

# Exclusive B-meson decays beyond leading power

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# Heavy hadron decays: a multi-scale problem

NP scale	EW scale	Heavy quark scale	Intermediate scale	Hadronization scale
TeV or beyond	$m_W$	$m_b(m_c)$	$\sqrt{m_b \Lambda}, m_c$	$\Lambda$
$\mathcal{L}_{NP}$	$\mathcal{L}_{SM} + \mathcal{L}_{D>4}$	$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i C_i O_i + \sum_i C'_i O'_i$	SCET-I	SCET-II/HQET

$$\langle f | O_i | B \rangle = \mathcal{A}_{LP} + \mathcal{O}\left(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_f}\right)$$

- Factorization** {
- Perturbation: matching, resummation, evolution
  - Nonperturbation: Lattice simulation, sum rules...

# Overview of power corrections in exclusive B decays

## ◆ $B \rightarrow \gamma \ell \nu$ : higher twist, Heavy quark expansion, hadronic photon...

Beneke et al, 1804.14962; Cui et al, 2023.16436

## ◆ $B_q \rightarrow \gamma \gamma$ : higher twist, Heavy quark expansion, hadronic photon...

YLS etc., 2009.02723, Qin etc. *PRL*. 131 (2023) 9, 091902, Beneke et al 2008.12494; Li et al, 2205.05528

## ◆ Annihilation diagrams in nonleptonic B decays

Lü, YLS, Wang, Wang, NPB990(2023)116175, Stillger, PHD Thesis, DOI: [10.25358/openscience-11523](https://doi.org/10.25358/openscience-11523)

## ◆ Long distance quark loop in FCNC semileptonic decays

A. Khodjamirian etc., 1006.4945, N. Gubernari etc., 2011.09813

## ◆ Heavy-to-light form factors from light-cone sum rules

Cui, Huang, YLS, Wang, Wang, *JHEP* 03 (2023) 140, Gao, Meißner, YLS, Li, *PRD* 112 (2025) 1, 1

Gao, Huber, Ji, Wang, Wang, Wei, *JHEP* 05 (2022) 024, Cui, Huang, Wang, Zhao, *Phys.Rev.D* 108 (2023) 7, 071504

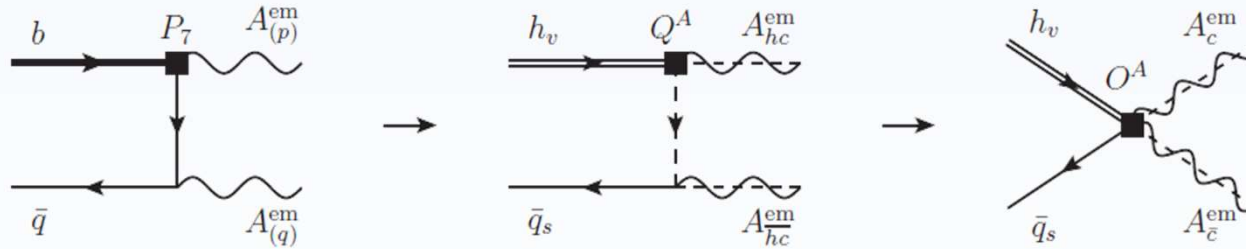
## ◆ Power correction to HQET B meson decay constant: see Yan-Bing Wei's talk

## ◆ .....

# QCD factorization in B meson decays

- Example  $B_q \rightarrow \gamma\gamma$

YLS, Wang Wei, 2009.02723



Integrate out b-mass:  
hard function

Integrate out hard-collinear propagator:  
jet function

- Factorization formula @ leading power

$$\mathcal{A}_{LP} \sim \frac{G_F}{\sqrt{2}} \lambda_{CKM} f_B m_B C(m_b, \mu) \int_0^\infty \frac{d\omega}{\omega} J(\sqrt{E}\omega, \mu) \phi_B^+(\omega, \mu)$$

Soft dynamics

- Power corrections: HQE, higher twist...

