Implications of the latest JUNO's measurement

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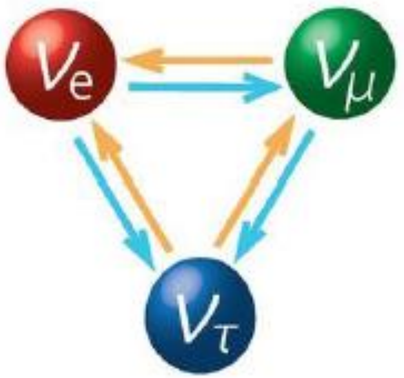

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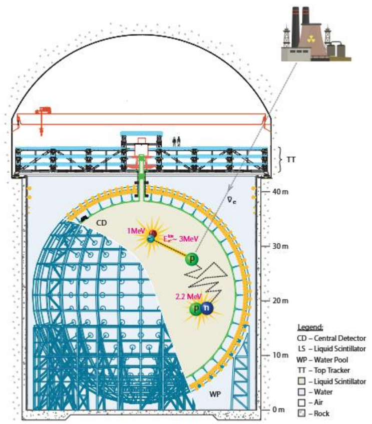
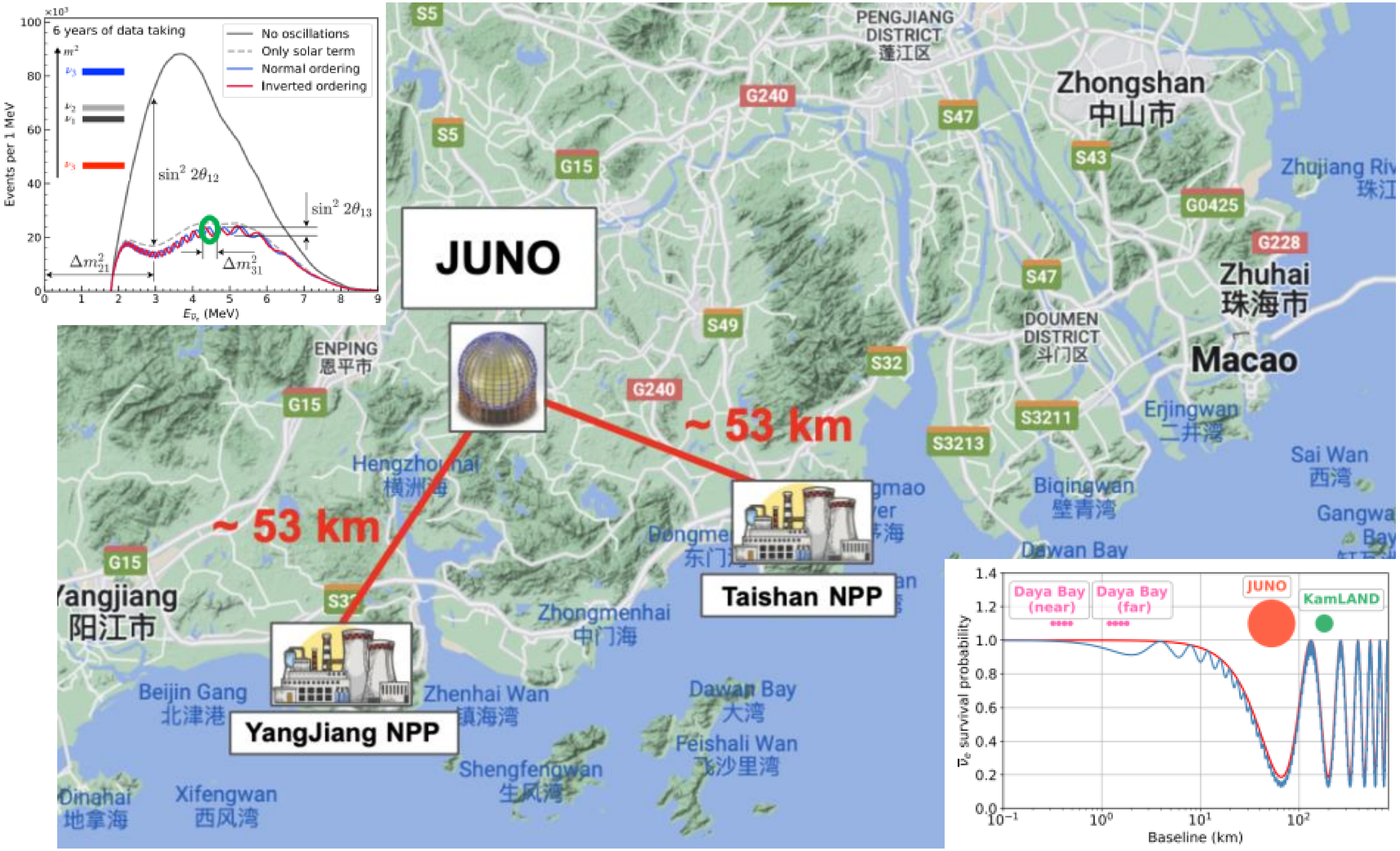
2026年4月24 日—4月28日，重庆

In collaboration with Cai-Chang Li, Jun-Nan Lu and S.T. Petcov, arXiv: 2512.03809

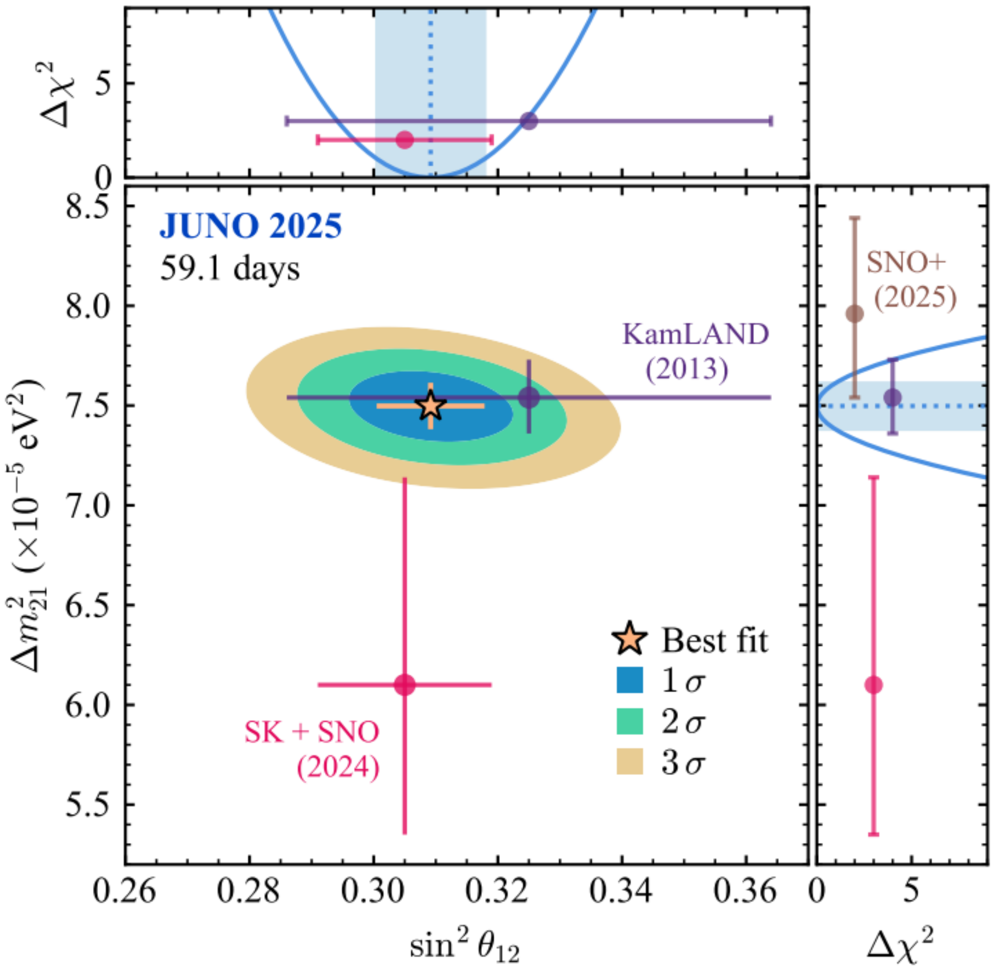
Jiangmen Underground Neutrino Observatory

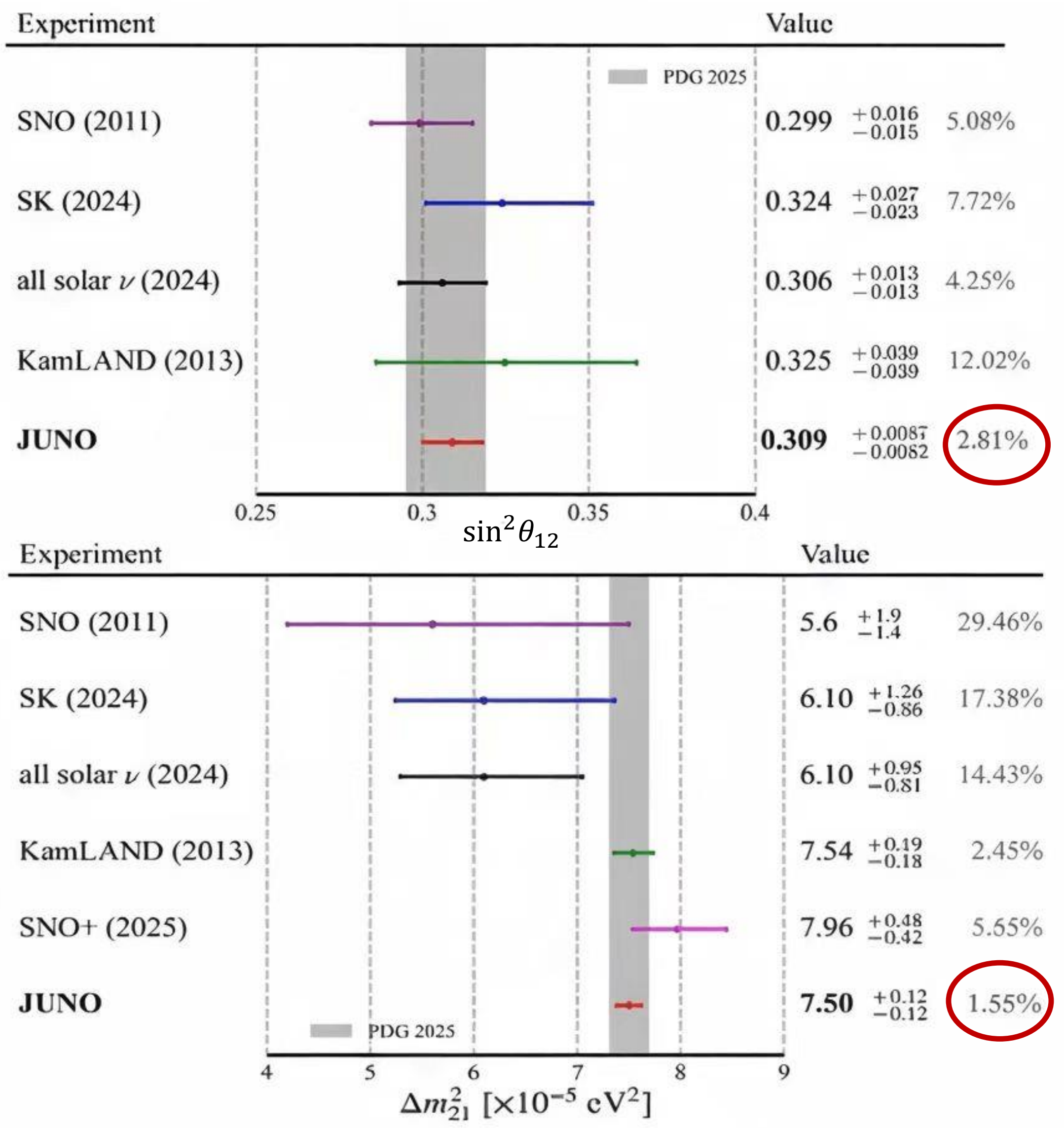
$$P_{\overline{\nu}_e \to \overline{\nu}_e}(L,E) = 1 - \sin^2 2\theta_{12} \cos^4\theta_{13} \sin^2\frac{\Delta m^2_{21} L}{4E} - \sin^2 2\theta_{13}\left(\cos^2\theta_{12}\sin^2\frac{\Delta m^2_{31} L}{4E} + \sin^2\theta_{12}\sin^2\frac{\Delta m^2_{32} L}{4E}\right)$$





