Relativistic calculations of nuclear matrix elements for neutrinoless double beta decay

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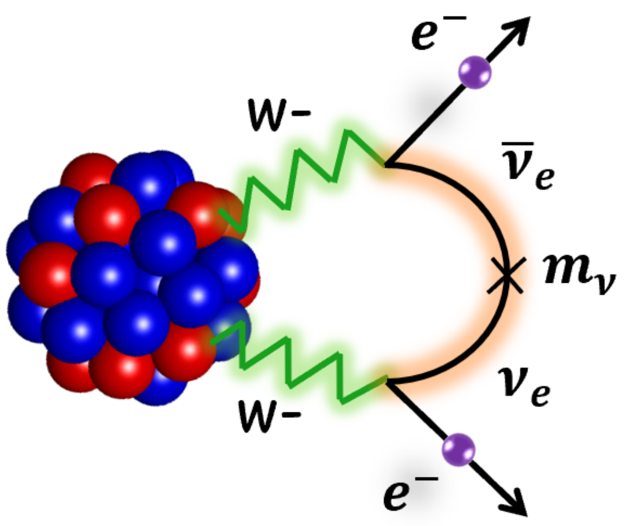
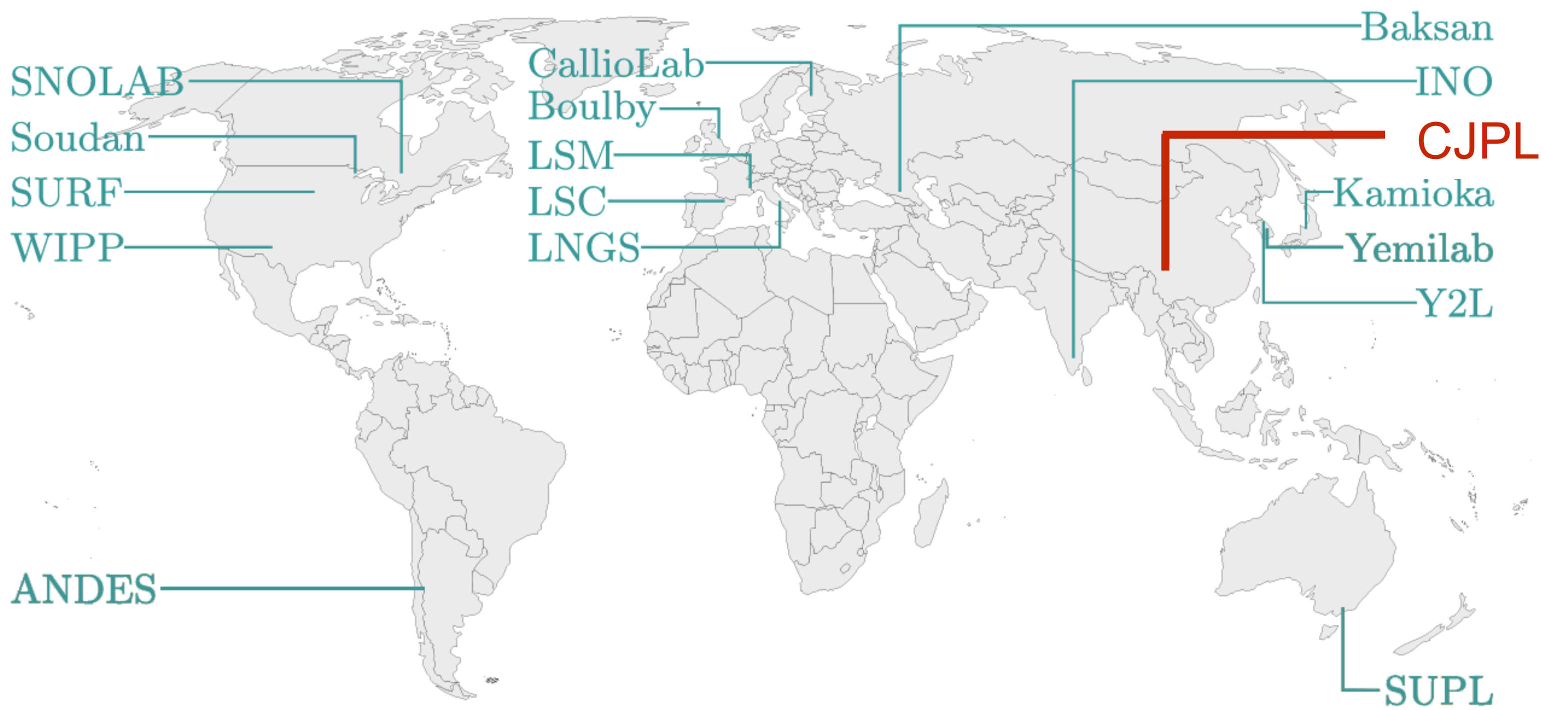
Neutrinoless double-beta decay

- Neutrinoless $\beta\beta$ decay ($0\nu\beta\beta$): $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$
  - ✓ Lepton number violation
  - ✓ Majorana nature of neutrinos
  - ✓ Neutrino mass scale and hierarchy
  - ✓ matter-antimatter asymmetry
  - ✓ …



- $0\nu\beta\beta$ search in worldwide experimental facilities

Avignone, Elliott, Engel, Rev. Mod. Phys. 80, 481 (2008)



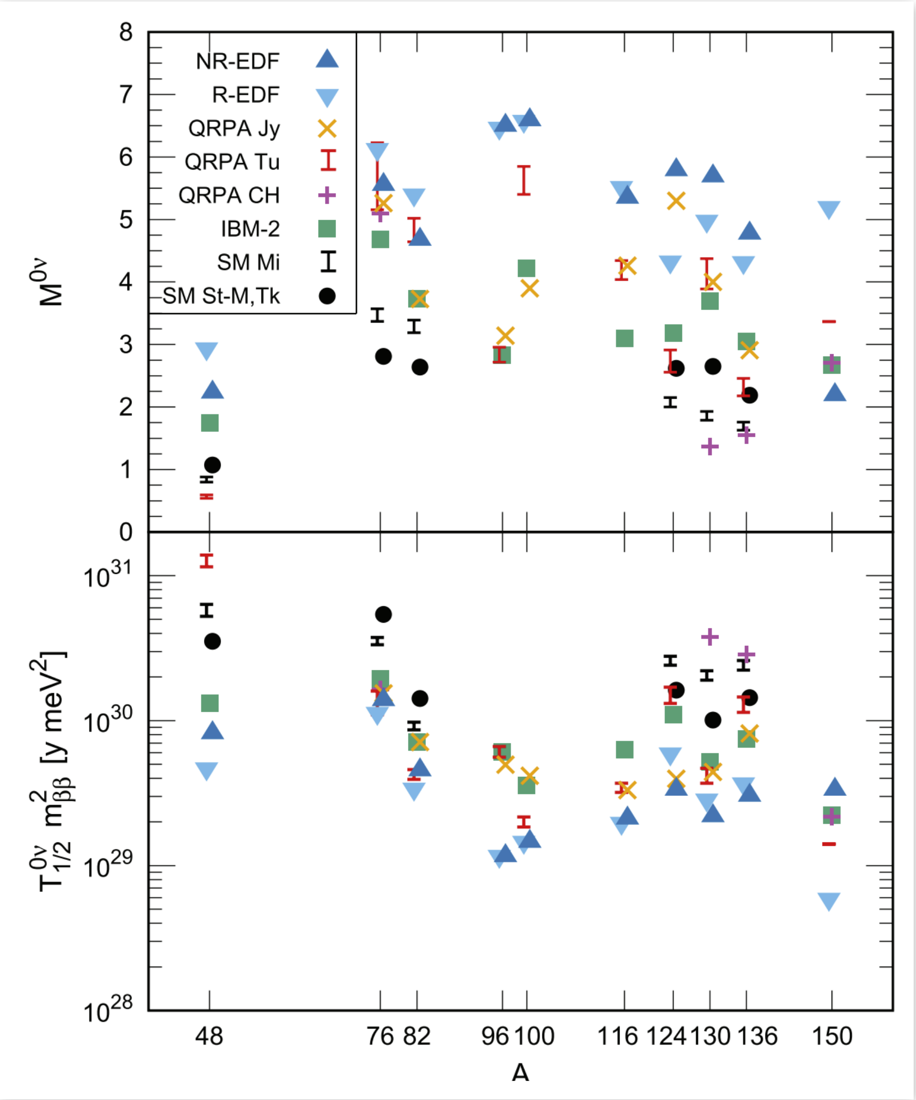
Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)

$0\nu\beta\beta$ decay occurs only if neutrinos are Majorana particles!

Nuclear matrix elements (NMEs)

Nuclear matrix elements encode the impact of the nuclear structure on the **decay half-life**, crucial to interpreting the $0\nu\beta\beta$ experimental limits on the **effective neutrino mass**.

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} \,|\, M^{0\nu} \,|^2 \, \langle m_{\beta\beta} \rangle^2$$



Nuclear matrix element

$$M^{0\nu} = \langle \Psi_f | \, \hat{O}^{0\nu} \, | \Psi_i \rangle$$

- Different **nuclear models** for $\Psi_{f,i}$
- **Decay operators**

Engel and Menéndez, Rep. Prog. Phys. 80, 046301 (2017)

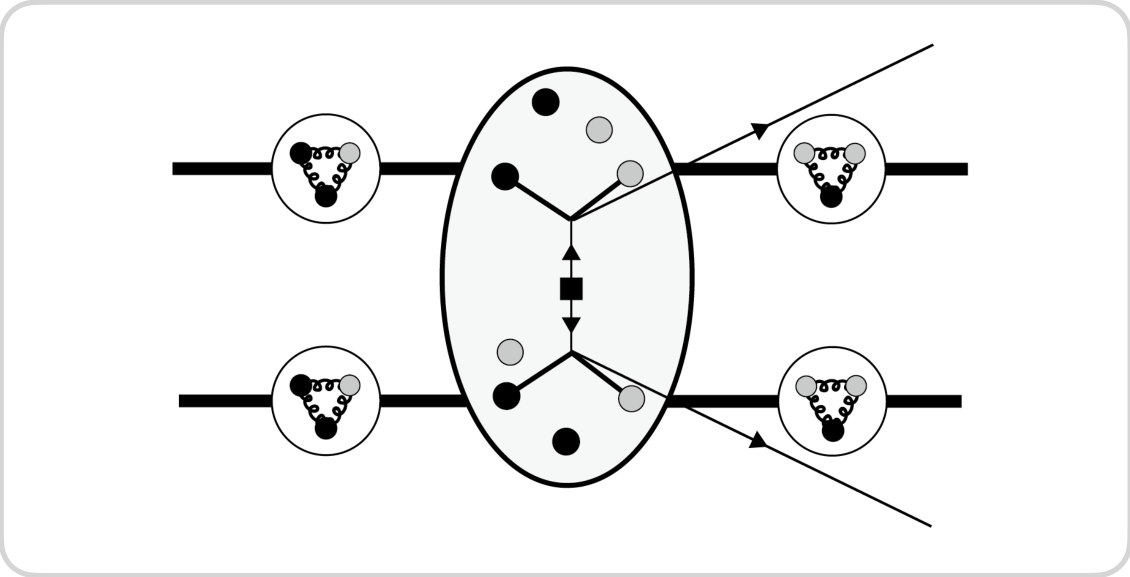
Outline

- Nuclear matrix elements of $0\nu\beta\beta$ decay
- $0\nu\beta\beta$ decay of two neutrons
- $0\nu\beta\beta$ decay of candidate nuclei
- Summary

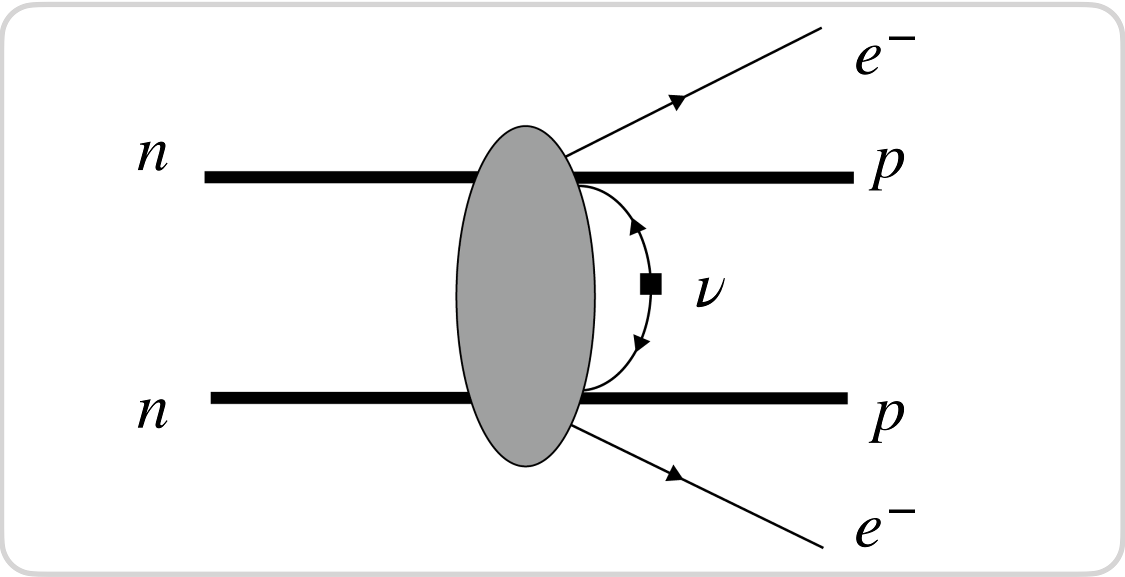
$0\nu\beta\beta$ decay operator

- Nucleons are the relevant degrees of freedom in nuclei.
- The hadronic decay operator needs to be derived from the quark-gluon level.

Quark-gluon level



Hadronic level



- Derivation of $0\nu\beta\beta$ decay operator

Standard mechanism

Avignone, Elliott, Engel,
Rev. Mod. Phys. 80, 481 (2008)
Cirigliano, Dekens, Mereghetti, Walker-Loud,
PRC 97, 065501 (2018)
…

Non-standard mechanism

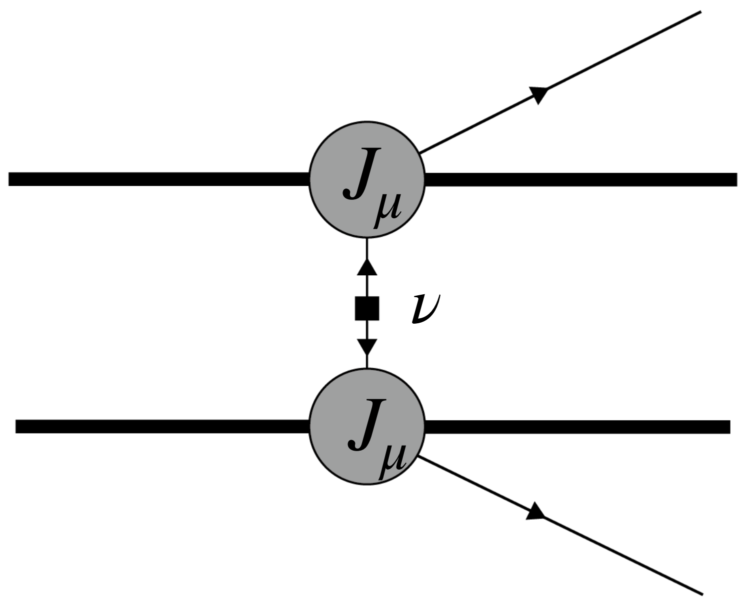
Cirigliano, Dekens, Mereghetti, Walker-Loud,
JHEP 12, 97 (2018)
Dekens, de Vries, Fuyuto, Mereghetii, Zhou,
JHEP 06, 97 (2020)
…

Decay operators for standard mechanism

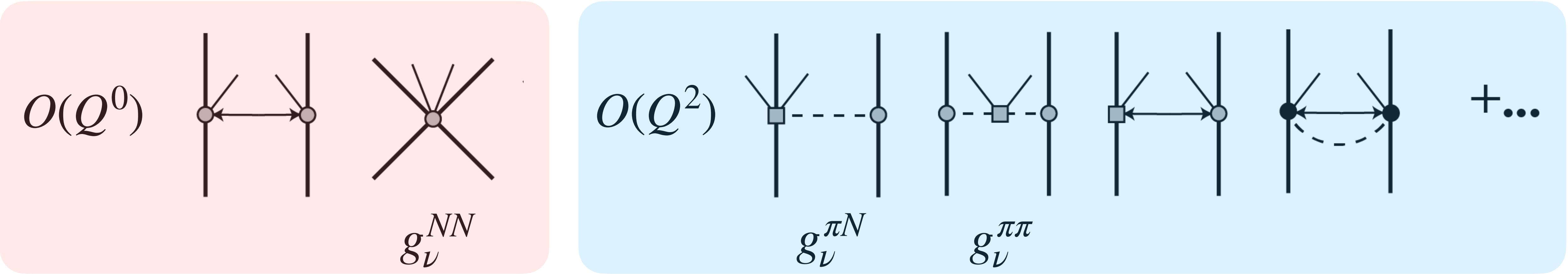
- Impulse approximation +

  phenomenological nucleon currents

  Avignone, Elliott, and Engel, Rev. Mod. Phys. 80, 481 (2008)



