Cosmological Implications of the Slingshot Effect

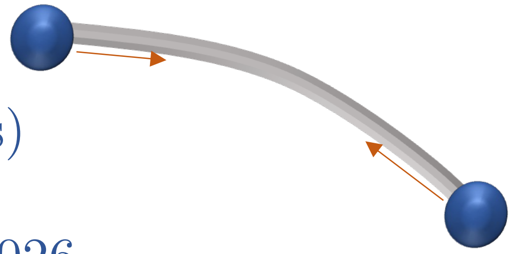
Maximilian Bachmaier
In collaboration with: Gia Dvali
Juan Sebastián Valbuena-Bermúdez
Michael Zantedeschi

Next Frontiers in Magnetic Monopole Searches
in Beijing (September 2026) (Online Talk)

EHU
LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MAX PLANCK GESELLSCHAFT
MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Cofinanciado por la Unión Europea
AGENCIA ESTATAL DE INVESTIGACIÓN

The Magnetic Monopole Problem - Solutions

Inflation (Guth 1979)
BUT: GUT phase transition can happen after inflation[1]



Magnetic Monopoles connected by Strings (Langacker and Pi 1980)
Numerical Studies: Hindmarsh, Rummukainen, Weir 2016 (Monopole Necklaces)
Dvali, Valbuena-Bermudez, Zantedeschi 2022
Bachmaier, Dvali, Valbuena-Bermudez, Zantedeschi 2023, 2026

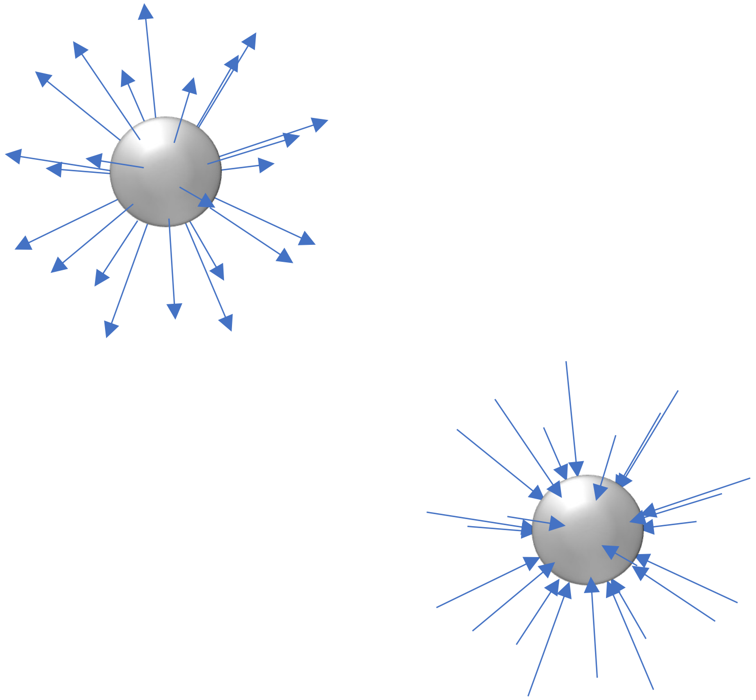
reduction in the number of monopoles → relevant for monopole searches

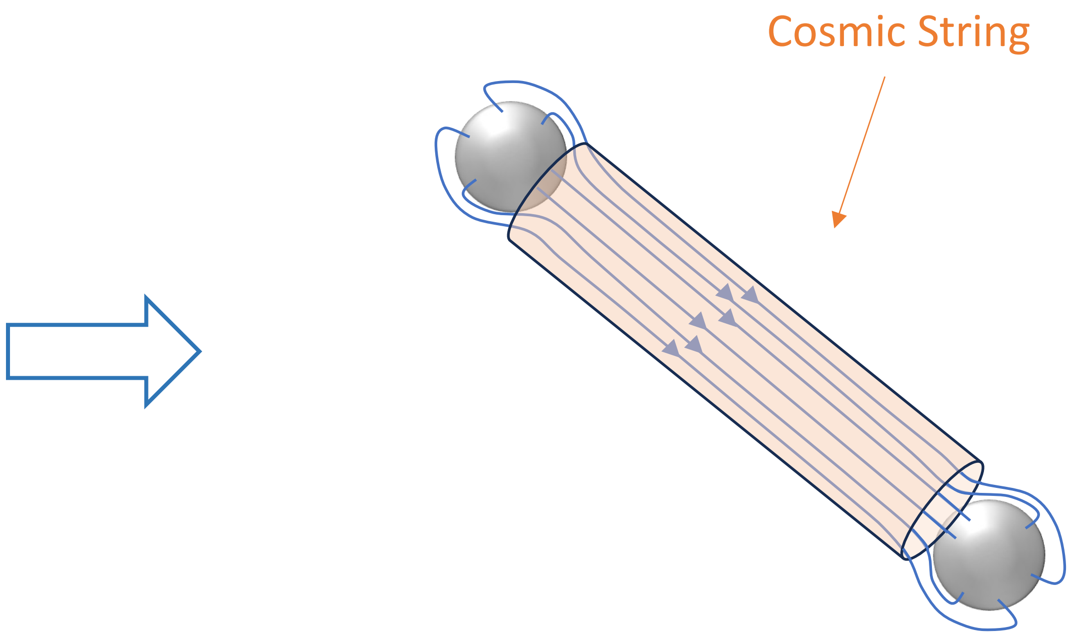
[1]see for example G. Dvali, H. Tye – Brane Inflation (1998)

Magnetic Monopoles Connected by a String

First Phase Transition: $G \to \ldots \times U(1)$

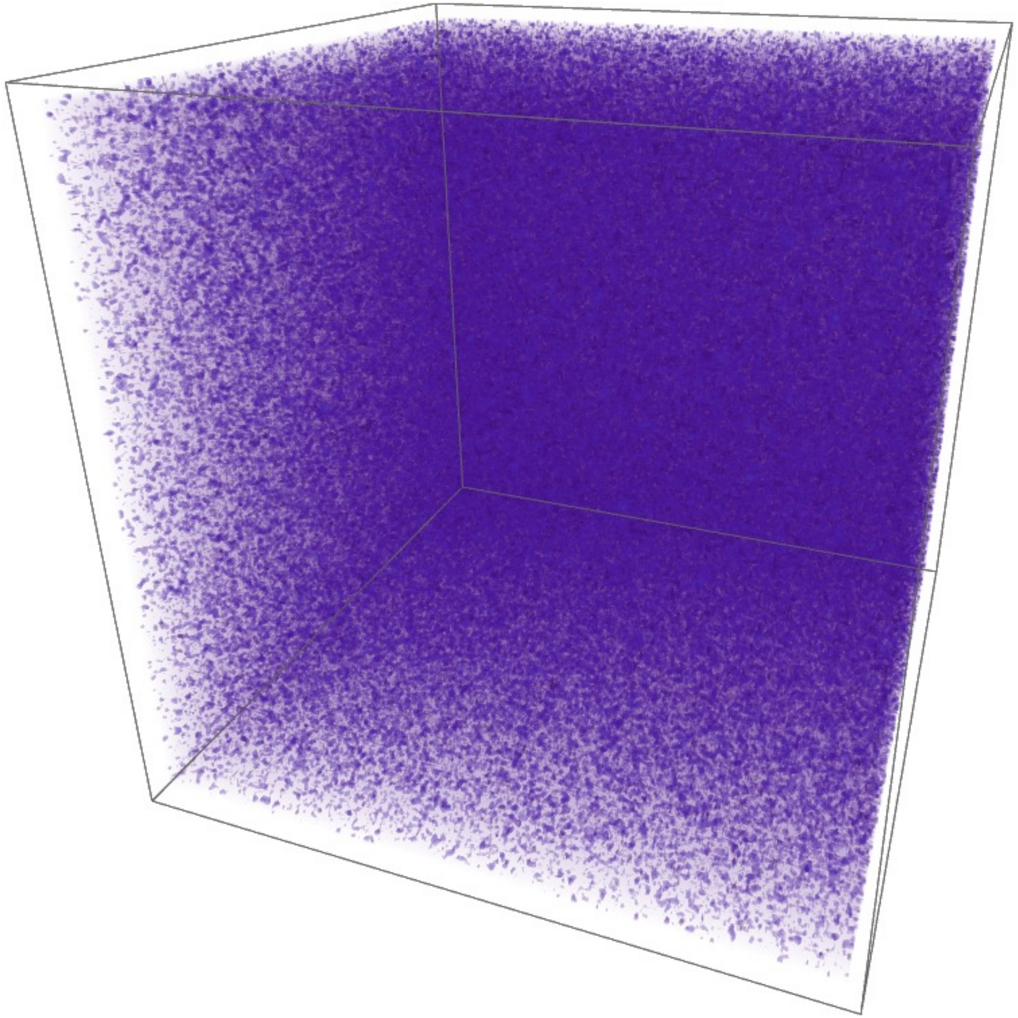
Second Phase Transition: $\ldots \times U(1) \to \ldots \times \cancel{U(1)}$



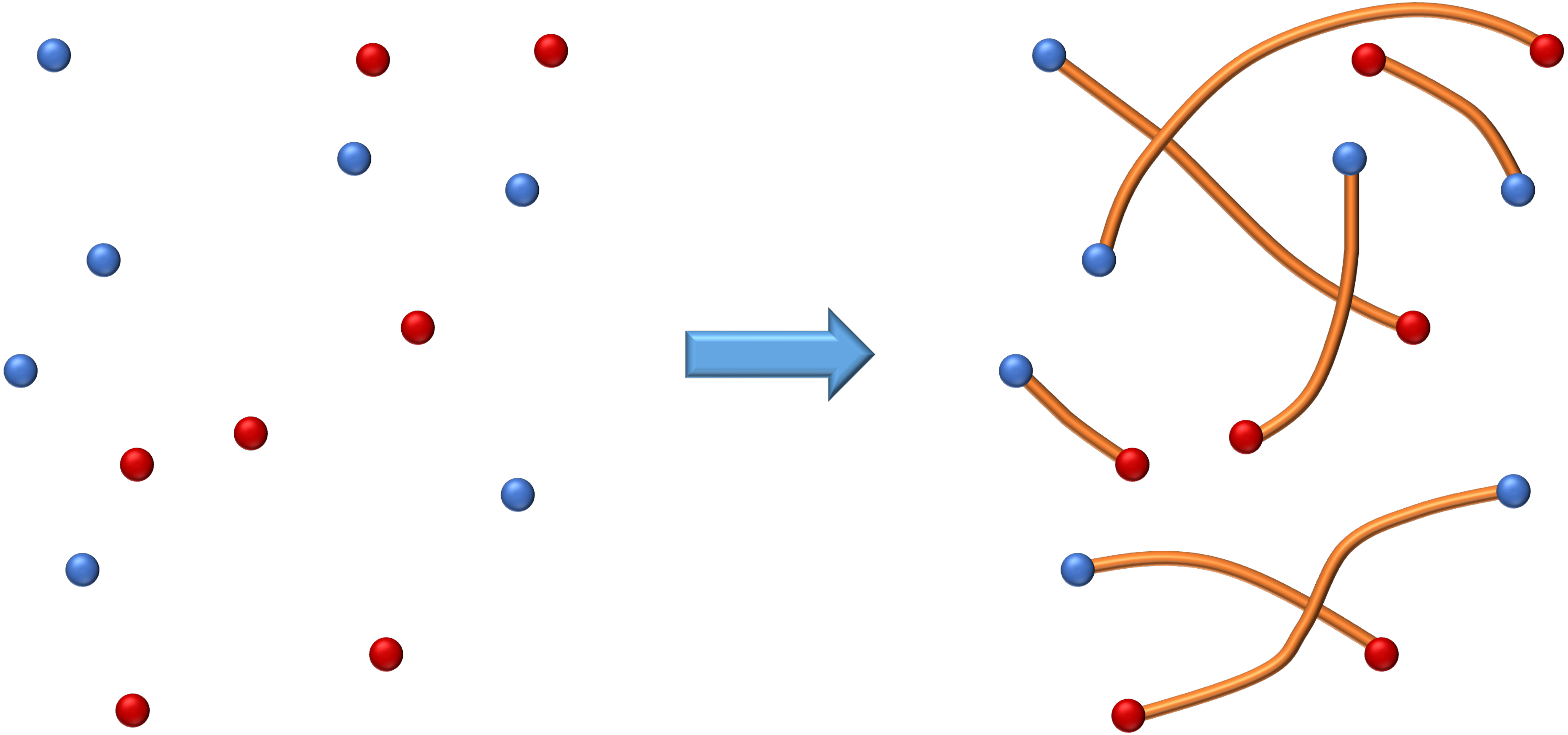


P. Langacker, S. Pi (1980)
X. Martin, A. Vilenkin (1997)
G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2022)

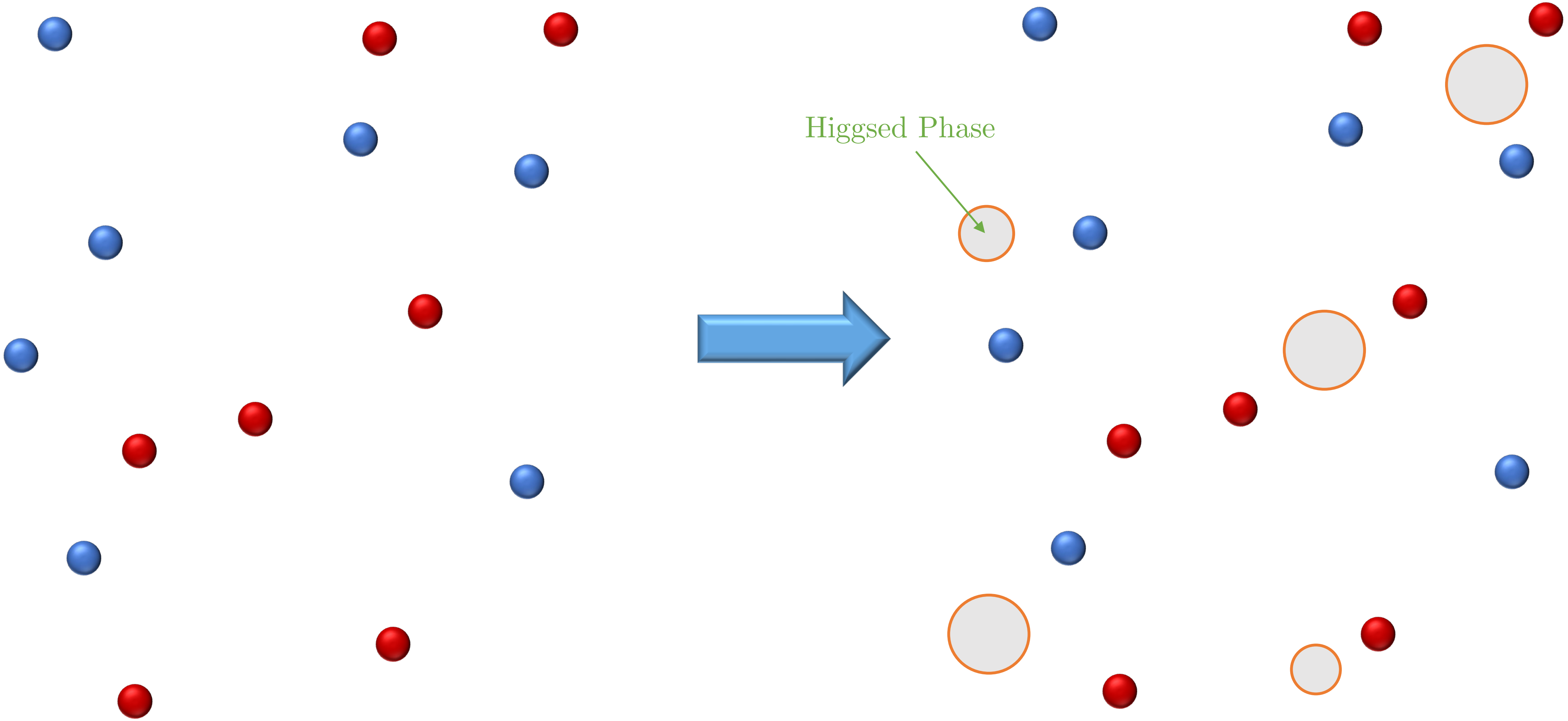
Magnetic Monopoles Connected by a String