The first result of JUNO 59.1 days data





$$\sin^2\theta_{12} = 0.3092 \pm 0.0087,$$

$$\Delta m^2_{21} = (7.5 \pm 0.12) \times 10^{-5}\,\mathrm{eV}^2$$

[JUNO Collaboration,2511.14593]

Status of lepton mixing matrix

Standard parametrization of lepton mixing matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

[PDG 2025]

θ_{12} & θ_{13} $\qquad$ θ_{13}

$$|U|_{3\sigma} = \begin{pmatrix} 0.810 \to 0.834 & 0.532 \to 0.568 & 0.144 \to 0.156 \\ 0.259 \to 0.498 & 0.514 \to 0.678 & 0.652 \to 0.756 \\ 0.280 \to 0.512 & 0.488 \to 0.659 & 0.637 \to 0.743 \end{pmatrix}$$

[NuFIT 6.1]

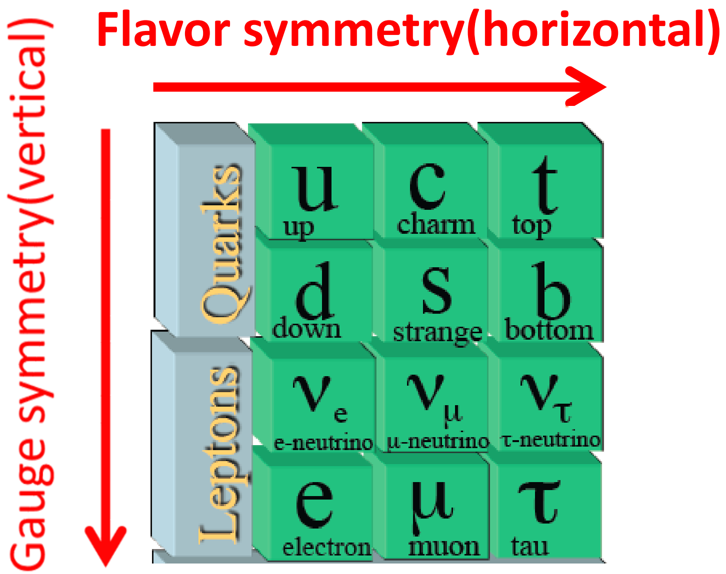
θ_{23} & δ_{CP} $\qquad$ θ_{23} & θ_{13}

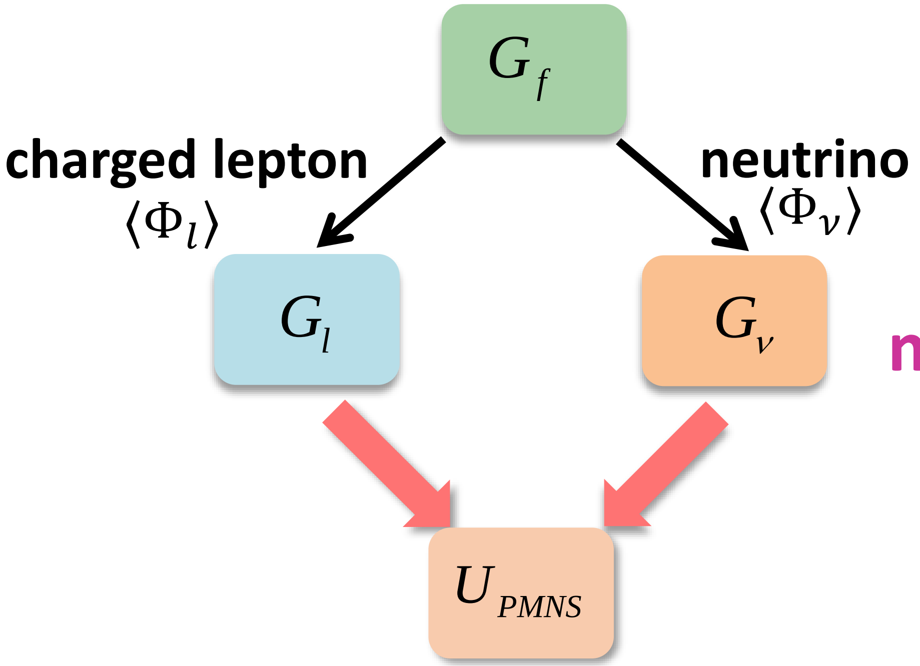
- **First row is determined by θ_{12} and θ_{13}, 1σ range at the sub-percent level**
- Both from reactor experiments such as **Daya Bay & JUNO**

Underlying mechanism of θ_{12} and neutrino mixing : flavor symmetry

Symmetry is an efficient tool for reducing number of free parameters

- **Flavor symmetry:** relating three families e-family ⟺ muon-family ⟺ tau-family





model independent

Lepton mixing arises from the mismatch between the residual subgroups G_l and G_ν

$$\mathcal{L}_m = -Y^e_{ij}(\langle\Phi_e\rangle)\overline{L_i}He_{Rj} - \frac{1}{2}Y^\nu_{ij}(\langle\Phi_\nu\rangle)\overline{L^c_i}HH^TL_j + \ldots$$

[Reviews: Feruglio,Romanino, 1912.06028, Rev. Mod. Phys.; Ding, King, 2311.09282, Rept. Prog. Phys.; Ding,Valle, 2402.16963, Phys. Rept.]

Patterns of flavor symmetry breaking and lepton mixing

①Lepton mixing is **fully** determined by flavor symmetry G_f, **i.e.** $\boldsymbol{G_l > Z_2}$ **&** $\boldsymbol{G_\nu > Z_2}$

$$U=\frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2}\cos\vartheta & 1 & -\sqrt{2}\sin\vartheta \\ -\sqrt{2}\cos(\vartheta-\pi/3) & 1 & \sqrt{2}\sin(\vartheta-\pi/3) \\ -\sqrt{2}\cos(\vartheta+\pi/3) & 1 & \sqrt{2}\sin(\vartheta+\pi/3) \end{pmatrix}$$

[Lindner et al., 1212.2411; King, Neder, Stuart, 1305.3200; Fonseca, Grimus, 1405.3678;Yao, Ding, 1505.03798; Ding,Valle,2402.16963 ....]

ϑ: discrete, fixed by groups G_f, G_l, G_ν

- Lepton mixing angles:

$$\sin^2\theta_{12}=\sec^2\theta_{13}/3\simeq 0.341,$$

$$\sin^2\theta_{23}=\frac{1}{2}\pm\frac{1}{2}\tan\theta_{13}\sqrt{2-\tan^2\theta_{13}}=0.605 \ \text{or} \ 0.395$$

✗

JUNO 3σ	$0.2831 \leq \sin^2\theta_{12} \leq 0.3353$
NuFIT v6.1 3σ	$0.2893 \leq \sin^2\theta_{12} \leq 0.3295$
	$0.435 \leq \sin^2\theta_{23} \leq 0.584$

- Dirac CP phase δ_{CP} is **conserved: sinδ_{CP}=0**

- **Larger** groups required, for example **|G_f|=648** for Majorana neutrinos

②Lepton mixing is **partially** determined by flavor symmetry G_f, i.e. $G_l > Z_2$ & $G_\nu = Z_2$

$$U = U^\circ(\theta_{12}^\circ, \theta_{13}^\circ, \theta_{23}^\circ, \delta^\circ) U_{ij}(\theta_{ij}^\nu, \delta_{ij}^\nu)$$

two free parameters: $\theta_{ij}^\nu, \delta_{ij}^\nu$

fixed by residual symmetries

free rotation in the *ij*-plane

Only **one column** of the lepton mixing matrix is determined by flavor symmetry breaking

TM1: $$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \times & \times \\ -\frac{1}{\sqrt{6}} & \times & \times \\ -\frac{1}{\sqrt{6}} & \times & \times \end{pmatrix}$$

[Xing, Zhou, hep-ph/0607302]

TM2: $$U = \begin{pmatrix} \times & \frac{1}{\sqrt{3}} & \times \\ \times & \frac{1}{\sqrt{3}} & \times \\ \times & \frac{1}{\sqrt{3}} & \times \end{pmatrix}$$

Sum rules:

$$\begin{cases} 3\cos^2\theta_{12}\cos^2\theta_{13} = 2, \\ \cos\delta_{CP} = \dfrac{(5\sin^2\theta_{13} - 1)\cot 2\theta_{23}}{2\sin\theta_{13}\sqrt{2 - 6\sin^2\theta_{13}}} \end{cases}$$

