Two-Loop Renormalization-Group Evolution for the Nucleon Distribution Amplitude

Based on arXiv: 2512.20471, with Yong-Kang Huang, Yao Ji, Yu-Ming Wang

施博轩

The 8th Workshop on Heavy Flavor Physics and QCD

Nankai University, 2026 年 4 月 26 日

Background & Motivations

Backgrounds

- LCDAs are essential non-perturbative inputs for hard exclusive processes in QCD, revealing hadron structure.

$$F_1(Q^2) = \frac{(4\pi\alpha_s)^2}{Q^4} \int [\mathcal{D}x] \int [\mathcal{D}y]\ \mathbb{T}_N(x_i, y_i, Q^2, \mu_F)\, \varphi_N(x_i, \mu_F)\ \varphi_N(y_i, \mu_F)\,,$$

- RG evolution of LCDAs are the key ingredient for resummation and the demonstration of the QCD factorization.
- Presence of a finite mass gap in anomalous dimensions.

V. M. Braun, S. E. Derkachov, G. P. Korchemsky, and A. N. Manashov, Nucl. Phys. B 553, 355 (1999), arXiv:hep-ph/9902375

Status of perturbative calculations

- One-loop evolution kernel known for more than 40 years.
- Two-loop RG evolution for light meson LCDAs have been available since the 1980s.
- Missing two-loop RG evolution kernel for nucleon LCDA.

Goal of This Work

- Compute the two-loop (NLO) evolution equation for nucleon LCDA in the evanescent operator scheme.
- Establish matching to the KM scheme.
- Construct explicit solution via conformal expansion in moment space.
- Enable phenomenological applications with improved precision.

Nucleon LCDA

- The leading-twist nucleon LCDA

$$
\begin{aligned}
&\left\langle 0 \left| \epsilon_{ijk} \left[q^{\uparrow}_{1,i}(\tau_1\, n)\; C \not{n}\; q^{\downarrow}_{2,j}(\tau_2\, n) \right] \not{n}\, q^{\uparrow}_{3,k}(\tau_3\, n) \right| N^{\uparrow}(P_1) \right\rangle \\
&= -\frac{f_N(\mu_F)}{2} (n\cdot P) \not{n}\, N^{\uparrow}(P_1) \int [\mathcal{D}u] \exp\left[-i\, n\cdot P_1 \sum_i u_i\, \tau_i \right] \varphi_N(u_i,\mu_F)\,,
\end{aligned}
$$

- Operator basis at two-loop

$$
\begin{aligned}
\mathcal{O}_1 &= [u^{\uparrow}\, C \not{n} u^{\downarrow}]\; \not{n} d^{\uparrow}\,, \\
\mathcal{O}_2 &= [u^{\uparrow}\, C \gamma_{\perp\alpha}\gamma_{\perp\beta} \not{n} u^{\downarrow}]\; \not{n} \gamma^{\beta}_{\perp}\gamma^{\alpha}_{\perp}\, d^{\uparrow}\,, \\
\mathcal{O}_3 &= [u^{\uparrow}\, C \gamma_{\perp\alpha}\gamma_{\perp\beta}\gamma_{\perp\rho}\gamma_{\perp\tau} \not{n} u^{\downarrow}]\; \not{n} \gamma^{\tau}_{\perp}\gamma^{\rho}_{\perp}\gamma^{\beta}_{\perp}\gamma^{\alpha}_{\perp}\, d^{\uparrow}\,,
\end{aligned}
$$

  $\mathcal{O}_1$ is physical, and $\mathcal{O}_{2,3}$ are evanescent and vanish in $D = 4$.

- By requiring that the renormalized evanescent operator vanishes in $D = 4$, we can constrain its renormalization scheme.

Renormalization Group Equation

- Renormalization of operators

$$\widetilde{\mathcal{O}}_1^{\rm ren}(x_i,\mu) = \sum_{k=1,2,3} \int [\mathcal{D}x'] \, \mathbb{Z}_{1k}(x_i, x_i', \mu) \, \widetilde{\mathcal{O}}_k^{\rm bare}(x_i') \, .$$

- RG equation:

$$\frac{d}{d\ln\mu} \widetilde{\mathcal{O}}_1^{\rm ren}(x_i,\mu) + \int [\mathcal{D}x'] \; \mathbb{H}(x_i, x_i', \mu) \, \widetilde{\mathcal{O}}_1^{\rm ren}(x_i', \mu) = 0 \, .$$

  where $\mathbb{H}$ is the evolution kernel.