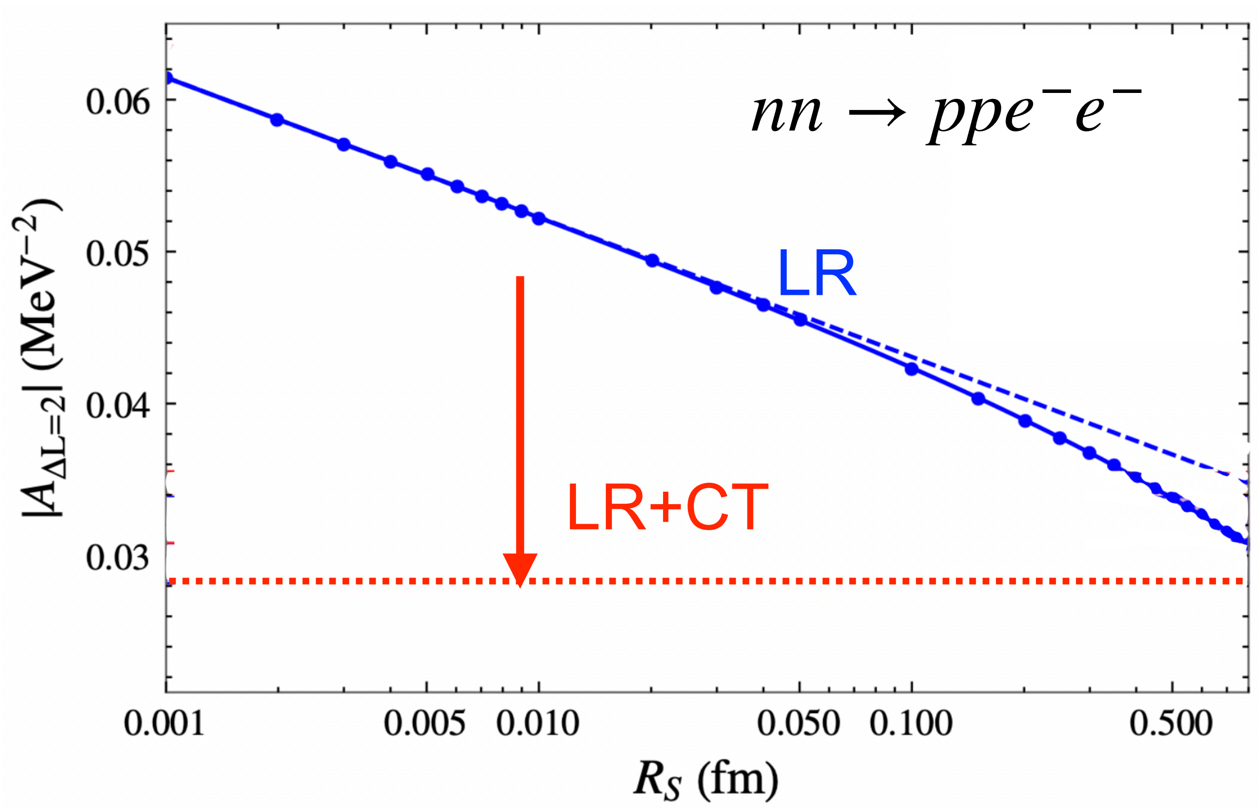
- **Chiral EFT framework**
  - Maps the quark-gluon level Lagrangian to chiral EFT Lagrangian
  - Perform low-energy expansion according to a power counting
  - Matching the low-energy constants (LECs) to Lattice QCD amplitudes



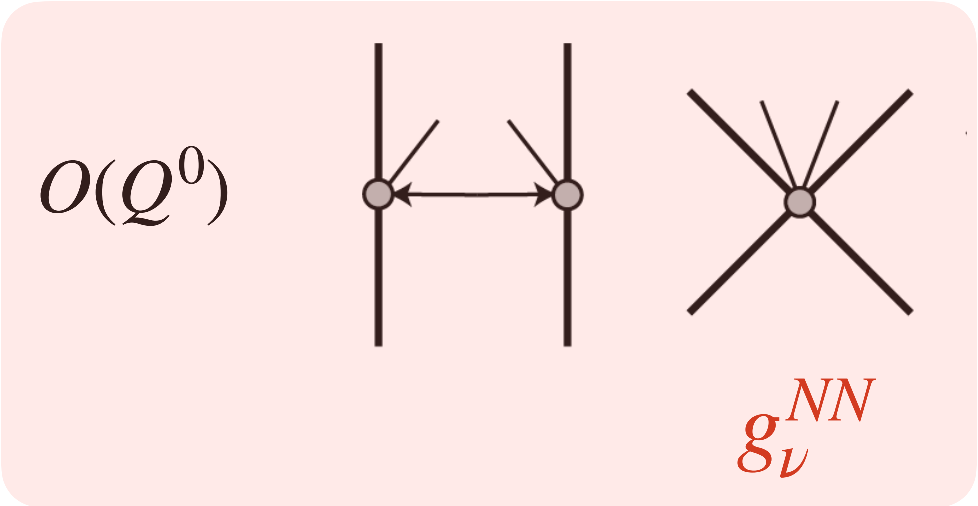
Cirigliano, Detmold, Nicholson, and Shanahan, Prog. Part. Nucl. Phys. 112, 103771 (2020)

Determine the LO decay operator

- The LO decay operator from chiral EFT includes already a short-range operator with **unknown** LEC g_ν^{NN}.



Cirigliano et. al., PRL 120, 202001 (2018)



How to determine its size ?

Determine the LO decay operator

- The LO decay operator from chiral EFT includes already a short-range operator with **unknown** LEC g_ν^{NN}.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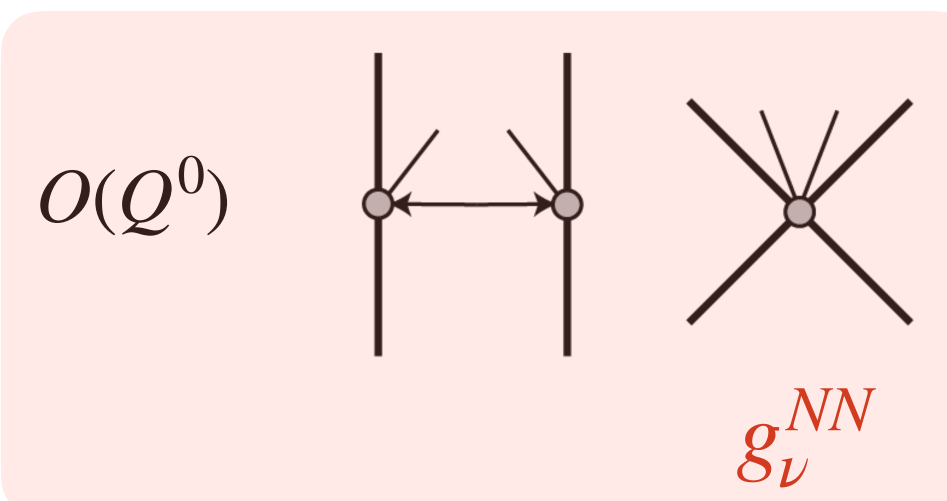
SYNOPSIS

A Missing Piece in the Neutrinoless Beta-Decay Puzzle

May 16, 2018 • *Physics* 11, s58

The inclusion of short-range interactions in models of neutrinoless double-beta decay could impact the interpretation of experimental searches for the elusive decay.

Cirigliano et. al., PRL 120, 202001 (2018)



How to determine its size ?

Matching to Cottingham model

- A generalized Cottingham model for $nn \to ppee$ amplitude

  Cirigliano, Dekens, de Vries, Hoferichter, and Mereghetti, PRL 126, 172002 (2021); JHEP 05, 289 (2021)

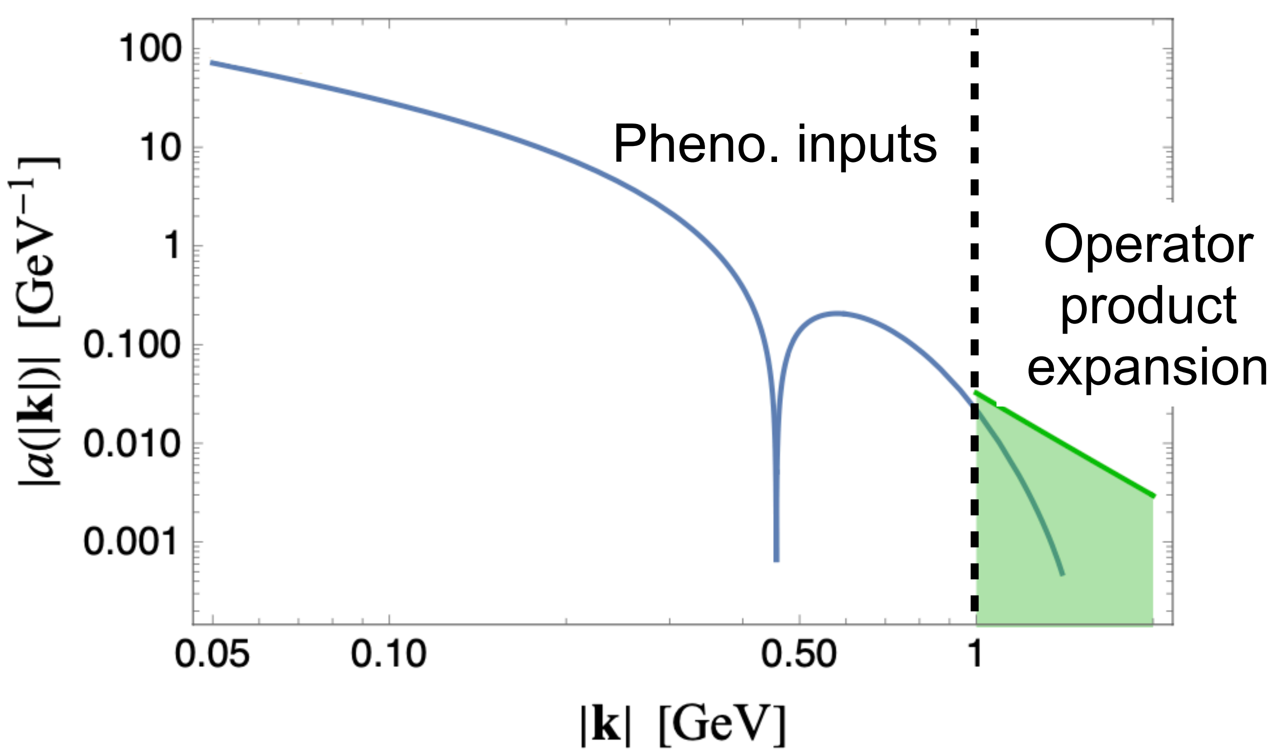
$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4} \frac{g_{\alpha\beta}}{k^2 + i\epsilon} \int d^4x \, e^{ik\cdot x} \langle pp | T\{ j_{\rm w}^\alpha(x) j_{\rm w}^\beta(0) \} | nn \rangle$$

$$= \int_0^\Lambda d|\mathbf{k}| \; a_<(|\mathbf{k}|) \; + \; \int_\Lambda^\infty d|\mathbf{k}| \; a_>(|\mathbf{k}|) \, ,$$

- Model assumptions and inputs：

  1. *Neglect inelastic intermediate states*
  2. *Phenomenological off-shell NN amplitudes*
  3. *Phenomenological weak form factors*
  4. *Separation of low- and high-energy region*
  5. *Unknown matrix element $\bar{g}_1^{NN}(\mu)$*

Low ← Λ → High



$$\Rightarrow \quad \tilde{\mathcal{C}}_1(\mu_\chi = M_\pi) \simeq 1.32(50)_{\rm inel}(20)_{V_S}(5)_{\rm par}$$

A model-free prediction, at least at LO,

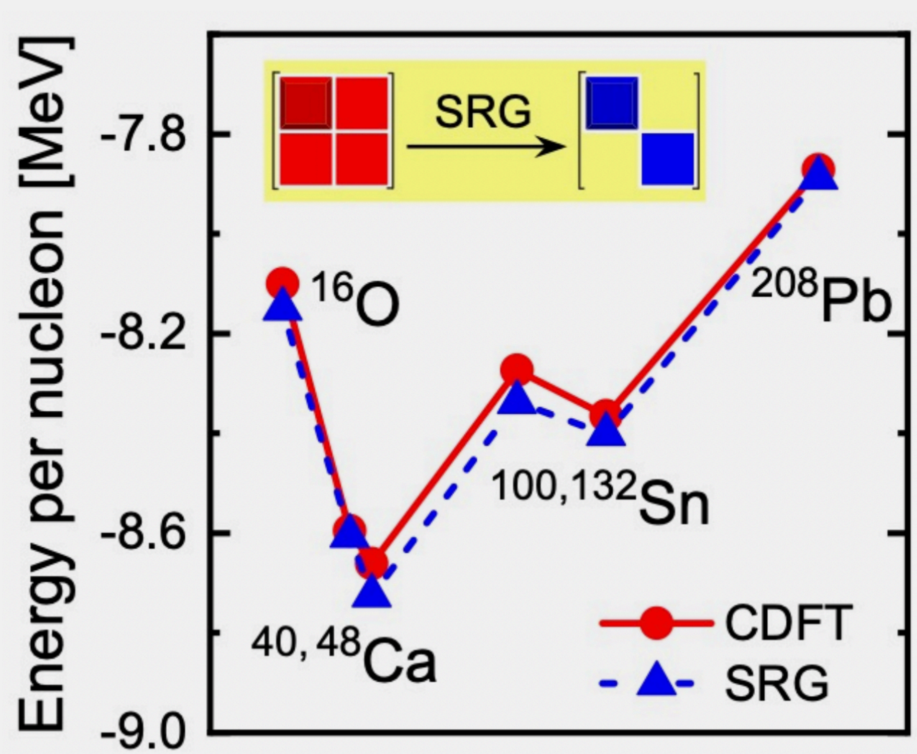
without the unknown contact term?

Relativistic effects in nuclear systems

Bridge the Rel. and Nonrel. DFTs

$$4\pi r^2\rho_v(r) = \rho_0 + \frac{d}{dr}\left[\frac{1}{4\tilde{M}^2}\frac{\kappa}{r}\rho_0\right] + \frac{d^2}{dr^2}\left[\frac{1}{8\tilde{M}^2}\rho_0\right] + O(\tilde{M}^{-3}),$$

Rel. Nonrel. (including high-order terms …)



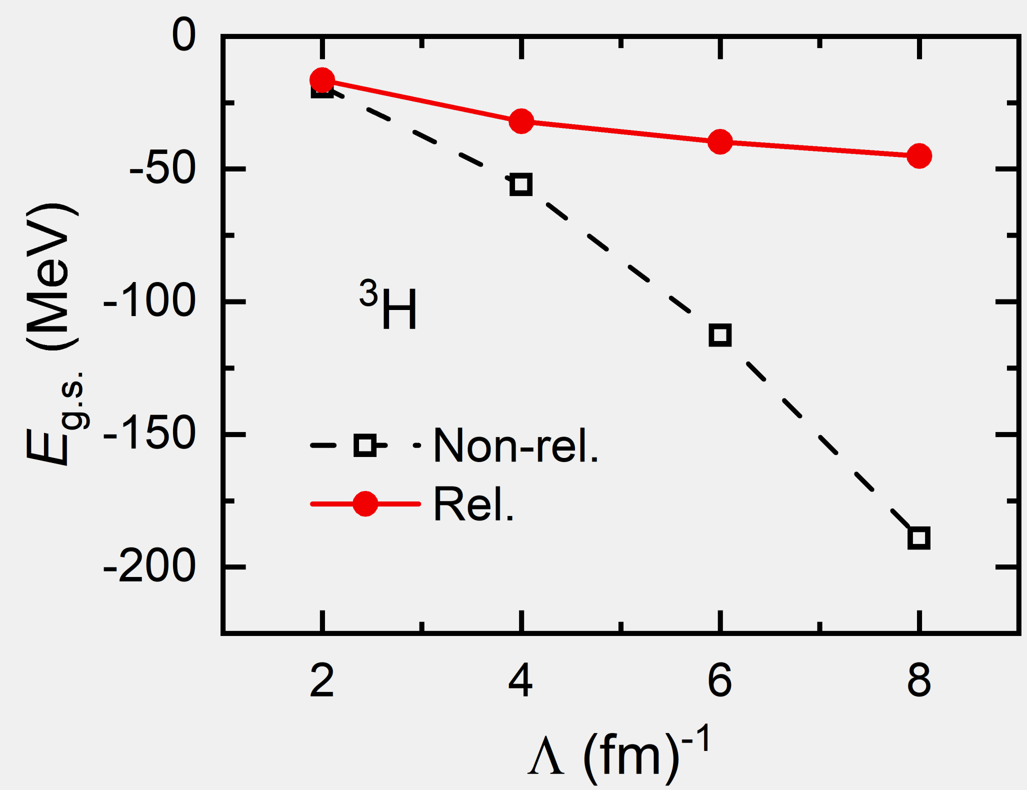
Ren and PWZ, PRC 102, 021301(R) (2020)
Editors' Suggestion

Rel. and Nonrel. VMC with LO forces

$$\left[\sum_{i=1}^{A} K_i + \sum_{i<} v(\boldsymbol{r}_{ij})\left(1 + v_t(\boldsymbol{r}_{ij}, \hat{\boldsymbol{p}}_{ij}^2) + v_b(\boldsymbol{r}_{ij}, \hat{\boldsymbol{P}}_{ij}^2)\right)\right]\Psi(\boldsymbol{R}) = E\Psi(\boldsymbol{R})$$

Nonrel Rel. corrections

Thomas collapse avoided !



Yang, PWZ, PLB 835, 137587 (2022)

The relativity brings high-order effects …

Relativistic framework for $0\nu\beta\beta$

- Manifestly Lorentz-invariant effective Lagrangian

Free

Strong

$$\mathscr{L}_{\Delta L=0} = \boxed{\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}^2 + \overline{\Psi}(\mathrm{i}\not{\partial} - M_N)\Psi} + \boxed{\frac{g_A}{2f_\pi}\overline{\Psi}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi + \sum_\alpha C_\alpha(\overline{\Psi}\Gamma\Psi)^2}$$

Weak

$$+\boxed{\frac{1}{2}\mathrm{tr}(l_\mu\vec{\tau})\cdot\partial_\mu\vec{\pi} + \frac{1}{2}\overline{\Psi}\gamma^\mu l_\mu\Psi - \frac{g_A}{2}\overline{\Psi}\gamma^\mu\gamma_5 l_\mu\Psi} + \cdots$$

Machleidt and Entem, Phys. Rep. 503, 1 (2011)

- Standard mechanism of $0\nu\beta\beta$: electron-neutrino Majorana mass

$$\mathscr{L}_{\Delta L=2} = -\frac{m_{\beta\beta}}{2}\nu_{eL}^T C\nu_{eL}, \quad C = \mathrm{i}\gamma_2\gamma_0$$

- Relativistic Kadyshevsky equation：

Kadyshevsky, NPB 6, 125 (1968)

$$T(\boldsymbol{p}',\boldsymbol{p};E) = V(\boldsymbol{p}',\boldsymbol{p}) + \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi^3)}V(\boldsymbol{p}',\boldsymbol{k})\frac{M^2}{\omega_k^2}\frac{1}{E-2\omega_k+\mathrm{i}0^+}T(\boldsymbol{k},\boldsymbol{p};E) \qquad \omega_k = \sqrt{M^2+\boldsymbol{k}^2}$$

For the interaction $V(\boldsymbol{p}',\boldsymbol{p})$, (1) neglect anti-nucleon d.o.f (2) only include the leading term in Dirac spinor (3) neglect retardation effects.

