# Bottom quark effects in Higgs boson pair production at the LHC

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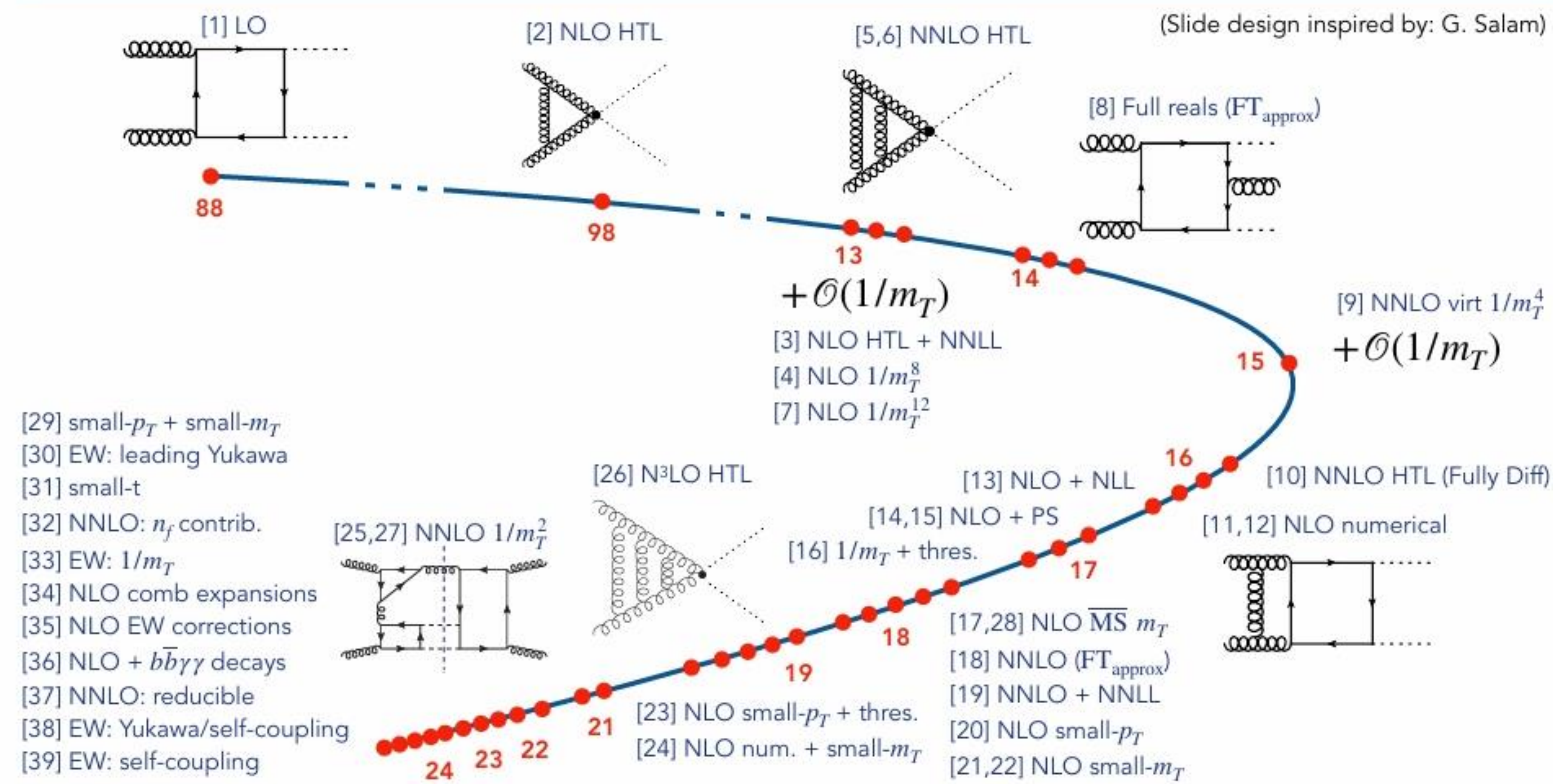
第八届全国重味物理与量子色动力学研讨会




[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23 [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner [39] Li, Si, Wang, Zhang, Zhao 24

**[Stephen Jones, Higgs Hunting 2024]**

# Outline

1. Motivation

2. Analytical calculation of the Master Integrals

3. Hadronic cross sections at the LHC

4. Leading logarithms in the amplitudes

5. Summary

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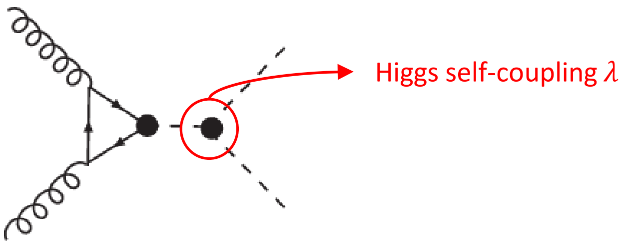
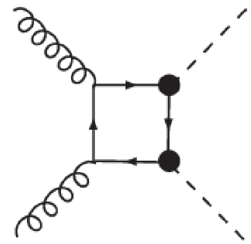
1. **Motivation**
2. **Analytical calculation of the master integrals**
3. **Hadronic cross sections at the LHC**
4. **Leading logarithms in the amplitudes**
5. **Summary**

*Based on: 2503.10051, 2509.06381 and work in progress*

# HH Production: QCD corrections



**NLO QCD:**

- Large-$m_t$ [Grigo, et al. '13]
- Threshold expansion [Gröber et al. '18]
- High-energy expansion [Davies et al. '18, '19]
- Small-$p_T$ expansion [Bonciani et al. '18]
- $m_h$ expansion [Xu Yang '18; Wang et al. '20]
- Small-t expansion [Davies et al. '23]
- Full $m_t$ dependence [Borowka et al. '16; Baglio et al. '20; Davies et al. '19]

**NNLO QCD:**

- Large-$m_t$ [Grigo et al. '15 ; Davies et al. '19; … ]
- Small-t expansion [Davies et al. '23; … ]

