Spin-orbit correlation within heavy quarkonium

Xianghui Cao

University of Science and Technology of China, Hefei, China

In collaboration with
Tian-Yang Hu (IMP), Si-Qi Xu (IMP), Qin-Tao Song (ZZU), Wei-Jie Du (IMP), Yang Li (USTC)

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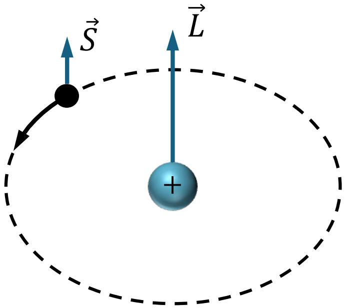
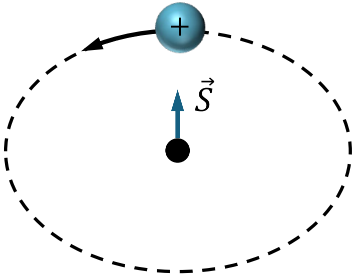
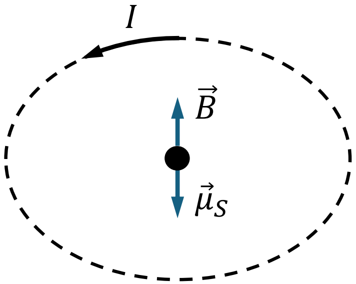
Spin-orbit correlation in atoms



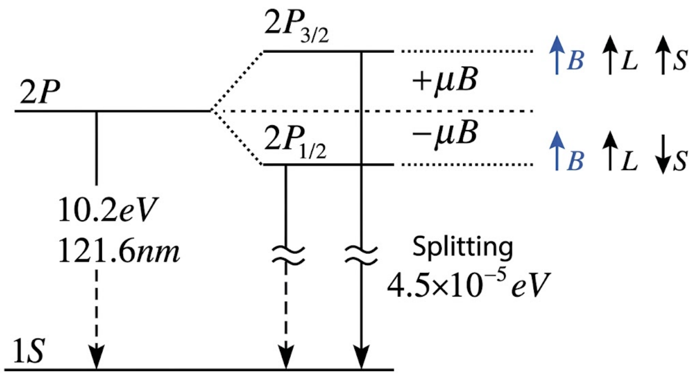
nuclei frame



electron frame



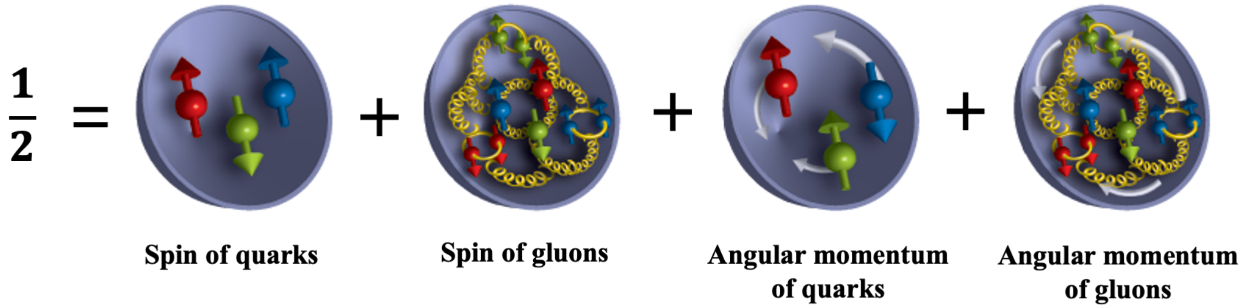
magnetic field from nuclei motion



$$H_{ls} = \frac{1}{2m_e^2c^2}\frac{Ze^2}{4\pi\epsilon_0 r^3}\vec{L}\cdot\vec{S}$$

Fine structure of hydrogen atom

Spin decomposition



- Ji's spin sum rule: [Ji:1996ek]
  (gauge invariant)

$$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2}$$

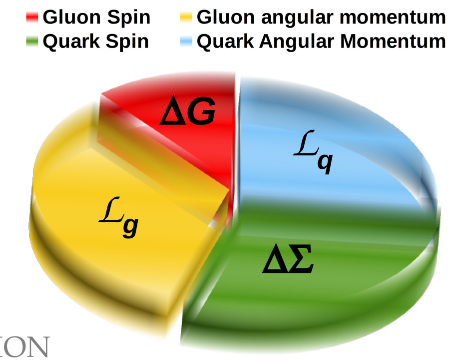
- Jaffe-Manohar spin sum rule: [Jaffe:1989jz]
  (infinite momentum frame, light-cone gauge)

$$\frac{1}{2}\Delta\Sigma + S_g + L_q + L_g = \frac{\hbar}{2}$$

Can quark spin and OAM couple in hadrons?

