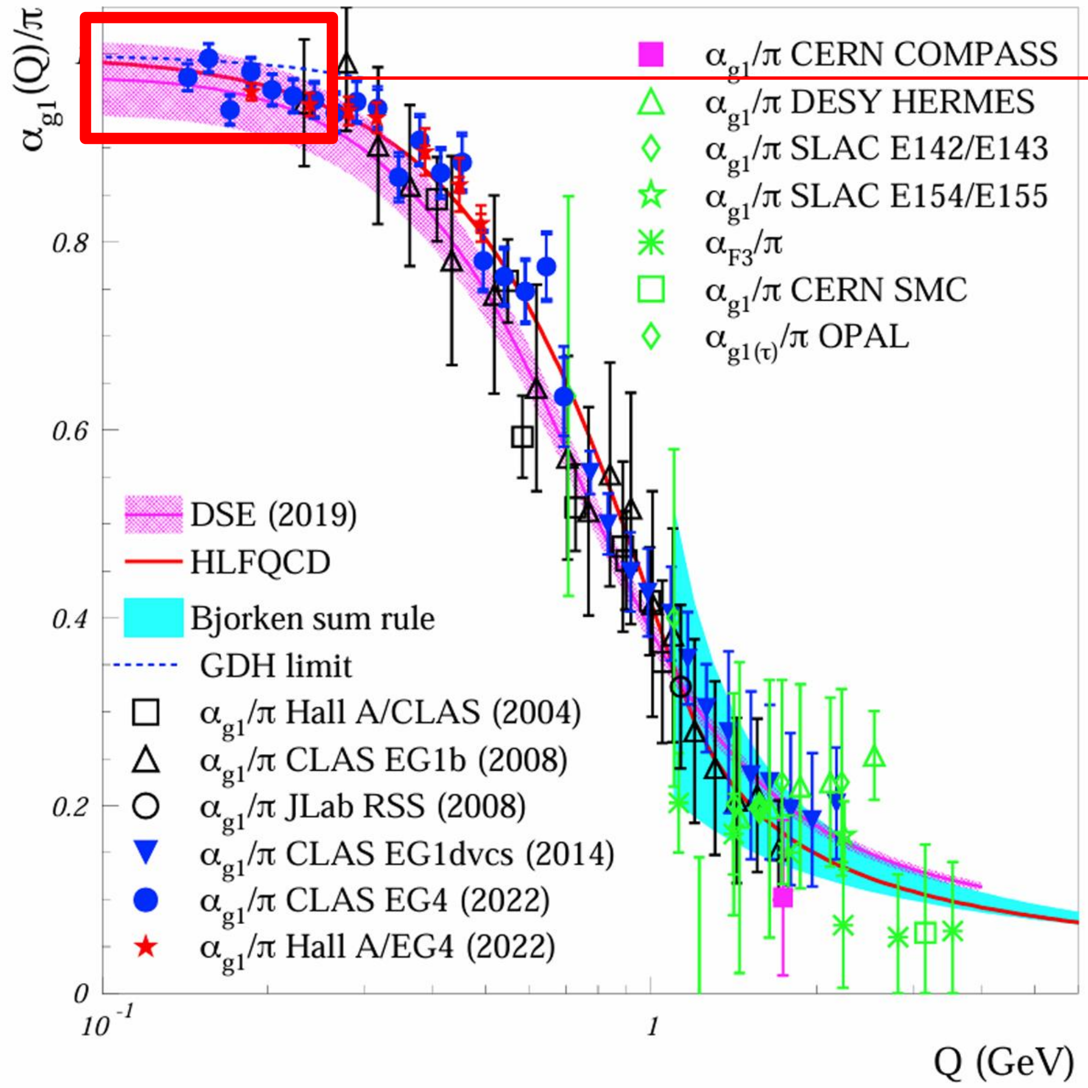
Gravitational form factors of pseudoscalar mesons in a contact interaction

Zanbin Xing (幸瓒彬）

Nankai University

第八届全国重味物理与量子色动力学研讨会，2026/04/27

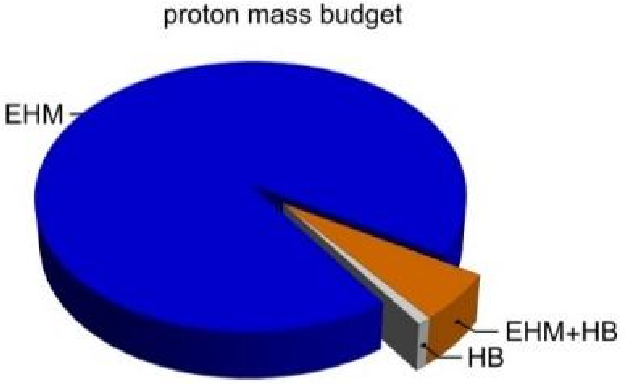
xingzb@mail.nankai.edu.cn



mass origin?

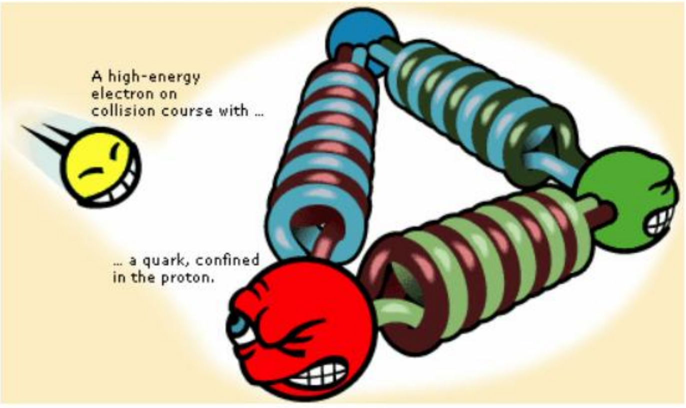
nonpertubative!

- confinement
- dynamical chiral symmetry breaking (DCSB)