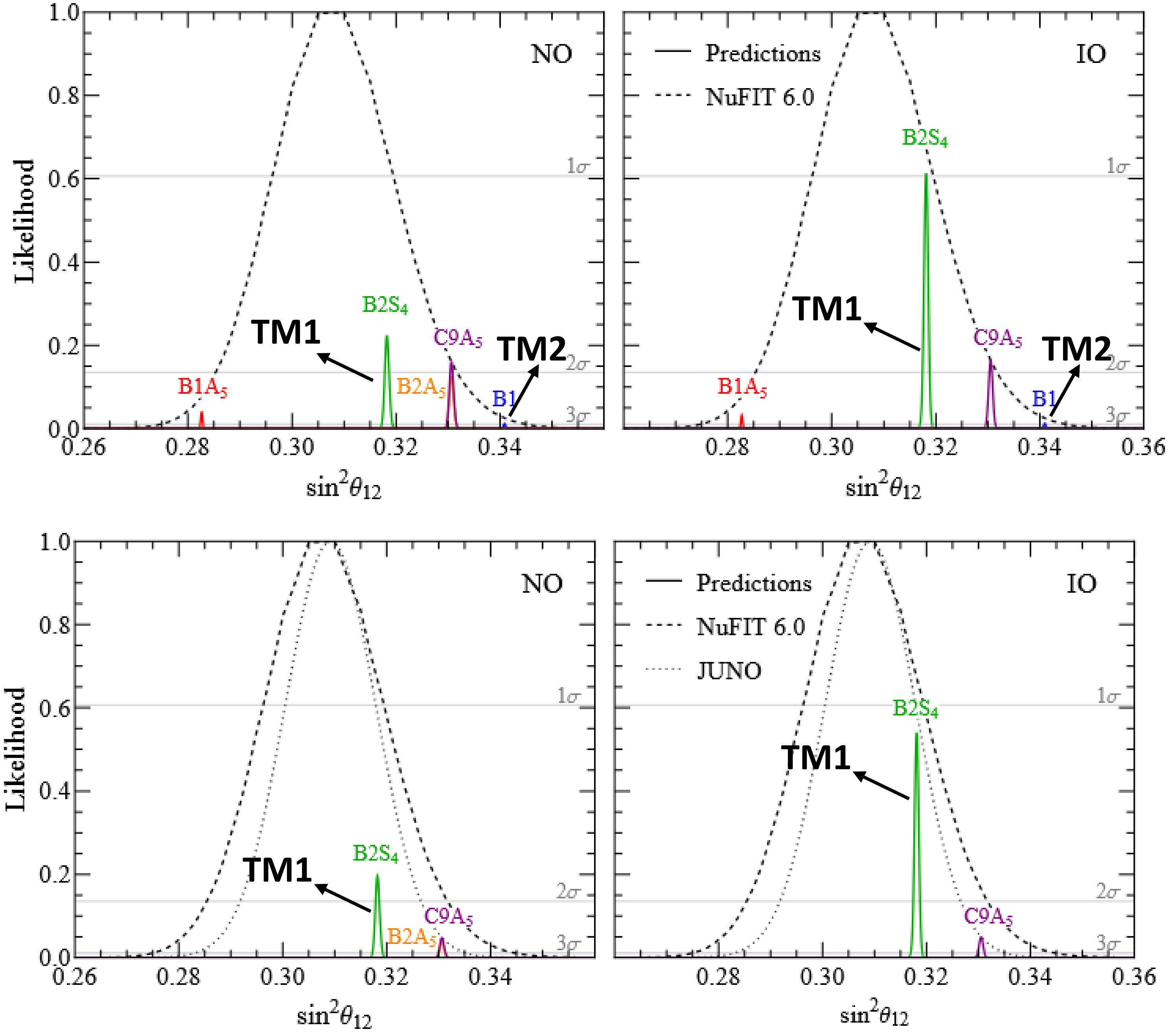
$$\begin{cases} 3\sin^2\theta_{12}\cos^2\theta_{13} = 1, \\ \cos\delta_{CP} = \dfrac{\cos 2\theta_{13}\cot 2\theta_{23}}{\sin\theta_{13}\sqrt{2 - 3\sin^2\theta_{13}}} \end{cases}$$

[Costa, King, 2307.13895; D. Zhang, 2511.15654; X.G. He, 2511.15978….]

Before the first JUNO measurement

δ_{CP} is less constrained

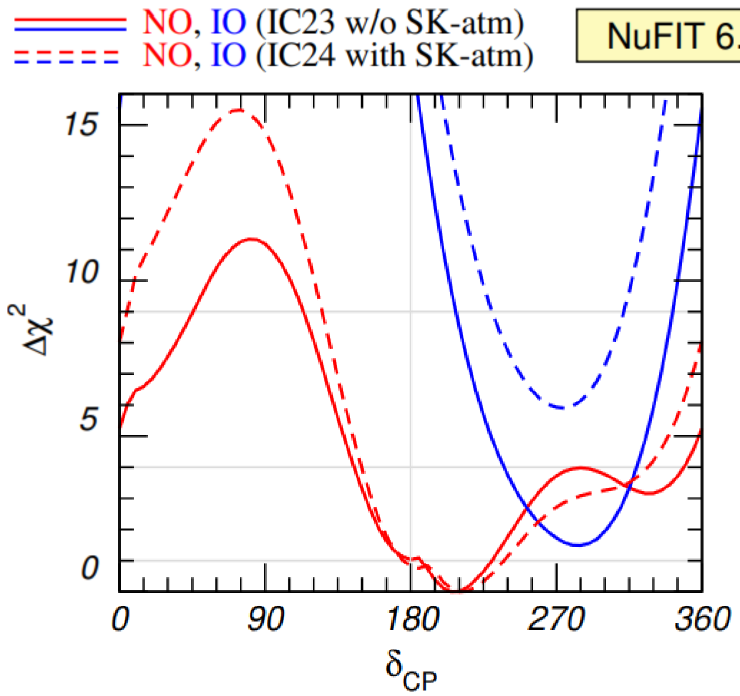
After the first JUNO measurement

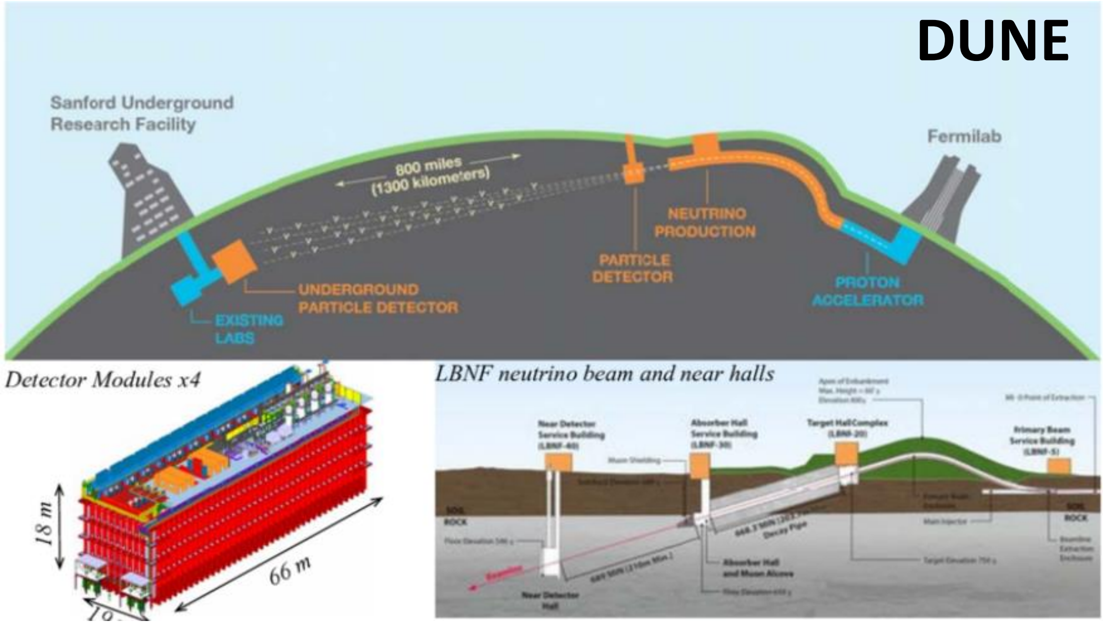


TM2 is disfavored by JUNO

[Petcov, Titov, 1804.00182,2511.19408]

Combine flavor symmetry with generalized CP symmetry





- **Flavor symmetry from CP symmetry**

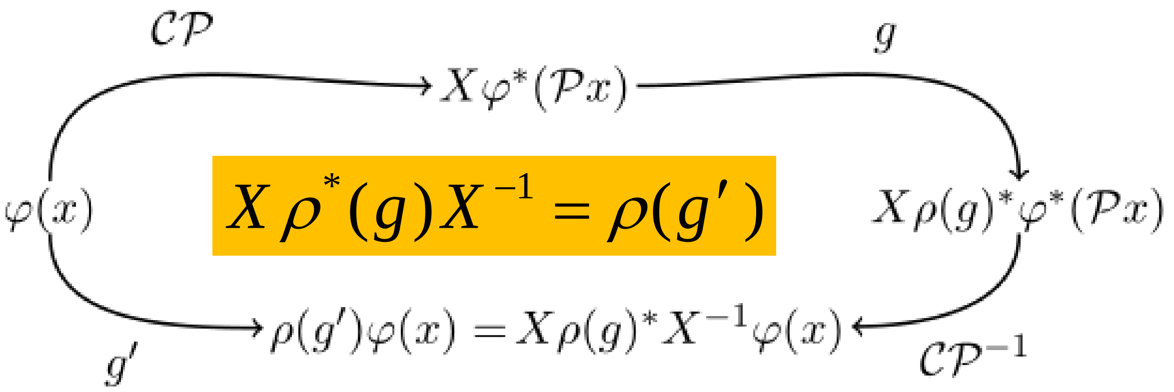
$$\varphi(x)\xrightarrow{\mathcal{CP}}X\varphi^*(\mathcal{P}x)\xrightarrow{\mathcal{CP}}XX^*\varphi(x)=\rho(g)\varphi(x),\ \ g\in G_f$$

Flavor symmetry → mixing angles

↕

CP symmetry → CP phases

"closure" relations have to hold!



[Feruglio, Hagedorn, Ziegler, 1211.5560; Holthausen, Lindner, Schmidt,1211.6953; Chen, Fallbacher, et al., 1402.0507]

Flavor and CP symmetry to lepton mixing

- Flavor + CP symmetries have rich symmetry breaking patterns, and the resulting lepton mixing matrix is determined up to few continuous free parameters.