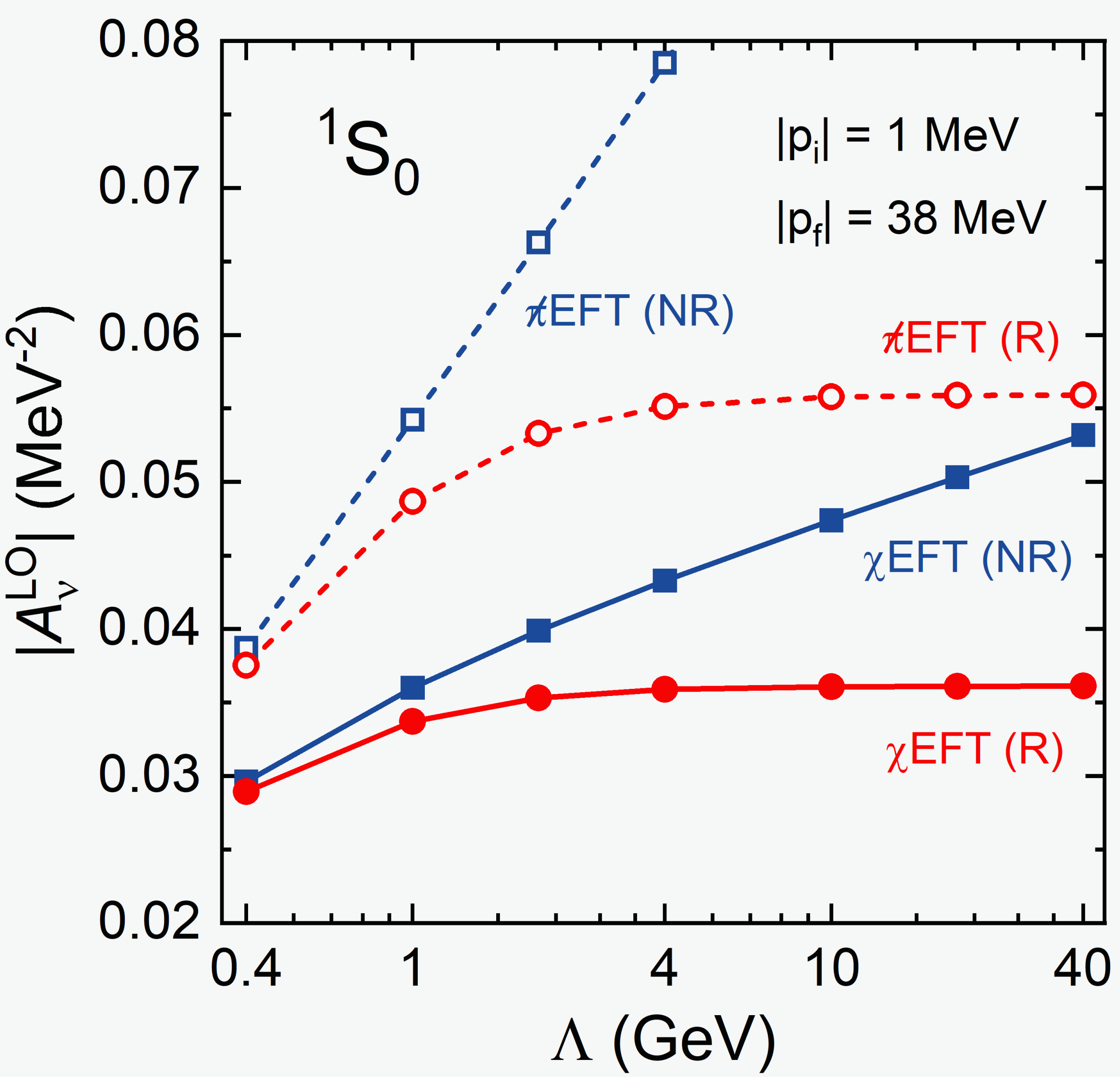
Modified Weinberg's approach in NN scattering: Epelbaum and Gegelia, PLB 716, 338 (2012)

Relativistic LO operator

- In contrast to nonrelativistic case, the unknown short-range operator is **NOT** needed at LO in the relativistic chiral EFT.

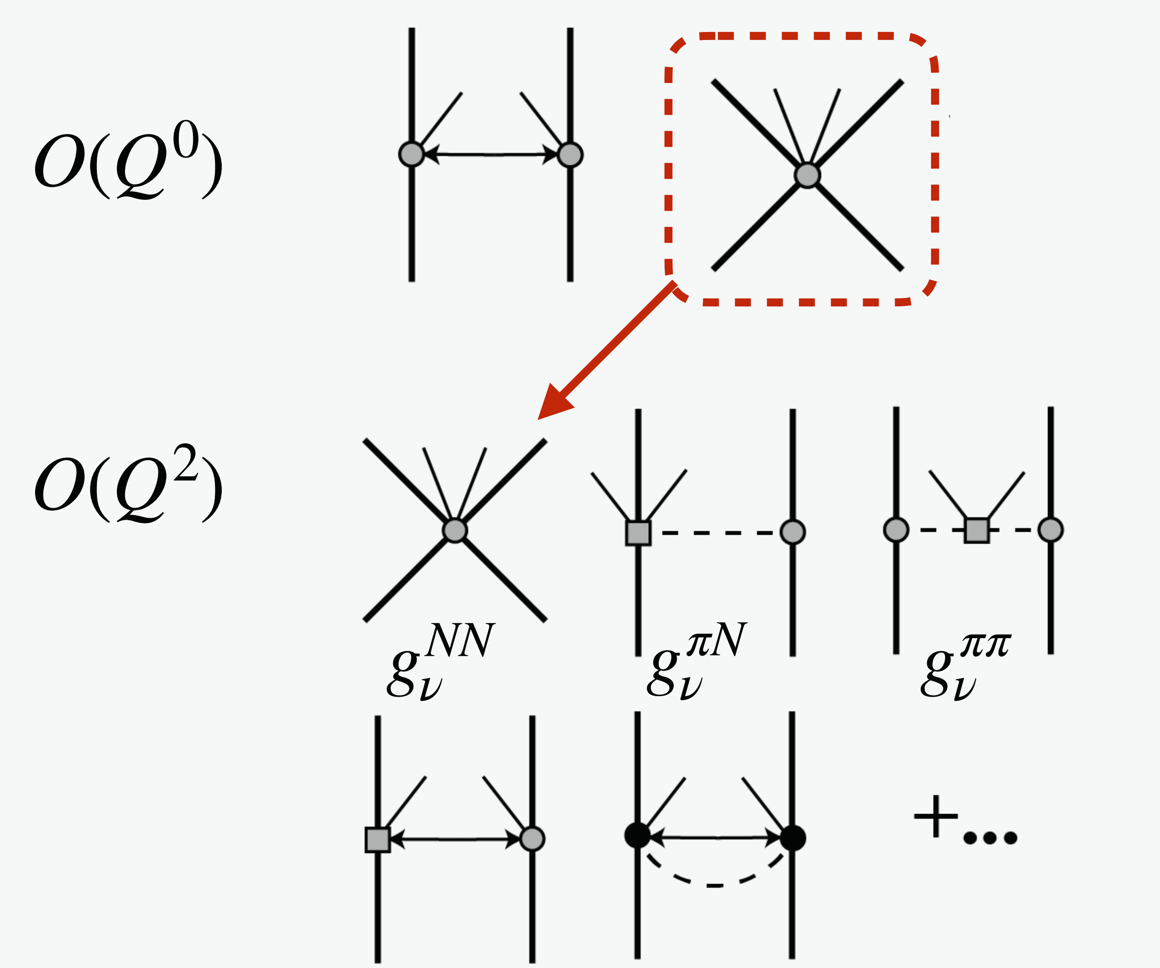
Yang and PWZ, PLB 855, 138782 (2024)

RG invariance at LO



Relativistic power counting

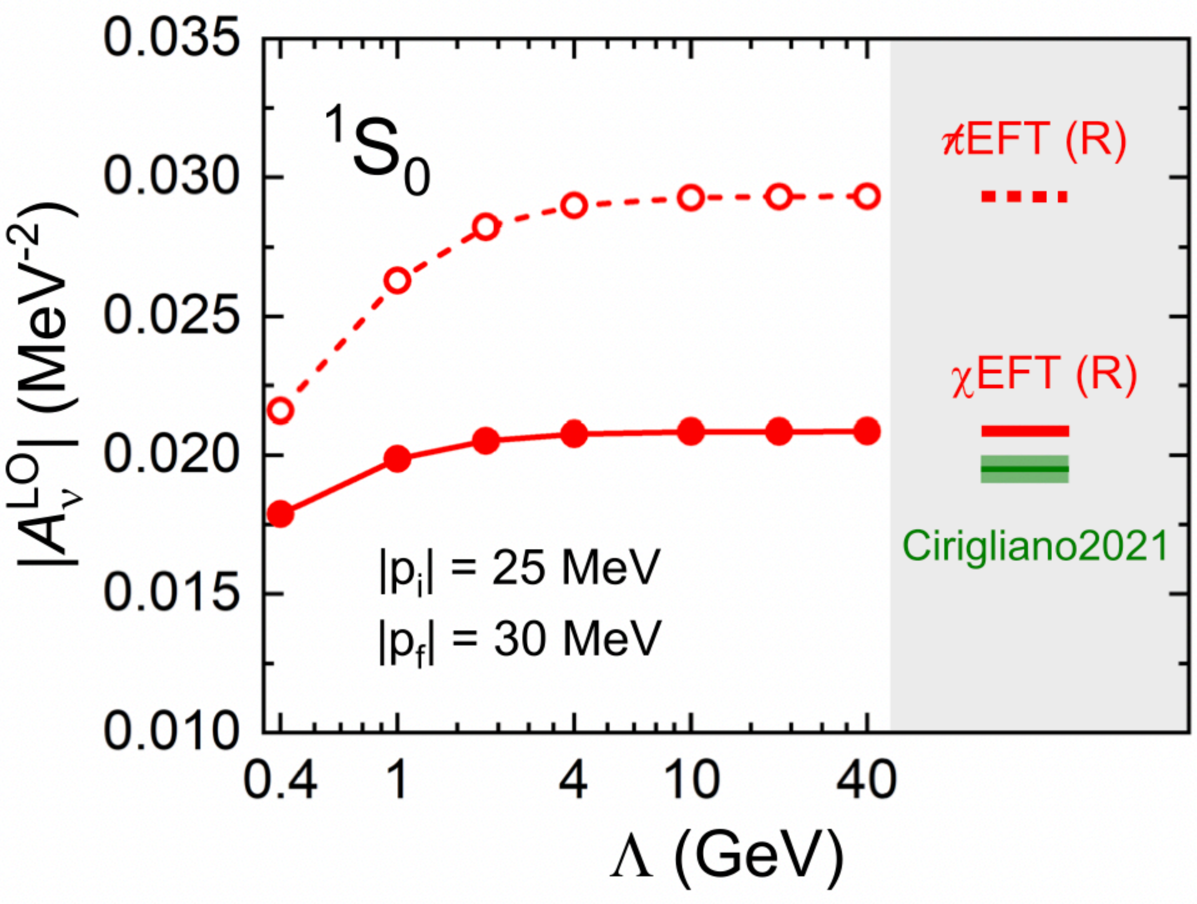
No Free Parameter at LO !

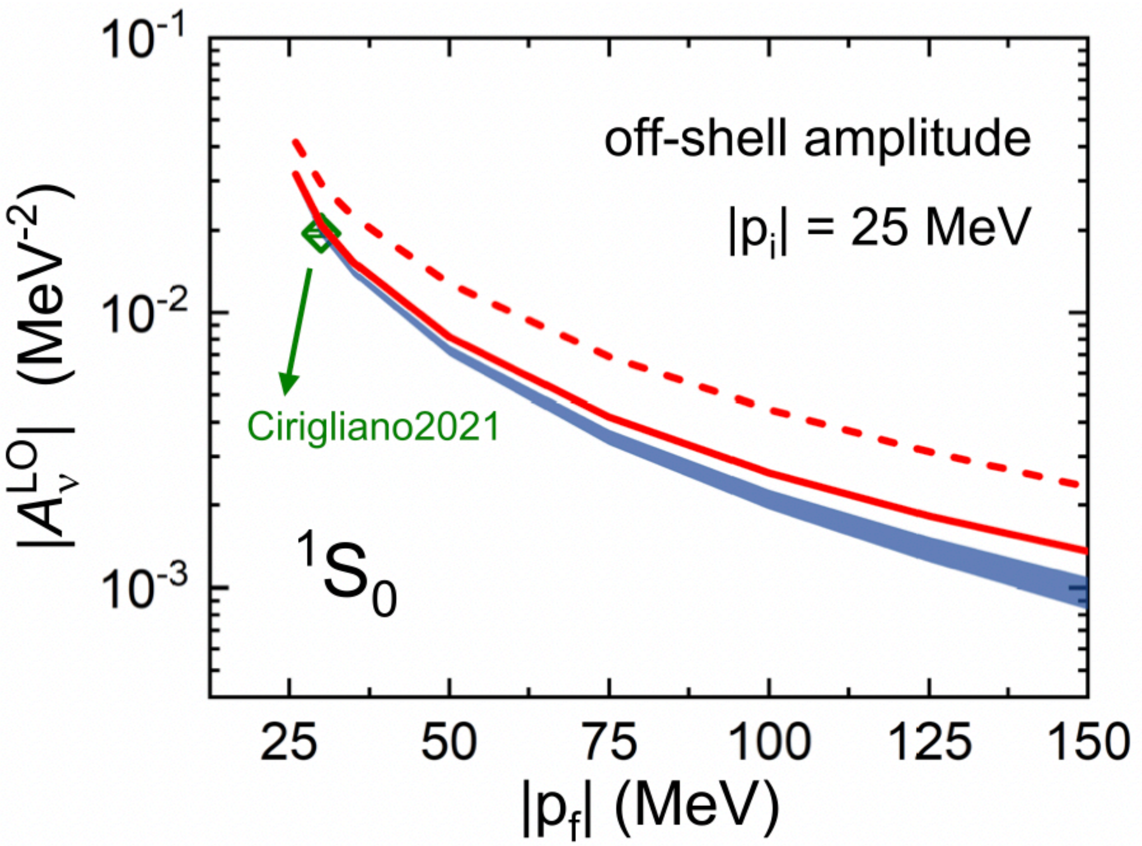


Comparison with existing results

- The $nn \to ppee$ amplitude obtained from the Cottingham model and the LO relativistic chiral EFT is consistent within 10%~40% .

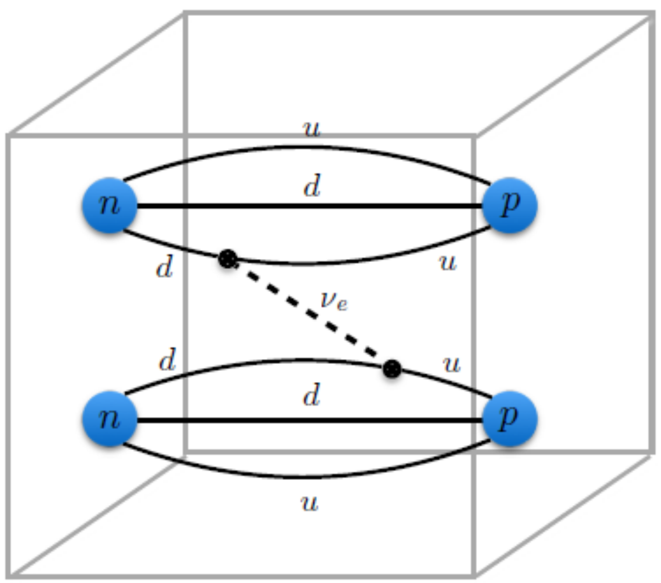
Yang and PWZ, PLB 855, 138782 (2024)
Cirigliano et al., PRL 126, 172002 (2021)





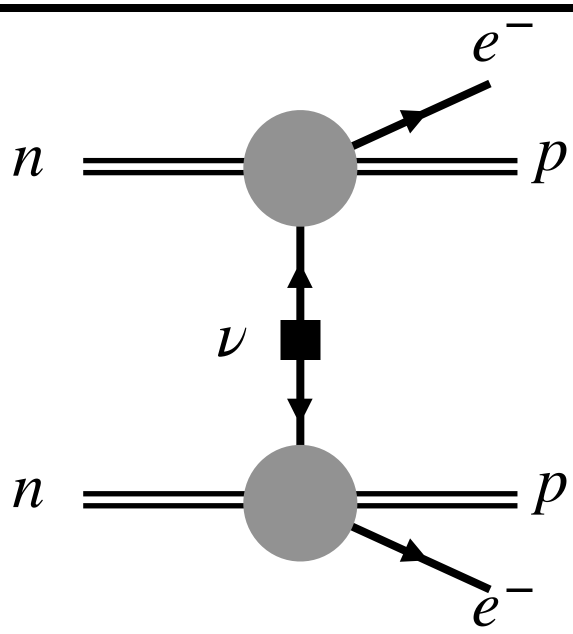
Benchmark with Lattice QCD

Benchmark with lattice QCD calculations of $0\nu\beta\beta$ decay will provide a stringent test on present framework



Lattice simulation of $0\nu\beta\beta$

Currently only possible at heavy quark (neutrino) masses



Rel.$\not\pi$EFT (χEFT) prediction

Fixed by a_{np}(, g_A) at lattice-QCD conditions

Benchmark with LQCD: LO at $m_\pi = 806\ \mathrm{MeV}$

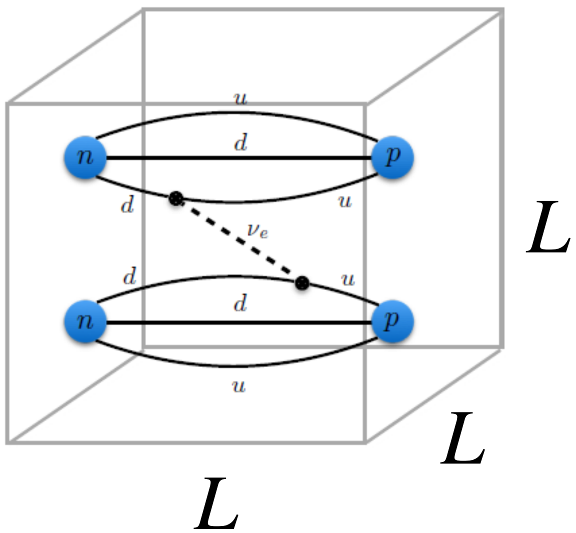
The prediction from LO relativistic pionless EFT is encouraging!

