

Toward Two-Loop EW Renormalization in the Full SM

Preliminary NNLO corrections for $H \rightarrow b\bar{b}$ as a benchmark process and beyond

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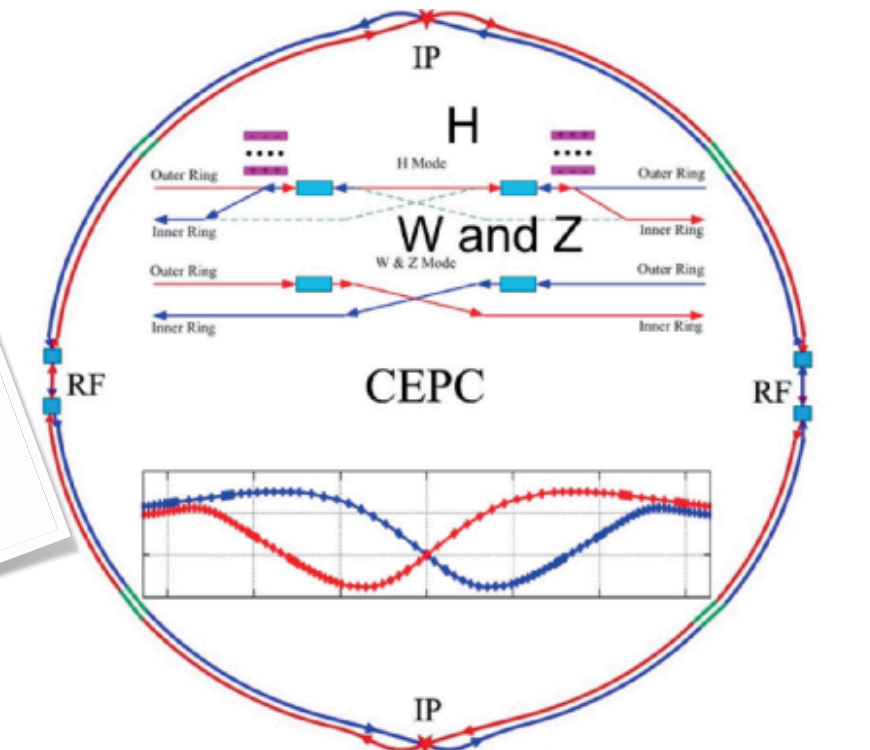
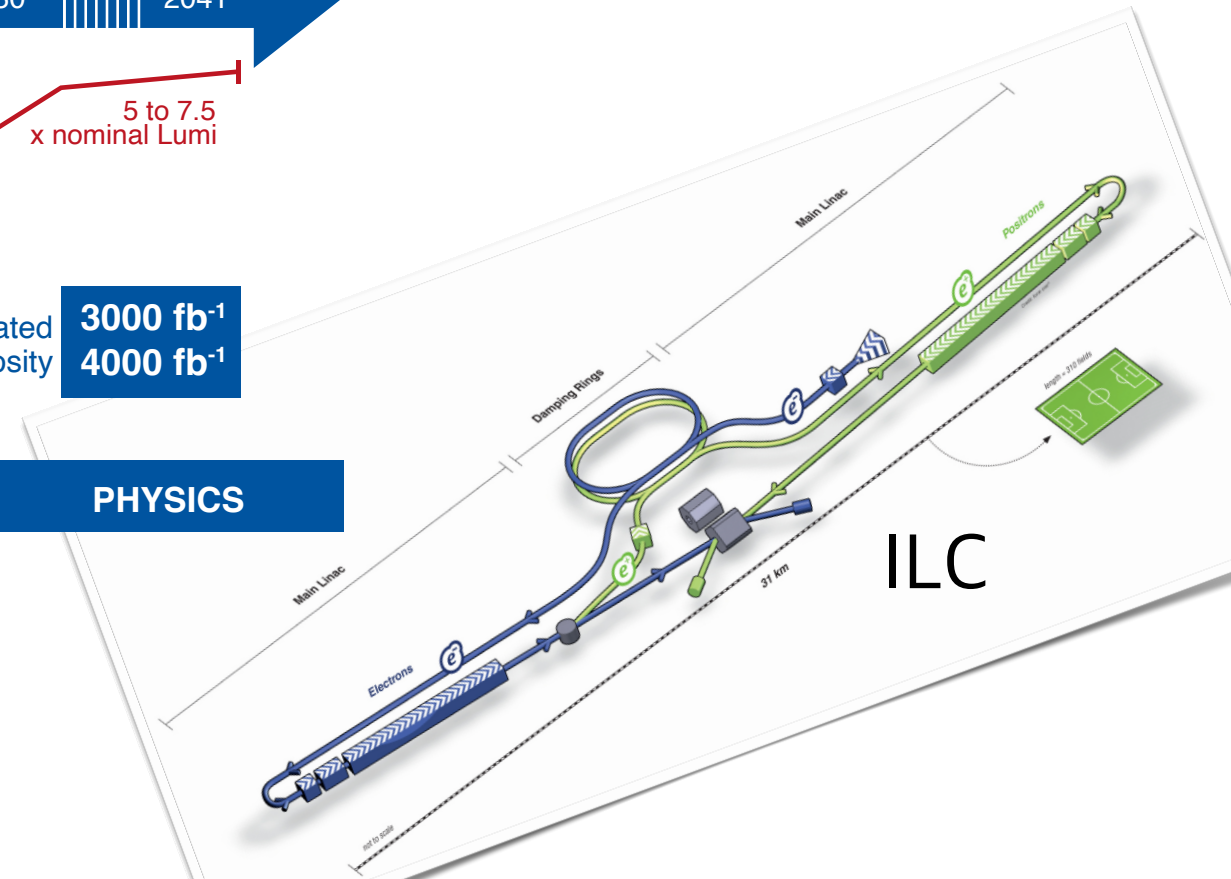
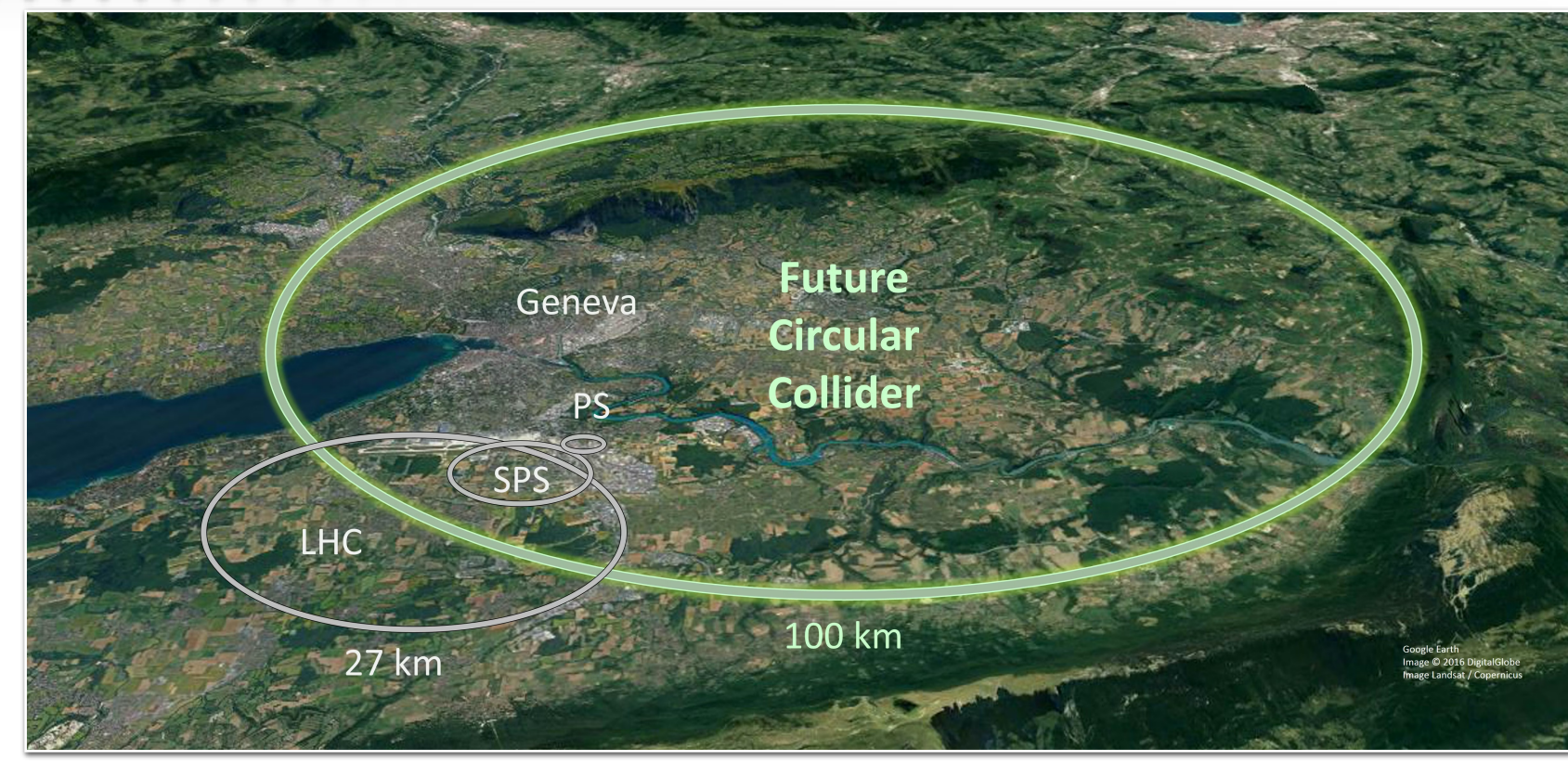
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Based on the ongoing works, papers coming soon