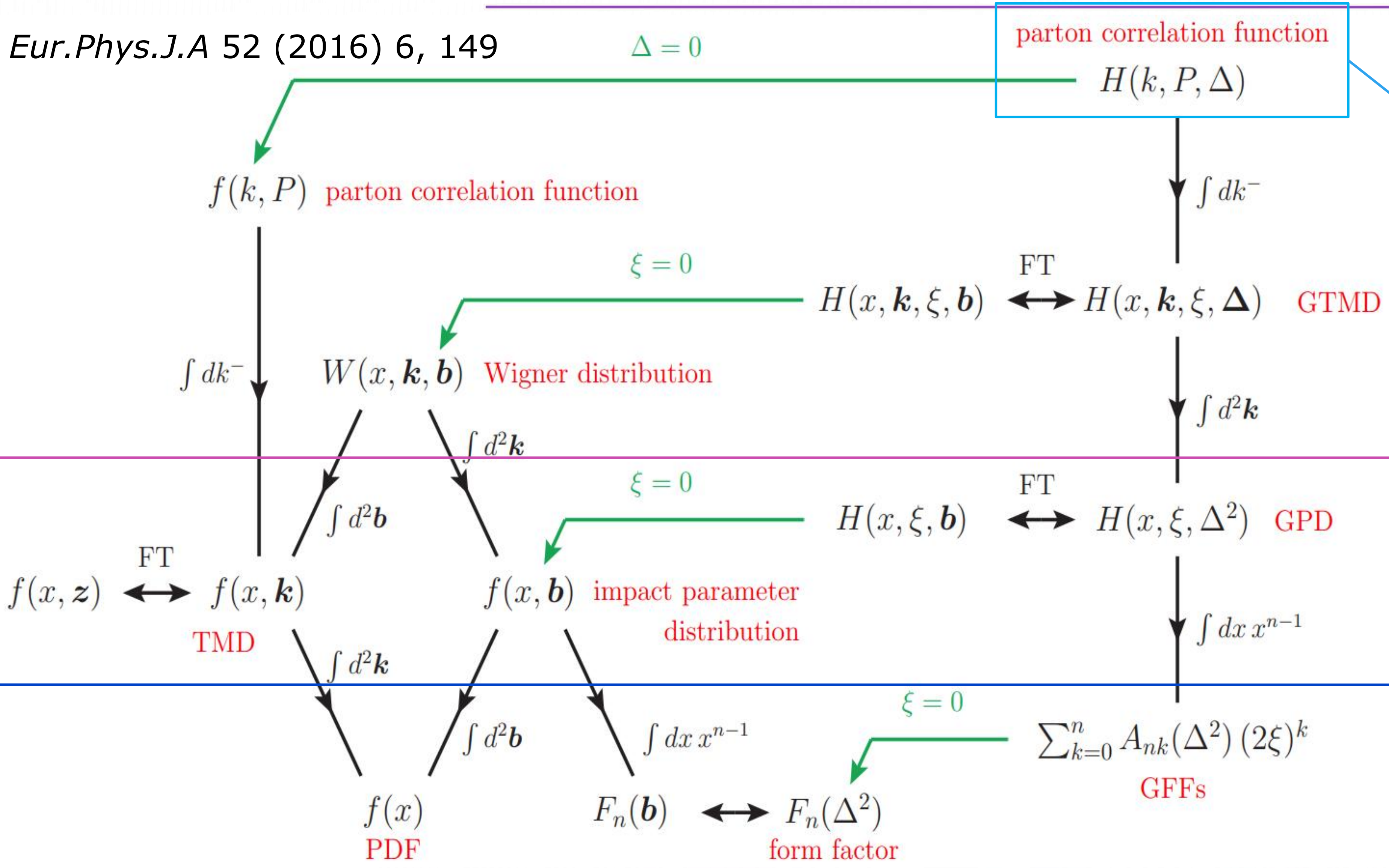


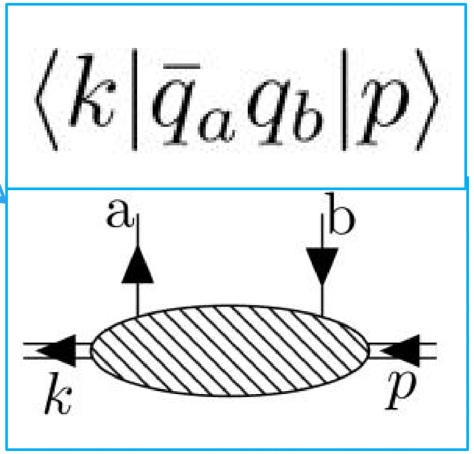
hadron physics

- spectrum
  multi-quark states, molecules, glueballs, ...
- hadron structures



Eur.Phys.J.A 52 (2016) 6, 149





generalized quark correlator

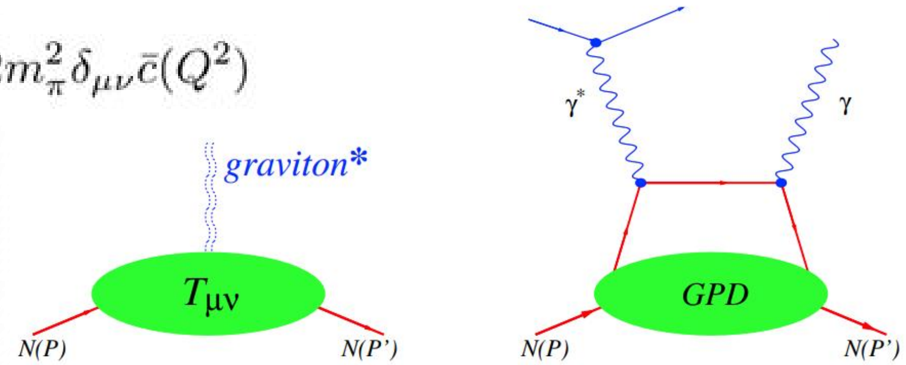
three dimensional structure of hadrons: GPD, TMD,...

one dimensional structure of hadrons: PDF, FF,...

- Characterize the matrix elements of energy momentum tensor (EMT)

$$\langle \pi(P+\frac{Q}{2})|\hat{T}^{\mu\nu}|\pi(P-\frac{Q}{2})\rangle \rightarrow 2P_\mu P_\nu A(Q^2) + \frac{1}{2}\left(Q_\mu Q_\nu - Q^2\delta_{\mu\nu}\right) D(Q^2) + 2m_\pi^2 \delta_{\mu\nu}\bar{c}(Q^2)$$

- Mass, spin and mechanical properties



Forces inside hadrons: pressure, surface tension, mechanical radius, and all that

Maxim V. Polyakov (St. Petersburg, INP and Ruhr U., Bochum), Peter Schweitzer (Connecticut U.) (May 17, 2018)

Published in: *Int.J.Mod.Phys.A* 33 (2018) 26, 1830025 • e-Print: 1805.06596 [hep-ph]

pdf DOI cite claim reference search 452 citations

$$\int_{-1}^{1} dx\, x\, H^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t)\,,$$

- EMT conservation → $\mathbf{0 = Q_\mu M^\pi_{\mu\nu} \sim \bar{c}(Q^2)}$
- Translation invariance → A(0)=1
- D-term defined as $\boldsymbol{D \equiv D(0)}$

em: $\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\|J^\mu_{\rm em}\|N\rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}$C	
			$\mu = 2.792847356(23)\mu_N$	
weak: PCAC	$\langle N'\|J^\mu_{\bf weak}\|N\rangle$	$\longrightarrow$	$g_A = 1.2694(28)$	
			$g_p = 8.06(55)$	
gravity: $\partial_\mu T^{\mu\nu}_{\bf grav} = 0$	$\langle N'\|T^{\mu\nu}_{\bf grav}\|N\rangle$	$\longrightarrow$	$m = 938.272013(23)$ MeV$/c^2$	
			$J = \frac{1}{2}$	
			$\boldsymbol{D = ?}$	

last unknown global property!!!