- quark spin $\Delta\Sigma \sim 30\%$
- quark orbital angular momentum (OAM) L_q
  (measurable in EIC and EicC)

[Accardi:2012qut]



Spin-orbit correlation in QCD

[Lorce:2014mxa, Lorce:2025ayr]

$$\hat{S}_z^q = \int \mathrm{d}^3x \frac{1}{2}\bar{\psi}\gamma^+\gamma_5\psi = \frac{1}{2}(\hat{N}^{q_R} - \hat{N}^{q_L}), \qquad \text{quark spin}$$

$$\hat{L}_z^q = \int \mathrm{d}^3x \frac{1}{2}\overline{\psi}\gamma^+(\vec{x} \times i\overleftrightarrow{D})_z\psi = \hat{L}_z^{q_R} + \hat{L}_z^{q_L}, \qquad \text{quark OAM}$$

$$\hat{C}_z^q = \int \mathrm{d}^3x \frac{1}{2}\bar{\psi}\gamma^+\gamma_5(\vec{x} \times \overleftrightarrow{D})_z\psi = \hat{L}_z^{q_R} - \hat{L}_z^{q_L} \qquad \text{quark spin-orbit correlation}$$

where q_R/q_L denotes the contribution of right/left-hand fermion and $\hat{N}$ is the quark number operator

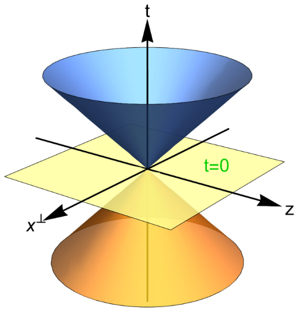
- Spin-orbit correlation (SOC) defined from parity-odd energy-momentum tensor (EMT):

$$\hat{C}_z^q = \int \mathrm{d}^3x (x^1\hat{T}_{q5}^{+2} - x^2\hat{T}_{q5}^{+1}), \qquad \hat{T}_{q5}^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^\mu\gamma_5 i\overleftrightarrow{D}^\nu\psi$$

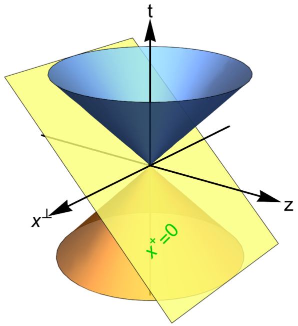
$$\hat{L}_z^q = \int \mathrm{d}^3x (x^1\hat{T}_{q}^{+2} - x^2\hat{T}_{q}^{+1}), \qquad \hat{T}_{q}^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^\mu i\overleftrightarrow{D}^\nu\psi$$

Light-front quantization

equal time quantization



light-front quantization



[Dirac:1949cp]

- ✔ Time: $t \equiv x^0$ | $t \equiv x^+ = x^0 + x^3$
- ✔ Hamiltonian: $H \equiv P^0$ | $H \equiv P^- = P^0 - P^3$
- ✔ Dispersion relation: $p^0 = \sqrt{\vec{p}^2 + m^2}$ | $p^- = (\vec{p}_\perp^2 + m^2)/p^+$

light-front coordinates

$$x^\pm = x^0 \pm x^3$$
$$\vec{x}^\perp = (x^1, x^2)$$

Light-front quantization is a Hamiltonian method of the quantum field theory

[Review: Brodsky:1997de]

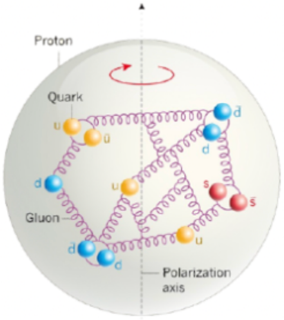
$$(P^+ \hat{P}^- - \vec{P}_\perp^2)|\psi_h\rangle = M_h^2 |\psi_h\rangle$$

Light-front wave functions

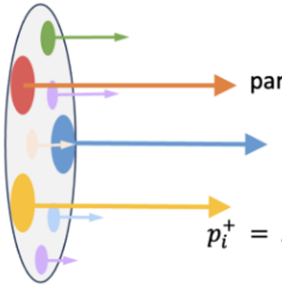
$$|\psi_h(P,j,\lambda)\rangle = \sum_{n=1}^{\infty}\int [\mathrm{d}x_i \mathrm{d}^2 k_{i\perp}]_n \psi_{h/n}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n)|\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$$

