NNLO mixed QCD-EW corrections to $t \to Wb$ process

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Based on the on-going works

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Inclusive $\Gamma(t \to Wb)$: Historical Calculations

Table: Inclusive top quark decay width: corrections from LO to N^{3}LO

Order	QCD	Mixed (QCD+EW)	Pure EW
NLO	-4.5% to -5% [Smith,Berman 85; Czarnecki 91]		$+0.5\%$ to $+1\%$ [Sirlin 85; Denner,Sack 91]
NNLO	-2.1% to -2.3% [Melnikov,Schulze 09; Chen,Guan,Ma 22]	*This work*	—
N^{3}LO	-0.6% to -0.7% [Chen,Guan,Ma 23; Yan,Wu.et 24]	—	—

Key Points:

- QCD corrections dominate at each order; corrections decrease with increasing order
- Mixed corrections at NNLO are comparable to N^{3}LO scale uncertainty
- Pure EW corrections at NLO are small but non-negligible

How about NNLO QCD-EW correction?

Why we need NNLO mixed corrections for top quark decay

1. Theoretical Precision:

- N^3LO QCD corrections reach **sub-percent** precision ($\pm 0.2\%$)
- NNLO EW-QCD mixed corrections estimated at 10^{-3}–10^{-2} (also sub-percent!)
- To achieve complete sub-percent theory precision $\Rightarrow$ need NNLO EW-QCD prediction

2. Experimental Precision:

- Future colliders will achieve sub-percent measurement precision
- HL-LHC: 4000 fb^{-1} $\Rightarrow$ $\sim$1% on $|V_{tb}|$
- FCC-ee: Clean environment, threshold scan $\Rightarrow$ $\sim$0.01% precision
- CEPC, ILC also probe top with high precision

Key Point

Completing NNLO EW-QCD calculation is essential for: (1) Complete sub-percent theory precision, (2) Meet the precision of future colliders, (3) Testing Standard Model correctness

Refs: HL-LHC: ATLAS-LHCb-TOP-CONF-2023; FCC-ee: arXiv:1905.03771; Top width: Phys. Rev. D 108 (2022) 054003

Challenges in calculating EW corrections

Why truncate at W (not full leptonic state)?

If we calculate $t \to W(\to \ell\nu)b$ with two leptons:

- Two-loop four-point diagrams required
- EW reactions involve Goldstone bosons and Ghosts
- Using t'Hooft-Feynman Gauge: massive Goldstone/Ghost integrals
- Using Landau Gauge: more tensor structures $\Rightarrow$ more tensor integrals

$\Rightarrow$ Much more computationally intensive

Three Major Challenges:

1. **Unstable External States**
   Unstable particles (t, W) cannot be treated as asymptotic states in conventional perturbation theory

In LSZ formula:

$$\tilde{Z} \lim_{s \to m_{os}} \frac{s - m_{os}^2}{s - m_{os}^2 + \Sigma_R(s)}$$

$$M_0^2 = m_{os}^2 + \delta m^2 \qquad \psi_B = Z_\psi^{1/2} \psi_R$$

1. **Maintaining Unitarity**
   S-matrix must remain unitary: $S^\dagger S = 1$

1. **Gauge Invariance**
   $SU(2)_L \times U(1)_Y$ gauge symmetry must be preserved at all orders

Refs: Willenbrock & Valencia (1991); Sirlin (1991); Kniehl & Sirlin (2002)

Comparison of Renormalization Schemes

Pole Scheme	Stuart (1991), Aeppli et al. (1993), Dittmaier and Schwan (2016)	Calculate total cross section for production-to-decay; gauge invariant
CMS	Denner et al. (1999), Denner et al. (2005), Denner and Dittmaier (2006)	Applicable to both resonance and non-resonance regions; gauge invariant
Traditional On-shell Scheme	Denner (1993)	Calculate unstable particle decay at NLO using standard on-shell renormalization
Complex Pole Scheme	Willenbrock and Valencia (1991), Sirlin (1991), Kniehl and Sirlin (2002)	Calculate decay width via imaginary part of self-energy at complex pole; gauge invariant

Complex Pole Scheme

Refs: Willenbrock & Valencia (1991), Sirlin (1991), Kniehl & Sirlin (2002),Dubovyk, et al.(2018),Willenbrock (2025)