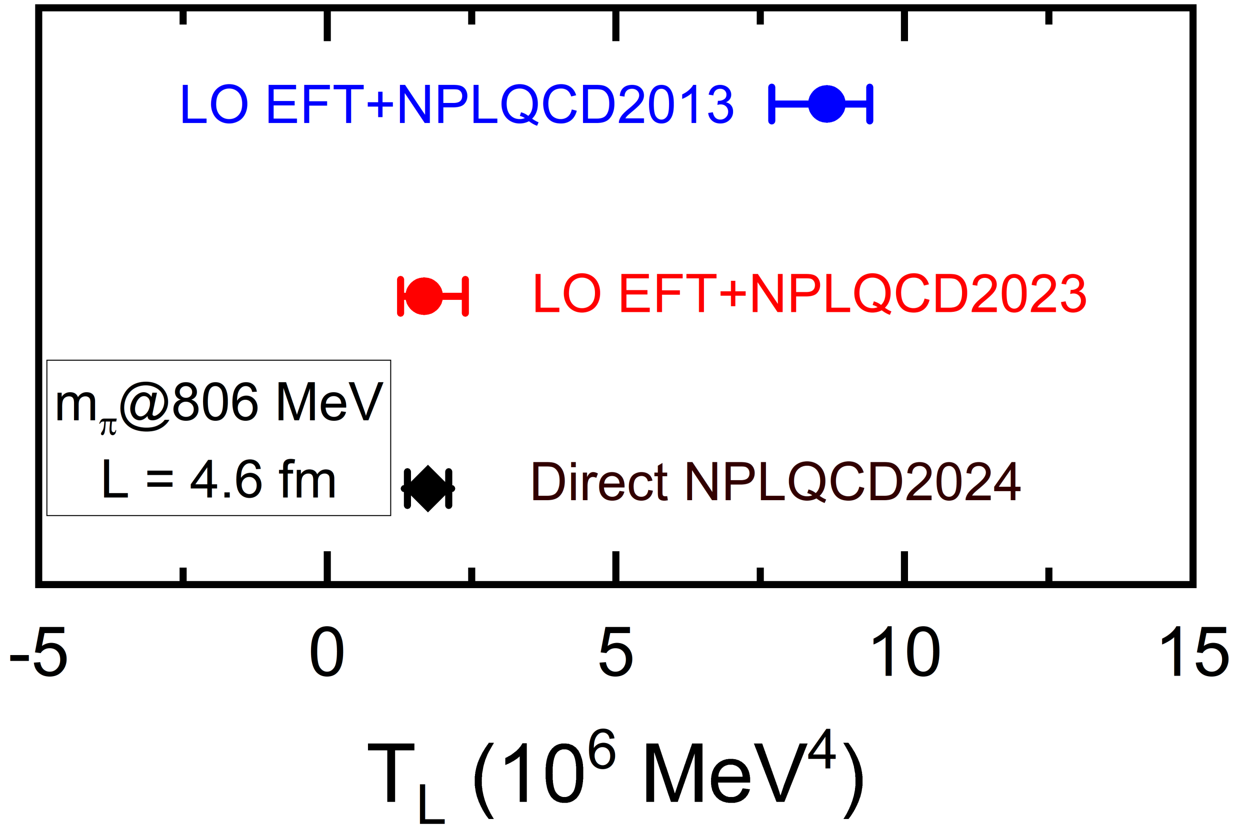
Yang and PWZ, PRD 111, 014507 (2025)

$$\mathcal{T}_L^{(M)}(E_f, E_i) = \int \mathrm{d}z_0 \int_L \mathrm{d}^3 z [\langle E_f, L | T[\mathcal{J}(z_0, \boldsymbol{z}) S_\nu(z_0, \boldsymbol{z}) \mathcal{J}(0)] | E_i, L\rangle]_L$$

LQCD: $|E, L\rangle$ is made from quarks in a box

REFT: $|E, L\rangle$ is made of two neutrons in a box





Inputs of EFT

$g_A = 1.27$

$m_\pi = 0.81\ \mathrm{GeV}$

$M_N = 1.64\ \mathrm{GeV}$

$E_{nn} = -17(4)\ \mathrm{MeV}$

$E_{nn} = -3.3(7)\ \mathrm{MeV}$

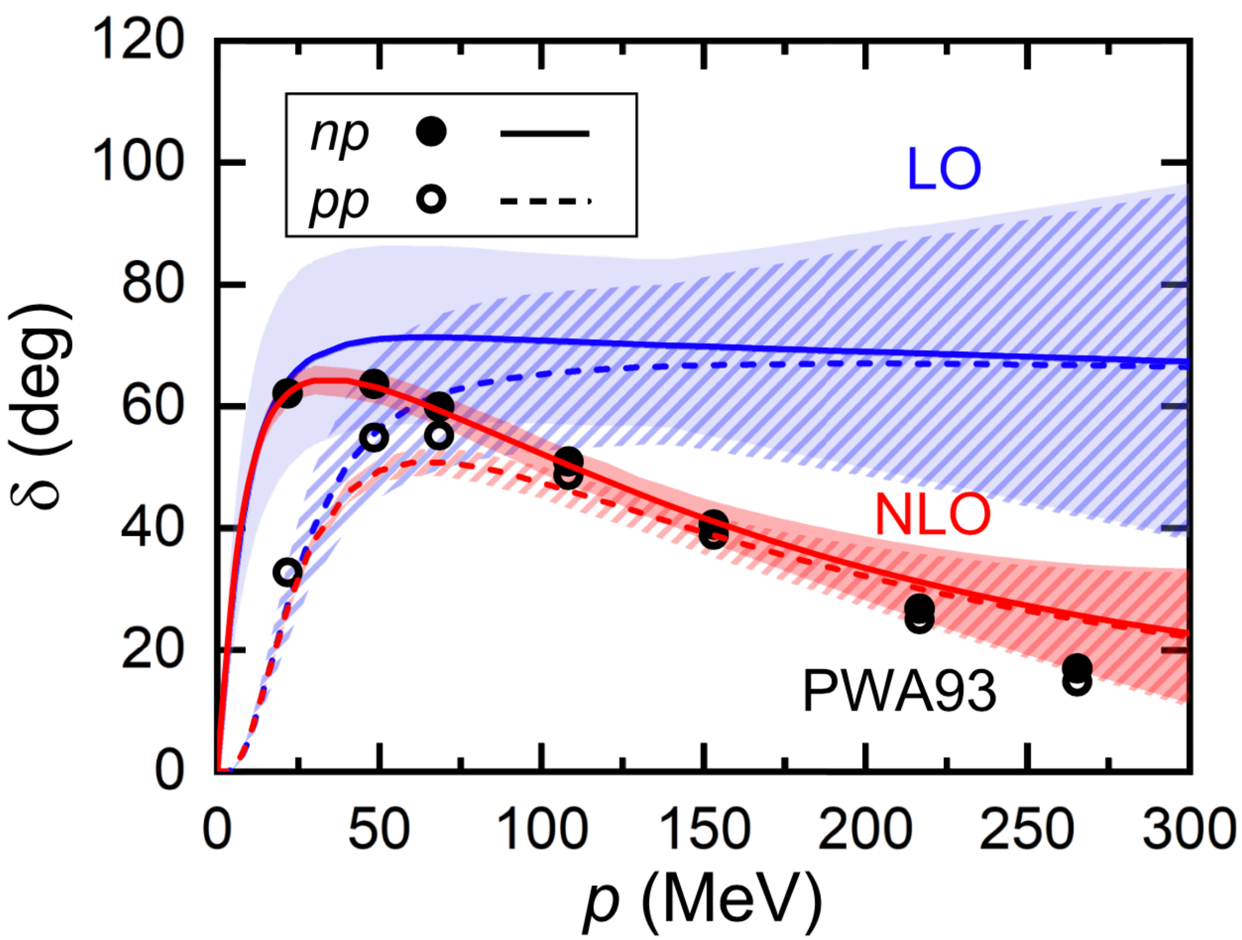
Beane et al. (NPLQCD), PRD 87, 034506 (2013)
Amarasinghe et al. (NPLQCD), PRD107,094508 (2023)
Davoudi et al. (NPLQCD), PRD 109, 114514 (2024)

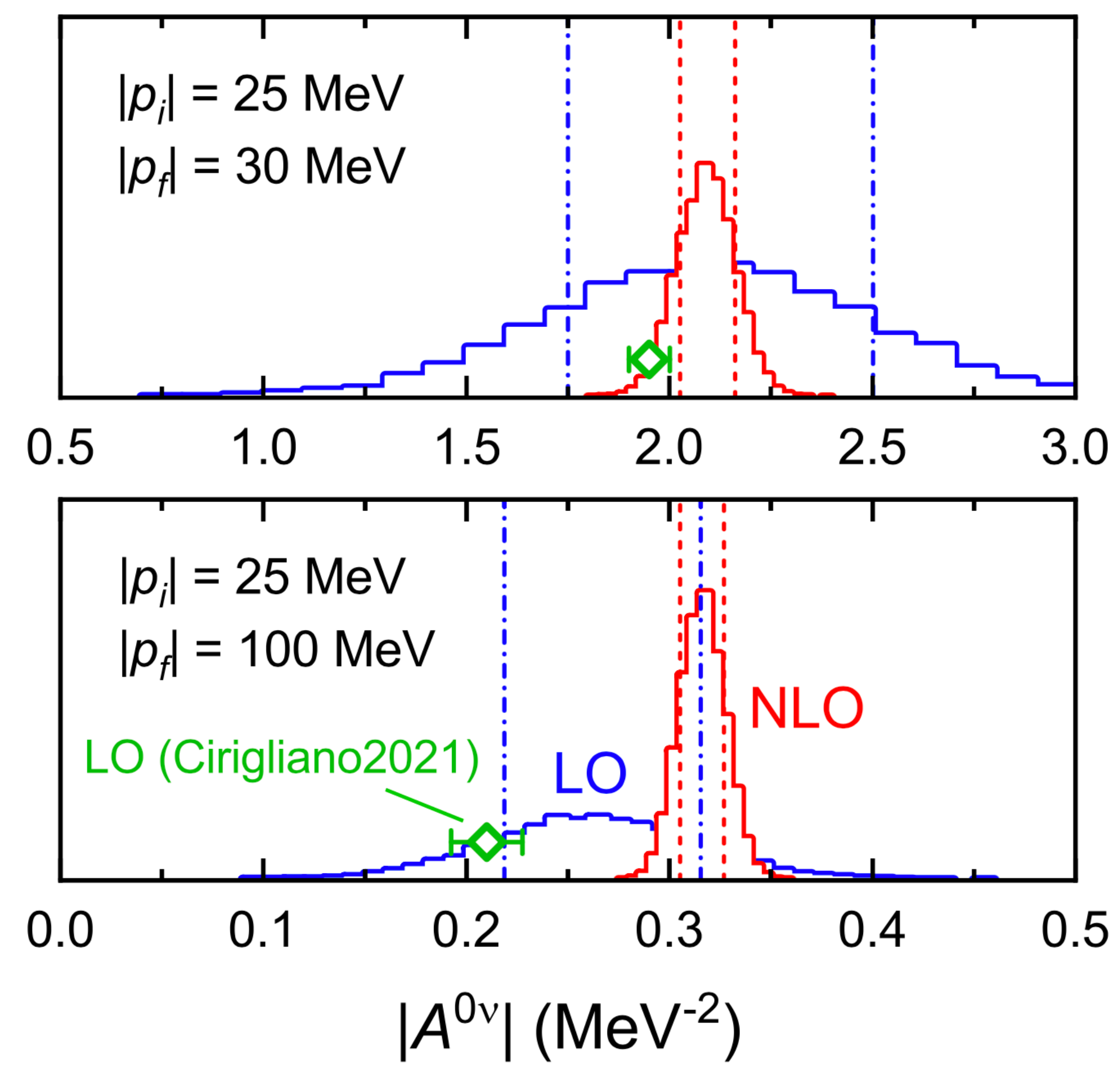
NLO prediction

The NLO effective-range corrections are included; significantly improves the description of phase shifts. The LECs are determined by a Bayesian approach.

Yang and PWZ, PRL 134, 242502 (2025)

$$\delta_{np}^{1S0} \longrightarrow P(\text{para.}) \longrightarrow P\left(A^{0\nu}\right)$$





Bayesian estimation of LECs in chiral EFT
Wesolowsk, Furnstahl, Melendez, and Phillips, JPG 46, 045102 (2019)

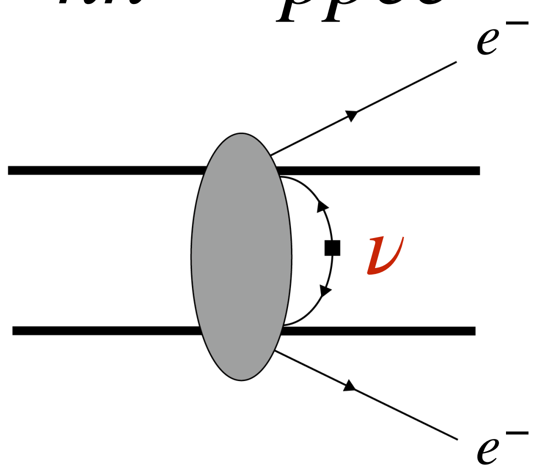
Validation

Replacing the neutrino exchange to γ exchange, the relativistic chiral EFT reproduces the CSB and CIB contributions to the *NN* scattering amplitude.

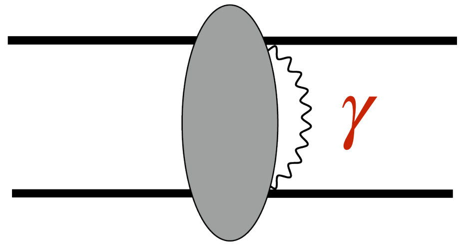
Prediction with NO free parameters.

Yang and PWZ, PRL 134, 242502 (2025)

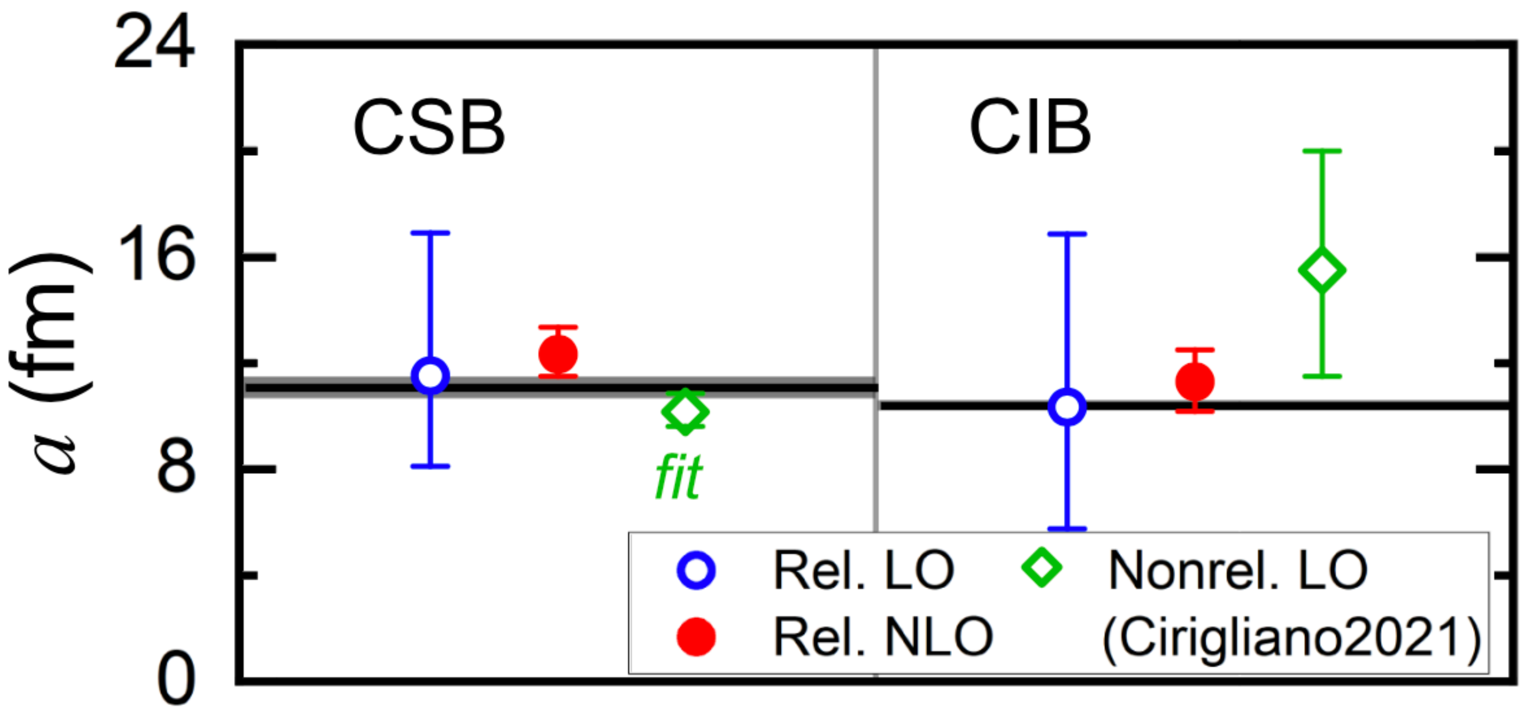
$nn \to ppee$



$NN \to NN$



Charge-Symmetry/
Independence Breaking



$$a_{\mathrm{CSB}} = a_{pp} - a_{nn} \qquad a_{\mathrm{CIB}} = \frac{a_{nn} + a_{pp}}{2} - a_{np}$$

Outline

- Nuclear matrix elements of $0\nu\beta\beta$ decay
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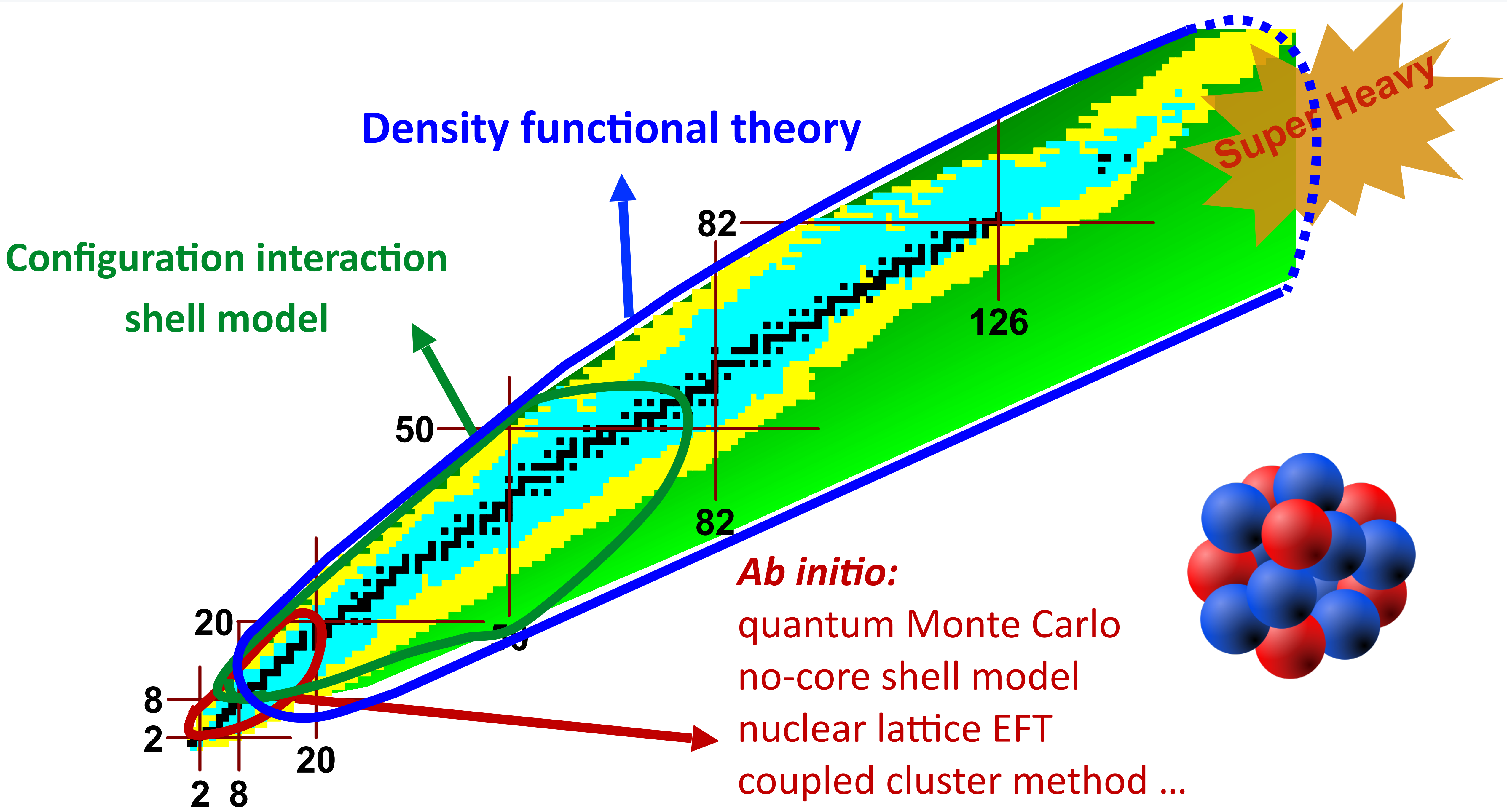
State-of-the-art theories for nuclei



It would be interesting to investigate the intersections between different theories for a unified and comprehensive description of nuclei.

