

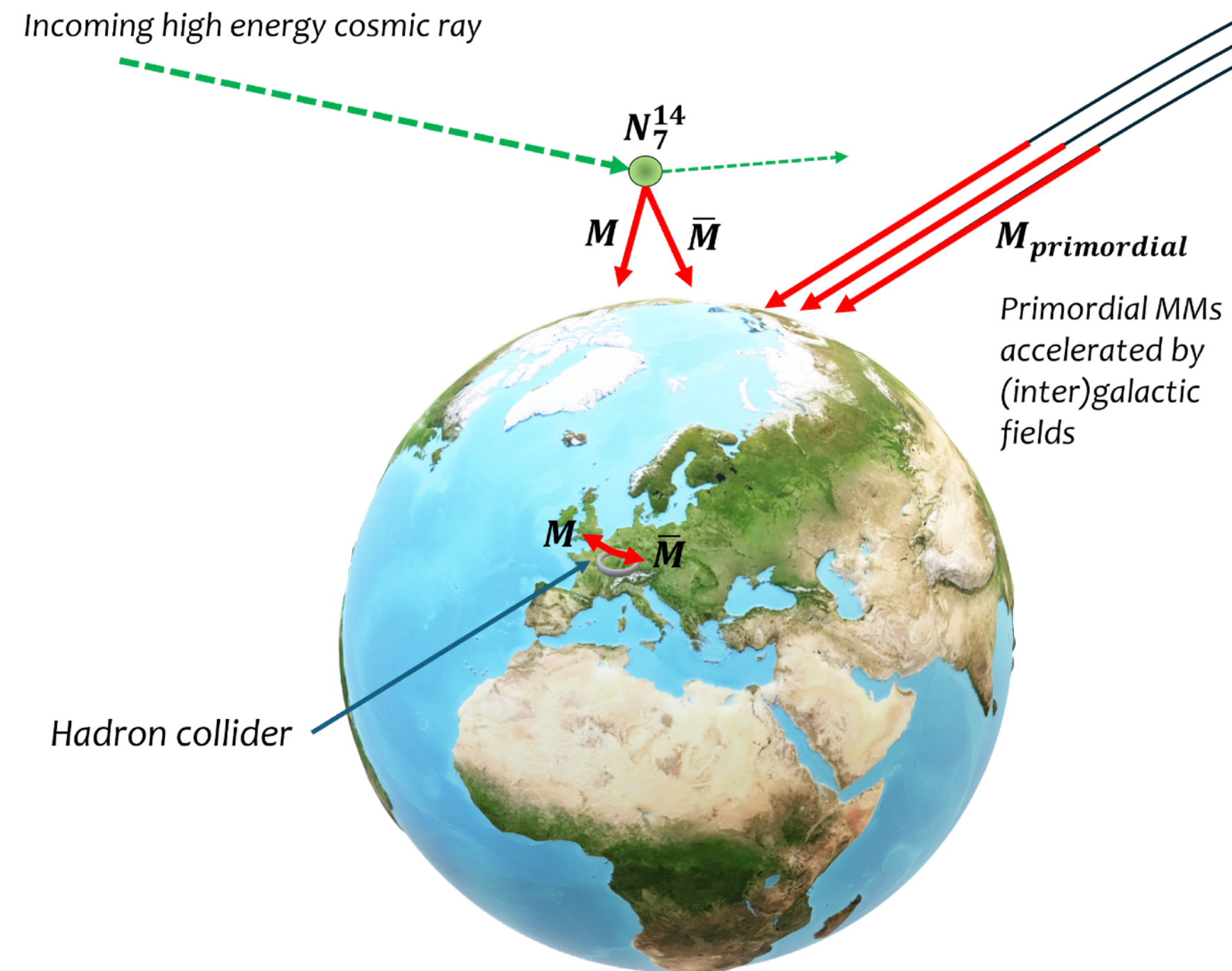
Cosmological Implications of GUT Monopoles for Gravitational Waves

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<https://indico.ihep.ac.cn/event/28485>

Homotopy group $\Leftrightarrow$ topological defects

- Kibble mechanism, 1976
 - — originally proposed for defects generated in a continuous phase transition.
- Topological defects depend on the homotopy groups (同伦群) of the manifold of degenerate vacua.

For symm breaking $G \rightarrow H$, degenerate vacua form a manifold $\mathcal{M} = G/H$, a homotopy group $\pi_k(\mathcal{M})$ is defined by the set of mapping $S^k \rightarrow \mathcal{M}$

- If $\pi_k(G/H) \neq 1$, $(2 - k)$ -dim topological defects form
 - $k = 2 \Rightarrow$ monopoles (单极子), 0-dim point in the core
 - $k = 1 \Rightarrow$ cosmic strings (宇宙弦), 1-dim string in the core
 - $k = 0 \Rightarrow$ domain walls (畴壁), 2-dim surface in the core

't Hooft-Polyakov monopole

't Hooft, NPB, 74; Polyakov, JEPT Lett., 74

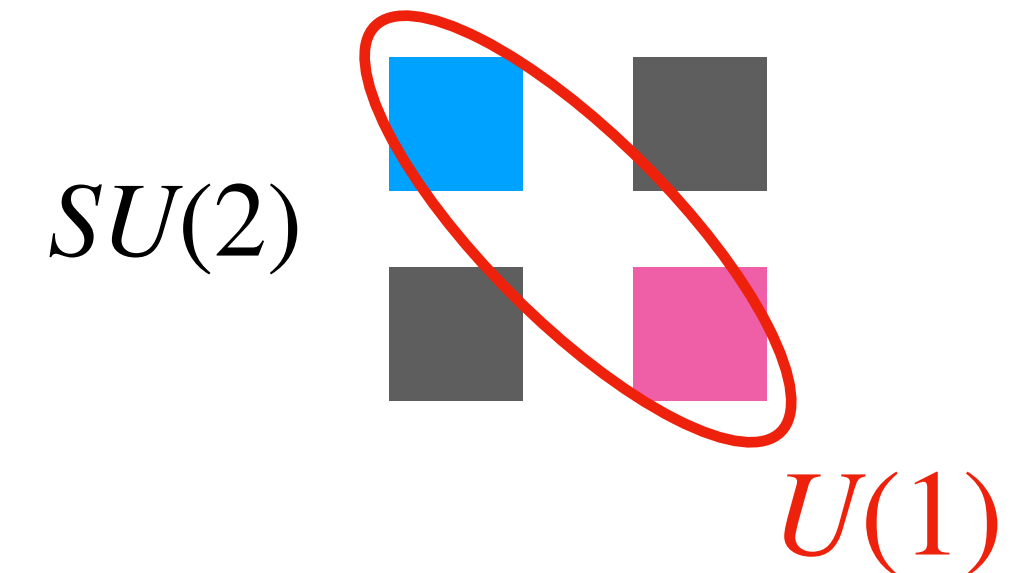
- A gauge $SU(2)$ symmetry with an adjoint scalar

$$\Phi = \frac{1}{2} \tau^a \Phi^a = \frac{1}{2} \begin{bmatrix} \Phi^3 & \Phi^1 - i\Phi^2 \\ \Phi^1 + i\Phi^2 & -\Phi^3 \end{bmatrix}$$

$$\mathcal{L} = -\frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \text{Tr} D_\mu \Phi D^\mu \Phi - V(\Phi)$$

- After Φ gains the VEV, $SU(2)$ is spontaneously broken to $U(1)$

$$\langle \Phi \rangle = U(\Omega) \frac{1}{2} \begin{bmatrix} v & 0 \\ 0 & -v \end{bmatrix} U^{-1}(\Omega)$$



- The Φ VEV does not have to be a constant in the gauge space

$$A_i^a = \epsilon_{iam} \hat{r}^m \left[\frac{1 - u(r)}{er} \right]$$

$$\Phi^a = \hat{r}^a h(r)$$

$$0 = h'' + \frac{2}{r} h' - \frac{2u^2 h}{r^2} + \lambda(v^2 - h^2)h$$

$$0 = u'' - \frac{u(u^2 - 1)}{r^2} - e^2 u h^2$$

$$\pi_2(SU(2)/U(1)) = \mathbb{Z}$$

in hedgehog gauge

- This field configuration leads to an energy condensation, referring to the monopole mass

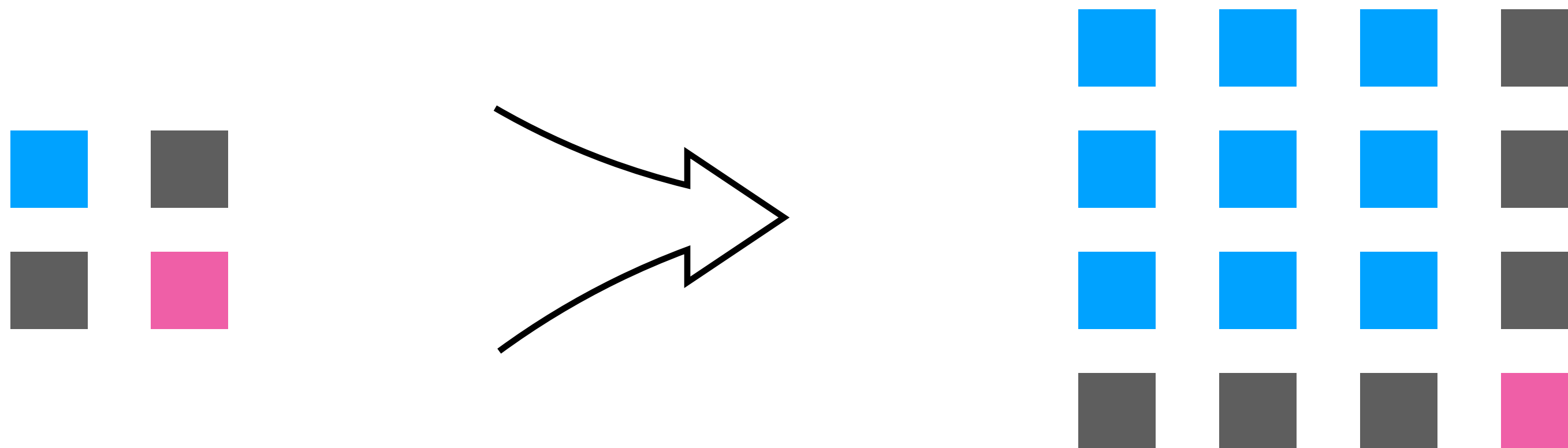
$$M_{\text{mono}} = \frac{4\pi v}{e} f(\lambda/e^2)$$

In the BPS limit, $\lambda/e^2 \rightarrow 0$, $f(\lambda/e^2) \rightarrow 1$

Most energy is restricted in the narrow radius $R \sim 1/v$

Pati-Salam monopoles

- Gauge symmetry $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$
- $SU(4)_c$ is spontaneously broken to $SU(3)_c \times U(1)_{B-L}$ $\pi_2\left(\frac{SU(4)}{SU(3) \times U(1)}\right) = \pi_2(CP^3) = \mathbb{Z}$
- The breaking of $SU(2) \rightarrow U(1)$ can be embedded into $SU(4) \rightarrow SU(3) \times (1)$
- Monopole arises from the breaking of Pati-Salam symmetry.
Its property is determined by the embedding of $SU(2)$ into the gauge space of $SU(4)$

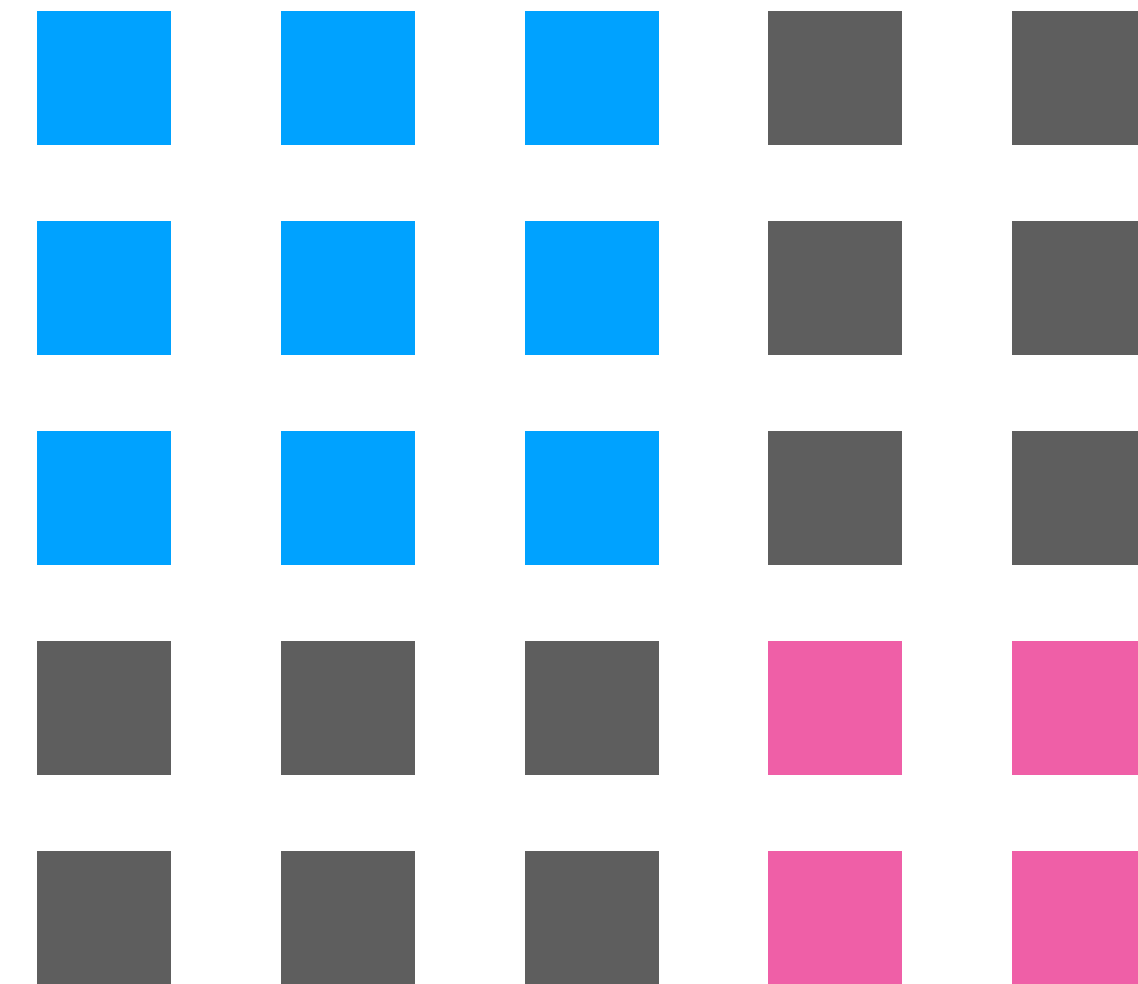


SU(5) monopoles

- Gauge symmetry $SU(5)$, broken to SM gauge symmetry directly via an adjoint 24-plet Higgs

