Towards a unified hadron-quark equation of state for neutron stars within the relativistic mean-field model

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Background & Motivation

- The Equation of State (EoS) of dense matter beyond nuclear saturation density lacks a first-principles description.
- Common strategy: Independently model hadronic and quark phases, then "paste" them together (Maxwell/Gibbs construction).
- **Problem**: Artificial separation; the phase transition is imposed, not derived.
- **Goal**: Develop **a unified framework** with a single Lagrangian where the hadron-quark transition emerges dynamically.

Overview of the EVA-01 Model

- Based on the DDRMF-SW4L parametrization, extended with a Polyakov-loop-inspired scalar field **Φ** to govern deconfinement.
- **Particle Content**:

  Hadrons: nucleons, hyperons, Δ resonances.

  Quarks: u, d, s.

  Leptons: e^{-}, μ^{-}, ν_e.

  Meson fields: σ, ω, ρ, ϕ, σ^*
- **Key Role of Φ**: Modulates effective masses.

  $\Phi \approx 0$ →Hadrons favored (low mass), quarks suppressed (high mass)

  $\Phi \approx 1$ →Quarks favored, hadrons suppressed.

Parameters tuned to satisfy nuclear saturation properties and astrophysical constraints.

Lagrangian and Field Equations

- Unified Lagrangian Density:

$$\mathcal{L} = \mathcal{L}_b + \mathcal{L}_q + \mathcal{L}_{meson} + \mathcal{L}_l - U_\Phi$$

$$\begin{aligned}\mathcal{L}_b \;=\; & \sum_b \overline{\psi}_b \big[\gamma_\mu \big(i\partial^\mu - g_{\omega b}\omega^\mu - g_{\phi b}\phi^\mu - \tfrac{1}{2} g_{\rho b} \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu \big) \\ & - \big(m_b - g_{\sigma b}\sigma - g_{\sigma^* b}\sigma^* + g_{\Phi b}\Phi^2 \big) \big] \psi_b \,, \end{aligned} \qquad (2)$$

$$\begin{aligned}\mathcal{L}_q \;=\; & \sum_q \overline{\psi}_q \big[\gamma_\mu \big(i\partial^\mu - g_{\omega q}\omega^\mu - g_{\phi q}\phi^\mu - \tfrac{1}{2} g_{\rho q} \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu \big) \\ & - \big(m_q - g_{\sigma q}\sigma - g_{\sigma^* q}\sigma^* + g_{\Phi q}(1 - \Phi) \big] \psi_q \,. \end{aligned} \qquad (3)$$

Lagrangian and Field Equations

- Effective Masses:

$$m_b^* = m_b - g_{\sigma b}\sigma - g_{\sigma^* b}\sigma^* + g_{\Phi b}\Phi^2$$

$$m_q^* = m_q - g_{\sigma q}\sigma - g_{\sigma^* q}\sigma^* + g_{\Phi q}(1 - \Phi)$$

Equations of Motion derived via variational principle.

Density-dependent couplings (especially for ρ meson) → **rearrangement term** ensures thermodynamic consistency.

$$g_{\rho i}(n_B) = g_{\rho i}(n_0) \exp\left[-a_\rho\left(\frac{n_B}{n_0} - 1\right)\right] ,$$