The 8th National Workshop on Heavy Flavor Physics and Quantum Chromodynamics

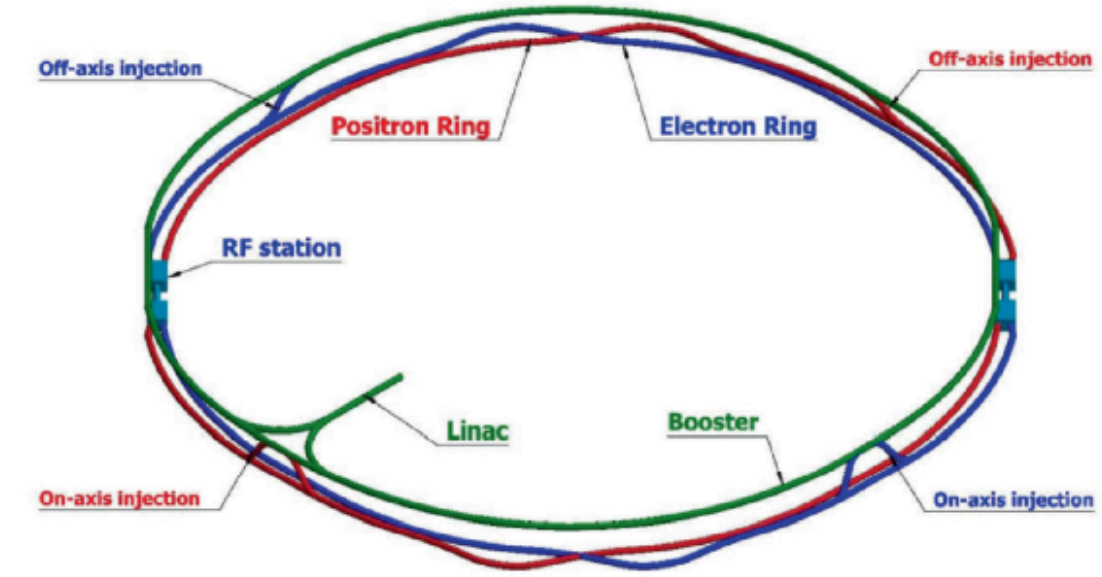
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Precision Era of Collider Physics