**$N^3LO$ QCD: Large-$m_t$** [Chen et al. '19; Ajjath et al. '22; Chen et al. '26]

**Top mass scheme uncertainties:**

[Baglio et al. '18, '20; Bagnaschi et al. '23; Jaskiewicz et al. '25]

## Loops and Leg in QFT '26

**Precise Higgs boson pair production**

*Bayreuth*

**Electro-weak corrections at NLO to Higgs pair production from gluon fusion**
*Augustin Vestner*

**gg->HH NLO EW corrections in the forward limit**
*Dominik Grau*
*Bayreuth* 15:00 - 15:30

**Three-loop virtual corrections to Higgs pair production: the $n_h^2$ contribution**
*Marco Vitti*

**Renormalisation of the general Two-Higgs-Doublet Model**
*Heidi Rzehak*
*Bayreuth* 16:00 - 16:30

16:30 - 17:00

**Analytic electroweak corrections at high energies**
*Hantian Zhang*
*Bayreuth* 17:00 - 17:30

**Electroweak corrections to Higgs boson pair production: The quark channel**
*Philipp Oscar Rendler*

**Three-loop QCD corrections to gg->HH at leading power in the high-energy limit.**
*Ajjath Abdul Hameed*

# Why bottom?

**Top-Bottom Interference Contribution to Fully Inclusive Higgs Production**

Michał Czakon (RWTH Aachen U.), Felix Eschment (RWTH Aachen U.), Marco Niggetiedt (Garching, Max Planck Inst.), Rene Poncelet (Cracow, INP), Tom Schellenberger (RWTH Aachen U.)
Dec 15, 2023

6 pages
Published in: *Phys.Rev.Lett.* 132 (2024) 21, 211902
Published: May 23, 2024
e-Print: 2312.09896 [hep-ph]

**Top-bottom interference effects in Higgs plus jet production at the LHC**

Jonas M. Lindert (Durham U., IPPP), Kirill Melnikov (KIT, Karlsruhe, TTP), Lorenzo Tancredi (KIT, Karlsruhe, TTP), Chris Wever (KIT, Karlsruhe, TTP)
Mar 10, 2017

6 pages
Published in: *Phys.Rev.Lett.* 118 (2017) 25, 252002
Published: Jun 22, 2017
e-Print: 1703.03886 [hep-ph]

## ✓ Enhanced by logarithms

**At one-loop level the bottom quark contribution to single Higgs production is suppressed by $m_b^2/m_H^2 \approx 0.0014$ but enhanced by $\ln^2(m_b^2/m_H^2) \approx 43.05$ relative to the top quark contribution.**

**Higgs pair production at the LO ($\sqrt{s}$ = 13 TeV):**

| $M_{HH}^{max}$ | $\sigma_t$(fb) | $\sigma_{t,b}$(fb) | $\frac{\sigma_{t,b}-\sigma_t}{\sigma_t}$ |
|---|---|---|---|
| 300 | 0.229(3) | 0.242(6) | 5.7% |
| 350 | 1.38(0) | 1.41(3) | 2.2% |
| 400 | 4.33(2) | 4.37(7) | 0.92% |

→ **NLO QCD corrections?**

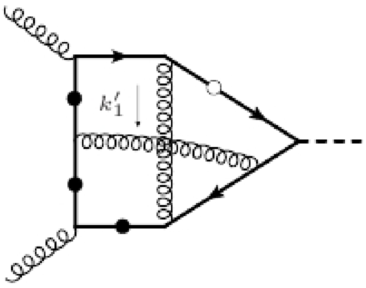
# Why analytical?

**Besides the well-known advantages, analytical results leave us a chance to understand the underlying (leading) logarithmic structures in the amplitudes.**

### Exact quark-mass dependence of the Higgs-gluon form factor at three loops in QCD

Micha L. Czakon (Aachen, Tech. Hochsch.), Marco Niggetiedt (Aachen, Tech. Hochsch.)
Jan 9, 2020

27 pages
Published in: *JHEP* 05 (2020) 149



### Higgs boson production and quark scattering amplitudes at high energy through the next-to-next-to-leading power in quark mass

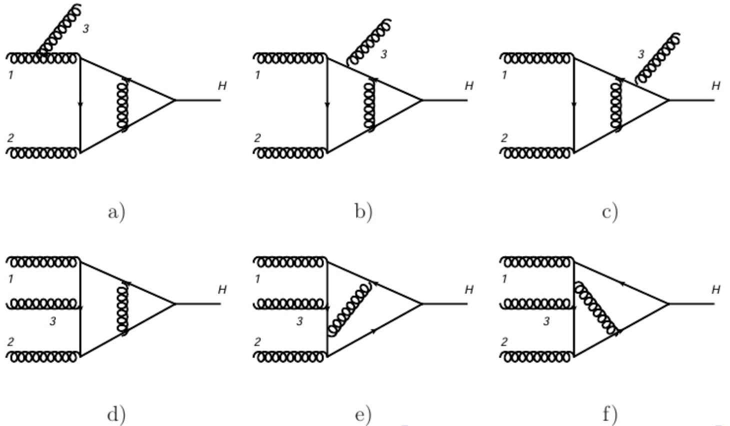
Tao Liu (Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Sneh Modi (Alberta U.), Alexander A. Penin (Alberta U.)
Nov 2, 2021

21 pages
Published in: *JHEP* 02 (2022) 170

### Light quark mediated Higgs boson production in association with a jet at the next-to-next-to-leading order and beyond

Tao Liu (Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Alexander A. Penin (Alberta U.), Abdur Rehman (Alberta U.)
Feb 28, 2024

18 pages
Published in: *JHEP* 04 (2024) 031
Published: Apr 5, 2024
e-Print: 2402.18625 [hep-ph]




[Melnikov, Penin '17]

