



Factorization and resummation for jets and heavy quark production in SCET

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The first workshop on the physics of small- x at the LHC
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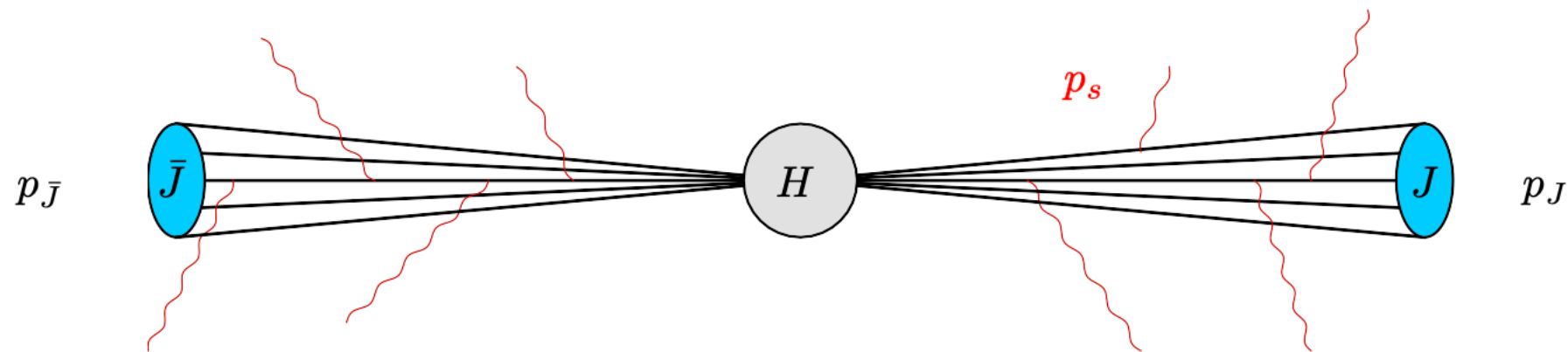
我们主要想了解近期理论方面的一些新进展，特别是有没有可能提出一些新的观测量，使得实验上能够测量 small-x 物理。我们对以下方向都很感兴趣。

- (1) isolated / prompt photons
- (2) Jpsi
- (3) neutral mesons (π^0 , η)
- (4) forward jets / dijets
- (5) correlation observables (γ -hadron, γ -jet, π^0 - π^0 , π^0 -hadron, jet-jet)
- (6) UPC

Soft-Collinear Effective Theory

Bauer, Pirjol, Stewart *et.al.* '01, '02; Beneke, Diehl *et.al.* '02

- Soft-Collinear Effective Theory (SCET) is the effective field theory for processes with energetic particles such as jet production at high-energy colliders.



- Typically such processes involve a scale hierarchy

$$Q^2 = (p_J + p_{\bar{J}})^2 \gg p_J^2 \sim p_{\bar{J}}^2 \gg p_s^2$$

- In SCET, the physics associated with the hard scale Q^2 is integrated out and absorbed into Wilson coefficients.
- SCET involves two different types of fields, collinear and soft fields to describe the physics associated with the two low-energy scales p_J^2 and p_s^2 .