CEPC collider ring (100km)

a) CEPC different working modes



Sub-percent precision

- LHC has established Higgs and electroweak measurements as a central part of collider physics.
- HL-LHC will bring a factor of 10 increase of the data set, up to 4000 fb⁻¹, at least 15 M Higgs per year.
- Future e^+e^- colliders aim at a new level of precision, with FCC-ee targeting 10⁶ Higgs bosons.
- An unique opportunity for precision studies of the electroweak sector is waiting ahead!

Perturbative QCD & EW: Where we stand?

See X. Chen's Talk

NNLO QCD

- NNLO revolution since 2015, great progress have been made during the past decade
- $2 \rightarrow 2$: $VV, HH, VH, V/H+\text{jet}, \text{dijet}, t\bar{t}, \dots$
- $2 \rightarrow 3$: $3\text{jet}, \gamma\gamma\gamma, b\bar{b}H, b\bar{b}W, t\bar{t}H, t\bar{t}W, \dots$
- $2 \rightarrow 4$: $e^+e^- \rightarrow 4\text{jet}, \dots$
- Amplitude frontier: massless 2-loop 6-point, massive external 3-loop 5-point, more massive internal, ...
- Typical correction: 2–15%; Scale uncertainty: 3–8%

Gehrmann et al. '14; Cascioli et al. '14; Grazzini et al. '14-'16; Gehrmann-De Ridder et al. '16; Boughezal et al. '15-'16; Catani et al. '12; Ferrera et al. '11,'15; de Florian et al. '16; Czakon et al. '13, Chawdhry et al. '20, Czakon et al. '21, Catani et al. '23, Buonocore et al. '23, Biello et al. '25, ...

N3LO QCD

- Inclusive: Higgs production, Drell-Yan, $VH, e^+e^- \rightarrow t\bar{t}$, top decay, ...
- Differential: ggF Higgs, Drell-Yan, $H \rightarrow b\bar{b}, e^+e^- \rightarrow 2\text{jet}$, heavy-to-light semileptonic decays, ...
- Amplitude frontier: 3-loop 5-point, massive external ...
- QCD is no longer defined only by NNLO maturity, but increasingly by N3LO precision, higher multiplicity, and massive final states.

Anastasiou et al. '15 '16, Mistlberger '18, Duhr et al. '19, Duhr et al. '20, Cieri et al. '18, Billis et al. '21, Chen et al. '21, Camarda et al. '21, Chen et al. '21, Chen et al. '22, Chen et al. '25, Simone et al. '25, Chen et al. '26, ...

Perturbative QCD & EW: Where we stand?

NLO EW

- Mature and largely automated
- MadGraph5_aMC@NLO, OpenLoops 2, RECOLA2, ...
- Higher multiplicity: $V + jets$, off-shell $t\bar{t}/t\bar{t}H$, VBS $VVjj$, triboson and triboson+ jj
- NLO EW is already part of standard precision phenomenology.

Frederix et al. '18, Buccioni et al. '19, Denner et al. '17, Kallweit et al. '15-'16, Denner & Pellen '16, Denner et al. '17-'24, Denner, Dittmaier et al. '19; Dittmaier et al. '20, Grazzini et al. '20, ...

Mixed QCD-EW

- Advancing benchmark by benchmark
- Decay: $HZ\gamma, H\gamma\gamma, \dots$
- Lepton colliders: HZ, W^+W^-, ZZ, \dots
- Hadron colliders: Neutral/Charged current Drell-Yan, ...
- Already percent/per-mille relevant for flagship precision processes

Dittmaier, Huss, Schwinn '14 – '15, Gong/Sun et al. '17, Buonocore/Bonciani et al. '21 – '22, Armadillo et al. '22, Armadillo et al. '24 – '25, Z. Li et al. '24, Feng, Jia '25, Z. Li et al. '25, ...

NNLO pure EW

- Sparse and process-specific
- Parameters: $\Delta r, M_W$, EW mixing angle, EWPO, ...
- Decay: muon decay, Z decay
- $2 \rightarrow 2$ frontier: Møller Scattering, $e^+e^- \rightarrow ZH$ fermionic contribution

Awramik, Czakon, Freitas et al. '03 – '06, Freitas '14, Dubovyk et al. '19; Freitas and Song '23; Chen and Ma et al. '22, ...

Compared with QCD, the sparse region is still full-SM beyond-NLO EW precision for benchmark collider processes.

Why Is the NNLO EW challenging?

Structural obstacles

- ▶ Large computation: $\mathcal{O}(10^{3\sim 4})$ diagrams, $\mathcal{O}(10^{2\sim 3})$ families
- ▶ γ^5 : chiral EW couplings make its treatment in dimensional regularization nontrivial
- ▶ Multi-massive internal propagators: $M_Z, M_W, M_H, M_t, \dots$ and kinematic variables generate thresholds and elliptic structures
- ▶ **Unstable-particle: complex masses, ...**

Renormalization systematics

- ▶ Existing two-loop EW renormalization studies provide important ingredients, but the **full SM still lacks a mature, unified, reusable framework** tailored to benchmark process phenomenology.
- ▶ In particular, **a complete set of Renormalization Constants in a directly usable and consistent form** is not yet standard in the literature.
- ▶ **A central goal of this series work is to construct and provide the required full-SM two-loop EW renormalization constants and reliable framework.**

At NNLO EW, the bottleneck is not only the process amplitude itself, but also the lack of a reusable full-SM two-loop renormalization framework.