# Master Integrals

**MIs: 11 (one-loop, 2 families)**
**177 (two-loop planar, 10 families)**
**49 (two-loop non-planar, 3 families)**

- **Scales:**

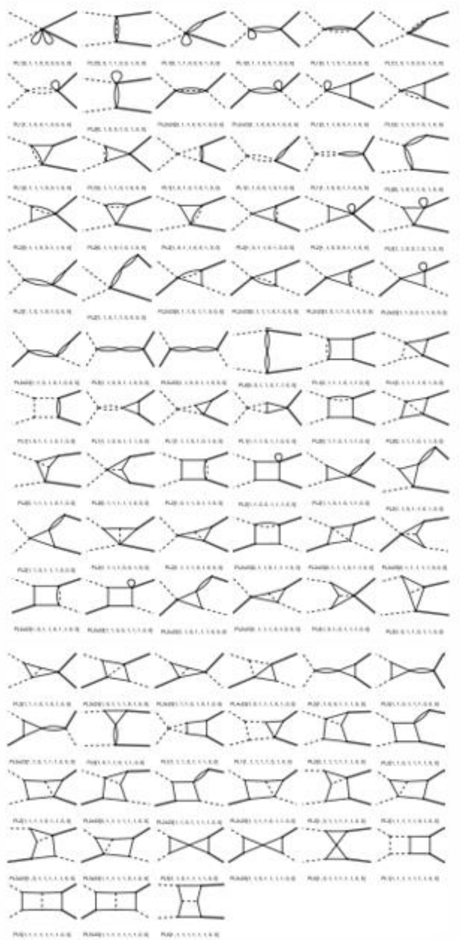
$$s = (p_1 + p_2)^2, u = (p_2 + p_3)^2, \ m_b^2, \ m_H^2$$

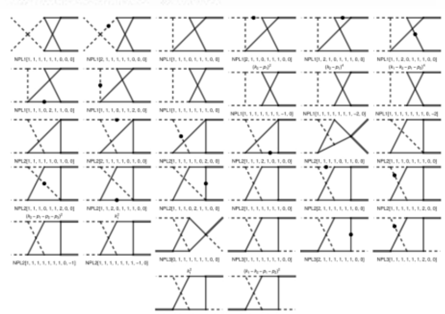
- **Hierarchy of scales:**

$$s, u, m_H^2 \gg m_b^2$$

- **Dimensionless variables $\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\kappa}$:**

$$s = m_H^2 \frac{(1+x)^2}{x}, \quad u = -m_H^2 z, \quad m_b^2 = m_H^2 \kappa$$





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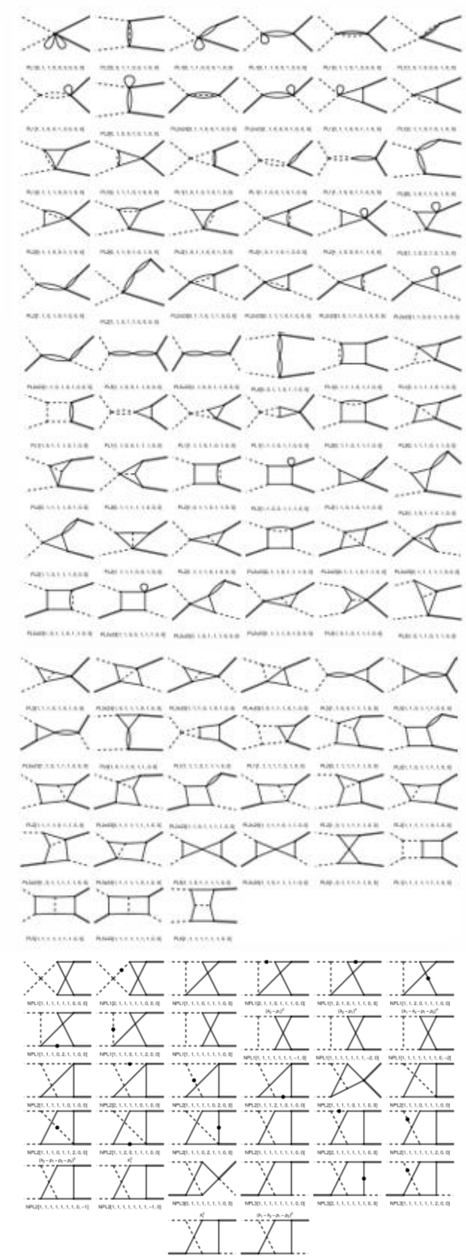
- **DEs:**

$$\partial_k \, I_i(x, z, \kappa, \epsilon) = A_{ij}^k(x, z, \kappa, \epsilon) I_j(x, z, \kappa, \epsilon), \qquad k \in \{x, z, \kappa\}$$

- **Ansatz for MIs** [Melnikov, et al. '16; Davies et al. '17, '18] **:**

$$I_i(x, z, \kappa, \epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^{2} \sum_{k=0}^{2} c_{n,j,k}^i(x, z, \epsilon) \kappa^{n-j\epsilon} \log^k \kappa$$



# Master Integrals

$$I_i(x,z,\kappa,\epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^{2} \sum_{k=0}^{2} c^i_{n,j,k}(x,z,\epsilon)\kappa^{n-j\epsilon} \log^k \kappa$$

- ✓ $\kappa$-differential equations
- ✓ Linear equations for $c^i_{n,j,k}$
- ✓ $x, z$-differential equations
- ✓ Fit boundary constants

1. Insert into $\kappa$-DEs

$$\partial_\kappa I_i(x,z,\kappa,\epsilon) = A^\kappa_{ij}(x,z,\kappa,\epsilon) I_j(x,z,\kappa,\epsilon)$$

2. Require the coefficients of $\kappa^{n-j\epsilon} \log^k \kappa$ are independent, leaving a linear system of equations.