Soft-Collinear Effective Theory

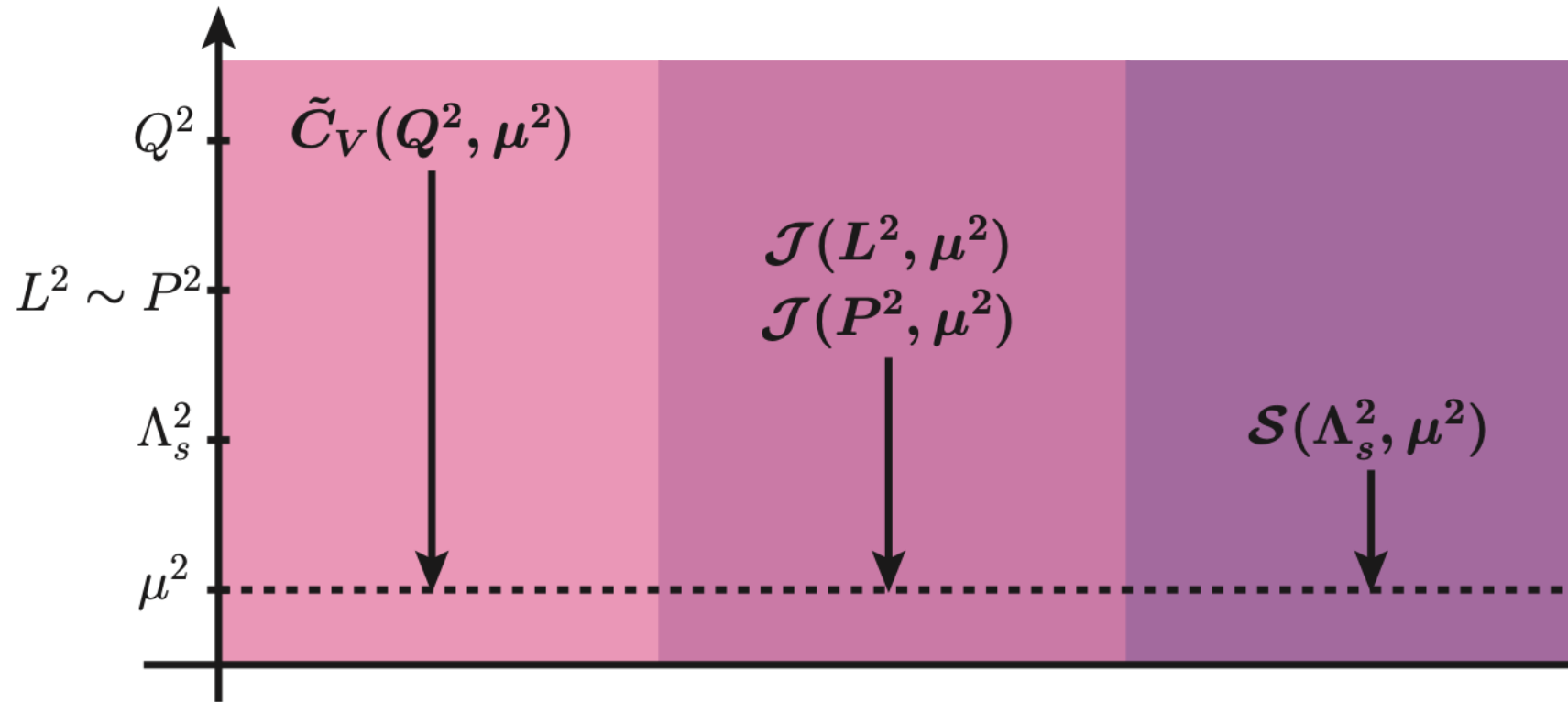
- The result of a SCET analysis of a jet cross section is often a factorization theorem

$$\sigma = H \cdot J \otimes \bar{J} \otimes S \qquad Q^2 = (p_J + p_{\bar{J}})^2 \gg p_J^2 \sim p_{\bar{J}}^2 \gg p_s^2$$

- Hard function; Jet function; Soft function
- The theorem is obtained after expanding in the ratios of the scales and holds at leading power.
- Each of the functions is only sensitive to a single scale.
- The individual functions furthermore fulfill renormalization group (RG) equations. By solving RG equations one can resum the large perturbative logarithms $\alpha^n \ln^m(Q^2/p_J^2)$

Resummation by RG

- Schematic representation of the scale separation and of the calculational procedure in renormalization group improved perturbation theory.