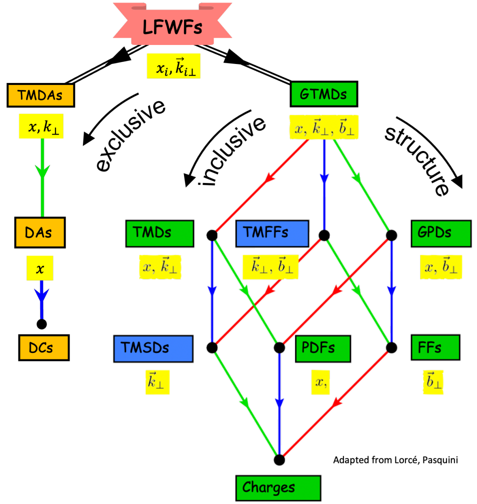
- Light-front physics measures hadron structures in high-energy scattering experiments
- Light-front wave functions (LFWFs) provide full information about the hadron structure
- LFWF representation offers simple microscopic interpretations

[Lorce:2011kd]



$P_z \to \infty$





Microscopic interpolation of form factors

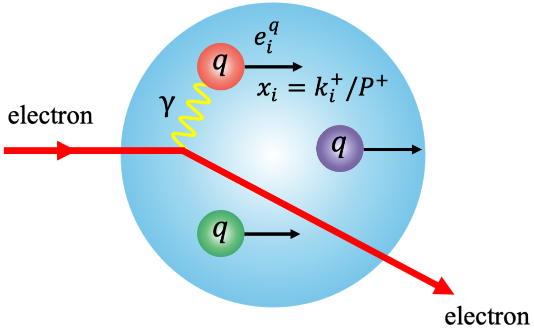
- Drell-Yan-West formula for charge form factor: [Drell:1969km, West:1970av, Brodsky:1980zm]

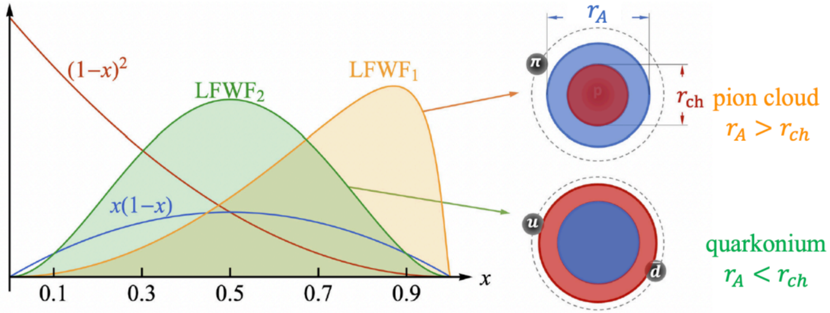
$$\rho_{\rm ch}(r_\perp) = \int [{\rm d}x_i {\rm d}r_{i\perp}]_n |\tilde\psi_n(\{x_i, \vec r_{i\perp}\})|^2 \sum_j e_j \delta^{(2)}(r_\perp - r_{j\perp}) \equiv \langle \sum_j e_j \delta^{(2)}(r_\perp - r_{j\perp}) \rangle$$

- Brodsky-Hwang-Ma-Schmidt formula for gravitational form factor A: [Brodsky:2000ii]

$$\mathcal{A}(r_\perp) = \int [{\rm d}x_i {\rm d}r_{i\perp}]_n |\tilde\psi_n(\{x_i, \vec r_{i\perp}\})|^2 \sum_j x_j \delta^{(2)}(r_\perp - r_{j\perp}) \equiv \langle \sum_j x_j \delta^{(2)}(r_\perp - r_{j\perp}) \rangle$$

Matter density $\mathcal{A}(r_\perp)$ mainly samples the valence partons $x_j \sim O(1)$; wee parton $x_j \ll 1$ contributions supressed





[Hackett:2023nkr] $r^\pi_{\rm ch} \sim 1.6 r^\pi_A$

Microscopic interpolation of spin-orbit correlation

$$\mathcal{C}^z_{LS}(r_\perp) = \langle \sum_j \delta^{(2)}(r_\perp - r_{i\perp}) L^z_j S^z_j \rangle$$

- Spin-orbit correlation defined from quantum many-body formula
- The orbit angular momentum operator $L^z_j = -(\vec{r}_{j\perp} \times i\overleftrightarrow{\nabla}_{j\perp})_z/2$
- The quantum average is defined as

$$\langle \hat{O} \rangle = \int [\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}] \psi^*_n(\{x_i, \vec{r}_{i\perp}\}) \hat{O} \psi_n(\{x_i, \vec{r}_{i\perp}\})$$

Heavy meson on the light front

- Effective Hamiltonian in the $q\bar{q}$ Fock sector [Li:2017mlw, Tang:2018myz]

