A Simple Formalism for Quantum Solitons

Next Frontiers in Magnetic Monopole Searches
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The First Monopole Search

People have been searching for magnetic monopoles since at least 1894

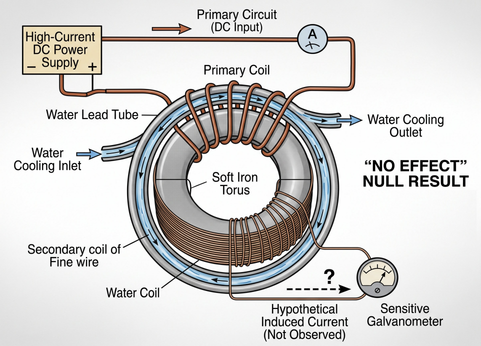

Figure: AI-generated interpretation of experiment performed by Pierre Curie, described in J. Phys. Theor. Appl., 1894, 3 (1) 415-417.

Monopole Discoveries

Since then, monopoles have been discovered many times, each with less confident wording

The Decomposition of Water by the So-Called Permanent Magnet and the Measurement of the Intensity of the Magnetic Current

FELIX EHRENHAFT
New York, New York

April 28, 1944

THE magnetic current manifests itself in many phenomena. The polar movement[1] of bodies in an homogeneous magnetic field, reversing their direction of movement with the reversal of the field, showing that they bear an excess of north or south magnetic charge, constitutes a magnetic current. Thus the Peregrinus experiment becomes

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Evidence for Detection of a Moving Magnetic Monopole

P. B. Price* and E. K. Shirk*
Physics Department, University of California, Berkeley, California 94720

and

W. Z. Osborne† and L. S. Pinsky‡
Physics Department, University of Houston, Houston, Texas 77004

(Received 4 August 1975)

A very heavy particle passed through a balloon-borne stack of Cherenkov film, emulsion, and Lexan sheets. In 33 Lexan sheets it produced tracks expected of either a nucleus with $125 \lesssim Z \lesssim 137$ and $\beta \lesssim 0.92$ or a magnetic monopole with $g = 137e$. Its track structure in emulsion indicated it was moving downward with $\beta = 0.5^{+0.1}_{-0.05}$ and was either a nucleus with $Z \approx 80$ or a monopole with $g = 137e$. These facts strongly favor identification of the particle as a magnetic monopole of strength $g = 137e$ and mass $> 200 m_p$.

First Results from a Superconductive Detector for Moving Magnetic Monopoles

Blas Cabrera
Physics Department, Stanford University, Stanford, California 94305

(Received 5 April 1982)

A velocity- and mass-independent search for moving magnetic monopoles is being performed by continuously monitoring the current in a 20-cm^2-area superconducting loop. A single candidate event, consistent with one Dirac unit of magnetic charge, has been detected during five runs totaling 151 days. These data set an upper limit of 6.1×10^{-10} cm^{-2} sec^{-1} sr^{-1} for magnetically charged particles moving through the earth's surface.

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Observation of an unexplained event from a magnetic monopole detector

A. D. Caplin, M. Hardiman*, M. Koratzinos & J. C. Schouten*

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A single event was recorded by our 0.18 m^2 inductive magnetic monopole detector, in a total effective observation time of 8,242 hours. None of the monitoring channels that record vibration, radiofrequency interference, and so on, indicated any extraneous cause for this event. An astrophysical estimate of the maximum number flux of monopoles suggests that the chances of a monopole having caused the event are less than 1 in 10^3. Alternative explanations for the candidate are considered, but none of them seem likely.

Formal Theory Evolution

At the same time, our theoretical models of magnetic monopoles have evolved

1) Maxwell (1873): "It is impossible, either by magnetization, or by breaking magnets, or by any other means, to procure a magnet whose poles are unequal. "
2) Vaschy (1890): Classical charge/current
3) Dirac (1931): Classical charge that lifts to QFT
4) 't Hooft (1974) and Polyakov (1974): Extended Classical Solution
5) Mandelstam (1975) and 't Hooft (1981): QFT monopoles with no classical avatar
6) Seiberg and Witten (1994): QFT monopoles with no classical avatar that can be deformed to QFT monopoles with a semiclassical limit

Progress in Phenomenology

Evolution in formal theory has been connected to evolution in experimental methods by evolution in phenomenology

Vinciarelli (1972) and Cornwall, Jackiw and Tomboulis (1974): Nonlinear field theory solutions (I will call them solitons) such as monopoles, are described by *coherent states*

This formal theory result was incorporated into phenomenology by Drukier and Nussinov (1982) who argued that the direct production of a specific coherent state is suppressed by an exponential factor

The conclusion was revolutionary: Monopole production is dominated instead by unsuppressed channels, such as the Schwinger process

Goal of this talk

In this talk I will note other significant developments in formal theory since 1974