Scope of this talk

- Two-loop electroweak renormalization in the on-shell scheme.
- First benchmark application to $H \rightarrow b\bar{b}$ in the full Standard Model.
- Preliminary results with $\alpha(0)$ and m_b^{OS} as input choices.
- Focus: inclusive NNLO EW and mixed QCD-EW corrections.
- Benchmark-level implementation, final phenomenological setup coming soon.

This setup establishes a consistent full-SM two-loop EW renormalization framework before moving to more phenomenology-oriented input schemes

Workflow of this work

2-loop EW renormalization



Renormalized amplitudes validations



Benchmark: $H \rightarrow b\bar{b}$ decay width

Why $H \rightarrow b\bar{b}$

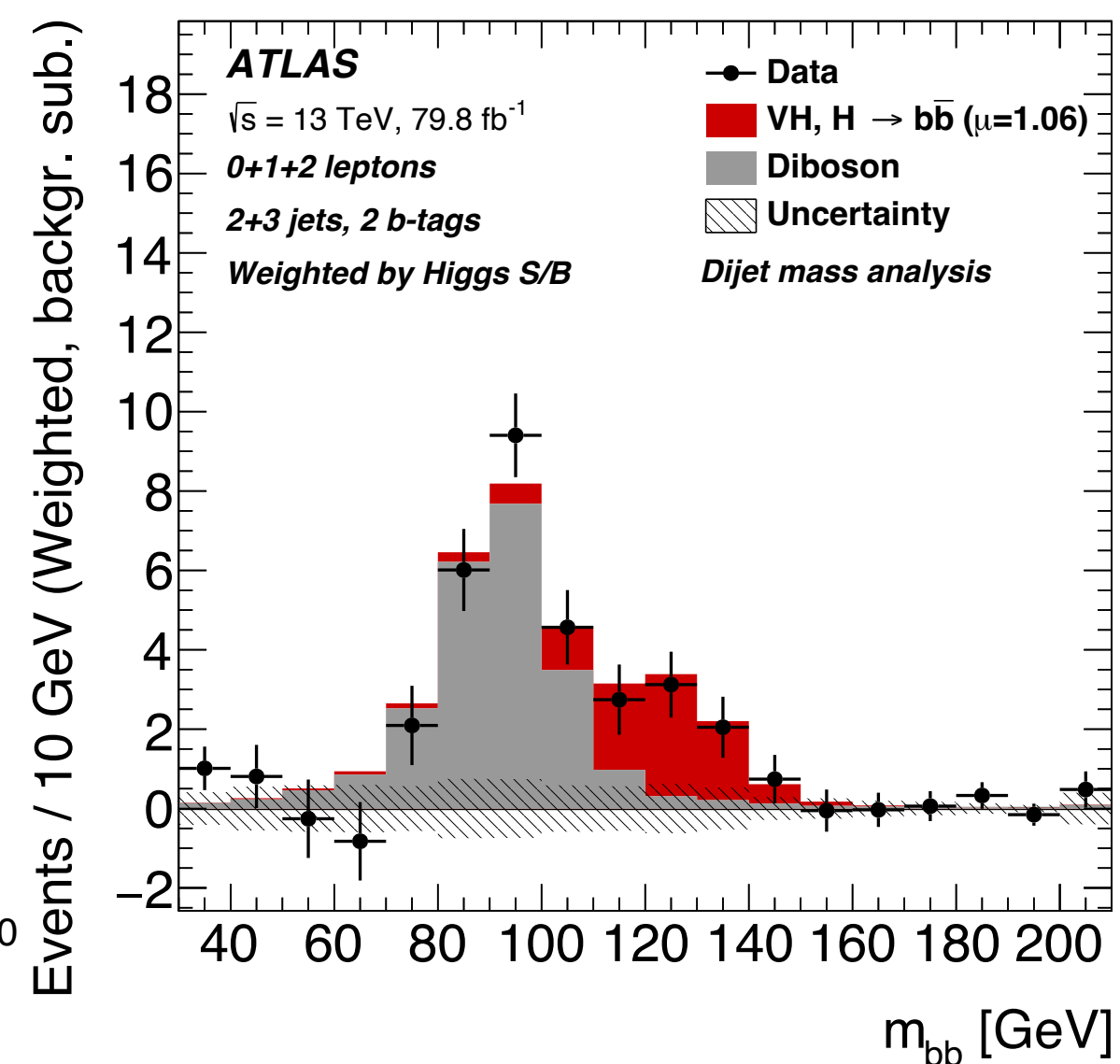
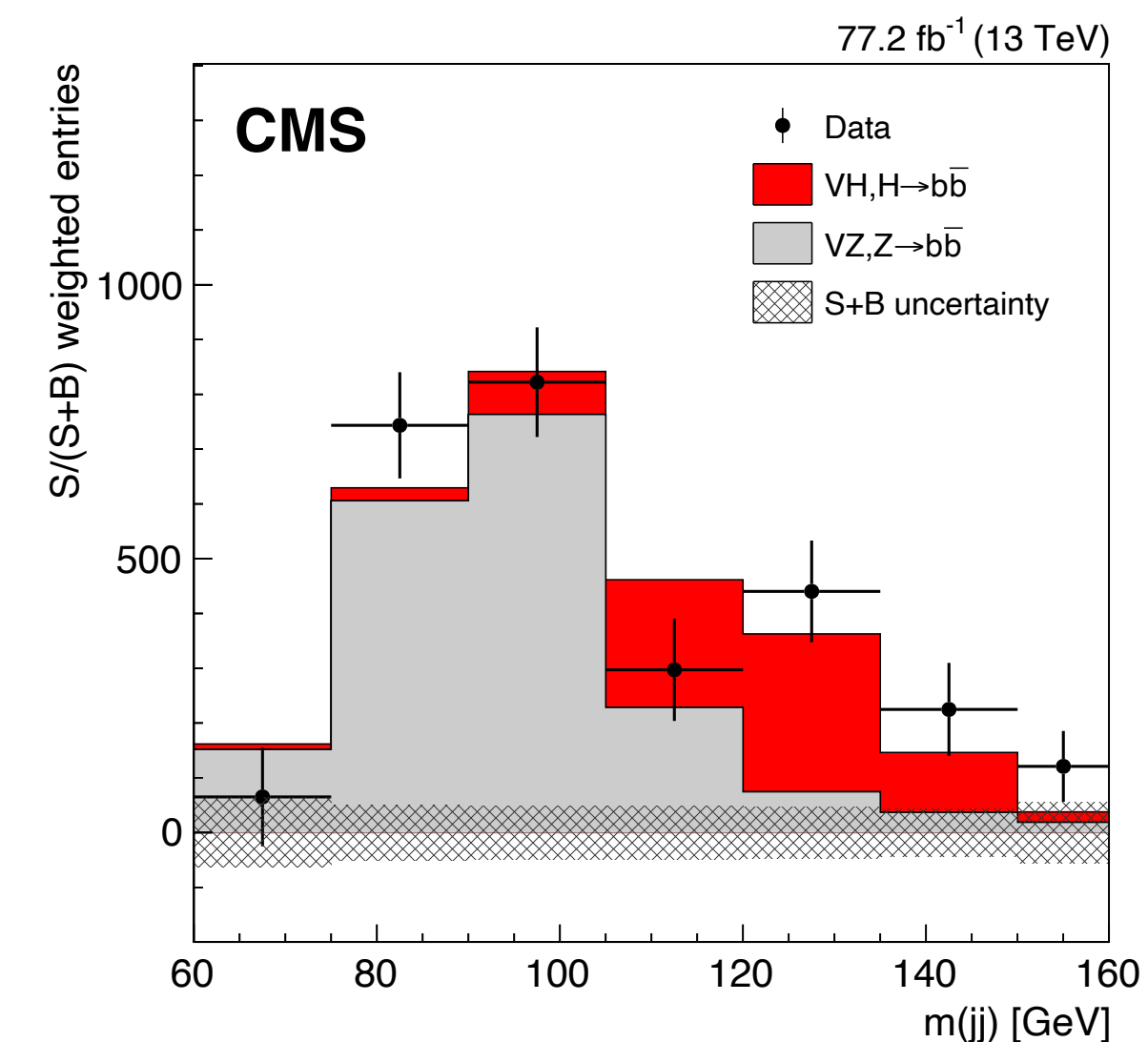
H DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	WW^*	$(25.7 \pm 2.5) \%$	
Γ_2	ZZ^*	$(2.80 \pm 0.30) \%$	
Γ_3	$\gamma\gamma$	$(2.50 \pm 0.20) \times 10^{-3}$	
Γ_4	bb	$(53 \pm 8) \%$	
Γ_5	e^+e^-	$< 3.0 \times 10^{-4}$	95%
Γ_6	$\mu^+\mu^-$	$(2.6 \pm 1.3) \times 10^{-4}$	
Γ_7	$\tau^+\tau^-$	$(6.0^{+0.8}_{-0.7}) \%$	
Γ_8	$Z\gamma$	$(3.4 \pm 1.1) \times 10^{-3}$	