南開大學 Nankai University

known constraints from soft pion theorem

D=-1

for Nambu-Goldstone boson in the chiral limit

another constraints?

$$D^{\Phi} = -1/3$$

for a non-free **point-like scalar** particle

Energy-momentum tensor in Φ^4 theory at one loop

Brean Maynard (Connecticut U.) (Jun 13, 2024)

Published in: *Phys.Rev.D* 111 (2025) 7, 076001 • e-Print: 2406.08857 [hep-ph]

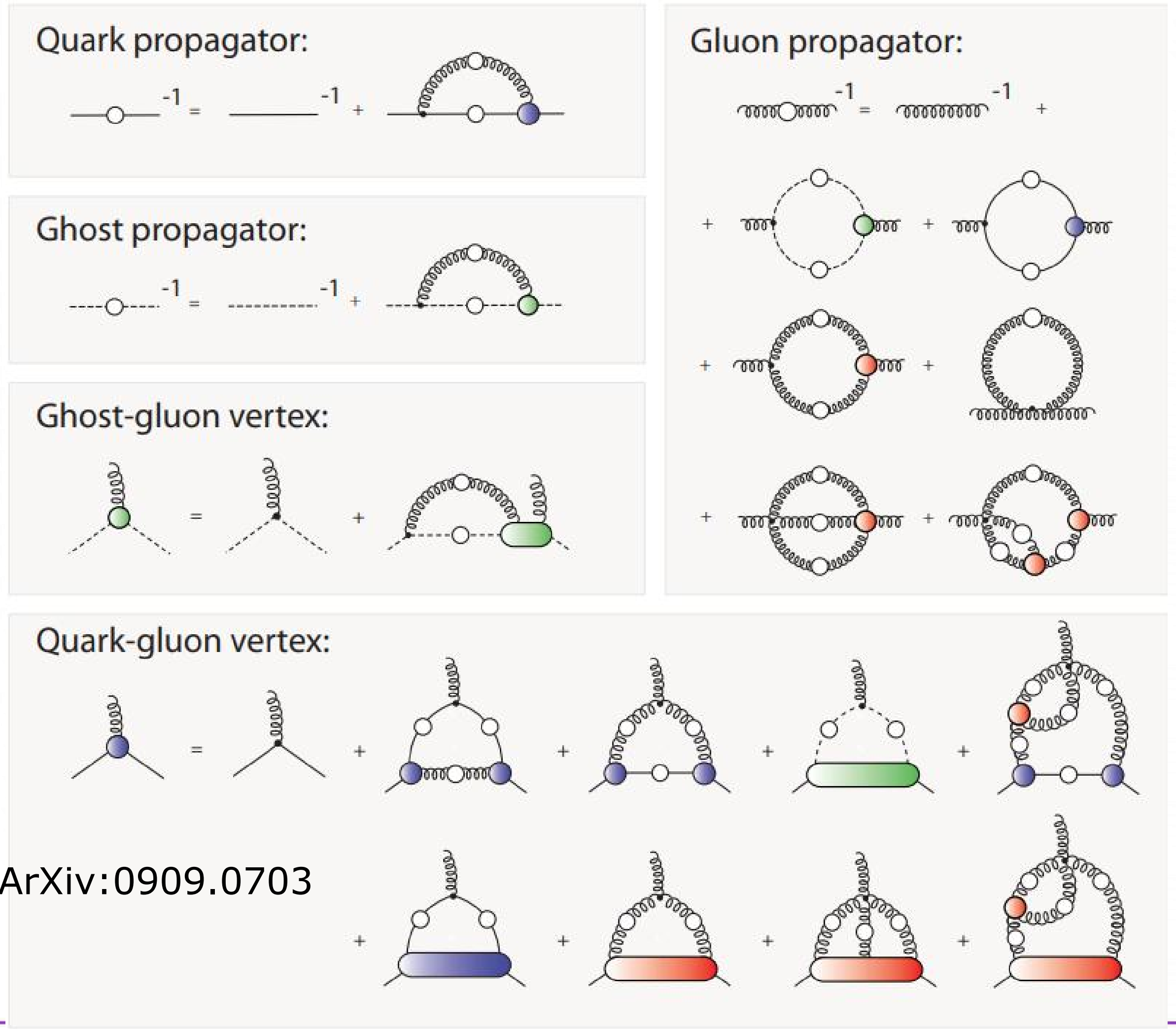
A companion work in Φ^3 theory confirms this result

- independent to the coupling strength even if it’s infinitesimally small
- It may potentially be a general result in scalar theories.