Density functional theory

The many-body problem is mapped onto a one-body problem

Hohenberg-Kohn Theorem

The exact ground-state energy of a quantum mechanical many-body system is a universal functional of the local density.

$$E[\rho] = T[\rho] + U[\rho] + \int V(\mathbf{r})\rho(\mathbf{r})\,\mathrm{d}^3\mathbf{r}$$

Kohn-Sham DFT

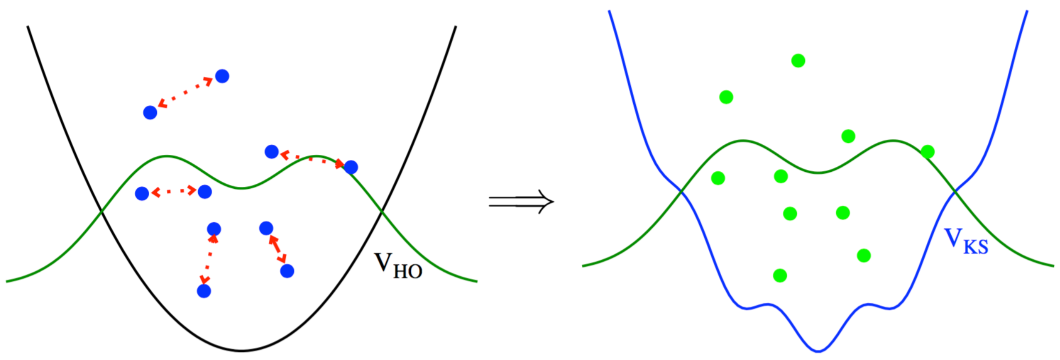


Figure from Drut PPNP 2010

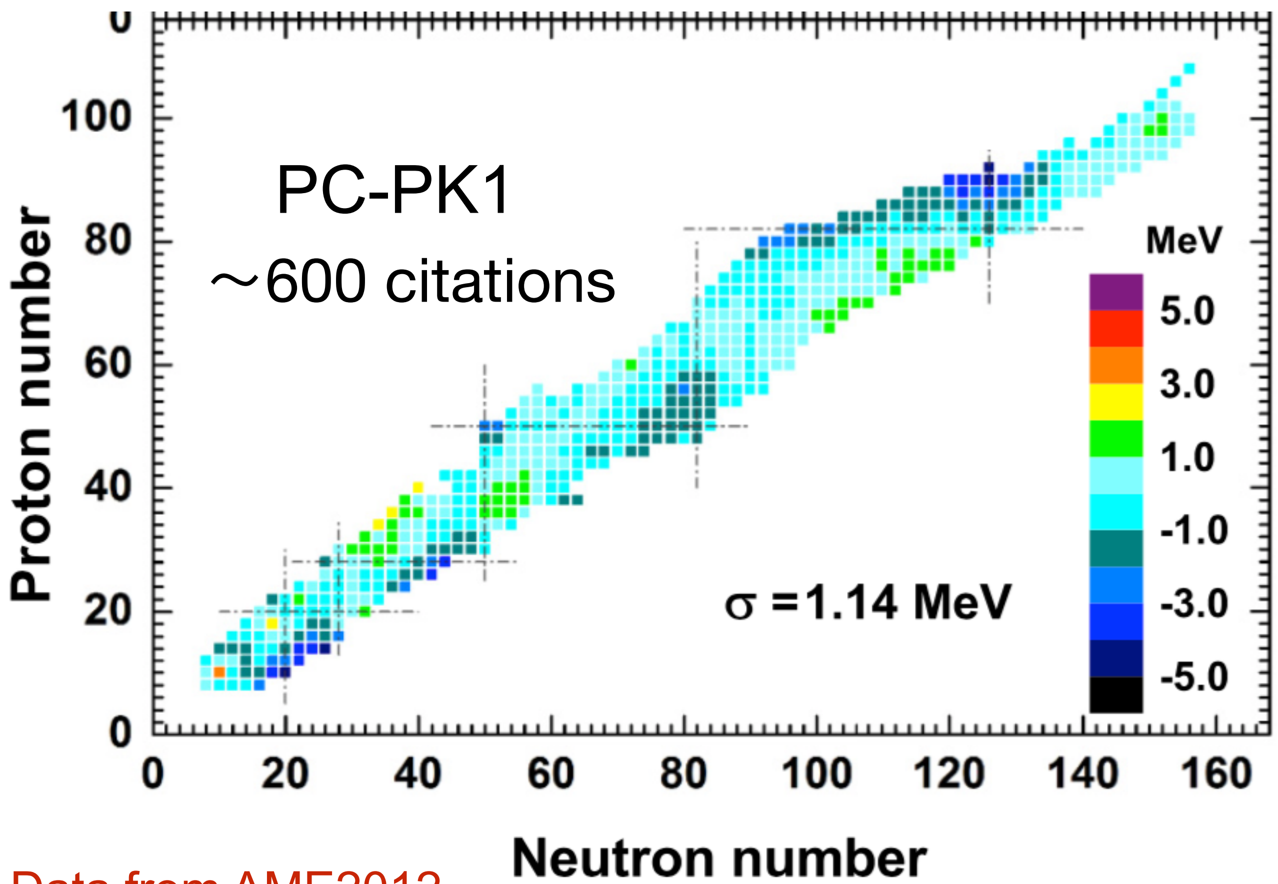
$$T[\rho] \doteq \sum_{i=1}^{N} \left\langle \varphi_i \left| -\frac{\hbar^2}{2m}\nabla^2 \right| \varphi_i \right\rangle$$

$$E[\rho] \Rightarrow \boxed{\hat{h} = \frac{\delta E}{\delta \rho}} \Rightarrow \boxed{\hat{h}\varphi_i = \varepsilon_i \varphi_i} \Rightarrow \boxed{\rho = \sum_{i=1}^{A} |\varphi_i|^2}$$

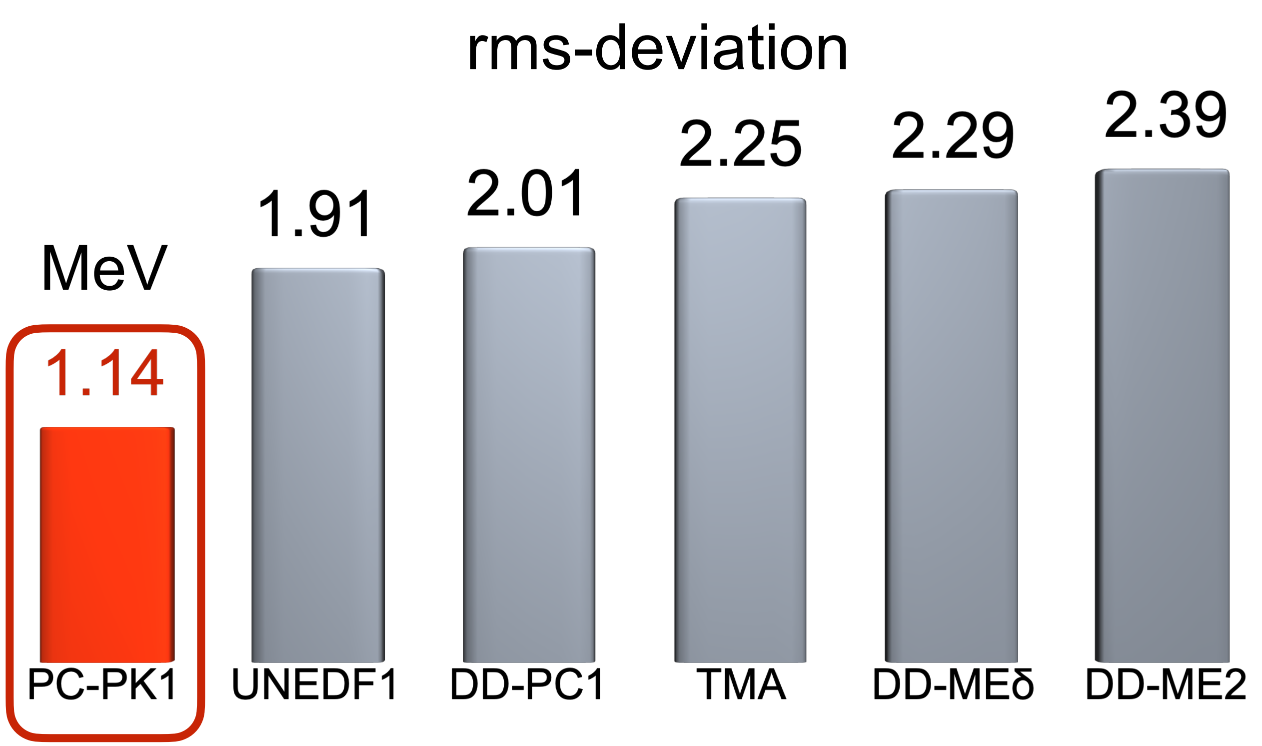
The practical usefulness of the Kohn-Sham theory depends entirely on whether an **Accurate Energy Density Functional** can be found!

Covariant density functional：PC-PK1

Mass Differences: M_{cal} - M_{exp}



Data from AME2012



PWZ, Li, Yao, Meng, PRC 82, 054319 (2010)

Lu, Li, Li, Yao, Meng, PRC 91, 027304 (2015)

http://nuclearmap.jcnp.org

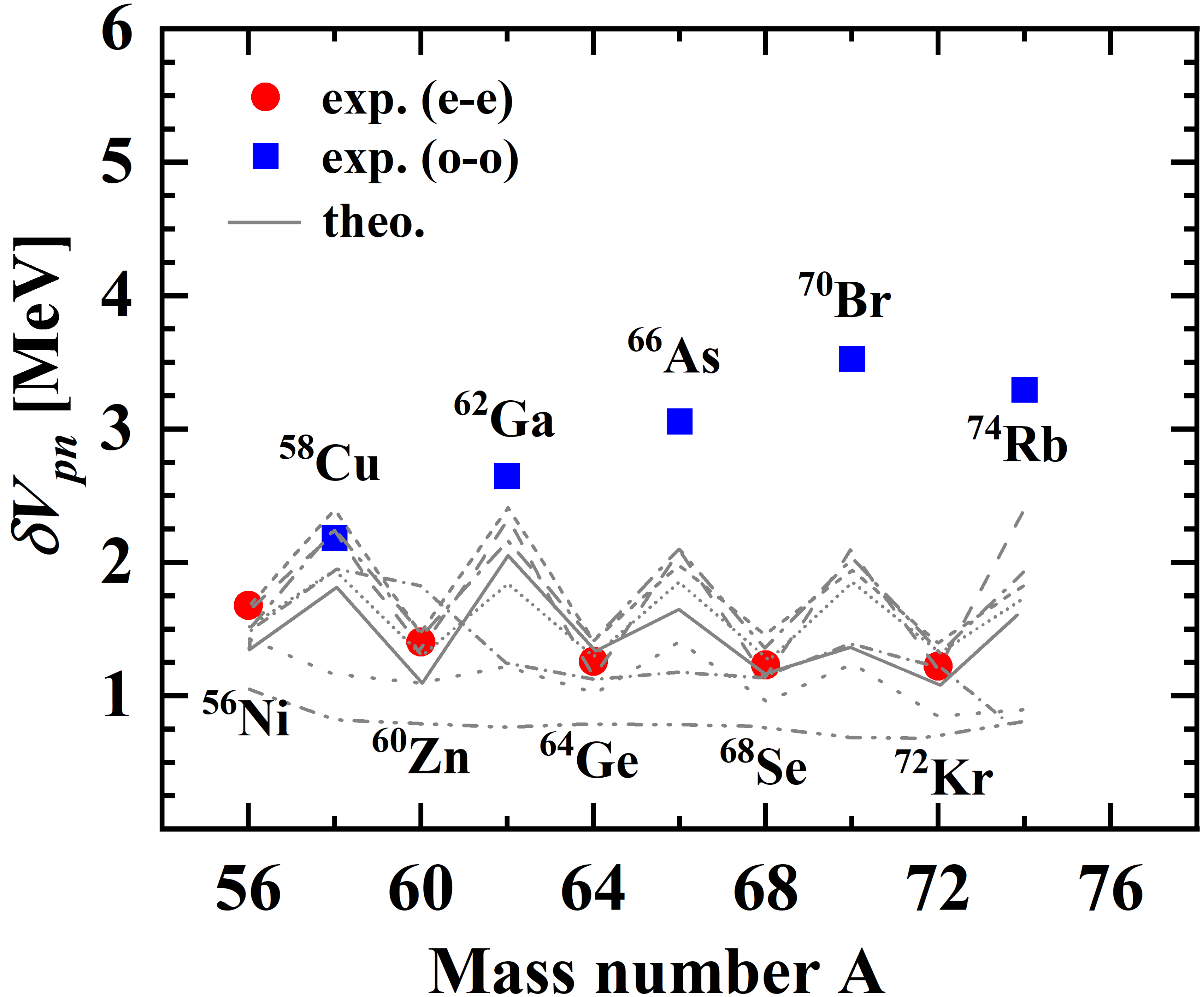
Yang, Wang, PWZ, Li, PRC 104, 054312 (2021)

Yang, PWZ, Li, PRC 107, 024308 (2023)

Among the best density-functional description for nuclear masses!

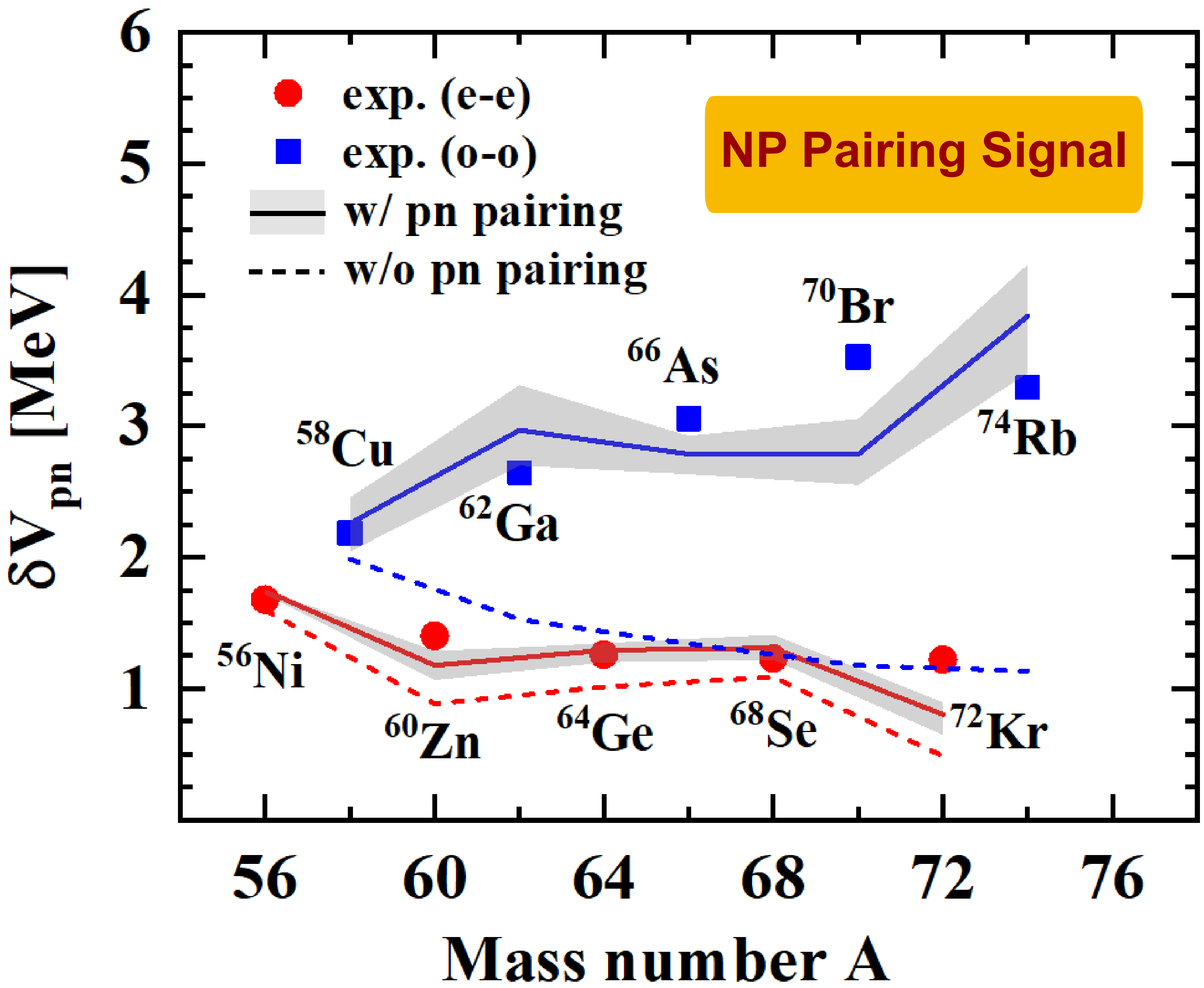
Second-order differences of binding energy

This bifurcation in the double binding energy differences δV_{pn} cannot be reproduced by existing mass models.