3. Solve this system, arriving at independent $c^i_{n,j,k}$ with their number equal to that of MIs.

4. Expand $c^i_{n,j,k}$ in powers of $\epsilon$

$$c^i_{n,j,k}(x,z,\epsilon) = \sum \epsilon^r c^{i,(r)}_{n,j,k}(x,z)$$

# Master Integrals

$$I_i(x,z,\kappa,\epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^{2} \sum_{k=0}^{2} c_{n,j,k}^{i}(x,z,\epsilon)\kappa^{n-j\epsilon} \log^k \kappa$$

- ✓ $\kappa$-differential equations
- ✓ Linear equations for $c_{n,j,k}^{i}$
- ✓ $x, z$-differential equations
- ✓ Fit boundary constants

5. Insert into $x, z$-DEs

$$\partial_{x(z)} I_i(x,z,\kappa,\epsilon) = A_{ij}^{x(z)}(x,z,\kappa,\epsilon) I_j(x,z,\kappa,\epsilon)$$

6. Require the coefficient of $\epsilon^r \kappa^{n-j\epsilon} \log^k \kappa$ are independent, leaving a system of differential equations

7. Solve this system and express the results in terms of multiple polylogarithms (MPLs)

$$G(a_1,\ldots,a_n;z) = \int_0^z \frac{dt}{t-a_1} G(a_2,\ldots,a_n;t)$$

# Master Integrals

$$I_i(x,z,\kappa,\epsilon) = \sum_{n=n_{min}}^{n_{max}} \sum_{j=0}^{2} \sum_{k=0}^{2} c^i_{n,j,k}(x,z,\epsilon)\kappa^{n-j\epsilon}\log^k\kappa$$

- ✓ $\kappa$-differential equations
- ✓ Linear equations for $c^i_{n,j,k}$
- ✓ $x, z$-differential equations
- ✓ Fit boundary constants

8. Fit boundary conditions by numerical approach
   - Using PLSQ algorithm

Specific point

↙ ↘

Numerical results (AMFlow) ⇆ MPLs' results (GiNac)

- Require that we can get stable expression from results of at least 30 digits
  - Set smaller values of $\kappa$, e.g., $\kappa = 10^{-25}$
  - Increase the expansion power of $\kappa$

**MIs have been expanded to a sufficient order in $\kappa$ to ensure that each scalar integral in the amplitude is expanded to at least $m_b^2$ order.**

# Crosschecks on the MIs

- ✓ **One loop results are consistent with the results of** $package$-X **[Patel '16]**
- ✓ $c^{i}_{0,0,0}(x,z,\kappa)$ **agree with the massless internal case**
- ✓ **Set different groups of numerical values to** $x,\ z,\ \kappa$ **and compare with the results obtained by AMFlow**
- ✓ **Choose another basis of MIs and check their differential equations**

# Crosschecks on the MIs

- ✓ One loop results are consistent with the results of $package$-X [Patel '16]
- ✓ $c^{i}_{0,0,0}(x, z, \kappa)$ agree with the massless internal case
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| Family | PL5 | NPL1 | NPL2 | NPL3 |
|---|---|---|---|---|
| File size | 30 MB | 2.5 GB | 10.1 GB | 18.7 GB |

Method of finite fields for simplifications during our calculation!

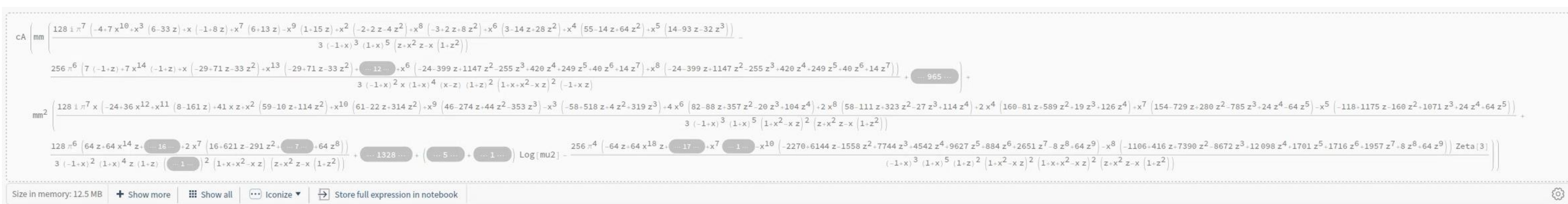
# Two-loop amplitudes

- **We reproduced the results in the literature if bottom quarks are replaced by top quarks.** [Davies, Mishima, Steinhauser, Wellmann '19]

- **UV/IR finite amplitude at two loops were obtained numerically after renormalization and IR subtractions.**

$$\frac{1}{\epsilon^2} \propto G[\{0\},x] + G\left[\{0\}, \frac{1+x^2}{x} - z\right] - G[\{-i\},x] - G[\{i\},x] - G\left[\left\{\frac{1+x^2}{x}\right\}, z\right] = 0$$

$$\frac{1}{\epsilon} \propto$$




$$\mathrm{cA}\left(\mathrm{mm}\left(\left(0.\times10^{-73}+0.\times10^{-73}\,i\right)+\left(0.\times10^{-76}+0.\times10^{-78}\,i\right)\mathrm{Log}[\mathrm{mu2}]\right)+\mathrm{mm}^2\left(\left(0.\times10^{-73}+0.\times10^{-73}\,i\right)+\left(0.\times10^{-75}+0.\times10^{-75}\,i\right)\mathrm{Log}[\mathrm{mm}]+\left(0.\times10^{-76}+0.\times10^{-78}\,i\right)\mathrm{Log}[\mathrm{mm}]^2+\left(0.\times10^{-76}+0.\times10^{-78}\,i\right)\mathrm{Log}[\mathrm{mu2}]\right)\right)$$

$$x \to \frac{1}{8}, z \to 4$$

- **MPLs of high weight are cancelled in the final amplitude. The number of MPLs have been reduced to about 2000 from about 56000.**

# Hadronic cross section

## ggxy: A flexible library to compute gluon-induced cross sections

Joshua Davies (Liverpool U., Dept. Math.), Kay Schönwald (Zurich U.), Matthias Steinhauser (KIT, Karlsruhe, TTP), Daniel Stremmer (KIT, Karlsruhe, TTP)

