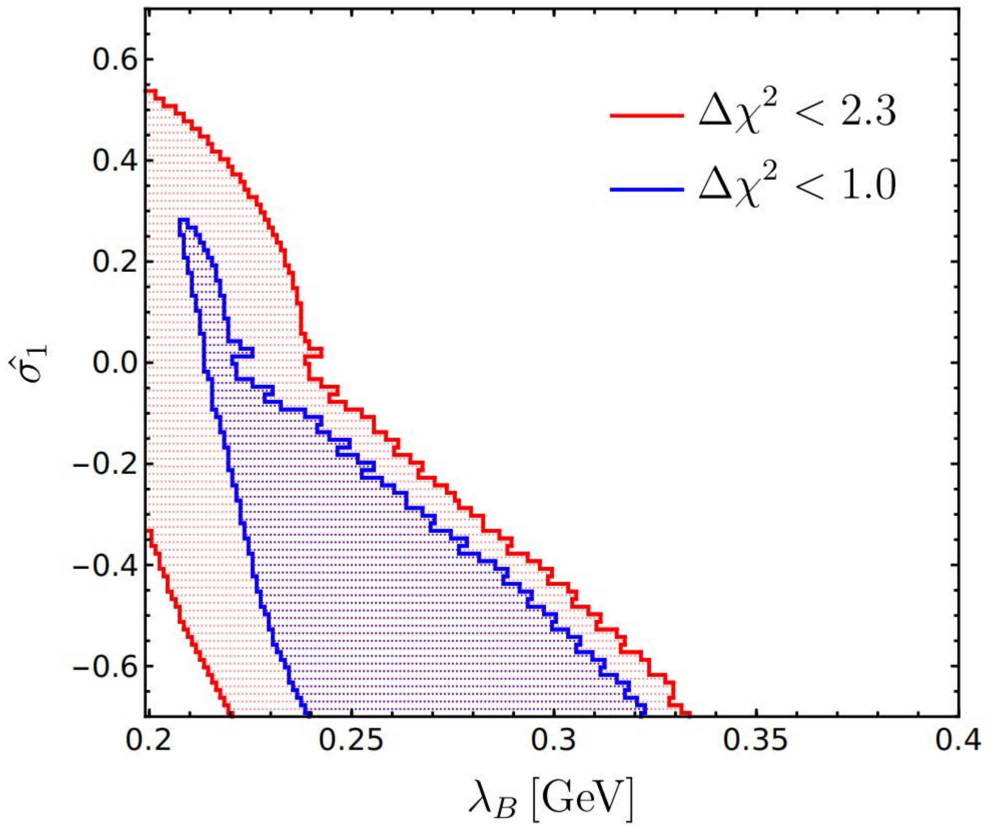
$\lambda_B > 200$ MeV

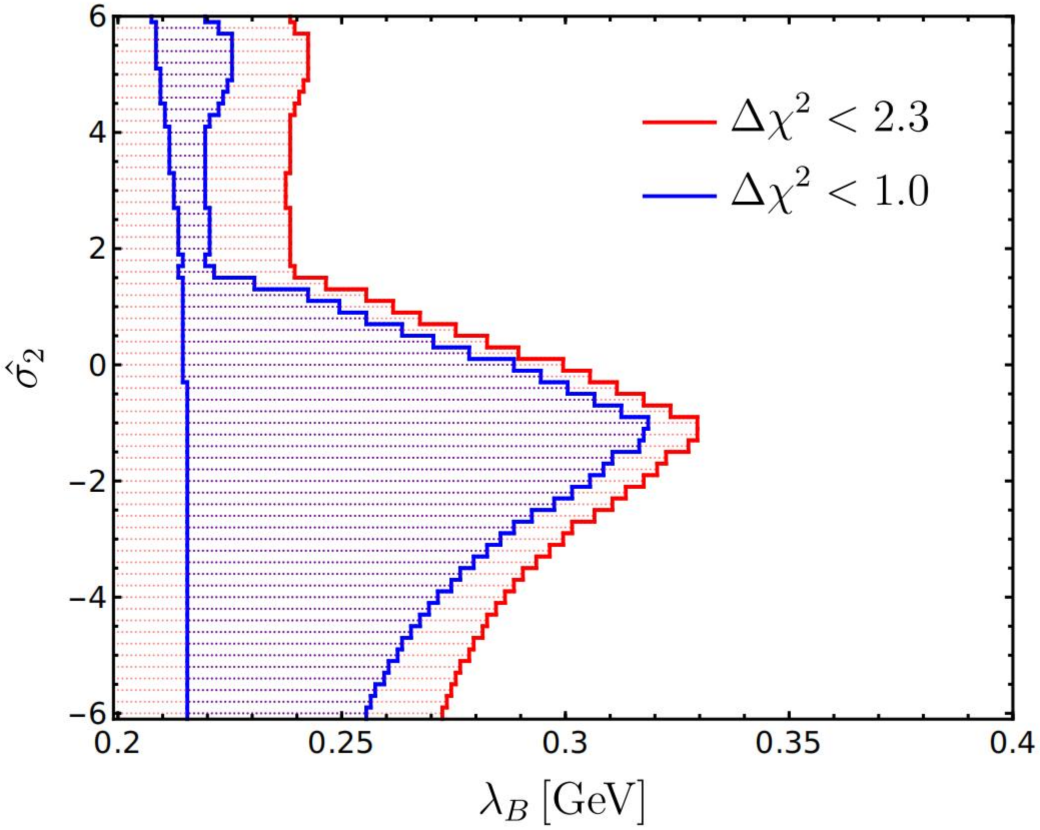
$\hat{\sigma}_1 \in [-0.7,\ 0.7]$

$\hat{\sigma}_2 \in [-6,\ 6]$

Analysis and Results

$$\Delta\chi^2 = \chi^2_{\min}(\lambda_B, \hat{\sigma}_1) - \chi^2_{\min}$$





$$\lambda_B = [208, 324]\,\text{MeV}, \qquad \hat{\sigma}_1 = [-0.7, 0.27].$$

we implement a global-fit strategy in which the B-meson LCDA are treated as fit parameters and are determined simultaneously with the BCL parameters and Vub from a combined set of lattice QCD inputs and experimental data across several decay channels.

It is complementary to direct theoretical determinations of λ_B, and it can be straightforwardly extended to incorporate λadditional decay modes and more q^2 points form factors, enabling an more comprehensive global analysis in the future.