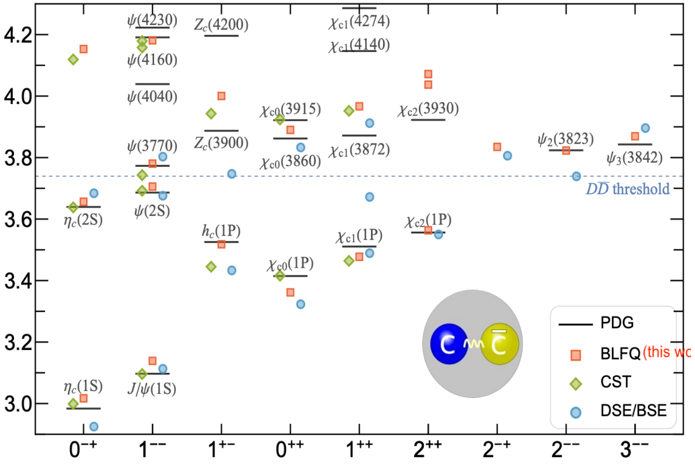
kinetic energy — confinement

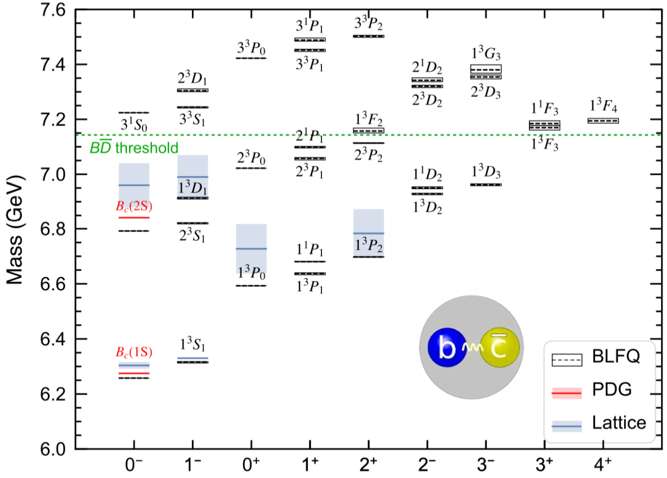
$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x(x(1-x)\partial_x)$$

$$-\frac{C_F 4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}')$$

one gluon exchange

- Two/three parameters $(m_q, m_{\bar{q}}, \kappa)$ are fixed by fitting the mass spectrum





Parity-odd energy-momentum tensor on light front

- Hadron matrix elements for spin-0 hadrons: [Carbonell:1998rj, Cao:2024rul]

$$\langle P+\frac{1}{2}\Delta|\hat{T}^{[\mu\nu]}_{q5}(0)|P-\frac{1}{2}\Delta\rangle = i\varepsilon^{\mu\nu\Delta P}\tilde{F}_q(\Delta^2)+M^2\frac{\varepsilon^{\mu\nu P\omega}}{\omega\cdot P}\tilde{A}_1(\Delta^2)+M^2\frac{i\varepsilon^{\mu\nu\Delta\omega}}{\omega\cdot P}\tilde{A}_2(\Delta^2)$$

$$\langle P+\frac{1}{2}\Delta|\hat{T}^{\{\mu\nu\}}_{q5}(0)|P-\frac{1}{2}\Delta\rangle = \frac{iP^{\{\mu}\epsilon^{\nu\}P\Delta\omega}}{\omega\cdot P}\tilde{S}_1(\Delta^2)+\frac{\Delta^{\{\mu}\varepsilon^{\nu\}P\Delta\omega}}{\omega\cdot P}\tilde{S}_2(\Delta^2)+\frac{i\omega^{\{\mu}\varepsilon^{\nu\}P\Delta\omega}}{(\omega\cdot P)^2}\tilde{S}_3(\Delta^2)$$

where $P=(p+p')/2$, $\Delta=p'-p$, $\varepsilon^{\mu\nu\Delta P}=\varepsilon^{\mu\nu\rho\sigma}\Delta_\rho P_\sigma$. M is the hadron mass. $\omega^\mu=(1,0,0,-1)$ is a null vector indicating the light-front direction

- $\tilde{A}_i(\Delta^2)$ and $\tilde{S}_i(\Delta^2)$ are spurious form factors appears due to the violation of the full Lorentz symmetry
- SOC receives contributions from both physical + spurious form factors

[Lorce:2025ayr]

$$\hat{C}^q_z=\int \mathrm{d}^3x(x^1\hat{T}^{+2}_{q5}-x^2\hat{T}^{+1}_{q5})$$

$$\langle P+\frac{1}{2}\Delta|\hat{T}^{+i}_{q5}|P-\frac{1}{2}\Delta\rangle = iP^+\epsilon^{ij}\Delta^j_\perp(\tilde{F}_q(\Delta^2_\perp)+\tilde{S}_1(\Delta^2_\perp))$$

Light-front wave function representation

- SOC density:

$$C_z^q(r_\perp) = r_\perp^1 \hat{\mathcal{T}}_{q5}^{+2}(\vec{r}_\perp) - r_\perp^2 \hat{\mathcal{T}}_{q5}^{+1}(\vec{r}_\perp)$$