Complex Pole Definition: Consider a $e^+e^- \to (Z/\gamma) \to$ (*final states*) 4-point amplitude with Dyson-resummed intermediate Z propagator, it acquires a pole in complex plane

$$\mathcal{M} \sim \frac{1}{s-\mu^2}$$

Then define this pole as:

$$\bar{s} = \left(m_1 - i\frac{\Gamma_1}{2}\right)^2 = m_2^2 - im_2\Gamma_2 \ , \qquad (1)$$

Which is **Gauge Invariant** the relation between Gauge invariant

mass and observable mass at some approximation are:

$$m_2 = M_{ob}/\sqrt{1+\Gamma_{ob}^2/M_{ob}^2} \ ; \quad \Gamma_2 = \Gamma_{os}/\sqrt{1+\Gamma_{ob}^2/M_{ob}^2} \qquad (2)$$

The renormalized Green function is

$$\begin{aligned} G_R^{2-point}(s) &= \frac{1}{Z(s-M_0^2+\Sigma_B(s))} \\ &= \frac{1}{Z(s-m_2^2-\delta m_2^2+\Sigma_B(s))} \end{aligned} \qquad (3)$$

We choose renormalization condition as:

$$\delta m_2^2 = \mathrm{Re}\Sigma_B(s)\Big|_{s=\bar{s}}$$

So that when $s=\bar{s}$, via Denominator of $G_R^{2-point}(\bar{s})$, we have the decay width

$$\Gamma_2 = \mathrm{Im}\left[\Sigma_B(\bar{s})\right]/m_2 \qquad (4)$$

Physical Significance and problems:

- Width: $\Gamma = \mathrm{Im}\,\Sigma(\bar{s})/m_2$ when expanded around the real momentum input $s=m_2^2$, possesses Pole. *Kniehl Sirlin (2002)*
- Complexity in Unstable fermion(SM is parity-nonconserving). *Kniehl&Sirlin (2008),Kniehl (2014)*

On-shell Scheme

Denner (1993)

Key Assumption: Since decay time $\tau_{\text{decay}} \ll$ lifetime τ of unstable particle(top and W), we treat **top quark and W** as having an **Asymptotic state** and can be used as external state.

- Follow the traditional on-shell presciption used for stable particle to Process the unstable particle renormalization
- For propagator of unstable particle, take the pole of the real-part of the Demominator as renormalized mass

On-shell Renormalization Conditions:

$$\mathrm{Re}\,\Sigma(p^2)\Big|_{p^2=M_{os}^2} = 0 \quad \text{(mass)} \tag{5}$$

$$\mathrm{Re}\,\frac{d\Sigma(p^2)}{dp^2}\Big|_{p^2=M_{os}^2} = 0 \quad \text{(wave function)} \tag{6}$$

Mass Counterterm definition:

$$M_0^2 = M_{os}^2 + \delta M^2 \tag{7}$$

$$\delta M^2 = \mathrm{Re}\,\Sigma_B(M_{os}^2) \tag{8}$$

Field Renormalization:

$$\psi_0 = \sqrt{Z_2}\,\psi \tag{9}$$

$$Z_2 = \frac{1}{1 + \mathrm{Re}\,\frac{d\Sigma_B(p^2)}{dp^2}\Big|_{p^2=M_{os}^2}} \tag{10}$$

Key Features:

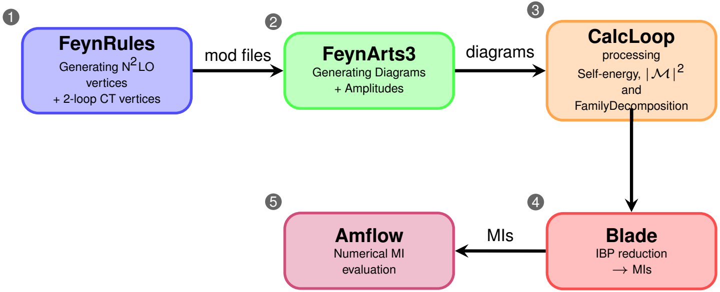
- Simple and straightforward implementation
- Renormalization of unphysical particle such as Ghost and GoldStone do not influence Final result
- Closely follows stable particle renormalization
- Works well at NLO(Gauge invariant)

possible issue

may break gauge invariance at NNLO.

