



# Compton Scattering Total Cross Section at NNLO and Resummation of LL

Cheng-Tai Tan

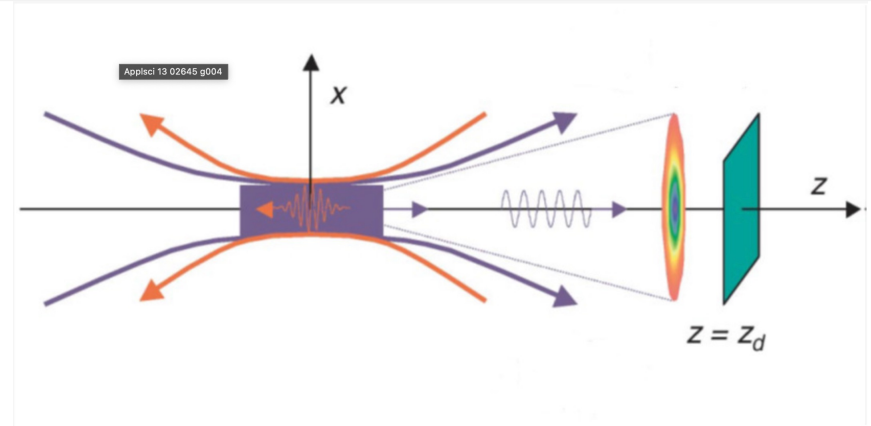
@ Chongqing 第八届全国重味物理与量子色动力学研讨会

Based on PRL 136.021802(2026)

In collaboration with Hai Tao Li, Yan-Qing Ma, Jian Wang and Hong-Fei Zhang

# Background

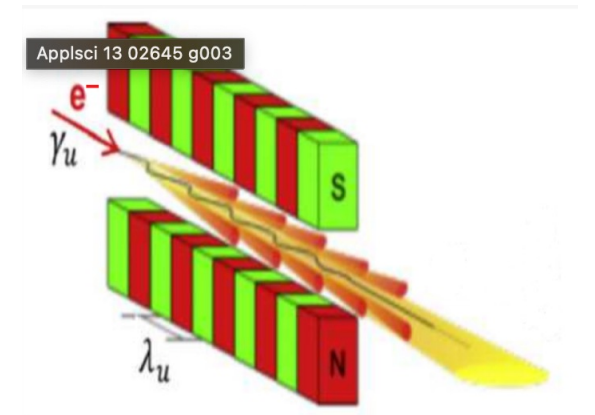
- Compton scattering  $e^- + \gamma \rightarrow e^- + \gamma + X$
- Long history: Dates back to more than a century ago.
- Broad applications: medical physics, astronomy,
- Particle colliders: FCC, ILC, CEPC and muon collider.
- Clean benchmark, Beam diagnostics, luminosity measurements and polarization monitoring



- **Double Logarithms** at NLO: (R.N.Lee, M.D.Schwartz, X.Y.Zhang 2021)

$$\sigma_{\text{tot}} \sim \frac{2\pi\alpha^2}{s} \ln\left(\frac{s}{m^2}\right) \left[1 + \frac{\alpha}{6\pi} \ln^2\left(\frac{s}{m^2}\right) + \dots\right]$$

- 8.2% at  $\sqrt{s} = 1\text{GeV}$  and 30% at  $\sqrt{s} = 1\text{TeV}$ .
- Does the NNLO contribution still offer a significant improvement?
- What is the origin of these double logarithms and how can one resum them to all orders?