Second-Order Phase Transition



First-Order Phase Transition



The Model

We consider an $SU(2)$ gauge theory with the following potential

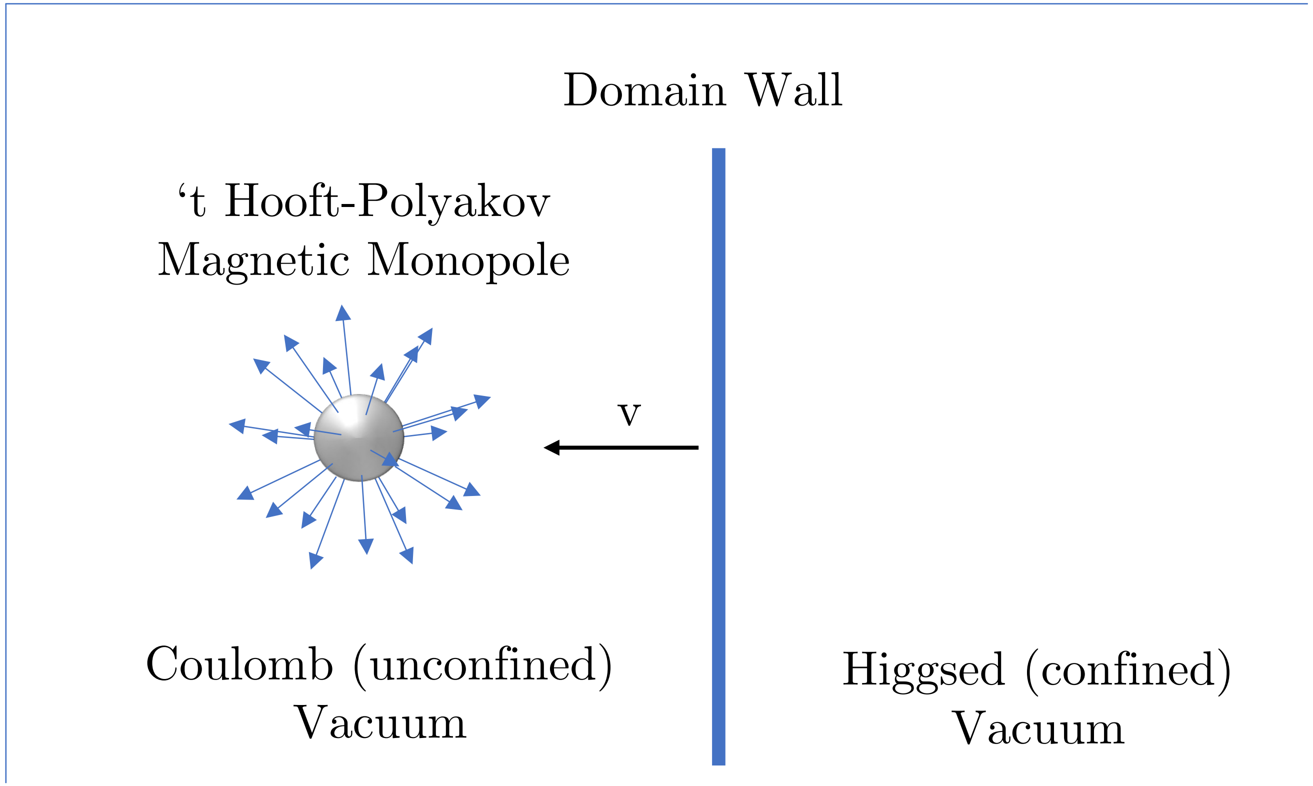
$$V(\phi) = \lambda_\phi \left(\phi^a\phi^a - v_\phi^2\right)^2 + \lambda_\psi \left(\psi^\dagger\psi - v_\psi^2\right)^2 \left(\psi^\dagger\psi\right) + \beta\psi^\dagger\phi\psi$$

ϕ: $SU(2)$ adjoint
ψ: $SU(2)$ fundamental

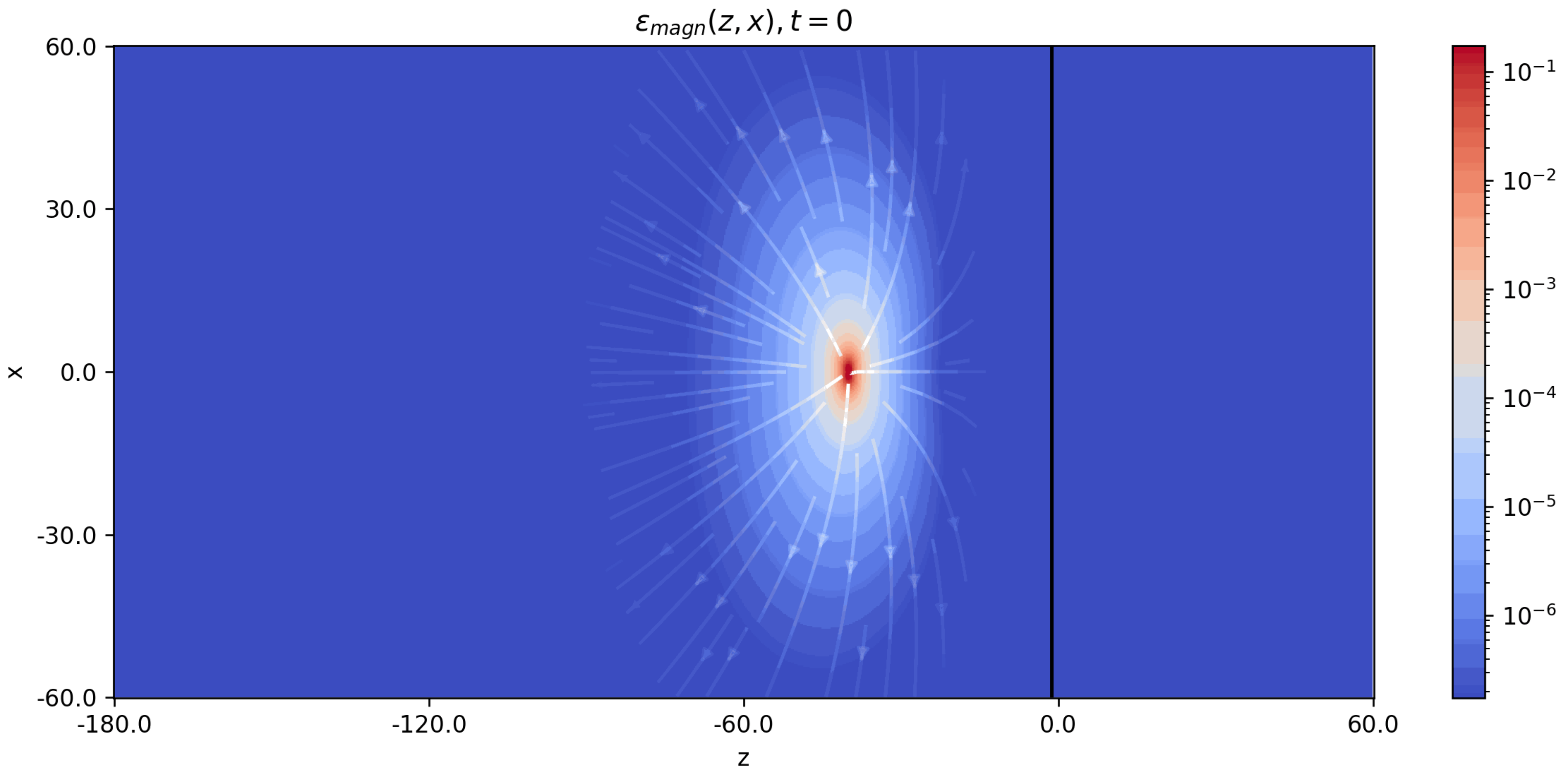
$\langle\phi^a\phi^a\rangle = v_\phi^2 \rightarrow SU(2)$ breaks down to $U(1)$
$\rightarrow$ Magnetic Monopoles