Calculation Workflow

Calculation tools for NNLO mixed QCD-EW corrections



FeynRules
Christensen *et al.*,
Comput. Phys. Commun. 181 (2010)

FeynArts3
Hahn*et al.*
Comput. Phys. Commun. 140 (2001)

CalcLoop
Yan-Qing.Ma,
https://gitlab.com/multiloop-pku/calcloop

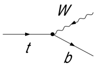
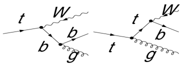
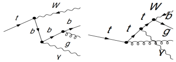
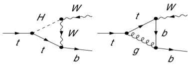
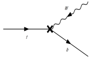
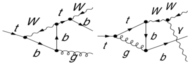
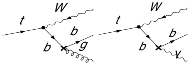
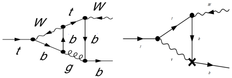
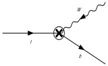
Blade
Guan *et al.*,
arXiv:2405.14621

Amflow
Liu & Ma,
Comput. Phys. Commun. 283 (2023)

Workflow

Deriving Feynman Rules → Diagram generation → Amplitude processing→ FamilyDecomposition → IBP reduction → Numerical evaluation

Feynman Diagrams in This Calculation

Loop Order	Virtual	Real-emission 1 particle	Real-emission 2 particle
Tree	EW tree graph: 1 	emission 1 gluon tree: 2 	emission 1gluon 1photon: 10 
1L-Order	EW bare: 18 QCD bare: 1  CT Vertex: 1 	em-gluon 1l: 94 em-photon 1l: 10  gluon ct: 6 photon ct: 14 	—
2L-Order	mix bare: 112 1l×1lct(All): 154  CT tree: 1 	—	—

Preliminary Numerical Results: Decay Width Corrections

Table: **Preliminary** Decay width corrections and K-factors for $t \to Wb$ at different orders (in GeV)

	t'Hooft Feynman Gauge		Landau Gauge	
	decay width correction	**K-factor**	**decay width correction**	**K-factor**
LO	$\Gamma_{LO} = 1.421003090$			
NLO QCD	-0.1331689986	-0.09371478465	-0.1331689986	-0.09371478465
NLO EW	$+0.07752185511$	$+0.05455431847$	$+0.07752185511$	$+0.05455431847$
NNLO mix	$+0.00009596167623$	$+0.00006753094129$	$+0.00009596167623$	$+0.00006753094129$

Parameters used in t'Hooft-Feynman Gauge and Landau Gauge:

$m_t = 172.4$ GeV $\quad m_b = 4.78$ GeV $\quad M_W = 80.3692$ GeV $\quad \alpha = 1/137 \quad \alpha_s = 0.119$

Table: Counter term comparison between t'Hooft-Feynman Gauge and Landau Gauge: At 1-loop Order first 3 same (✓), rest differ (×)

	dZe1	dSW1	dCW1	dMWsq1	dZW1	dMZsq1	dZZZ1	dZAZ1	dZZA1	dZAA1	dZtL1	dMt1	dZbL1	dZbR1	dMb1
comparing res	✓	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×

2-loop cts between these 2 gauge are totally different

Analysis, Summary and Outlooks

	decay width correction	K-factor	(%)
LO		$\Gamma_{LO} = 1.421003090$	
NLO QCD	-0.1331689986	-0.09371478465	-9.37
NLO EW	$+0.07752185511$	$+0.05455431847$	$+5.46$
NLO total	-0.05564714349	-0.03916046618	-3.92
NNLO mix	$+0.00009596167623$	$+0.00006753094129$	$+0.0068$

Preliminary Analysis:

1. NLO corrections vs. literature:

- NLO QCD and NLO EW individually slightly larger than literature values
- Sum of QCD + EW $\approx$ literature NLO total correction
- Source: different input parameters, α scheme (not G_μ scheme), and non-perturbative effects not included

2. NNLO mix $\Rightarrow$ gauge invariant? Two interpretations:

(a) Our scheme may have gauge-dependent mass; however,ours using of parameters leads to cancellation between mass difference (gauge-dependent) and width correction

(b) Mixed correction itself may be gauge invariant; only pure EW corrections show gauge dependence — consistent with $H \to b\bar{b}$ observations

Summary and Outlooks

- Using traditional On-shell scheme to calculate NNLO t $\to$ W + b inclusive decay width
- Our result for mixed is much smaller than usual estimates: $\mathcal{O}(K_{\text{QCD}} \times K_{\text{EW}})$
- Needs more Validation
- If validated, it means $\to K_{QCD} + K_{EW}$ gives a better approximation than $K_{QCD} \times K_{EW}$.

Thanks for your attention!!!

Any Questions?