$$SU(5) \rightarrow S(U(3) \times U(2)) \simeq \frac{SU(3)_c \times SU(2)_L \times U(1)_Y}{Z_3 \times Z_2}$$

$$\pi_2 = Z$$

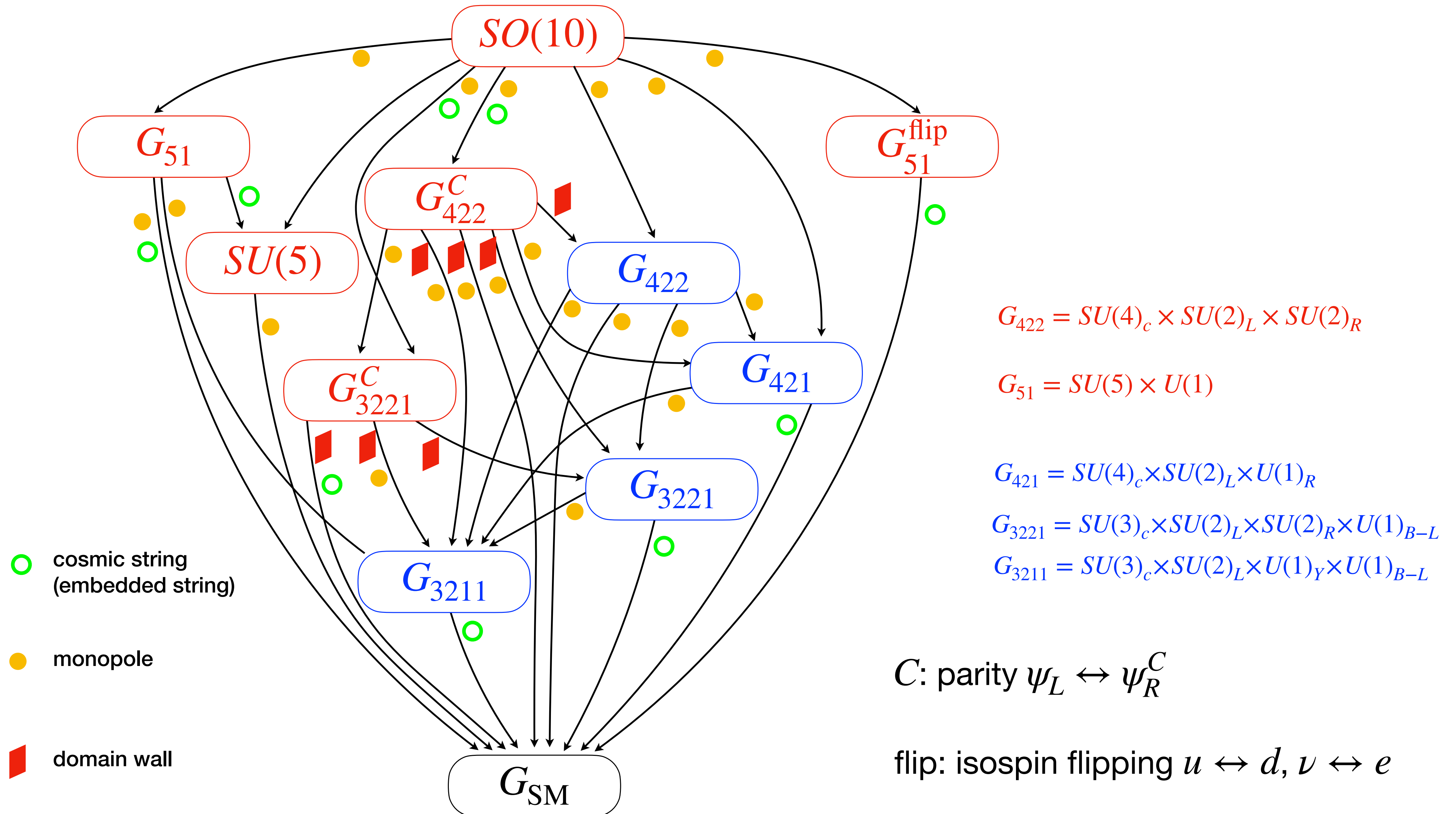


SO(10) monopoles: two scales

$$SO(10) \rightarrow SU(4)_c \times SU(2)_R \times SU(2)_L$$

$$SU(4)_c \rightarrow SU(3)_c \times U(1)_{B-L}$$

Monopoles is a general prediction of GUTs



GUT monopole problem

't Hooft, NPB, 1974; Zeldovich & Khlopov, PLB, 1978; Preskill, 1979

- Monopoles as topological defects are produced during cosmological phase transition, referring to spontaneous symmetry breaking of certain non-Abelian gauge symmetry.
- Typical distance between monopoles is around the Hubble length scale at phase transition, $L_m \sim H_\star^{-1}$. Thus, the initial number density $n_\star \sim H_\star^3$.
- Monopoles, once they are produced, evolve as matter during Hubble expansion. The number density today is given by

$$\dot{n} = -3Hn - Dn^2 \quad \xrightarrow{H \gg Dn} \quad n_{\text{mono}}(t_0) = \left(\frac{a(t_\star)}{a(t_0)} \right)^3 n_\star$$

- GUT monopole problem: GUT monopoles $M_{\text{mono}} \sim M_{\text{GUT}}/\alpha_{\text{GUT}} \gtrsim 10^{16}$ GeV, $H_\star \sim 0.1 M_{\text{GUT}}^2/M_{\text{pl}}$. Their energy density fraction $\Omega_{\text{mono}} = M_{\text{mono}} n(t_0)/\rho_c$ is given by

$$\Omega_{\text{mono}} = \frac{8\pi G M_{\text{mono}} H_\star^3}{3H_0^2(1+z_{\text{Rh}})^3} \sim 10^{14} \left(\frac{T_{\text{Rh}}}{10^{15} \text{ GeV}} \right)^4 \gg 1$$

- Inflation is the standard way to solve the GUT monopole problem

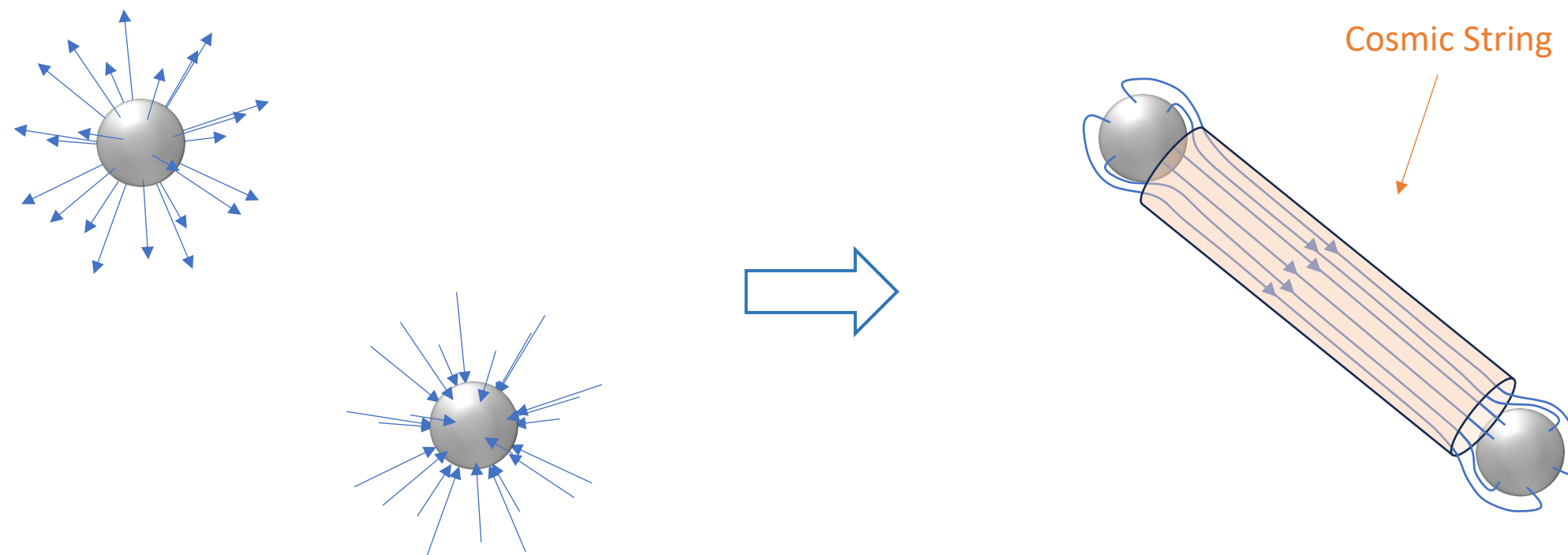
Solving monopole problem via slingshot effect?

⇒ Maximilian Bachmaier's talk

Magnetic Monopoles Connected by a String

First Phase Transition: $G \rightarrow \dots \times U(1)$

Second Phase Transition: $\dots \times U(1) \rightarrow \dots \times \cancel{U(1)}$



P. Langacker, S. Pi (1980)
X. Martin, A. Vilenkin (1997)
G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2022)

Maximilian Bachmaier

NFMMS 2026 (12.09.2026)

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$$G_{3221} = SU(3)_c \times SU(2)_L \times \textcolor{red}{SU(2)}_R \times U(1)_{B-L}$$

$\pi_2 = Z$ ↓ monopole

$$G_{3211} = SU(3)_c \times SU(2)_L \times \textcolor{red}{U(1)}_R \times U(1)_{B-L}$$

$\pi_1 = Z$ ↓ string

$$G_{\text{SM}} = SU(3)_c \times SU(2)_L \times \textcolor{red}{1} \times U(1)_{B-L}$$

$\pi_2 = \pi_1 = 0$

The slingshot effect does not apply to PS or SU(5) monopoles.

$$\pi_2(G_{422}/G_{\text{SM}}) = Z$$

$$\pi_1(G_{422}/G_{\text{SM}}) = 0$$

Monopoles are point-like objects, how do they affect GWs?

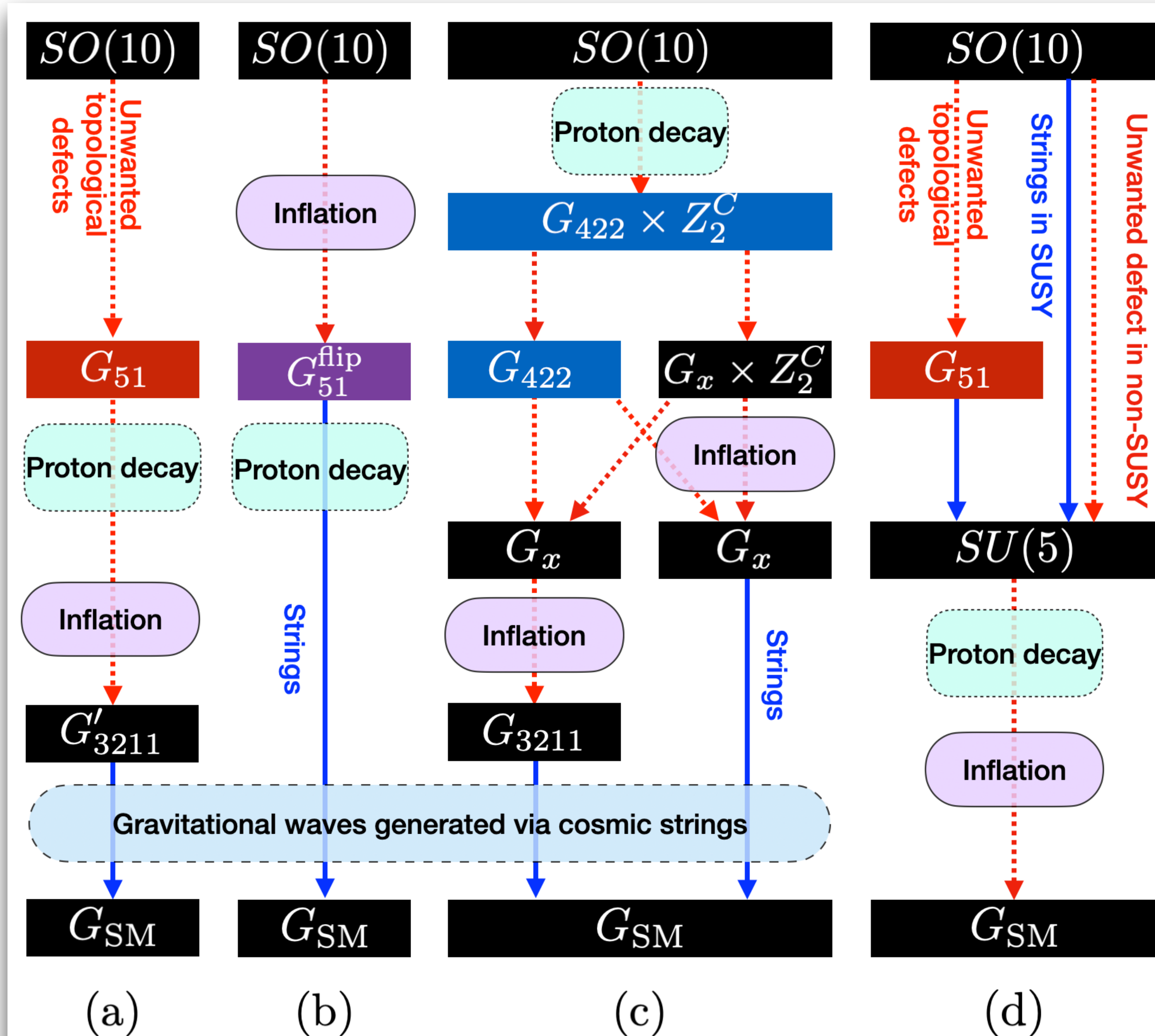
- **GWs via cosmic strings:**

It may influence cosmic string evolution, and thus change the spectrum of GWs via cosmic strings

- **GWs via GUT phase transition:**

Solving the monopole problem does not require the inflation fully after GUT phase transition. If phase transition happens during inflation, inflated GW spectrum is predicted.

