Revisiting the Dynamics of Fermions near Magnetic Monopoles and the Callan–Rubakov Effect

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MANDATE workshop, IHEP, Beijing 09.14.2026

Why fermion dynamics matters for a monopole search

A GUT monopole in matter is more than a heavy, slow, highly ionizing track. The fermion sector adds

- matter particles are fermions with electric charge
- catalysis of nucleon decay at strong rates,
- chiral fermions leads to half-particle states
- ionizations and many other effects

Plan

1. Charges around a monopole
2. Fermions around a monopole
3. Callan–Rubakov and the half-particles
4. Signals from the sky

1894 Curie: free magnetism

1896 Birkeland, Poincaré: cone orbits

1931 Dirac: $eg = n/2$

1974 't Hooft, Polyakov: GUT monopoles

1980 Jackiw: conformal symmetry

1981–83 Rubakov, Callan: catalysis, half-particles

1992–94 monopoles meet gravity: magnetic black holes

2020–26 electroweak black holes; half-particles reconsidered

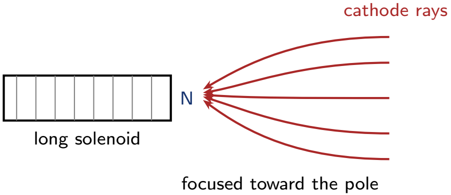
Charges around a monopole: Curie, Birkeland, Poincaré, Dirac

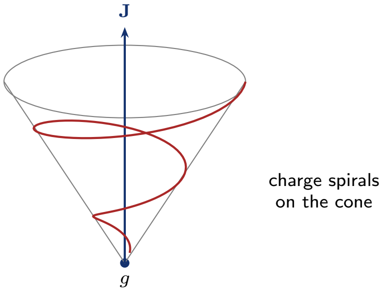
- Pierre Curie (1894): electric–magnetic symmetry does not forbid free magnetic charges
- Birkeland (1896): cathode rays are focused near the pole of a long solenoid; Poincaré (1896): that is a charge moving around a magnetic pole $\mathbf{B} = g\hat{\mathbf{r}}/r^2$— the orbit lies on a cone, and the angular momentum has $\mathbf{J} = \mathbf{r} \times m\dot{\mathbf{r}} - eg\hat{\mathbf{r}}$. (Thomson 1904)
- Dirac (1931): charge quantization

$$eg = \frac{n}{2}, \qquad g_D = \frac{e}{2\alpha} \approx 68.5\, e$$

For the search

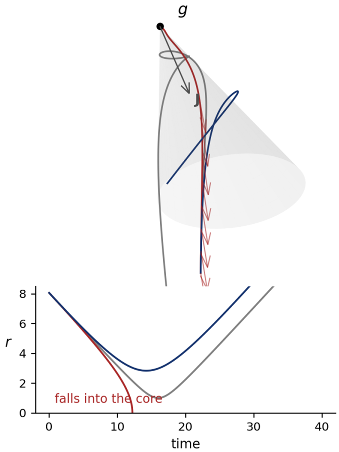
A Dirac monopole ionizes like ~ 4700 protons — the handle of every direct search.





A spinning charge around a monopole

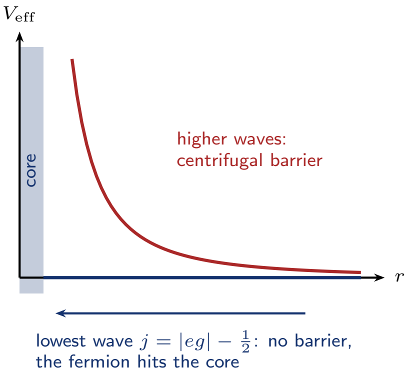
- Give the charge a spin $\mathbf{S}$ and a moment $\boldsymbol{\mu} = \gamma\mathbf{S}$: an extra force $\nabla(\boldsymbol{\mu}\cdot\mathbf{B})$, and the spin precesses, $\dot{\mathbf{S}} = \gamma\,\mathbf{S}\times\mathbf{B}$
- Conserved are $E = \boldsymbol{\pi}^2/2m - \boldsymbol{\mu}\cdot\mathbf{B}$ and $\mathbf{J} = \mathbf{r}\times\boldsymbol{\pi} - eg\,\hat{\mathbf{r}} + \mathbf{S}$; the problem is still scale-free, so the conformal symmetry survives with Casimir $\mathbf{L}^2 - 2m\gamma g\hat{\mathbf{r}}\cdot\mathbf{S}$
- $\gamma = e/m$ ($g = 2$: the Dirac electron, and the W): in $d(\mathbf{S}\cdot\boldsymbol{\pi})/dt$ the Lorentz force and the precession cancel exactly; For spin $\frac{1}{2}$, $Q = \boldsymbol{\sigma}\cdot\boldsymbol{\pi}$ is conserved with $H = Q^2/2m$, and so is its conformal partner — D'Hoker–Vinet's supersymmetry
- Orbits: spin along the field gives an attractive $1/r^2$ potential and the charge spirals into the core; against the field it is repelled; $\mathbf{S}\cdot\hat{\mathbf{r}}$ stays locked — the classical face of the missing centrifugal barrier (next slide)