Disconnected Vacuum Manifold for ψ
$\rightarrow$ Domain Walls

Breaking Pattern: $SU(2) \xrightarrow{\phi} U(1) \xrightarrow{\psi} 1$

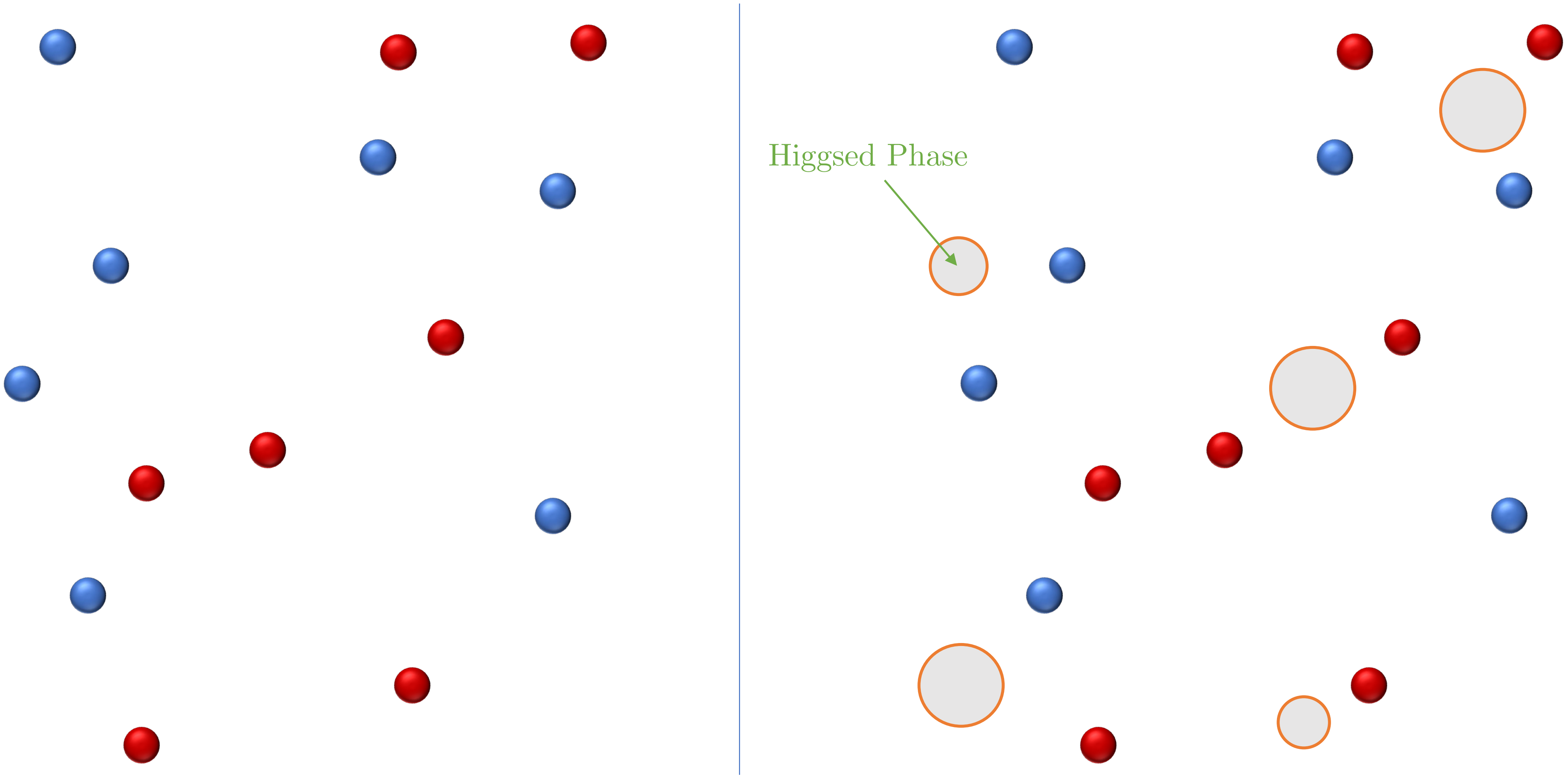


Confinement Slingshot

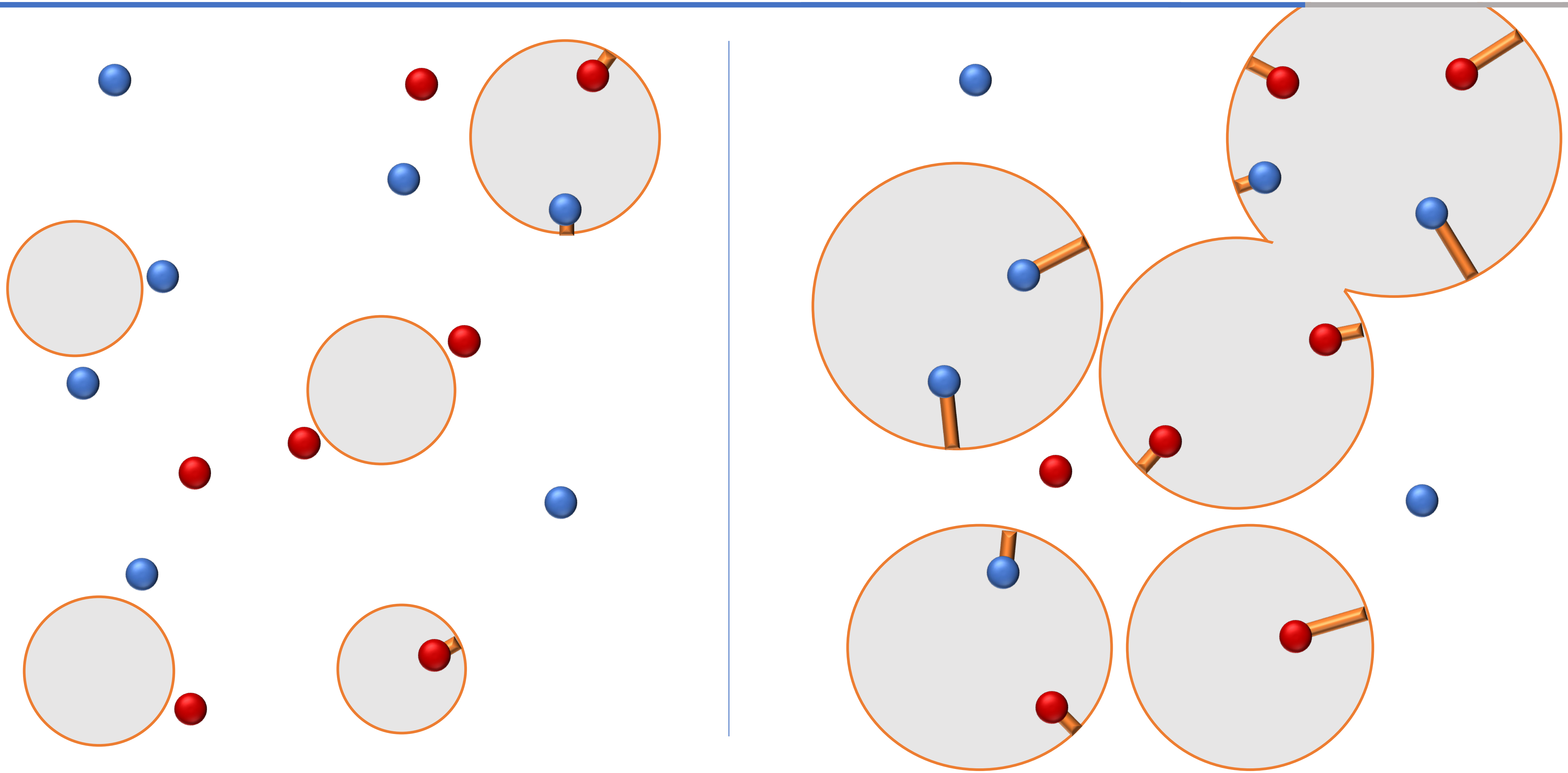


M. B., G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2023)

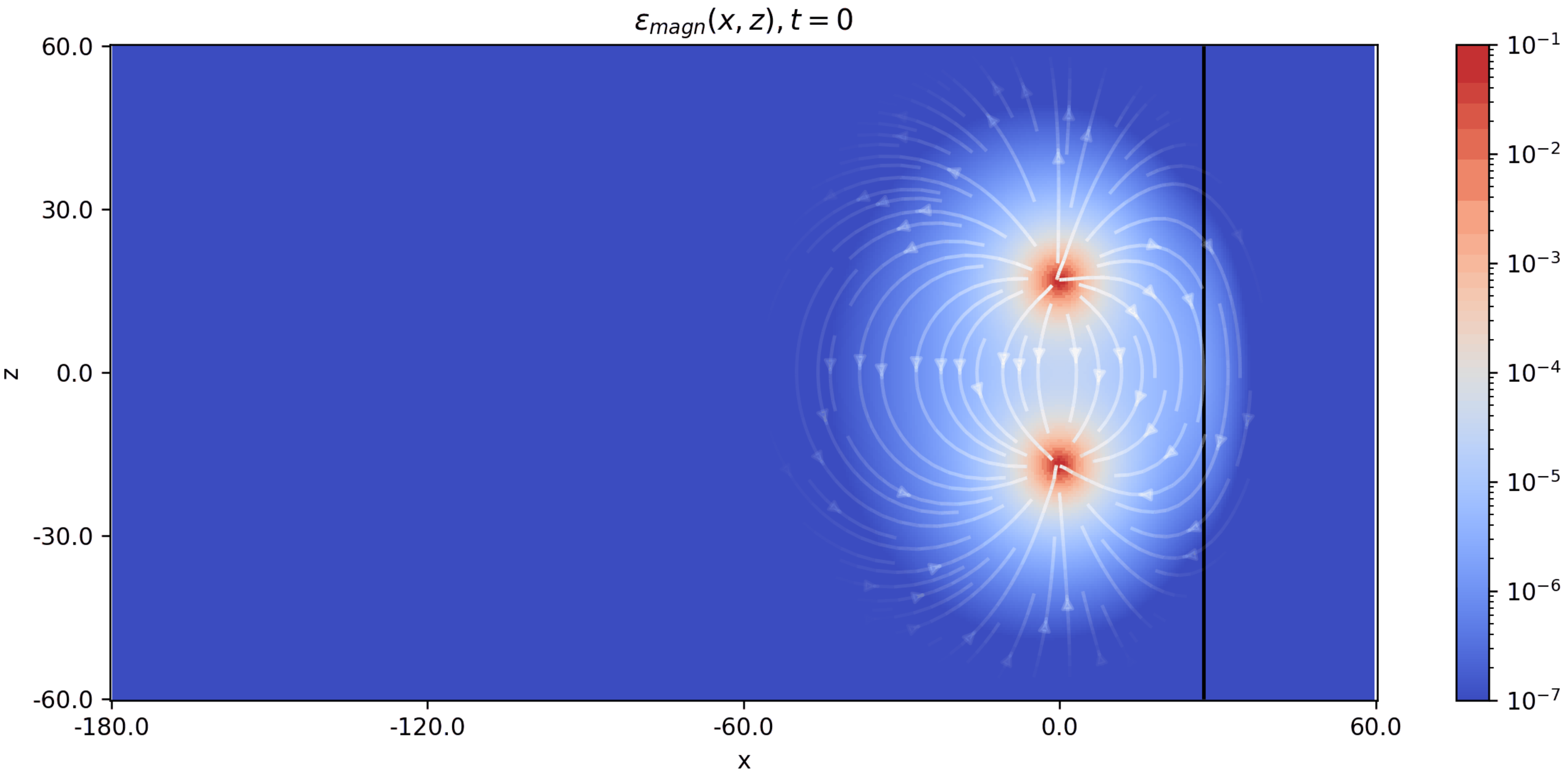
First-Order Phase Transition



First-Order Phase Transition

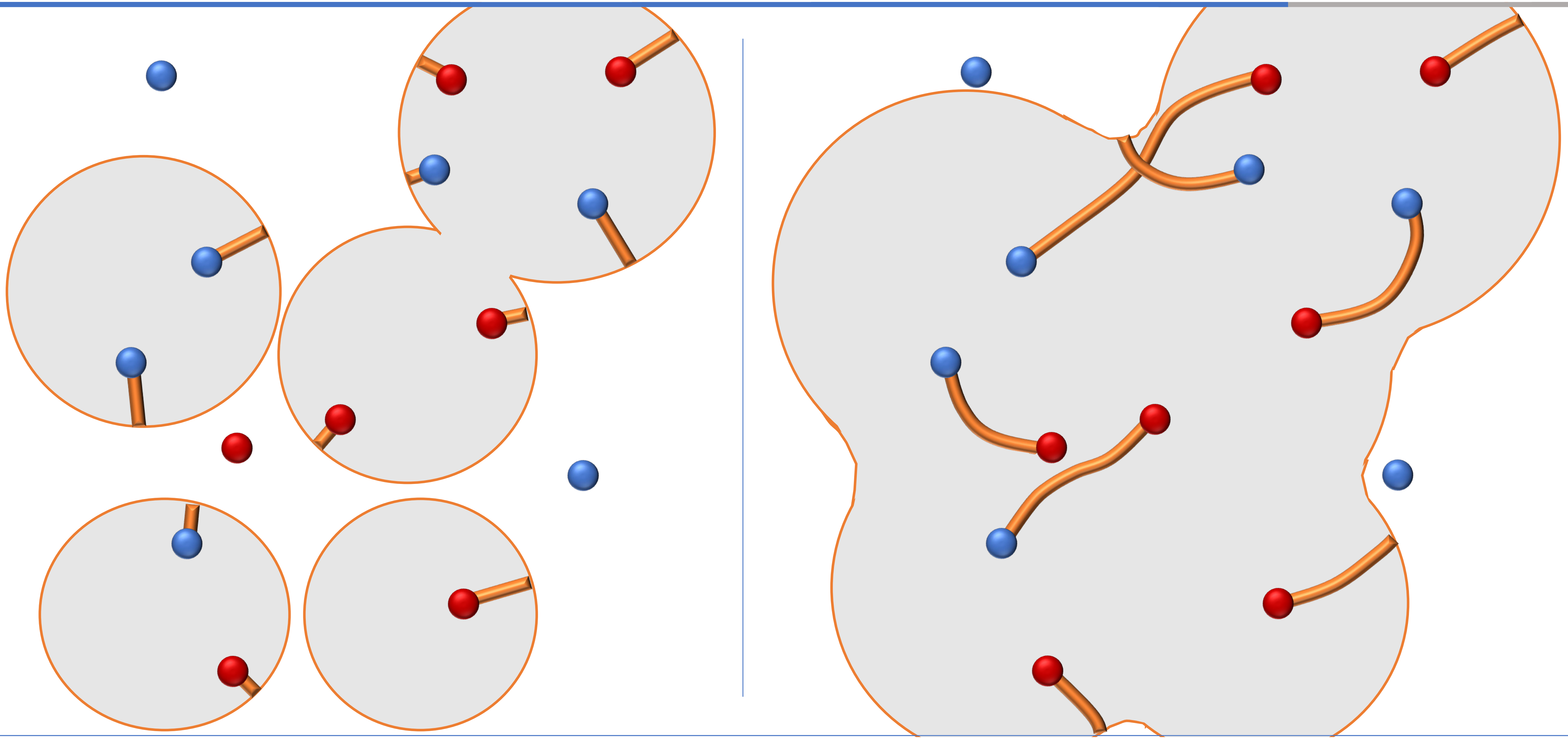


Multiple Confinement Slingshots

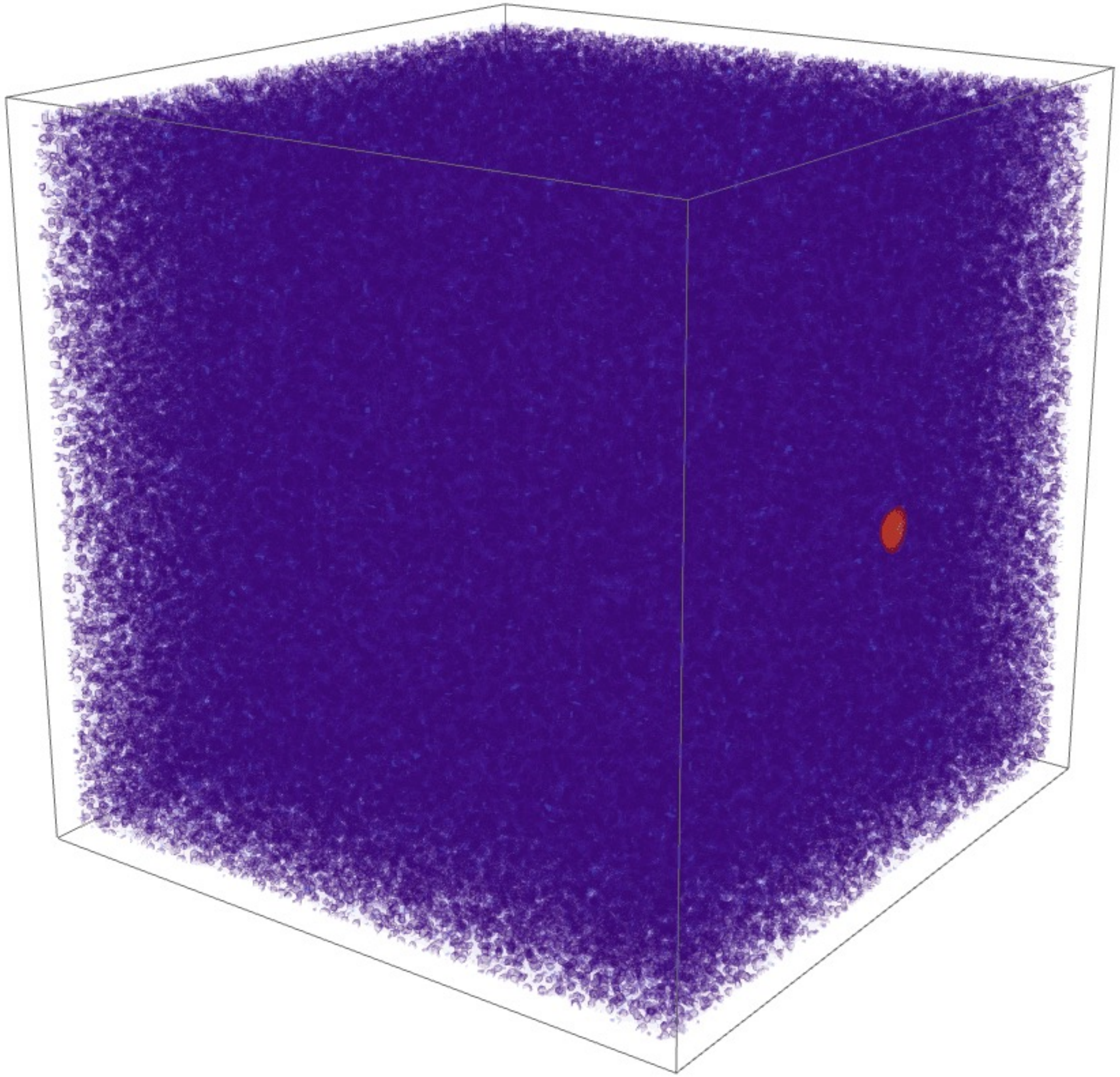


M. B., G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2026)

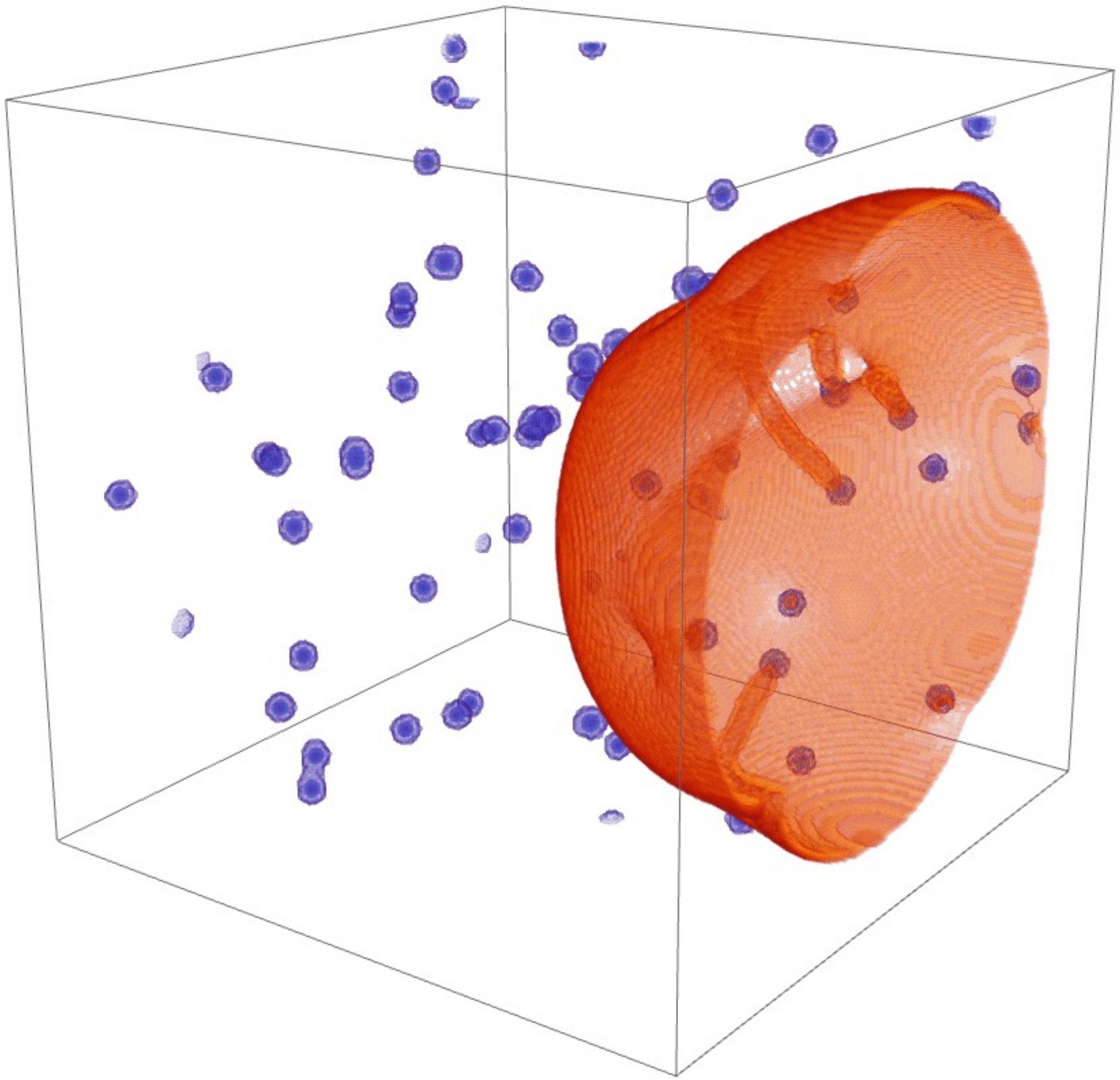
First-Order Phase Transition



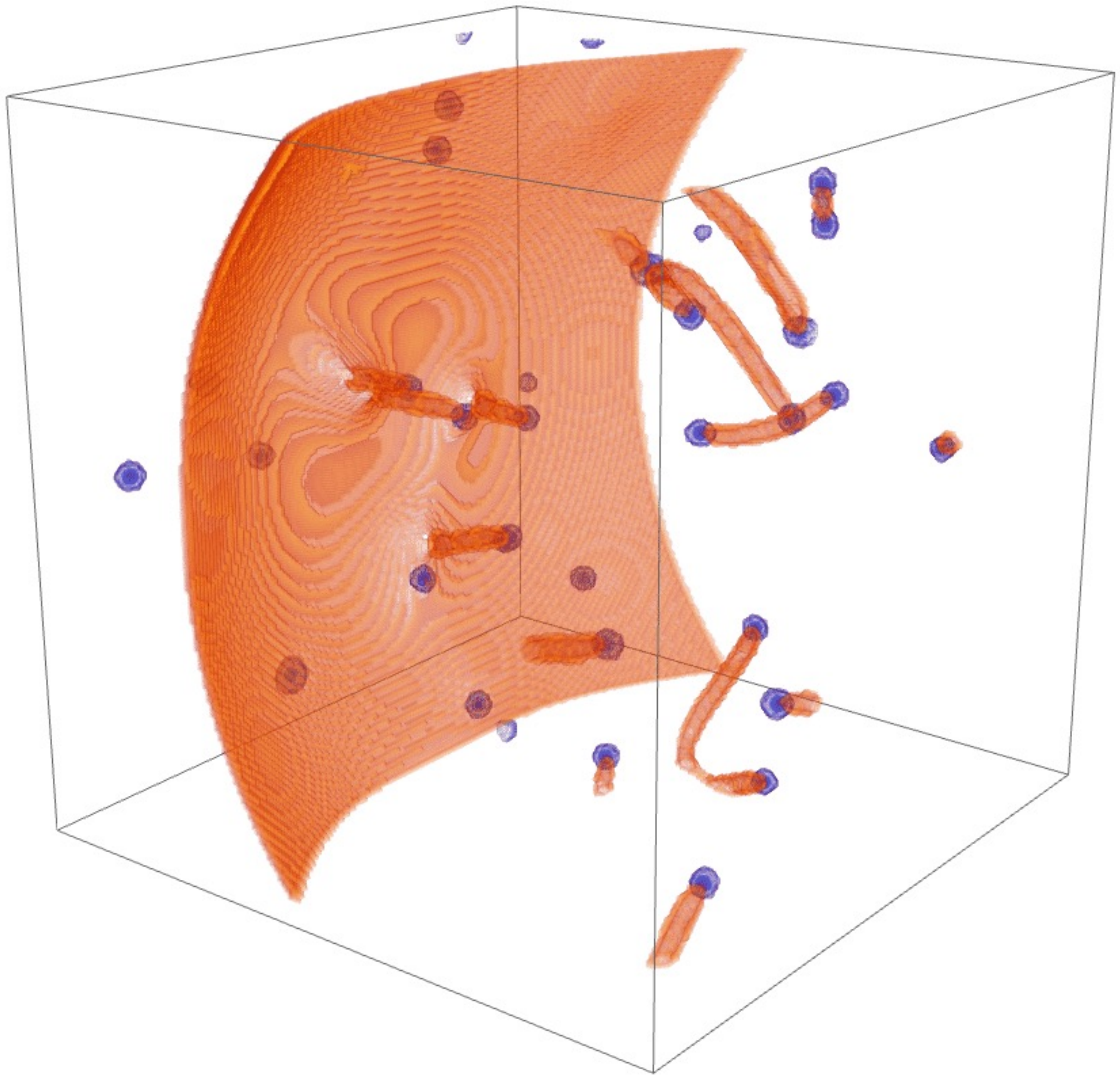
First-Order Phase Transition



First-Order Phase Transition



First-Order Phase Transition

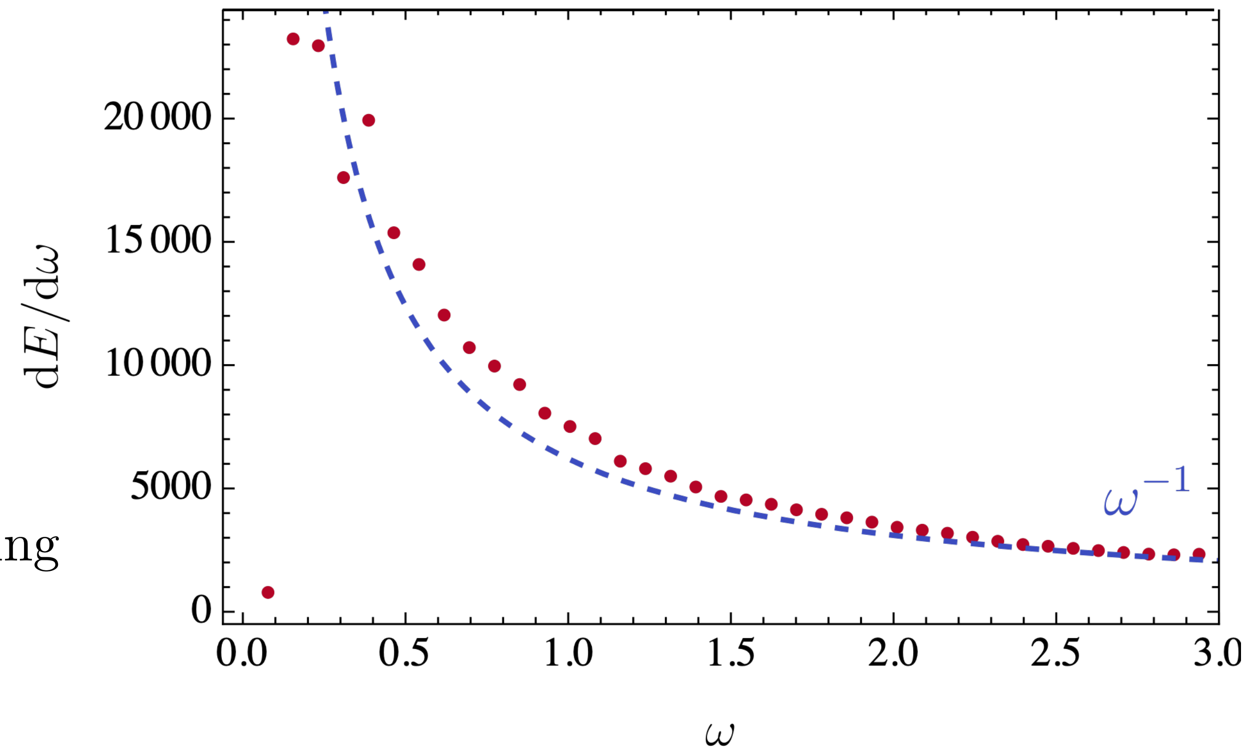


Confinement Slingshot – Gravitational Waves

The slingshot effect leads to the emission of gravitational waves

The energy spectrum decays with ω^{-1}

→ Similar to two monopoles connected by a string



Martin and Vilenkin (1996): GW spectrum for monopole dumbbell scales as $\dfrac{\mathrm{d}P_{\mathrm{GW}}}{\mathrm{d}\omega} \sim \dfrac{4\mu^2}{M_{\mathrm{P}}^2}\dfrac{1}{\omega}$

Confinement Slingshot – Gravitational Waves