Jun 4, 2025

28 pages

Published in: *Comput.Phys.Commun.* 320 (2026) 109933

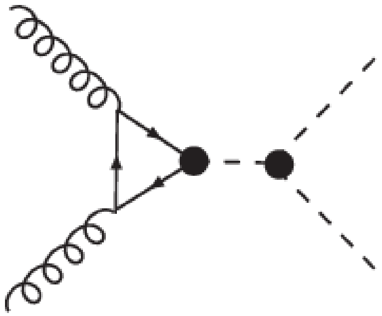
Published: Nov 14, 2025

e-Print: 2506.04323 [hep-ph]

DOI: 10.1016/j.cpc.2025.109933 (publication)

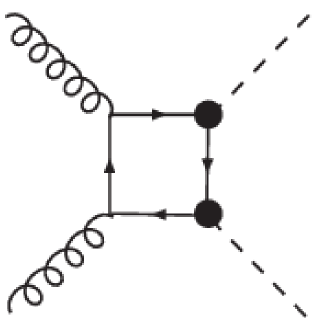
Main revision: five massless flavors → four massless flavors

# Leading Logarithms



QCD corrections can be analyzed in the same way as single Higgs production.

[Liu, Penin '17,'18; Anastasiou, Penin '20; Liu, Modi, Penin '21]
[Liu, Neubert, Schnubel, Wang '22]



Two form factors for box diagrams.

Analytical results of top quark contributions in the high-energy limit show that the maximal powers of $\ln\rho$ in form factor $F_{box1}^{1,C_F}$ are 1 and 4 at leading and next-to-leading power, respectively.

[Davies, Mishima, Steinhauser, Wellmann '19]

No $C_A - C_F$ color structure after IR subtraction.

# High-energy limit

$$m_h^2 \ll m_t^2 \ll s, |t| \quad \rho = \frac{m_t^2}{s}$$

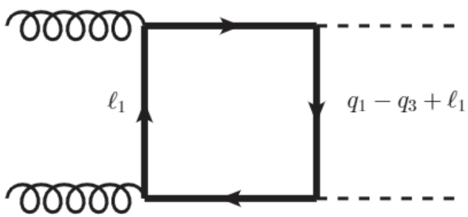
**Sudakov parameterization:** $\ell_1 = q_1 u_1 + q_2 v_1 + \ell_{1\perp}$

$$\frac{1}{\ell_1^2 - m_t^2} \approx -i\pi\, \delta(2q_{12}u_1 v_1 + \ell_{1\perp}^2 - m_t^2)$$

$$\frac{1}{(q_1 + \ell_1)^2 - m_t^2} \approx \frac{1}{2q_{12}v_1}$$

$$\frac{1}{(q_2 - \ell_1)^2 - m_t^2} \approx \frac{-1}{2q_{12}u_1}$$

$$(q_3 \cdot \ell_1)^2 = \frac{1}{2} q_{3\perp}^2 \ell_{1\perp}^2 = m_t^2 \times \frac{q_{13}q_{23}}{q_{12}}$$




Four symmetric configurations

# High-energy limit

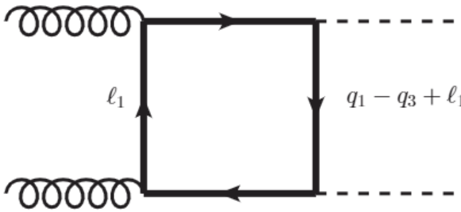
$$m_h^2 \ll m_t^2 \ll s, |t| \quad \rho = \frac{m_t^2}{s}$$

**Sudakov parameterization:** $\ell_i = q_m u_i + q_n v_i + \ell_{i\perp}$

$$\frac{1}{\ell_i^2 - m_t^2} \approx -i\pi\, \delta\left(2q_{mn} u_1 v_1 + \ell_{i\perp}^2 - m_t^2\right)$$

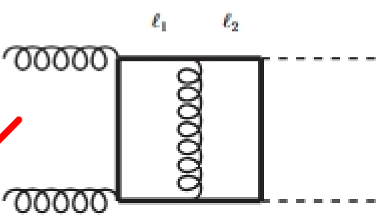
$$\frac{1}{(q_m + \ell_i)^2 - m_t^2} \approx \frac{1}{2q_{mn} v_i}$$

$$\frac{1}{(q_n - \ell_i)^2 - m_t^2} \approx \frac{-1}{2q_{mn} u_i}$$

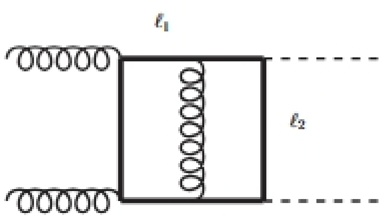



**Two-loop abelian diagrams:**

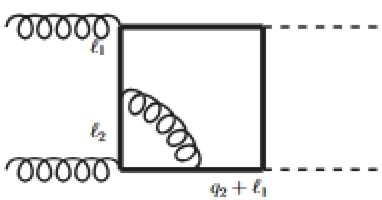
$$\frac{1}{(\ell_1 - \ell_2)^2} \approx \frac{1}{-2q_{13}\left(u_1 v_2 + v_2 u_1\right) + \ell_{1\perp}^2 + \ell_{2\perp}^2 + \ \ldots}$$



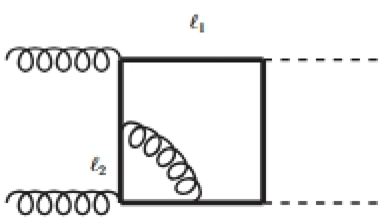

(a₁)



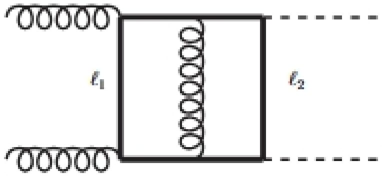

(a₂)