- The Higgs boson is central stage of the Standard Model and future precision collider programmes.
- $H \rightarrow b\bar{b}$ is **the dominant decay channel** of Higgs boson
- High precision experimental measurements in CMS and ATLAS.
- Future electron colliders are expected to substantially sharpen the extraction of y_b .

The most important is that it can help us to validate all kinds of EW RCs!

Since it is both dominant and experimentally accessible, $H \rightarrow b\bar{b}$ is a natural benchmark for precision Higgs physics! ! !



State-of-the-Art $H \rightarrow b\bar{b}$

Perturbative QCD

- Inclusive width up to $\mathcal{O}(y_b^2\alpha_s^4)$ in the massless-bottom approximation.
- Massive-bottom differential observables up to $\mathcal{O}(y_b^2\alpha_s^2)$.
- Top-induced / large- m_t differential contributions up to $\mathcal{O}(\alpha_s^3)$.
- Higgs decay jet rates and event shapes; three jets at NNLO; two-jet rate inferred at N3LO.
- Recent $\mathcal{O}(\alpha_s^4)$ with top-Yukawa-induced massive-bottom progress.

See Y.F. Wang's Talk

Missing ingredients

- Consistent two-loop EW renormalization in the full SM
- NNLO mixed QCD-EW corrections
- NNLO pure QCD-EW corrections

Mixed QCD-EW
 $\mathcal{O}(\alpha_s\alpha)$

NNLO EW
 $\mathcal{O}(\alpha^2)$

$$\Gamma(H \rightarrow b\bar{b}) = \Gamma_{\text{LO}} \left[1 + \delta_{\text{QCD}}^{\text{NLO}} + \delta_{\text{EW}}^{\text{NLO}} + \delta_{\text{QCD}}^{\text{N}^k\text{LO}} + \delta_{\text{mix}}^{\text{NNLO}} + \delta_{\text{EW}}^{\text{NNLO}} + \dots \right]$$

On-Shell Renormalization

Input parameters:

$$\alpha_0 = \frac{e_0^2}{4\pi} = \frac{g_0^2 g_0'^2}{4\pi (g_0^2 + g_0'^2)} \quad M_{W,0} = \frac{1}{4} g_0^2 v_0^2$$

$$M_{Z,0} = \frac{1}{4} (g_0^2 + g_0'^2) v_0^2 \quad M_{H,0} = \lambda_0 \frac{v_0^2}{2}$$