## Distribution amplitudes of B meson

- The definition of leading twist B meson DA

$$\langle 0 | \bar{q}_\beta(z) [z, 0] h_{v\alpha}(0) | \bar{B}(v) \rangle = -\frac{i}{4} F_B m_B \left[ \frac{1 + v \cdot \gamma}{2} \left\{ 2\tilde{\phi}^+(t, \mu) + \frac{\tilde{\phi}^-(t, \mu) - \tilde{\phi}^+(t, \mu)}{2} z \cdot \gamma \right\} \gamma_5 \right]_{\alpha\beta}$$

- The Evolution: Lange-Neubert equation Lange, Neubert, 2003

$$\mu \frac{d}{d\mu} \phi^+(\omega, \mu) = - \left[ \Gamma_{\text{cusp}}(\alpha_s) \ln \frac{\omega}{\mu} + \gamma_+(\alpha_s) \right] \phi^+(\omega, \mu) - \omega \int_0^\infty d\eta \Gamma_+(\omega, \eta, \alpha_s) \phi^+(\eta, \mu)$$

- The evolution equation at two loops: Braun, Ji, Manashov, 2019
- Solution to the LN equation: **dual space**: Bell, Feldmann, YMW, Yip, 2013:
- Solution to the evolution equation @ two loop level: Galda, Neubert, 2020
- Modelling the B meson LCDA: Grozin, Neubert, 1997; KKQT, 2001; Braun etc. 2003
- Lattice simulation via LaMET: see Qi-An Zhang's talk

## Higher twist

- Two-particle LCDAs: Braun, Ji, Manashov, 2017

$$\langle 0 | \bar{q}(z) \Gamma[z, 0] h_v(0) | \bar{B}(v) \rangle = -\frac{i}{4} F_B \int_0^\infty d\omega e^{-i\omega \cdot x} \{ 2 \text{Tr}[\gamma_5 \Gamma P_+] [\phi^+(\omega) + x^2 g^+(\omega)] + \text{Tr} \left[ \gamma_5 \Gamma P_+ \frac{\gamma \cdot x}{v \cdot x} \right] [(\phi^+ - \phi^-) + x^2 (g^+ - g^-)] \}$$

- Three-particle LCDAs:

$$\begin{aligned} & \langle 0 | \bar{q}(nz_1) g G_{\mu\nu}(nz_2) \Gamma h_v(0) | \bar{B}(v) \rangle \\ &= -\frac{i}{4} F_B \text{Tr} \{ \gamma_5 \Gamma P_+ [(v_\mu \gamma_\nu - v_\nu \gamma_\mu) (\Psi_A - \Psi_V) - i \sigma_{\mu\nu} \Psi_V - (n_\mu v_\nu - n_\nu v_\mu) X_A \\ &+ (n_\mu \gamma_\nu - n_\nu \gamma_\mu) (W + Y_A) - i \epsilon_{\mu\nu\alpha\beta} n^\alpha v^\beta \gamma_5 \tilde{X}_A + i \epsilon_{\mu\nu\alpha\beta} n^\alpha \gamma^\beta \gamma_5 \tilde{Y}_A] - (n_\mu v_\nu - n_\nu v_\mu) \gamma \cdot n W \\ &+ (n_\mu \gamma_\nu - n_\nu \gamma_\mu) \gamma \cdot n Z \} \end{aligned}$$

- Evolution at large  $N_c$  limit
- Twist expansion is not equivalent to power expansion

Examples: Correlation functions in B-meson LCSR

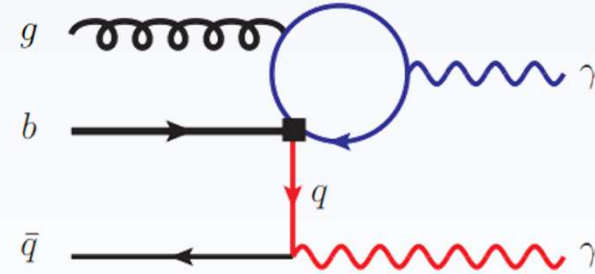
Wang, YLS, 1506.00667

Annihilation contribution in  $B \rightarrow (\pi, K) \ell^+$

Huang, YLS, Wang, Wang, *PRL*. 134 (2025) 9, 091901

## Long distance quark loop

- Soft gluon emitted from quark loop
- Power counting of charm quark mass



Qin, YLS, Wang, Wang, *PRL*. 131 (2023) 9, 091902

$$\Lambda_{QCD} \ll m_c \sim m_b \quad \text{or} \quad m_c^2 \approx m_b \Lambda_{QCD}:$$