谢谢！

B介子LCDA和光锥求和规则方法介绍

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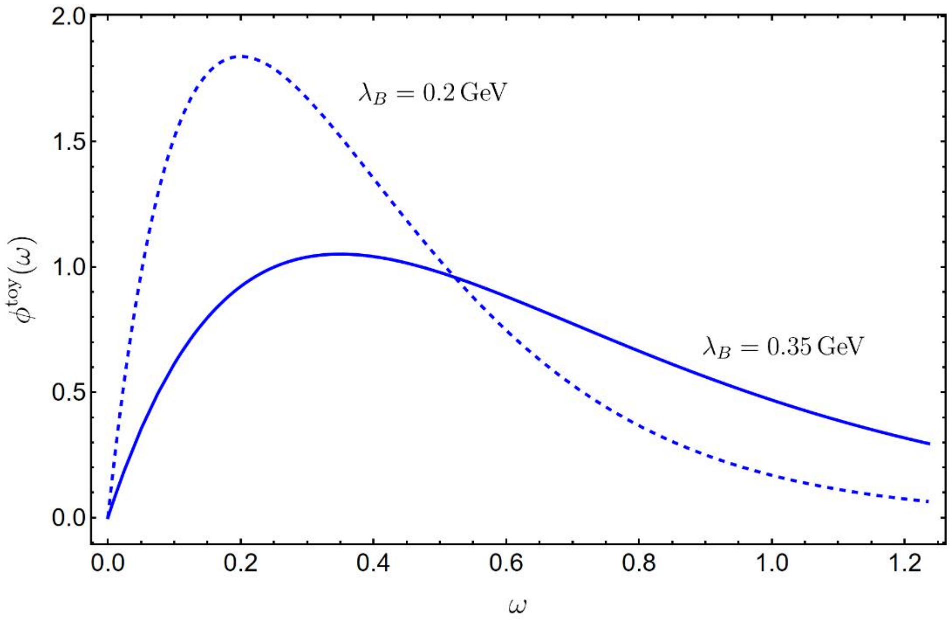
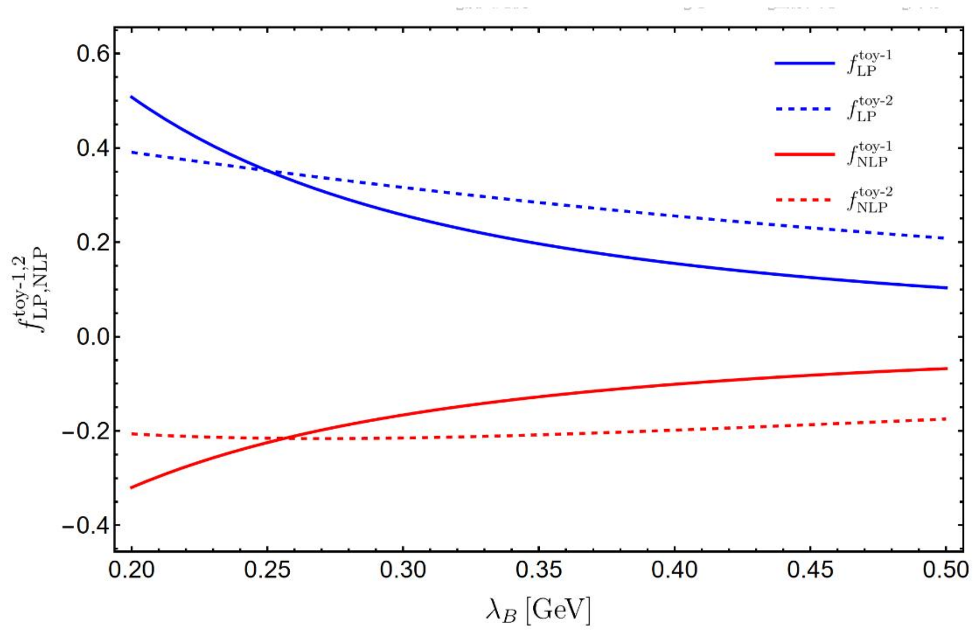
以一个toy model为例，解释抽取λ_B的可行性

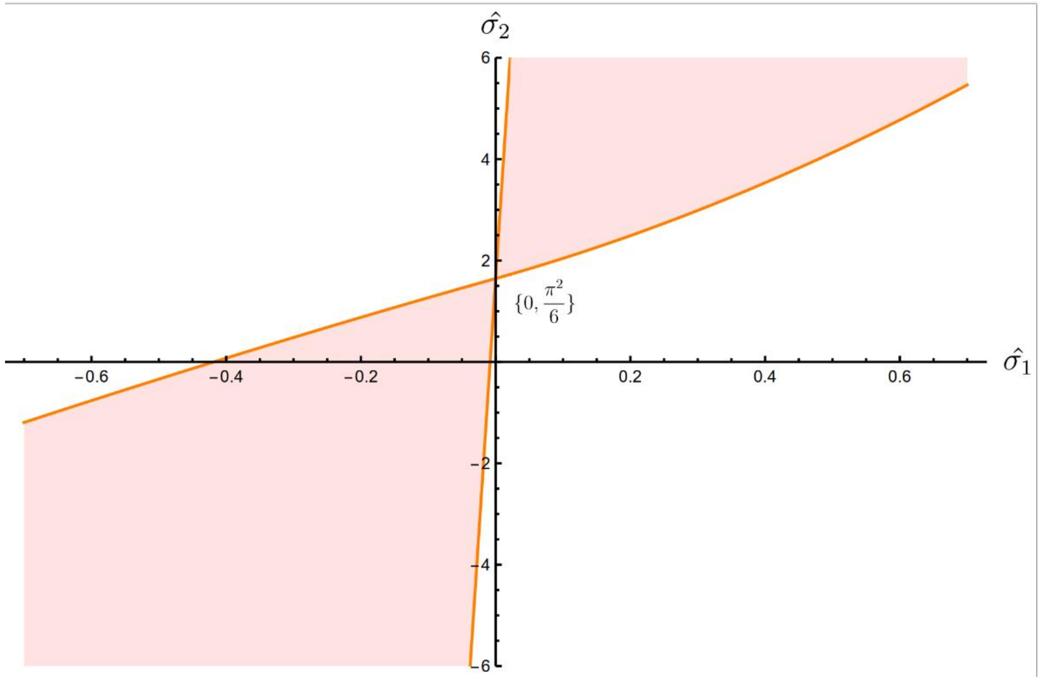
$$f_{\mathrm{LP}}^{\mathrm{toy}} \propto \int_0^{w_s} \mathrm{dw}\ \mathrm{Exp}\left[-\frac{w}{w_M}\right] \phi_B^+(w) \qquad f_{\mathrm{NLP}}^{\mathrm{toy}} \propto -\int_0^{w_s} \mathrm{dw}\ \frac{w}{n\cdot p} \mathrm{Exp}\left[-\frac{w}{w_M}\right] \phi_B^+(w)$$