- Anomalous dimension

$$\mathbb{H}(x_i, x_i', \mu) = \sum_{k=1,2,3} \int [\mathcal{D}x''] \, \mathbb{Z}_{1k}(x_i, x_i'', \mu) \, \frac{d\mathbb{Z}_{k1}^{-1}(x_i'', x_i', \mu)}{d\ln\mu} \, .$$

- Master formulae

$$\begin{aligned} \mathbb{H}^{(0)} =& 2\, \mathbb{Z}_{11}^{(1,1)} \, , \\ \mathbb{H}^{(1)} =& 4\, \mathbb{Z}_{11}^{(2,1)} - 2\, \mathbb{Z}_{12}^{(1,1)} \otimes \mathbb{Z}_{21}^{(1,0)} . \end{aligned}$$

- The evolution kernel $\mathbb{H}$ does not depend on $\mathcal{O}_3$.

Computation

- The correlation function

$$\langle \mathcal{O}_i \rangle \equiv \langle 0 | \mathcal{O}_i(x_1, x_2, x_3) | q_1(x_1') q_2(x_2') q_3(x_3') \rangle$$
$$= \sum_\ell \left(\frac{Z_{\alpha_s} \alpha_s}{4\pi} \right)^{(\ell)} \sum_j A_{ij}^{(\ell)} \langle O_j \rangle^{(0)} .$$

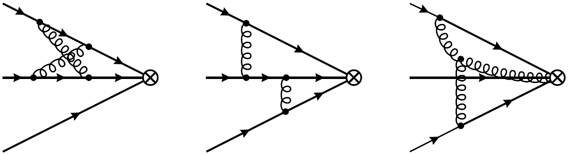
- Master formulae for $\mathbb{Z}_{ij}$

$$\mathbb{Z}_{ij}^{(1)} = - \left[A_{ij}^{(1)} \right]_{\text{div}} ,$$
$$\mathbb{Z}_{ij}^{(2)} = - \left[A_{ij}^{(2)} + Z_{ik}^{(1)} \otimes A_{kj}^{(1)} + Z_{\alpha_s}^{(1)} A_{ij}^{(1)} \right]_{\text{div}} .$$

- We need: $A_{11}^{(2)}, A_{ij}^{(1)}$.
- UV divergences: dimensional regularization.
- IR divergences: regulated by a finite mass m_{IR} for all internal quarks and gluons.

Computation

- 70 diagrams in a general covariant gauge.
  Sample diagrams:



(a) (b) (c)

- 15 of them do not contribute due to:
  - Vanishing color factor.
  - Containing only $1/\epsilon^2$ divergences.
  - Purely evanescent.
- Standard reduction of the Feynman amplitudes:
  - PV reduction to handle Dirac algebras and retain independence structure in D-dimension.
  - 11 operators found for open spinors, 3 operators leaves after projection in the nucleon case.
  - We identified 17 families and 20 master integrals after IBP reduction.

Computation

- Sample amplitude

$$A = \int [d\ell_1][d\ell_2] F(\ell_1, \ell_2, p_1, p_2) \frac{1}{n\cdot \ell_1} [c_1 \delta(X_3 + n\cdot\ell_1)\delta(X_1 - n\cdot\ell_1 - n\cdot\ell_2) + (c_2 - c_1)\delta(X_3)\delta(X_1 - n\cdot\ell_1 - n\cdot\ell_2) - c_2\delta(X_3)\delta(X_1 - n\cdot\ell_2)] \,.$$

- The discontinuity

$$\delta(X_i - n\cdot\ell) = \frac{1}{i\pi}\left[\frac{1}{X_i - n\cdot\ell - i0} - \frac{1}{X_i - n\cdot\ell + i0}\right] = \mathrm{Disc}_{X_i}\left[\frac{1}{X_i - n\cdot\ell}\right] \,.$$