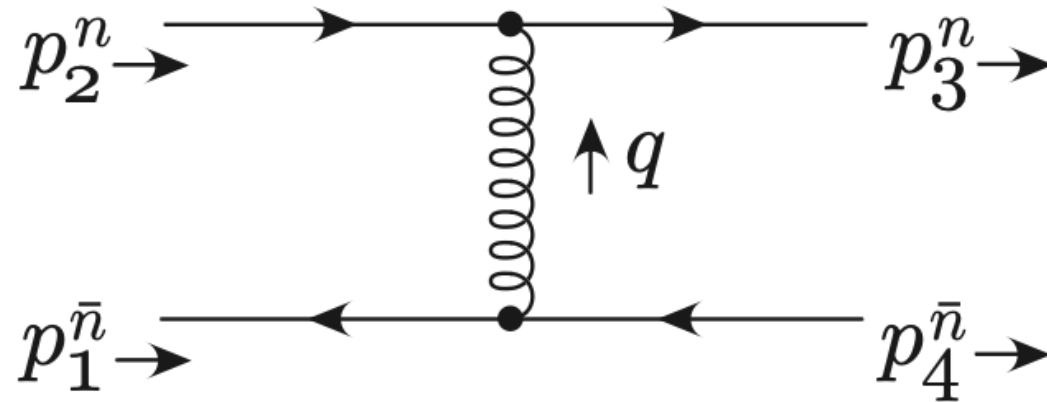
$$\frac{d}{d \ln \mu} \tilde{C}_V(Q^2, \mu) = \left[C_F \gamma_{\text{cusp}}(\alpha_s) \ln \frac{Q^2}{\mu^2} + \gamma_V(\alpha_s) \right] \tilde{C}_V(Q^2, \mu)$$

$$\frac{d}{d \ln \mu} \mathcal{J}(L^2, \mu^2) = - \left[C_F \gamma_{\text{cusp}}(\alpha_s) \ln \frac{L^2}{\mu^2} + \gamma_J(\alpha_s) \right] \mathcal{J}(L^2, \mu^2) ,$$

$$\frac{d}{d \ln \mu} \mathcal{S}(\Lambda_s^2, \mu^2) = \left[C_F \gamma_{\text{cusp}}(\alpha_s) \ln \frac{\Lambda_s^2}{\mu^2} + \gamma_S(\alpha_s) \right] \mathcal{S}(\Lambda_s^2, \mu^2) ;$$

Small x factorization in SCET

- SCET with Glauber operator Rothstein, Stewart '16



$$p_{\bar{n}} \sim \sqrt{s} \left(\underbrace{1}_{p^+}, \underbrace{\lambda^2}_{p^-}, \underbrace{\lambda}_{p_\perp} \right)$$

Glauber Modes

$$q_G^\mu = q'^\mu \sim \sqrt{s} (\lambda^2, \lambda^2, \lambda)$$

q_G

$$p_n \sim \sqrt{s} \left(\underbrace{\lambda^2}_{p^+}, \underbrace{1}_{p^-}, \underbrace{\lambda}_{p_\perp} \right)$$

$$\mathcal{L}_G = \sum_{n_i, n_j} \mathcal{O}_{n_i} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{n_i} + \sum_{n_i} \mathcal{O}_{n_i} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{n_i}$$

$$S_G^{(n_i s)} = 8\pi\alpha_s \sum_{ij} \int d^4x \int d^4z \int \frac{d^4q}{(2\pi)^4} \frac{e^{iq \cdot (x-z)}}{\mathbf{q}_\perp^2} \mathcal{O}_{n_i}^{iA}(x) \mathcal{O}_s^{j n_i A}(z)$$

$$\begin{array}{c} \bullet \\ \vdots \\ \bullet \end{array} \begin{array}{l} \mathcal{O}_s^{i n A} \\ \mathcal{O}_n^{i A} \end{array}$$

$$\mathcal{O}_s^{n_i, qA} = \bar{\psi}_S^{n_i} \mathbf{T}_i^A \frac{\not{n}}{2} \psi_S^{n_i}, \quad \mathcal{O}_s^{n_i, gA} = \frac{1}{2} \mathcal{B}_{S\perp\mu}^{n_i B} (if^{ABC}) \frac{n_i}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{n_i C\mu},$$

$$\mathcal{O}_{n_i}^{qA} = \bar{\chi}_{n_i} \mathbf{T}_i^A \frac{\not{n}}{2} \chi_{n_i}, \quad \mathcal{O}_{n_i}^{gA} = \frac{1}{2} \mathcal{B}_{n\perp\mu}^B (if^{ABC}) \frac{\bar{n}_i}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{C\mu},$$