$G_l \rtimes H^l_{CP}$	$G_\nu \rtimes H^\nu_{CP}$	U	# parameters
Z_n	$K_4 \times CP$	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	0 not viable
Z_n	$Z_2 \times CP$	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta) P_\nu Q_\nu$	1
$Z_2 \times CP$	$K_4 \times CP'$	$Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	
$Z_2 \times CP$	$Z_2 \times CP'$	$Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$	2
Z_2	$K_4 \times CP$	$Q_l^\dagger P_l^T U_{23}^\dagger(\theta_l, \delta_l) \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	
Z_n	CP	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu O_3(\theta_1, \theta_2, \theta_3) Q_\nu$	3 low predictivity
CP	$K_4 \times CP'$	$Q_l^\dagger O_3^T(\theta_1, \theta_2, \theta_3) \Sigma_l^\dagger \Sigma_\nu Q_\nu$	
Z_2	$Z_2 \times CP$	$Q_l^\dagger P_l^T U_{23}^\dagger(\theta_l, \delta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$	

[Ding,Valle,2402.16963]

- The lepton mixing angles as well as Dirac and Majorana CP phases can be predicted by residual symmetry, neutrino masses can be either NO or IO.

Lepton mixing with one parameter from flavor&CP symmetry breaking

[Feruglio et al., 1211.5560, Ding et al., 1303.6180]

$G_f \rtimes H_{CP}$

charged lepton: $Z_n^{g_l}, n \geq 3$ or K_4

neutrino: $Z_2^{g_\nu} \times X_\nu$

$\Sigma_l^\dagger \rho_3(g_l) \Sigma_l = \text{diag}(e^{i\alpha_e}, e^{i\alpha_\mu}, e^{i\alpha_\tau})$

$X_\nu = \Sigma_\nu \Sigma_\nu^T,$

$\Sigma_\nu^\dagger \rho_3(g_\nu) \Sigma_\nu = \pm\text{diag}(1, -1, -1)$

$U_l = \Sigma_l P_l Q_l$

$U_\nu = \Sigma_\nu R_{23}(\theta) Q_\nu$

$$U_{PMNS}(\theta) = Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta) P_\nu Q_\nu$$

Q_ν: CP parity of neutrinos

- Lepton mixing matrix is determined up to row and column permutations P_l and P_ν, and one column of mixing matrix is fixed by residual symmetry.
- All mixing angles and CP phases depend on a single real parameter $\theta \in [0, \pi)$

Letpon mixing patterns of one free parameter

- **Lepton mixing matrix from $\Delta(6n^2)\rtimes H_{CP}$**

Case I: $G_l = Z_3^{ac^s d^t}, G_\nu = Z_2^{bc^x d^x}, X_\nu = c^\gamma d^{-2x-\gamma}, \quad s,t,x,\gamma = 0,1,,\dots,n-1$

$$U^I = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2}\sin\varphi_1 & e^{i\varphi_2} & \sqrt{2}\cos\varphi_1 \\ \sqrt{2}\cos\left(\varphi_1-\frac{\pi}{6}\right) & -e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1-\frac{\pi}{6}\right) \\ \sqrt{2}\cos\left(\varphi_1+\frac{\pi}{6}\right) & e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1+\frac{\pi}{6}\right) \end{pmatrix} R_{23}(\theta)$$

$$3\cos^2\theta_{12}\cos^2\theta_{13} = 2\sin^2\varphi_1$$

$$\cos\delta_{CP} \approx \frac{(3\cos 2\theta_{12}-2)\cot 2\theta_{23}}{3\sin 2\theta_{12}\sin\theta_{13}}$$

Case II: $G_l = Z_3^{ac^s d^t}, G_\nu = Z_2^{c^{n/2}}, X_\nu = c^\gamma d^\delta, \quad s,t,\gamma,\delta = 0,1,,\dots,n-1$

$$U^{II} = \frac{1}{\sqrt{3}}\begin{pmatrix} e^{i\varphi_3} & 1 & e^{i\varphi_4} \\ \omega e^{i\varphi_3} & 1 & \omega^2 e^{i\varphi_4} \\ \omega^2 e^{i\varphi_3} & 1 & \omega e^{i\varphi_4} \end{pmatrix} R_{13}(\theta)$$

TM2

$$3\sin^2\theta_{12}\cos^2\theta_{13} = 1$$

$$\cos\delta_{CP} = \frac{\cos 2\theta_{13}\cot 2\theta_{23}}{\sqrt{3\cos^2\theta_{13}-1}\,\sin\theta_{13}}$$

Discrete parameters: $\varphi_1 = \frac{s-x}{n}\pi,\ \varphi_2 = \frac{2t-s-3(\gamma+x)}{n}\pi,\ \varphi_3 = \frac{\gamma+\delta+2s}{n}\pi,\ \varphi_4 = \frac{2\delta-\gamma+2t}{n}\pi$

➢ Lepton mixing matrix from $A_5 \rtimes H_{CP}$

Case III: $G_l = Z_5^T, G_\nu = Z_2^{T^3ST^2ST^3}, X_\nu = S$

$$U^{III} = \begin{pmatrix} -i\sqrt{\frac{\phi_g}{\sqrt{5}}} & \sqrt{\frac{1}{\sqrt{5}\phi_g}} & 0 \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta)$$

GR2

$$\sin^2\theta_{12} = \frac{3-\phi_g}{5\cos^2\theta_{13}}$$
$$\delta_{CP} = \pm\frac{\pi}{2},$$
$$\alpha_{21}, \alpha_{31} = 0, \pi$$

Case IV: $G_l = K_4^{(ST^2ST^3S,\, TST^4)}, G_\nu = Z_2^S, X_\nu = T^3ST^2ST^3$

$$U^{V} = \frac{1}{2\sqrt{3}} \begin{pmatrix} \sqrt{3}-1 & 2e^{-i\varphi} & -(\sqrt{3}+1)e^{\frac{3\pi i}{4}} \\ -\sqrt{3}-1 & 2e^{-i\varphi} & (\sqrt{3}-1)e^{\frac{3\pi i}{4}} \\ 2 & 2e^{-i\varphi} & 2e^{\frac{3\pi i}{4}} \end{pmatrix} R_{13}(\theta)$$

$$\cos^2\theta_{12} = \frac{1+\phi_g}{4\cos^2\theta_{13}}$$
$$\delta_{CP}, \alpha_{21}, \alpha_{31} = 0, \pi$$

➢ Lepton mixing matrix from $\Sigma(168) \rtimes H_{CP}$

Case V: $G_l = Z_3^{ST^4ST^2S}, G_\nu = Z_2^S, X_\nu = 1$

$$U^{III} = \begin{pmatrix} -i\sqrt{\frac{\phi_g}{\sqrt{5}}} & \sqrt{\frac{1}{\sqrt{5}\phi_g}} & 0 \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta)$$

TM2

$$\varphi = \arctan(2-\sqrt{7}) \approx -0.183\pi$$

$$3\sin^2\theta_{12}\cos^2\theta_{13} = 1$$
$$\cos\delta_{CP} = \frac{\cos 2\theta_{13}\cot 2\theta_{23}}{\sqrt{3\cos^2\theta_{13}-1}\,\sin\theta_{13}}$$

Viable lepton mixing with one parameter

➢ Best-fit results of mixing angles and CP phases for all discrete groups $|G_f| \leq 168$

- 3σ interval of JUNO:

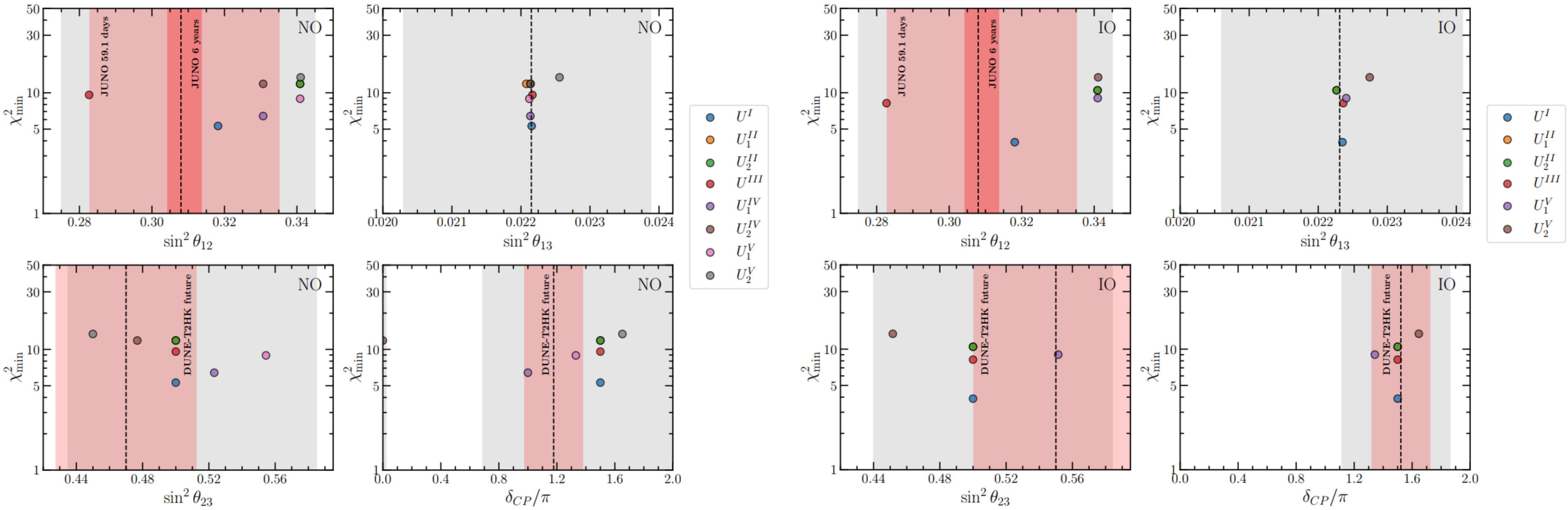
$$0.2831 \leq \sin^2\theta_{12} \leq 0.3353$$

- U^{I} can accommodate experiment data at 3σ for both NO and IO
- $U^{IV}_{1,2}$ can only be compatible with experiment data at 3σ for NO
- U^{III} predicts **GR2**, which marginally agrees with JUNO
- $U^{II}_{1,2}$ and $U^{V}_{1,2}$ predict **TM2,** they are disfavored at more than 3σ by JUNO