# Double Logs and Regge Limit

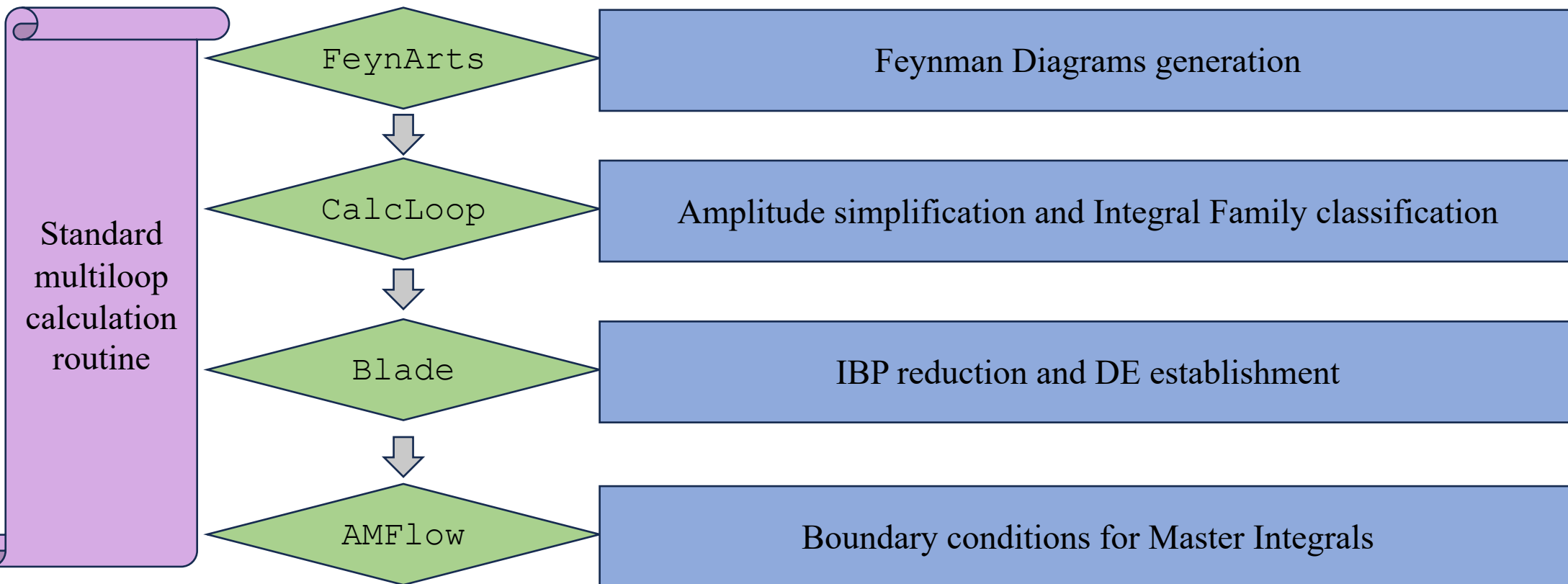
- The logarithm at Leading Order originates from the limit of **backward scattering**, corresponding to  $s \gg |t|$  (Regge limit).
- Origin of double logs?
  - Soft-collinear overlapping singularities: cancelled due to BN theorem.
  - Connection to DGLAP or BFKL: The LL in Regge Limit of QCD gluon is well-studied, which takes the form  $[\alpha_s \ln(s/(-t))]^n$ , is not applicable to the **double log structure** in QED Compton scattering.
- Resummation of double logs at amplitude level (V.G.Gorshkov, V.N.Gribov, L.N. Lipatov, G.V.Frolov 1966,1967)
- Resummation related to endpoint divergence
  - Bottom-quark induced  $h \rightarrow \gamma\gamma$  decay (J.Y.Hou, J.Wang, D.J.Zhang 2022, Z.L.Liu, B.Mecaj, M.Neubert, X.Wang 2019,2020 )
  - Muon-electron backward scattering (G.Bell P. Böer, T. Feldmann 2022)

$$F_1(\lambda) = \sum_{n=0}^{\infty} \left(\frac{\alpha_{\text{em}}}{2\pi}\right)^n F_1^{(n)}(\lambda) \simeq \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_{\text{em}}}{2\pi} \ln^2 \lambda^2\right)^n}{n!(n+1)!} = \frac{I_1\left(2\sqrt{\frac{\alpha_{\text{em}}}{2\pi} \ln^2 \lambda^2}\right)}{\sqrt{\frac{\alpha_{\text{em}}}{2\pi} \ln^2 \lambda^2}},$$

# NNLO Computation $e^-(p_1)\gamma(p_2) \rightarrow e^-(p_3)\gamma(p_4)\{\gamma(p_5)\gamma(p_6)\}$

QED NNLO corrections with full electron mass dependence.