- Integrating out quark loop:

$$\begin{aligned} & \mathcal{M}(g + b \rightarrow q + \gamma) \\ &= \frac{iG_F}{\sqrt{2}} \frac{eg_s}{4\pi^2} \sum V_{pb} V_{pq}^* \left\{ \left( C_2 - \frac{C_1}{2N_c} \right) Q_p [F(z_p) - 1] + 6C_6 \sum Q_{q'} [F(z_{q'}) - 1] \right. \\ & \left. + \left[ \left( C_3 - \frac{C_4}{2N_c} \right) + 16 \left( C_5 - \frac{C_6}{2N_c} \right) \right] Q_q [F(z_q) - 1] \right\} \bar{q}(\tilde{q}) \gamma_\beta P_L G_{\mu\alpha} \tilde{F}^{\mu\beta} b(v) \frac{p^\alpha}{(p - \ell)^2} \end{aligned}$$

- B-meson soft function

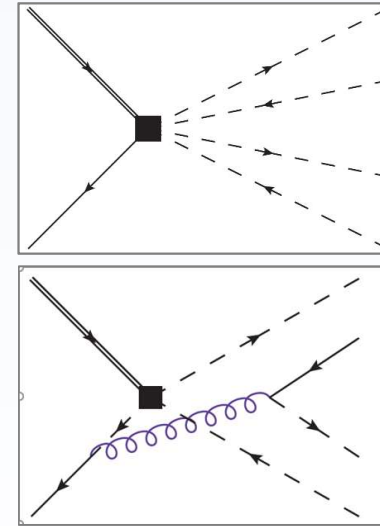
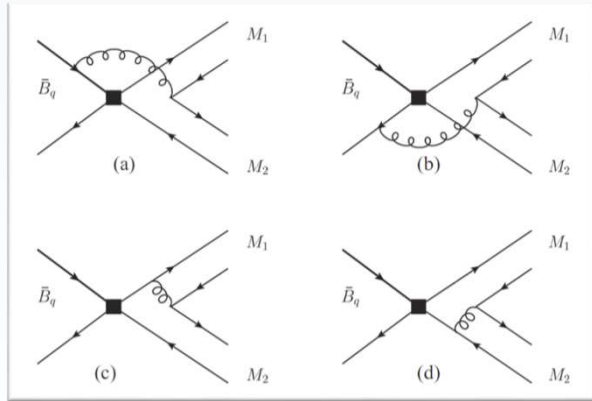
$$\begin{aligned} & \langle 0 | (\bar{q}_s S_n)(\tau_1 n) S_n^\dagger S_{\bar{n}}(0) S_{\bar{n}}^\dagger g_s G_{\mu\nu} S_{\bar{n}}(\tau_2 \bar{n}) \bar{n}^\nu n \cdot \gamma \gamma_\perp^\mu \gamma_5 S_{\bar{n}}^\dagger h_\nu(0) | \bar{B}_\nu \rangle \\ &= 2F_B(\mu) m_B \int_{-\infty}^{\infty} d\omega_1 d\omega_2 e^{-i(\omega_1 \tau_1 + \omega_2 \tau_2)} \Phi_G(\omega_1, \omega_2, \mu) \end{aligned}$$