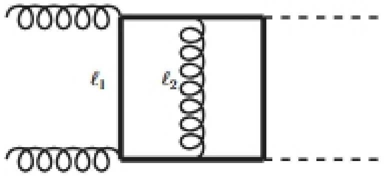

(b₁)




(b₂)




(a₃)




(a₄)

# High-energy limit

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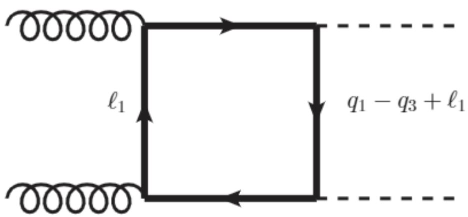
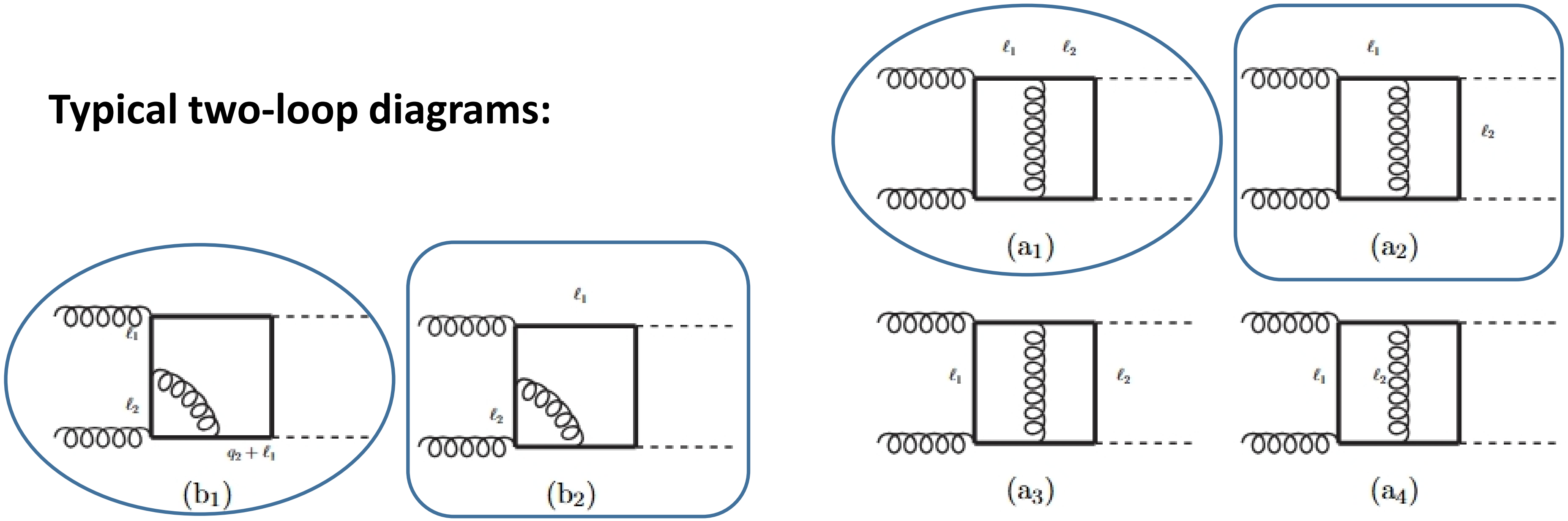



$$\frac{1}{\ell_i^2 - m_t^2} \approx -i\pi\, \delta\left(2q_{mn}u_1 v_1 + \ell_{i\perp}^2 - m_t^2\right)$$

$$\frac{1}{(q_m + \ell_i)^2 - m_t^2} \approx \frac{1}{2q_{mn}v_i}$$

$$\frac{1}{(q_n - \ell_i)^2 - m_t^2} \approx \frac{-1}{2q_{mn}u_i}$$

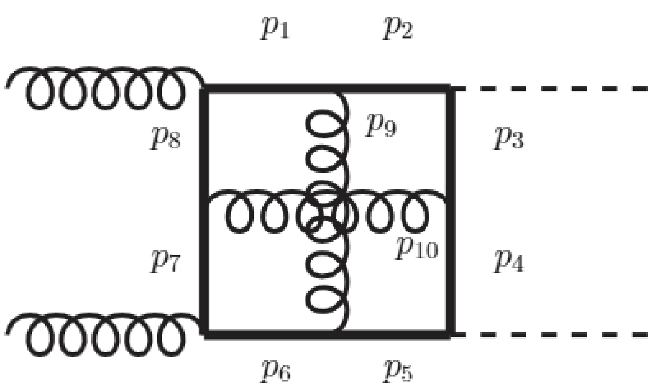
**Typical two-loop diagrams:**



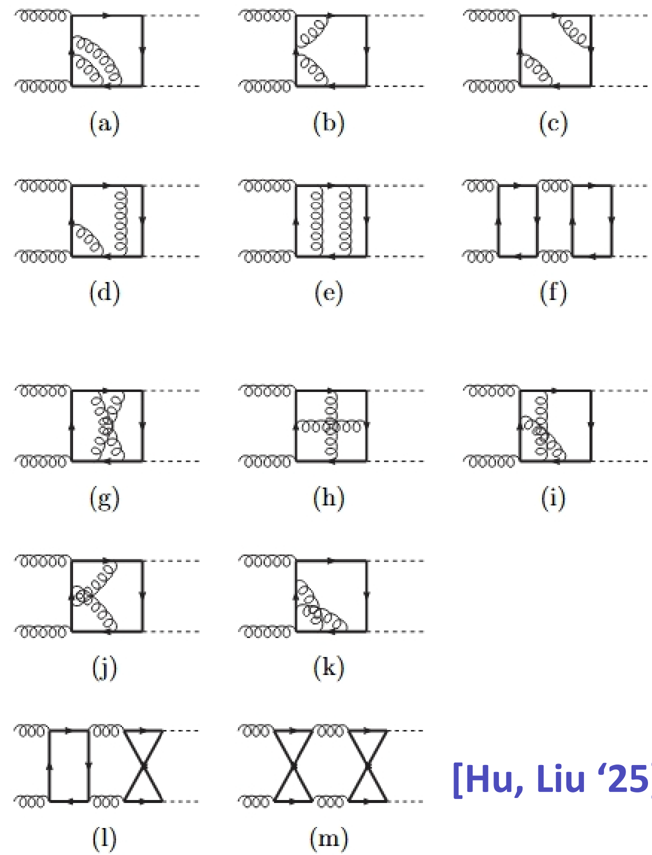
# One three-loop example

$$m_h^2 \ll m_t^2 \ll s, |t| \quad \rho = \frac{m_t^2}{s}$$

**After obtaining the correct LLs at two loops, we considered the three-loop abelian corrections.**




36 non-vanishing configurations!