Renormalized parameters:

$$m_{H,0} = m_H + \delta m_H, \quad m_{Z,0} = m_Z + \delta m_Z,$$

$$m_{W,0} = m_W + \delta m_W, \quad m_{f,0} = m_f + \delta m_f,$$

$$e_0 = Z_e e = (1 + \delta Z_e) e,$$

Renormalized Fields:

$$H_0 = Z_H^{1/2} H = \left(1 + \frac{1}{2} \delta Z_H\right) H, \quad W_0^\pm = Z_W^{1/2} W^\pm = \left(1 + \frac{1}{2} \delta Z_W\right) W^\pm,$$

$$f_0^L = (Z_f^L)^{1/2} f^L = \left(1 + \frac{1}{2} \delta Z_f^L\right) f^L, \quad f_0^R = (Z_f^R)^{1/2} f^R = \left(1 + \frac{1}{2} \delta Z_f^R\right) f^R,$$

Mixing $\gamma - Z$

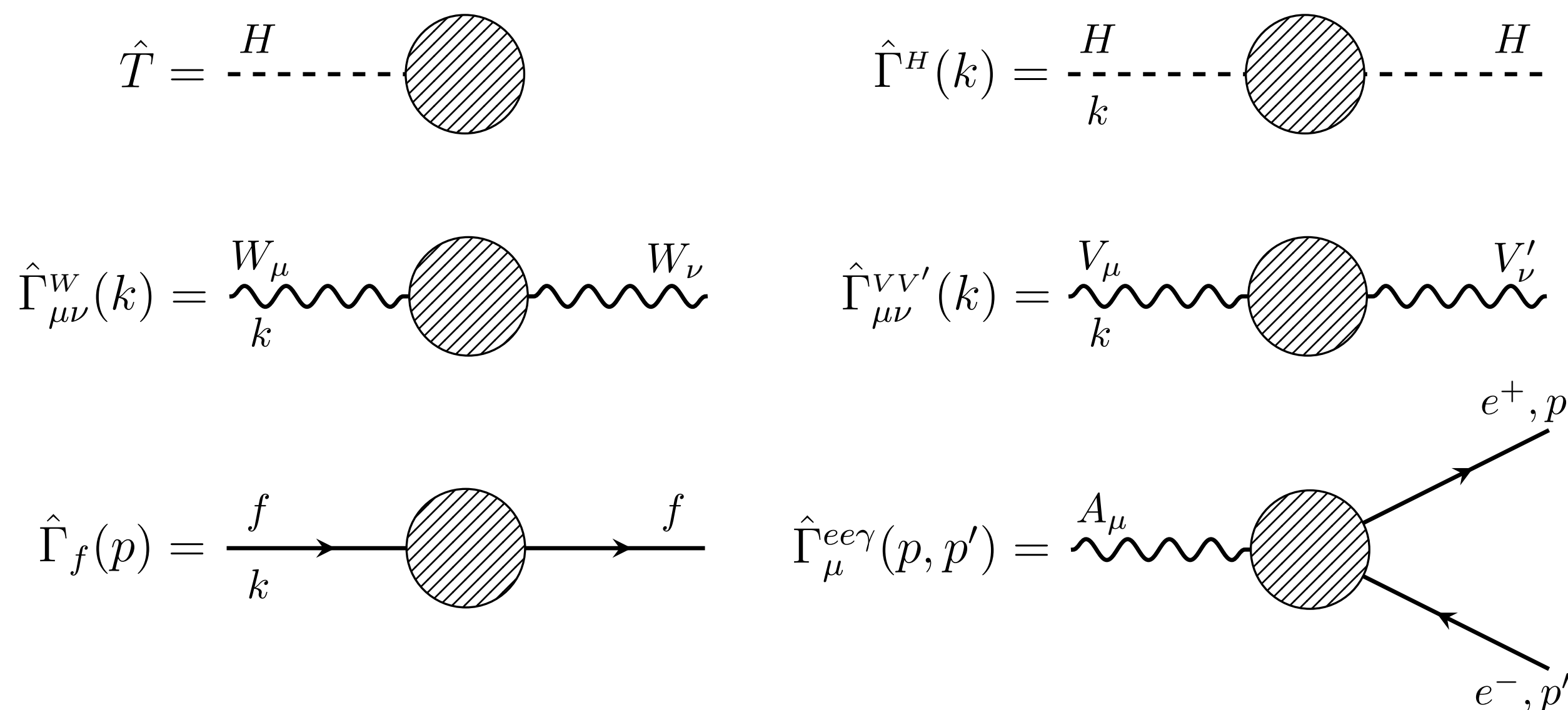
$$\begin{pmatrix} A_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} Z_{AA}^{1/2} & Z_{AZ}^{1/2} \\ Z_{ZA}^{1/2} & Z_{ZZ}^{1/2} \end{pmatrix} \begin{pmatrix} A_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2} \delta Z_{AA} & \frac{1}{2} \delta Z_{AZ} \\ \frac{1}{2} \delta Z_{ZA} & 1 + \frac{1}{2} \delta Z_{ZZ} \end{pmatrix} \begin{pmatrix} A_0 \\ Z_0 \end{pmatrix}.$$

On-Shell Renormalization

OS renormalization conditions:

- Renormalized propagator has a simple pole at physical mass with unit residue
- Renormalized vertex equals its tree-level vertex

1PI green functions:



OS RC projected onto physical states:

- Pole relation:

$$\begin{aligned} \widetilde{\text{Re}} \hat{\Gamma}^H(k) \Big|_{k^2=m_H^2} &= 0, & \widetilde{\text{Re}} \hat{\Gamma}_{\mu\nu}^W(k) \varepsilon^\nu(k) \Big|_{k^2=m_W^2} &= 0, \\ \widetilde{\text{Re}} \hat{\Gamma}_{\mu\nu}^{AA}(k) \varepsilon^\nu(k) \Big|_{k^2=0} &= 0, & \widetilde{\text{Re}} \hat{\Gamma}_f(p) u(p) \Big|_{p^2=m_f^2} &= 0, \end{aligned}$$

...