No double logs with three charged particle production (R.Lee, A.Lyubiyakin, V.A.Stotsky 2020)



# Frobenius series solutions

From the differential equations of the Master Integrals(MIs)

$$\frac{d\vec{I}}{d\tau} = M(\epsilon, \tau) \cdot \vec{I}, \quad \tau = \frac{m^2}{s}$$

The solution of the MIs can be represented by piecewise **Frobenius series solutions**

$$I(\epsilon, \tau) = \sum_{a,b} \tau^a \ln^b(\tau) \left( \sum_{n=0}^{\infty} C_{abn}(\epsilon) \tau^n \right)$$

where  $a = a_0 + a_1\epsilon$ ,  $2a_0, a_1 \in \mathbb{Z}$  and  $b$  is a non-negative integer.

The coefficients are fixed by the boundary condition at  $\tau = 1/530$ , and checked at another regular point  $\tau = 49/53$ .

# Renormalization

- IR divergences cancel after real and virtual contributions are combined together.
- UV divergence is renormalized in the **on-shell** scheme.
- Two methods to double check:
  - Calculate counterterm Feynman diagrams.
  - Replace bare parameters with renormalized ones.
- Instead of reconstructing  $\epsilon$  dependence, we only take two values of  $\epsilon$ ,  
 $\epsilon = \pm 1/1000$  , resulting in an error of  $\mathcal{O}(10^{-6})$  at the cross section level.

(H.-Y. Bi et al. 2024)

# Fixed Order Results


- The cross section in the full physical region  $\tau \in (0, 1)$  is obtained in terms of a piecewise power series with up to 16 segments.

- Threshold Limit ( $x = (1 - \tau)/\tau$ )

$$\sigma(x) = \frac{\pi\alpha^2}{m^2} \left( \frac{8}{3} - \frac{8}{3}x + \frac{52}{15}x^2 - \frac{133}{30}x^3 + \frac{572}{105}x^4 \right) + \frac{\alpha^3}{m^2} x^2 \left( -\frac{16}{9} \ln x + \frac{4}{3}x \ln x - \frac{25}{9}x^2 \ln x \right. \\ \left. + \frac{7}{15} - \frac{113}{90}x + \frac{203}{125}x^2 \right) + \frac{\alpha^4}{\pi m^2} x^2 (0.814x^2 \ln^2 x + 0.797x^2 \ln x - 2.05 + 7.16x - 18.6x^2) + \mathcal{O}(x^5),$$

- High-energy limit

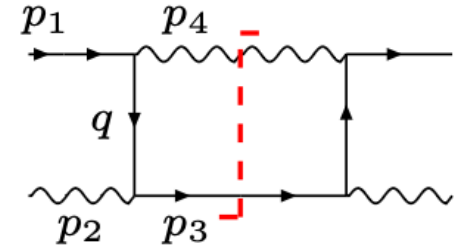
$$\sigma(\tau) = \frac{\pi\alpha^2}{s} [(2L + 1) + \tau(-6L + 17) + \tau^2(-30L + 32) + \tau^3(-70L + 48) + \tau^4(-126L + 64) + \mathcal{O}(\tau^5)] \\ + \frac{\alpha^3}{s} \left[ \left( \frac{1}{3}L^3 - \frac{1}{2}L^2 + \frac{17}{4}L - 9.5016 \right) + \tau(2L^3 + 13L^2 - 36.5340L + 0.55139) + \right. \\ \left. + \tau^2 \left( \frac{38}{3}L^3 + \frac{151}{2}L^2 - 89.091L + 47.062 \right) + \tau^3 \left( \frac{157}{3}L^3 + \frac{344}{3}L^2 - 220.07L + 149.73 \right) + \mathcal{O}(\tau^4) \right] \\ \left. + \frac{\alpha^4}{\pi s} \left[ \left( \frac{1}{48}L^5 - 0.149L^4 + 1.34L^3 - 1.17L^2 - 16.6L + 23.8 \right) \right. \right. \\ \left. \left. + \tau(1.13L^5 - 6.54L^4 - 44.6L^3 + 442L^2 - 36.0L - 2.64 \times 10^3) + \mathcal{O}(\tau^2) \right] \right]$$