Gravitational radiation density parameter:

$$\Omega_{\mathrm{GW}} = \frac{1}{\rho_{\mathrm{c}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln f} \xrightarrow[\text{Using Martin-Vilenkin formula}]{} \Omega_{\mathrm{GW}}^{\mathrm{peak}} \sim \frac{\Lambda^4}{M_{\mathrm{P}}^2}\, t_{\mathrm{sl}}\, n_{\mathrm{M}} \left(\frac{a(T_{\mathrm{sl}})}{a(T_0)}\right)^4 \frac{1}{\rho_{\mathrm{c}}}$$

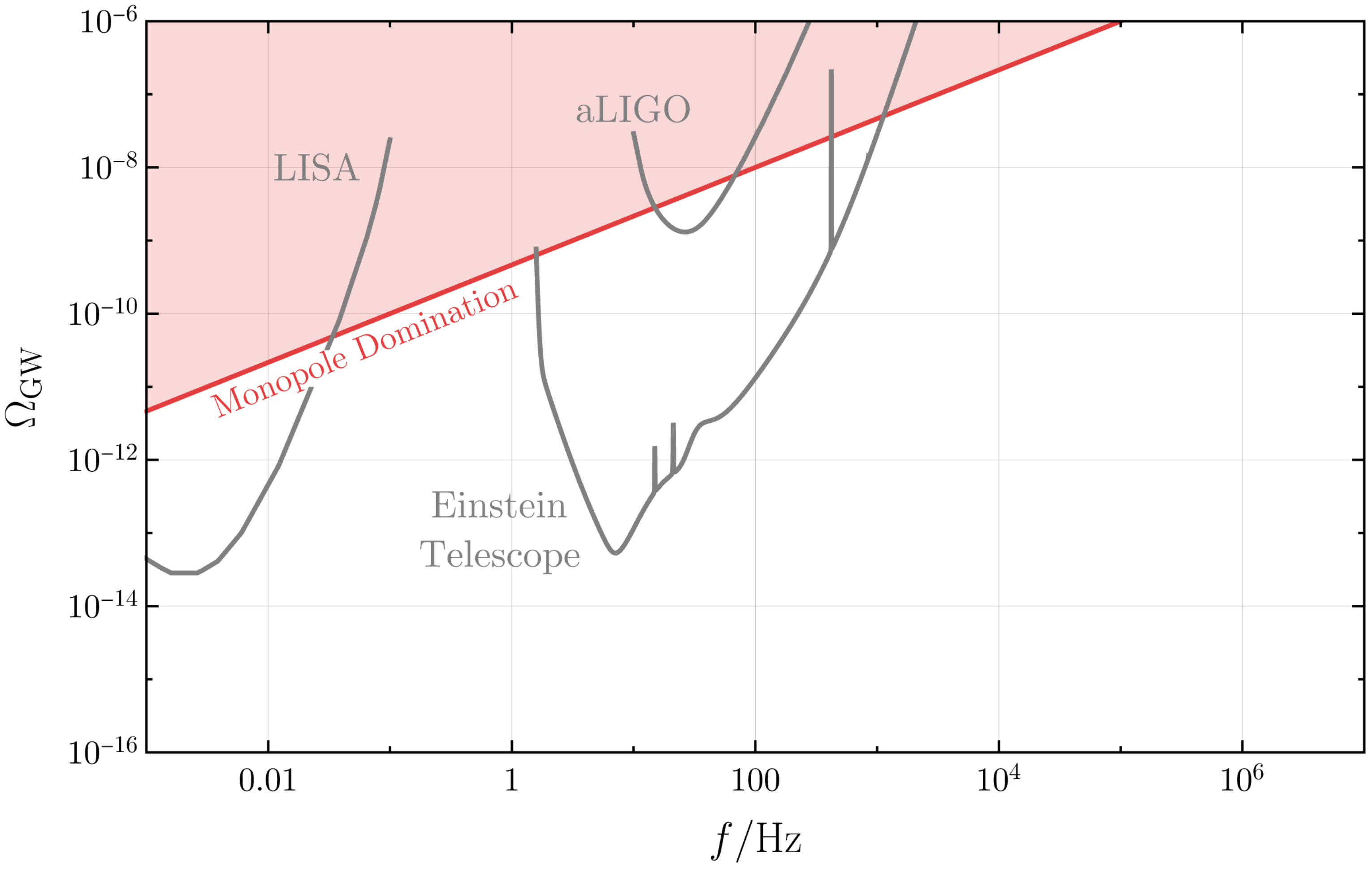
$$f^{\mathrm{peak}} \sim \frac{1}{t_{\mathrm{sl}}} \frac{a(T_{\mathrm{sl}})}{a(T_0)}$$

Avoiding domain wall domination: $T_{\mathrm{sl}} \gtrsim \left(\frac{\Lambda}{M_{\mathrm{P}}}\right)^{\frac{3}{2}} M_{\mathrm{P}}$

Avoiding monopole domination: $n_{\mathrm{M}} \lesssim \frac{T_{\mathrm{sl}}^4}{M}$

M: monopole mass
n_{M}: monopole density at slingshot time
Λ: string scale
T_{sl}: temperature at slingshot time
T_0: temperature of Universe today
ρ_{c}: critical energy density of Universe
t_{sl}: slingshot duration

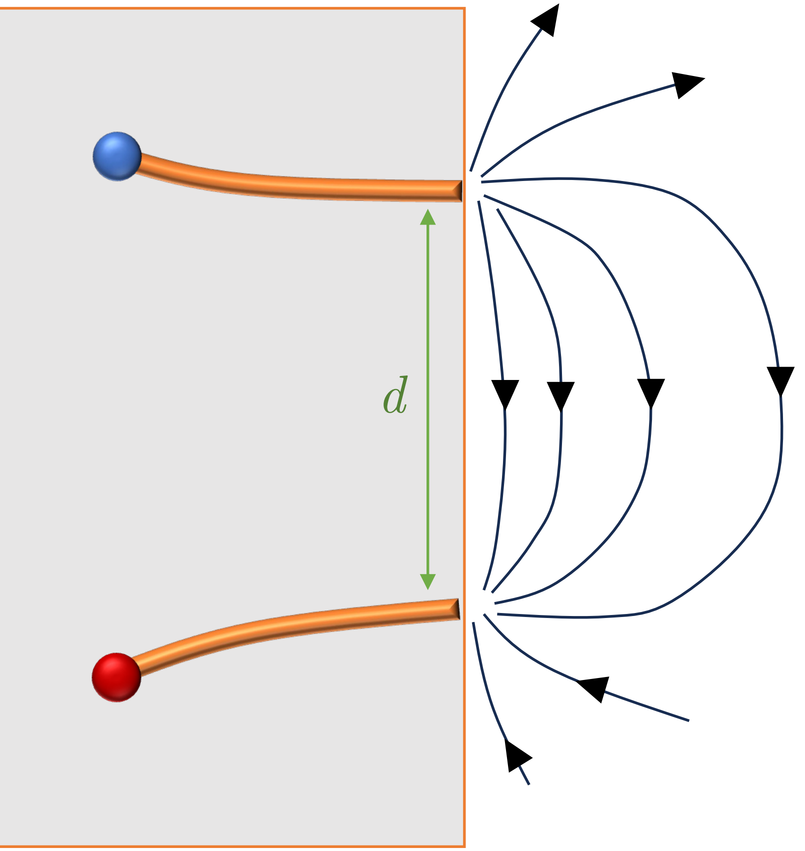
Confinement Slingshot – Gravitational Waves

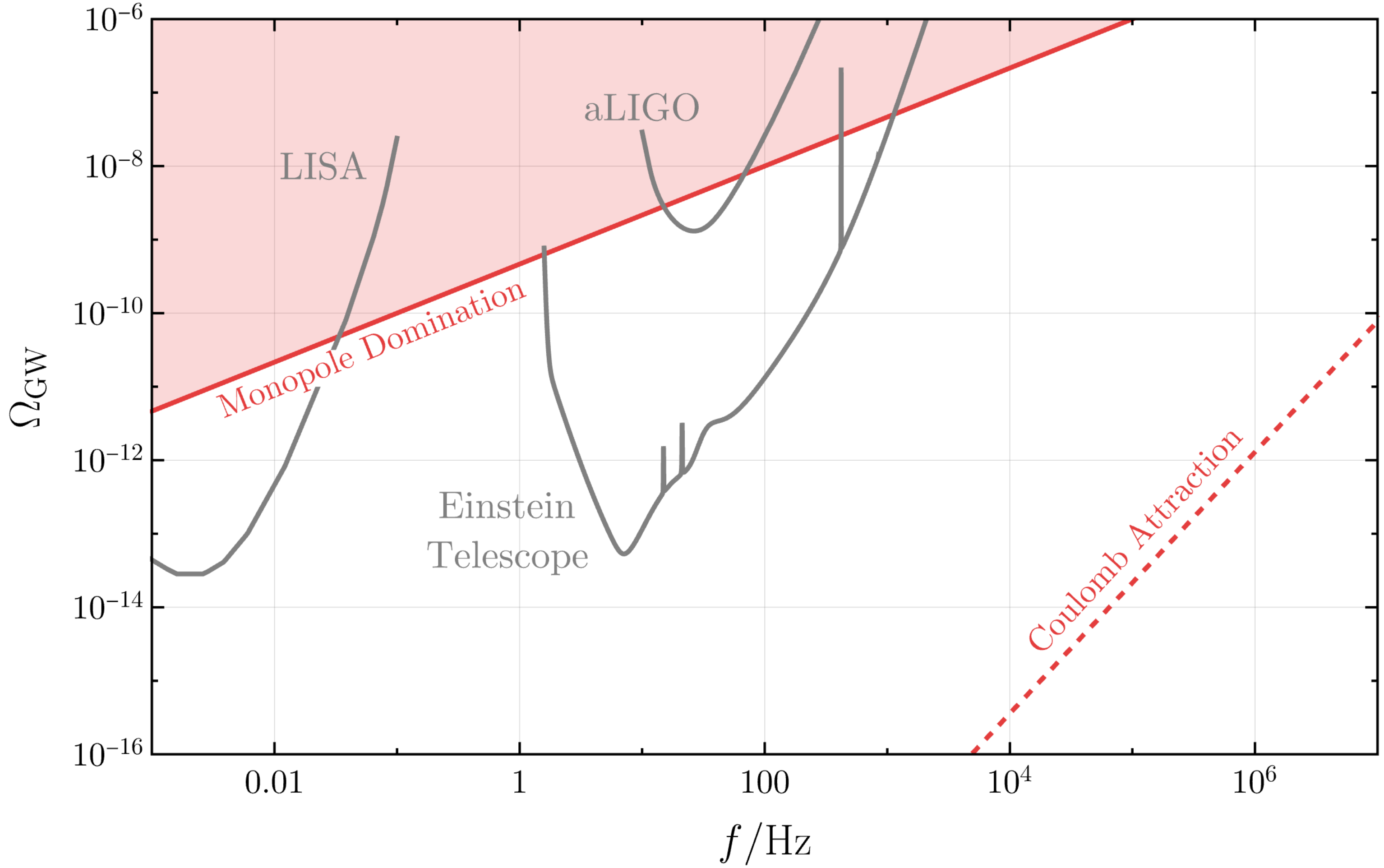


Colored region is excluded

$M \sim 10^{16}\mathrm{GeV}$

Confinement Slingshot – Gravitational Waves





BUT: there can be friction, angular momentum, etc.