- Residue relation:

$$\begin{aligned} \lim_{k^2 \rightarrow m_H^2} \frac{1}{k^2 - m_H^2} \widetilde{\text{Re}} \hat{\Gamma}^H(k) &= i, & \lim_{k^2 \rightarrow m_W^2} \frac{1}{k^2 - m_W^2} \widetilde{\text{Re}} \hat{\Gamma}_{\mu\nu}^W(k) \varepsilon^\nu(k) &= -i \varepsilon_\mu(k), \\ \lim_{k^2 \rightarrow 0} \frac{1}{k^2} \widetilde{\text{Re}} \hat{\Gamma}_{\mu\nu}^{AA}(k) \varepsilon^\nu(k) &= -i \varepsilon_\mu(k), & \lim_{p^2 \rightarrow m_f^2} \frac{p + m_f}{p^2 - m_f^2} \widetilde{\text{Re}} \hat{\Gamma}_f(p) u(p) &= i u(p), \end{aligned}$$

...

Coupling renormalization condition:

- Electrical charge is defined as the full $e^+e^-\gamma$ -coupling for on-shell external particles in the Thomson limit:

$$\bar{u}(p) \Gamma_\mu^{ee\gamma}(p, p) u(p) \Big|_{p^2=m_e^2} = i e \bar{u}(p) \gamma_\mu u(p)$$

Two-Loop EW RCs results

RCs in terms of self energy:

$$\delta Z_{(2)}^{HH} = - \left. \frac{\partial \Sigma_{(2)}^{HH}(k^2)}{\partial k^2} \right|_{k^2=M_H^2}, \quad \delta M_{H,(2)}^2 = \text{Re} \left(\Sigma_{(2)}^{HH}(M_H^2) \right) - \delta Z_{(1)}^H \delta M_{H,(1)}^2$$

$$\delta Z_{(2)}^{WW} = - \left. \frac{\partial \Sigma_{(2)}^{WW}(k^2)}{\partial k^2} \right|_{k^2=M_W^2}, \quad \delta M_{W,(2)}^2 = \text{Re} \left(\Sigma_{(2)}^{WW}(M_W^2) \right) - \delta Z_{(1)}^W \delta M_{W,(1)}^2$$

$$\delta Z_{(2)}^{AA} = - \left. \frac{\partial \Sigma_{(2)}^{AA}(k^2)}{\partial k^2} \right|_{k^2=0} - \frac{1}{4} \left(\delta Z_{(1)}^{ZA} \right)^2,$$

$$\delta Z_{e,(2)} = - \frac{1}{2} \delta Z_{(2)}^{AA} - \frac{s_w}{2c_w} \delta Z_{(2)}^{ZA} + \left(\delta Z_{e,(1)} \right)^2 + \frac{1}{8} \left(\delta Z_{(1)}^{AA} \right)^2 - \frac{1}{2c_w^3} \delta Z_{(1)}^{ZA} \delta s_{w,(1)},$$

$$\delta Z_{(2)}^{l,L} = - \text{Re} \left(\Sigma_{(2)}^{l,L}(m_l^2) + m_l^2 \frac{\partial}{\partial p^2} \left(\Sigma_{(2)}^{l,L}(p^2) + \Sigma_{(2)}^{l,R}(p^2) + 2\Sigma_{(2)}^{l,S}(p^2) \right) \right) \Big|_{p^2=m_l^2},$$

$$\delta Z_{(2)}^{l,R} = - \text{Re} \left(\Sigma_{(2)}^{l,R}(m_l^2) + m_l^2 \frac{\partial}{\partial p^2} \left(\Sigma_{(2)}^{l,L}(p^2) + \Sigma_{(2)}^{l,R}(p^2) + 2\Sigma_{(2)}^{l,S}(p^2) \right) \right) \Big|_{p^2=m_l^2},$$

Derivative of self energy:

► For $p^2 \neq 0$:

$$\frac{\partial}{\partial p^2} I(\alpha_1, \dots, \alpha_n; p^2 \neq 0) = \frac{1}{2p^2} p^\mu \frac{\partial}{\partial p^\mu} I(\alpha_1, \dots, \alpha_n; p^2)$$

► For $p^2 = 0$:

$$\frac{\partial}{\partial p^2} I(\dots; p^2 = 0) = \frac{1}{2D} \frac{\partial^2}{\partial p_\mu \partial p^\mu} I(\dots; p^2) \Big|_{p^2=0}$$

One should note that the derivative has to be implemented either at the integrand level, or at the integral/master-integral level while retaining the full p^2 -dependence before taking the on-shell limit.

Unstable particles in EW Renormalization

The issue

- ▶ Unstable particles are not strict S-matrix asymptotic states.
- ▶ Only appear through resonances reconstructed from decay products
- ▶ Fixed-order propagators become singular near $p^2 = M^2$; a finite width is needed
- ▶ Naive Dyson resummation spoil order-by-order gauge cancellations.

Gauge-invariant object: $s_p = \bar{M}^2 - i\bar{M}\bar{\Gamma}$.

Known prescriptions

**More discussion
see Han's talk**

- ▶ Pole scheme/ pole approximation: gauge-invariant pole residue.
- ▶ Complex-mass scheme: complex masses and couplings, unitarity issues, preserving ST identities.
- ▶ Fermion-loop / EFT: useful, but process- or region-dependent.