DYSON-SCHWINGER EQUATIONS

Nankai University

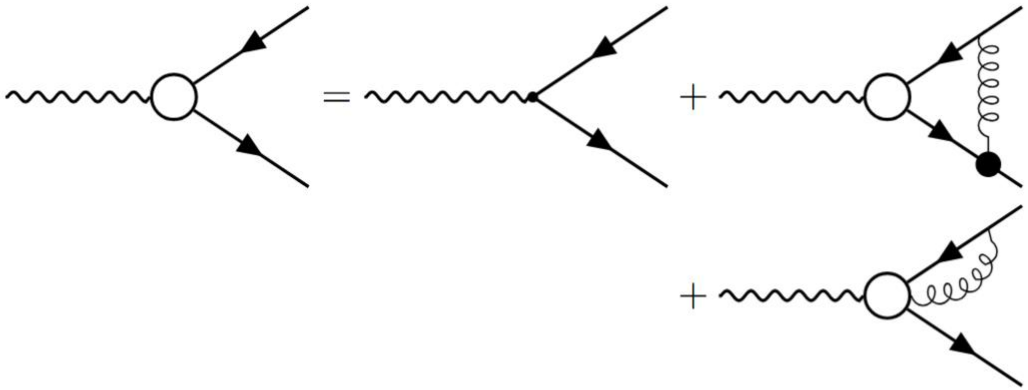
QCD green functions



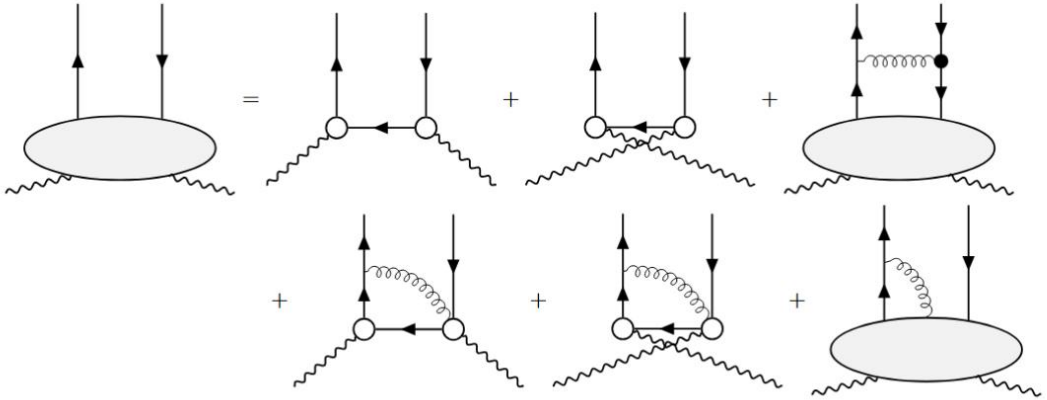
ArXiv:0909.0703

current correlation functions

insertion of one current

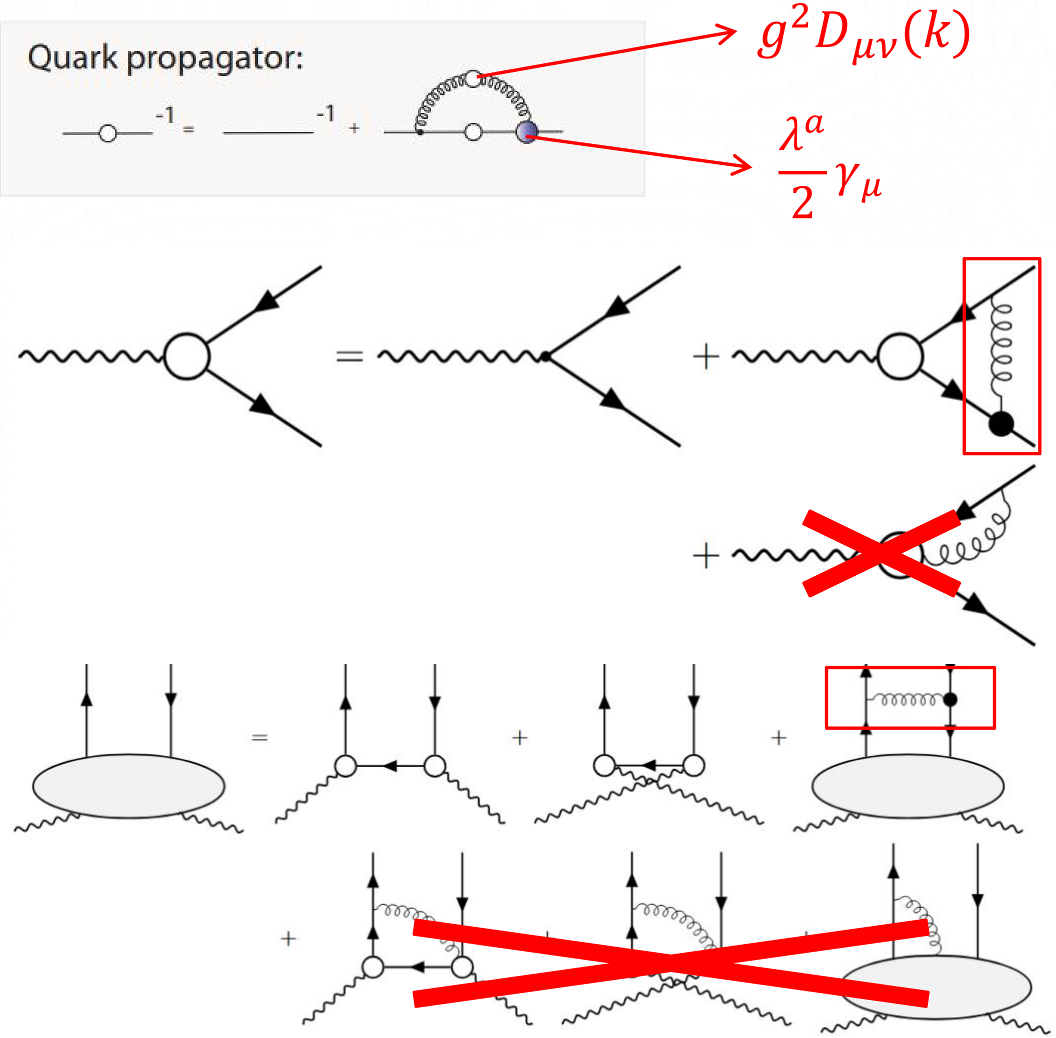


insertion of two external currents



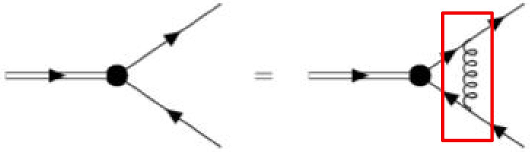
……

rainbow ladder truncation

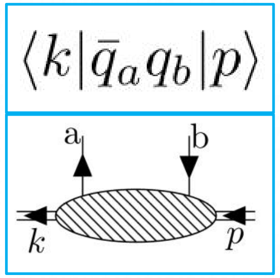


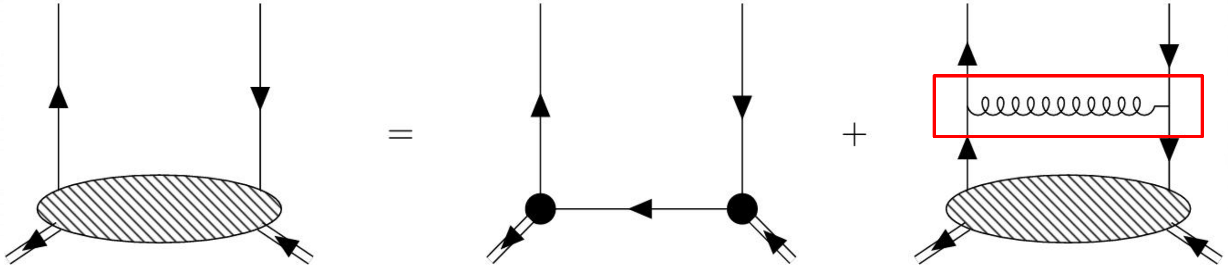
Polology ---- Weinberg QFT

Bethe Salpeter equation



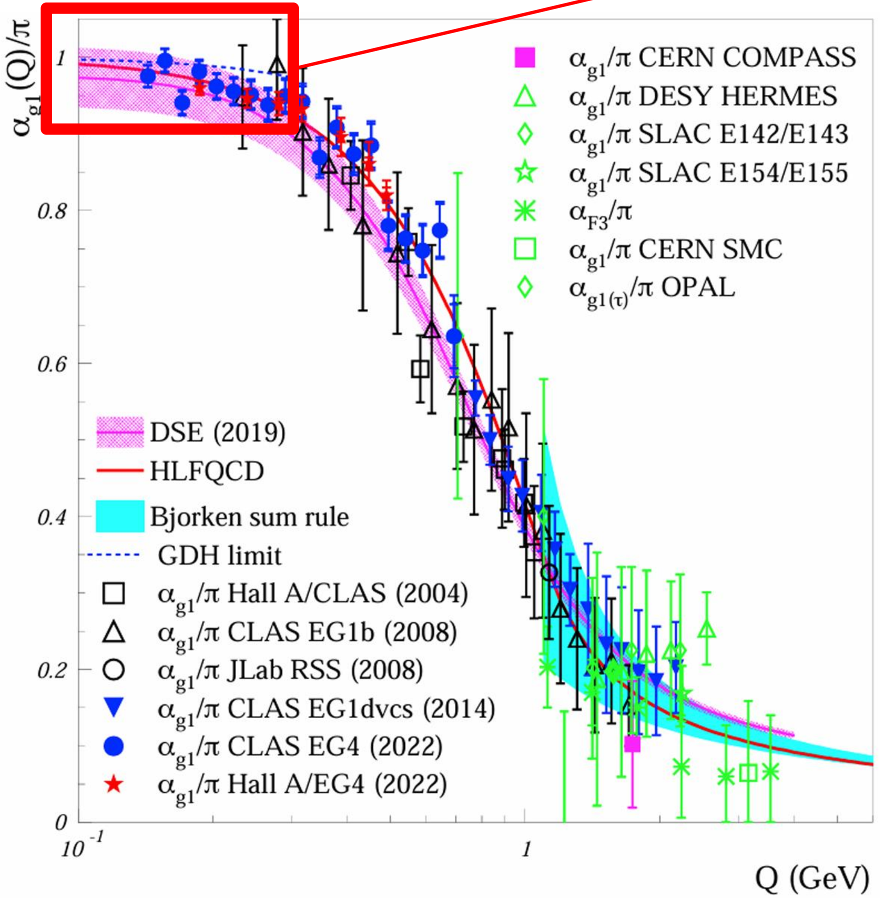
Equation for generalized quark correlator!!