$1/(96\pi^2)$  suppression 

- 0.2% at  $\sqrt{s} = 1\text{GeV}$  and 2.5% at  $\sqrt{s} = 1\text{TeV}$

# Region Analysis of the Leading Logarithms(Leading Order)

- At LO, LL comes from t-channel diagram in the backward scattering limit.



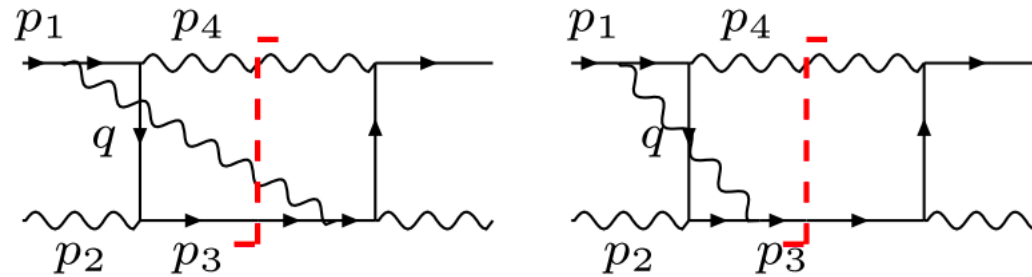
- Four regions: **hard** ( $h$ ):  $\sqrt{s}(1,1,1)$  , **collinear** ( $c$ ):  $\sqrt{s}(1,\lambda^2,\lambda)$  , **anticollinear** ( $\bar{c}$ ):  $\sqrt{s}(\lambda^2,1,\lambda)$  , **soft** ( $s$ ):  $\sqrt{s}(\lambda,\lambda,\lambda)$  where  $\lambda = \sqrt{\tau}$ .
- Endpoint divergences in the (anti)collinear and soft regions:
  - Imposing analytic regulator  $v^\eta / (n_+ q + n_- q + i0)^\eta$
  - The imaginary part of the forward scattering loop amplitude comes from the combination of hard and anticollinear region:

$$\begin{aligned} & \text{Im} \left[ \left( \frac{\mu^2}{-s - i0} \right)^\epsilon \frac{-1}{\epsilon^2} + \left( \frac{\mu^2}{m^2} \right)^\epsilon \left( \frac{\nu}{-\sqrt{s} + i0} \right)^\eta \frac{1}{\epsilon\eta} \right] \\ & = -\pi \left[ \ln \frac{\mu^2}{s} - \ln \frac{\mu^2}{m^2} \right] = -\pi \ln \tau. \end{aligned}$$

- First applying the Cutkosky rule: anticollinear mode turns to **Glauber mode** ( $G$ ):  $\sqrt{s}(\lambda^2,\lambda^2,\lambda)$

# NLO Analysis

- Double logs produced by diagrams with additional photon radiated from external electrons.

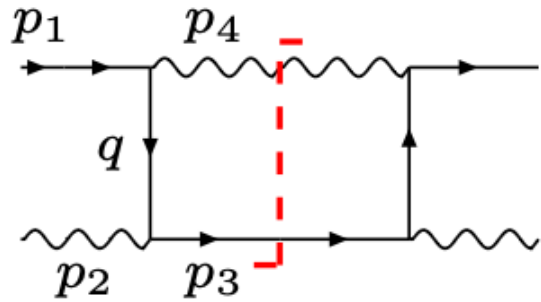


- Only uncanceled diagram contributing to LL is

$$= \frac{\alpha^3}{s} \left( \frac{L}{\epsilon} + \frac{1}{3}L^3 + \frac{1}{2}L^2 + L + \frac{5}{2} - \frac{2\pi^2}{3} - 2\zeta_3 + \mathcal{O}(\tau) \right)$$