grey: spinless, on the cone; red: spin along $\mathbf{B}$; blue: against $\mathbf{B}$ (same b, v; $\mu = (e/m)S$)

Quantum motion: the lowest partial wave reaches the core

- Monopole harmonics (Wu–Yang): $j \geq |eg| - \frac{1}{2}$ for spin $\frac{1}{2}$ — the "missing $\frac{1}{2}$"
- Lowest wave: no centrifugal barrier, so the fermion of $eg = \pm 1/2$ reaches $r = 0$ and needs a boundary condition at the core (Kazama–Yang–Goldhaber 1977)
- Same hidden symmetries as before: Jackiw's conformal $\mathrm{SO}(2,1)$, supersymmetric for $\gamma = e/m$ (D'Hoker–Vinet 1984–85)
- Orbits never close $\Rightarrow$ aperiodic, beamed radiation — kept for Part 4

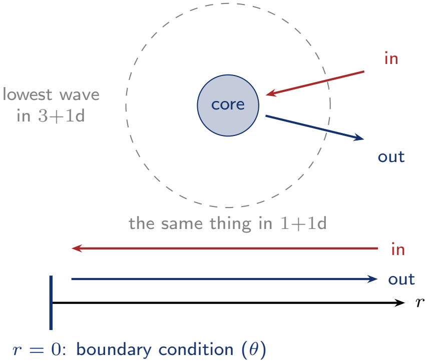


Highlight

Everything that follows happens in this lowest partial wave.

Fermions on a monopole: zero modes, θ, dyons

- In the $j = 0$ wave the $3{+}1$d fermion of $eg = \pm 1/2$ becomes a $1{+}1$d fermion on the half-line $r > 0$; the core is a boundary condition, and in-going and out-going waves have opposite helicity
- The boundary condition is not unique: a parameter θ (Rossi, Callias, Goldhaber, Yamagishi) — and the vacuum charge $Q = -e\theta/2\pi$ makes the monopole a dyon (Witten 1979)

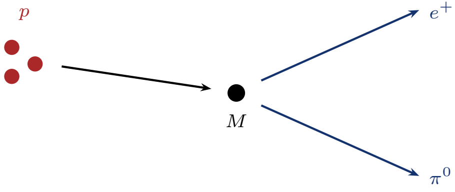


The three halves so far

$eg = \pm 1/2$, $s = 1/2$, and $j = |eg| - \frac{1}{2} = 0$ around a monopole, $Q = -e\theta/2\pi$ on a dyon — all boundary and zero-mode physics.

Callan–Rubakov: monopoles catalyse nucleon decay

- Rubakov (1981–82), Callan (1982–83): in the lowest wave the quark reaches the GUT core; anomaly plus boundary condition violate B
- No $1/M_X^2$ suppression: a strong-interaction cross-section, $\sigma_{\rm cat}\,\beta \sim 10^{-27}\,{\rm cm}^2$

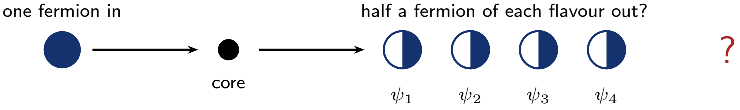


$p + M \to M + e^+ + \pi^0$
the monopole survives: a catalyst

For the search

A slow GUT monopole leaves a trail of nucleon decays along its track — MACRO looked for this.