Nankai University 南開大學

gluon propagator $g^2 D_{\mu\nu}(k) \to \delta_{\mu\nu} \frac{4\pi\alpha}{m_G^2}$



regularization of irreducible loop integrals

Yue-Liang Wu, Int.J.Mod.Phys.A 18 (2003) 5363-5420

$$I_{-2\alpha}(\mathcal{M}^2) = \int_q \frac{1}{(q^2+\mathcal{M}^2)^{\alpha+2}},$$

$$I^{\mu\nu}_{-2\alpha}(\mathcal{M}^2) = \int_q \frac{q_\mu q_\nu}{(q^2+\mathcal{M}^2)^{\alpha+3}},$$

$$I^{\mu\nu\rho\sigma}_{-2\alpha}(\mathcal{M}^2) = \int_q \frac{q_\mu q_\nu q_\rho q_\sigma}{(q^2+\mathcal{M}^2)^{\alpha+4}}$$

$$I_{-2\alpha R}(\mathcal{M}^2) = \int_{\tau_{uv}^2}^{\tau_{ir}^2} d\tau \frac{\tau^{\alpha-1}}{\Gamma(\alpha+2)} \frac{e^{-\tau\mathcal{M}^2}}{16\pi^2}.$$

$$I^{\mu\nu}_{-2\alpha R}(\mathcal{M}^2) = \frac{\Gamma(\alpha+2)}{2\Gamma(\alpha+3)} \delta_{\mu\nu} I_{-2\alpha R}(\mathcal{M}^2),$$

$$I^{\mu\nu\rho\sigma}_{-2\alpha R}(\mathcal{M}^2) = \frac{\Gamma(\alpha+2)}{4\Gamma(\alpha+4)} S_{\mu\nu\rho\sigma} I_{-2\alpha R}(\mathcal{M}^2).$$

symmetry preserving——First time!!

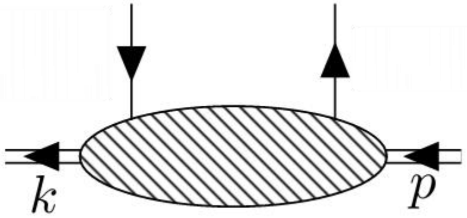
Phys.Rev.D 107 (2023), 014019

consistent conditions that automatically preserves Ward-Green-Takahashi Identities(WGTIs)

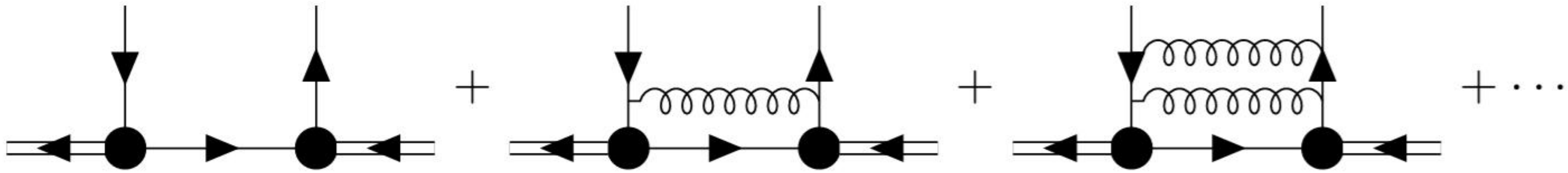
$$iP_\mu \Gamma^{ab}_{5\mu}(P,k) = S_a^{-1}(k_+) i\gamma_5 + i\gamma_5 S_b^{-1}(k_-) - i(m_a+m_b)\Gamma^{ab}_5(P,k)$$

$$iP_\mu \Gamma^{ab}_{\mu}(P,k) = S_a^{-1}(k_+) - S_b^{-1}(k_-) - (m_a-m_b)\Gamma^{ab}_I(P,k)$$

CONTACT INTERACTION INTERPRETATION

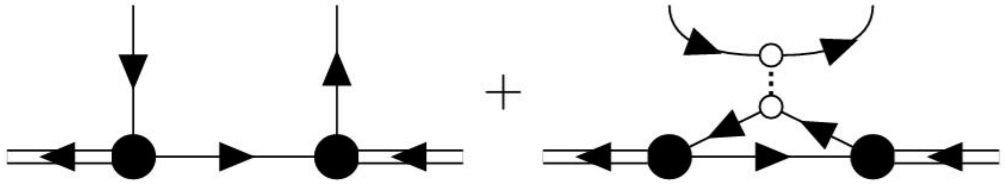


ladder approximation →

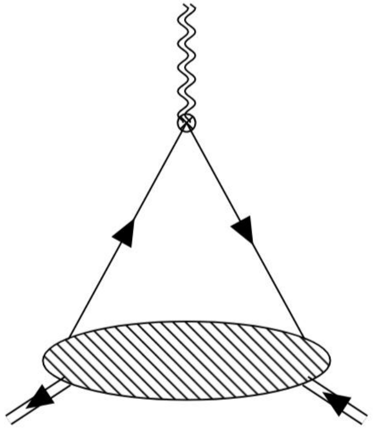


$\langle k|\bar{q}_a q_b|p\rangle$

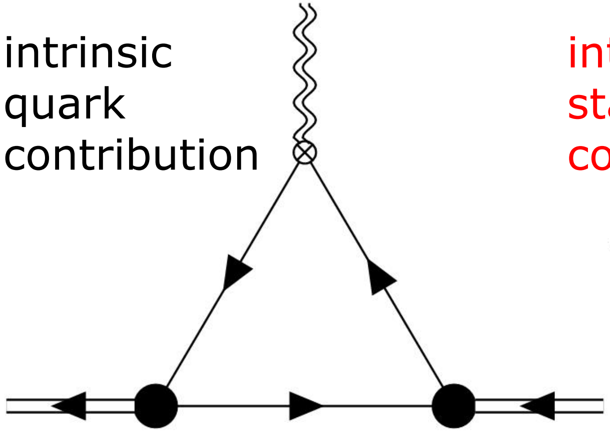
contact interaction →



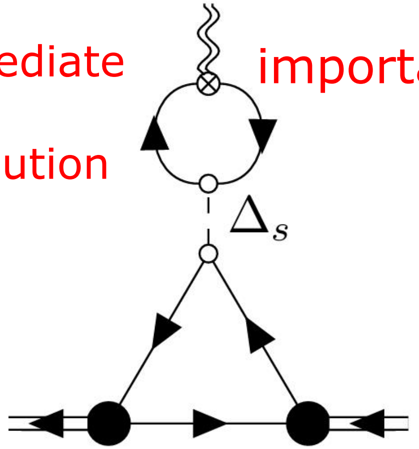
$\langle\pi(P+\frac{Q}{2})|\hat{T}^{\mu\nu}|\pi(P-\frac{Q}{2})\rangle$ $\quad \hat{T}^{\mu\nu}=\frac{i}{2}\bar{q}(\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}+\gamma^{\nu}\overleftrightarrow{\partial}^{\mu})q+\ldots$