- Parity-odd EMT density:

$$\hat{\mathcal{T}}_{q5}^{\mu\nu} = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2 2p^+} e^{-i\vec{\Delta}_\perp\cdot\vec{r}_\perp} \langle p'|\hat{T}_{q5}^{\mu\nu}(0)|p\rangle$$

- Light-front wave function representation for quark sectors:

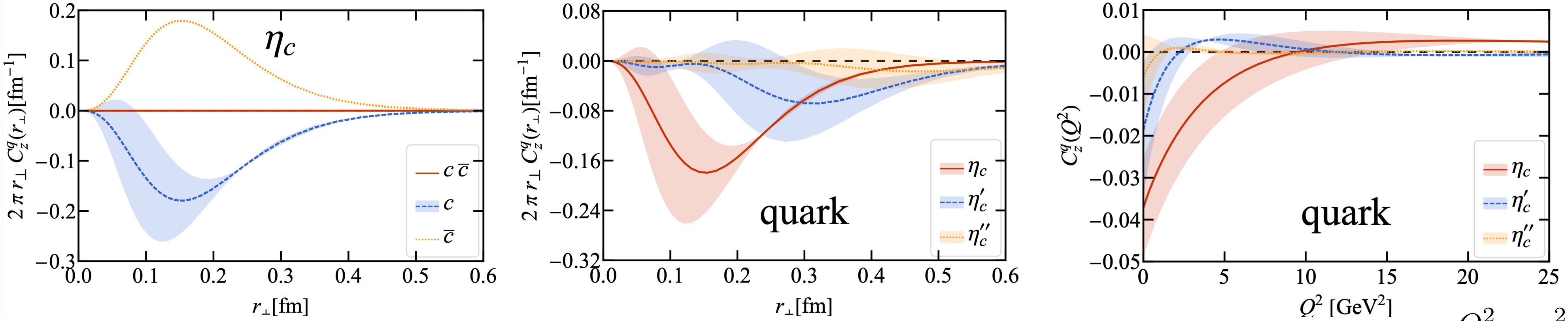
$$\tilde{\mathcal{F}}^q(\Delta_\perp^2) = \tilde{F}^q(\Delta_\perp^2) + \tilde{S}_1(\Delta_\perp^2)$$

$$C_z^q(r_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{r}_\perp} [\tilde{\mathcal{F}}^q(\Delta_\perp^2) + \Delta_\perp^2 \frac{\mathrm{d}\tilde{\mathcal{F}}^q(\Delta_\perp^2)}{\mathrm{d}\Delta_\perp^2}]$$

$$= \sum_{s,\bar{s}} \int [\mathrm{d}x\mathrm{d}^2r_\perp] \tilde{\psi}_{s\bar{s}}^*(x,\vec{r}_\perp) \sum_{j=q,\bar{q}} \delta^{(2)}(r_\perp - r_{j\perp}) \eta_j \underbrace{s_j(\vec{r}_{j\perp} \times (-i\overleftrightarrow{\nabla}_{j\perp}))_z}_{L_j^z S_j^z} \tilde{\psi}_{s\bar{s}}(x,\vec{r}_\perp)$$

