# Energy and lnA reconstruction method based on superposition model

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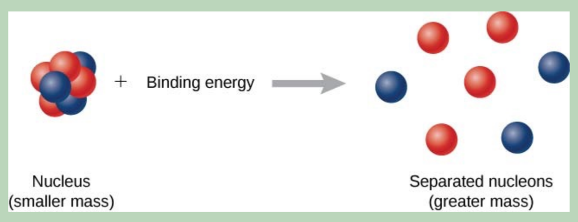
**Abstract:** 1. A novel method for reconstructing energy and logarithm mass (lnA) based on a superposition model is introduced. Energy and lnA are reconstructed using two universal, composition- and energy-independent calibration lines. For zenith angle below 40 degree, the energy and lnA biases are within $\pm$5% and $\pm$0.3, respectively, across all compositions.
2. The method uses particle densities—measured by LHAASO's electromagnetic and muon detectors at a fixed distance from the shower axis—rather than integrated particle counts in annular bands. The density-based approach improves resolution for both energy and lnA, especially for heavy nuclei.
3. The hadronic model dependencies of energy and lnA are also reported. These dependencies scale with lg(E/A) and are nearly independent of primary composition.

## 1. Energy and lnA reconstruction based on superposition model

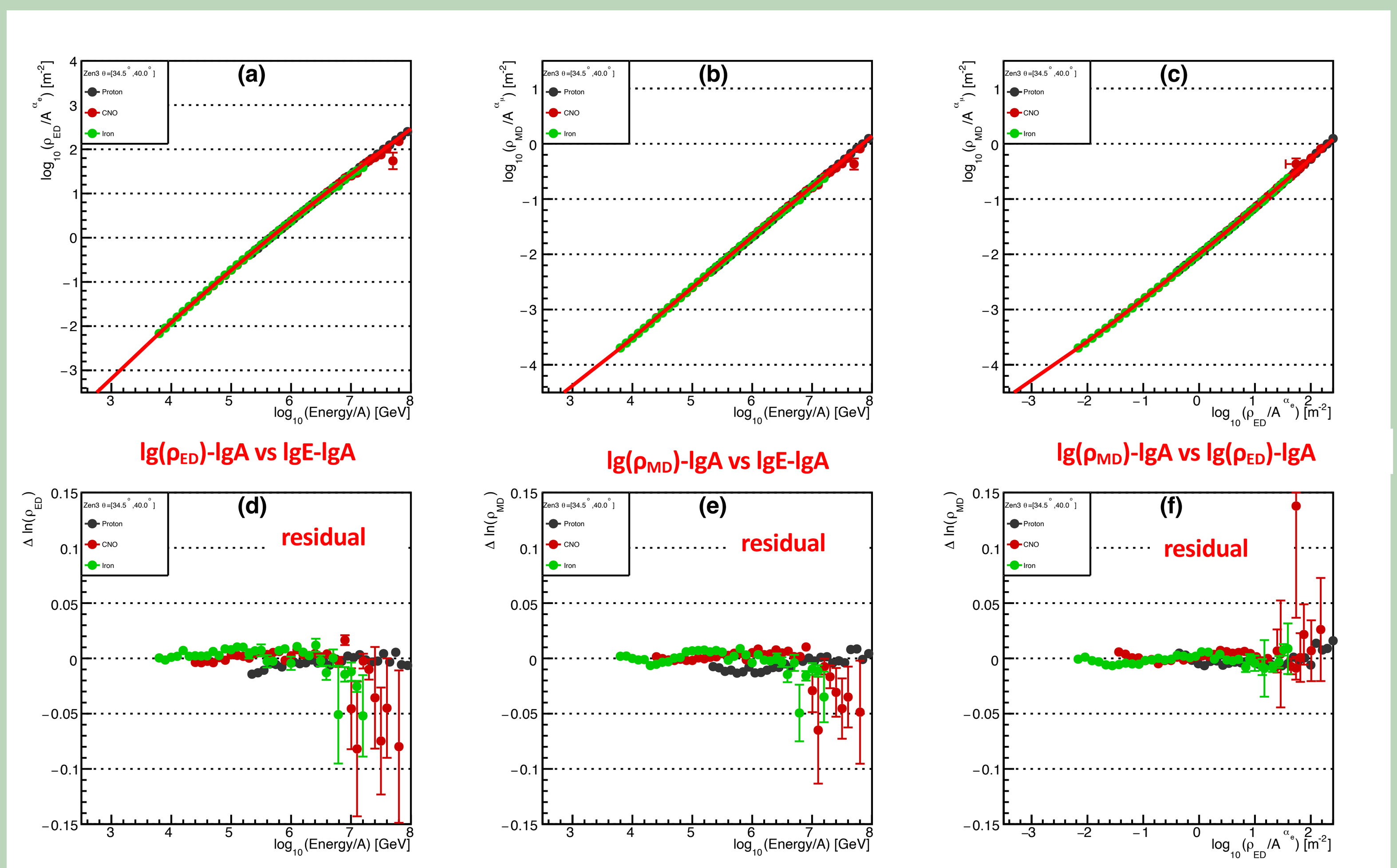
### 1.1 Super position model:
atomic nucleus (A,E) = A protons, each with an energy of E/A when transferred energy >> binding energy