→



+

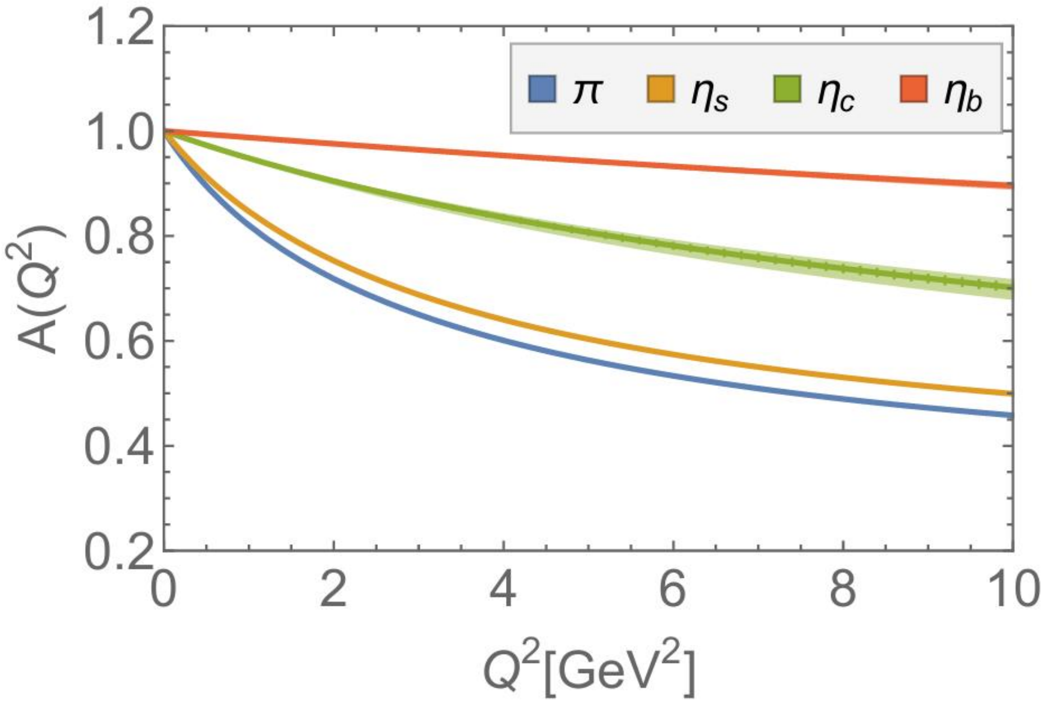


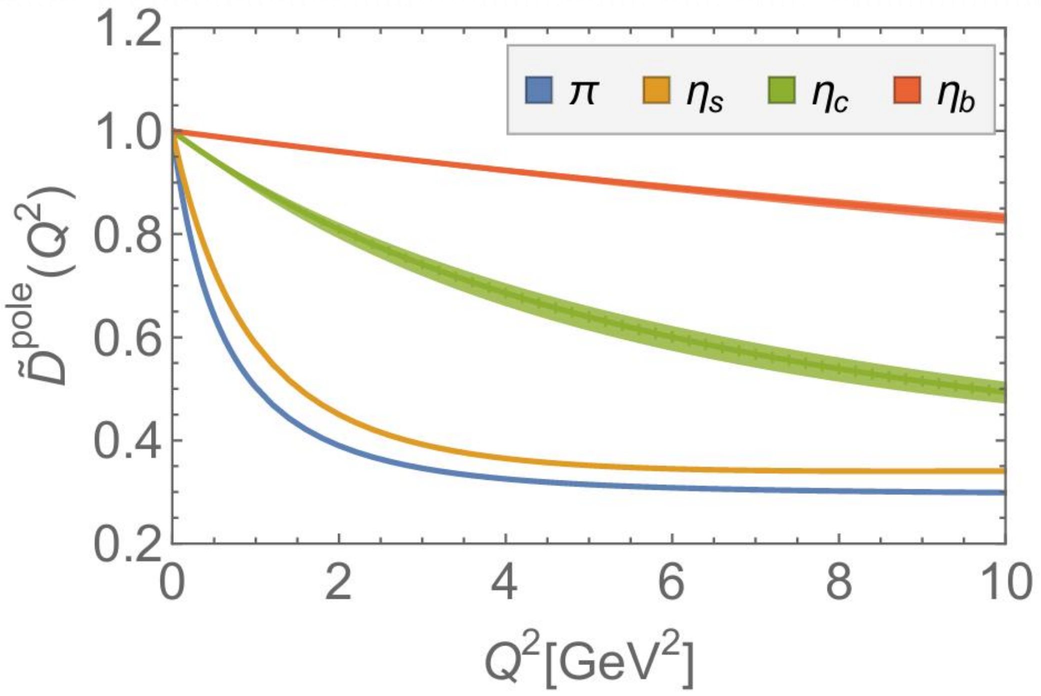
important for

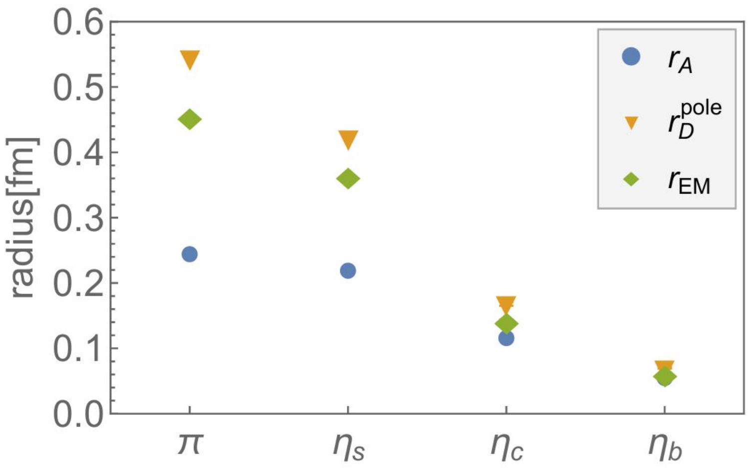
soft pion theorem !