- Normalization

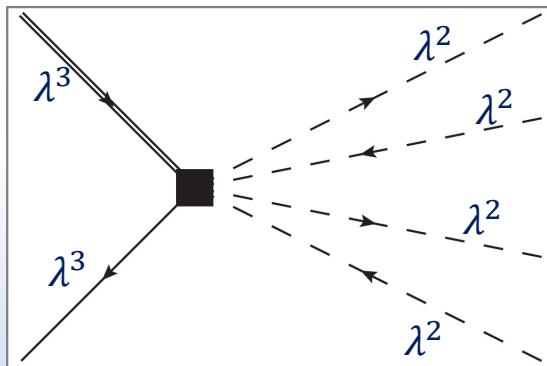
$$\begin{aligned} \int_{-\infty}^{\infty} d\omega_1 \Phi_G(\omega_1, \omega_2, \mu) &= \int_0^{\infty} d\omega_1 \Phi_4(\omega_1, \omega_2, \mu) \\ \int_{-\infty}^{\infty} d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) &= \int_0^{\infty} d\omega_2 \Phi_5(\omega_1, \omega_2, \mu) \\ \int_{-\infty}^{\infty} d\omega_1 d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) &= \frac{\lambda_E^2 + \lambda_H^2}{3} \end{aligned}$$

# Annihilation diagrams in nonleptonic decays

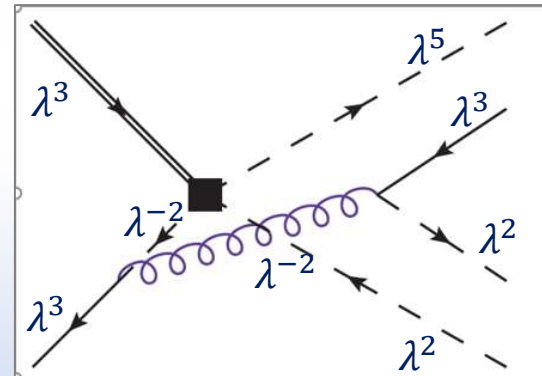
- Integrating out hard modes



- Power counting in effective theory



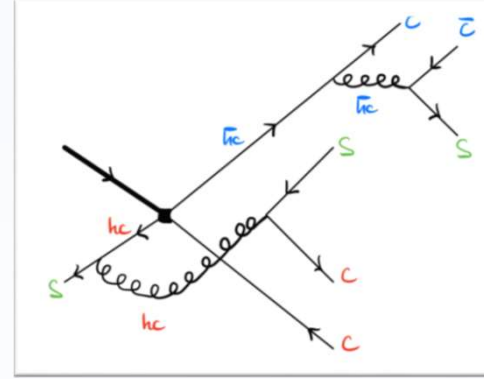
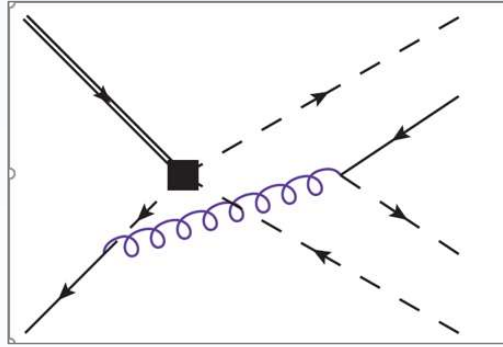
$\sim \lambda^{14}$



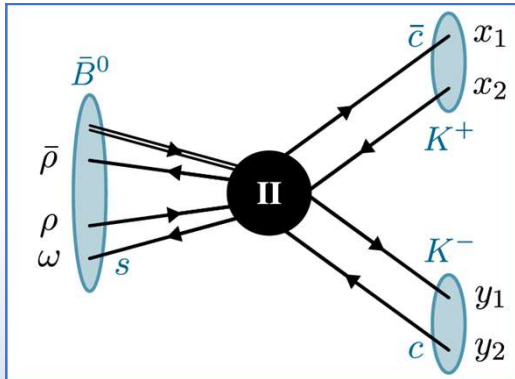
$\sim \lambda^{14}$

# Annihilation diagrams in nonleptonic decays

- Integrating out hard-collinear modes



- 4-particle B meson soft function Stillger, PHD Thesis, DOI: [10.25358/openscience-11523](https://doi.org/10.25358/openscience-11523)