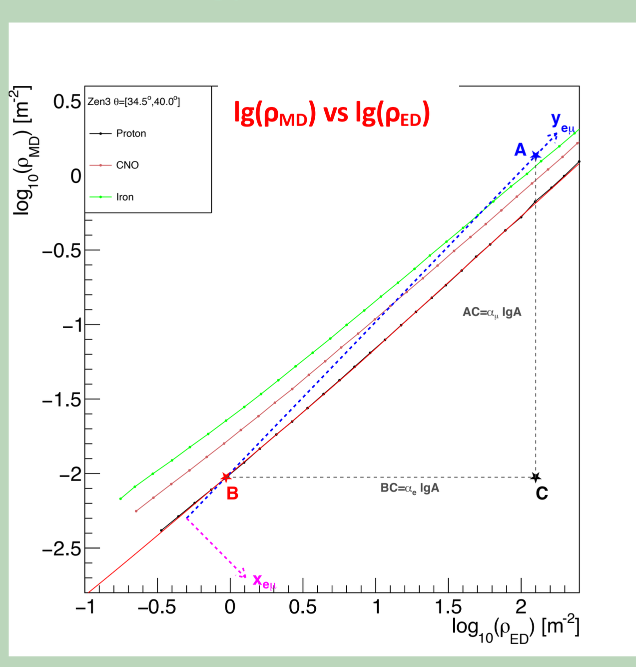

$$N_{e/\mu}^{(A,E)} = A N_{e/\mu}^{(P,E/A)}$$

$$lg(N_{e/\mu}^{(A,E)}/A) \quad vs \quad lg(E/A) = lg(N_{e/\mu}^{(P,E/A)}/1) \quad vs \quad lg(E/A)$$

MC simulation verification using density@100m from ED ($\rho_{ED}$) and density@150m from MD($\rho_{MD}$):




### 1.2 lnA and energy reconstruction based on $\rho_{ED}$ and $\rho_{MD}$ :




Define $x_{e\mu}$ & $y_{e\mu}$ such that $y_{e\mu}^A - y_{e\mu}^B = lgA$

$\alpha_e$ & $\alpha_\mu$ are constants very close to 1.

$x_0,y_0$ are the origin of $x_{e\mu}$ & $y_{e\mu}$ frame :

$$y_{e\mu} = \frac{\alpha_e}{\alpha_e^2+\alpha_\mu^2}(lg(\rho_{ED})-x_0) + \frac{\alpha_\mu}{\alpha_e^2+\alpha_\mu^2}(lg(\rho_{MD})-y_0)$$

$$x_{e\mu} = \frac{\alpha_\mu}{\alpha_e^2+\alpha_\mu^2}(lg(\rho_{ED})-x_0) - \frac{\alpha_e}{\alpha_e^2+\alpha_\mu^2}(lg(\rho_{MD})-y_0)$$

Then:

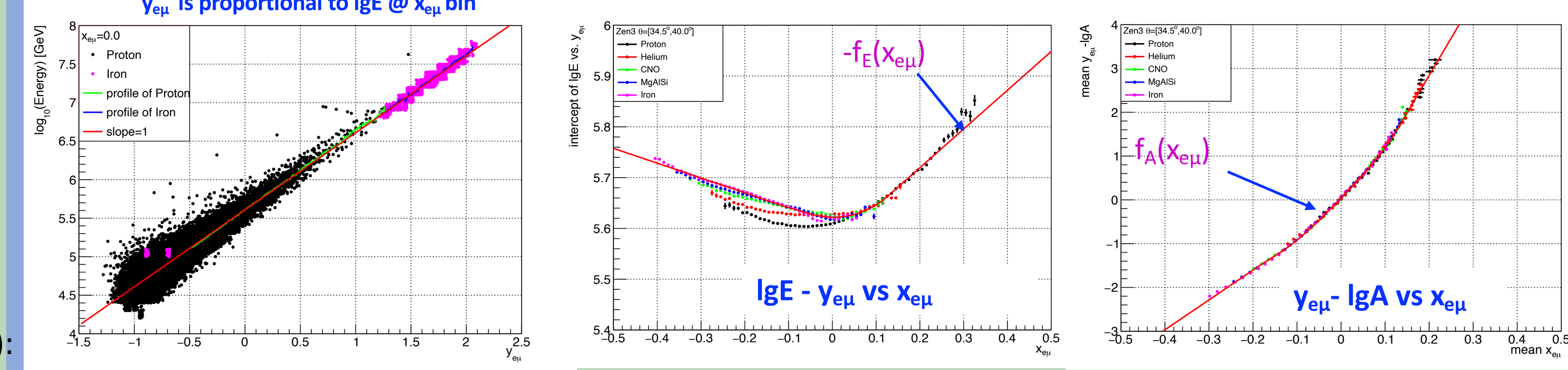
$$y_{e\mu} - lgA = f_A(x_{e\mu})$$

$$y_{e\mu} - lgE = y_{e\mu} - lgA - lg(E/A) = f_E(x_{e\mu})$$

function $f_A$&$f_E$ are the only systematics for E and lnA reconstruction. They are derived from MC.