I will argue that some of these may lead to important phenomenology, but are not currently considered in the monopole search literature

I will then introduce a new formalism, created by our group in Lanzhou, which is suitable for such calculations

The formalism is simple to use, but not simple enough to explain in 15 minutes

Goal of the talk: Members of the audience may be motivated to learn our formalism to perform these calculations, or suggest that their students or postdocs do so

Theoretical Development: The Squeeze

Shortly after it was argued that solitons correspond to coherent states in QFT:

Coleman (1975): In more than 1+1 dimensions, coherent states corresponding to solitons have an infinite energy density

Coleman concluded that, beyond 1+1 dimensions, solitons do **NOT** correspond to coherent states in QFT

Recently this problem has been resolved: Solitons correspond to *squeezed coherent states* in QFT, up to perturbative corrections

Implications for phenomenology: Clearly the squeeze affects their interactions. Also, unsqueezed coherent states would quickly radiate away their energy.

Theoretical Development: Monopoles are Soft

Drukier and Nussinov exploited the fact that ‘t Hooft-Polyakov monopoles are extended objects.

But it is also known that they are soft and fluffy

The phenomenology and the underlying theory appeared in two back to back PRL papers:

1) G. Fodor and I. Racz. What does a strongly excited ’t Hooft-Polyakov magnetic monopole do? Phys. Rev. Lett., 92:151801, 2004

2) P. Forgacs and M. S. Volkov. Resonant excitations of the ’t Hooft-Polyakov monopole. Phys. Rev. Lett., 92:151802, 2004.

Paper 2 shows that the ‘t Hooft Polyakov monopoles enjoy an infinite number of Feshbach quasinormal modes.

As a result, when perturbed arbitrarily, a long-living breather excitation is formed, which was observed in paper 1.

Extended to the non-BPS case by Russel and Schroers (2010)

And So?

What could this mean for MANDATE?

1) The infinite number of quasinormal modes means that there is a huge space of quantum states corresponding to quantum monopoles.
   Drukier and Nussinov's suppression factor should be multiplied by the corresponding large multiplicity of these states.

2) While a monopole travels through the primordial plasma, the Earth's atmosphere, or a detector it will be continually perturbed.
   As a result, these quasinormal modes will be excited.
   One may expect the breather to be excited, and then to slowly radiate

   More generally, as monopoles interact with their environment, the internal degrees of freedom will be excited and de-excited.

   Clearly a full picture of monopole creation, propagation and detection needs to account for the resulting phenomenology

Current practice

How are monopole-matter interactions treated today?

While there has been progress in developing methods in recent years, to my knowledge the standard approach remains S. D. Wick, T. W. Kephart, T. J. Weiler, and P. L. Biermann, Astropart. Phys. 18, 663 (2003).

They write:

> $Z \sim \alpha_M / \alpha \sim 1/2\alpha \simeq 68$. We will follow others and treat the electromagnetic interaction of the monopole with matter semiclassically, i.e., viewing the monopole as a classical source of radiation, while treating the matter–radiation interaction quantum-

At low energies, monopole interactions such as ionization are usually treated approximating monopoles to be classical, point particles.

How?

Summary so far: As monopoles travel from their creation to the detector, internal modes are constantly excited and de-excited. This affects their interactions and the radiation that they emit.

How can such processes be calculated?

At the Institute of Modern Physics in Lanzhou, we have developed a formalism called *Linearized Soliton Perturbation Theory* (LSPT).

It provides a full QFT treatment of the interactions of a single soliton (such as a monopole or Q-ball) with perturbative matter.

It does not treat soliton production, only interactions of a single soliton with matter.

LSPT Achievements Thus Far

So far we have largely applied LSPT to kinks and domain walls. Here we have, for the first time in theories without integrability, calculated:

1) Elastic soliton-meson scattering amplitudes
2) Inelastic Stokes and anti-Stokes scattering amplitudes, in which a meson excites or de-excites and internal excitation of a kink or domain wall
3) Meson multiplication, in which a meson strikes a kink and two are emitted
4) Decay rates of unstable excitations and the shapes of the corresponding resonances in elastic scattering

Recent Results

Our current focus is on oscillons.

It was widely believed that oscillons in QFT rapidly decay via emission of pairs of elementary quanta.

We used LSPT to show that this conclusion resulted from the fact that authors used *coherent* states of oscillons, which indeed emit pairs of quanta as they relax to the true *squeezed coherent* ground state

We concluded that relaxed oscillons in QFT are longer-lived than had been believed

Needless to say, monopole states are also squeezed and it would be interesting to see what implications this has

Current Research

We have now turned our attention to Q-balls

The workshop website suggests that Q-ball observation is a potential science goal for MANDATE

Our goal is to study Q-ball interaction with matter, with an emphasis on the superradiance which was discovered by the Nottingham-Hefei collaboration of Saffin, Xie and Zhou, Phys.Rev.Lett. 131 (2023) 11, 11

So far the superradiance has only been observed classically, it remains to be seen how virtual particles in the vacuum will interact with Q-balls

The motivating goal of our program is the application of LSPT to magnetic monopoles, and so MANDATE fits in perfectly

Consider a classical field theory with a field $\phi(x,t)$ and its momentum $\pi(x,t)$.