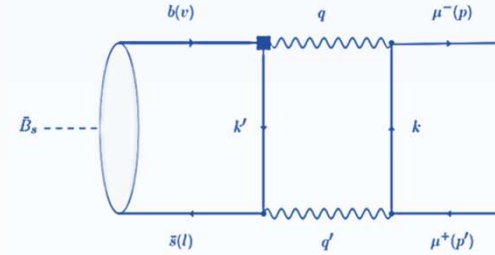
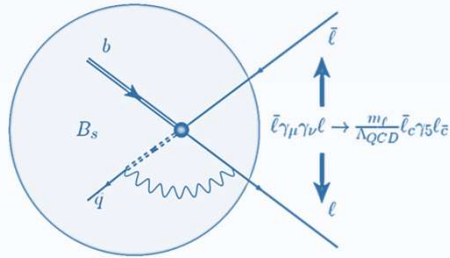


$$\langle 0 | (\bar{q}_s S_{\bar{n}})(\tau_2 \bar{n}) \bar{n} \cdot \gamma n \cdot \gamma \gamma_{\perp}^{\mu} S_{\bar{n}}^{\dagger} S_n (S_n^{\dagger} q_s)(s n) (\bar{q}_s S_n)(\tau_1 n) \gamma_{\perp}^{\mu} \bar{n} \cdot \gamma n \cdot \gamma (1 - \gamma_5) S_{\bar{n}}^{\dagger} h_{\nu} | \bar{B}_{\nu} \rangle$$

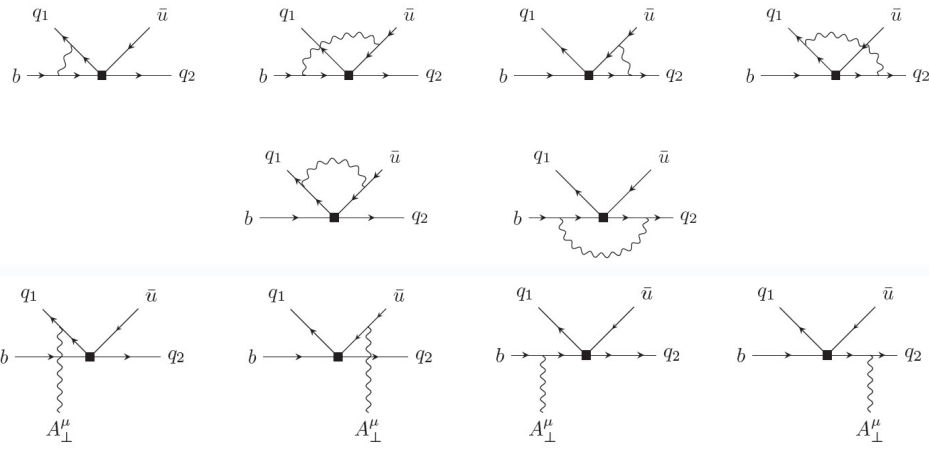
$$= i \tilde{f}_B \int d\tilde{\omega} d\rho d\bar{\rho} e^{-i\tilde{\omega}\tau_1 + \bar{\rho}\tau_2 + \rho} \phi_4^{\perp}(\tilde{\omega}, \bar{\rho}, \rho)$$

# Structure dependent QED corrections

- Power enhancement and double logarithmic enhancement in  $B_{(s)} \rightarrow \mu^+ \mu^-$  : Beneke et al 1908.07111



- QED corrections to hadronic B decays



- ◆ QCD×QED Factorization Beneke et al 2008.10615

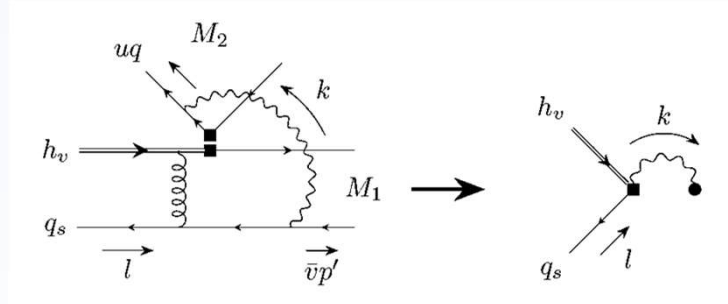
$$\begin{aligned}
 & \langle M_1 M_2 | Q_i | \bar{B}_v \rangle \\
 &= im_B^2 \left\{ \mathcal{F}_{Q_2}^{BM_1}(0) \int_0^1 du T_{i,\otimes}^{II}(u) \mathcal{F}_{M_2} \phi_{M_2}(u) \right. \\
 & \left. + \int_{-\infty}^{\infty} d\omega \int_0^1 dudv T_{i,\otimes}^{II}(u, v, \omega) \mathcal{F}_{M_2} \phi_{M_2}(u), \mathcal{F}_{M_2} \phi_{M_2}(u) \right\}
 \end{aligned}$$