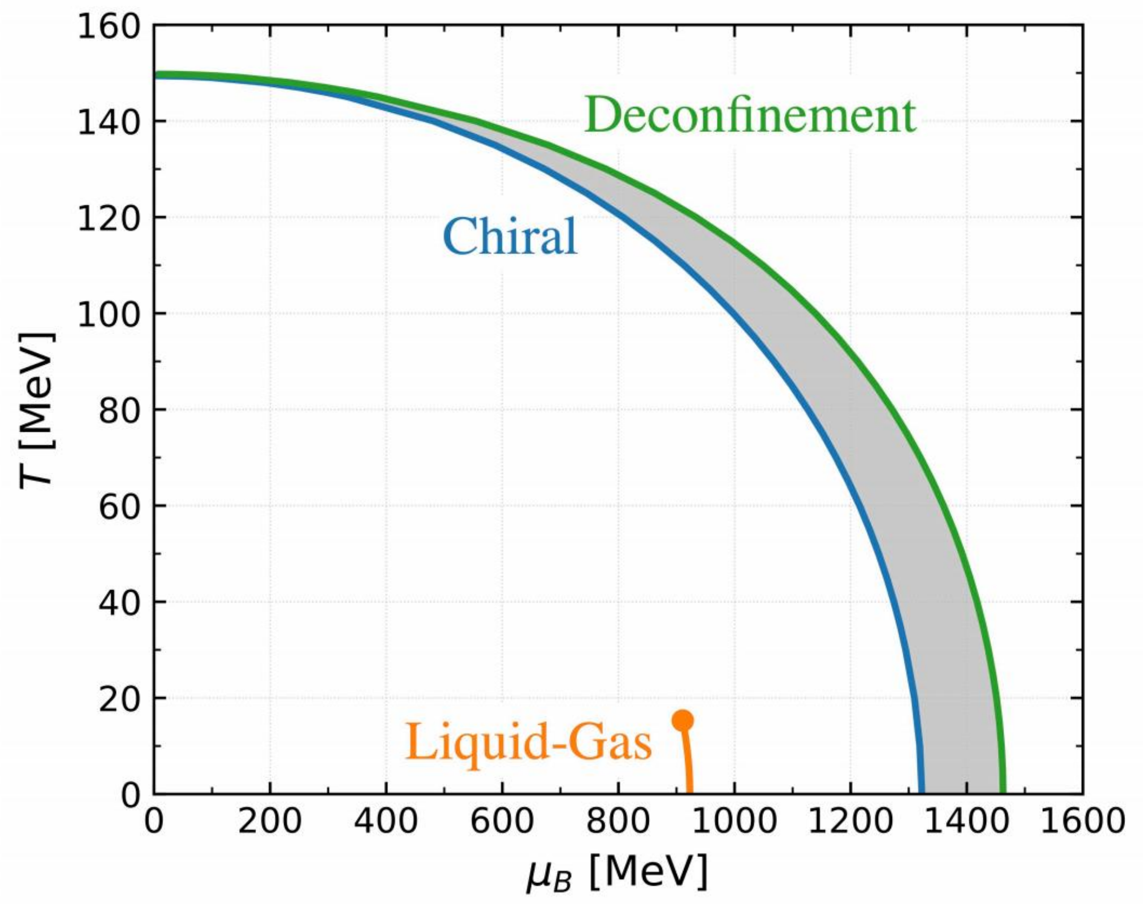
Phase Diagram Results

- Three identified first-order phase transitions:
  1. Nuclear Liquid-Gas transition (low density).
  2. Deconfinement transition (hadron $\rightarrow$ quark matter).
  3. Chiral Symmetry Restoration transition (within the quark phase).

Deconfinement and chiral transitions are separated, opening a possible "quarkyonic" region.

No Critical End Point (CEP) found in the studied range. The transition remains first-order up to T ~ 150 MeV.

Phase Diagram



- Green curve: Deconfinement boundary.
- Blue curve: Chiral transition (within quark phase, thermodynamically subleading).
- Orange curve: Nuclear liquid-gas coexistence line with its critical point.
- Highlight the region between green and blue curves.

Proto-Neutron Star Evolution Snapshots

- Three isentropic snapshots model key PNS thermal evolution stages:

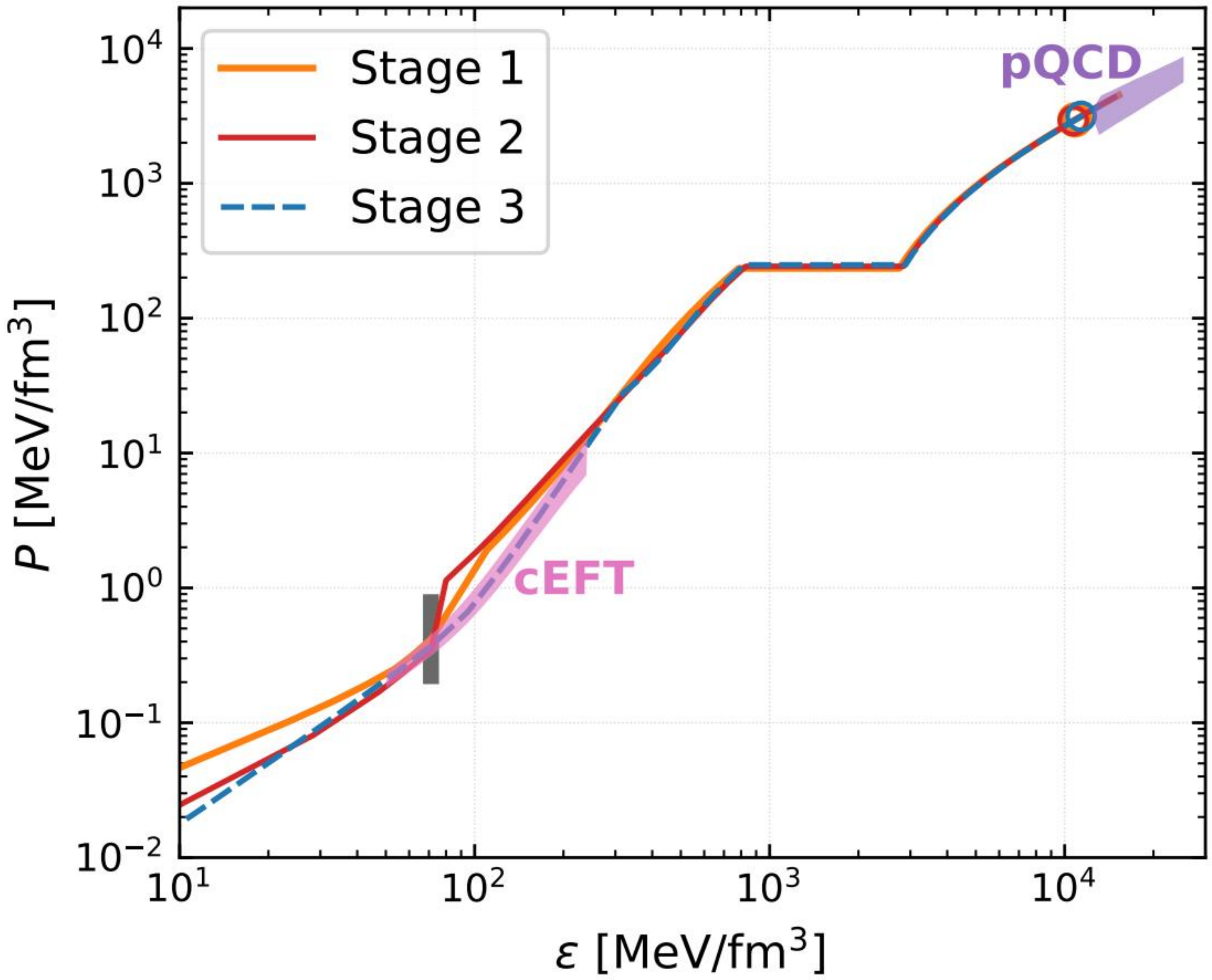
Stage 1: Early PNS (~1-2 s). High trapped lepton fraction ($Y_{l_e} = 0.4$), entropy per baryon s≈ 1.

Stage 2: Hot post-neutrino-diffusion phase (~10-15 s). Neutrino-free, muons present, s≈ 2.

Stage 3: Cold, catalyzed Neutron Star. T=0, in full β-equilibrium.