- Recast the amplitude

$$A = -\, c_1 \left[\frac{1}{X_3}\mathrm{Disc}_{X_{1,3}}\left\{\int [d\ell_1][d\ell_2] F(\ell_1, \ell_2, p_1, p_2) \frac{1}{X_3 + n\cdot\ell_1}\frac{1}{X_1 - n\cdot\ell_1 - n\cdot\ell_2}\right\}\right]_{+,X_3} + c_2\delta(X_3)\left[\mathrm{Disc}_{X_1}\left\{\int [d\ell_1][d\ell_2] F(\ell_1, \ell_2, p_1, p_2) \frac{1}{X_1 - n\cdot\ell_1 - n\cdot\ell_2}\frac{1}{X_1 - n\cdot\ell_2}\right\}\right] \,.$$

Anomalous Dimension in Evanescent Operator Scheme

With all pieces at hand, we finally arrive at our desired anomalous dimension in the evanescent operator scheme (EO scheme)

$$\begin{aligned}\mathbb{H}^{(1)} =& \Bigg\{ \left(\frac{C_F}{N_c-1}\right) C_A \left[\mathbb{V}_{\text{LC}}^{(1),\, C_F C_A}(\chi_i)\,\delta(x_1-x_1')\,\delta(x_2-x_2') + \mathbb{V}_{\text{2P}}^{(1),\, C_F C_A}(x_1,x_2,x_1',x_2',\chi_i)\,\delta(x_3-x_3')\right] \\ &+ \left(\frac{C_F}{N_c-1}\right) \beta_0 \left[\mathbb{V}_{\text{LC}}^{(1),\, C_F \beta_0}(\chi_i)\,\delta(x_1-x_1')\,\delta(x_2-x_2') + \mathbb{V}_{\text{2P}}^{(1),\, C_F \beta_0}(x_1,x_2,x_1',x_2',\chi_i)\,\delta(x_3-x_3')\right] \\ &+ \left(\frac{C_F}{N_c-1}\right)^2 \Bigg[\mathbb{V}_{\text{LC}}^{(1),\, C_F^2}(\chi_i)\,\delta(x_1-x_1')\,\delta(x_2-x_2') + \mathbb{V}_{\text{2P}}^{(1),\, C_F^2}(x_1,x_2,x_1',x_2',\chi_i)\,\delta(x_3-x_3') \\ &\qquad + \mathbb{V}_{\text{3P}}^{(1),\, C_F^2}(x_i,x_i',\eta_i,\kappa_i,\chi_i)\Bigg]\Bigg\} + \{x_1 \leftrightarrow x_3,\, \chi_i \leftrightarrow \eta_i\} + \{x_2 \leftrightarrow x_3,\, \chi_i \leftrightarrow \kappa_i\}\,.\end{aligned}$$

- $\mathbb{V}_{\text{2P}}$ have similar structure with ERBL kernels.
- $\mathbb{V}_{\text{3P}}$ is typical three-particle term.
- The parameter sets $\{\eta_i,\, \chi_i,\, \kappa_i\}$ come from symmetry breaking induced by the projection.
- We have verified the result by reproducing some existing results, including
  - The leading-twist pion LCDA.
  - Transversely polarized ρ-meson $\Phi_\perp$.
  - First few local anomalous dimensions.

Anomalous Dimension in KM Scheme

- The operator is defined with open spinors

$$\mathcal{O}^{\alpha\beta\gamma}_{\rm bare}(x_i) = u^{\alpha}(x_1)\, u^{\beta}(x_2)\, d^{\gamma}(x_3)\,.$$

- The renormalized finite operator is defined as

$$\mathcal{O}^{\alpha\beta\gamma} = \mathcal{Z}^{\alpha\beta\gamma}_{\alpha'\beta'\gamma'} \otimes \mathcal{O}^{\alpha'\beta'\gamma'}_{\rm bare}\,.$$