Order	Case	(φ_1,φ_2)	$\theta_{\rm bf}/\pi$	$\chi^2_{\rm min}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	α_{21}/π (mod 1)	α_{31}/π (mod 1)
NO	U^{I}	$(-\frac{\pi}{2},\frac{\pi}{2})$	0.9170	5.308	0.318	0.02215	0.5	1.5	0	0
	U^{II}_1	$(0,\pi)$	0.1918	11.862	0.341	0.02208	0.5	1.5	0	0
	U^{II}_2	$(0,\pi)$	0.3083	11.859	0.341	0.02214	0.5	1.5	0	0
	U^{III}	—	0.0560	9.600	0.283	0.02217	0.5	1.5	0	0
	U^{IV}_1	—	0.9053	6.414	0.331	0.02214	0.523	1	0	0
	U^{IV}_2	—	0.9053	11.856	0.331	0.02214	0.477	0	0	0
	U^{V}_1	—	0.5715	8.921	0.341	0.02212	0.554	1.331*	0.161*	0.559*
	U^{V}_2	—	0.5746	13.427	0.341	0.02256	0.450	1.652	0.837	0.439
IO	U^{I}	$(-\frac{\pi}{2},\frac{\pi}{2})$	0.9166	3.887	0.318	0.02235	0.5	1.5	0	0
	U^{II}_1	$(0,\pi)$	0.1915	10.471	0.341	0.02227	0.5	1.5	0	0
	U^{II}_2	$(0,\pi)$	0.3085	10.471	0.341	0.02227	0.5	1.5	0	0
	U^{III}	—	0.0563	8.188	0.283	0.02236	0.5	1.5	0	0
	U^{V}_1	—	0.5736	9.012	0.341	0.02241	0.551	1.343*	0.162*	0.561*
	U^{V}_2	—	0.5758	13.416	0.341	0.02274	0.452	1.646	0.837	0.438

[Ding,Li,Lu,Petcov,2512.03809]

Survey of symmetry predictions with one parameters



Some patterns compatible with NuFIT 6.0 are exclude by JUNO first result.

6 years of running of JUNO can exclude these one-parameter mixing patterns.

[Ding,Li,Lu,Petcov,2512.03809]

A viable example with one parameter

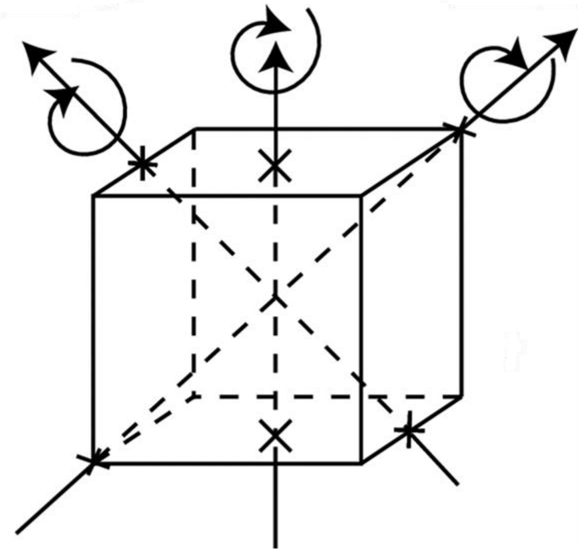
- The minimal group that can realize viable lepton mixing is $\Delta(24) \cong S_4$

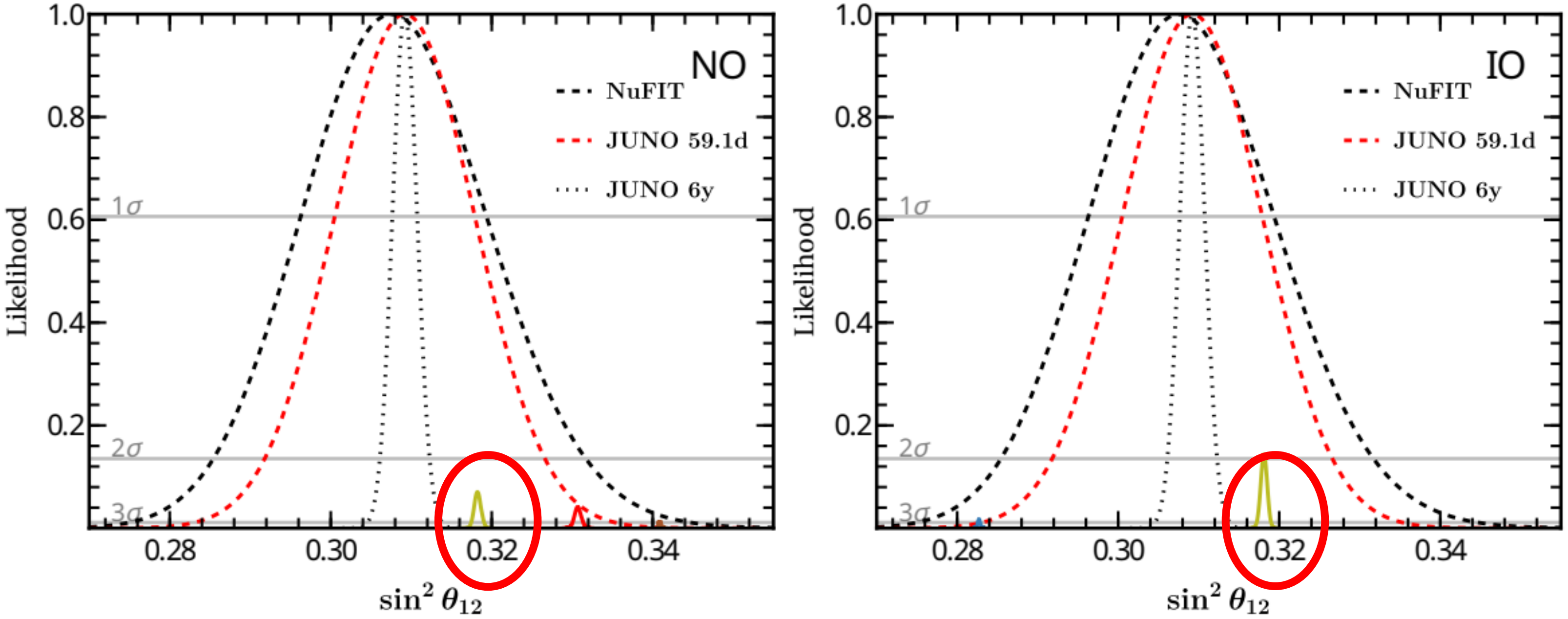
TM1 [Xing, Zhou,hep-ph/0607302]

$$U^I\left(\varphi_1 = -\frac{\pi}{2}, \varphi_2 = \frac{\pi}{2}\right) = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{i}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{i}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{i}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{23}(\theta)$$

$$\sin^2\theta_{12} = 1 - \frac{2}{3\cos^2\theta_{13}} \approx 0.318,$$
$$\theta_{23} = \frac{\pi}{4}, \delta_{CP} = \pm\frac{\pi}{2},$$
$$\alpha_{21}, \alpha_{31} = 0, \pi$$

maximal θ_{23} and δ_{CP}





Potentially ruled out by JUNO 6y

Lepton mixing with two parameters from flavor&CP symmetry breaking

[Lu, Ding, 1610.05682, 1806.02301; Li, Lu, Ding, 1706.04576; Lu, Ding, 1901.07414]

$$G_f \rtimes H_{CP}$$

charged lepton → $Z_2^{g_l} \times X_l$

neutrino → $Z_2^{g_\nu} \times X_\nu$

$$X_l = \Sigma_l \Sigma_l^T, \quad \Sigma_l^\dagger \rho_3(g_l) \Sigma_l = \pm \mathrm{diag}(1,-1,-1)$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T, \quad \Sigma_\nu^\dagger \rho_3(g_\nu) \Sigma_\nu = \pm \mathrm{diag}(1,-1,-1)$$

$$U_l = \Sigma_l R_{23}(\theta_l) P_l Q_l \qquad U_\nu = \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

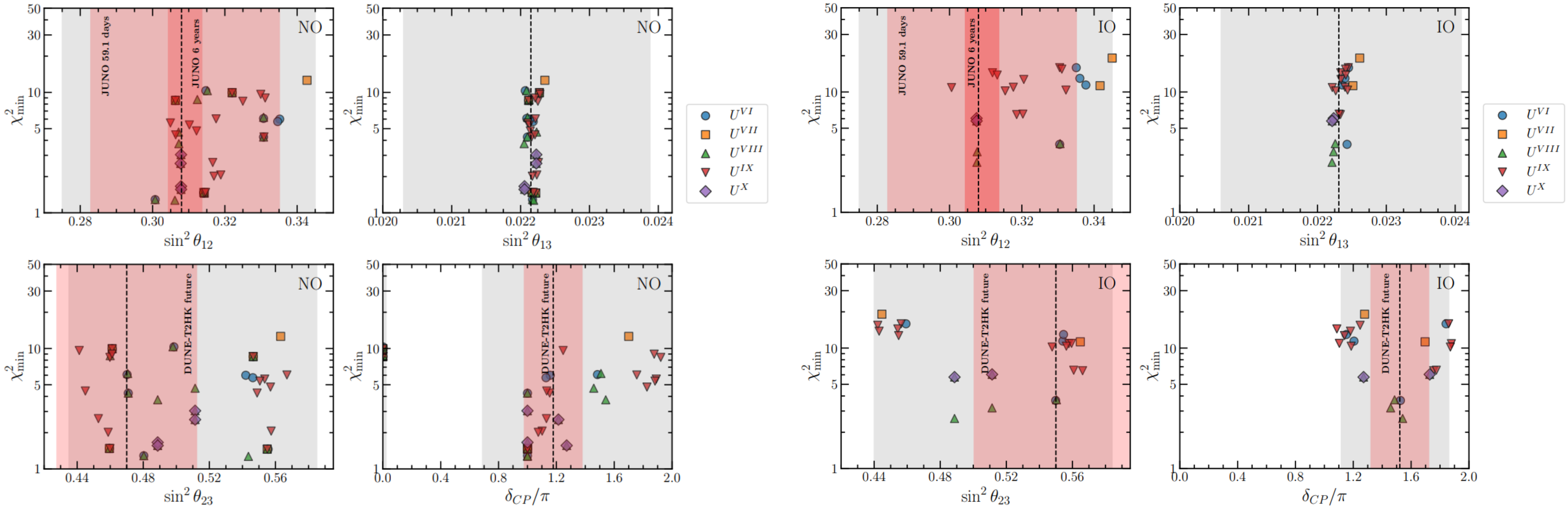
$$U(\theta_l, \theta_\nu) = Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