$\eta_j = \pm 1$ for quarks/antiquarks

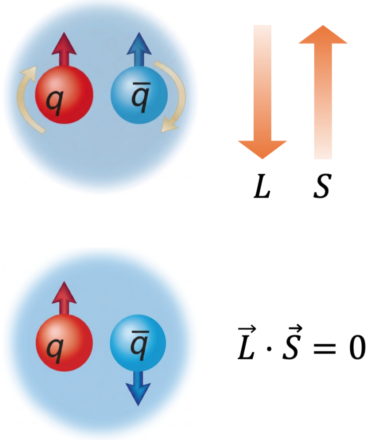
Spin-orbit correlation: η_c



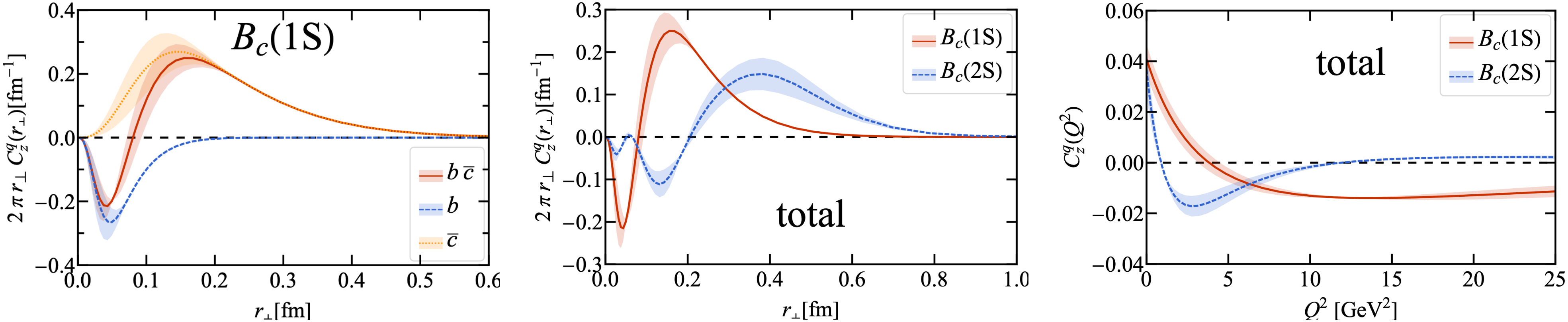
$Q^2 = q_\perp^2$

- Quark spin and OAM are anti-aligned in spin-0 hadrons
- $C_z^q(0) = 0$ is enforced by charge conjugation (C) symmetry
- Magnitude of SOC decreases as the radial excitation increases
- The SOC form factor is defined as:

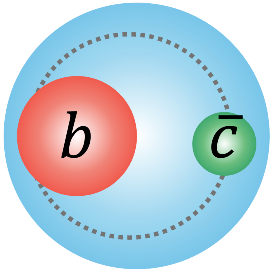
$$C_z^q(Q^2) = \int \mathrm{d}^2 r_\perp e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} C_z^q(r_\perp)$$



Spin-orbit correlation: B_c

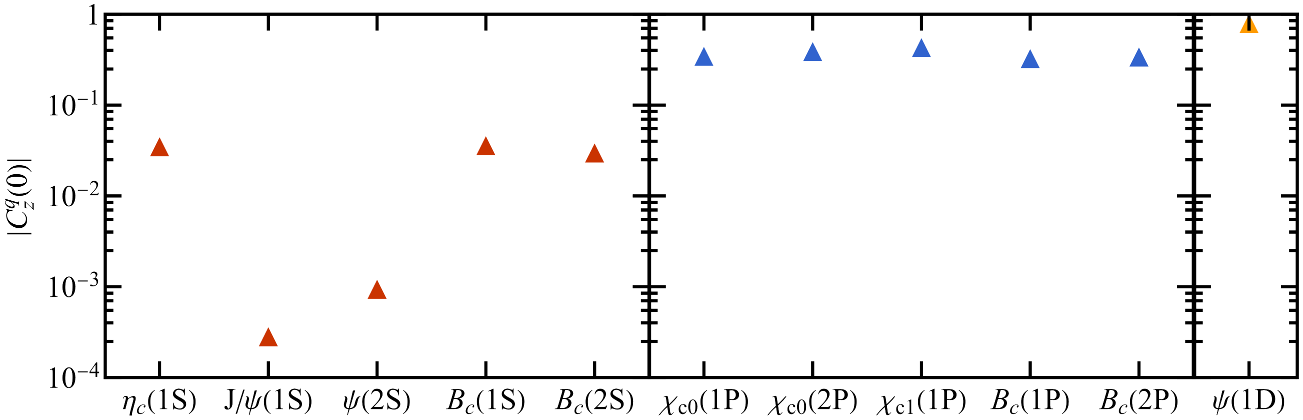


- b quark dominates at small $r_\perp$, while $\bar{c}$ quark dominates at large $r_\perp$
- The global spin and OAM are parallel
- Spin-0 hadrons have non-zero spin-orbit correlations

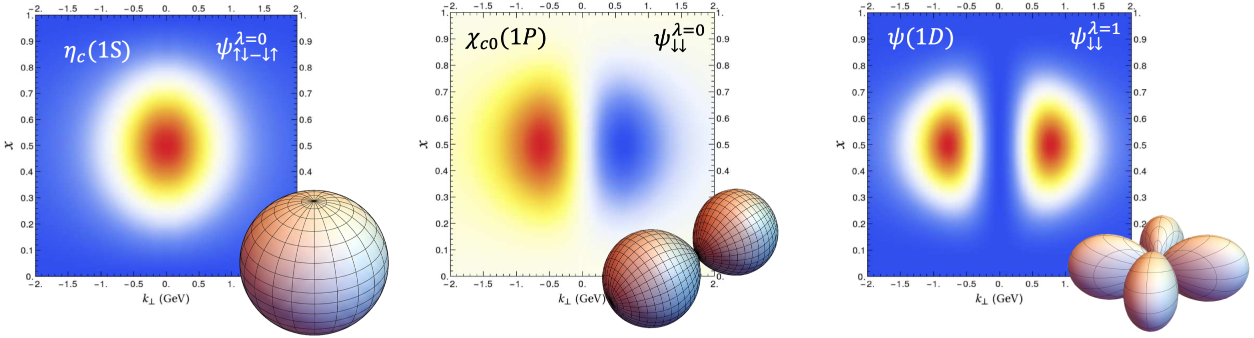


$m_b \sim 3m_{\bar{c}}$

Summary of SOC in heavy mesons



angular excitations



Summary

- Spin-orbit correlations provide a novel probe of hadron spin structures
- We obtain a non-perturbative quantum many-body formula for spin-orbit correlations under the light-front Hamiltonian method
- We apply this formula to heavy mesons solved from an effective Hamiltonian
- This formula can be further extended to proton and other hadrons