- 11 Dirac basis under D-dimension.
- The renormalized physical operator is given by projection under 4-dimension.
- The matching between EO-scheme and KM-scheme

$$\Phi^{\rm KM}_{N}(x_i,\mu) = \int [\mathcal{D}x'_i]\; \mathbb{K}_N(x_i, x'_i, \mu)\; \Phi^{\rm EO}_{N}(x'_i,\mu),$$

$$\mathbb{K}_N(x_i, x'_i, \mu) = \delta(x_1 - x'_1)\, \delta(x_2 - x'_2) + \sum_{m=1}^{\infty} \left(\frac{\alpha_s(\mu)}{4\pi}\right)^m \mathbb{K}^{(m)}_N(x_i, x'_i)\,.$$

Anomalous Dimension in KM Scheme

- NLO matching kernel (sufficient for two-loop matching)

$$\mathbb{K}_N^{(1)}(x_i, x_i') = \left(\frac{C_F}{N_c - 1}\right) \Bigg\{ \left[\frac{3}{x_1' + x_2'} \left(\frac{x_1}{x_1'}\,\theta(x_1' - x_1) + \frac{x_2}{x_2'}\,\theta(x_2' - x_2)\right)\right]_+ \delta(x_3 - x_3') + \left[\frac{1}{x_2' + x_3'} \left(\frac{x_2}{x_2'}\,\theta(x_2' - x_2) + \frac{x_3}{x_3'}\,\theta(x_3' - x_3)\right)\right]_+ \delta(x_1 - x_1') - \left[\frac{1}{x_1' + x_3'} \left(\frac{x_1}{x_1'}\,\theta(x_1' - x_1) + \frac{x_3}{x_3'}\,\theta(x_3' - x_3)\right)\right]_+ \delta(x_2 - x_2') + \frac{3}{2}\,\delta(x_1 - x_1')\,\delta(x_2 - x_2') \Bigg\}.$$

- NNLO matching kernel (for three-loop matching)

$$\mathbb{K}_N^{(2)}(x_i, x_i') = \frac{1}{4} \left\{ \left[\mathbb{H}^{(1)}(x_i, x_i')\,\big|_{\eta_i \to \eta_i',\ \kappa_i \to \kappa_i',\ \chi_i \to \chi_i'}\right] - \left[\mathbb{H}^{(1)}(x_i, x_i')\,\big|_{\eta_i \to 0,\ \kappa_i \to 0,\ \chi_i \to 0}\right] \right\} - \left(\frac{C_F}{N_c - 1}\right)^2 \Bigg\{ \Bigg[\frac{7}{2}\,\frac{1}{x_1' + x_2'} \Bigg(\left(\frac{x_1}{x_2'}\,\ln\frac{x_1'}{x_1' + x_2'} + \frac{x_2}{x_1'}\,\ln\frac{x_2}{x_1' + x_2'} + \frac{x_1}{x_1'}\right) \theta(x_1' - x_1) + \left(\frac{x_2}{x_1'}\,\ln\frac{x_2'}{x_1' + x_2'} + \frac{x_1}{x_2'}\,\ln\frac{x_1}{x_1' + x_2'} + \frac{x_2}{x_2'}\right) \theta(x_2' - x_2) \Bigg) \delta(x_3 - x_3') \Bigg] + \left[x_1 \leftrightarrow x_3,\ x_1' \leftrightarrow x_3'\right] - \frac{1}{7}\left[x_2 \leftrightarrow x_3,\ x_2' \leftrightarrow x_3'\right] + \Delta\mathbb{K}_N^{(2)}(x_i, x_i') \Bigg\}.$$

ADMs in Moment Space

- Conformal expansion of φ_N

$$\varphi_N(x_i,\mu) = x_1\,x_2\,x_3 \sum_{M=0}^{\infty}\sum_{m=0}^{M} \mathcal{N}_{Mm}\,\Psi_{Mm}(\mu)\,\mathcal{P}_{Mm}(x_i)\,.$$