Summary

The slingshot effect can appear in first-order phase transitions involving magnetic monopoles

The slingshot effect leads to the emission of gravitational waves
$\rightarrow$ observable? restrictions on models? ...

The slingshot effect can modify strongly the number of the today's magnetic monopoles $\rightarrow$ relevant for direct monopole searches

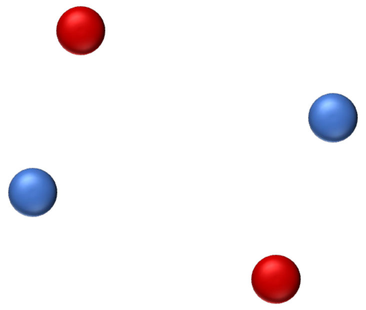
Thank you!

More Videos on YouTube

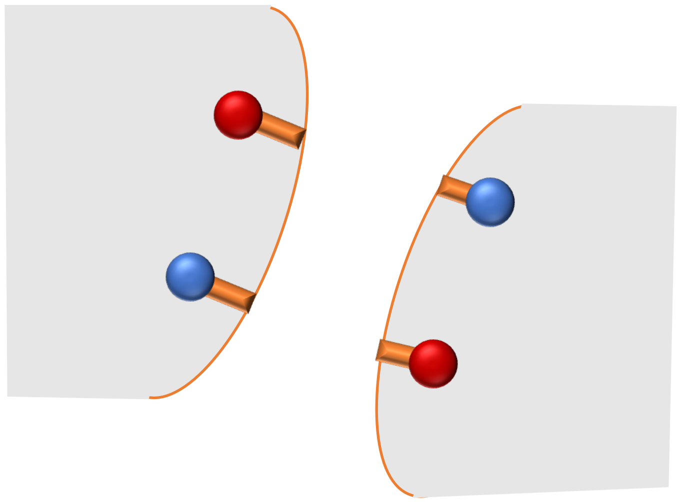
Maximilian Bachmaier

Backup

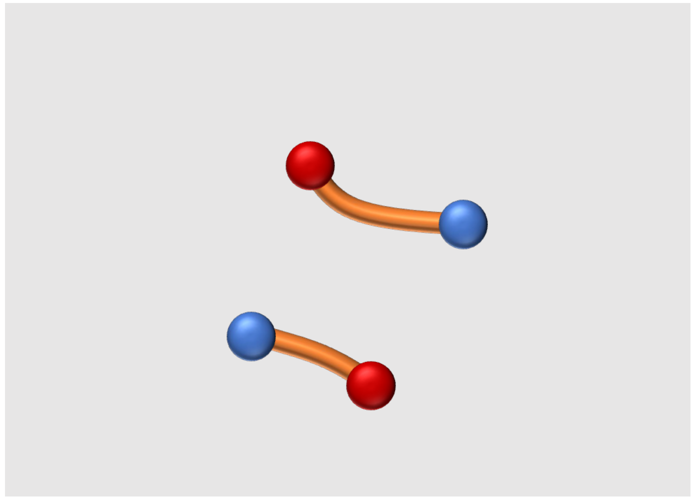
Confinement Slingshot – Gravitational Waves



$M \gtrsim T \gtrsim \Lambda$



$T \sim T_{\rm sl} \sim \Lambda$

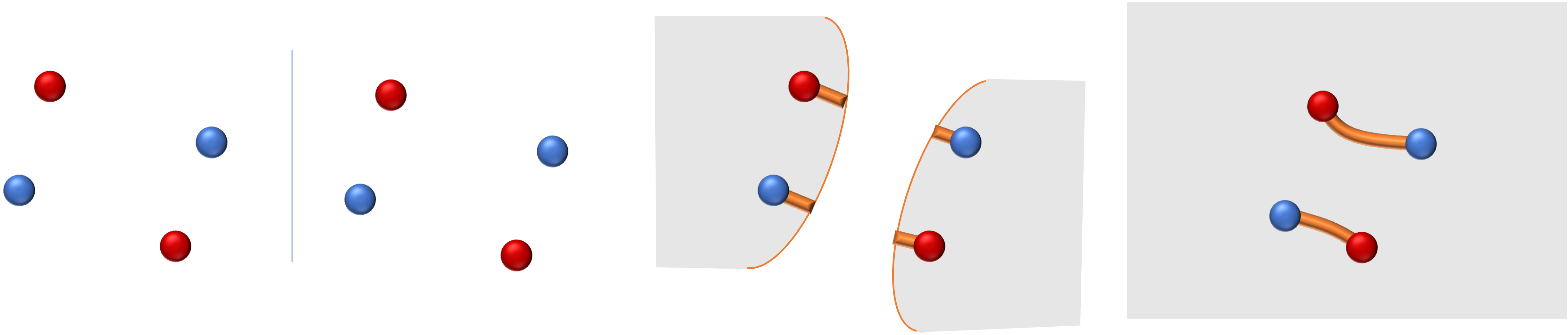


$T < T_{\rm sl}$

$\rightarrow$ GW signal far below experimental reach

M: monopole mass
Λ: string scale
$T_{\rm sl}$: temperature at slingshot time

Confinement Slingshot – Gravitational Waves

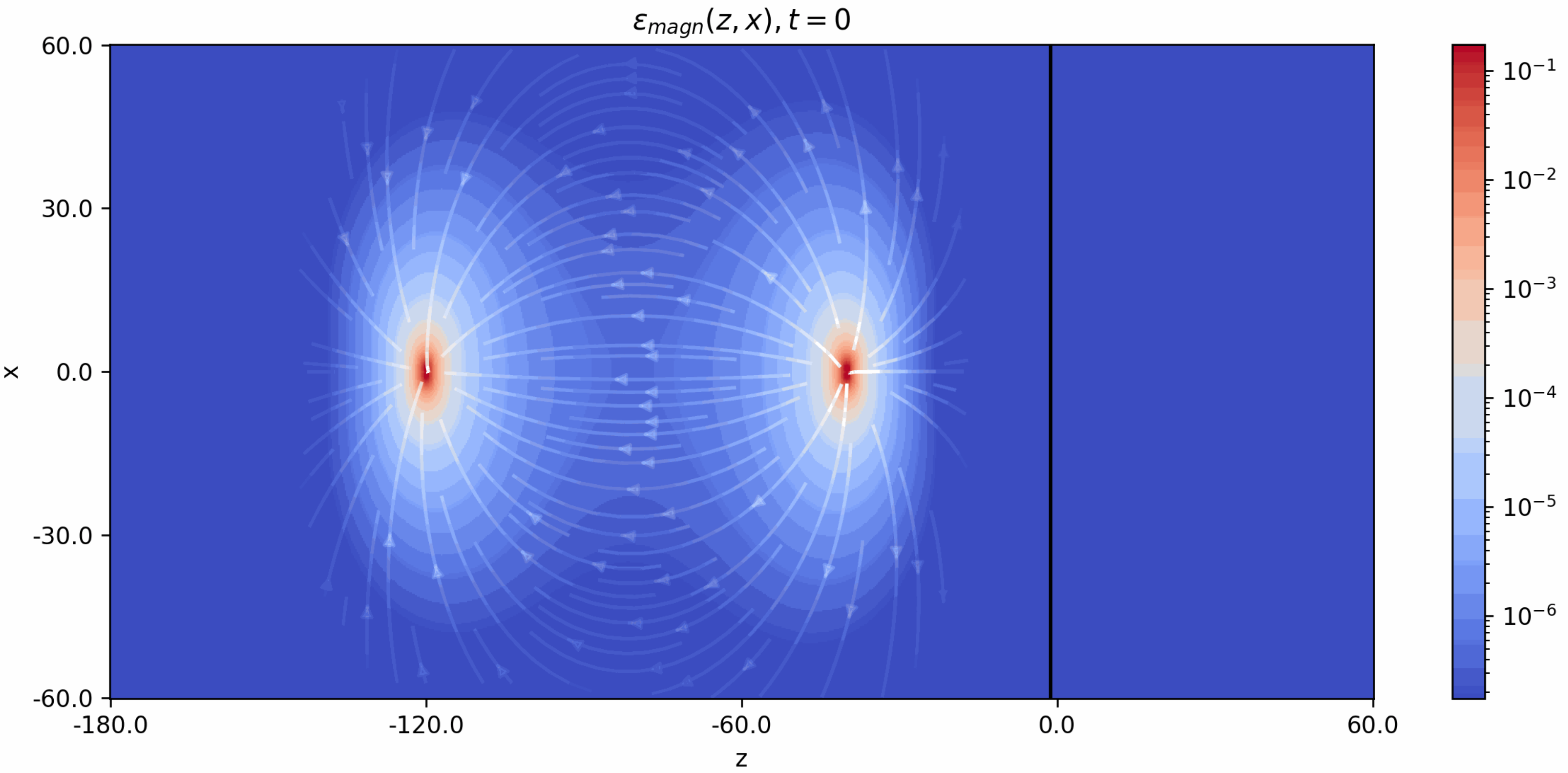


$M \gtrsim T \gtrsim \Lambda$ | $\Lambda \gtrsim T \gtrsim T_{\mathrm{sl}}$ | $T \sim T_{\mathrm{sl}}$ | $T < T_{\mathrm{sl}}$

We assume a delayed slingshot (e.g. through supercooling)

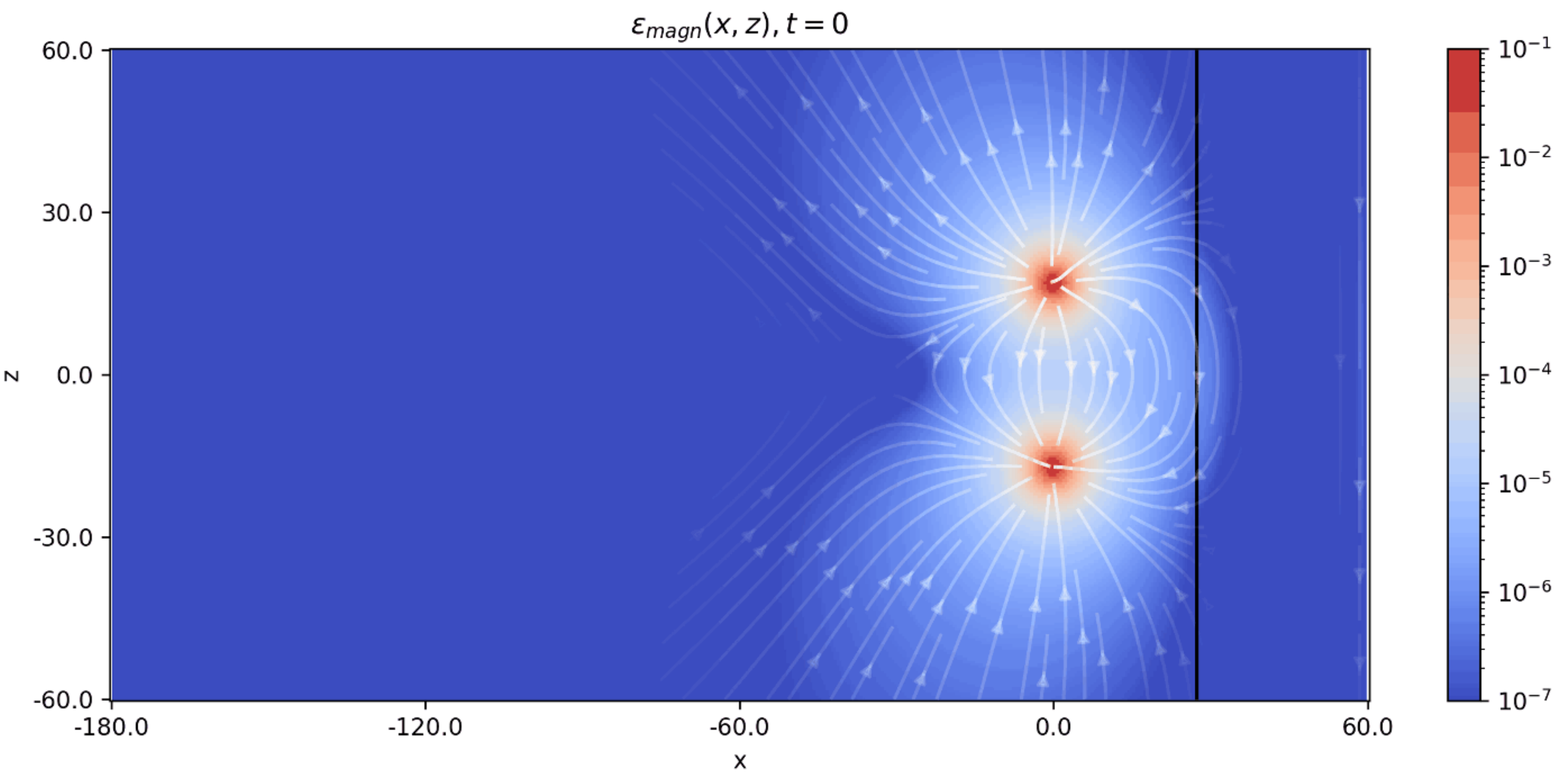
M: monopole mass
Λ: string scale
T_{sl}: temperature at slingshot time

Confinement Slingshot – Two Monopoles



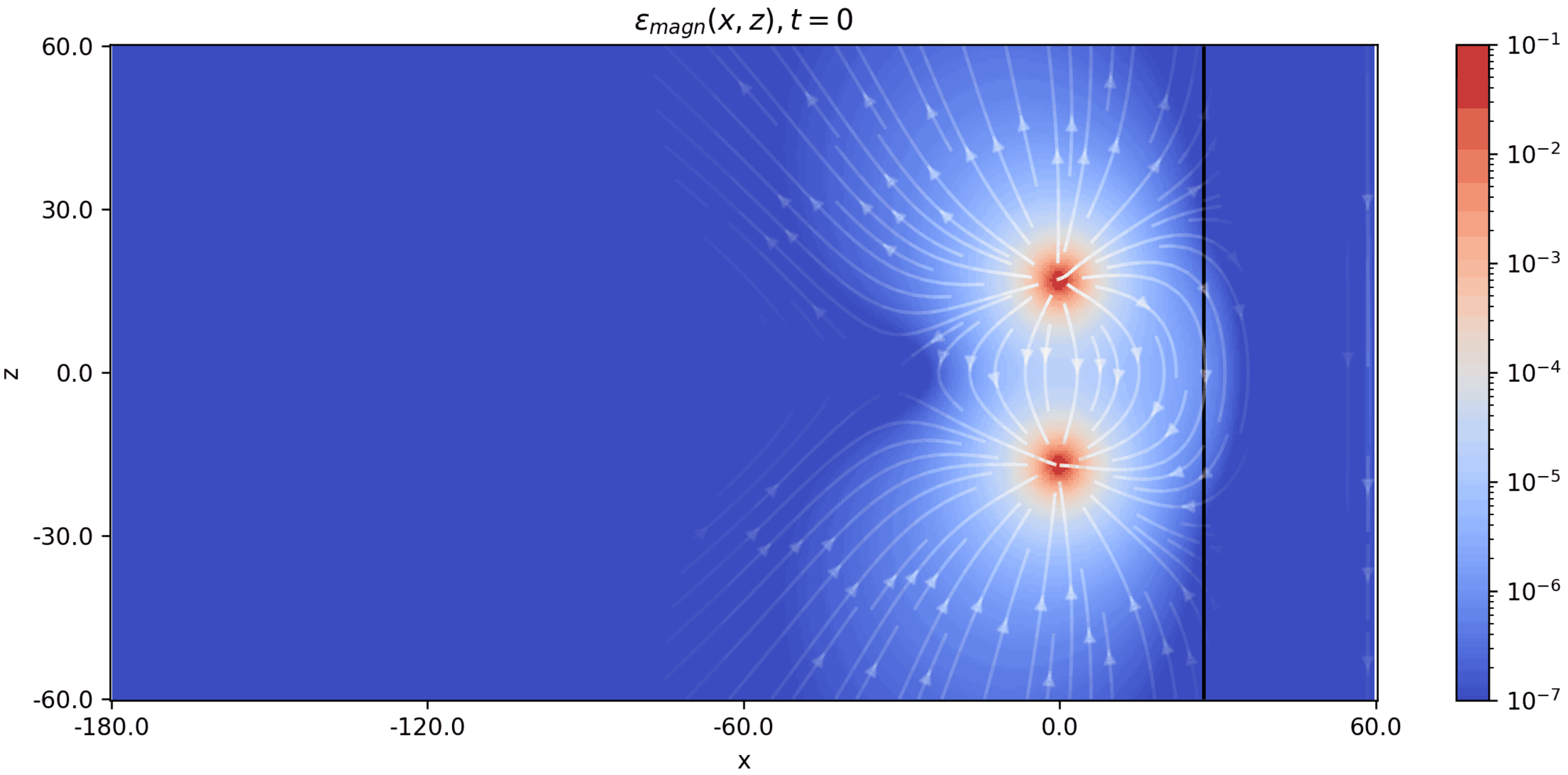
M. B., G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2023)