Exp: M. Wang et al., PRL 130, 192501 (2023)

Second-order differences of binding energy

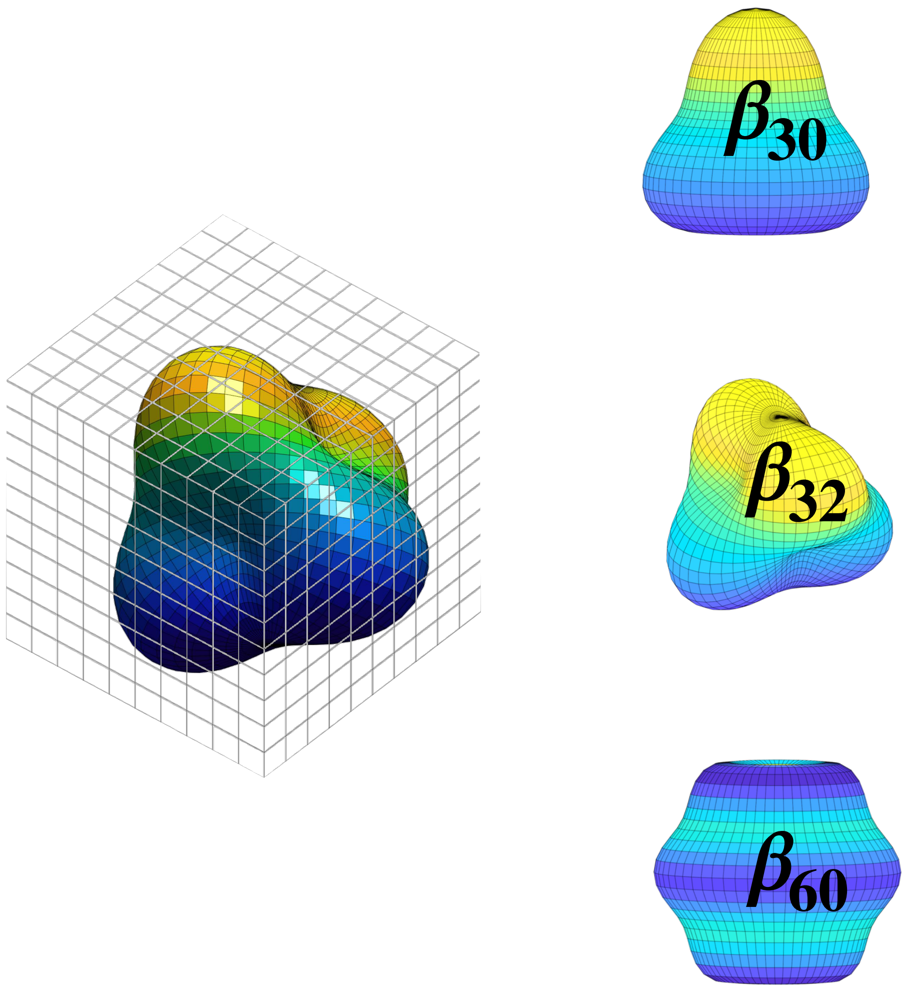


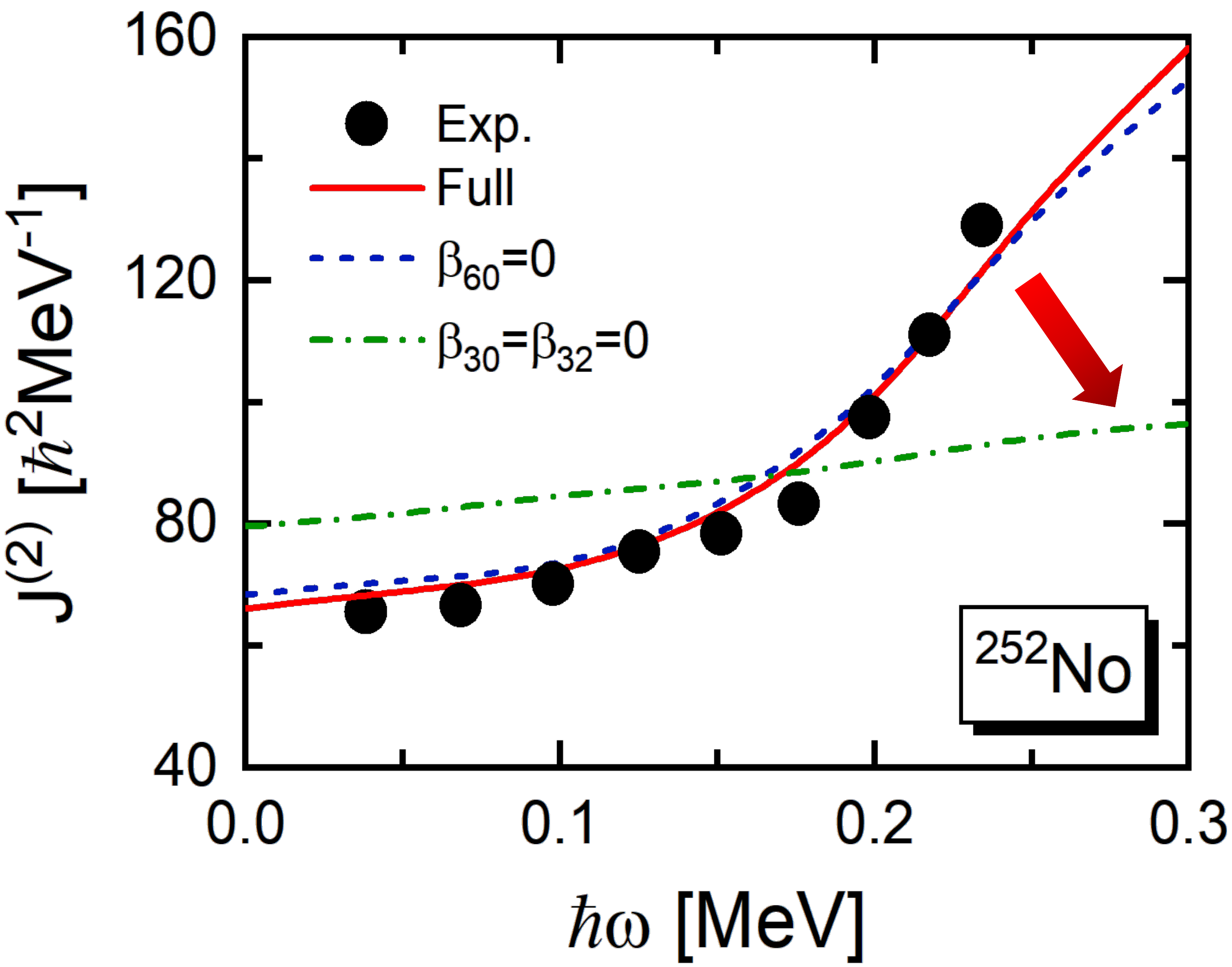
Exp: M. Wang et al., PRL 130, 192501 (2023)

Wang, Wang, Xu, **PWZ**, Meng, PRL 132, 232501 (2024)

Second-order differences of energy spectra

- **Lattice RDFT:** solving the RDFT in a 3D lattice space
- no spatial symmetry restriction; **full deformation space**





Xu, Wang, Wang, Ring, **PWZ**, PRL 133, 022501 (2024)

Relativistic **C**onfiguration-interaction **D**ensity functional theory

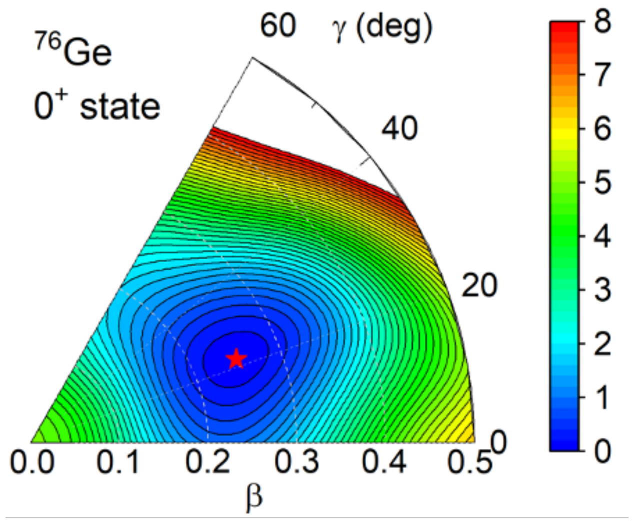
DFT	Shell Model
✔ Universal density functionals	✘ Non-universal effective interactions
Symmetry broken	No symmetry broken
Single config. fruitful physics	Single config. little physics
No Configuration mixing	Configuration mixing
✔ Applicable for almost all nuclei	✘ intractable for deformed heavy nuclei
✘ No spectroscopic properties	✔ spectroscopy from multi config.

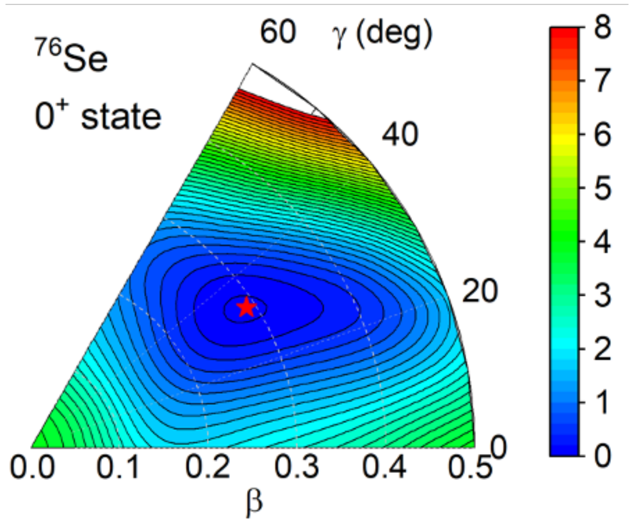
ReCD:

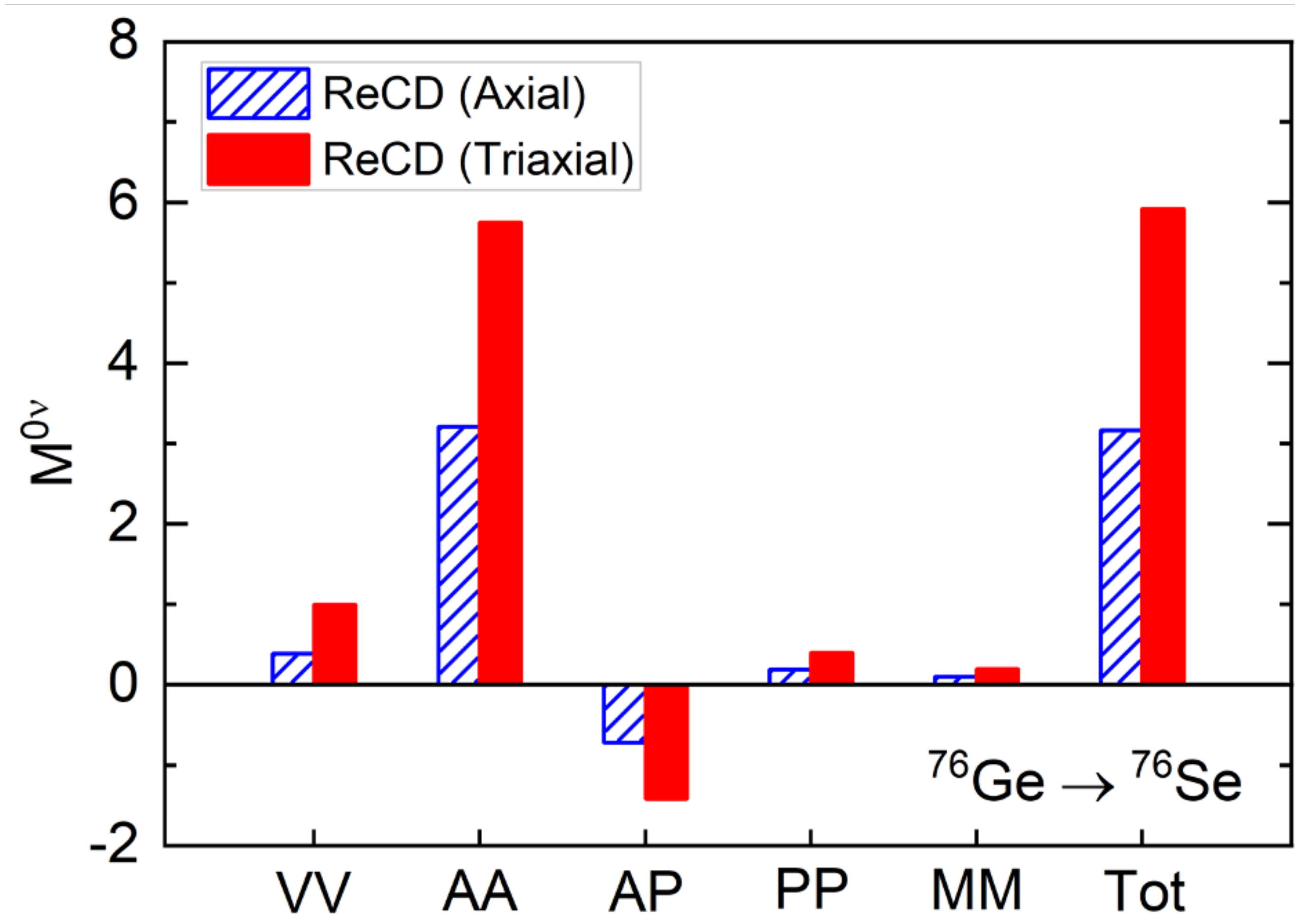
a method combining the advantages from both Shell model and DFT

ReCD method for $0\nu\beta\beta$

Both axial and triaxial degrees of freedom are included in the ReCD theory



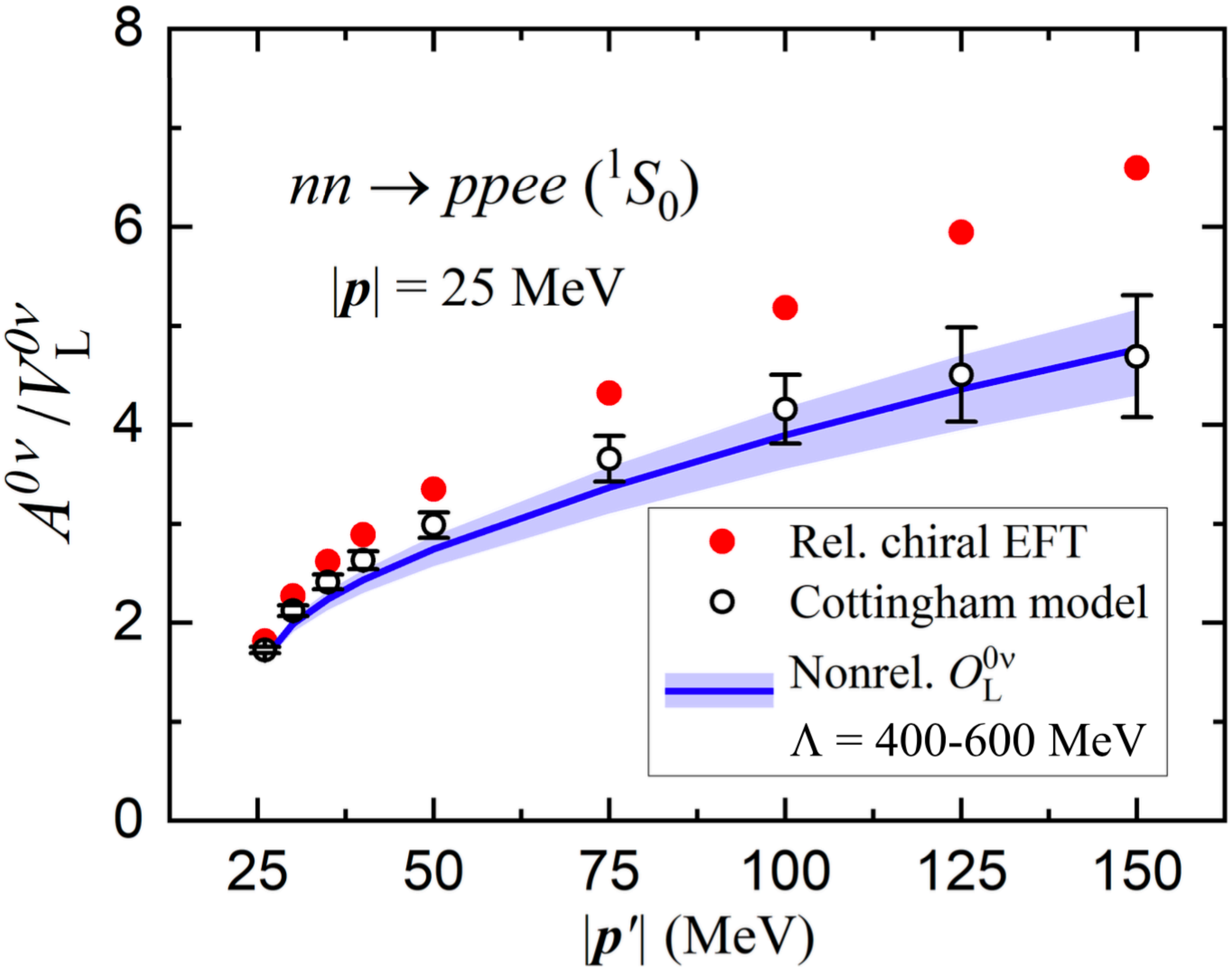




Wang, PWZ, Meng, Sci. Bull. 69, 2017-2020 (2024)