$$D^{PS}_{c.l.}=-1$$

RESULTS: FORM FACTORS

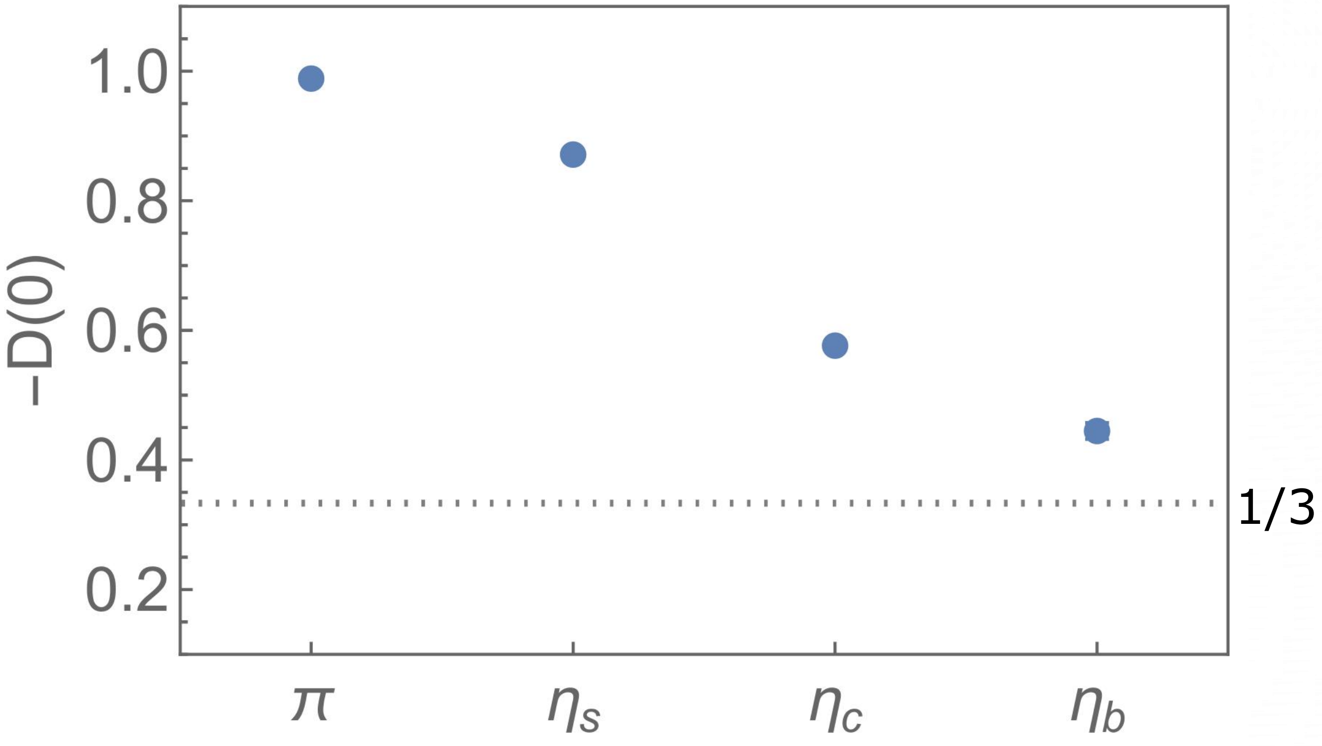






As mass increases, the form factors become harder, and the radius become smaller and converge -- the quarkonia tends to exhibit characteristics of a

point-like particle



$$D^{\pi} = -0.989 \ , \ D^{\eta_s} = -0.871 \, ,$$
$$D^{\eta_c} = -0.576(3) \ , \ D^{\eta_b} = -0.445(13) \, .$$

1. -D decrease with increasing mass

2. Pseudo scalar D term within the range:

!

$$D^{PS} \in (-1, -1/3)$$

massless infinite mass

Basis light front quantization for charmonia

Phys.Rev.D 109 (2024) 11, 114024

$$D^{\rm PS} \sim -5$$

?

limited results for heavy quarkonia D-term
not easy to explain

- The generalized quark correlator is described in a consistent manner, the soft pion theorem is well reproduced after considering the scalar pole contribution.

- The symmetry preserving framework also provides reliable supports to the gravitational form factors of heavy quarkonia. And the quarkonia tends to behave like a point-like particle with increasing mass.

- Two possible limit and one confirmed tension for the D-term:

$$D^{PS} \in (-1, -1/3)$$

massless (−1) infinite mass (−1/3)

VS

$$D^{\mathrm{PS}} \sim -5$$

We are trying and welcome collegues to clarify the situation!

Thank you!

Nankai University

Irreducible loop integrals (Wu)

$$I_{-2\alpha}(\mathcal{M}^2) = \int_q \frac{1}{(q^2+\mathcal{M}^2)^{\alpha+2}},$$

$$I^{\mu\nu}_{-2\alpha}(\mathcal{M}^2) = \int_q \frac{q_\mu q_\nu}{(q^2+\mathcal{M}^2)^{\alpha+3}},$$

$$I^{\mu\nu\rho\sigma}_{-2\alpha}(\mathcal{M}^2) = \int_q \frac{q_\mu q_\nu q_\rho q_\sigma}{(q^2+\mathcal{M}^2)^{\alpha+4}}$$

$$\begin{aligned} I_{-2\alpha}(\mathcal{M}^2) &= \int_q \frac{1}{(q^2+\mathcal{M}^2)^{\alpha+2}} \\ &= \int_q \int_0^\infty d\tau \frac{\tau^{\alpha+1}}{\Gamma(\alpha+2)} e^{-\tau(q^2+\mathcal{M}^2)} \\ &= \int_0^\infty d\tau \frac{\tau^{\alpha-1}}{\Gamma(\alpha+2)} \frac{e^{-\tau\mathcal{M}^2}}{16\pi^2} \\ \to I_{-2\alpha R}(\mathcal{M}^2) &= \int_{\tau_{uv}^2}^{\tau_{ir}^2} d\tau \frac{\tau^{\alpha-1}}{\Gamma(\alpha+2)} \frac{e^{-\tau\mathcal{M}^2}}{16\pi^2}. \end{aligned}$$

$$\begin{aligned} I^{\mu\nu}_{-2\alpha}(\mathcal{M}^2) &= \int_q \frac{q_\mu q_\nu}{(q^2+\mathcal{M}^2)^{\alpha+3}} \\ &= \int_q \int_0^\infty d\tau q_\mu q_\nu \frac{\tau^{\alpha+2}}{\Gamma(\alpha+3)} e^{-\tau(q^2+\mathcal{M}^2)} \\ &= \int_q \int_0^\infty d\tau \delta_{\mu\nu} \frac{q^2}{4} \frac{\tau^{\alpha+2}}{\Gamma(\alpha+3)} e^{-\tau(q^2+\mathcal{M}^2)} \\ &= \delta_{\mu\nu} \int_0^\infty d\tau \frac{\tau^{\alpha-1}}{\Gamma(\alpha+3)} \frac{e^{-\tau\mathcal{M}^2}}{32\pi^2} \\ \to I^{\mu\nu}_{-2\alpha R}(\mathcal{M}^2) &= \delta_{\mu\nu} \int_{\tau_{uv}^2}^{\tau_{ir}^2} d\tau \frac{\tau^{\alpha-1}}{\Gamma(\alpha+3)} \frac{e^{-\tau\mathcal{M}^2}}{32\pi^2}, \end{aligned}$$

Consistency conditions

$$I^{\mu\nu}_{-2\alpha R}(\mathcal{M}^2) = \frac{\Gamma(\alpha+2)}{2\Gamma(\alpha+3)} \delta_{\mu\nu} I_{-2\alpha R}(\mathcal{M}^2),$$

$$I^{\mu\nu\rho\sigma}_{-2\alpha R}(\mathcal{M}^2) = \frac{\Gamma(\alpha+2)}{4\Gamma(\alpha+4)} S_{\mu\nu\rho\sigma} I_{-2\alpha R}(\mathcal{M}^2).$$

Gauge and chiral symmetry

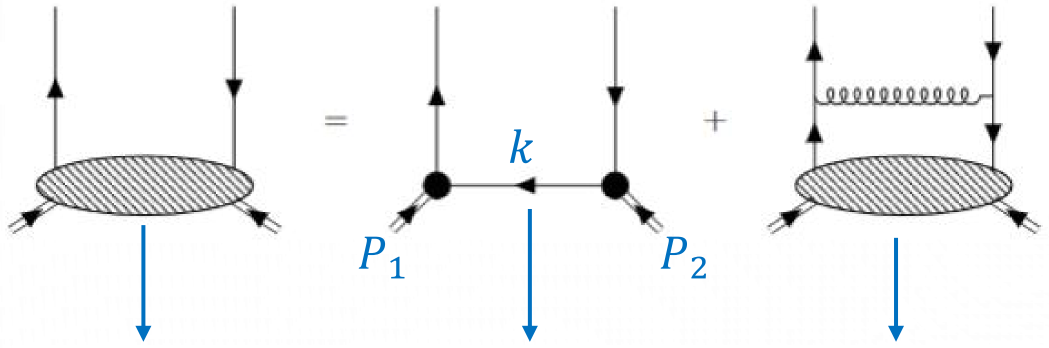
Symmetry preserving contact interaction treatment of the kaon

Zanbin Xing and Lei Chang

Phys. Rev. D **107**, 014019 (2023) - Published 19 January 2023

$$0 = \frac{8}{3m_G^2} \int_0^1 du \{Q^2 I_2(\omega_2) - 2Q_\mu Q_\nu I_2^{\mu\nu}(\omega_2)\}$$

[22] Yue-Liang Wu. Symmetry principle preserving and infinity free regularization and renormalization of quantum field theories and the mass gap. *Int. J. Mod. Phys. A*, 18:5363–5420, 2003. doi:10.1142/S0217751X03015222.
[23] Yue-Liang Wu. Symmetry preserving loop regularization and renormalization of QFTs. *Mod. Phys. Lett. A*, 19: 2191–2204, 2004. doi:10.1142/S0217732304015361.



$$F(k,P_1,P_2) = F_0(k,P_1,P_2) + \Sigma^F(P_1,P_2)$$

$\Delta_g = \frac{4}{3m_g^2}$, $q_1 = q + P_1$, $q_2 = q - P_2$

$$\Sigma^F(P_1,P_2) = -\Delta_g \int_q \gamma_\alpha S(q_1) F(q,P_1,P_2) S(q_2)\gamma_\alpha$$

Solve $F(k,P_1,P_2)$?