GWs via cosmic strings are widely predicted in GUTs



Unwanted topological defects:
monopoles and domain walls

In any breaking chains, inflation has to be introduced to inflate unwanted defects

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$G_{51} = SU(5) \times U(1)_X$$

$$G_{51}^{flip} = SU(5)_{flip} \times U(1)_{flip}$$

$$Z_2^C: \psi_L \leftrightarrow \psi_R^c$$

$$G_x = G_{421} \text{ or } G_{3221}$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$

$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

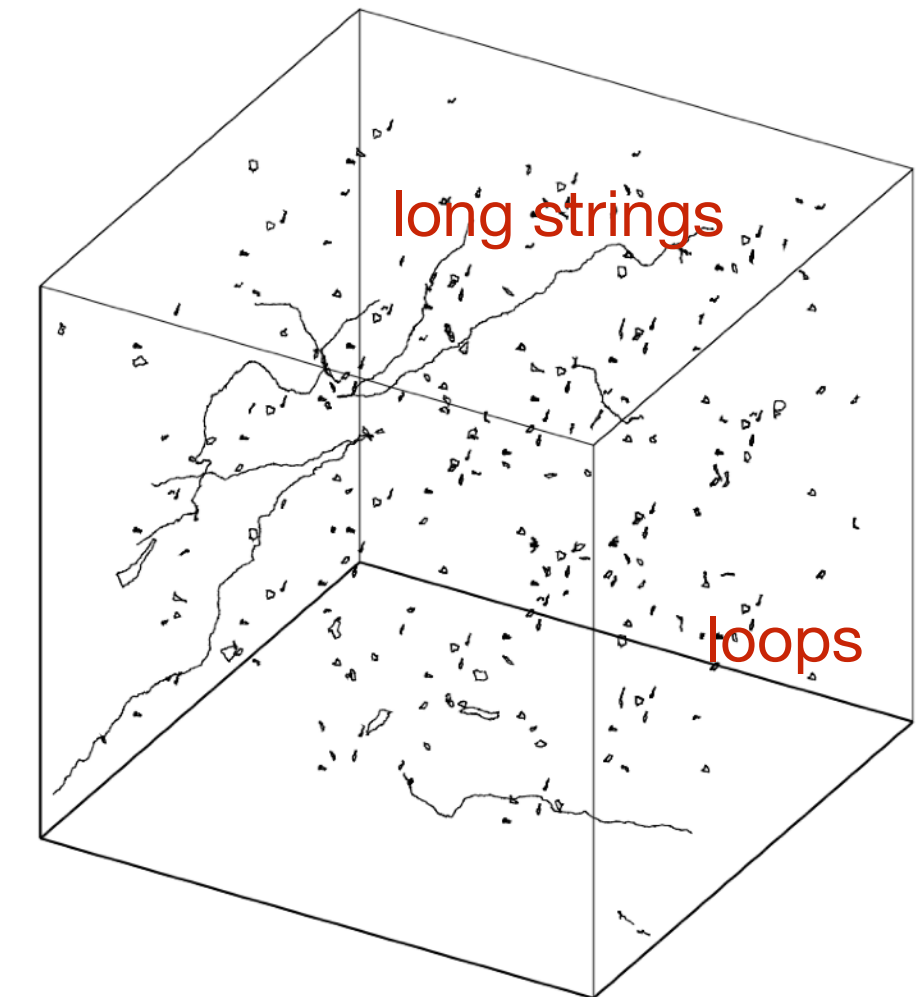
King, Pascoli, Turner, YLZ, 2005.13549

Mechanism of GWs via cosmic strings in GUTs

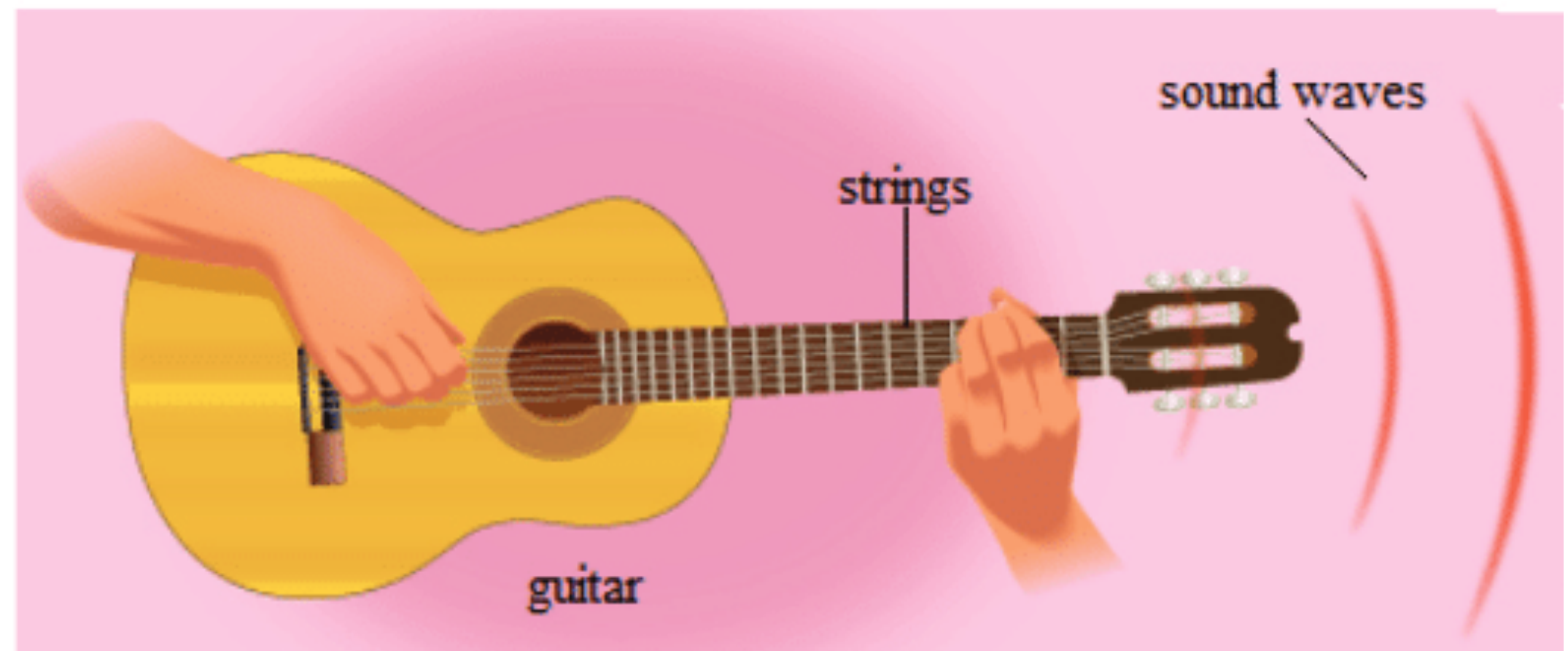
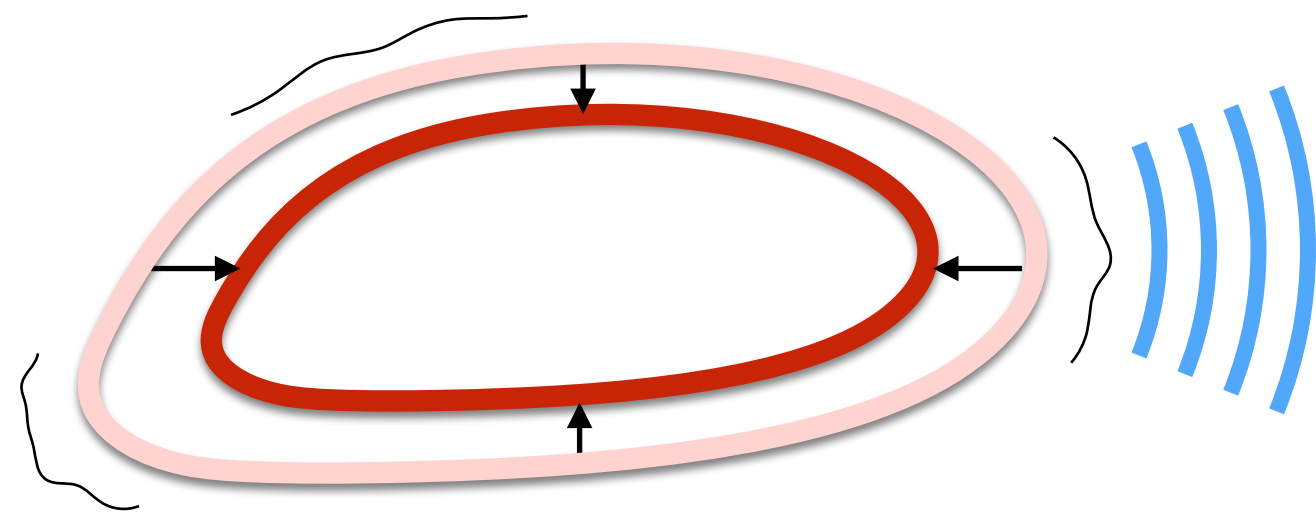
GW via cosmic strings

$$\pi_1(U(1)) = \mathbb{Z}$$

- Most GUTs include $U(1)_{B-L}$, denotes the scale as M_{B-L}
- Spontaneous breaking of $U(1)_{B-L}$ generates cosmic strings.
- Strings intersect and intercommute to form loops and cusps
- Loops oscillates via gravitational radiation