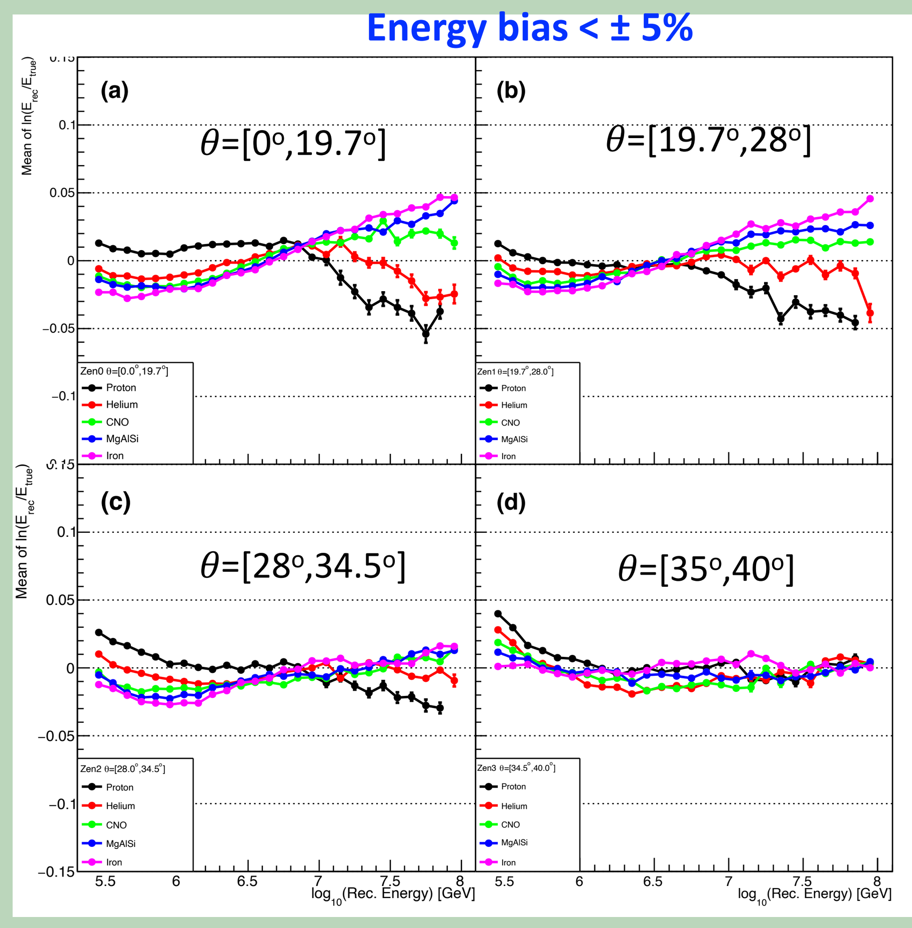
Shifting the data point A in x-axis (and y-axis) by lgA at a 45° angle to intercept the red curve at B. |AB| is ∝ lgA.
In the meanwhile, E/A is constant along $y_{e\mu}$ axis, |AB| is also ∝ lgE.