Expanding mass RCs in Γ

Propagator complex pole:

- ▶ On-shell mass: $\delta\bar{M}^2 = \text{Re} \Sigma (\bar{M}^2 - i\bar{M}\bar{\Gamma})$
- ▶ Decay width: $\bar{\Gamma} = \frac{1}{\bar{M}} \text{Im} \Sigma (\bar{M}^2 - i\bar{M}\bar{\Gamma})$

Additional finite terms @ NNLO:

$$\begin{aligned} \delta\bar{M}_{W,(2)}^2 &= \text{Re} \left(\Sigma_{T,(2)}^{WW} (M_W^2) \right) - \delta Z_{(1)}^W \delta M_{W,(1)}^2 \\ &\quad + \text{Im} \left\{ \Sigma_{T,(1)}^{W/} (M_W^2) \right\} \text{Im} \left\{ \Sigma_{T,(1)}^W (M_W^2) \right\} \\ \delta\bar{M}_{Z,(2)}^2 &= \text{Re} \left(\Sigma_{T,(2)}^{ZZ} (M_Z^2) \right) - \delta Z_{(1)}^{ZZ} \delta M_{Z,(1)}^2 + \frac{M_Z^2}{4} \left(\delta Z_{(1)}^{AZ} \right)^2 \\ &\quad + \frac{\left(\text{Im} \left\{ \Sigma_{T,(1)}^{Z/} (M_Z^2) \right\} \right)^2}{M_Z^2} + \text{Im} \left\{ \Sigma_{T,(1)}^{ZZ/} (M_Z^2) \right\} \text{Im} \left\{ \Sigma_{T,(1)}^{ZZ} (M_Z^2) \right\} \end{aligned}$$

Technical Remarks and Checks

Renormalization outputs

- **A complete set of full-SM two-loop EW renormalization constants has been obtained.**
- All two-loop RCs are expressed compactly in terms of master integrals.
- **Results are available in both Feynman-'t Hooft gauge and Landau gauge.**
- Sub-loop renormalization is included consistently in all two-loop self-energies.
- Compact formulae will be released soon.

Validation

- Known NLO OS formulae reproduced
- Selected two-loop RCs compared with literature
- UV poles cancel in the benchmark processes
- NLO EW and mixed QCD-EW contributions pass gauge-parameter checks

Reusable result

Compact MI representation of full-SM two-loop EW and mixed QCD-EW renormalization constants in Feynman-'t Hooft and Landau gauges

Preliminary Numerical Results for $H \rightarrow b\bar{b}$

Contribution	width/correction [MeV]	K -factor
LO width	+5.38355944	
NLO QCD	-1.917859831	-0.356244
NLO EW	+0.1507268410	+0.0279976
NNLO mix	-0.02869168938	-0.0053295
NNLO EW (FtH)	+0.01803554076	+0.00335013
NNLO EW (LG)	+0.0206	+0.00382647

$$\Gamma = \Gamma_{\text{LO}} \left(1 + K_{\text{QCD}}^{\text{NLO}} + K_{\text{EW}}^{\text{NLO}} + K_{\text{mix}}^{\text{NNLO}} + K_{\text{EW}}^{\text{NNLO}} + \dots \right)$$

Take-home:

beyond-NLO EW pieces are small in absolute size, but already live at the sub-percent level of the Higgs width.

Explicit gauge dependence of width in traditional on-shell scheme.

Benchmark process setup

- On-shell renormalization scheme.
- $\alpha = \alpha(0) = \frac{1}{137}$.
- $\alpha_s = \alpha(M_H) = 0.112677$.
- $m_b = m_b^{\text{os}} = 4.78 \text{ GeV}$.
- All particles masses are taken from PDG.

Gauge benchmark

- Feynman-'t Hooft:
 - $\Delta\Gamma = 0.01804 \text{ MeV}, \quad K = 0.00335$
- Landau:
 - $\Delta\Gamma = 0.0206 \text{ MeV}, \quad K = 0.00383$

Implications and Interpretation

Take-home message

- NLO QCD gives the dominant contribution.
- **Mixed QCD-EW and pure NNLO EW terms are both sub-percent.**
- **Their sizes are comparable to high-order pure-QCD corrections** such as $O(\alpha_s^3)$ and $O(\alpha_s^4)$ effects in the $\overline{\text{MS}}$ -scheme phenomenology.

Important lesson @ NNLO

- ***Mixed QCD-EW and NNLO EW partially cancel*** in this setup.
- This cancellation is not an a priori assumption, but an outcome observed after the calculation.
- Computing ***only the NNLO mixed or the pure EW term is not sufficient for*** a reliable sub-percent width prediction.

Precision motivation

- To get a reliable result for the Higgs decay width at sub-percent-level precision, both NNLO mixed QCD-EW and pure NNLO EW effects have to be controlled.
- The current $\alpha(0) + m_b^{\text{OS}}$ setup is a benchmark validation of the full-SM two-loop EW framework before moving to the final phenomenological setup.

Summary and Outlook

Summary

- ▶ A complete two-loop on-shell EW renormalization setup has been constructed in the full Standard Model.
- ▶ A complete two-loop on-shell EW renormalization setup has been constructed in the full Standard Model.
- ▶ The framework has been applied to $H \rightarrow b\bar{b}$ as a first benchmark process.
- ▶ Mixed QCD-EW and pure NNLO EW effects are sub-percent but relevant for Higgs-width precision.

Outlook

- ▶ Release the compact two-loop EW renormalization constants soon.
- ▶ Move toward G_μ -type input schemes and improved bottom-Yukawa treatment.
- ▶ Quantify gauge dependence before quoting the final precision prediction.
- ▶ **Apply the framework to $t \rightarrow Wb$: mixed QCD-EW corrections will be discussed in H.-Y. Han's talk, while the pure NNLO EW part will be released soon.**
- ▶ Longer-term target: extend the framework toward more challenging $2 \rightarrow 2$ scattering processes.

Thanks for your attention.