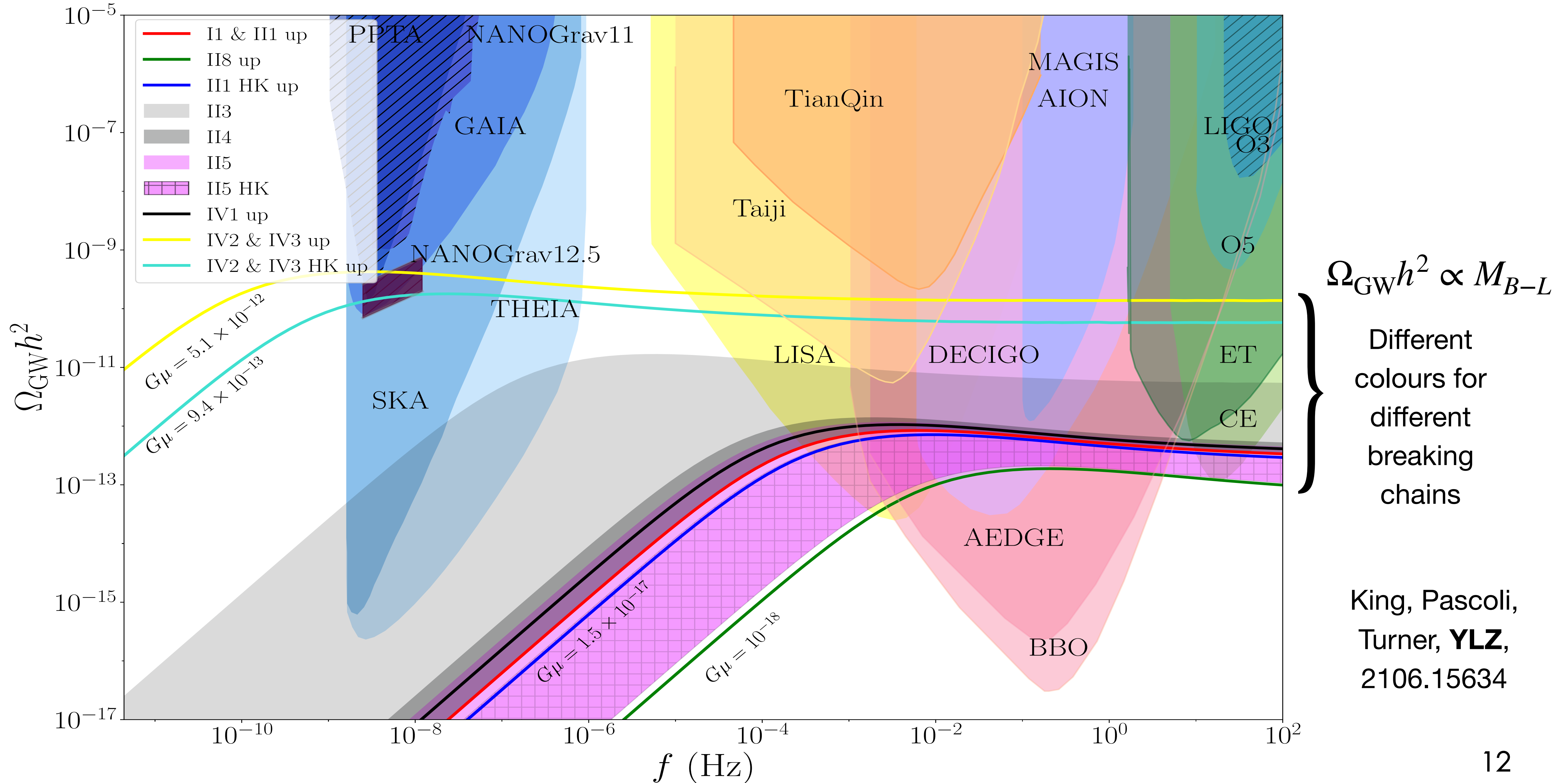
Vanchurin, Olum, Vilenkin, 0511159



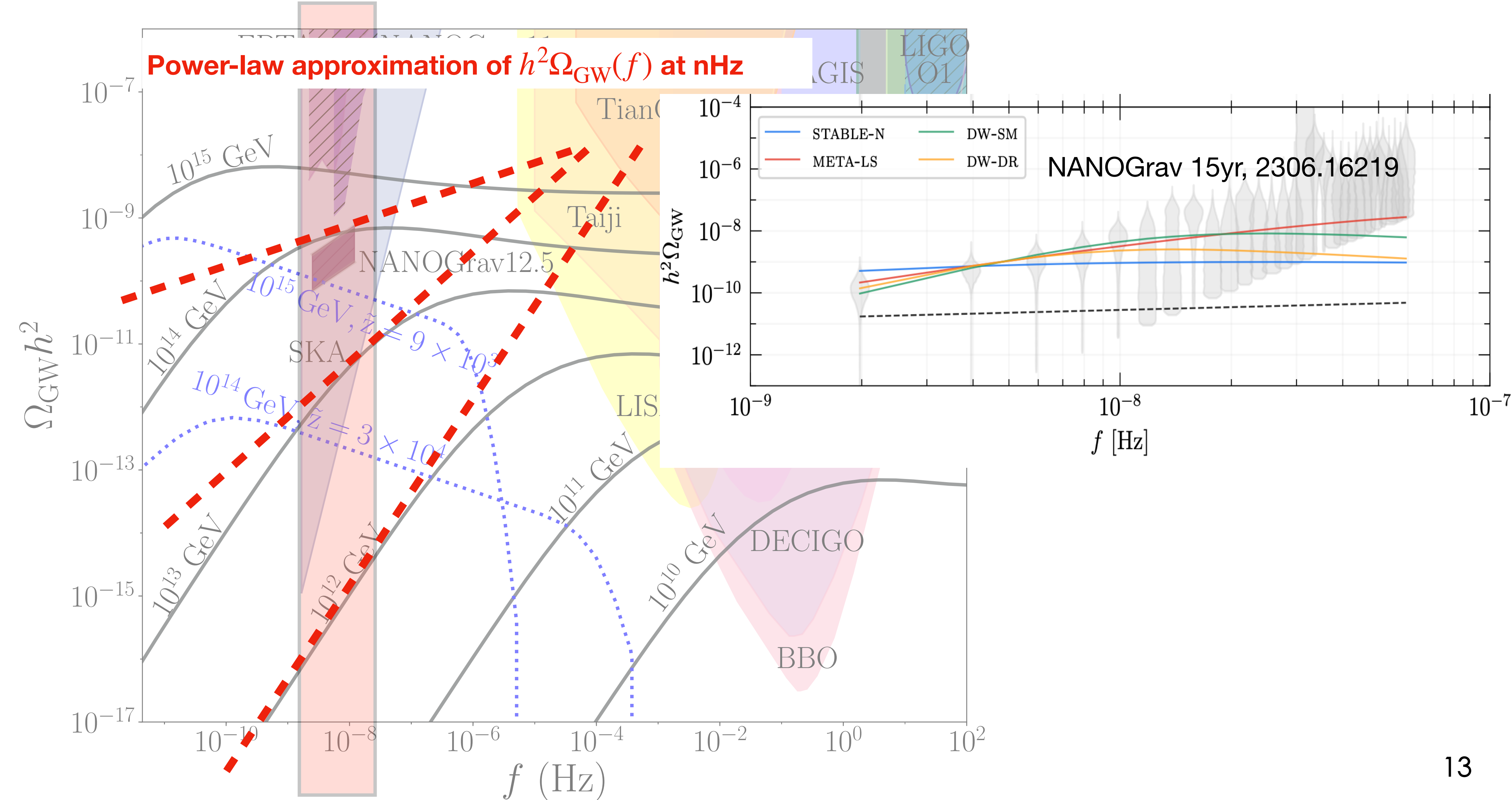
Another mechanism: GW via GUT phase transition?

— —require technique developments to measure high-frequency GW, see e.g., 2011.12414, 2310.06607

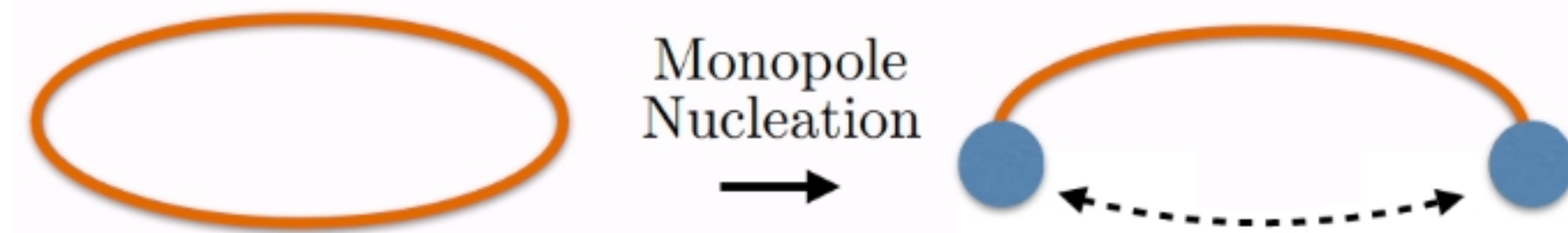
Predictions of GWs via cosmic strings in non-SUSY GUTs



Tensions between NANOGrav and GWs via cosmic strings



Metastable strings: decaying via monopole-antimonopol pairs

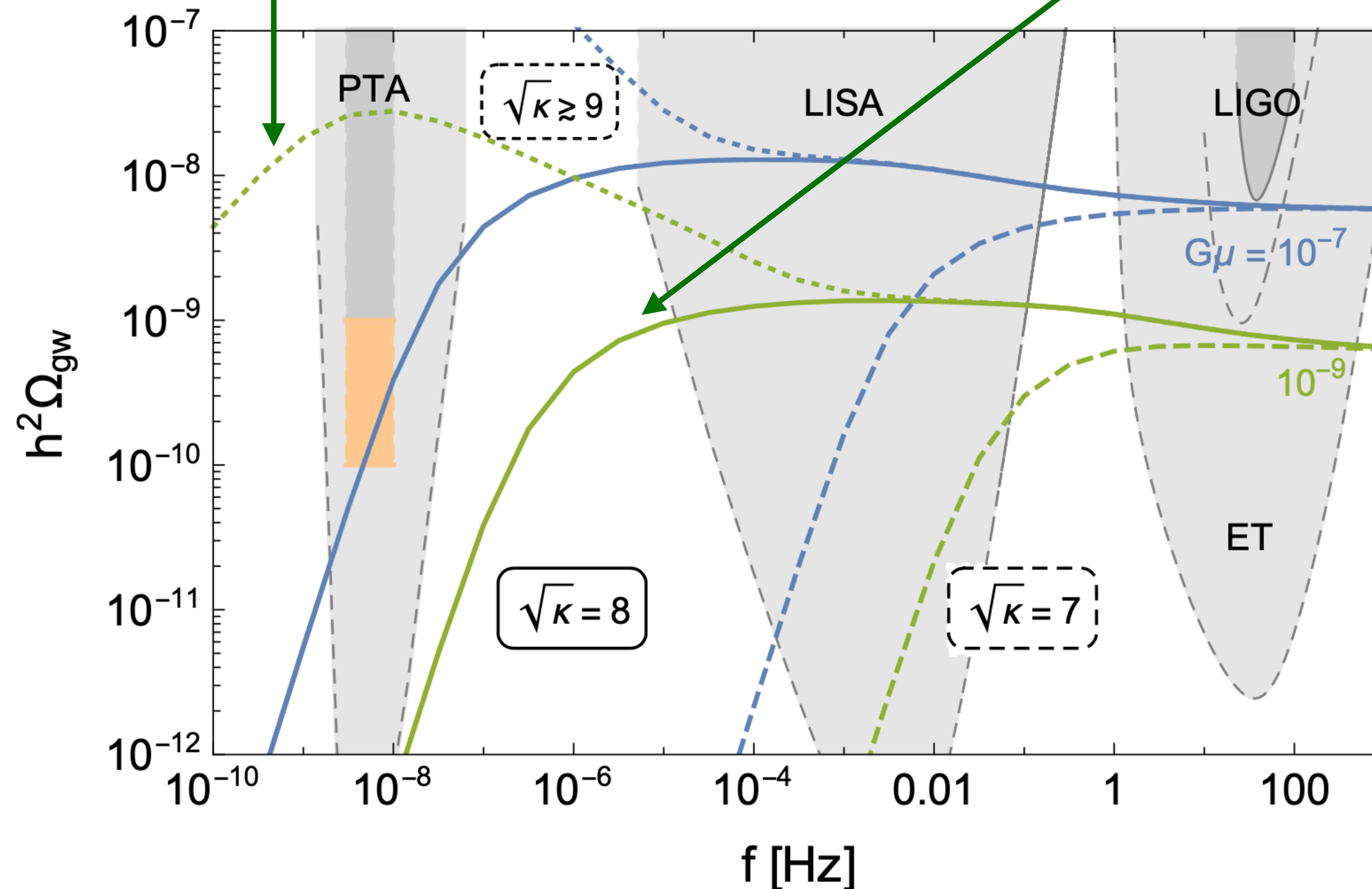


Vilenkin, NPB(1982); Preskill, Vilenkin, 9209210;
Leblond, Shlaer, Siemens, 0903.4686; Monin, Voloshin,
0808.1693; Buchmuller, Domcke, Schmitz, 2107.04578

$n(l, t)$ of loops from NG strings

$n(l, t) e^{-\Gamma_d l t}$ of metastable loops

$$\Gamma_d = \frac{\mu}{2\pi} e^{-\pi\kappa}$$



$$\sqrt{\kappa} = \frac{m_{\text{monopole}}}{\sqrt{\mu_{\text{string}}}} \sim \alpha_{\text{GUT}}^{-1/2} \frac{M_{\text{GUT}}}{M_{B-L}}$$

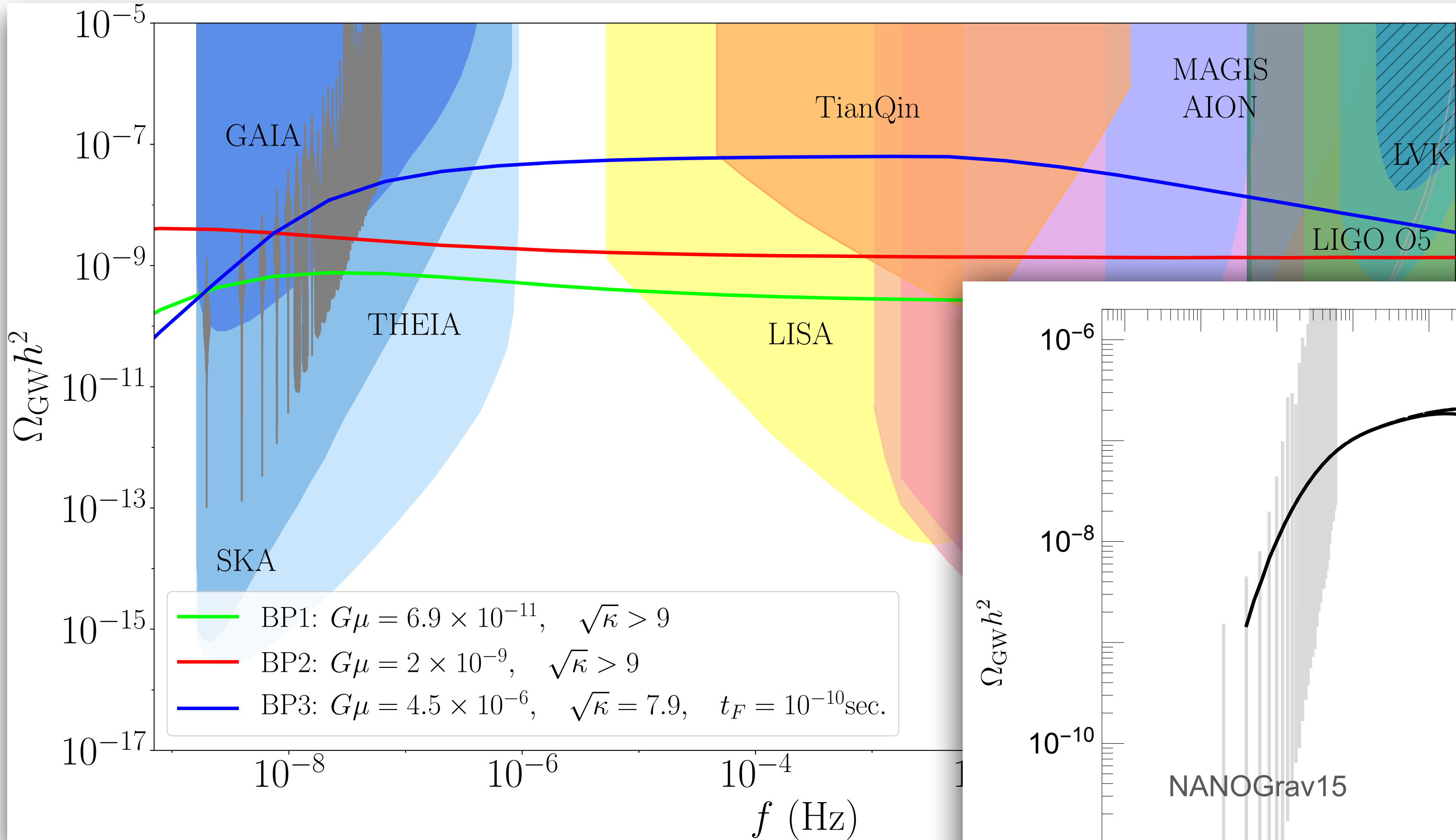
$$\sqrt{\kappa} \simeq (8,9) \Rightarrow M_{\text{GUT}} \sim M_{B-L}$$

Buchmuller, Domcke, Schmitz, 2307.04691

A GUT inflation separates the GUT breaking and B-L breaking in the time scale is required.