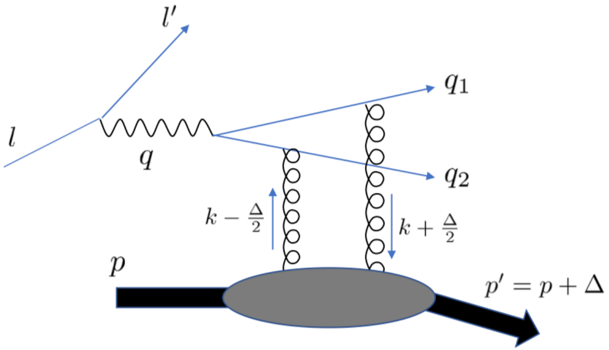
Based on: arXiv: 26XX.XXXXX

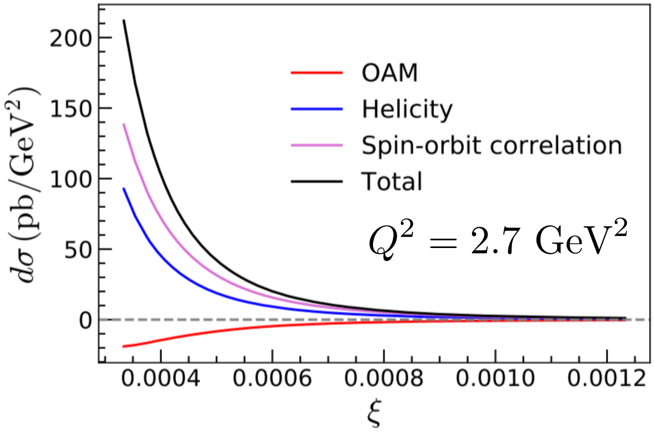
Thank you!

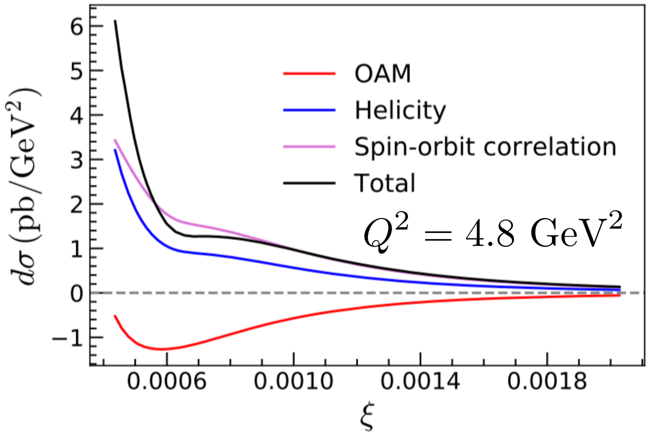
Backup Slides

Spin-orbit correlation in experiments

- Spin-orbit correlations are non-negligible in exclusive di-jet production [Bhattacharya:2024sck]