- Arbitrary row and column permutations P_l and P_ν. Residual symmetries fix one entry to be $(\Sigma_l^\dagger \Sigma_\nu)_{11}$
- All mixing angles and CP phases are expressed in terms of two free angles $\theta_{l,\nu} \in [0, \pi)$

Survey of symmetry predictions with two free parameters

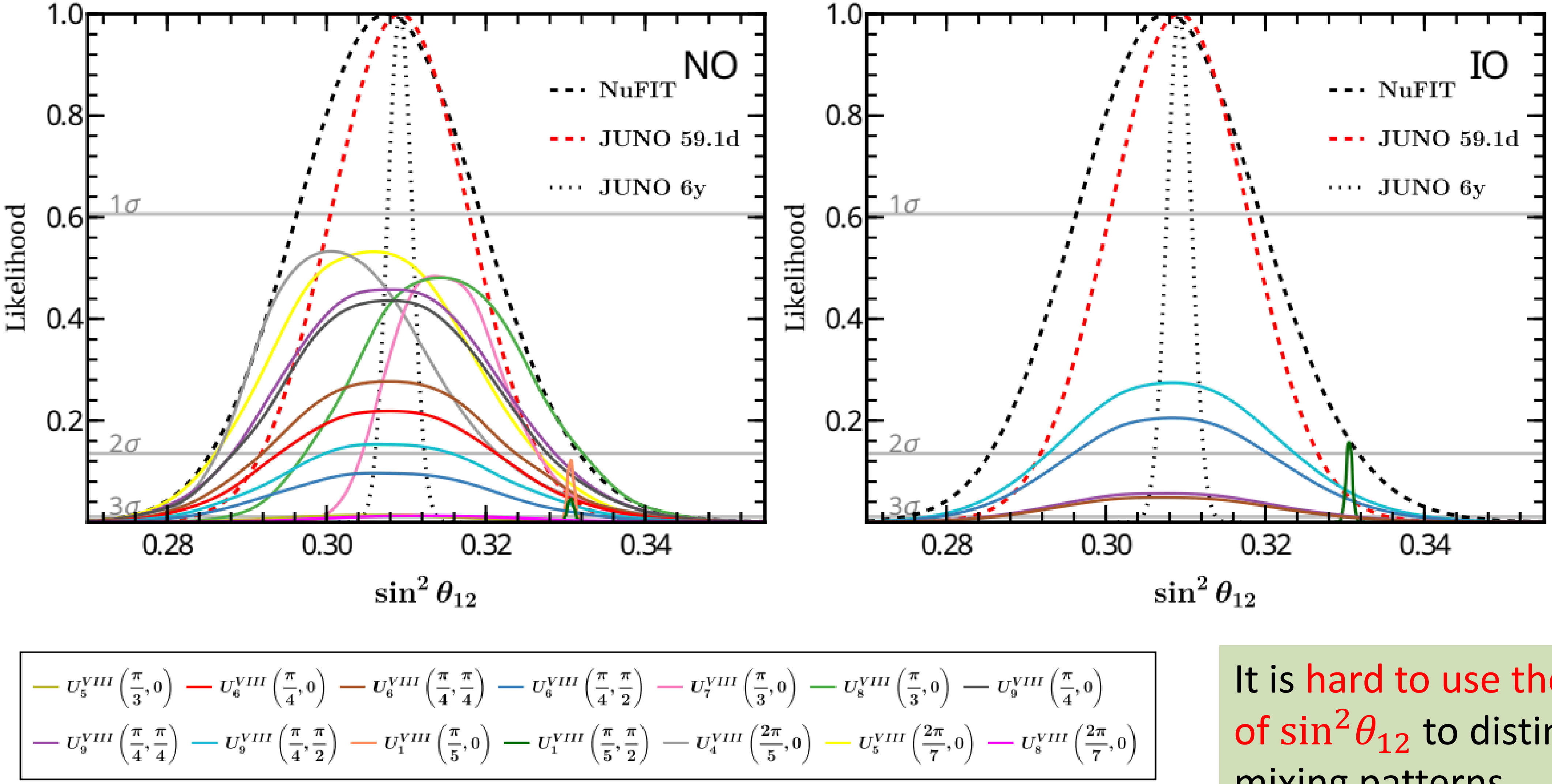
- Best fit values of mixing angles and CP phase

[Ding,Li,Lu,Petcov,2512.03809]



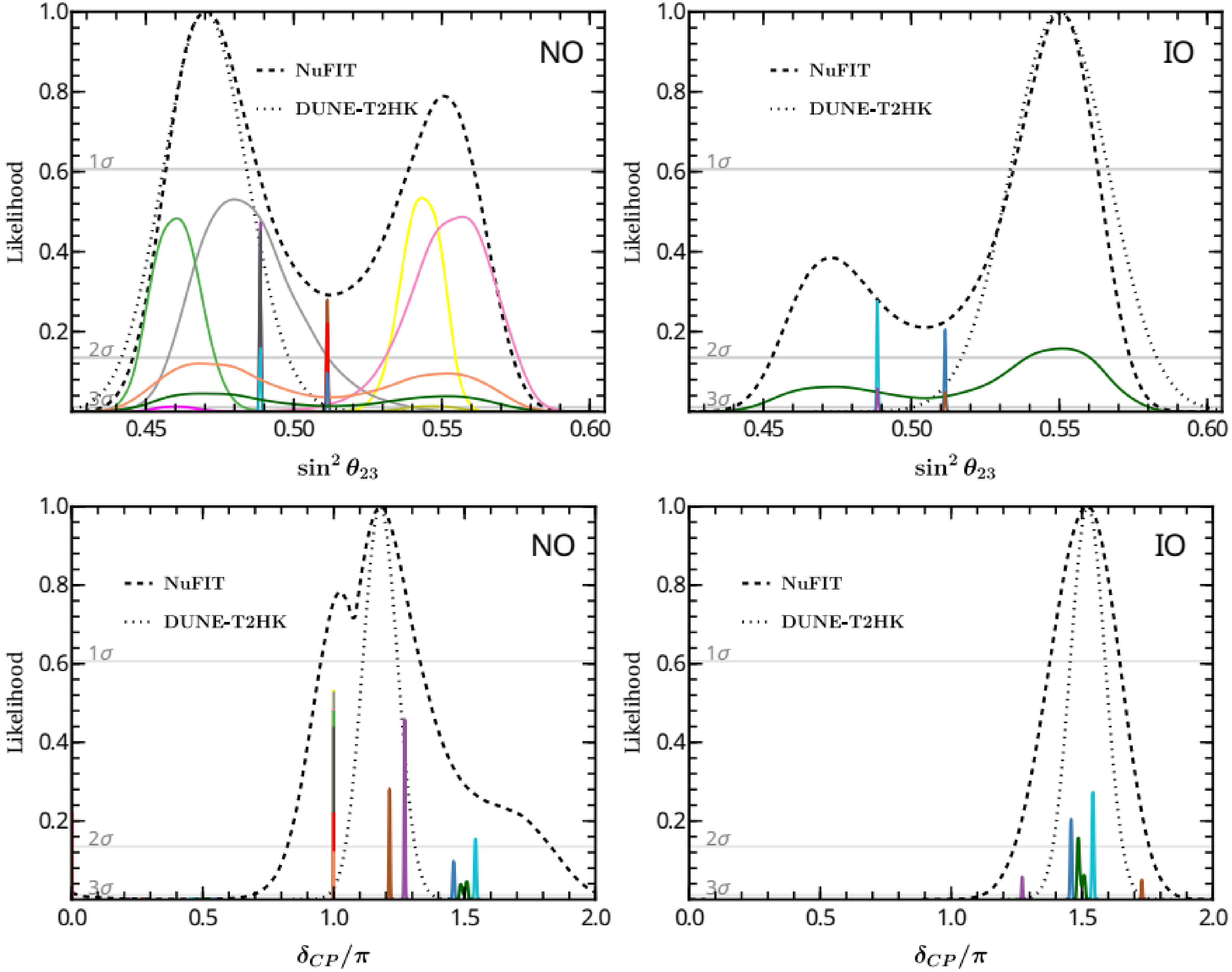
A number of two-parameter mixing patterns are consistent with JUNO's first results and NuFIT.

Predictions for solar neutrino mixing angle



It is hard to use the predictions of $\sin^2\theta_{12}$ to distinguish these mixing patterns.

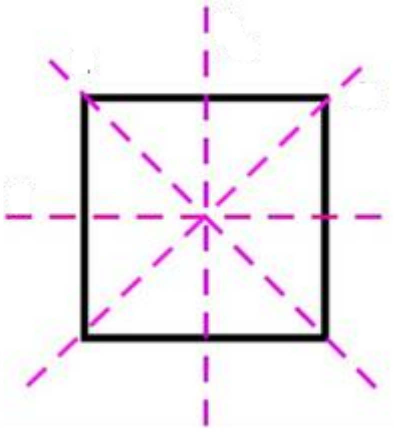
Predictions for atmospheric neutrino mixing angle and CP phase



- The predictions of $\sin^2\theta_{23}$ and δ_{CP} are narrow and different among these mixing patterns.
- The synergy of JUNO, DUNE and T2HK data can provide an exhaustive test of discrete flavor and CP symmetries.