The short-range uncertainty in NME

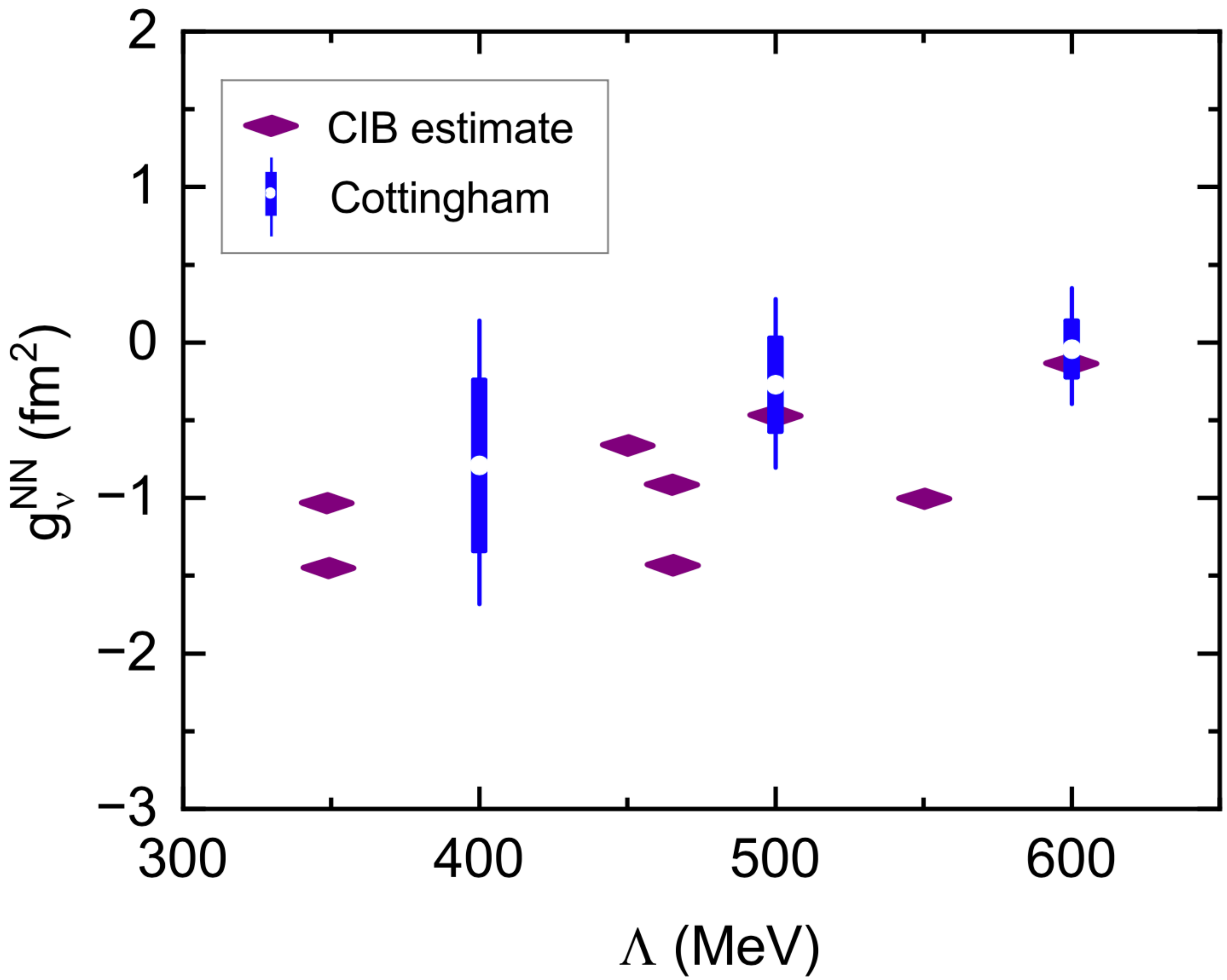
Wang, Yang, PWZ, submitted



The nonrelativistic results depend on cutoff, introducing a short-range term with unknown coupling g_ν^{NN}

Determine the g_ν^{NN}

- Thick bars: systematic error of the Cottingham model
- Thin bars: errors after considering the dependence on final momentum |p'|



CIB estimates from

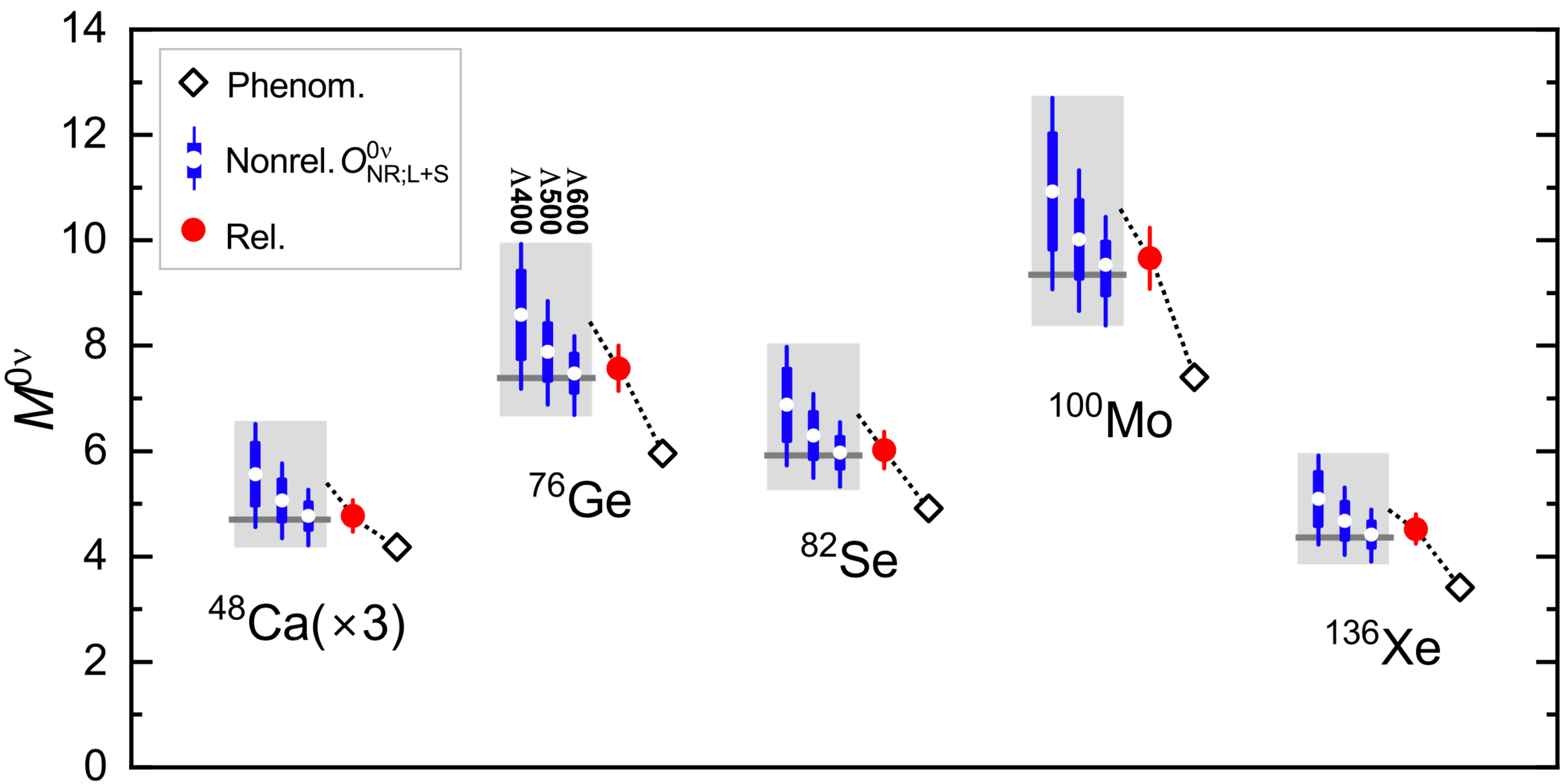
Jokiniemi, Soriano, Menéndez, PLB 823, 136720 (2021)

Cirigliano, et al., PRC 100, 055504 (2019)

Wang, Yang, PWZ, submitted

NMEs based on relativistic decay operator

ReCD theory ⇒ wavefunctions; Relativistic EFT ⇒ decay operator



Wang, Yang, PWZ, submitted

The relativistic NMEs are free from LO short-range uncertainty

The relativistic NMEs lie within the uncertainty range of nonrelativistic NMEs

Outline

- Nuclear matrix elements of $0\nu\beta\beta$ decay
- $0\nu\beta\beta$ decay of two neutrons
- $0\nu\beta\beta$ decay of candidate nuclei
- Summary

Summary

A **model-free** prediction of the $nn \to ppee$ decay amplitude is obtained by the **relativistic framework based on chiral EFT**…

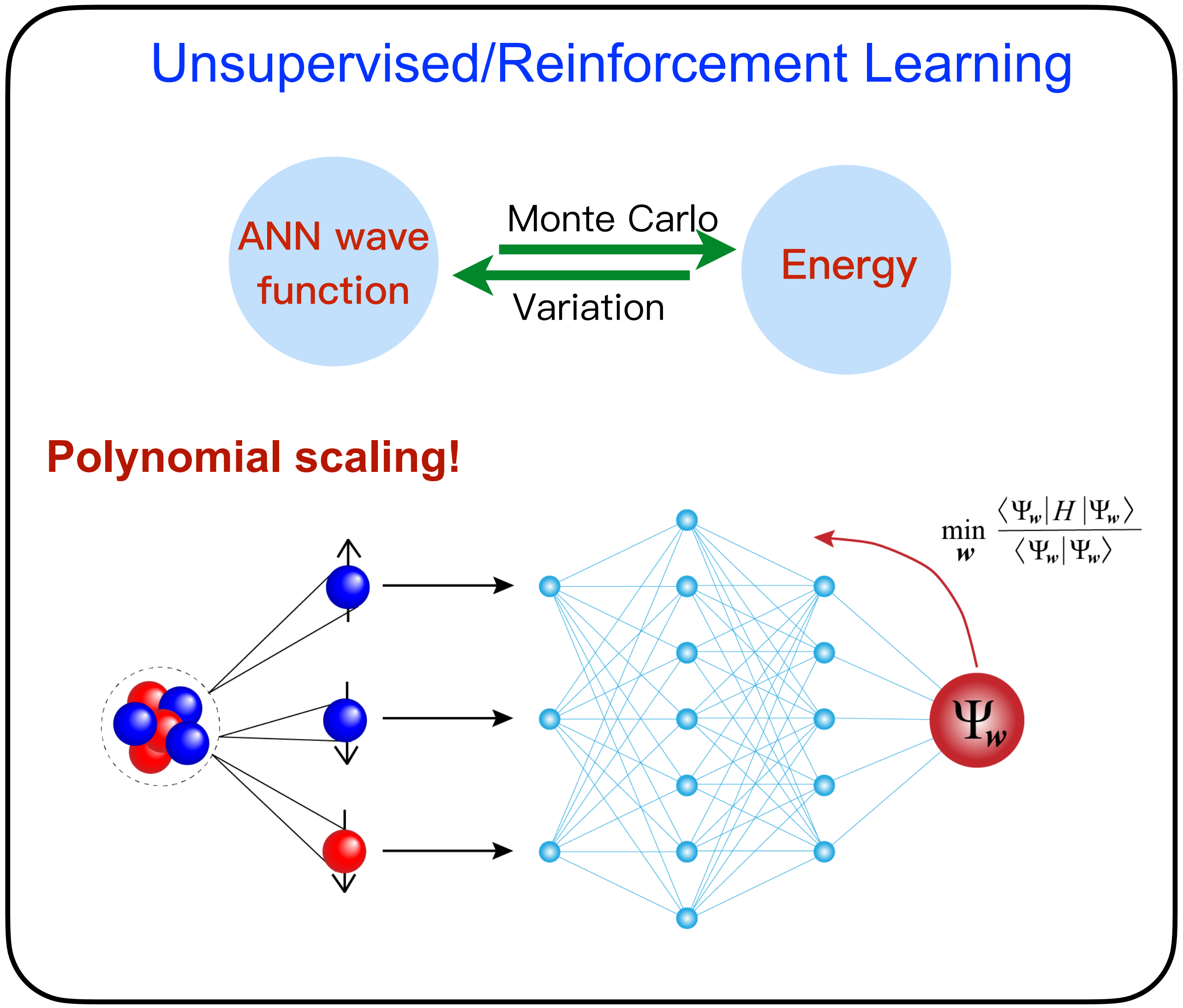
- No contact term is needed for renormalization at LO.
- First NLO and the most accurate prediction for $nn \to ppee$ decay amplitude.
- Reproducing the charge-dependent *NN* scattering data.
- ReCD: combining the advantages of both Shell model and DFT.
- Importance of the triaxial deformation in the NME of ^{76}Ge.
- NMEs for candidate nuclei free from the LO short-range uncertainty.

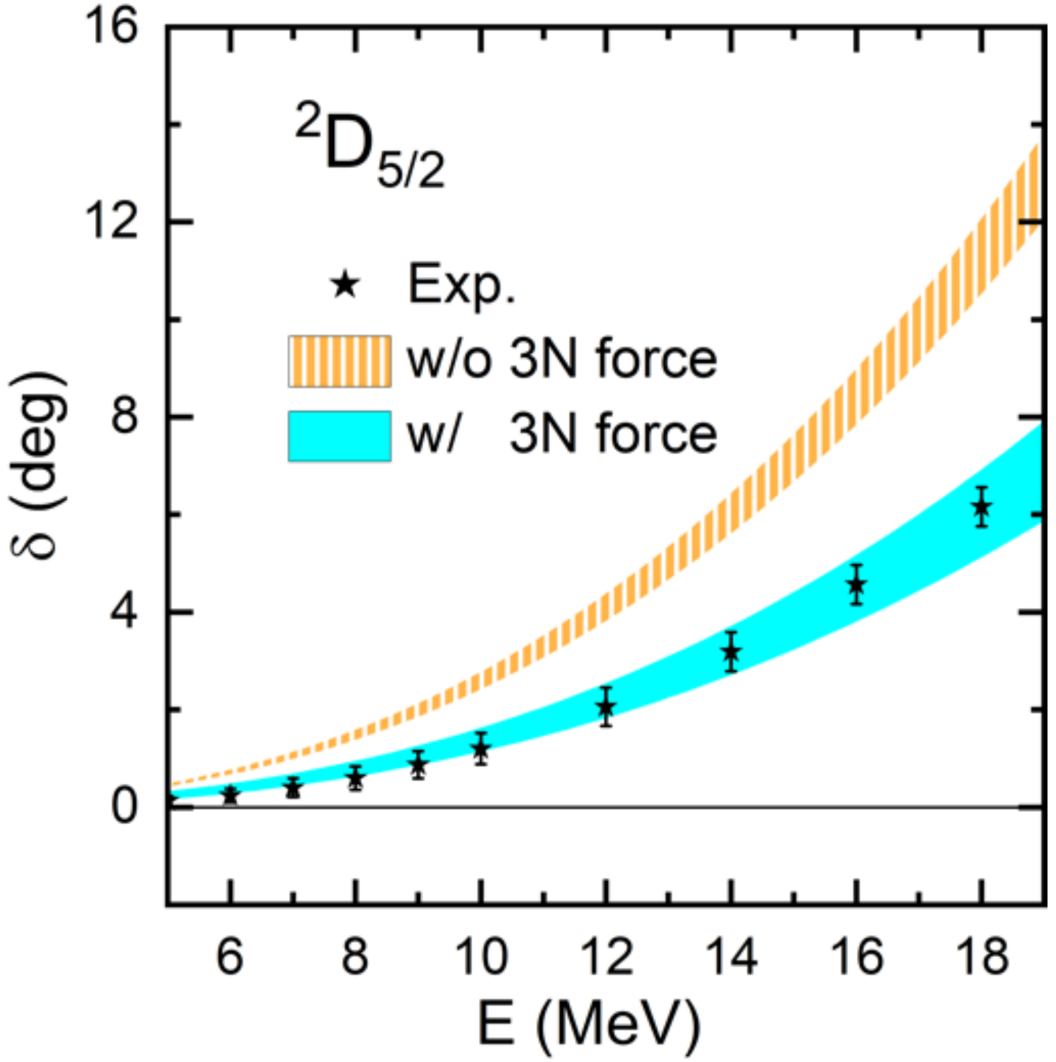
Decay operator: benchmark with lattice QCD …;

Many-body wave functions: Ai-driven ab initio calculations …

Neural-network–based nuclear ab initio calculations

- **FeynmanNet:** Variational Monte Carlo + Deep Neural-Networks
- a **high-precision** and **sign-problem free** nuclear ab initio method





Yang and **PWZ**, PLB 835, 137587 (2022)
Yang and **PWZ**, PRC 107, 034320 (2023)
Yang, Epelbaum, Meng, Meng, **PWZ**, PRL 135, 172502 (2025)

$N\alpha$ scattering as a sensitive probe of 3N forces

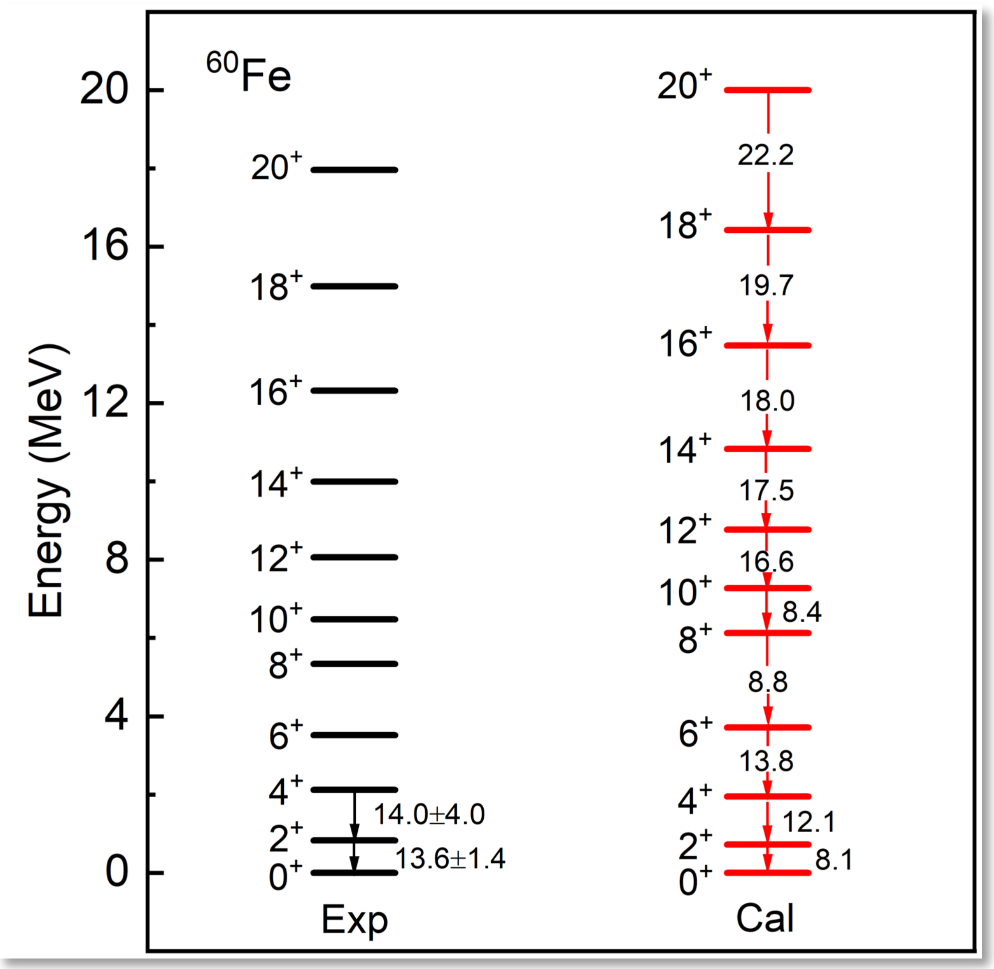

Thank you.