- ◆ QED-Generalized form factors and Distribution amplitudes

- ◆ No power enhancement, numerically small

# QED-generalized distribution amplitudes

- Definition:



$$\frac{\langle 0 | (\bar{q}_s^{(q)} S_n^q) (tn) n \cdot \gamma \gamma_5 S_n^{q\dagger} h_\nu(0) S_n^{\dagger Q_{M_1}} S_{\bar{n}}^{Q_{M_2}} | \bar{B}_\nu \rangle}{\langle 0 | S_n^{\dagger Q_{M_1}} S_{\bar{n}}^{Q_{M_2}} | 0 \rangle} = iF(\mu) \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \Phi_{B,\otimes}(\omega, \mu)$$

$$S_n^q(x) = \exp \left\{ -iQ_q e \int_0^\infty n \cdot A_s(x + sn) \right\} P \exp \left\{ -ig_s \int_0^\infty n \cdot G_s(x + sn) \right\}$$

- Renormalization scale evolution Beneke et al 2204.09091.

## Other discussions on B-meson soft functions

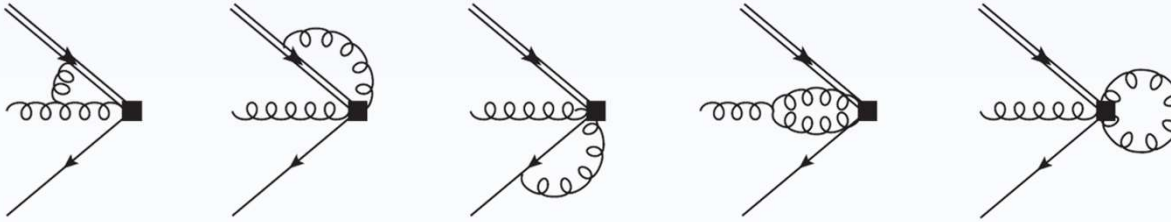
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- General discussion in FCNC processes Melikhov, 2208.04907, 2302.13673
- Charming loops in  $B_s \rightarrow \gamma\gamma, B_s \rightarrow \gamma\ell^+\ell^-$  Belov et al, 2309.00358; 2404.01222
- Multiparticle contribution Melikhov, 2507.23441

# Evolution of B-meson soft function $\Phi_G(\omega_1, \omega_2, \mu)$

- Feynman diagrams for Renormalization constant

Huang, Ji, YLS, Wang, Wang, *PRL*. 133 (2024) 17, 171901



- Difference from LCDA

$$\begin{aligned} \Pi_e^{(1)} = & \frac{\alpha_s}{4\pi} \frac{C_A}{2\pi i} \Gamma(\epsilon) \bar{q}(ig_s) n \cdot \gamma (\bar{n} \cdot q \gamma_\perp^\mu - \bar{n}^\mu q_\perp \cdot \gamma) \gamma_5 h_\nu T^a \left\{ \prod_{i,j=1,2}^{i,j=2,1} \left\{ \theta(\omega_i) \left( \left[ \frac{\theta(\omega_i - \omega'_i)}{\omega_i - \omega'_i} \right]_{\oplus} + \left[ \frac{\theta(\omega'_i - \omega_i)}{\omega'_i (\omega'_i - \omega_i)} \right]_{+} \omega'_i \right) + \right. \right. \\ & \left. \left. \theta(-\omega_i) \left( \left[ \frac{\theta(\omega'_i - \omega_i)}{\omega_i - \omega'_i} \right]_{\oplus} + \left[ \frac{\theta(\omega_i - \omega'_i)}{(\omega'_i - \omega_i)} \right]_{+} \right) \omega'_i + i\pi \delta(\omega'_i - \omega_i) [\theta(\omega'_j - \omega_j) - \theta(\omega_j - \omega'_j)] \right\} - 2\pi i \delta(\omega'_1 - \omega_1) \delta(\omega'_2 - \right. \\ & \left. \omega_2) \left[ \frac{1}{\epsilon} + \ln \frac{\mu^2}{|\omega_1 \omega_2|} + \frac{i\pi}{2} \right] \right\} \end{aligned}$$