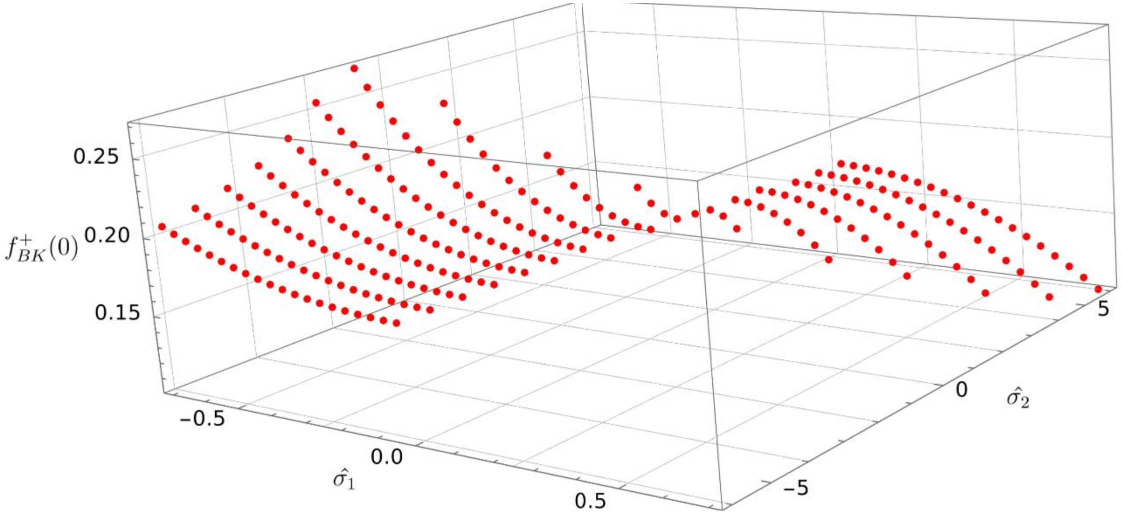
$$\phi_B^+(\omega) = \frac{\omega}{\lambda_B^2} e^{-\omega/\lambda_B}$$