The half-particles: a forty-year puzzle



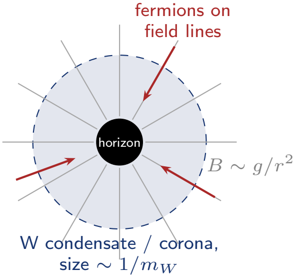
- Callan (1983): the out-state carries fractional fermion number in every flavour — "semitons"
  $\bar{e}_L + M \to \frac{1}{2}(\bar{e}_R d_L^3 u_R^1 u_R^2) + M$
- Polchinski's rotor (1984); the unitarity puzzle (Maldacena–Ludwig 1997); boundary CFT (Affleck–Sagi 1994)
- 2023–24: the out-state lives in a twisted sector, organized by a non-invertible symmetry (van Beest, Boyle Smith, Delmastro, Komargodski, Tong)
- Dissent: dyon bound states carry the deficit (Brennan); AdS_2 / instanton picture (Csáki et al.)

Highlight

Is the out-state a state at all? Two different answers — and the question is being reconsidered.

Monopoles with a horizon: electroweak magnetic black holes

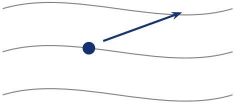
- 1992 (Lee–Nair–Weinberg): 'tHooft-Polyakov monopole and magnetic black hole are one family: above a critical Higgs scale $\sim M_P$ the monopole is an extremal black hole, below it small black holes sit inside the monopole. A small Reissner–Nordström hole is unstable — the W condenses into hair — and can evaporate completely, leaving a monopole
- 1994 (Lee–Weinberg): no topology needed: an Abelian theory with a massive W has hairy magnetic black holes, gravity hiding the singular core. Magnetic charge $2g_D$ so spin-1 W has a $j = 0$ wave and its hair is spherical (Ridgway–Weinberg 1995). The electroweak case is the Cho–Maison monopole (1997).
- 2020–25: electroweak symmetry restored near the horizon, order-Q massless 2d fermions on the field lines (Maldacena): Callan–Rubakov with a horizon for a core. Bai–Korwar; Gervalle–Volkov: the extremal hole is a Cho–Maison monopole around a tiny black hole, $M < |Q|$



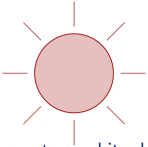
For the search

Primordial electroweak magnetic black holes (Planck to Earth mass): another highly ionizing relic, catalysis without a GUT core.

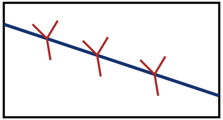
Monopoles from the sky: where things stand



Galactic field
Parker bound $F \lesssim 10^{-15}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$



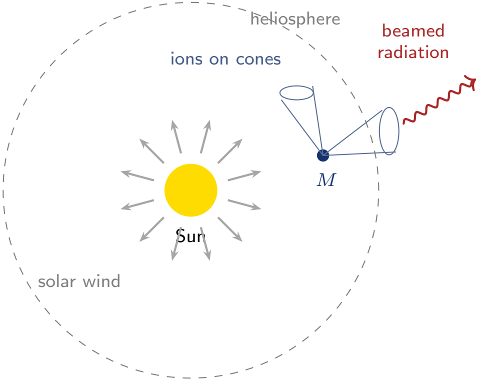
Neutron stars, white dwarfs
catalysis heating $\Rightarrow$ luminosity bounds



Direct searches
slow, heavily ionizing track
$+$ catalysis vertices (MACRO)

- Direct limits: MACRO, IceCube, ANTARES, Baikal, SLIM, ...
- Which window is MANDATE's — slow Galactic ($\beta \sim 10^{-3}$) or relativistic light monopoles? The low-scale example is the electroweak (Cho–Maison) monopole: 4–10 TeV, charge $2g_D$ so four times the Dirac ionization, hunted by MoEDAL and ATLAS

New signals: radiation from ions on Poincaré cones



- A monopole in the heliosphere captures solar-wind ions (p, α) onto cone orbits (Part 1)
- They radiate aperiodically, beamed along the cone axis — no cyclotron line; spectrum set by ion speed and impact parameter

What a dedicated search could target

slow ionizing track + catalysis vertices; beamed radiation from heliospheric monopoles

Summary

1. Cone orbits

Curie to Dirac; the missing $\frac{1}{2}$; the lowest wave reaches the core

2. Fermions

chirality flip on monopoles; θ and the dyon

3. Catalysis

strong-interaction rate; the half-particles still open; a horizon can replace the core

4. From the sky

catalysis remnants; beamed radiation from ions on cones; electroweak magnetic black holes

Outlook: We hope to report the resolution on the half-particles in Callan-Rubakov effect in near future.

Thank you for listening

Backup: key references

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