- ◆ The argument will be evaluated from positive value to negative value
- ◆ The anomalous dimension must be complex

## Evolution of B-meson soft function

- Anomalous dimension

$$\Gamma_G = \frac{\alpha_s}{\pi} \left\{ \left[ C_F \left( \ln \frac{\mu}{\omega_1 - i0} - \frac{1}{2} \right) + C_A \left( \ln \frac{\mu}{\omega_2 - i0} + \frac{i}{2} \pi \right) \right] \delta(\omega_1 - \omega'_1) \delta(\omega_2 - \omega'_2) - C_F H_+(\omega_1, \omega'_1) \delta(\omega_2 - \omega'_2) \right. \\ \left. - C_A H_+(\omega_2, \omega'_2) \delta(\omega_1 - \omega'_1) + C_A \left( \frac{\omega_2}{\omega_2'^2} \right) [\theta(\omega_2) \theta(\omega'_2 - \omega_2) - \theta(-\omega_2) \theta(\omega_2 - \omega'_2)] \delta(\omega_1 - \omega'_1) + \Delta \Gamma_G \right\},$$

$$\Delta \Gamma_G = \frac{i}{4} \frac{C_A}{\pi} [H_+(\omega_1, \omega'_1) - H_-(\omega_1, \omega'_1) - 2i\pi \delta(\omega_1 - \omega'_1)] [H_+(\omega_2, \omega'_2) - H_-(\omega_2, \omega'_2) - 2i\pi \delta(\omega_2 - \omega'_2)].$$

- Solution: Laplace transformation

$$\tilde{\Phi}_G^{>(<), >(<)}(\eta_1, \eta_2, \mu) \equiv \int_0^\infty \frac{d\omega_1}{\omega_1} \int_0^\infty \frac{d\omega_2}{\omega_2} \left( \frac{\mu}{\omega_1} \right)^{\eta_1} \left( \frac{\mu}{\omega_2} \right)^{\eta_2} \Phi_G^{>(<), >(<)}(\pm\omega_1, \pm\omega_2, \mu)$$

$$\left( \frac{d}{d \ln \mu} - \eta_1 - \eta_2 \right) \tilde{\Phi}_G(\eta_1, \eta_2, \mu) = -\frac{\alpha_s(\mu)}{\pi} \left[ \tilde{\Gamma}_G^{(0)} + \tilde{\Gamma}_G^{(1)} + \left( \frac{i}{4\pi} \right) \tilde{\Gamma}_G^{(2)} \right] \tilde{\Phi}_G(\eta_1, \eta_2, \mu)$$

$$\tilde{\Phi}_G(\eta_1, \eta_2, \mu) = \exp[V + 2\gamma_E(a_1 + a_2)] \left( \frac{\mu}{\mu_0} \right)^{\eta_1 + \eta_2} \hat{U}^{-1}(\eta_1, \eta_2) \text{diag}(1, 1, 1, e^{-i\pi a_2}) \hat{U}(\eta_1 + a_1, \eta_2 + a_2) \\ \times \frac{\Gamma(1 - \eta_1) \Gamma(1 + \eta_1 + a_1)}{\Gamma(1 + \eta_1) \Gamma(1 - \eta_1 - a_1)} \frac{\Gamma(2 - \eta_2) \Gamma(1 + \eta_2 + a_2)}{\Gamma(1 + \eta_2) \Gamma(2 - \eta_2 - a_2)} \tilde{\Phi}_G(\eta_1 + a_1, \eta_2 + a_2, \mu_0)$$