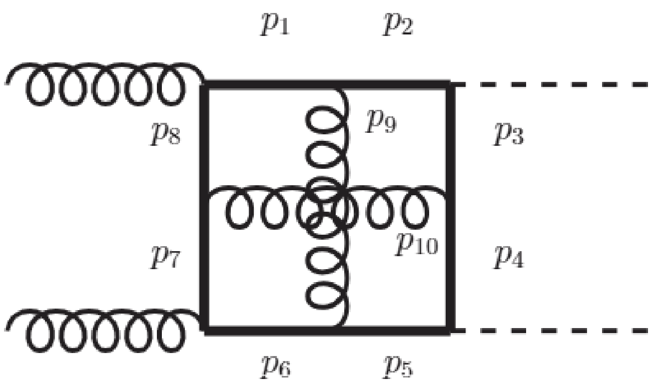

| Soft propagators of logarithmic configurations | Constraints on $u_i$ and $v_i$ |
|---|---|
| $p_{1,2,7}$ | $u_3 > u_1 > u_2,\quad u_3 > v_2 > v_1,\quad v_3 > v_1$ |
| $p_{1,2,10}$ | $u_1 > u_2,\quad u_3 > u_2,\quad v_3 > v_1,\quad v_2 > v_1$ |
| $p_{1,3,10}$ | $u_2 > u_3 > u_1,\quad u_2 > v_3 > v_2,\quad v_1 > v_2$ |
| $p_{1,9,4}$ | $u_1 > u_2 > u_3,\quad v_3 > v_2 > v_1,\quad u_1 > v_2,\quad v_3 > u_2$ |
| $p_{1,9,6}$ | $u_1 > u_2 > u_3,\quad v_3 > v_2 > v_1,\quad u_1 > v_2,\quad v_3 > u_2$ |
| $p_{1,9,5}$ | $u_1 > u_2 > u_3,\quad v_3 > v_2 > v_1,\quad v_3 > u_1$ |

[Hu, Liu '25]

# One three-loop example

$$m_h^2 \ll m_t^2 \ll s, |t| \quad \rho = \frac{m_t^2}{s}$$

**After obtaining the correct LLs at two loops, we considered the three-loop abelian corrections.**




36 non-vanishing configurations!

| Soft propagators of logarithmic configurations | Constraints on $u_i$ and $v_i$ |
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| $p_{1,2,7}$ | $u_3 > u_1 > u_2,\quad u_3 > v_2 > v_1,\quad v_3 > v_1$ |
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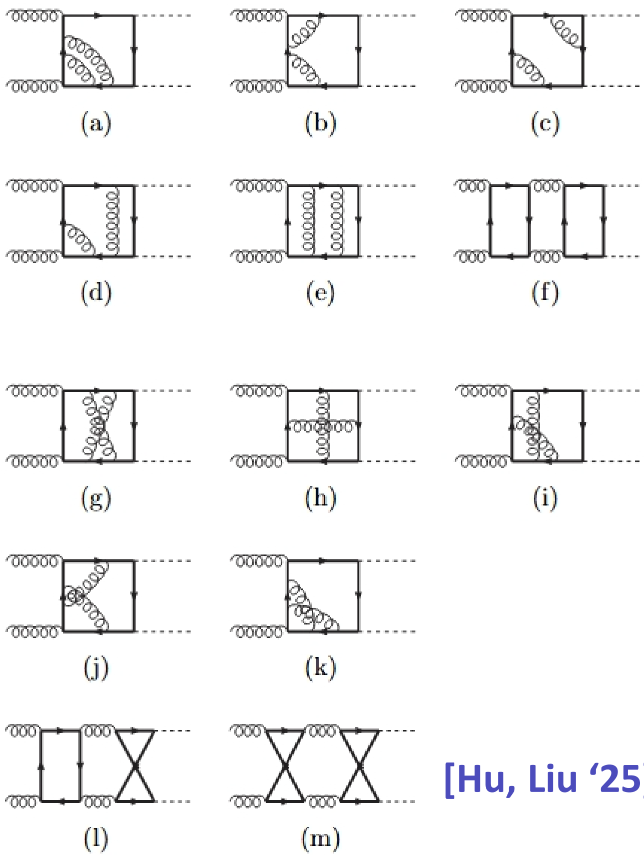


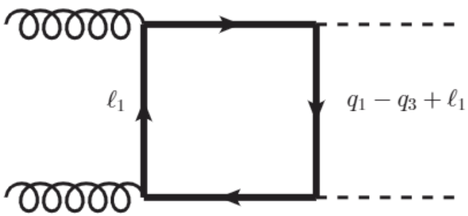

[Hu, Liu '25]

**More leading logarithmic configurations at higher loops?**

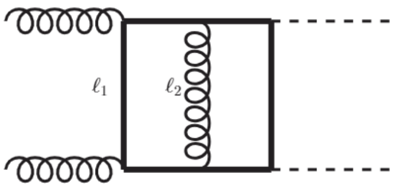
# Bottom contribution

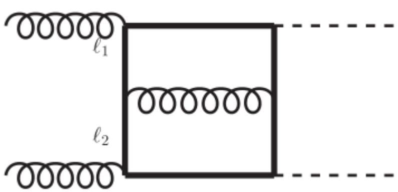
$$m_b^2 \ll m_h^2, s, |t|$$

$$\kappa = \frac{m_b^2}{m_h^2}\ ,\ \alpha_s^n \ln^{2n}\kappa$$

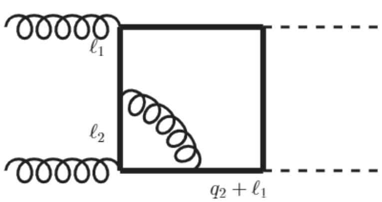



One LL configuration.