### 1.3 Function $f_A$&$f_E$ derived from MC data

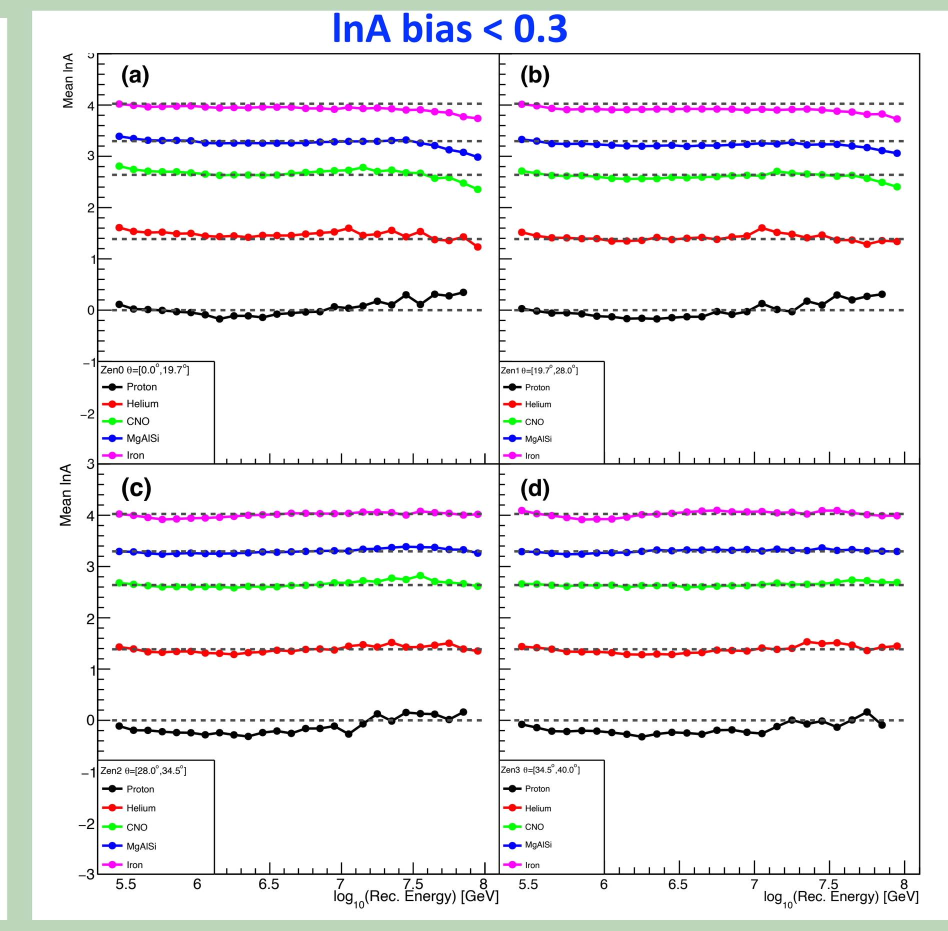



### 1.4 Result (bias