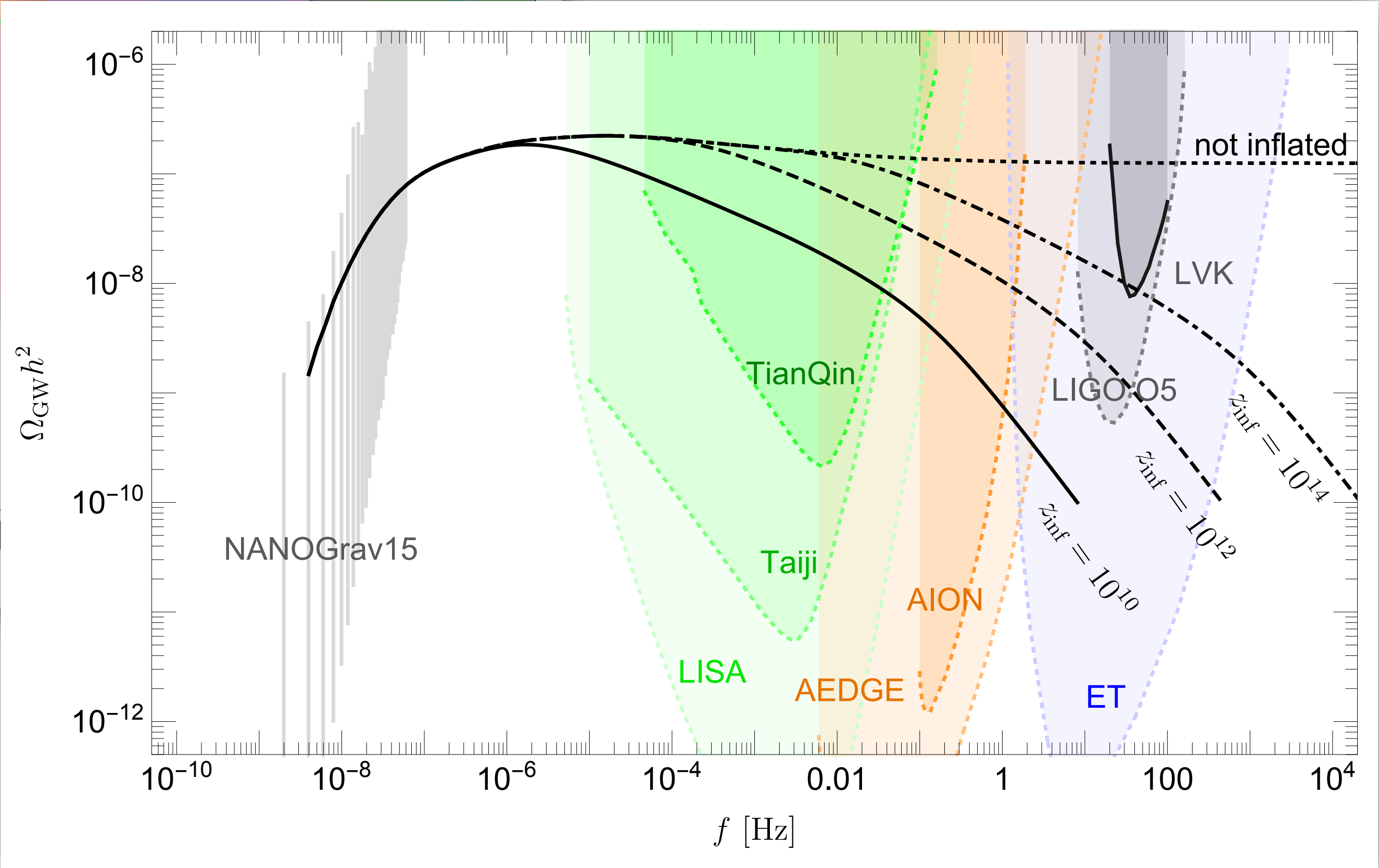
Antusch, Hinze, Saad, Steiner, 2307.04595

See also Lazarides, Maji, Moursy, Shafi, 2308.07094



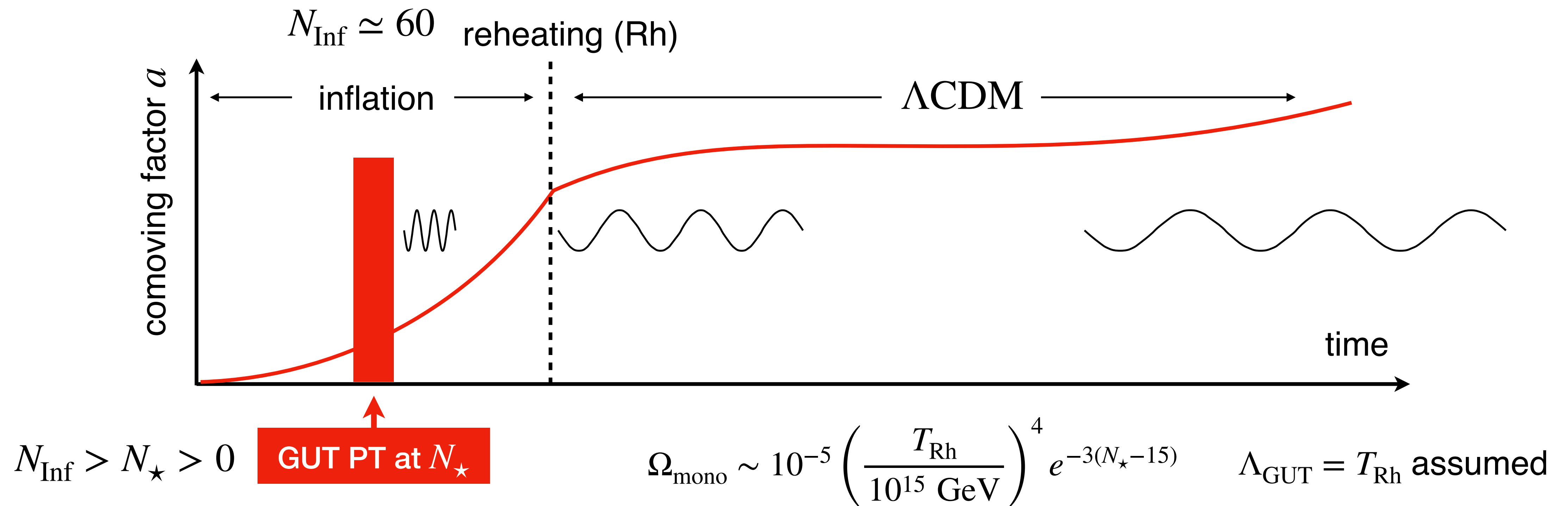
GWs in SUSY SO(10) GUTs
Fu, King, Marsili, Pascoli, Turner,
YLZ, 2308.05799

GWs in flipped SU(5)
King, Leontaris, **YLZ**,
2311.11857



GUT breaking during inflation

- Locating the GUT PT during inflation can also solve the monopole problem.
And if the PT is first order ...



- Explicit model constructions, underway ...

SU(5) inflation, Vilenki, Shafi, PRL, 1984

Smooth hybrid inflation, Lazarides, Panagiotakopoulos, hep-ph/9506325

Shifted hybrid inflation in PS model, Jeannerot, Khalil, Lazarides, Shafi, hep-ph/0002151, ...

GWs from phase transition during inflation: formalism

- The mechanism was originally proposed in [An, Lyu, Wang, Zhou, 2009.12381, 2201.05171]
- We repeat and generalise the formalism to PT happens at any e-folds to the end of inflation (=reheating), and interpret the formalism in a more transparent way (in my opinion)

- Conformal frame $ds^2 = a^2(\tau) [d\tau^2 - d\mathbf{x}^2], dt = a d\tau$

- GWs as perturbation of the metric $ds^2 = a^2(\tau) [d\tau^2 - (\delta_{ij} + h_{ij}(\tau, \mathbf{x})) dx^i dx^j]$

- EOM of GWs $h''_{ij}(\tau, \mathbf{x}) + 2a(\tau)H h'_{ij}(\tau, \mathbf{x}) - \nabla^2 h_{ij}(\tau, \mathbf{x}) = 16\pi G_N a^2(\tau) \sigma_{ij}(\tau, \mathbf{x})$

- Fourier Trans. $\tilde{h}''_{ij}(\tau, \mathbf{k}) + 2a(\tau)H \tilde{h}'_{ij}(\tau, \mathbf{k}) + k^2 \tilde{h}_{ij}(\tau, \mathbf{k}) = 16\pi G_N a^2(\tau) \tilde{\sigma}_{ij}(\tau, \mathbf{k})$

- GW metric proportional from τ' to τ ($\tau' < \tau$) is given by

$$h_1^{\text{Inf}}(\tau, \mathbf{k}) = \cos k\tau + k\tau \sin k\tau$$

$$h_2^{\text{Inf}}(\tau, \mathbf{k}) = \sin k\tau - k\tau \cos k\tau$$

$$\tilde{h}_{ij}(\tau, \mathbf{k}) = 16\pi G_N \int d\tau' \theta(\tau - \tau') a^2(\tau') \tilde{\sigma}_{ij}(\tau', \mathbf{k}) \times \mathcal{G}(\tau, \tau'; \mathbf{k})$$

Green function: $\mathcal{G}(\tau, \tau'; \mathbf{k}) = \left[\frac{\partial h_1}{\partial \tau} - \frac{h_1}{h_2} \frac{\partial h_2}{\partial \tau} \right]_{\tau=\tau'}^{-1} h_1(\tau, \mathbf{k}) + \left[\frac{\partial h_2}{\partial \tau} - \frac{h_2}{h_1} \frac{\partial h_1}{\partial \tau} \right]_{\tau=\tau'}^{-1} h_2(\tau, \mathbf{k})$

GWs from phase transition during inflation: formalism

- GW propagation in Inflation (Inf) era

$$\tilde{h}_{\mu\nu}(\tau^{\text{Inf}}, \mathbf{k}) = C_{\mu\nu,1}^{\text{Inf}} h_1^{\text{Inf}}(\tau^{\text{Inf}}, \mathbf{k}) + C_{\mu\nu,2}^{\text{Inf}} h_2^{\text{Inf}}(\tau^{\text{Inf}}, \mathbf{k})$$

$$h_1^{\text{Inf}}(\tau, \mathbf{k}) = \cos k\tau + k\tau \sin k\tau$$

$$h_2^{\text{Inf}}(\tau, \mathbf{k}) = \sin k\tau - k\tau \cos k\tau$$

- GW propagation in Radiation Domination (RD) era

$$\tilde{h}_{\mu\nu}(\tau^{\text{RD}}, \mathbf{k}) = C_{\mu\nu,1}^{\text{RD}} h_1^{\text{RD}}(\tau^{\text{RD}}, \mathbf{k}) + C_{\mu\nu,2}^{\text{RD}} h_2^{\text{RD}}(\tau^{\text{RD}}, \mathbf{k})$$

$$h_1^{\text{RD}}(\tau, \mathbf{k}) = \frac{\cos k\tau}{k\tau} \quad h_2^{\text{RD}}(\tau, \mathbf{k}) = \frac{\sin k\tau}{k\tau}$$

- Matching between the end of Inflation and the beginning of RD (ignoring the influence of reheating)

$$\tilde{h}_{ij}^{\text{Inf}}(\tau^{\text{Inf}}, \mathbf{k}) \Big|_{\text{Rh}} = \tilde{h}_{ij}^{\text{RD}}(\tau^{\text{RD}}, \mathbf{k}) \Big|_{\text{Rh}} \quad \partial_t \tilde{h}_{ij}^{\text{Inf}}(\tau^{\text{Inf}}, \mathbf{k}) \Big|_{\text{Rh}} = \partial_t \tilde{h}_{ij}^{\text{RD}}(\tau^{\text{RD}}, \mathbf{k}) \Big|_{\text{Rh}}$$

- GW energy density
$$\rho_{\text{GW}} = \frac{1}{32\pi G_{\text{N}} a^2(t)} \int_{T_\tau} \frac{d\tau}{T_\tau} \int \frac{d^3\mathbf{k}}{(2\pi)^3 V} \left| \dot{h}_{ij}(\tau, \mathbf{k}) \right|^2$$

And the ratio to the critical energy density

$$h^2 \Omega_{\text{GW}}(f) = \frac{h^2}{\rho_c} \frac{d\rho_{\text{GW}}}{d \log k} \Big|_{t=t_0, k=2\pi a_0 f}$$

- PS. Matching from RD to Matter Domination (MD) and further to Vacuum Domination (VD) eras are also considered. These periods induce no new effect, thus ignored in the discussion.