- The RGE of the moments $\Psi_{Mm}(\mu)$

$$\frac{d}{d\ln\mu}\varphi_N + \mathbb{H}\otimes\varphi_N = 0 \qquad \Rightarrow \qquad \sum_{Q=0}^{M}\sum_{q=0}^{Q}\left[\frac{d}{d\ln\mu}\delta_{MQ}\delta_{mq} + \Gamma_{Mm,\,Qq}(\mu)\right]\Psi_{Qq}(\mu) = 0\,,$$

where the non-diagonal anomalous dimension matrix $\Gamma_{Mm,Qq}(\mu)$ can be given by

$$\Gamma_{Mm,\,Qq}(\mu) = \mathcal{N}_{Qq}\int[\mathcal{D}x]\int[\mathcal{D}x']\,x_1'\,x_2'\,x_3'\,\left[\mathcal{P}_{Mm}(x_i)\,\mathbb{H}(x_i,x_i',\mu)\,\mathcal{P}_{Qq}(x_i')\right]\,.$$

- We show the ADM for $M \leq 3$ in this work.
- In principle, one can obtain $\Gamma_{Mm,Qq}$ for arbitrary entry $\{Mm,\,Qq\}$.

ADMs in Moment Space

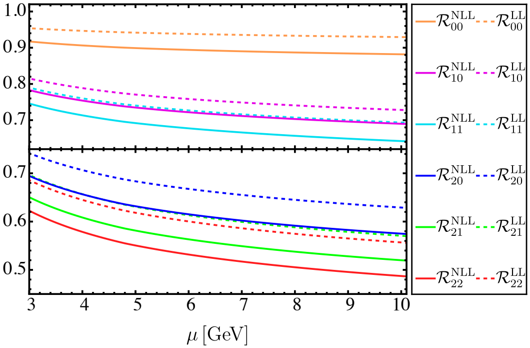
The general solution to the RGE can be cast in the form of

$$\Psi_{Mm}(\mu) = \sum_{Q=0}^{M} \sum_{q=0}^{Q} \left[\mathbb{U}(\mu,\, \mu_0)\right]_{Mm,\, Qq} \Psi_{Qq}(\mu_0)\,,$$
$$\mathbb{U}(\mu,\, \mu_0) = \mathbb{T}_\mu \exp\left[-\int_{\mu_0}^{\mu} \frac{d\nu}{\nu}\, \Gamma(\nu)\right]\,.$$

In the NLL approximation

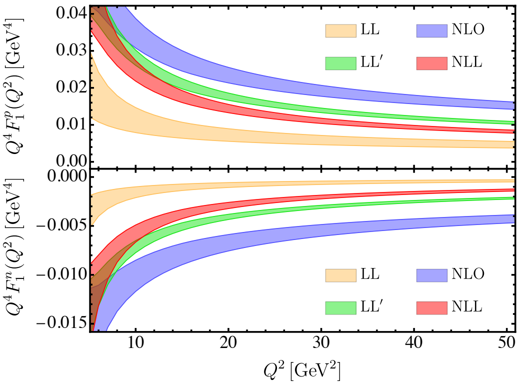
$$\mathbb{U}^{\mathrm{NLL}}(\mu,\, \mu_0) = \left(\mathbb{I} - \frac{\alpha_s(\mu)}{4\,\pi}\mathbb{J}^{(1)}\right)\left[\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\Gamma^{(0)}}{2\,\beta_0}}\right]\left(\mathbb{I} + \frac{\alpha_s(\mu_0)}{4\,\pi}\,\mathbb{J}^{(1)}\right)\,.$$

Numerical Implications



- NLL corrections to Ψ_{Mm} significantly affect LL predictions at intermediate scales, with numerical impacts around $\mathcal{O}(20\%)$.
- Two-loop QCD evolution for the nucleon distribution amplitude induces larger effects than for the π-meson and B-meson leading-twist amplitudes.
- Higher conformal spins experience more pronounced corrections from NLL evolution.

Numerical Implications



- The perturbative uncertainties from varying the renormalization and factorization scales in the preferred intervals, $\nu^2 = \mu^2 = \langle x \rangle Q^2$ with $1/6 \leq \langle x \rangle \leq 1/2$.
- Numerical effect of NLL resummation compared with LL' is about $\mathcal{O}(10\%) \sim \mathcal{O}(20\%)$ in $Q^2 \in \{5, 50\}\mathrm{GeV}^2$.

Thank you for your attention!