Multiple Confinement Slingshots (maximal Twist)



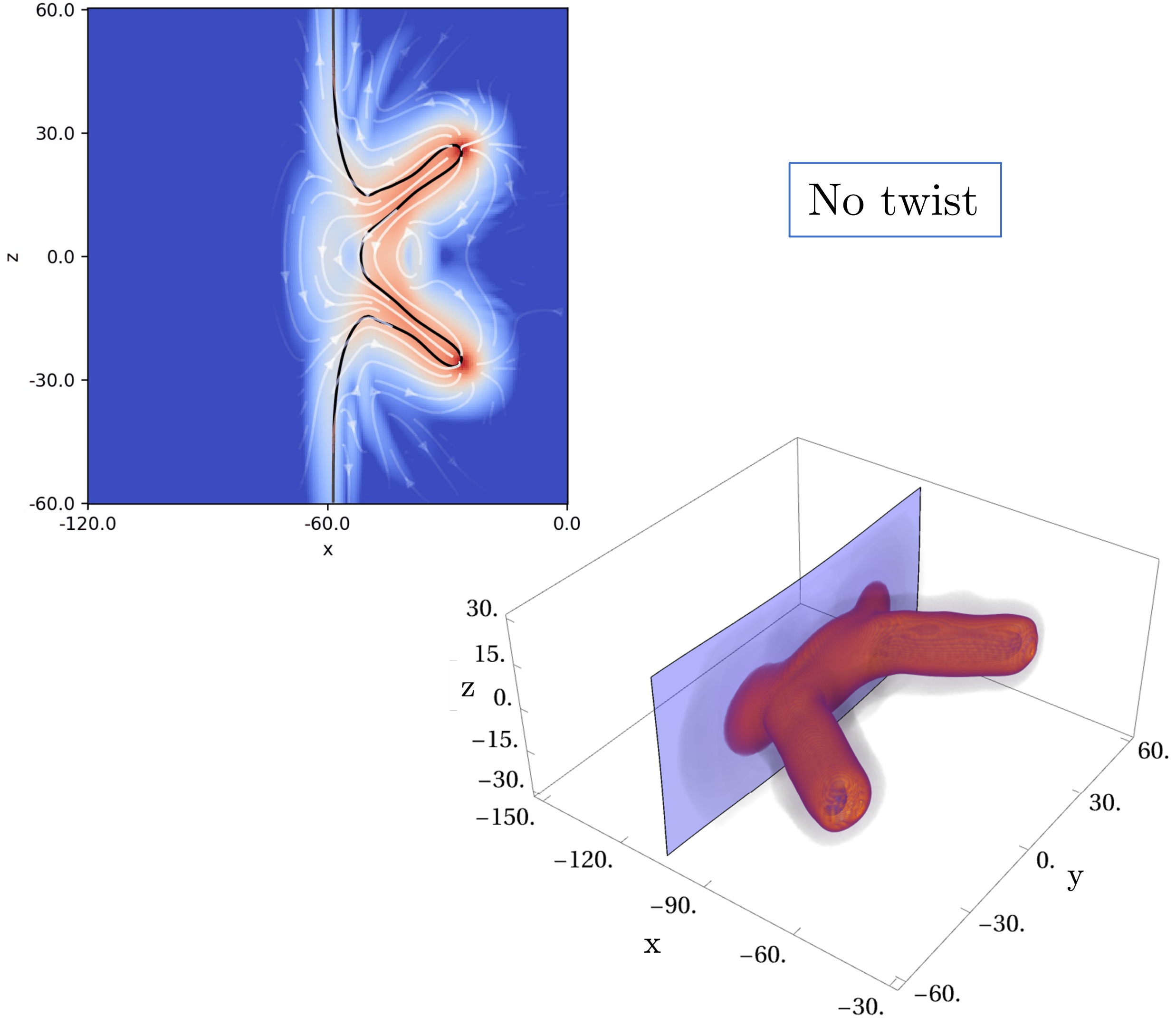
M. B., G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2026)

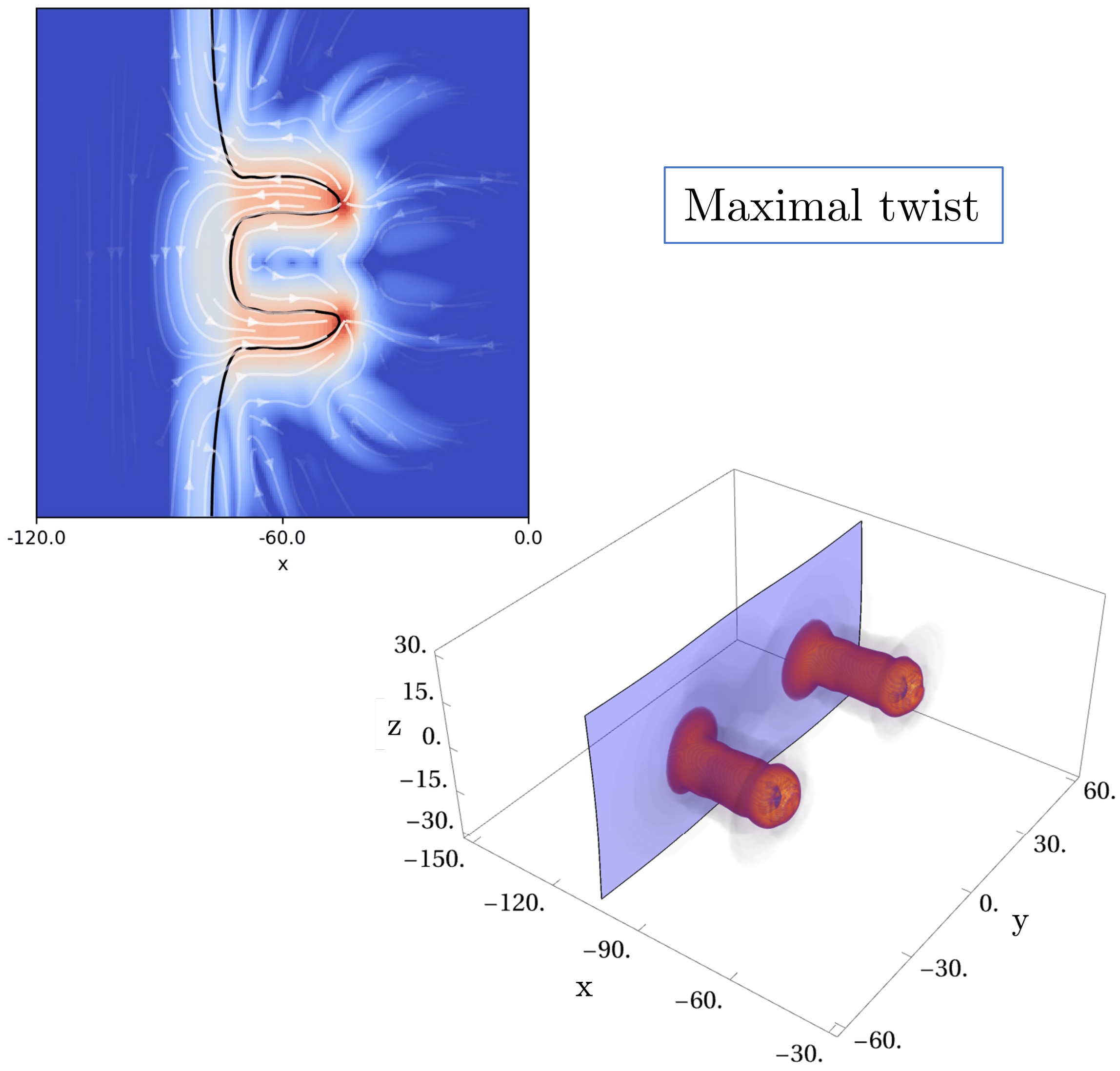
Multiple Confinement Slingshots (maximal Twist)



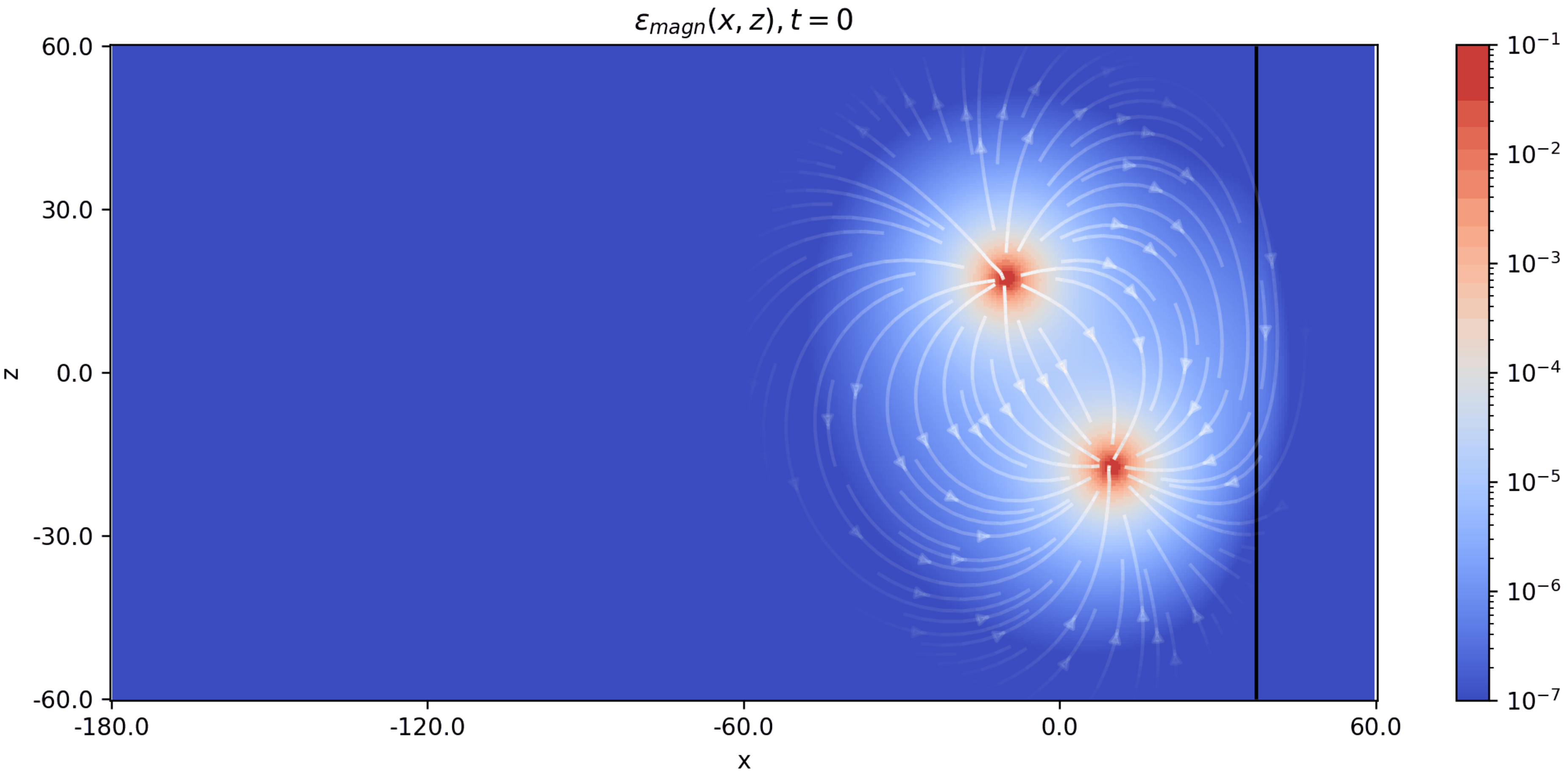
M. B., G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2026)

Why the strings bend?



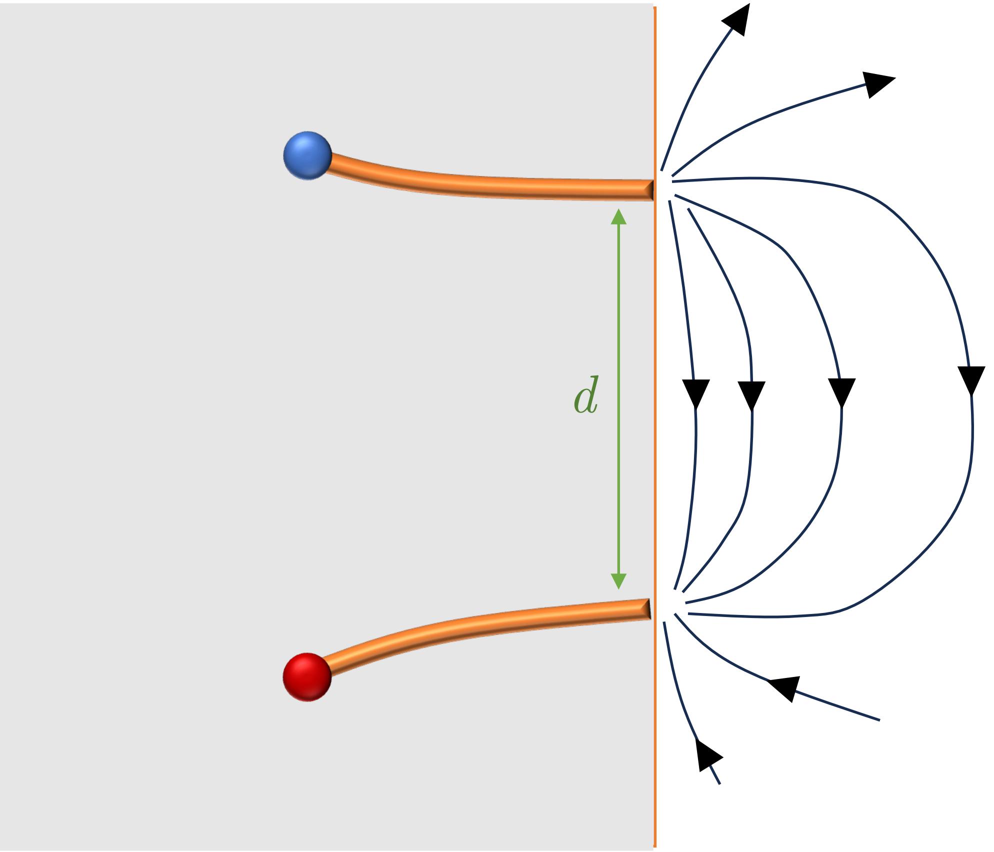


Multiple Confinement Slingshots (rotated)



M. B., G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2026)

Attraction between Slingshots



Flux tube disappears for $d \gg \Lambda^{-1}$.