GW spectrum: inflated vs uninflated

Inflated GW

Uninflated GW

$$h^2 \Omega_{\text{GW}}(f) = h^2 \widetilde{\Omega}_{\text{GW}}(f e^{N_\star}) \times S(f)$$

redshift
deformation

$$h^2 \widetilde{\Omega}_{\text{GW}}(\tilde{f}) \equiv \frac{h^2}{\rho_c} \frac{d\rho_{\text{GW}}^{\text{flat}}}{d \log k} \times \frac{a_{\text{Rh}}^4}{a_0^4}$$

Instant-source approximation

$$S(f) = S_0(f) + S_1(f)$$

$$S_0(f) = \left\{ \frac{\cos[y(1-\epsilon)]}{y^2} - \frac{\sin[y(1-\epsilon)]}{y^3} \right\}^2 \quad y = \frac{2\pi a_0 f}{a_\star H_\star}$$

Derived in [An et al, 2009.12381]

$$S_1(f) = y\epsilon \times \left\{ \left[\frac{1}{y^2} + 2\epsilon - 1 \right] \frac{\sin[2y(1-\epsilon)]}{y^4} - \left[\frac{2-\epsilon}{y^2} + \epsilon \right] \frac{\cos[2y(1-\epsilon)]}{y^3} + \frac{\epsilon^3}{y} \left(\frac{1}{y^2} + 1 \right) \right\}$$

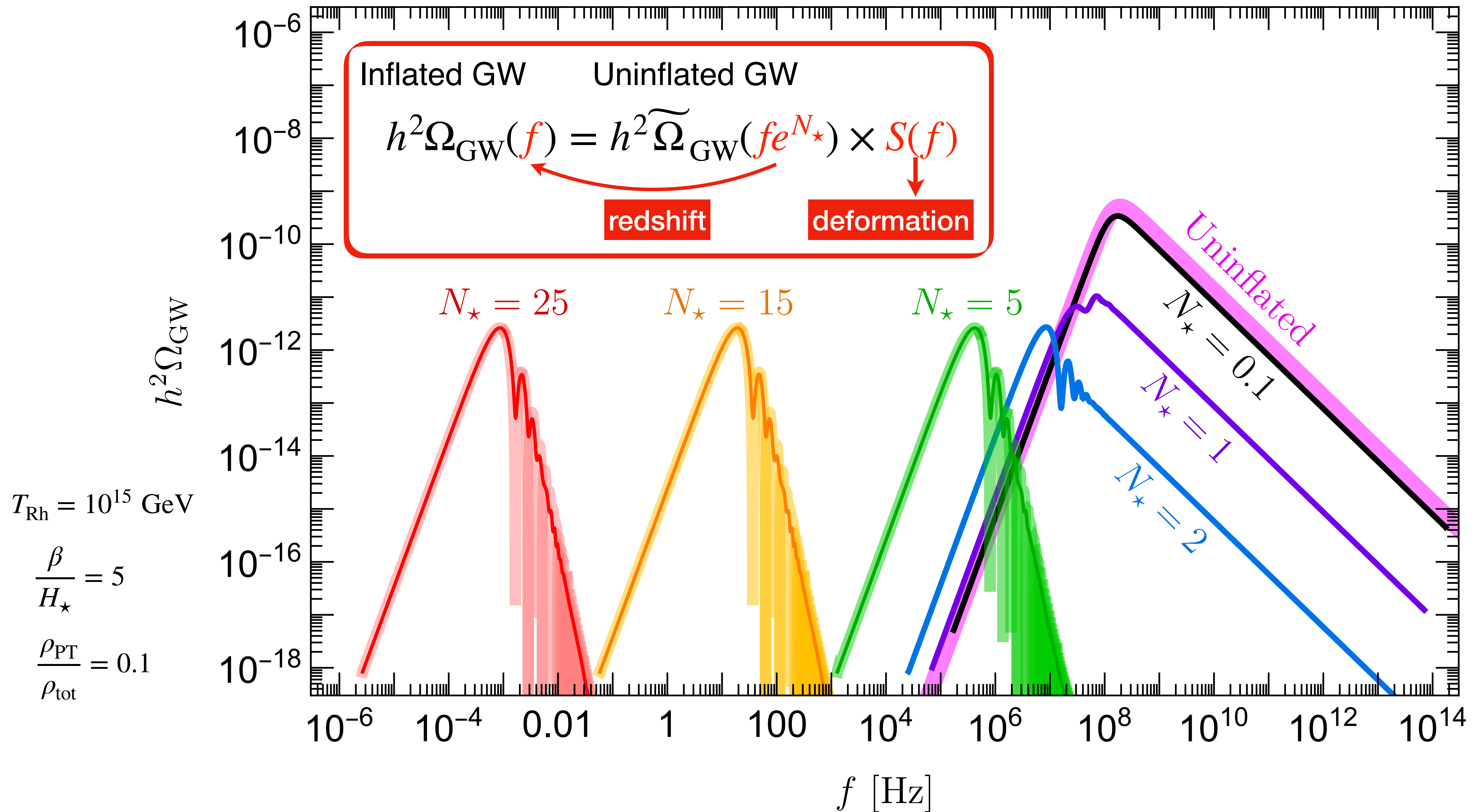
Our correction

Transitory-source approximation

$$S(f) \rightarrow \bar{S}(f) = \frac{1}{\Delta_y} \int_{\bar{y}-\Delta_y/2}^{\bar{y}+\Delta_y/2} dy S(f) \Big|_{\bar{y}=\frac{2\pi a_0 f}{a_\star H_\star}}$$

$$\Delta_y = \frac{a_\star \Delta_\tau}{1/H_\star} \bar{y}$$

GW spectrum: inflated vs uninflated



Three bands of frequencies

Infrared (IR)

Ultraviolet (UV)

Far Ultraviolet (FUV)

Wave length

$$\lambda_{\star} = \frac{2\pi a_{\star}}{k} \gg H_{\star}^{-1}$$

$$\lambda_{\star} \ll H_{\star}^{-1} \ll \lambda_{\star} \frac{a_{\text{Rh}}}{a_{\star}}$$

$$H_{\star}^{-1} \gg \lambda_{\star} \frac{a_{\text{Rh}}}{a_{\star}}$$

A quantity with subscript $\star$ means its value during phase transition, which happens during the inflation

The Hubble rate is the same as that during inflation $H_{\star} = H_{\text{Inf}}$

The coming factor is much smaller than that at reheating $a_{\star} = a_{\text{Rh}} e^{-N_{\star}}$

Co-moving momentum

$$k \ll a_{\star} H_{\star}$$

$$a_{\star} H_{\star} \ll k \ll a_{\text{Rh}} H_{\star}$$

$$k \gg a_{\text{Rh}} H_{\star}$$

Frequency today

$$f \ll \frac{a_{\star} H_{\star}}{2\pi a_0}$$

$$\frac{a_{\star} H_{\star}}{2\pi a_0} \ll f \ll \frac{a_{\text{Rh}} H_{\star}}{2\pi a_0}$$

$$f \gg \frac{a_{\text{Rh}} H_{\star}}{2\pi a_0}$$

GW propagation to the radiation domination (RD)

$$h_{\text{RD}}^{\text{IR}} \simeq \frac{1}{k} \frac{a_{\text{Rh}}^2}{a(t)} \times \frac{1}{3} \sin\left(k\tau - \frac{k}{a_{\text{Rh}} H_{\star}}\right)$$

$$h_{\text{RD}}^{\text{UV}} \simeq \frac{-1}{k} \frac{a_{\text{Rh}}^2}{a(\tau)} \frac{\cos[y(1 - \epsilon)]}{y^2} \sin(k\tau - y\epsilon)$$

$$h_{\text{RD}}^{\text{FUV}} \simeq \frac{1}{k} \frac{a_{\star}^2}{a(t)} \sin[k\tau + y(1 - 2\epsilon)]$$

Deformation function

$$S(f)^{\text{IR}} \simeq \frac{1}{9}$$

$$S(f)^{\text{UV}} \simeq \frac{\cos[2y(1 - \epsilon)] + 1}{2y^4}$$

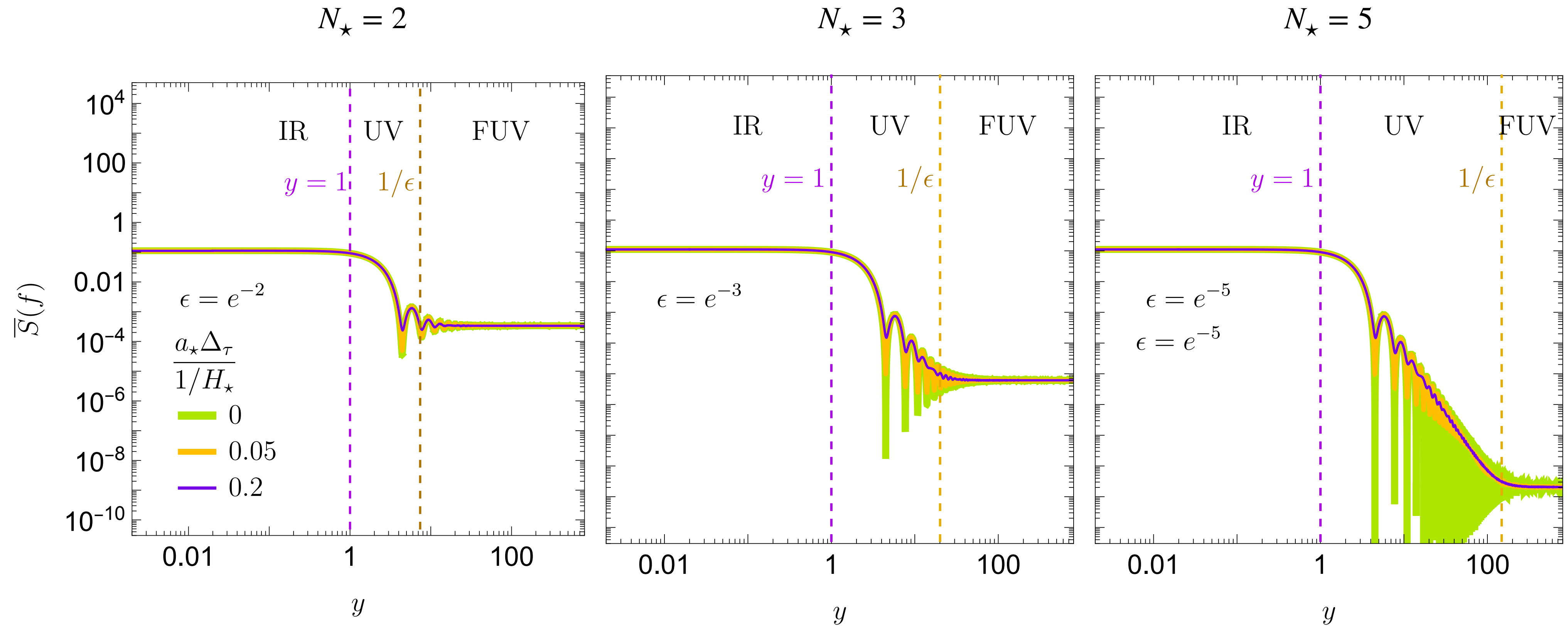
$$S(f)^{\text{FUV}} \simeq \frac{a_{\star}^4}{a_{\text{Rh}}^4}$$