Toy-1. $w_s = 0.13, w_M = 0.24;$
Toy-2. $w_s = 1.14, w_M = 0.85;$

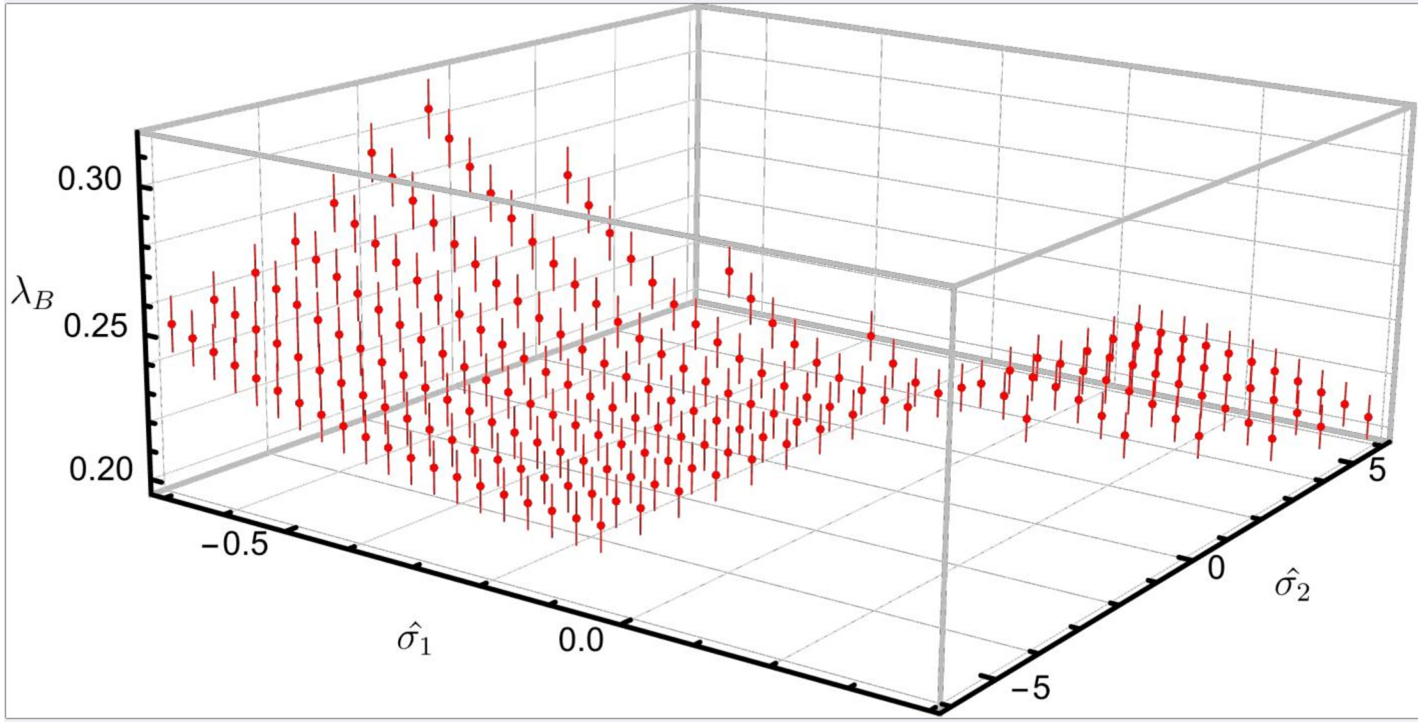





$\hat{\sigma}_2$
$\hat{\sigma}_1$
$\{0, \frac{\pi^2}{6}\}$
−0.6
−0.4
−0.2
0.2
0.4
0.6
6
4
2
−2
−4
−6


$f^+_{BK}(0)$
0.25
0.20
0.15
−0.5
0.0
0.5
$\hat{\sigma}_1$
5
0
−5
$\hat{\sigma}_2$

强幺正限
更多的道
更多的q2
更多的拟合参数



$$\lambda_B = 223(6)^{+33}_{-23}\,\text{MeV} \quad \text{for} \quad f^+_{B\pi}(0) = 0.248(9), \ [76]$$
$$\lambda_B = 211(6)^{+39}_{-11}\,\text{MeV} \quad \text{for} \quad f^+_{BK}(0) = 0.331(11), \ [89]$$
$$\lambda_B = 254(7)^{+44}_{-52}\,\text{MeV} \quad \text{for} \quad f^T_{BK}(0) = 0.330(10). \ [89]$$