A viable example with two parameters

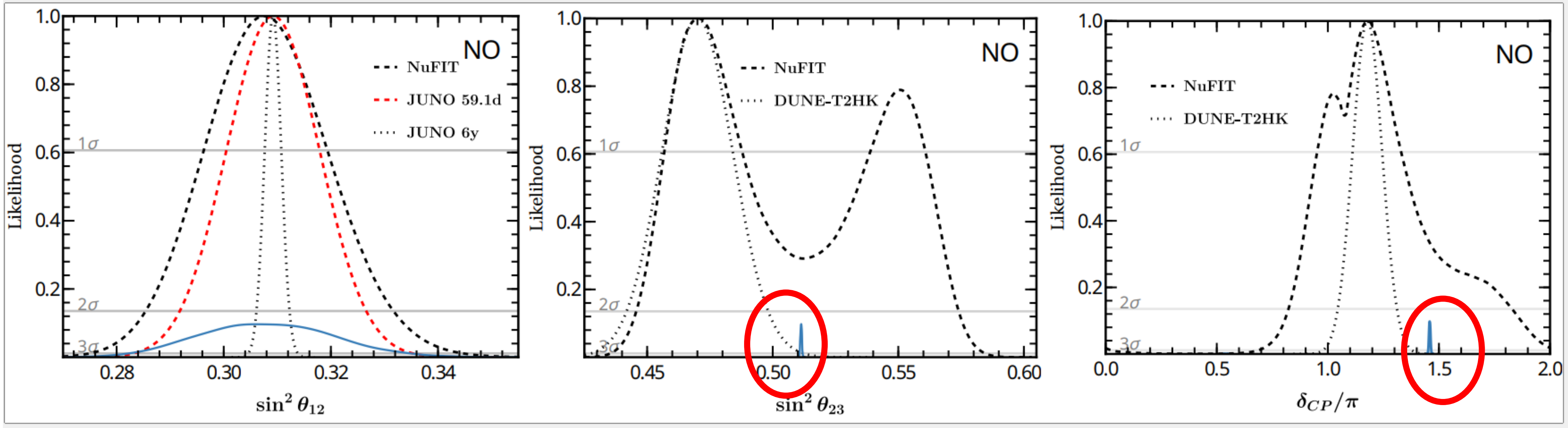
- The minimal group that can yield viable mixing with non-trivial CP phases is $\boldsymbol{D_4}$



$$U_6^{VIII}\Big|_{\varphi_1=\frac{\pi}{4},\,\varphi_2=\frac{\pi}{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\, i c_l s_\nu - c_\nu s_l & \sqrt{2}\, i c_l c_\nu + s_l s_\nu & -s_l \\ -c_\nu & s_\nu & 1 \\ \sqrt{2}\, i s_l s_\nu + c_l c_\nu & \sqrt{2}\, i c_\nu s_l - c_l s_\nu & c_l \end{pmatrix} \Rightarrow 2\cos^2\theta_{13}\sin^2\theta_{23} = 1$$

At the best fit point $\theta_l \approx 0.0676\pi, \theta_\nu \approx 0.684\pi$,

$$\sin^2\theta_{12} \approx 0.307,\ \ \sin^2\theta_{13} \approx 0.02223,\ \ \sin^2\theta_{23} \approx 0.511,$$
$$\delta_{CP} \approx 1.458\pi,\ \ \alpha_{21} \approx 0.791\pi \ (\mathrm{mod}\ \pi),\ \ \alpha_{31} \approx 0.854\pi \ (\mathrm{mod}\ \pi)$$



- No sharp prediction for θ_{12}
- $\theta_{23} > 45°$, $\delta_{CP} \sim 1.5\pi$

Summary

Case	Fixed column	Case	Fixed entry
U^{I}	$\sqrt{\frac{2}{3}}\left(\sin\varphi_1, \cos(\varphi_1-\frac{\pi}{6}), \cos(\varphi_1+\frac{\pi}{6})\right)^T$	U^{VI}	$\frac{\phi_g}{2}, \frac{1}{2}, \frac{1}{2\phi_g}$
U^{II}	$\frac{1}{\sqrt{3}}(1,1,1)^T$	U^{VII}	$\frac{1}{2}$
U^{III}	$\left(\sqrt{\frac{1}{\sqrt{5}\phi_g}}, \sqrt{\frac{\phi_g}{2\sqrt{5}}}, \sqrt{\frac{\phi_g}{2\sqrt{5}}}\right)^T$	U^{VIII}	$\cos\varphi_1$
U^{IV}	$\frac{1}{2}(\phi_g, \phi_g-1, 1)^T$	U^{IX}	$\frac{1}{2}$
U^{V}	$\frac{1}{\sqrt{3}}(1,1,1)^T$	U^{X}	$\frac{1}{\sqrt{2}}$

- Flavor and CP symmetries can strongly constrain lepton mixing in a **model-independent** way.
  - One-parameter mixing patterns enforced by $G_l = Z_n, G_\nu = Z_2 \times CP$
    - —One column of the mixing matrix is fixed by residual symmetry
    - —The minimal viable flavor symmetry is $\boldsymbol{S_4}$
    - —The first JUNO results have already **ruled out some of them**
  - Two-parameter mixing patterns enforced by $G_l = Z_2 \times CP, G_\nu = Z_2 \times CP$
    - —One entry of the mixing matrix is fixed by residual symmetry
    - —The minimal viable flavor symmetry is $\boldsymbol{D_4}$
- The synergy of high precision JUNO, DUNE and T2HK data can provide an exhaustive evidence for the origin of neutrino mixing.

Backup

The choices of flavor and CP symmetries

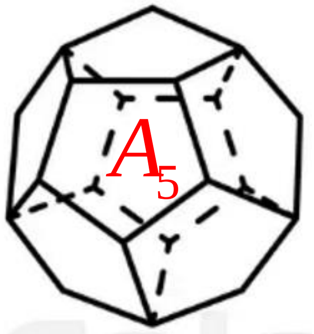
- A systematical analysis of the discrete flavor group G_f up to order 2000 found that all viable mixing patterns with one parameter can be obtained by considering:

$$A_5 \rtimes H_{CP}, \quad \Sigma(168) \rtimes H_{CP}, \quad \Delta(6n^2) \rtimes H_{CP}$$

[Yao, Ding,1606.05610]

- $A_5 \cong \Gamma_5$ is the group of even permutations of five objects,

$$S^2 = T^5 = (ST)^3 = 1$$



- 3-dim representation

$$\mathbf{3}: \quad S = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & -\phi_g & \phi_g - 1 \\ -\sqrt{2} & \phi_g - 1 & -\phi_g \end{pmatrix}, \quad T = \mathrm{diag}(1, \omega_5, \omega_5^4),$$

$$\phi_g = (1+\sqrt{5})/2, \quad \omega_5 = e^{2\pi i/5}$$

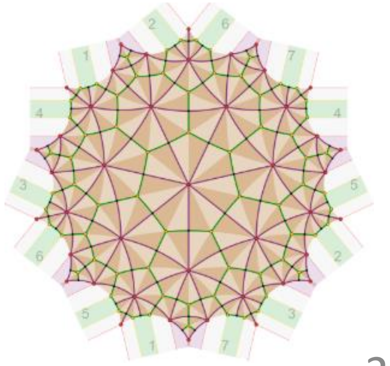
- $\Sigma(168) \cong \Gamma_7$ is a non-Abelian subgroup of $SU(3)$ of order 168,

$$S^2 = (ST)^3 = T^7 = (ST^3)^4 = 1$$

- 3-dim representation

$$\mathbf{3}: \quad S = \frac{2}{\sqrt{7}} \begin{pmatrix} -s_2 & -s_1 & s_3 \\ -s_1 & s_3 & -s_2 \\ s_3 & -s_2 & -s_1 \end{pmatrix}, \quad T = \mathrm{diag}(\omega_7, \omega_7^2, \omega_7^4),$$

$$\omega_7 = e^{2\pi i/7}, \quad s_n = \sin\frac{2n\pi}{7}$$



- CP symmetry H_{CP}: $\boldsymbol{X_r = \rho_r(g)}$, $g \in A_5$ or $\Sigma(168)$

CP transformations are of the same form as flavor symmetry transformations in the chosen basis

- $\Delta(6n^2), n \in \mathbb{N}$ is an infinite series of non-Abelian finite subgroup of $SU(3)$, it is isomorphic to $(Z_n \times Z_n) \rtimes S_3$ with the multiplication rules:

$$a^3 = b^2 = (ab)^2 = 1,$$
$$c^n = d^n = 1, \qquad cd = dc,$$
$$aca^{-1} = c^{-1}d^{-1}, \quad ada^{-1} = c, \quad bcb^{-1} = d^{-1}, \quad bdb^{-1} = c^{-1}$$

$S_3 =< a, b >$

$Z_n \times Z_n =< c, d >$

$\rtimes$

— $n = 1,\ \Delta(6) \cong S_3 \rightarrow$ symmetry of **equilateral triangle**

— $n = 2,\ \Delta(24) \cong S_4 \rightarrow$ symmetry of **cube**

- 3-dim irreducible representations: [Escobar,Luhn,2008]

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad b = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta^{-1} \end{pmatrix}, \quad \eta \equiv e^{2\pi i/n}$$

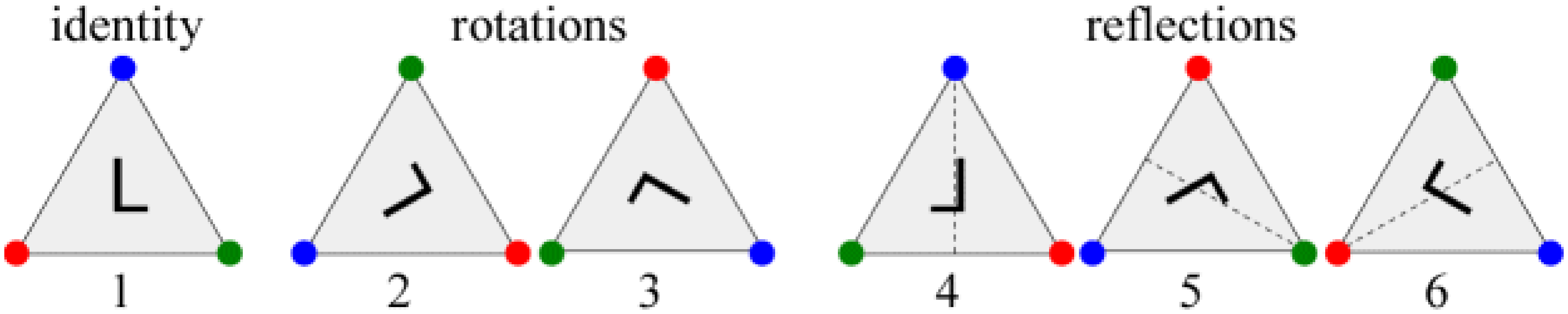
- $\boldsymbol{H_{CP}}$: CP and flavor symmetry transformations are of the **same form**

Dihedral group D_n

- The dihedral group D_n is the symmetry group of a n-sided regular polygon, it has two generators R and S

$$R^n = S^2 = (RS)^2 = 1$$

$R \rightarrow$ rotation, $S \rightarrow$ reflection



- Irreducible representations : 1-dim, 2-dim

$$1_1: \quad R = S = 1, \qquad 1_2: \quad R = 1, \quad S = 1,$$

$$1_3: \quad R = -1, \; S = 1, \qquad 1_4: \quad R = S = -1 \quad (even \quad n),$$

$$2_j: \; R = \begin{pmatrix} e^{2\pi ij/n} & 0 \\ 0 & e^{2\pi ij/n} \end{pmatrix}, \qquad S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad j = 0,1,\ldots,\left[\frac{n-1}{2}\right]$$

CP and flavor symmetry transformations are of the same form in this basis.