Frozen

Oscillation and partial dilution

Dilution as radiation

Deformation function in three regimes



Source of GWs during phase transition

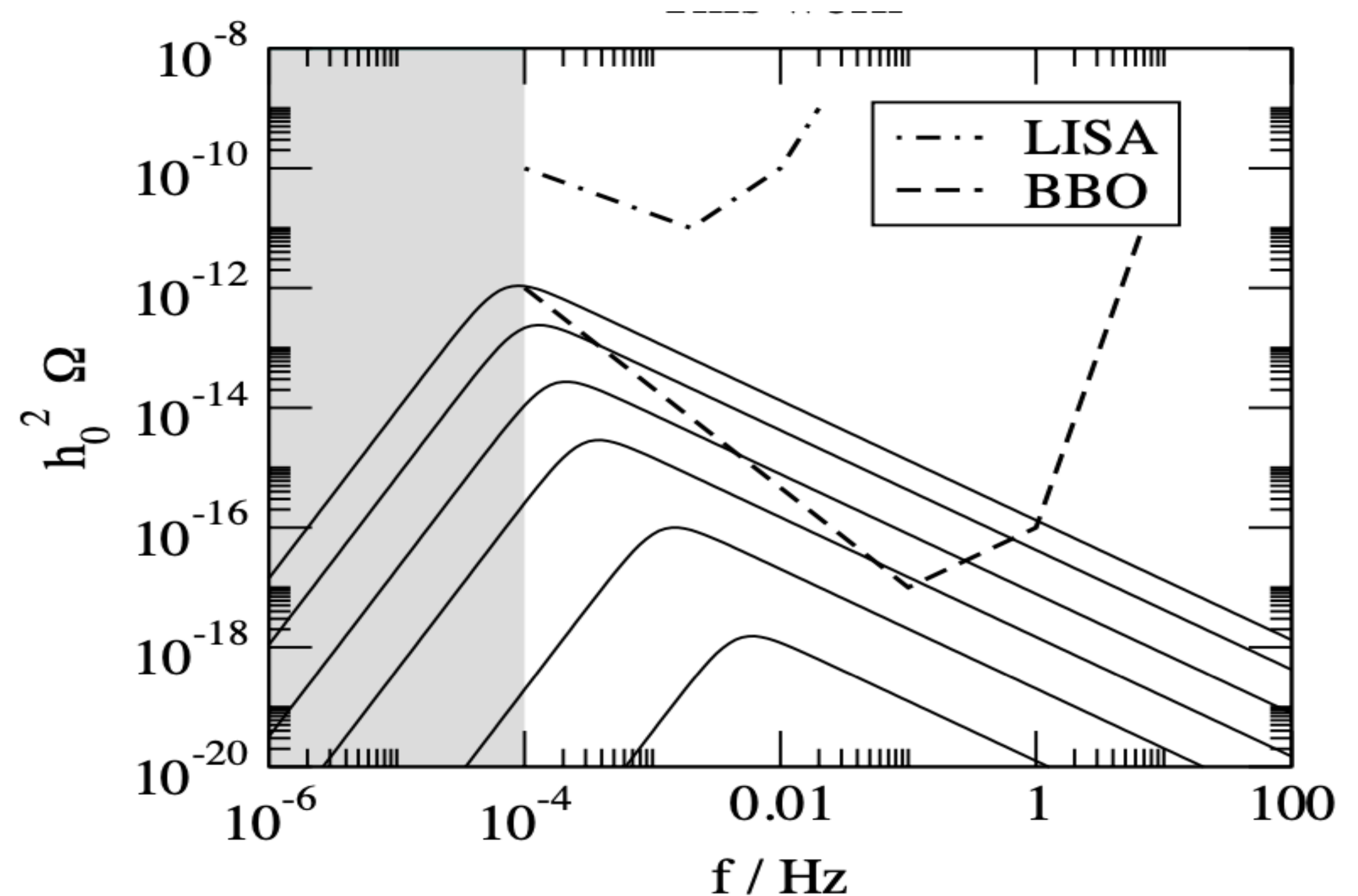
- GWs via bubble collisions in FOPT

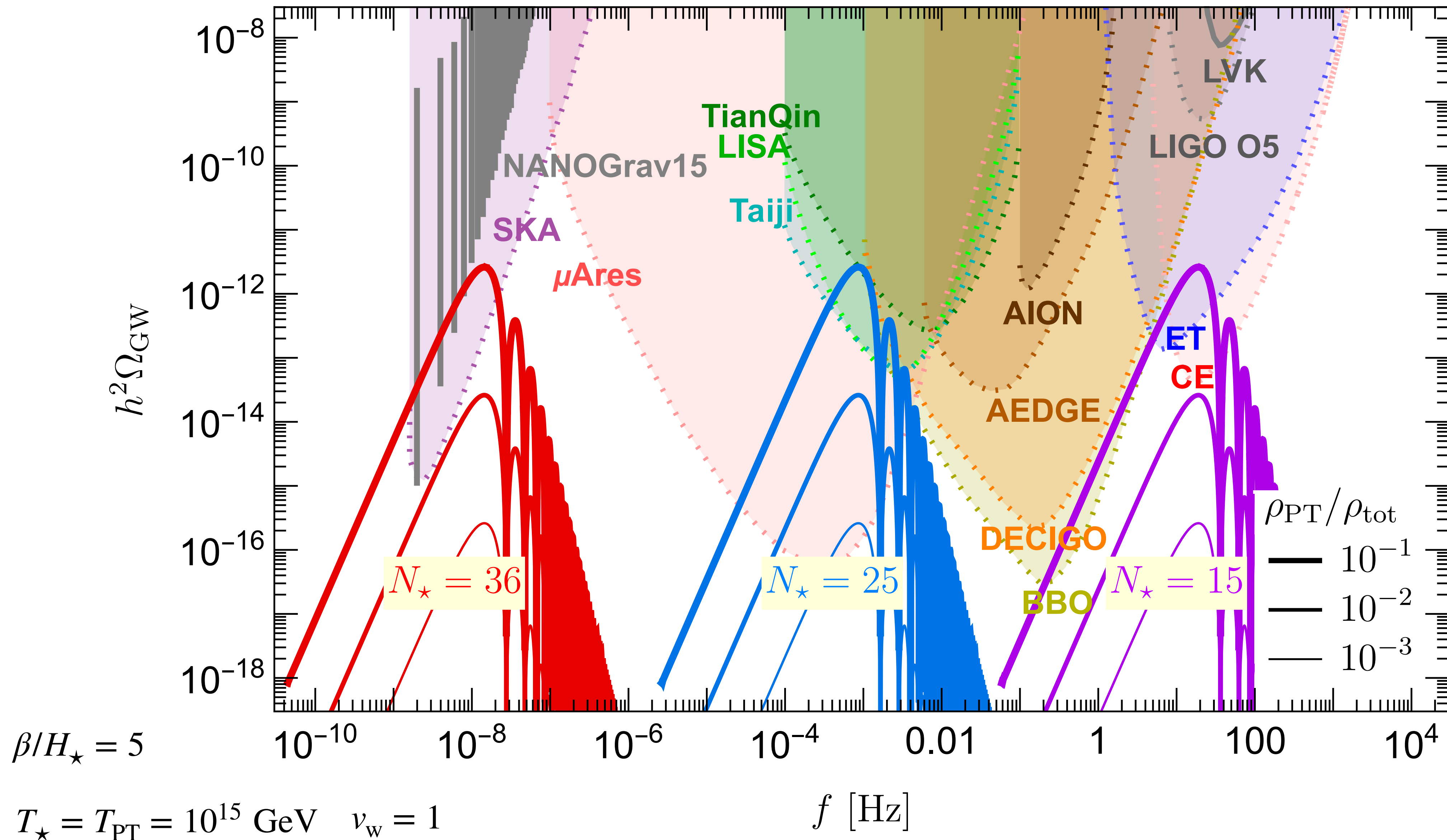
$$h^2 \widetilde{\Omega}_{\text{GW}}(\tilde{f}) = 1.27 \times 10^{-6} \times \left(\frac{H_\star}{\beta} \right)^2 \left(\frac{\rho_{\text{PT}}}{\rho_{\text{tot}}} \right)^2 \left(\frac{100}{g_\star} \right)^{1/3} \times \frac{(a+b)(\tilde{f}/\tilde{f}^{\text{peak}})^a}{a(\tilde{f}/\tilde{f}^{\text{peak}})^{a+b} + b}$$

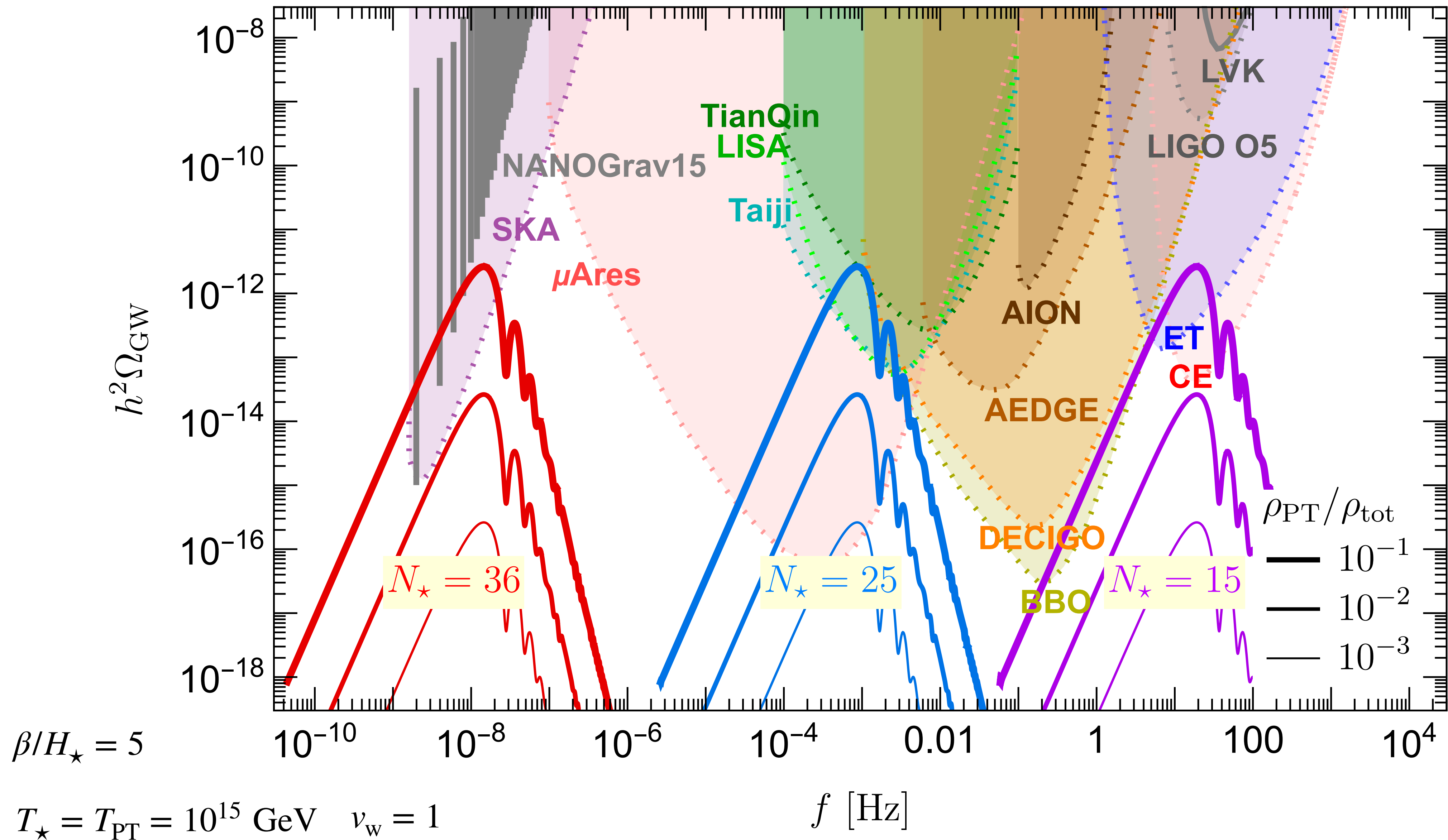
$$\tilde{f}^{\text{peak}} = 37.8 \text{ MHz} \times \left(\frac{\beta}{H_\star} \right) \left(\frac{T_\star}{10^{15} \text{ GeV}} \right) \left(\frac{g_\star}{100} \right)^{1/6}$$