# Evolution of B-meson soft function

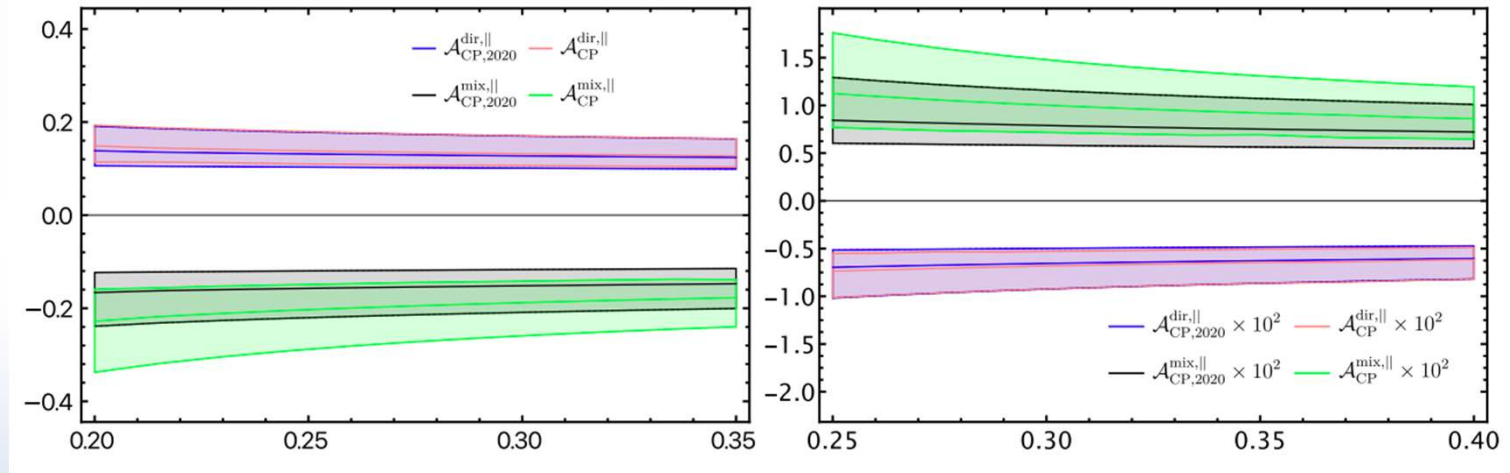
- Asymptotic behavior(exponential model)

$$\Phi_G(\omega_1 \rightarrow 0, \omega_2 \rightarrow 0, \mu) \propto \frac{\lambda_E^2 + \lambda_H^2}{6\omega_0^2} e^{V+2\gamma_E(a_1+a_2)} \left(\frac{\mu_0}{\omega_0}\right)^{a_1+a_2} \frac{1 - e^{-i\pi a_2}}{4\pi^2} \Gamma(1 + a_1)\Gamma(1 + a_2)$$

$$\Phi_G(\omega_1 \rightarrow \pm\infty, \omega_2 \rightarrow \pm\infty, \mu) \propto \frac{\lambda_E^2 + \lambda_H^2}{6\omega_0^2} e^{V+2\gamma_E(a_1+a_2)} \left(\frac{\mu_0}{\omega_0}\right)^{a_1+a_2} \left(\frac{\mu_0}{\pm\omega_1}\right)^{1+a_1} \left(\frac{\mu_0}{\pm\omega_2}\right)^{1+a_2}$$

- Factorization of quark loop contribution to  $B_q \rightarrow \gamma\gamma$  decays

- CP violation observables in  $B_q \rightarrow \gamma\gamma$  decays



Huang, Ji, YLS, Wang, Wang, *PRL*. 133 (2024) 17, 171901

## Summary

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- Power corrections are among the most essential subjects in heavy flavor physics research.
- Novel soft functions for B mesons arise in a variety of B-meson decay processes, and their scale evolution is investigated.
- More comprehensive study on power corrections are required.

Thanks for your patience !