$\rightarrow$ We assume a Coulomb attraction of the slingshot throats:

$$m\ddot{x} \sim \frac{1}{x^2}$$

Throat mass $m \sim \mu\Lambda^{-1}$
(Λ: confinement scale, $\mu \sim \Lambda^2$: string tension)

Solving the equation results in an attraction time

$$\boxed{\tau \sim \Lambda^{\frac{1}{2}} d^{\frac{3}{2}}}$$

M. B., G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2026)

Gravitational Wave Measurement in Simulations

The gravitational radiation spectrum can be calculated by Weinberg's formula:

$$\frac{\mathrm{d}E}{\mathrm{d}\Omega\,\mathrm{d}\omega} = \frac{G\,\omega^2}{2\pi^2}\Lambda_{ij,lm}(\hat{\mathbf{k}})T^{ij*}(\mathbf{k},\omega)T^{lm}(\mathbf{k},\omega)\,,$$

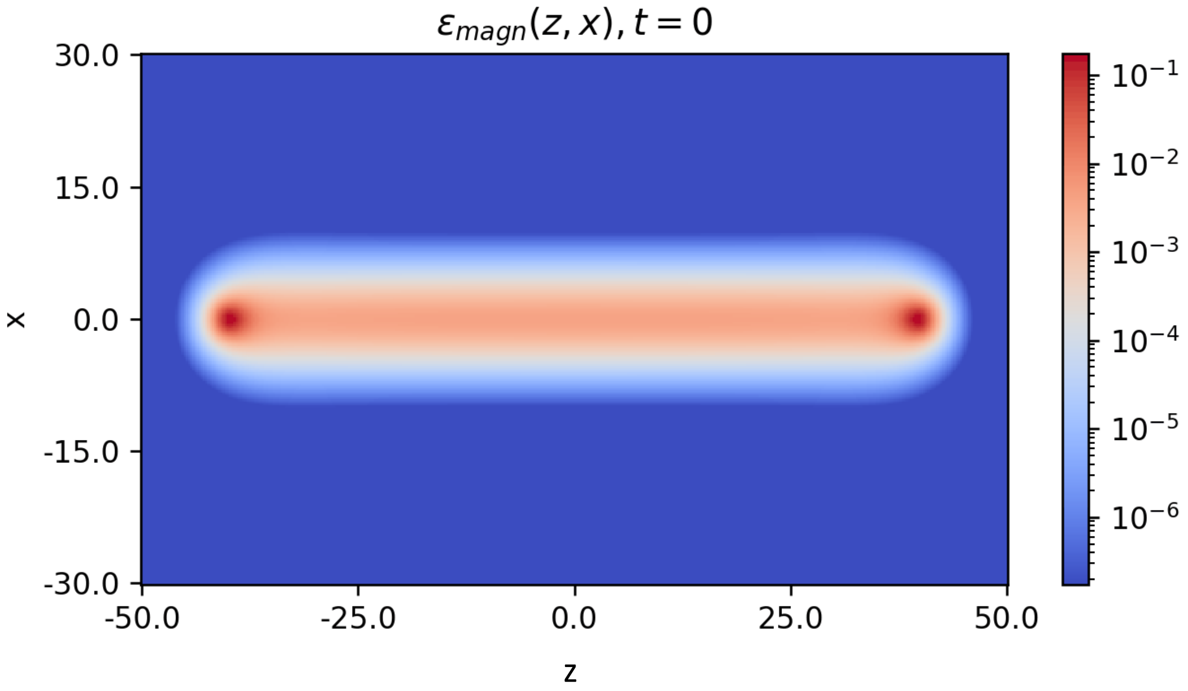
with

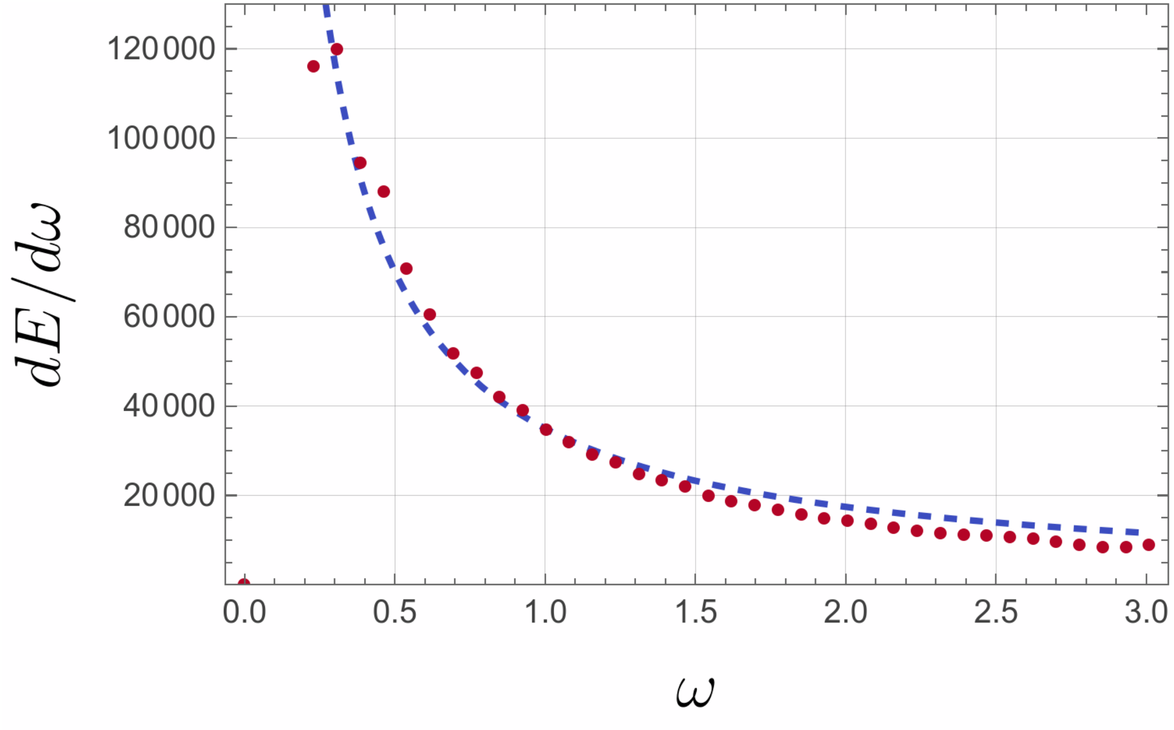
$$\Lambda_{ij,lm}(\hat{\mathbf{k}}) \equiv P_{il}(\hat{\mathbf{k}})P_{jm}(\hat{\mathbf{k}}) - \frac{1}{2}P_{ij}(\hat{\mathbf{k}})P_{lm}(\hat{\mathbf{k}})\,,$$

where $P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i\hat{k}_j$.

Martin and Vilenkin (1996): GW spectrum for monopole dumbbell scales as $\frac{\mathrm{d}P_{\mathrm{GW}}}{\mathrm{d}\omega} \sim \frac{4\mu^2}{M_{\mathrm{P}}^2}\frac{1}{\omega}$

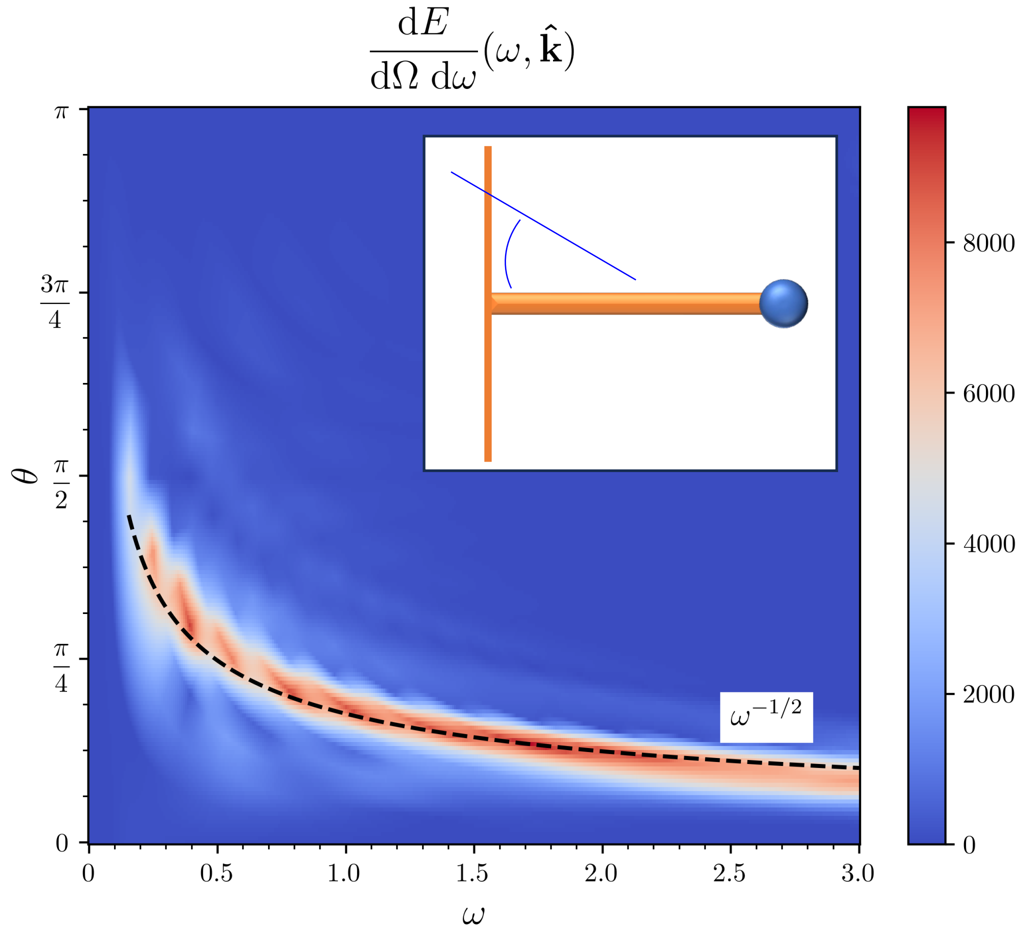
Monopole Dumbbell – Gravitational Waves





M. B. (2026) (PhD Thesis)

Confinement Slingshot – Gravitational Waves



The angle of emission depends on the frequency

Most of the radiation is emitted in the direction of acceleration

→ Again similar to two monopoles connected by a string [1]

[1] L. Leblond, B. Shlaer, X. Siemens (2009)

Initial Configuration – Domain Wall

We consider an $SU(2)$ gauge theory with the following potential

ϕ: $SU(2)$ adjoint
ψ: $SU(2)$ fundamental

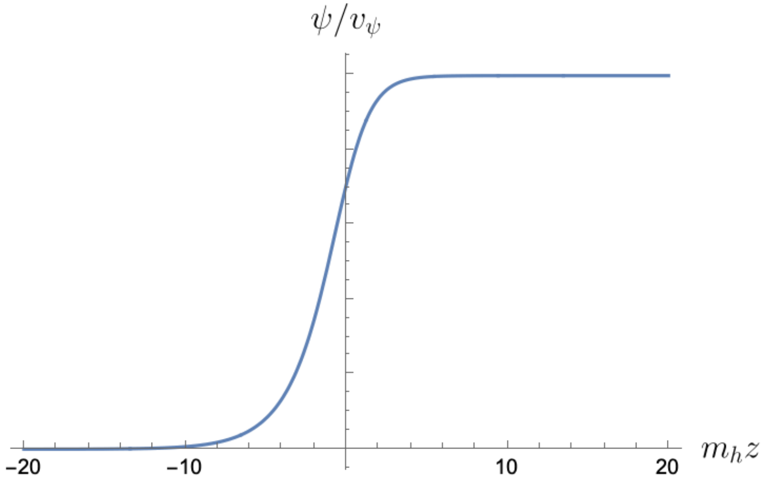
$$V(\phi) = \lambda_\phi \left(\phi^a\phi^a - v_\phi^2\right)^2 + \lambda_\psi \left(\psi^\dagger\psi - v_\psi^2\right)^2 \left(\psi^\dagger\psi\right) + \beta\psi^\dagger\phi\psi$$

This term needs to be minimized

$$\rightarrow \psi^\dagger\sigma^a\psi \sim \phi^a$$

The profile of the domain wall is

$$|\psi| = \frac{v_\psi}{\sqrt{1+e^{-m_h z}}}$$



Initial Configuration – Magnetic Monopole

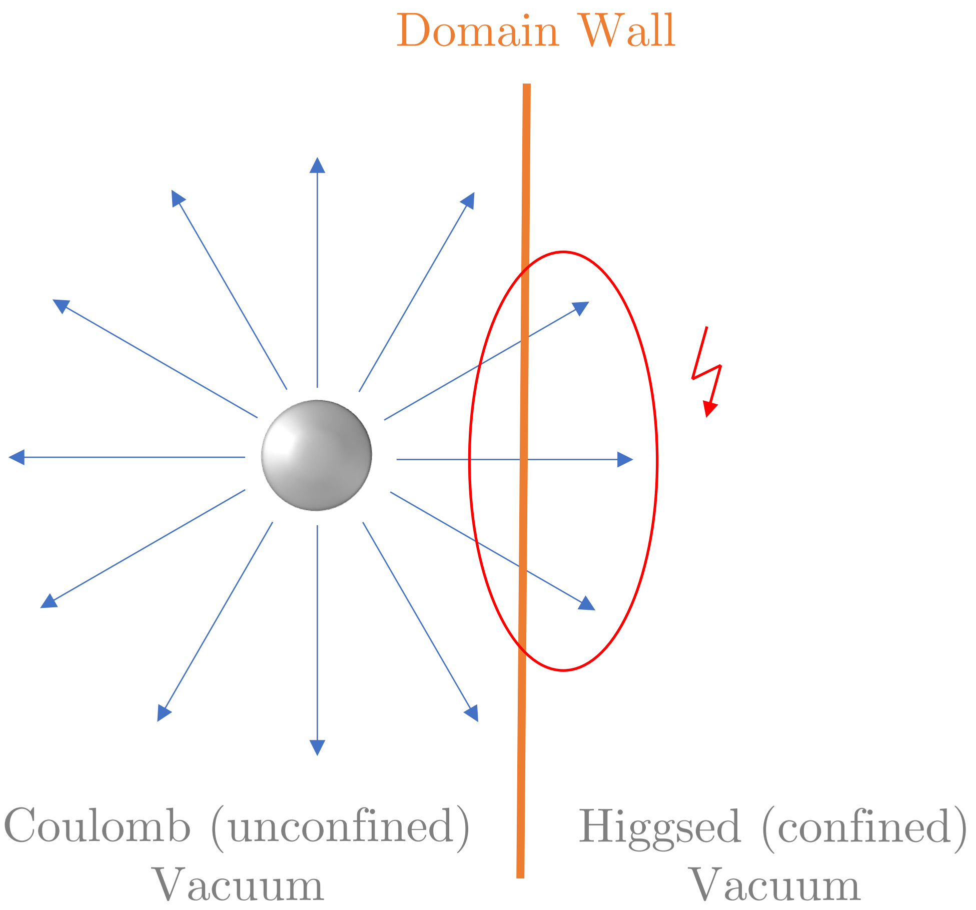
Ansatz for 't Hooft-Polyakov magnetic monopole:

$$W_i^a = \varepsilon_{aij} \frac{r_j}{r^2} \frac{1}{g} (1 - K(r))$$

$$W_t^a = 0$$

$$\phi^a = \frac{r^a}{r^2} \frac{1}{g} H(r)$$

Magnetic field needs to be repelled from the domain wall!