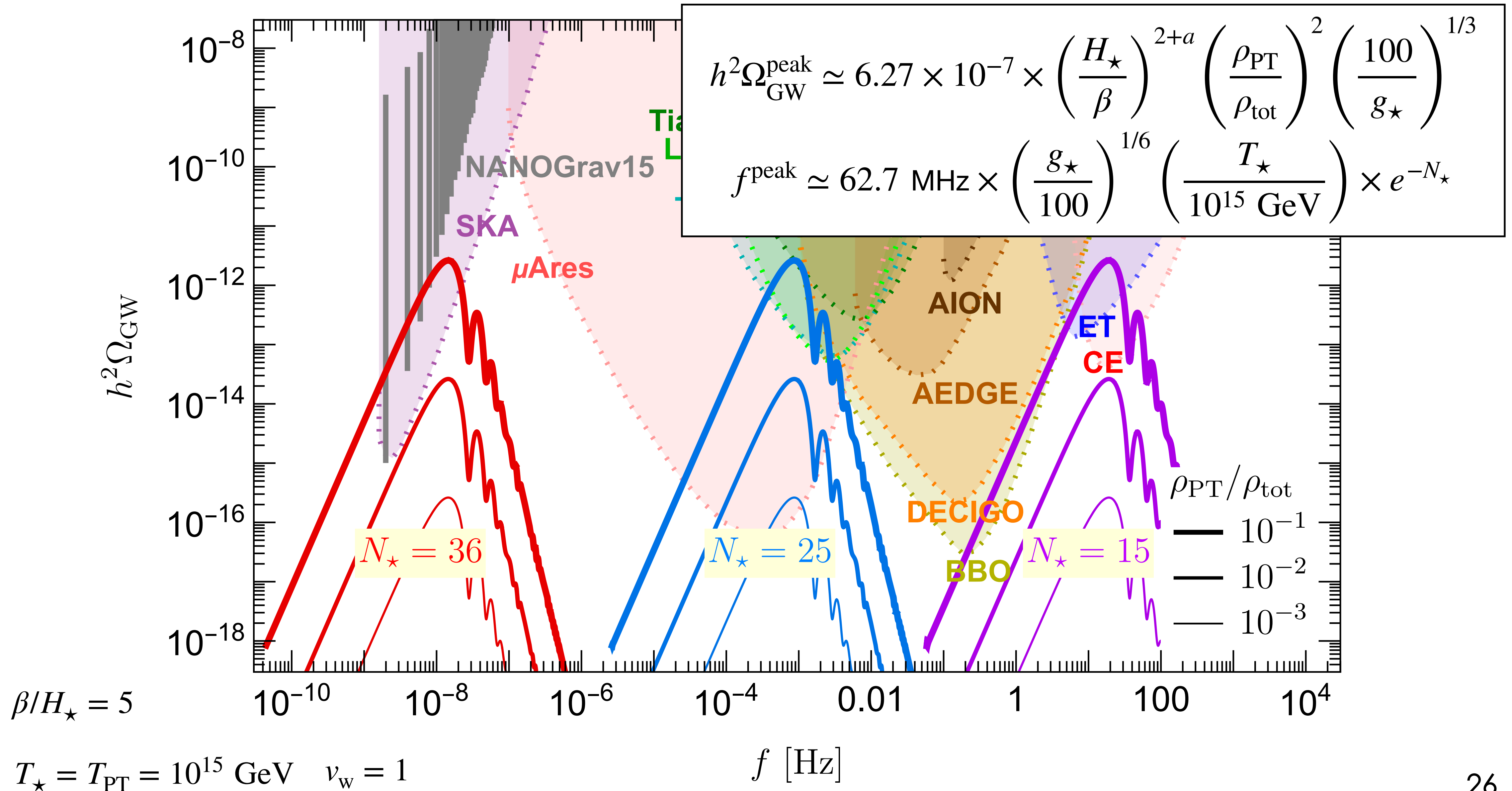
$$a = 2.8, b = 1$$

Kosowsky, Turner, Watkins, PRD, 92;
Huber, Konstandin, 0806.1828

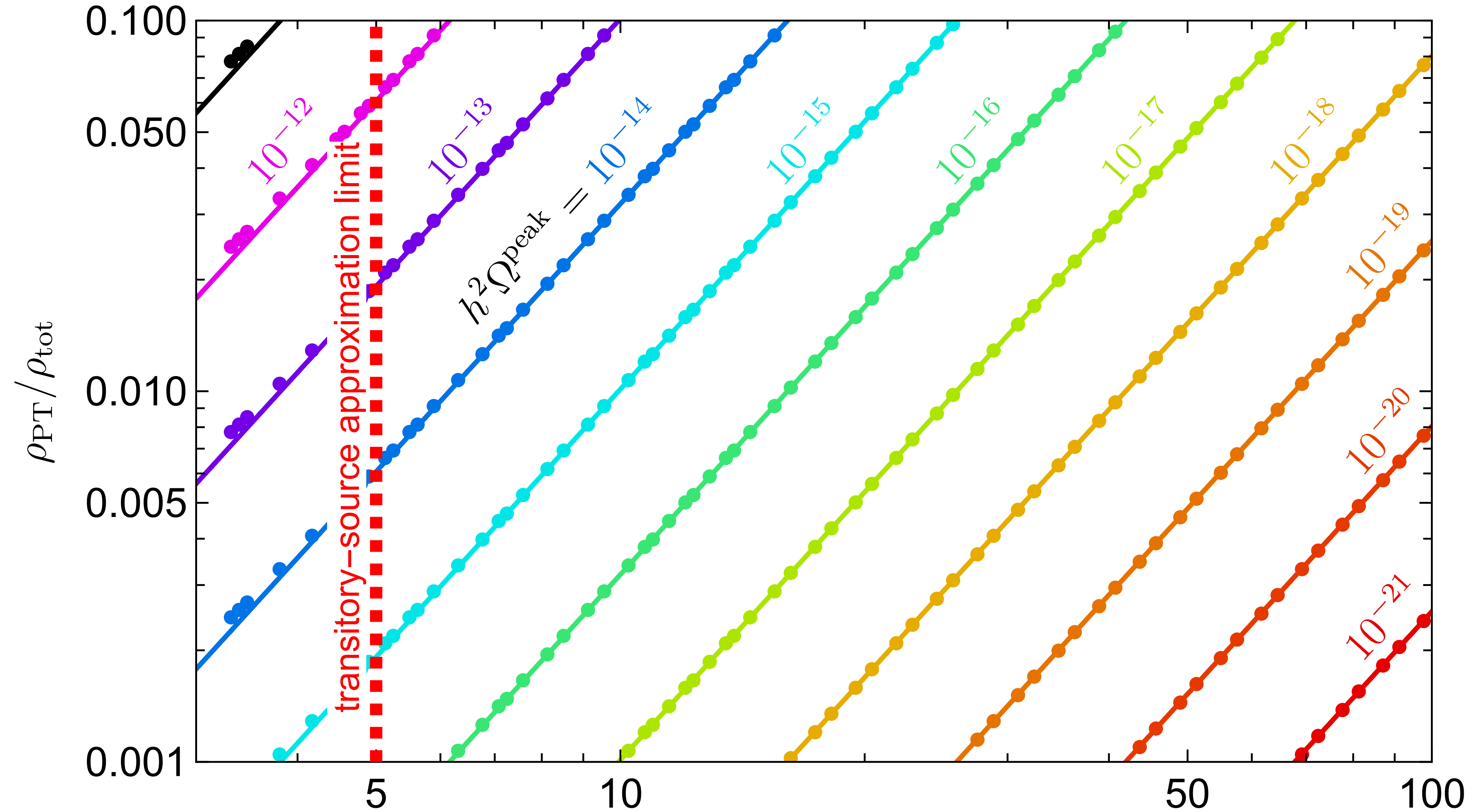








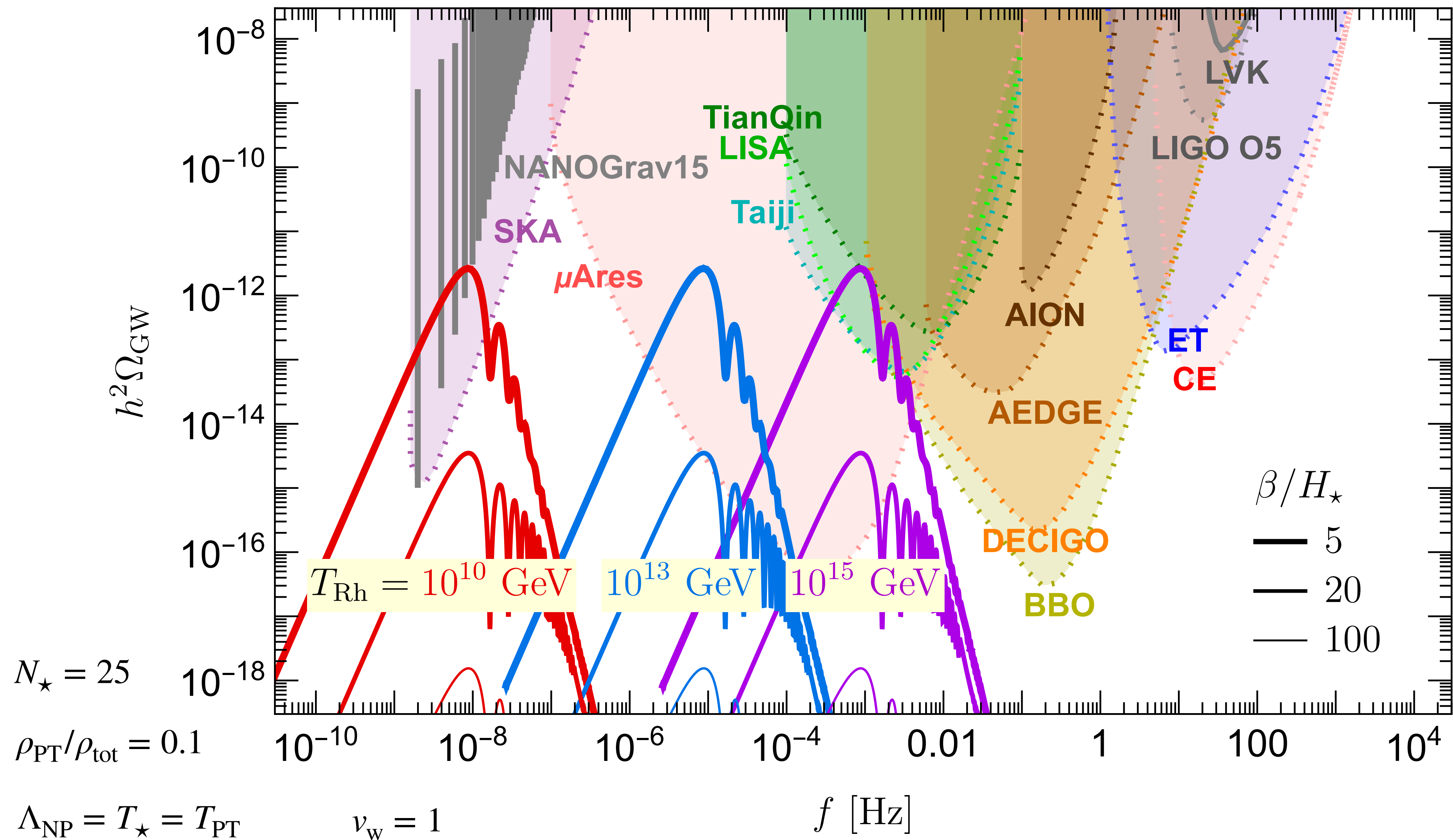
Testable region respect to the peak of GW spectrum



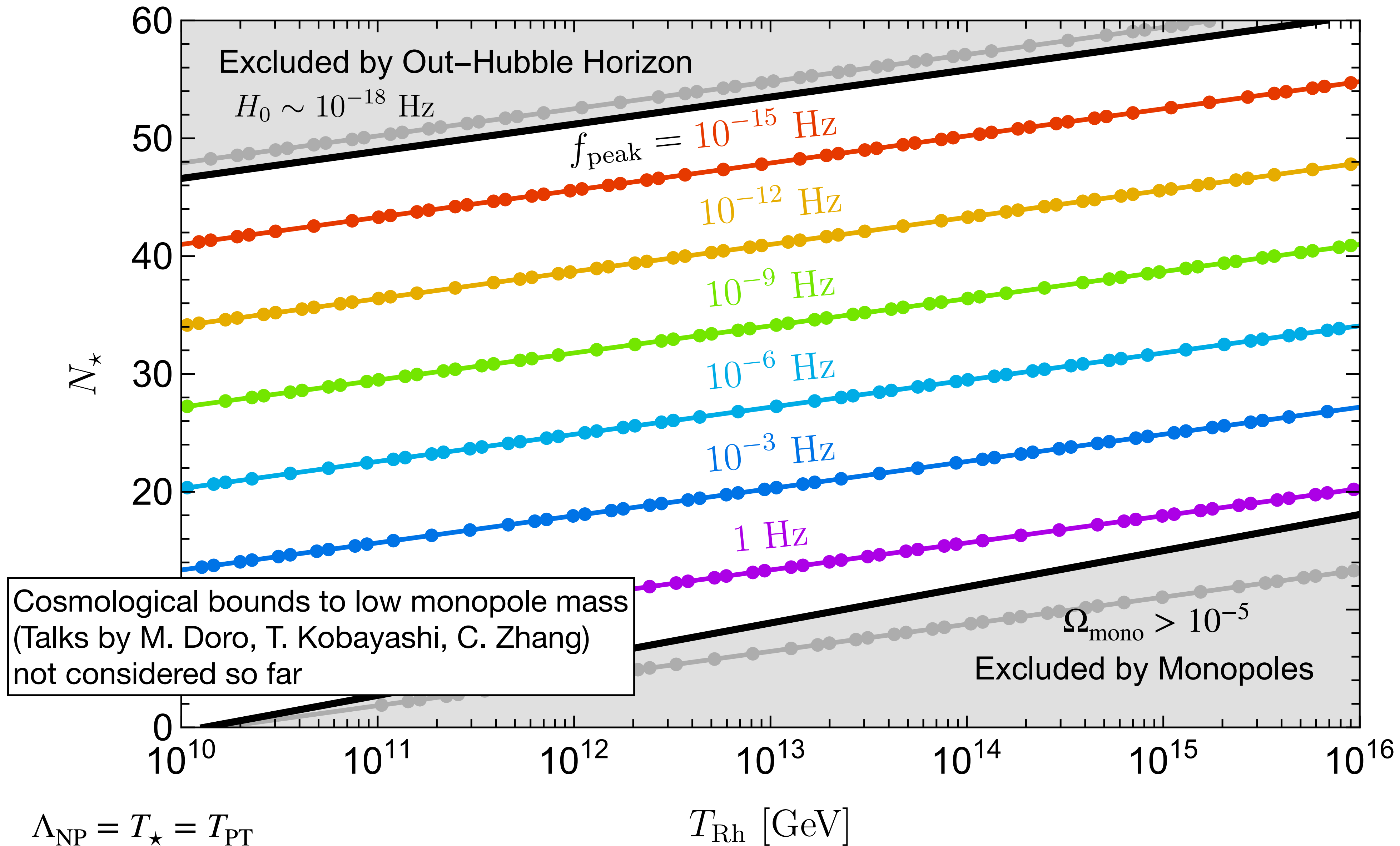
$$T_\star = T_{PT} = 10^{15} \text{ GeV} \quad v_w = 1$$

$$\beta/H_\star$$

Application to other phase transition (e.g. Pati-Salam)



Application to other phase transition (e.g. Pati-Salam)



Conclusion

- ☑ Monopoles are general predictions in GUTs. Inflation is required to solve the GUT monopole problem.
- ☑ As point-like objects, monopoles do not radiate GWs directly, but can influence GWs generated from phase transition or other topological defects.
- ☑ Monopoles play a role in GWs via cosmic strings.
 - String can be metastable via the decay to monopole-antimonopole pairs.
 - The process solves the tensions between NANOGrav and GWs via cosmic strings.
 - Requirement: scales for monopoles and strings are nearby, naturally released in SUSY GUTs and flipped SU(5).
- ☑ A scenario of GUT phase transition with testable GWs and without monopole problem
 - It happens during inflation and the phase transition is strong first order
 - We generalise the formalism to phase transition at any e-folds before the end of inflation
 - We specify three frequency bands: IR, UV and FUV.
 - Testable regimes: e-folds $15 \rightarrow 36$, $100 \text{ Hz} \rightarrow \text{nHz}$; $\rho_{\text{PT}}/\rho_{\text{tot}} > 10^{-3}$, $5 < \beta/H_{\star} < 100$

Thanks and questions ...