Small x factorization in SCET

- SCET factorization with Glauber operator Neill, Pathak, Stewart '23

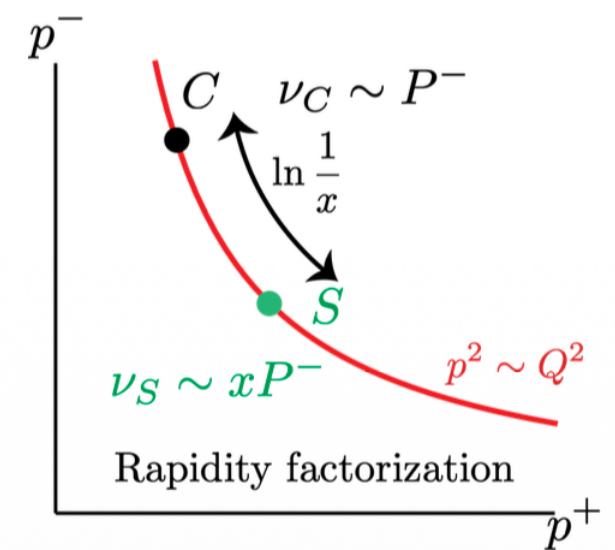
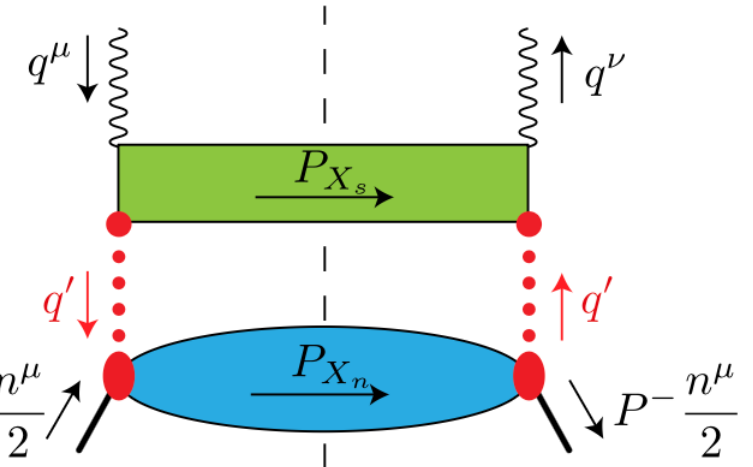
$$S_G^{ns} = 8\pi\alpha_s \sum_{i,j,A} \int d^d y \int d^d x \int \frac{d^d q'}{(2\pi)^d} \frac{e^{i(x-y)\cdot q'}}{\mathbf{q}'_{\perp}{}^2} \mathcal{O}_n^{iA}(x) \mathcal{O}_s^{j_n A}(y).$$

Factorization formula

$$W^{\alpha\beta}(q, P) = \int d^{d-2} q'_{\perp} S^{\alpha\beta}\left(q, q'_{\perp}, \frac{\nu}{x_b P^-}, \epsilon\right) C\left(q'_{\perp}, P, \frac{\nu}{P^-}, \epsilon\right) + \dots$$

Here small- x_b logs are resummed via *rapidity evolution* for $\nu_S \sim x_b P^-$ and $\nu_C \sim P^-$

$$\frac{\nu_S}{\nu_C} = x_b$$



Small x Factorization = Rapidity Factorization

Small x factorization in SCET

Comparison with previous work

The Glauber-SCET approach is special: it avoids many problems in the k_T -factorization framework.