and resolution):

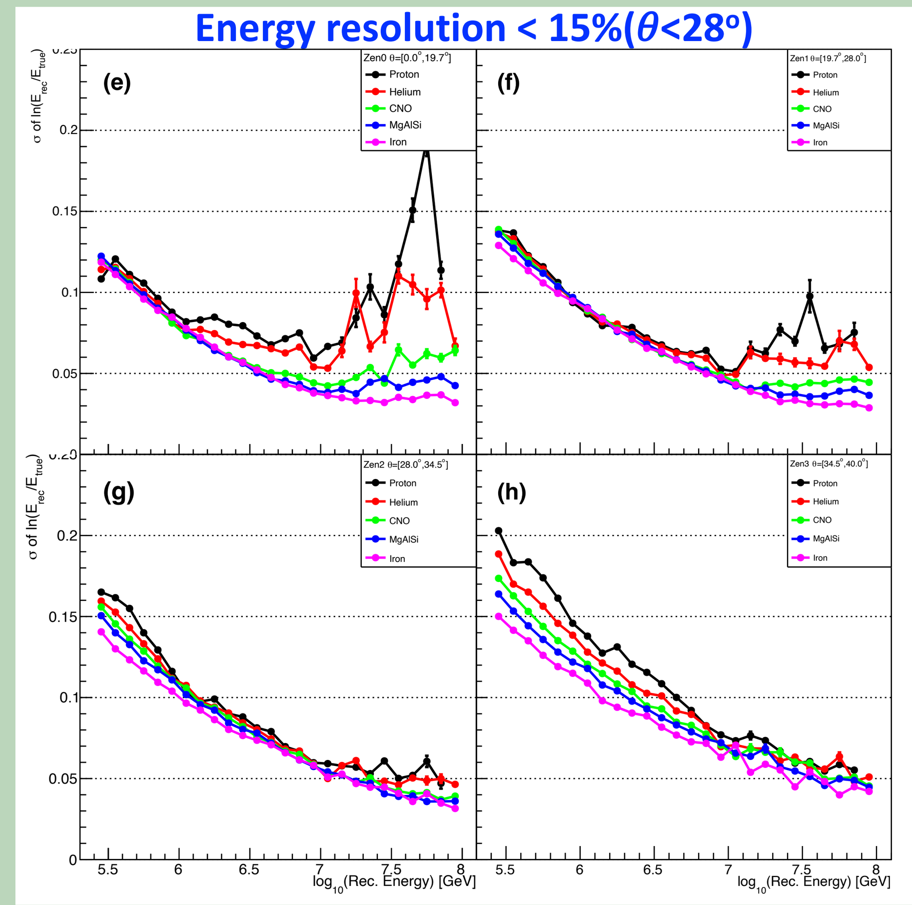
**Energy bias < ± 5%**



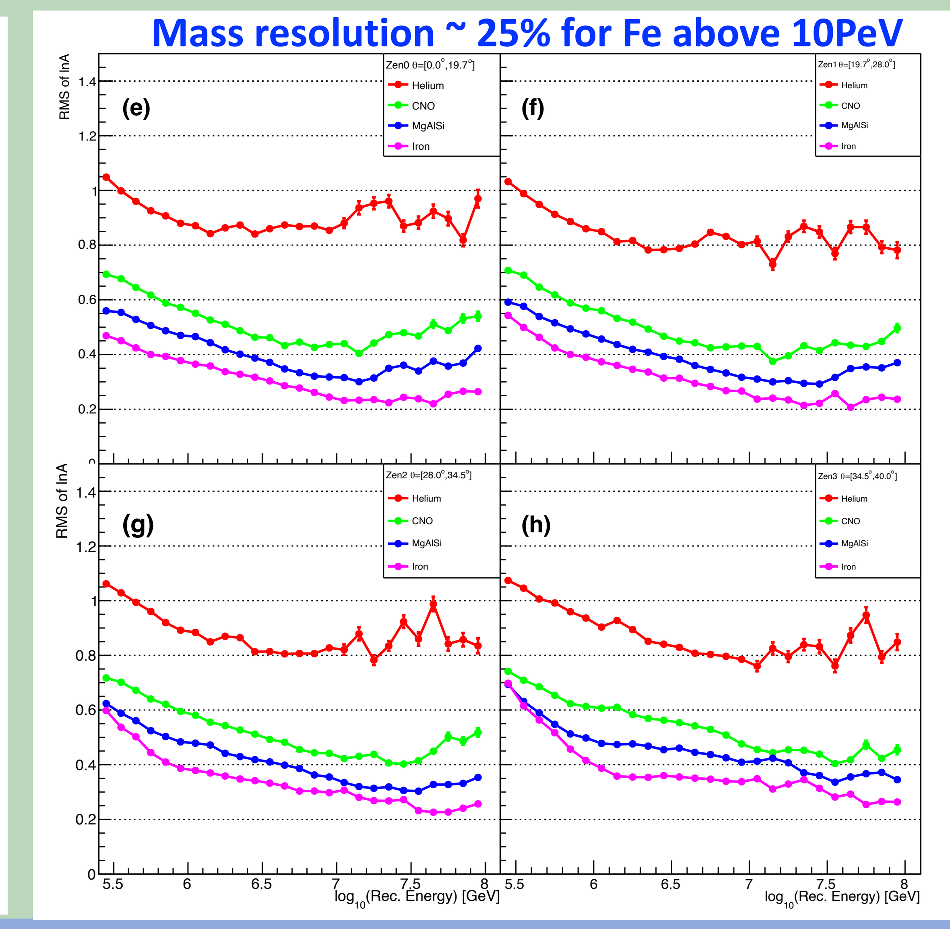