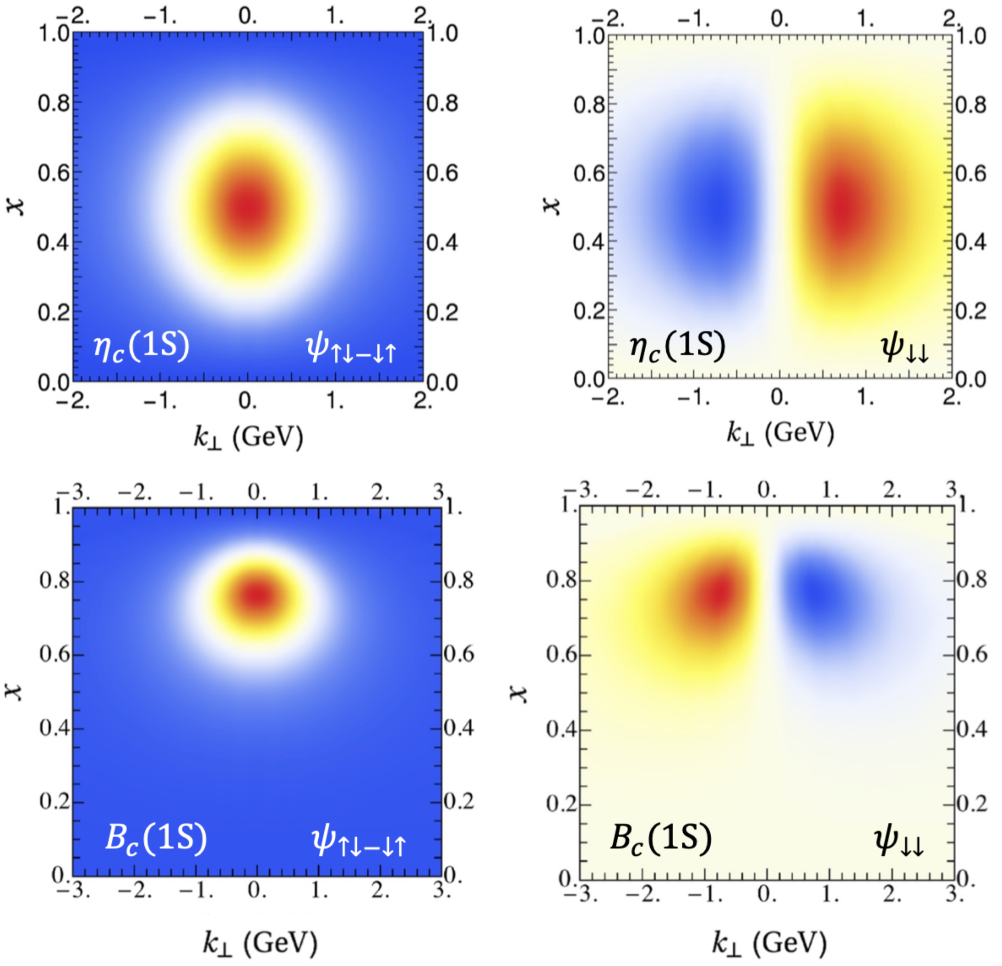


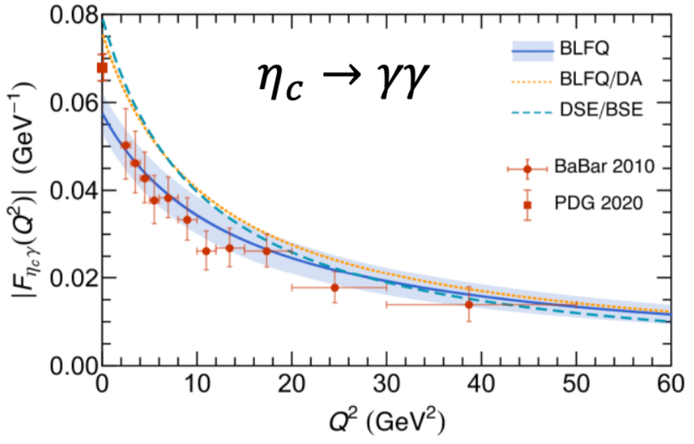


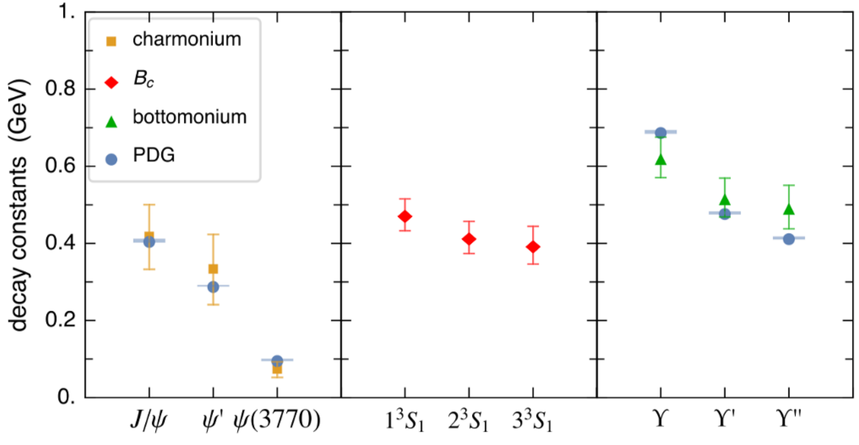


Light-front wave functions of heavy mesons

[Li:2021ejv, Tang:2018myz]







How to access SOC

- Relation with other form factors: [Lorce:2025ayr]

$$\tilde{F}^q(q^2) = \frac{1}{2}\left[-F^q(q^2) + \frac{m_q}{M}H^q(q^2)\right] = -\frac{1}{2}F^q(q^2) + \mathcal{O}(\frac{m_q}{M})$$

$$\langle p'|\bar{\psi}\gamma^\mu\psi(0)|p\rangle = 2P^\mu F^q(q^2)$$

$$\langle p'|\bar{\psi}i\sigma^{\mu\nu}\gamma_5\psi(0)|p\rangle = \frac{i\epsilon^{\mu\nu\Delta P}}{M}H^q(q^2)$$

- Relation with leading-twist GPDs:

$$C_z^q(x) = \frac{m_q}{M}\mathcal{H}_q^1(x,0,0) - \frac{1}{2}\mathcal{H}^q(x,0,0) - x\mathcal{G}_2(x,0,0)$$