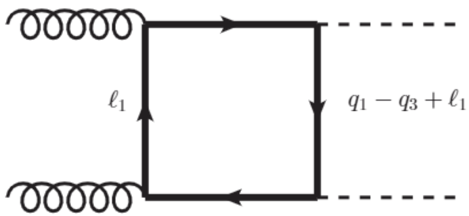



**Contribute only to one form factor**



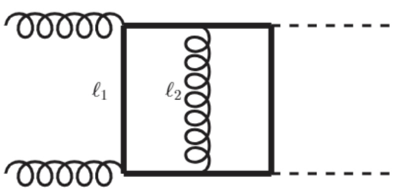
# Bottom contribution

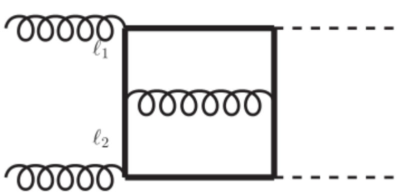


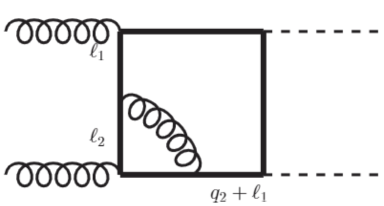

One LL configuration.

$$m_b^2 \ll m_h^2, s, |t|$$

$$\kappa = \frac{m_b^2}{m_h^2}\ ,\ \alpha_s^n \ln^{2n}\kappa$$




**Contribute only to one form factor**

$$\text{f0bx1} = \frac{2\,\text{mb2}^2\left(t^2 + 6\,t\,u + u^2 - 4\,\text{mh2}\,(t+u)\right)\text{Log}\left[\frac{\text{mb2}}{\text{mh2}}\right]^2}{t\,u\,(-2\,\text{mh2} + t + u)^2};$$

$$\text{f0bx2} = \frac{2\,\text{mb2}^2\left(\text{mh2}^2 - t\,u\right)\left(t^2 + u^2\right)\text{Log}\left[\frac{\text{mb2}}{\text{mh2}}\right]^2}{t^2\,u^2\,(-2\,\text{mh2} + t + u)^2};$$

$$\text{f1tri} = \frac{(\text{cA} - \text{cF})\,\text{mb2}\,(4\,\text{mb2} - 2\,\text{mh2} + t + u)\,\text{Log}\left[\frac{\text{mb2}}{\text{mh2}}\right]^4}{12\,(-2\,\text{mh2} + t + u)^2};$$

$$\text{f1bx1} = \frac{(\text{cA} - \text{cF})\,\text{mb2}^2\left(t^2 + 6\,t\,u + u^2 - 4\,\text{mh2}\,(t+u)\right)\text{Log}\left[\frac{\text{mb2}}{\text{mh2}}\right]^4}{12\,t\,u\,(-2\,\text{mh2} + t + u)^2};$$

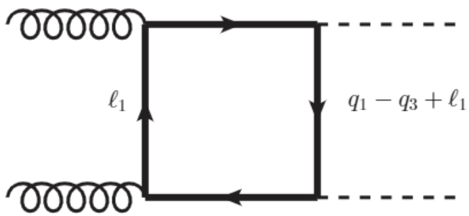
$$\text{f1bx2} = \frac{(2\,\text{cA} - 3\,\text{cF})\,\text{mb2}^2\left(\text{mh2}^2 - t\,u\right)\left(t^2 + u^2\right)\text{Log}\left[\frac{\text{mb2}}{\text{mh2}}\right]^4}{12\,t^2\,u^2\,(-2\,\text{mh2} + t + u)^2};$$

**Agree with what we get from calculations of MIs.**

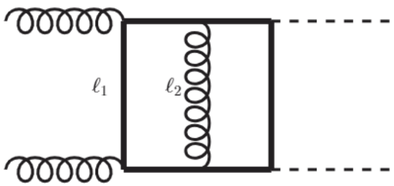
# Bottom contribution

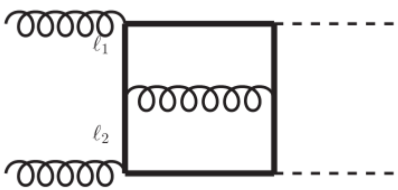
$$m_b^2 \ll m_h^2, s, |t|$$

$$\kappa = \frac{m_b^2}{m_h^2}\ ,\ \alpha_s^n \ln^{2n}\kappa$$

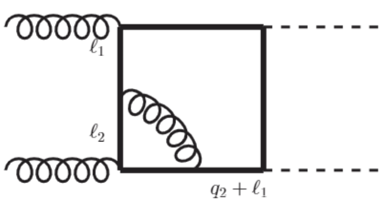



One LL configuration.




Contribute only to one form factor




**How about three loops?**

**If no new DL configurations, all-order results should not be a big problem.**

# Summary

- **All MIs have been calculated analytically through differential equations, and we also made several crosschecks on them.**
- **UV and IR finite two-loop amplitudes for Higgs pair production are obtained. Currently, the corresponding total and differential cross section are calculated with the help of C++ library ggxy.**
- **The leading logarithmic analysis for bottom mediated amplitudes looks similar but a little simpler than what we have done for top quarks in the high-energy limit.**

**Thanks for your attention!**