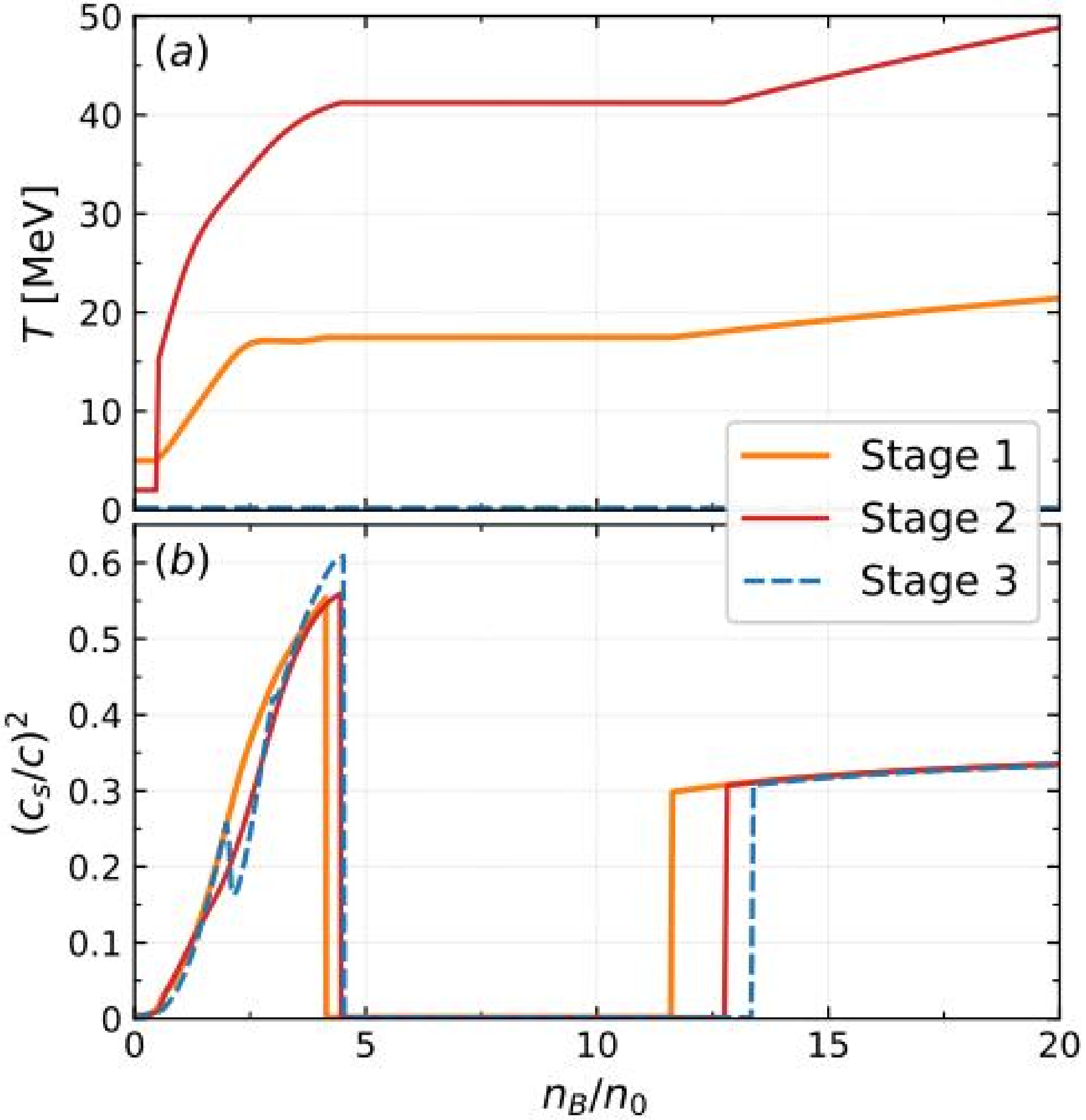
- Crust EoS: Sequentially cooled (T=5 MeV → 2 MeV → 0) to mimic realistic PNS cooling.

Equation of State Comparison



- **Cold EoS (Stage 3)** is consistent with cEFT (low density) and pQCD (asymptotic) constraints.
- **Hot EoS (Stages 1 & 2)** are stiffer due to thermal contributions.
- All show a **strong first-order phase transition** at $P_t \sim$ 220-240 MeV/fm³ with a large energy gap Δε ~ 2000 MeV/fm³.

Temperature & Speed of Sound Profiles



- **Temperature vs. Density:**

  Isothermal crust → Isentropic hadron phase (T increases) → Isentropic quark phase.