- Relevant regions:  $(q_1, q_2) \sim (h, h), (c, h), (h, \bar{c}), (c, G), (G, \bar{c})$

# Calculation of the ladder diagrams



$$\sigma_{\text{LO,LL}} = \int d^d q \delta_+(p_3^2) \delta_+(p_4^2) \frac{e^4}{4q^4} \text{Tr} [\not{p}_1 \gamma^{\mu_1} \not{q} \gamma^{\mu_2} \not{p}_3 \gamma_{\mu_2} \not{q} \gamma_{\mu_1}]$$

$$d^d q \delta_+(p_3^2) \delta_+(p_4^2) \sim \delta_+(q^2 - 2q^-) \delta_+(q^2 + 2q^+)$$

$$q^- = \frac{1}{2} q_T^2, \quad q^+ = -\frac{1}{2} q_T^2 \quad \longrightarrow \quad \text{Glauber mode}$$

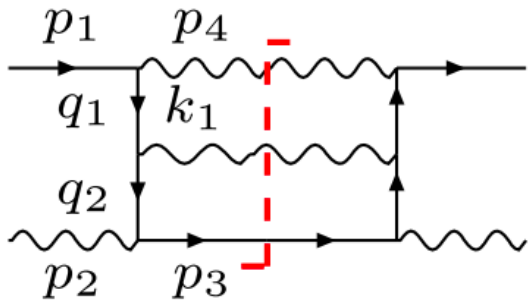
$$\sigma_{\text{LO,LL}} = \frac{2\alpha^2}{s} (2\pi) \int_{m^2}^1 \frac{d^2 q_T}{q_T^2} = -\frac{2\pi\alpha^2}{s} \ln \tau$$

Lower bound is from the propagator  $1/(q^2 - m^2)$

Similarly, the on-shell conditions at NLO leads to

$$q_1^- = \frac{1}{2} q_{1T}^2, \quad q_2^- = q_1^- + \frac{(q_{1T} - q_{2T})^2}{2(q_1^+ - q_2^+)}, \quad q_2^+ = -\frac{1}{2} q_{2T}^2$$

$$\sigma_{\text{NLO,LL}} = \frac{\alpha^3}{s} \int_{m^2}^1 dq_1^+ \int_{m^2}^{q_1^+} dq_{1T}^2 \int_{m^2}^{q_1^+} dq_{2T}^2 \frac{1}{q_{1T}^2 q_{2T}^2 (q_1^+ + \frac{1}{2} q_{2T}^2)} = -\frac{\alpha^3}{3s} \ln^3 \tau$$



# All Orders calculation

- The LL structure extends to all orders.
- Relevant regions are  $r_i = (c, \dots, c, G, \bar{c}, \dots, \bar{c})$

(J.F.Donoghue, B.K.El-Menoufi, G. Ovanesyan 2014)

- Ladder diagram with momenta satisfying strong ordering (overlap of all  $r_i$  regions)

$$p_{1+} > q_{1+} > \dots > q_{n+} > 0,$$

$$0 < -q_{1-} < \dots < -q_{n-} < p_{2-}.$$

contributes to LLs.

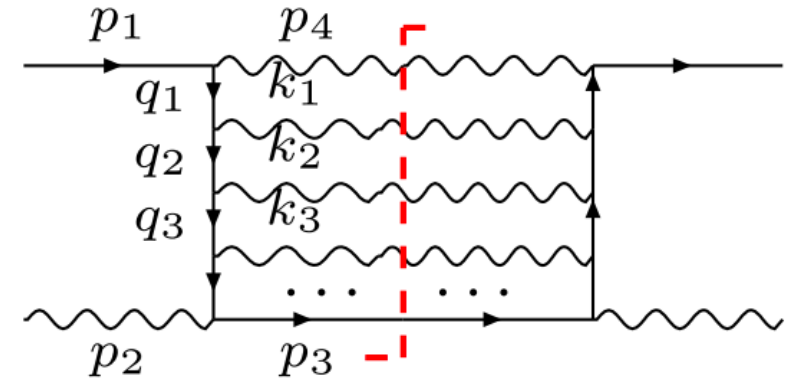
The calculation simplifies in this momentum scaling