Solve $\Sigma^F(P_1,P_2)$!

$$\Sigma^F(P_1,P_2) = \Sigma^{F_0}(P_1,P_2) - \Delta_g \int_q \gamma_\alpha S(q_1)\Sigma^F(P_1,P_2)S(q_2)\gamma_\alpha\,,$$

where the inhomogeneous term is

$$\Sigma^{F_0}(P_1,P_2) = -\Delta_g \int_q \gamma_\alpha S(q_1) F_0(q,P_1,P_2) S(q_2)\gamma_\alpha\,,$$

$$\Sigma^{F_0}(P_1,P_2) \;=\; \textstyle\sum_{i=1}^4 b_i T_i$$

$$\Sigma^F(P_1,P_2) = \sum_{i=1}^4 t_i T_i\,,$$

Why these basis?

$$T_i = \left\{\mathbb{1},\ \frac{-i}{M}\not{K},\ \frac{-i}{M}\not{Z},\ \frac{i}{M^2}\sigma_{\mu\nu}Z_\mu K_\nu\right\}$$

Nankai University

Pole structure:

Vaccum polarization of corresponding mesons

$$t_1(Z^2) = \frac{b_1(Z^2)}{1+\Delta_g f_s(Z^2)}, \qquad f_s(Z^2) = \mathrm{tr}\int_q S(q_1)S(q_2),$$

$$t_2(Z^2) = \frac{b_2(Z^2)}{1+\Delta_g f_v(Z^2)}, \qquad f_v(Z^2) = \frac{K_\mu K_\nu}{2K^2}\mathrm{tr}\int_q i\gamma_\mu^T S(q_1) i\gamma_\nu^T S(q_2),$$

Inhomogeneous term:

$$b_1(Z^2) = -\frac{1}{4}\Delta_g \mathrm{tr}\int_q T_1\gamma_\alpha S(q_1)F_0(q,P_1,P_2)S(q_2)\gamma_\alpha,$$

$$b_2(Z^2) = \frac{M^2}{4K^2}\Delta_g \mathrm{tr}\int_q T_2\gamma_\alpha S(q_1)F_0(q,P_1,P_2)S(q_2)\gamma_\alpha.$$

b_i vs corresponding form factors

$$iF_s(Z^2) = N_c \mathrm{tr}\int_q i\Gamma_I(Z)S(q_1)i^2F_0(q,P_1,P_2)S(q_2)$$

$$2K_\mu F_{\mathrm{em}}(Z^2) = N_c \mathrm{tr}\int_q i\Gamma_\mu(Z)S(q_1)i^2F_0(q,P_1,P_2)S(q_2)$$

$$N_c b_1(Z^2) = \Delta_g F_s^b(Z^2),$$

$$N_c b_2(Z^2) = -M\Delta_g F_{\mathrm{em}}^b(Z^2),$$

Solve $\Sigma^F(P_1, P_2)$!

$$\Sigma^F(P_1,P_2) = \Sigma^{F_0}(P_1,P_2) - \Delta_g \int_q \gamma_\alpha S(q_1)\Sigma^F(P_1,P_2)S(q_2)\gamma_\alpha,$$

where the inhomogeneous term is

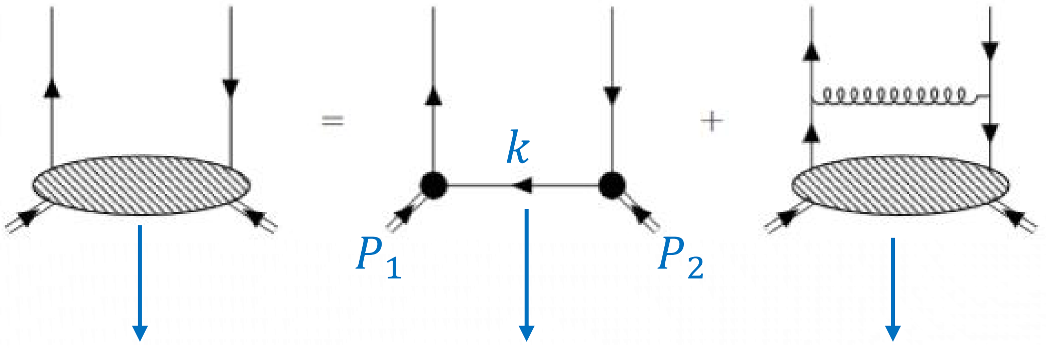
$$\Sigma^{F_0}(P_1,P_2) = -\Delta_g \int_q \gamma_\alpha S(q_1)F_0(q,P_1,P_2)S(q_2)\gamma_\alpha,$$

$$\Sigma^{F_0}(P_1,P_2) = \sum_{i=1}^4 b_i T_i$$

$$\Sigma^F(P_1,P_2) = \sum_{i=1}^4 t_i T_i,$$

$$T_i = \left\{\mathbb{1}, \frac{-i}{M}\not{K}, \frac{-i}{M}\not{Z}, \frac{i}{M^2}\sigma_{\mu\nu}Z_\mu K_\nu\right\}$$

π-π AMPLITUDE IN CI



$$F(k,P_1,P_2) = F_0(k,P_1,P_2) + \Sigma^F(P_1,P_2)$$

$$N_c\Sigma^F(P_1,P_2) = \mathbb{1} F_s^b(Z^2)\Delta_s(Z^2) + i\not{K} F_{\rm em}^b(Z^2)\Delta_v(Z^2)\,,$$

where Δ_s and Δ_v are given by

$$\Delta_s(Z^2) = \frac{1}{\Delta_g^{-1} + f_s(Z^2)}\,,$$
$$\Delta_v(Z^2) = \frac{1}{\Delta_g^{-1} + f_v(Z^2)}\,.$$

$$\Gamma_I \to \mathbb{1}$$
$$\Gamma_\mu \to \gamma_\mu$$

Some comments about $\Sigma^F(P_1,P_2)$:

Dirac structure:

- Probe selection, e.g. the bare photon probe will interact only with $i\gamma \cdot K$ so that the EM form factor only contains vector pole.

Dressing function:

- $\Delta_{s/v}$ exhibits scalar and vector meson pole and can be viewed as the corresponding effective meson propagator (up to Lorentz structure), respectively.
- $F^b_{s/em}$ is the pion scalar and vector form factor, except that the dressed detector is replaced with the bare one.