The ReCD method

Relativistic **C**onfiguration-interaction **D**ensity functional theory

a theory combining the advantages from both Shell model and DFT

1. **Covariant Density Functional Theory**
   a minimum of the energy surface
2. **Configuration space**
   multi-quasiparticle states
3. **Angular momentum projection**
   rotational symmetry restoration
4. **Shell model calculation**
   configuration mixing / interaction from CDFT



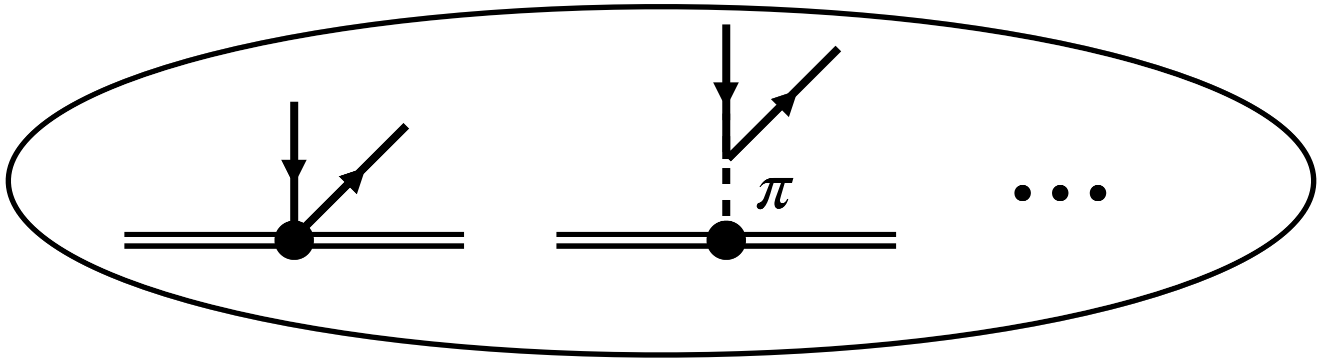
PWZ, Ring, Meng, PRC 94 (2016) 041301(R)

Wang, PWZ, Meng, PRC 105 (2022) 054311

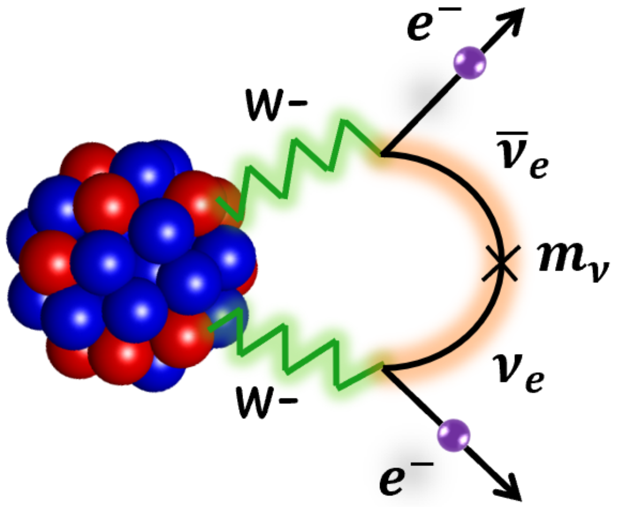
Future

Relativistic calculations of $0\nu\beta\beta$ nuclear matrix elements with LO and NLO chiral decay operators

Weak charged current



Nuclear-structure calculations



$$\langle p'|J^{\mu}_{\text{LO}}(x)|p\rangle = e^{iqx}\bar{u}(p')\left(g_V\gamma^{\mu} - g_A\gamma^{\mu}\gamma_5 + g_A\frac{2m_N q^{\mu}}{m_{\pi}^2 + \boldsymbol{q}^2}\gamma_5\right)u(p)$$

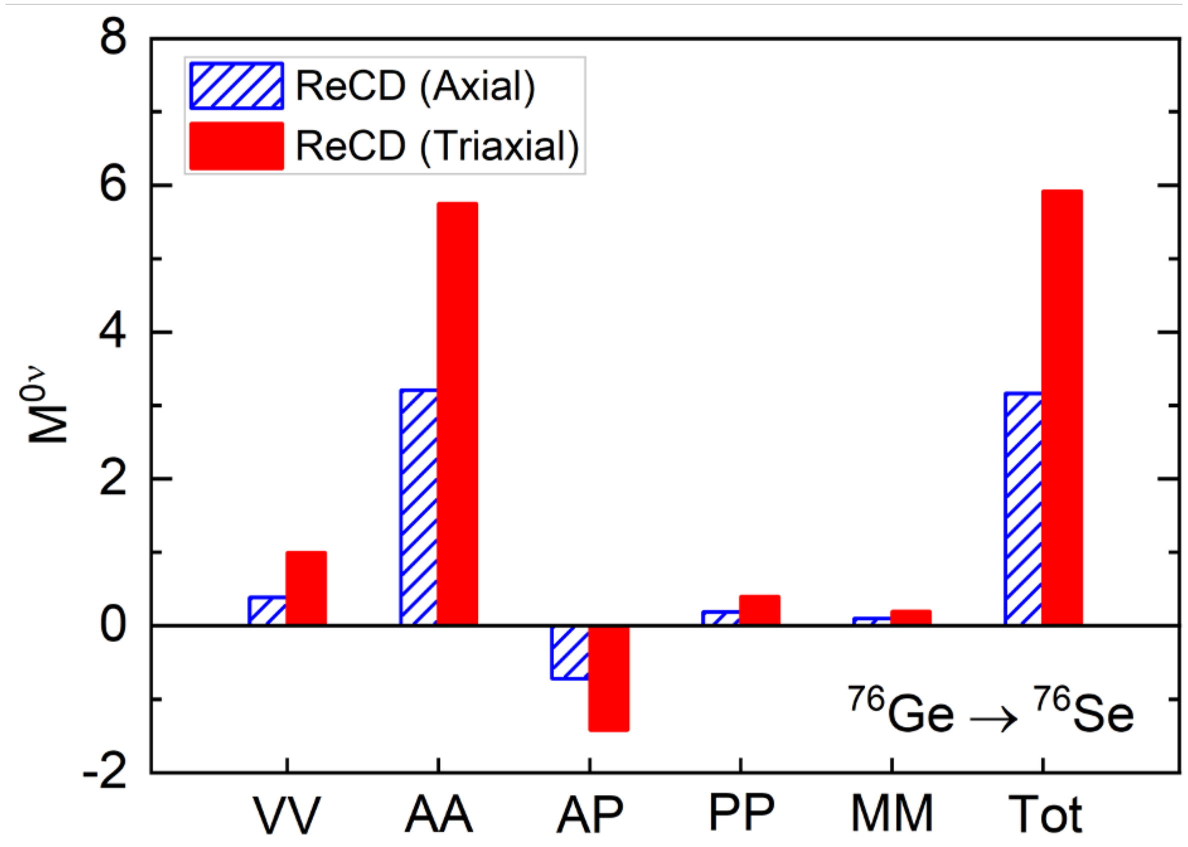
No need for contact term!

ReCD method for $0\nu\beta\beta$

Relativistic **C**onfiguration-interaction **D**ensity functional theory

a theory combining the advantages from both Shell model and DFT

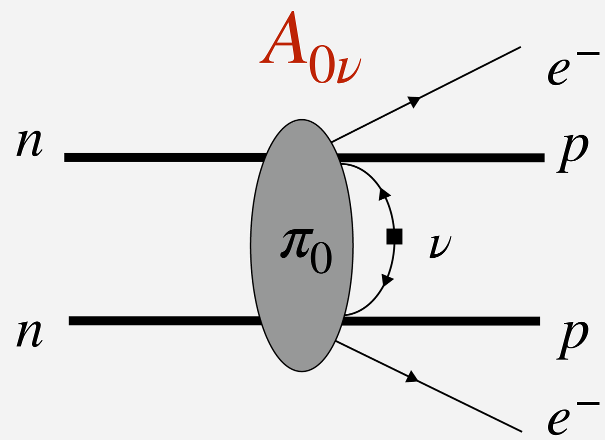
1. **Covariant Density Functional Theory**
   a minimum of the energy surface
2. **Configuration space**
   multi-quasiparticle states
3. **Angular momentum projection**
   rotational symmetry restoration
4. **Shell model calculation**
   configuration mixing / interaction from CDFT



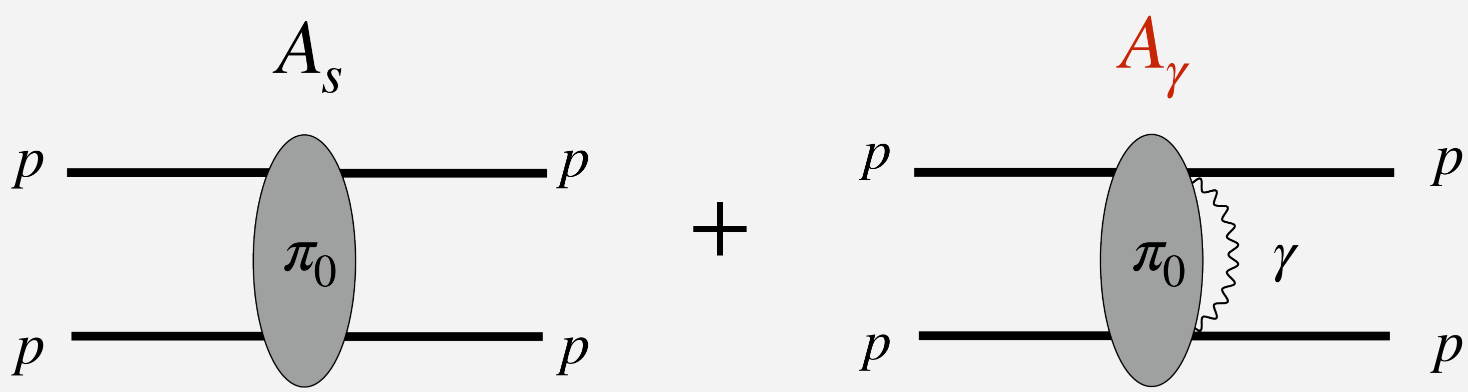
PWZ, Ring, Meng, PRC 94 (2016) 041301(R)
Wang, PWZ, Meng, arXiv:2304.12009

Neutrino and photon exchange

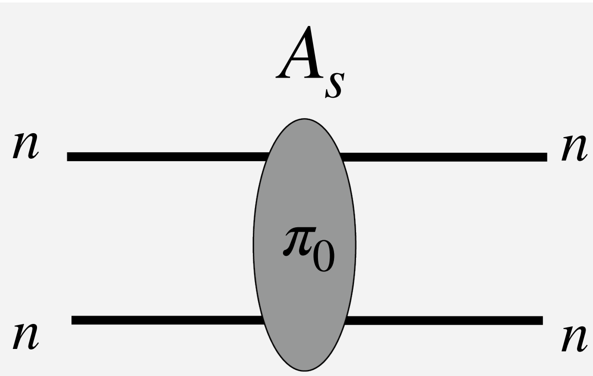
$nn \rightarrow ppee$



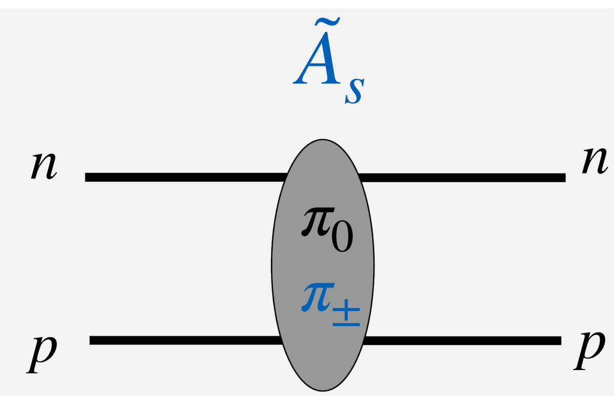
$pp \rightarrow pp$



$nn \rightarrow nn$



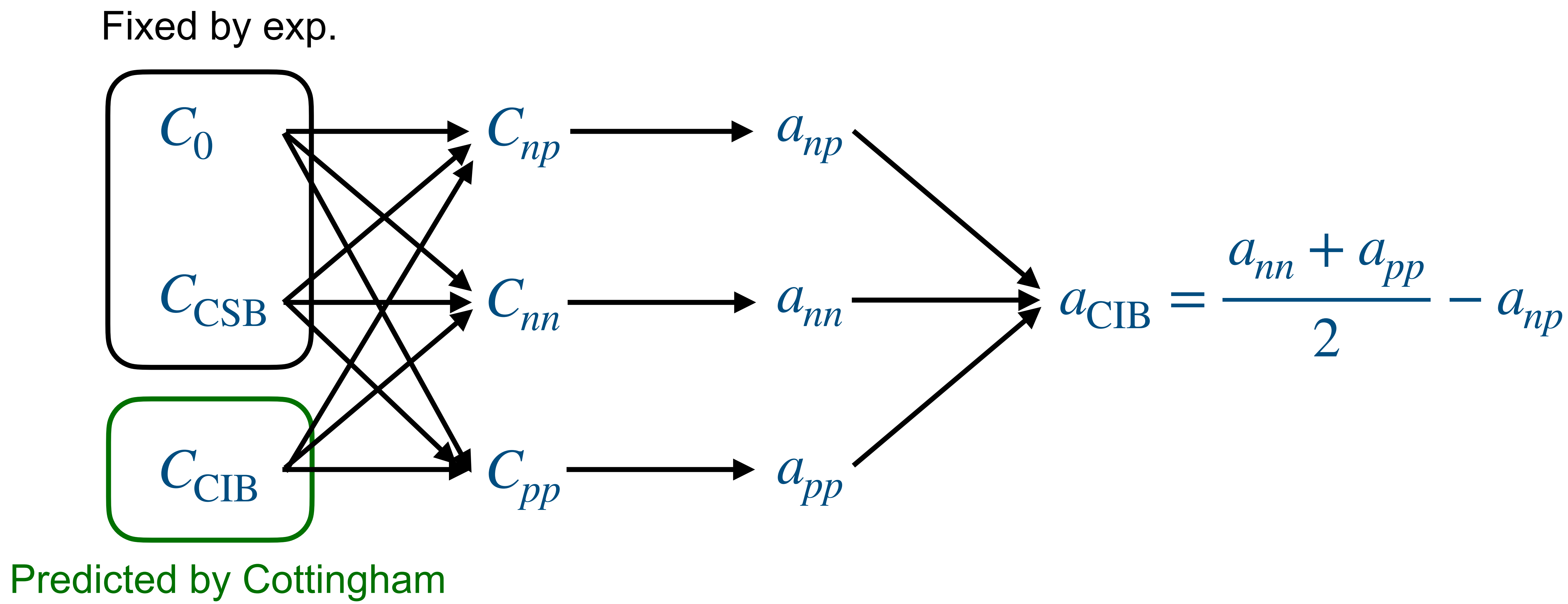
$np \rightarrow np$



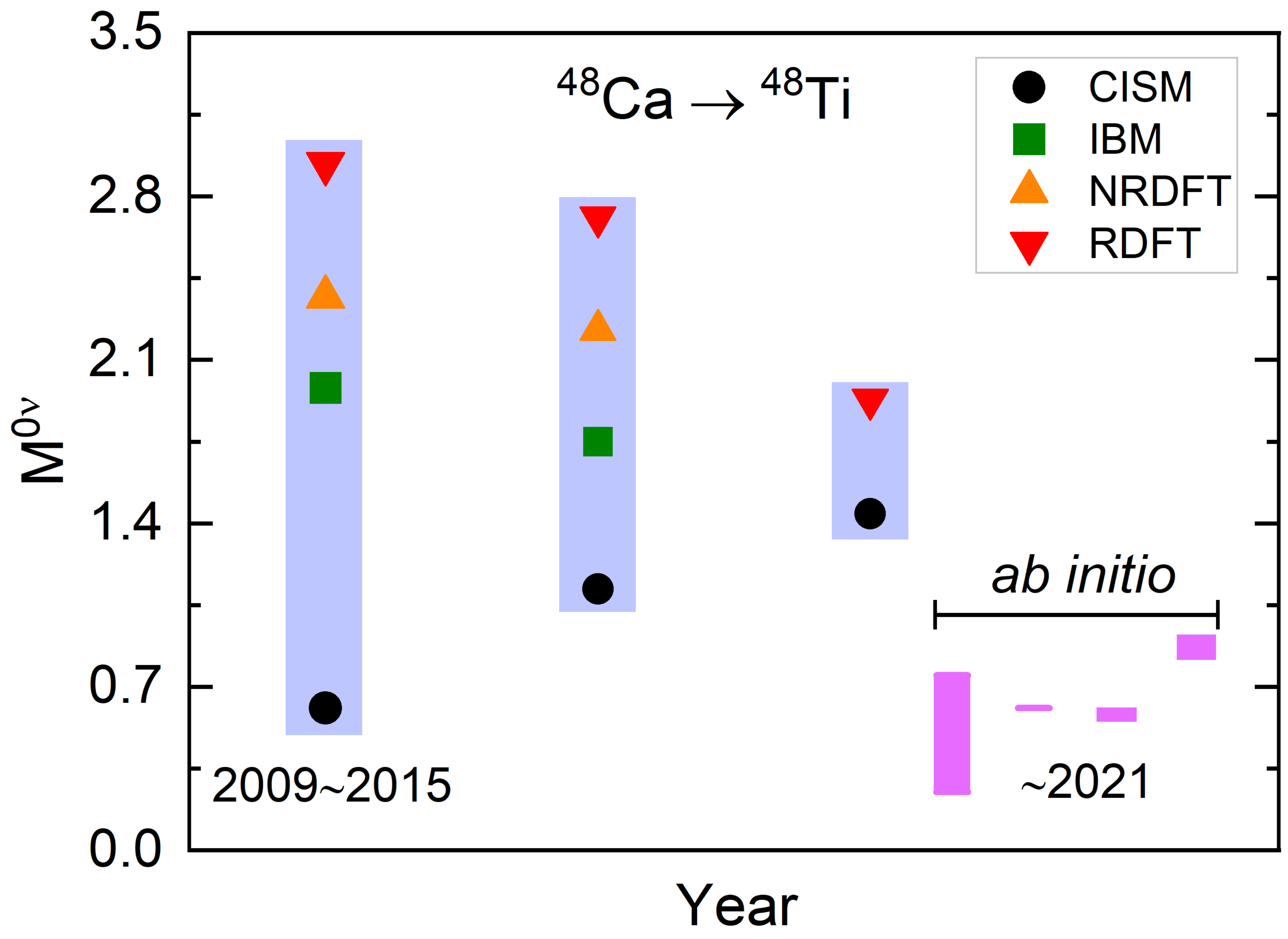
CSB: $A_{nn} - A_{pp} \rightarrow -A_\gamma$

CIB: $\frac{1}{2}(A_{nn} + A_{pp}) - A_{np} \rightarrow \frac{1}{2}A_\gamma + A_s - \tilde{A}_s$

Scattering length



To predict a_{CIB}, all the three contact terms in nn, np, pp must be known. The Cottingham model provides the value of one contact term and the other two are fixed by data. Here, the goal is to test the prediction of the Cottingham model for contact terms. In practice, all the three contact terms can be determined by the data.


3.5
2.8
2.1
1.4
0.7
0.0
$M^{0\nu}$
$^{48}Ca \rightarrow {}^{48}Ti$
CISM
IBM
NRDFT
RDFT
ab initio
2009~2015
~2021
Year