Letpon mixing patterns with two free parameters

- **Breaking of $A_5 \rtimes H_{CP}$ into $Z_2^{g_l} \times X_l$ in charged lepton and $Z_2^{g_\nu} \times X_\nu$ in neutrino**

Case VI:

$\{Z_2^{g_\nu}, X_\nu\}$	$\{Z_2^{g_l}, X_l\}$	$\Sigma = \Sigma_l^\dagger \Sigma_\nu$	fixed element
$\{Z_2^{TST^4}, T^3ST^2ST^3S\}$	$\{Z_2^S, 1\}$	$\Sigma_1^{VI} = \mathrm{diag}(i,-i,1)\, U_{RC}\, \mathrm{diag}(-1,1,-1)$	$\frac{\phi_g}{2}$
	$\{Z_2^S, T^3ST^2ST^3\}$	$\Sigma_2^{VI} = \mathrm{diag}(i,1,1)\, U_{RC}\, \mathrm{diag}(1,-1,1)$	
$\{Z_2^{T^2ST^3}, ST^2S\}$	$\{Z_2^S, 1\}$	$\Sigma_3^{VI} = \mathrm{diag}(1,-1,-i)\, P_{312} U_{RC}\, \mathrm{diag}(i,1,-1)$	$\frac{1}{2}$
	$\{Z_2^S, T^3ST^2ST^3\}$	$\Sigma_4^{VI} = \mathrm{diag}(i,1,1)\, P_{312} U_{RC}\, \mathrm{diag}(i,1,-1)$	
$\{Z_2^{T^4(ST^2)^2}, T^3ST^2ST^3S\}$	$\{Z_2^S, 1\}$	$\Sigma_5^{VI} = \mathrm{diag}(1,-1,-i)\, U_{RC} P_{213}\, \mathrm{diag}(i,1,1)$	$\frac{1}{2\phi_g}$
	$\{Z_2^S, T^3ST^2ST^3\}$	$\Sigma_6^{VI} = \mathrm{diag}(i,1,1)\, U_{RC} P_{213}\, \mathrm{diag}(i,1,1)$	

$$U_{RC} = \frac{1}{2}\begin{pmatrix} \phi_g & 1/\phi_g & 1 \\ 1/\phi_g & 1 & -\phi_g \\ 1 & -\phi_g & -1/\phi_g \end{pmatrix}$$

$\phi_g \equiv (1+\sqrt{5})/2$

- **Independent breaking patterns of $\Sigma(168)$ H_{CP}**

Case VII:

$\{G_l, X_l\} = \{Z_2^S, 1\}$, $\{G_\nu, X_\nu\} = \{Z_2^{T^2ST^5}, T^4\} \rightarrow \Sigma_1^{VII} = \mathrm{diag}(1, e^{-\frac{\pi i}{4}}, e^{\frac{\pi i}{4}})\Sigma_2^{VII}\mathrm{diag}(1, e^{\frac{\pi i}{4}}, e^{-\frac{\pi i}{4}})$

$\{G_l, X_l\} = \{Z_2^S, T^2ST^5ST^2\}$, $\{G_\nu, X_\nu\} = \{Z_2^{T^2ST^5}, T^4ST^5ST^4\} \rightarrow \Sigma_2^{VII} = \frac{1}{4}\begin{pmatrix} -2 & \sqrt{2(3+\sqrt{7})} & \sqrt{2(3-\sqrt{7})} \\ \sqrt{2(3+\sqrt{7})} & \sqrt{7}-1 & \sqrt{2} \\ \sqrt{2(3-\sqrt{7})} & \sqrt{2} & -\sqrt{7}-1 \end{pmatrix}$

One element is $\frac{1}{2}$ in the (21), (22), (31) or (32) entry

➢ Independent breaking patterns of $\Delta(6n^2)\rtimes H_{CP}$

	$\{Z_2^{g_\nu}, X_\nu\}$	$\{Z_2^{g_l}, X_l\}$	$\Sigma = \Sigma_l^\dagger \Sigma_\nu$	fixed element
Case VIII:	$\{Z_2^{bc^y d^y}, c^\beta d^{-2y-\beta}\}$	$\{Z_2^{bc^x d^x}, c^\alpha d^{-2x-\alpha}\}$	$\Sigma^{VIII} = \begin{pmatrix} \cos\varphi_1 & -i\sin\varphi_1 & 0 \\ -i\sin\varphi_1 & \cos\varphi_1 & 0 \\ 0 & 0 & e^{i\varphi_2} \end{pmatrix}$	$\cos\varphi_1$
Case IX:	$\{Z_2^{abc^y}, c^\beta d^{2y+2\beta}\}$	$\{Z_2^{bc^x d^x}, c^\alpha d^{-2x-\alpha}\}$	$\Sigma^{IX} = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{2}e^{i\varphi_4} & -1 \\ -1 & -\sqrt{2}e^{i\varphi_4} & 1 \\ -\sqrt{2}e^{i\varphi_3} & 0 & -\sqrt{2}e^{i\varphi_3} \end{pmatrix}$	$\frac{1}{2}$
Case X:	$\{Z_2^{c^{n/2}}, c^\beta d^\gamma\}$	$\{Z_2^{bc^x d^x}, c^\alpha d^{-2x-\alpha}\}$	$\Sigma^{X} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\varphi_6} & 0 & -1 \\ e^{i\varphi_6} & 0 & 1 \\ 0 & \sqrt{2}e^{i\varphi_5} & 0 \end{pmatrix}$	$\frac{1}{\sqrt{2}}$

$\alpha, \beta, \gamma, x, y = 0, 1, \ldots, n-1$

Discrete residual symmetry parameters:

$$\begin{cases} \varphi_1 = \dfrac{x-y}{n}\pi, \quad \varphi_2 = \dfrac{3(x-y+\alpha-\beta)}{n}\pi, \\ \varphi_3 = \dfrac{3\alpha+2(x+y)}{n}\pi, \quad \varphi_4 = -\dfrac{3\beta+2(x+y)}{n}\pi, \\ \varphi_5 = \dfrac{2x+3\alpha-2\beta+\gamma}{n}\pi, \quad \varphi_6 = -\dfrac{2x+\beta+\gamma}{n}\pi \end{cases}$$

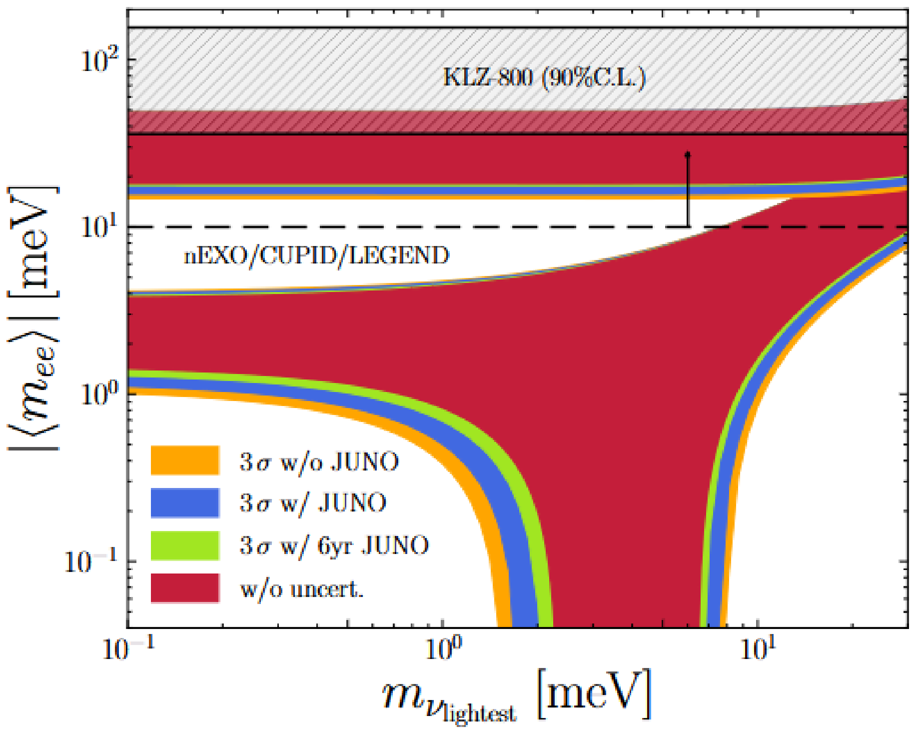
The mixing pattern Σ^{VIII} can also be obtained from the dihedral group $D_n \rtimes H_{CP}$

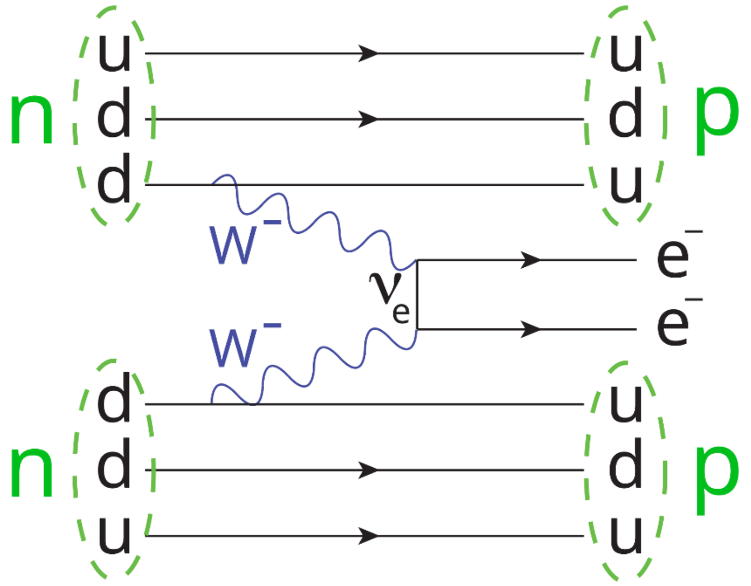
Implication for the $0\nu\beta\beta$ decays

- $0\nu\beta\beta$ decay is the most promising probe of neutrino nature: Majorana vs. Dirac. The decay amplitude is proportional to the effective neutrino mass

$$|\langle m_{ee}\rangle| = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_{21}} m_2 + s_{13}^2 e^{i\alpha_{31}} m_3|$$

- The lower boundaries of $|\langle m_{ee}\rangle|$ strongly depend on solar parameters Δm_{21}^2 and θ_{12}, whose uncertainties will be largely resolved by JUNO





[Ge,Kong,Lindner,Pinheiro, 2511.15391;
Ding, Kumar, Nath, Srivastava, Valle, 2511.22689]