**lnA bias < 0.3**



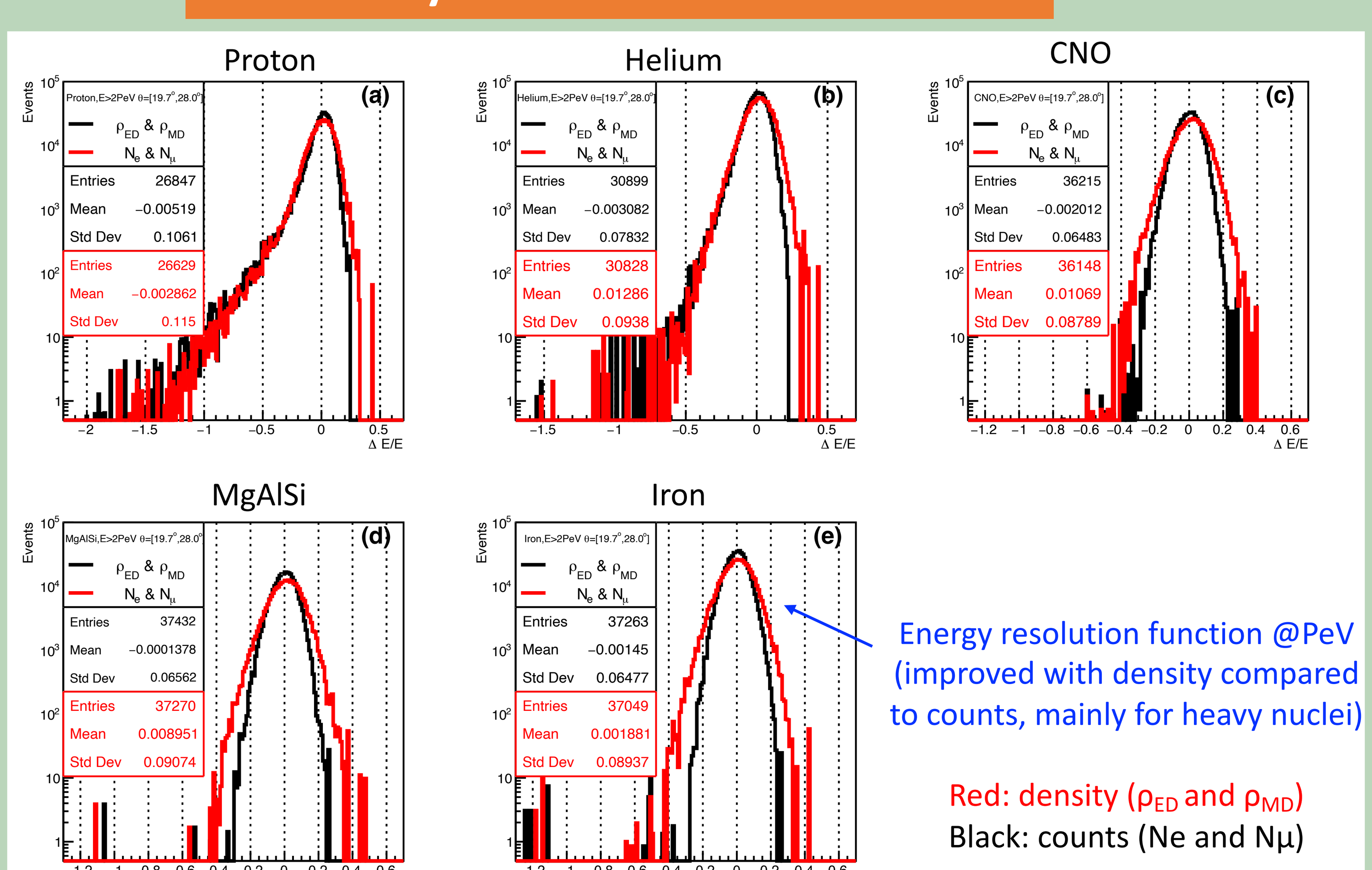
**Energy resolution < 15%($\theta$<28°)**