Imagine that the classical equations of motion have a solition

$$\phi(x,t) = f(x).$$

Then, in Schrodinger picture QFT, one may construct the displacement operator

$$\mathcal{D}_f = \mathrm{Exp}\left[-i\int dx\pi(x)f(x)\right]$$

The displacement operator shifts the field by the classical solution

$$\mathcal{D}_f^\dagger\phi(x)\mathcal{D}_f = \phi(x) + f(x)$$

Introduction to LSPT: Coherent States

The displacement operator $\mathcal{D}_f$ may be used to construct the state corresponding to the solition $f(x)$.

Let $|\Omega\rangle$ be the perturbative vacuum. Then the state

$$\mathcal{D}_f|\Omega\rangle$$

is a coherent state.
It resembles our soliton solution $f(x)$ because the expectation value of the field is the classical solution

$$\langle\Omega|\mathcal{D}_f^\dagger\phi(x)\mathcal{D}_f|\Omega\rangle = f(x).$$

However $\mathcal{D}_f|\Omega\rangle$ is not the ground state of the soliton, it is not even a Hamiltonian eigenstate.

To see this, define the soliton Hamiltonian H' by

$$H' = \mathcal{D}_f^\dagger H \mathcal{D}_f$$

Then acting the Hamiltonian on the coherent state

$$H\mathcal{D}_f|\Omega\rangle = \mathcal{D}_f H'|\Omega\rangle.$$

It is not an eigenstate because $|\Omega\rangle$ is not an eigenstate of H'.

Introduction to LSPT: The Squeeze

To obtain a Hamiltonian eigenstate, one needs to replace $|\Omega\rangle$ with an eigenstate of H'.

The soliton Hamiltonian H' is equal to the sum of commuting quantum harmonic oscillators (one for each normal mode), plus the soliton center of mass kinetic energy, plus perturbative interactions.

The ground state is thus, up to perturbative corrections, the simultaneous ground state of each quantum harmonic oscillator

This is a squeezed state $|0\rangle$

In conclusion, up to perturbative corrections, the soliton ground state is the squeezed coherent state

$$\mathcal{D}_f|0\rangle$$

Introduction to LSPT: Interactions

Time evolution on this state acts as

$$\mathcal{D}_f|0\rangle \longrightarrow \mathcal{D}_f e^{-iH't}|0\rangle$$

The nonperturbative part is in the $\mathcal{D}_f$ which is known and factors out, therefore we have reduced time evolution to a *perturbative calculation*. **This is the main advantage of our formalism.**

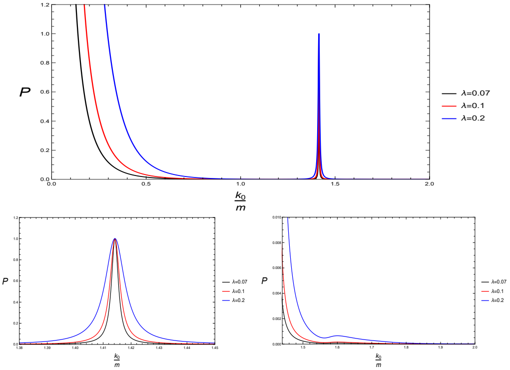
Perturbative mesons can be created with creation operators related to the usual creation operators by a Bogoliubov transform

Bogoliubov transforms also give discrete creation operators which generate bound excitations of the solitons

Conclusion: Writing the Hamiltonian in terms of these operators, one can calculate processes involving the interaction of a soliton with perturbative mesons using standard perturbation theory techniques, and the excitations of the internal modes are included automatically

Application: Kink-Meson Elastic Scattering

The probability of kink-meson elastic scattering as a function of the incoming meson momentum (Bayarsaikhan et al., PRD 114 (2026) 4, 045003)



Bottom-left: Zoomed in on the resonance in which the unstable twice-excited shape mode is created

Bottom-right: Zoomed in on the threshold at which the shape mode may be excited once

Summary

1) Q-balls and 't Hooft-Polyakov monopoles both enjoy a rich structure of internal excitations (Floquet modes, normal modes, quasinormal modes, zero modes and more)
2) As they travel from the production point to the detector, interacting with matter, these modes are continually excited and de-excited
3) Inevitably this will affect the observed velocity distributions of the Q-balls and monopoles, as well as the radiation/showers that they produce at detection
4) Our group has developed LSPT, a lightweight formalism that can be used to calculate the interactions of matter with a single Q-ball (today) or monopole (tomorrow)
5) This formalism is work in progress, and collaboration is more than welcome