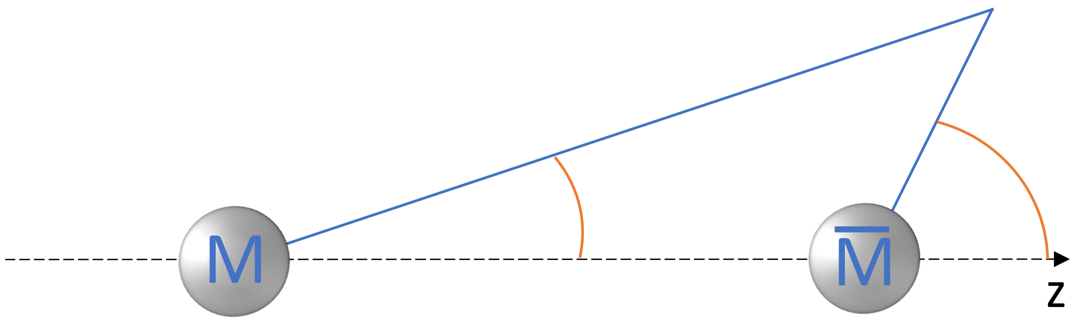
Initial Configuration – Magnetic Monopole

Ansatz for a monopole-antimonopole configuration[1]:

$$\hat{\phi}_1 = \left(\sin\bar{\theta}\cos\theta - \sin\theta\cos\bar{\theta}\cos\alpha\right)\cos\left(\varphi - \alpha/2\right) + \sin\theta\sin\alpha\sin\left(\varphi - \alpha/2\right)$$

$$\hat{\phi}_2 = \left(\sin\bar{\theta}\cos\theta - \sin\theta\cos\bar{\theta}\cos\alpha\right)\sin\left(\varphi - \alpha/2\right) - \sin\theta\sin\alpha\cos\left(\varphi - \alpha/2\right)$$

$$\hat{\phi}_3 = -\cos\theta\cos\bar{\theta} - \sin\theta\sin\bar{\theta}\cos\alpha$$



α: twist parameter

The gauge field is given by

$$W_\mu^a \sim \varepsilon_{abc}\hat{\phi}^b\partial_\mu\hat{\phi}^c$$

[1] T. Vachaspati, G. B. Field (1994) / A Saurabh, T. Vachaspati (2018)

Initial Configuration – Magnetic Monopole

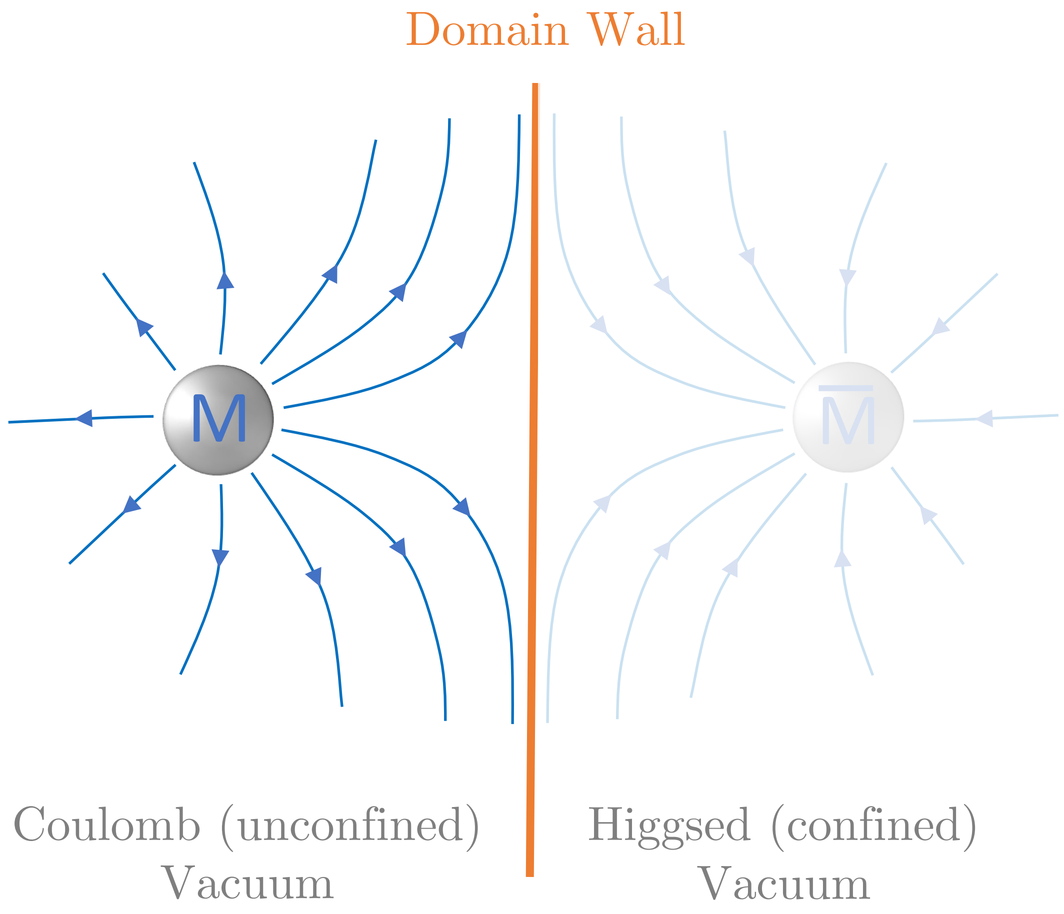
Ansatz for a monopole-antimonopole configuration[1]:

$$\begin{aligned}
\hat{\phi}_1 &= \left(\sin\bar{\theta}\cos\theta - \sin\theta\cos\bar{\theta}\cos\alpha\right)\cos\left(\varphi - \alpha/2\right) \\
&\quad + \sin\theta\sin\alpha\sin\left(\varphi - \alpha/2\right) \\
\hat{\phi}_2 &= \left(\sin\bar{\theta}\cos\theta - \sin\theta\cos\bar{\theta}\cos\alpha\right)\sin\left(\varphi - \alpha/2\right) \\
&\quad - \sin\theta\sin\alpha\cos\left(\varphi - \alpha/2\right) \\
\hat{\phi}_3 &= -\cos\theta\cos\bar{\theta} - \sin\theta\sin\bar{\theta}\cos\alpha
\end{aligned}$$

α: twist parameter

The gauge field is given by

$$W^a_\mu \sim \varepsilon_{abc}\hat{\phi}^b\partial_\mu\hat{\phi}^c$$

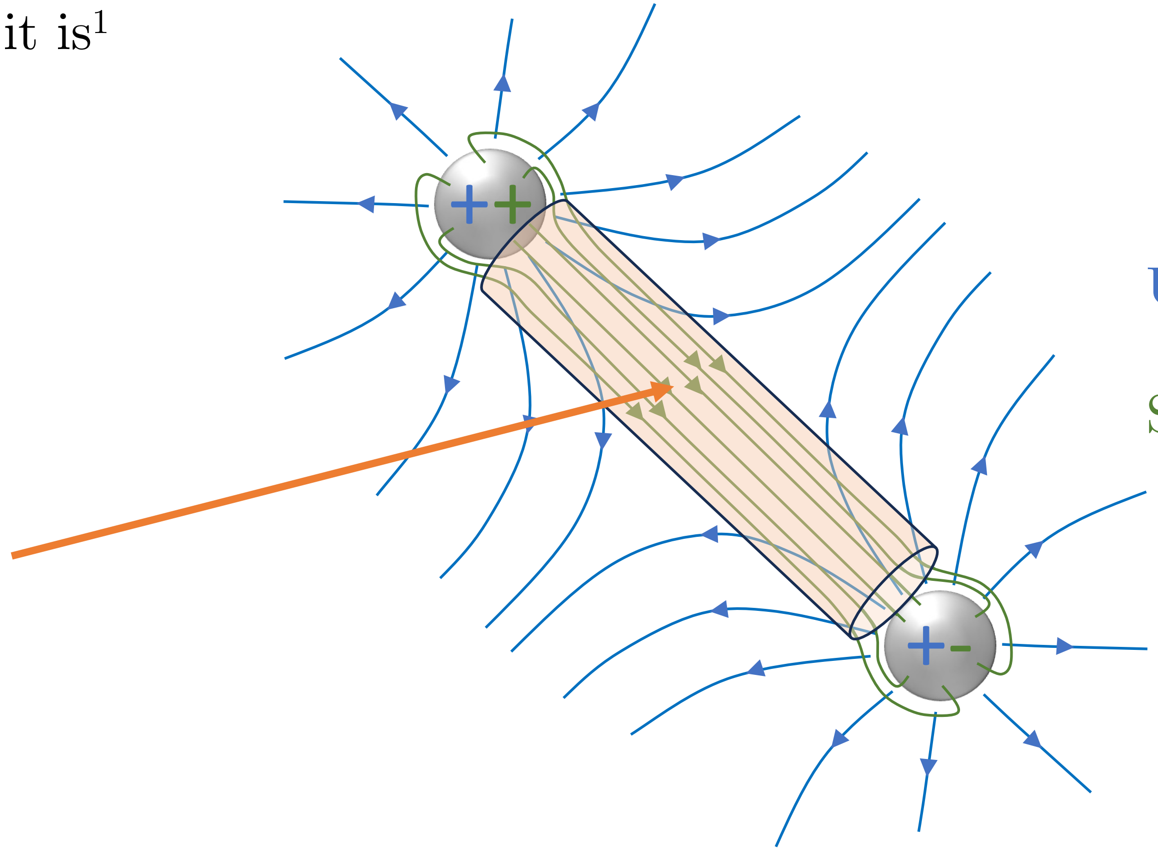


[1]T. Vachaspati, G. B. Field (1994) / A Saurabh, T. Vachaspati (2018)

Monopole Confinement – Outlook

In SU(2) it's not possible two connect two monopoles of same charge

But in an SU(3) theory it is[1]



[1] Y. Ng, T.W.B. Kibble, T. Vachaspati (2008)

Numerical Simulation

Python Package Numba:

- Translates Python and NumPy code into fast machine code
- Easy implementation of parallelization

Numerical Simulation – Axial Symmetry

Axial Symmetry:

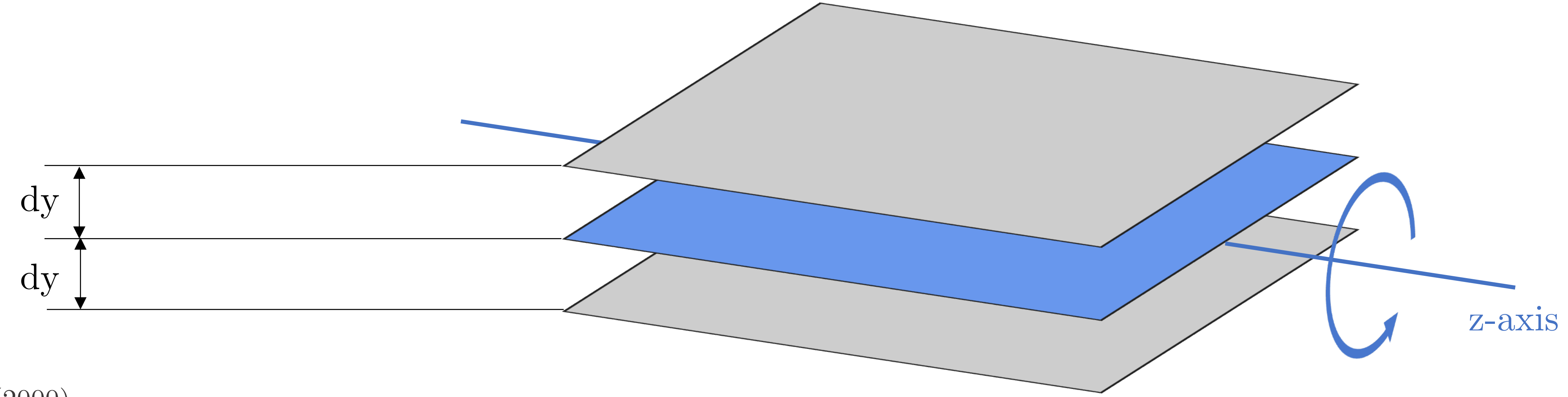
$$\phi^1 = xf_1 + yf_2 \qquad \phi^2 = yf_1 - xf_2 \qquad \phi^3 = f_3$$

$$W_x^1 = xyf_4 + y^2 f_5 + f_6 \qquad W_x^2 = -x^2 f_4 - xyf_5 + f_7 \qquad W_x^3 = xf_8 + yf_9$$

$$W_y^1 = y^2 f_4 - xyf_5 - f_7 \qquad W_y^2 = -xyf_4 + x^2 f_5 + f_6 \qquad W_y^3 = yf_8 - xf_9$$

$$W_z^1 = xf_{10} + yf_{11} \qquad W_z^2 = -xf_{11} + yf_{10} \qquad W_z^3 = 0$$

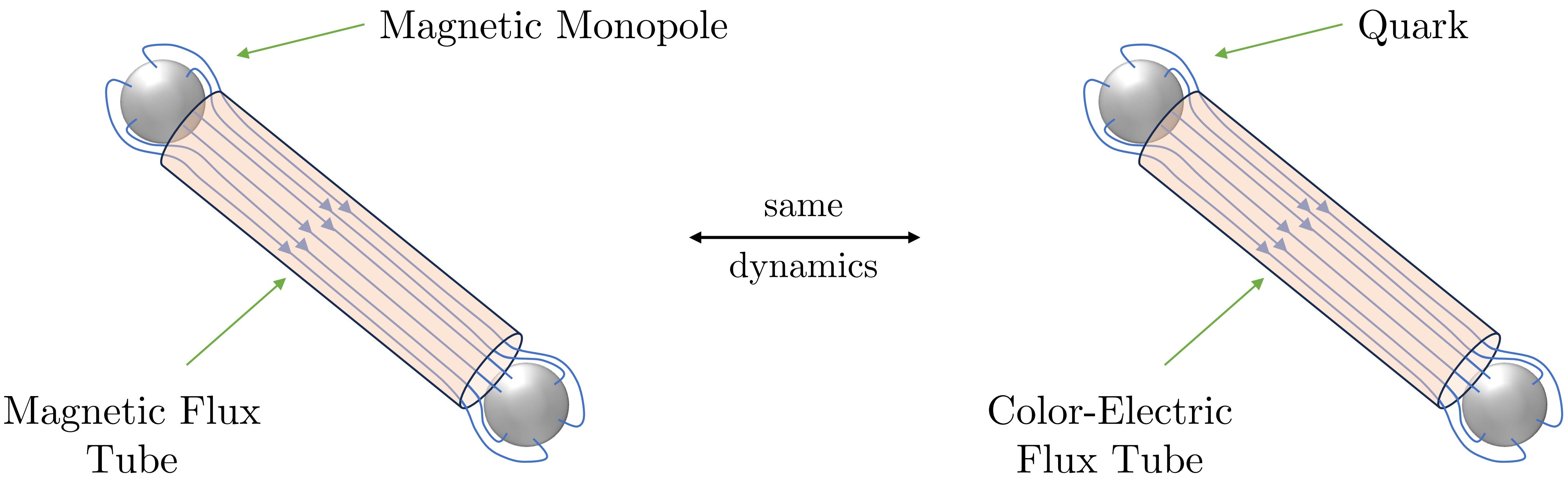
$$W_t^1 = xf_{12} + yf_{13} \qquad W_t^2 = -xf_{13} + yf_{12} \qquad W_t^3 = 0$$



L. Pogosian, T. Vachaspati (2000)

Monopoles Connected by Strings – Quark Confinement

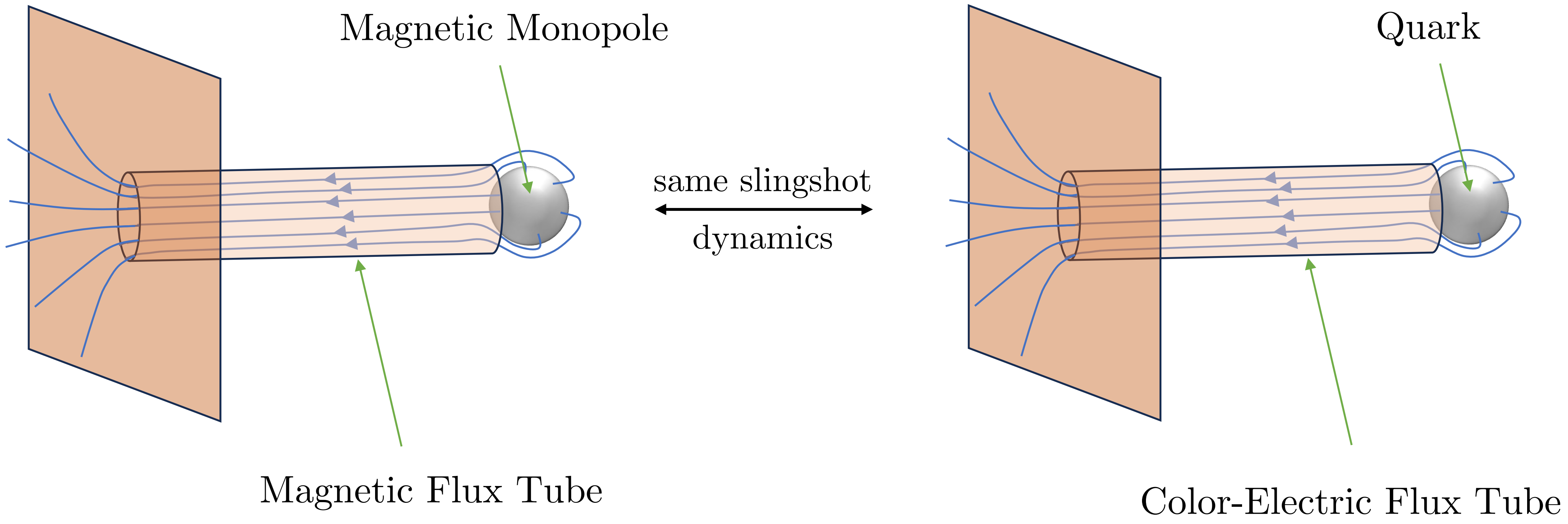
The similar slingshot effect is expected in a "dual" picture when a heavy quark crosses into a confined vacuum of QCD.



[1] G. 't Hooft (1974)
[2] G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2022)

Confinement Slingshot – Quark Confinement

The similar slingshot effect is expected in a "dual" picture when a heavy quark crosses into a confined vacuum of QCD.



(In the case of light quarks, quark-antiquark pairs can emerge and break the string.)