$$\delta(p_4^2) \approx \delta((n_+ p_1)(n_- p_1 - n_- q_1) - q_{1T}^2)$$

$$\delta(k_i^2) \approx \delta(-(n_+ q_i)(n_- q_{i+1}) - (q_i - q_{i+1})_T^2)$$

$$\delta(p_3^2 - m^2) \approx \delta(n_+ q_n n_- p_2 - q_{nT}^2 - m^2)$$

$$q_i^2 - m^2 \approx -q_{iT}^2 - m^2 - \left( \frac{n_+ q_i}{n_+ q_{i-1}} \right) q_{(i-1)T}^2$$



# All Orders Calculation

- The phase space integration is simplified to

$$\sigma_{\text{LL}}^{\text{N}^{n-1}\text{LO}} = \frac{4\pi^2}{s} \frac{\alpha^{n+1}}{(2\pi)^n} \int_1^{1/\tau} \frac{dx_1}{x_1} \prod_{j=2}^{n-1} \int_1^{x_{j-1}} \frac{dx_j}{x_j} \\ \times \int_1^{x_1} \frac{dy_1}{y_1} \prod_{i=2}^{n-1} \int_{\max\{1, \frac{x_i}{x_{i-1}} y_{i-1}\}}^{x_i} \frac{dy_i}{y_i} \int_1^{x_{n-1}} \frac{dy_n}{y_n},$$

where  $x_i = n_+ q_i \sqrt{s}/m^2$ ,  $y_i = (q_{iT}^2 + m^2)/m^2$ .

The integration bound is determined by the momentum scaling and thus the **dimensional regularization is not needed**.

- The integration can be carried out by constructing a recursive relation

$$\sigma_{\text{LL}}^{\text{N}^{n-1}\text{LO}} = -\frac{2\alpha^{n+1}}{(2\pi)^{n-2} s} \frac{1}{(n-1)!(n+1)!} \ln^{2n-1} \tau.$$