**Mass resolution ~ 25% for Fe above 10PeV**

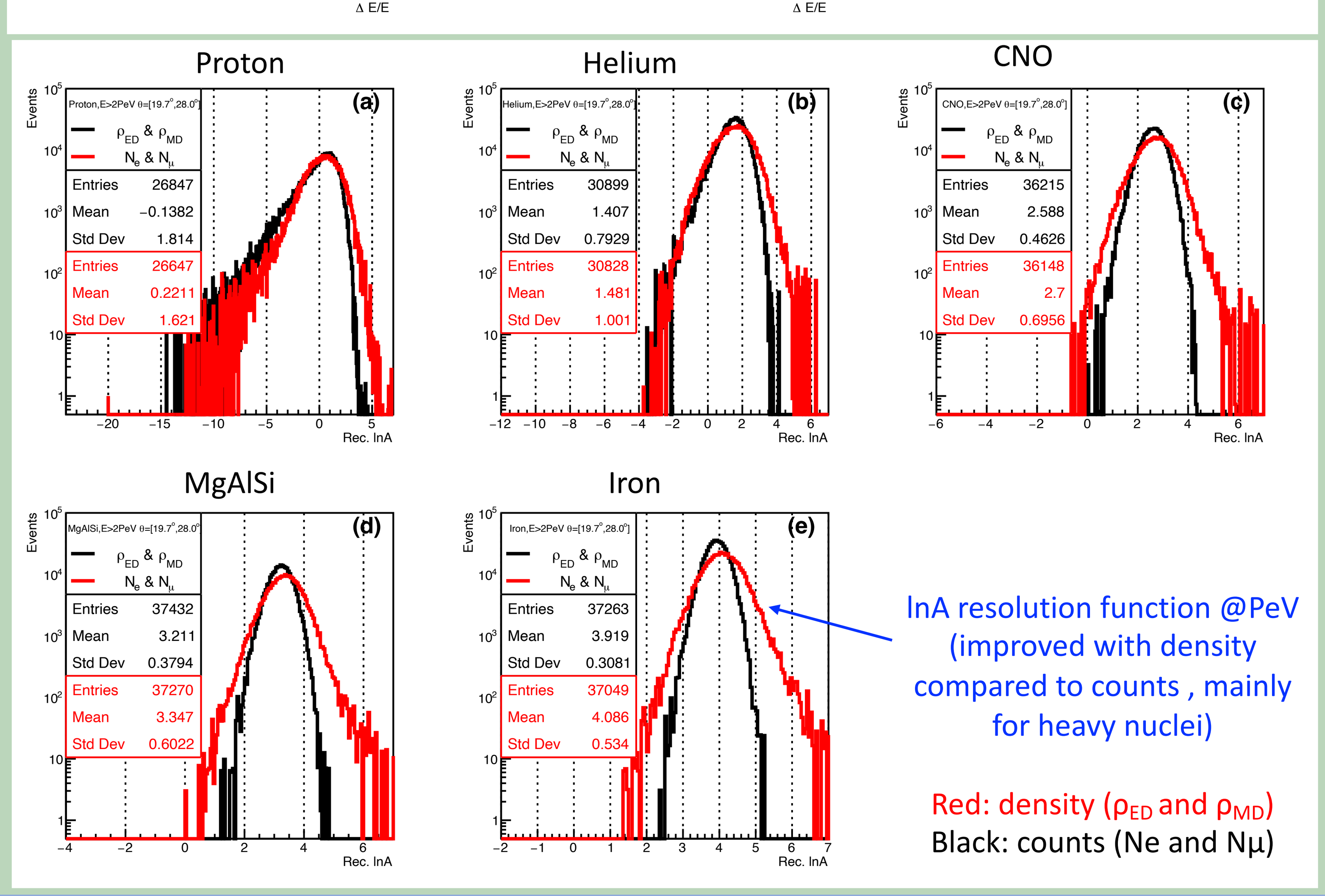


## 2. Density variable vs Count variable




Energy resolution function @PeV (improved with density compared to counts, mainly for heavy nuclei)

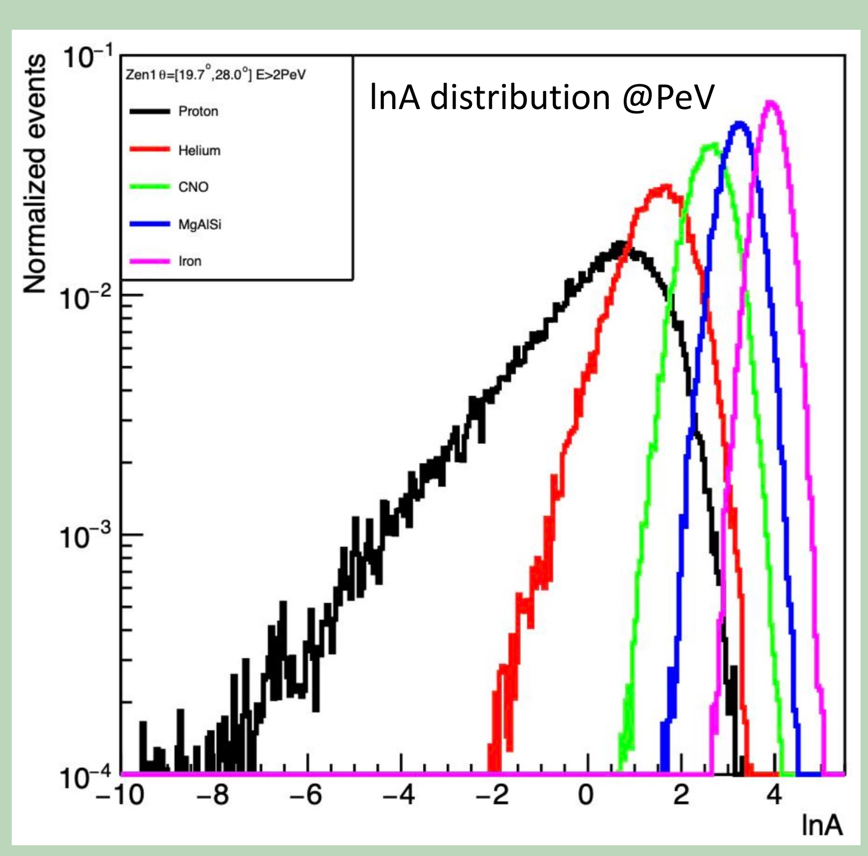
Red: density ($\rho_{ED}$ and $\rho_{MD}$)
Black: counts (Ne and Nμ)