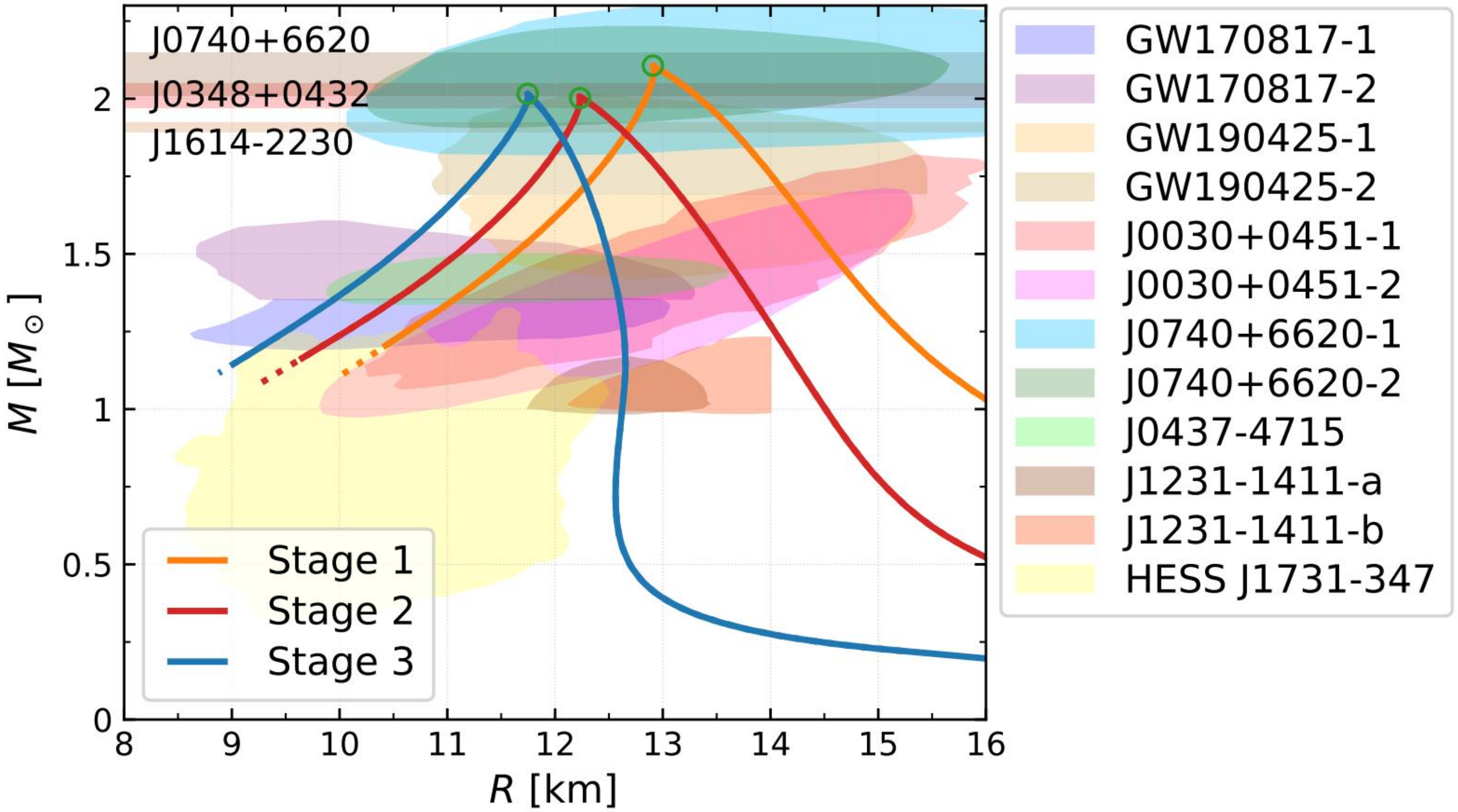
  Temperature is continuous at the phase boundary while Entropy density jumps

- **Speed of Sound squared**

  Hadron phase: Varies with stage; peaks in Stage 1.

  Quark phase: For all stages, converges toward the conformal limit $(C_s/C)^2$→ 1/3 at high density.

Mass-Radius Relations



Cold NS sequence (Stage 3) complies with modern constraints:

Massive pulsars.

mass-radius measurements.

Tidal deformability

- **Thermal Evolution:** Radii contract significantly from hot (Stages 1&2) to cold (Stage 3). Maximum mass slightly decreases from Stage 1 to Stage 2.

Thanks!