Validated at N3LO

# Resummation

- The LLs can be directly resummed as

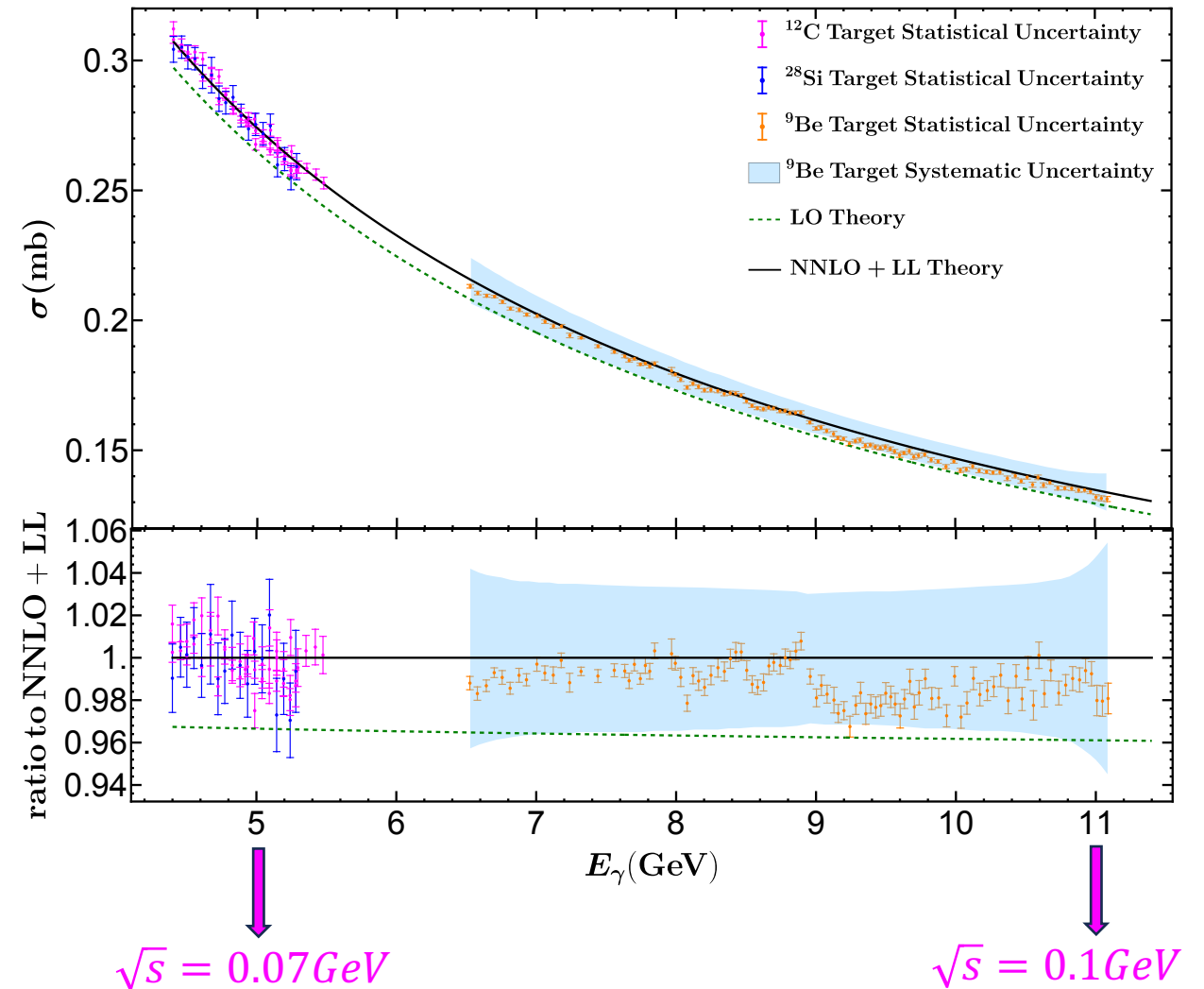
$$\begin{aligned}\sigma_{\text{LL}} &= -\frac{8\pi^2\alpha}{s \ln \tau} I_2 \left( \sqrt{\frac{2\alpha}{\pi}} \ln \tau \right) \\ &= \frac{2^{9/4} \pi^{7/4} \alpha^{3/4}}{s(-\ln \tau)^{3/2} \tau \sqrt{2\alpha/\pi}} \quad \text{as } \tau \rightarrow 0,\end{aligned}$$

For fixed  $m$ , the cross section scales as  $s^{-1+\sqrt{2\alpha}} / \log^{3/2}(\frac{s}{m^2})$  as  $s \rightarrow \infty$ .

For fixed  $s$ , the cross section becomes divergent as  $m \rightarrow 0$  since new IR divergences emerge in this limit.

# Numerical Comparison of theory and experiments

- the combined NNLO + LL resummation effects are minuscule (at the 0.05% level) and thus provides an exceptionally precise prediction.
- Compared with PrimEx(2019) and GlueX(2025) (at JLab) experimental data.



# Conclusion

- We performed **complete NNLO calculation** of the Compton-scattering total cross section with **full electron mass dependence**, revealing that the anticipated double-logarithmic enhancement is indeed present but numerically suppressed.
- We traced the origin of these logarithms to a specific kinematic configuration induced by a **Glauber electron exchange**, allowing us to resum the leading logarithms to all orders, yielding a **compact result** expressed in terms of a modified Bessel function.
- We validated our **combined NNLO+LL results** with high-precision data from PrimEx and GlueX collaborations.
- Resummation using SCET(Ongoing): the factorization formula has more complicated structure.
- The methodology developed here can be directly applied to other fundamental processes with **similar logarithmic structure arising from massive fermion exchanges**, e.g.  $e^+e^- \rightarrow \gamma\gamma$  and  $gg \rightarrow t\bar{t}$ .