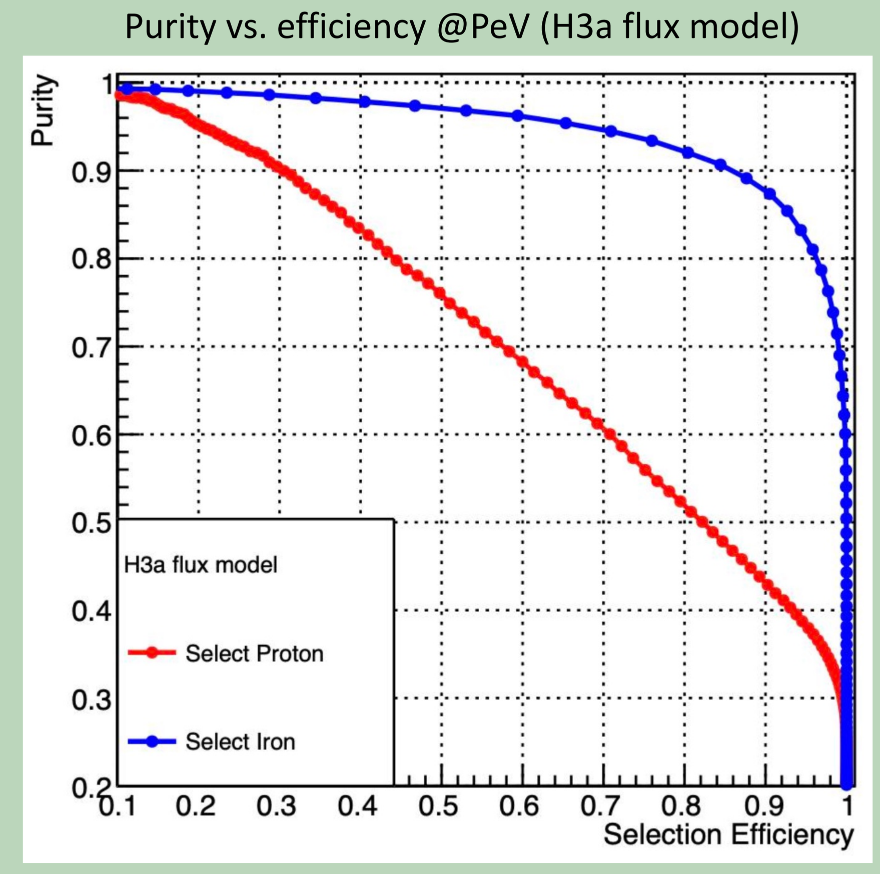

lnA resolution function @PeV (improved with density compared to counts , mainly for heavy nuclei)

Red: density ($\rho_{ED}$ and $\rho_{MD}$)
Black: counts (Ne and Nμ)

Discrimination capability @ PeV:

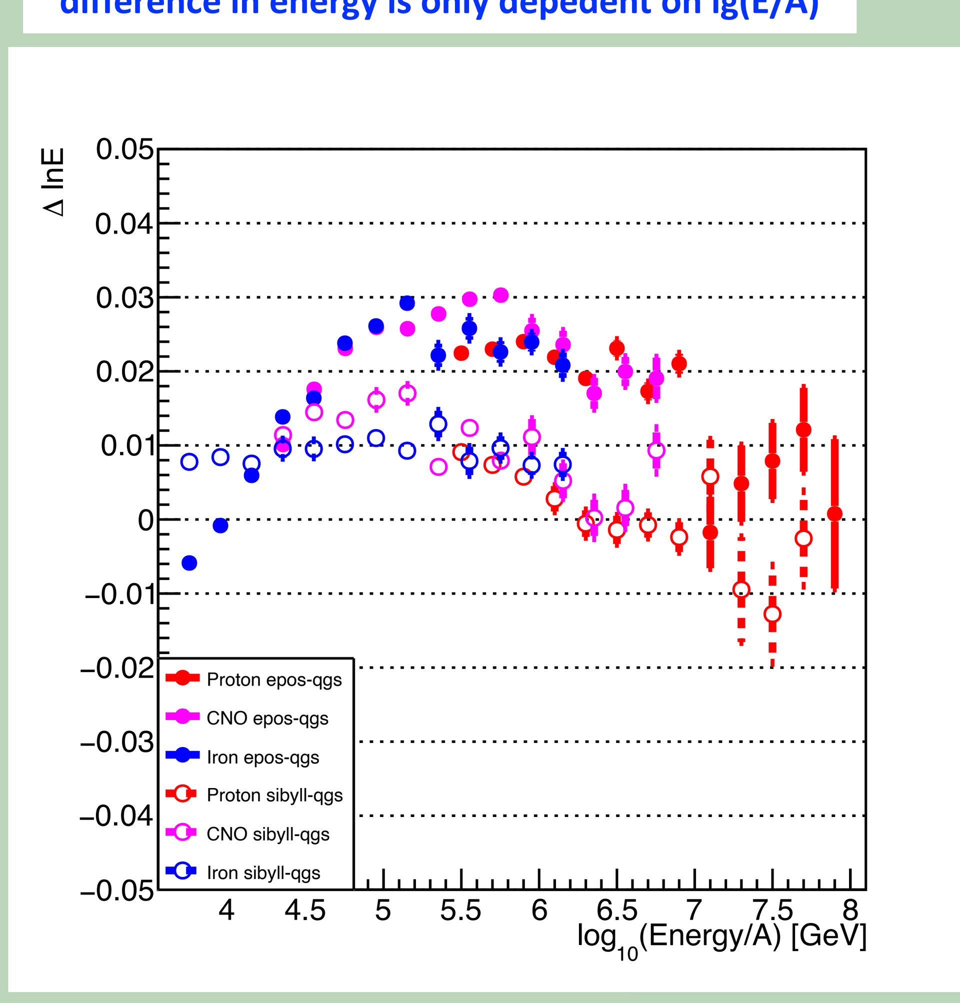



Purity vs. efficiency @PeV (H3a flux model)




## 3. Hadronic model dependence

**ΔlnE vs lg(E/A):**
**difference in energy is only depedent on lg(E/A)**



**ΔlnA vs lg(E/A)**
**difference in lnA is approximately linear with lg(E/A)**