Feature	k_T -Factorization	Glauber-SCET
<i>Gauge Invariance</i>	off-shell cross sections: gauge inv. only at LO	Constructed from gauge invariant Glauber ops.
<i>Power Counting</i>	Obscured due to attempting simultaneous twist and high-energy factorization	Manifest power counting from the start
<i>Objects</i>	Requires auxiliary Green's functions	Nothing beyond collinear and soft functions
<i>Isolating $1/\epsilon_{\text{IR}}$</i>	Additional non-trivial conversion for $1/\epsilon_{\text{IR}}$ in $\overline{\text{MS}}$	$S^{\alpha\beta}$ and C can be computed with $1/\epsilon_{\text{IR}}$ in $\overline{\text{MS}}$
<i>Handling collinear (non-BFKL) $1/\epsilon_{\text{IR}}$</i>	New non-BFKL $1/\epsilon_{\text{IR}}$ $\rightarrow$ new Green's functions	No mixing of BFKL and collinear $1/\epsilon_{\text{IR}}$ poles.

Slides from Aditya Pathak

Small x factorization in SCET

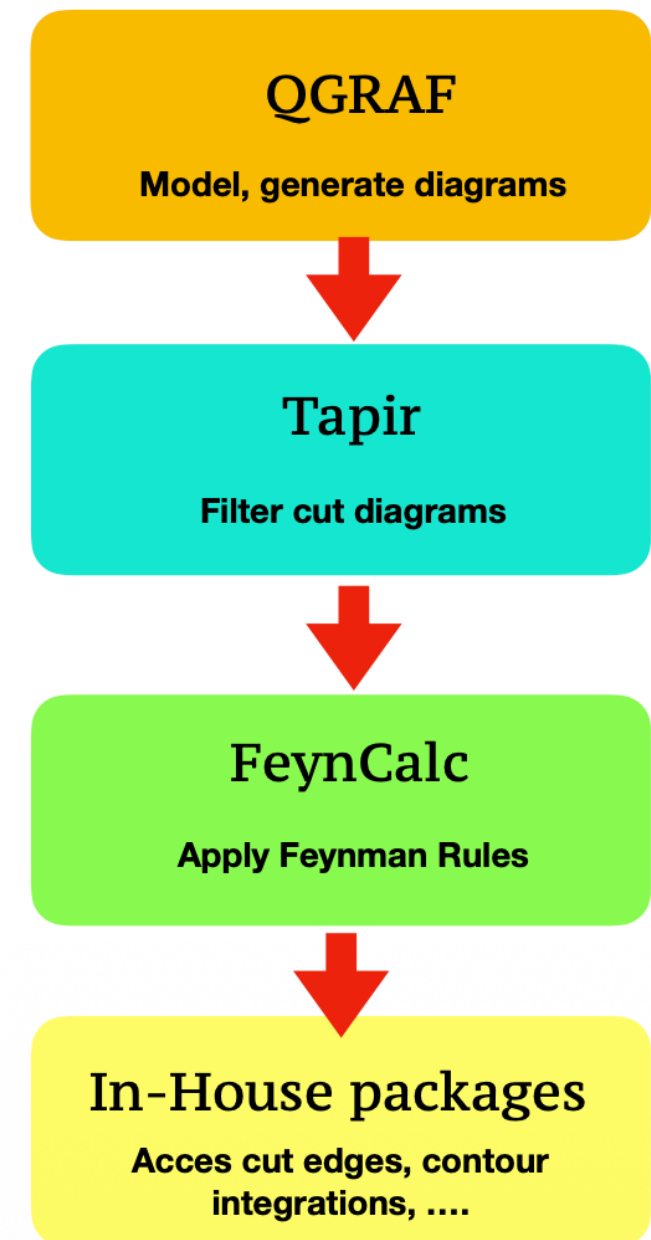
Need for automation

At two loops computing by hand becomes impractical. However, standard tools for Feynman Diagrams do not straightforwardly apply:

- > SCET Lagrangian is not Lorentz Invariant
- > Presence of Wilson lines leads to vertices with arbitrary valency
- > Nontrivial problem to represent operators: $\chi_n, \mathcal{B}_{n\perp}^\mu, \mathcal{O}_n, \dots$

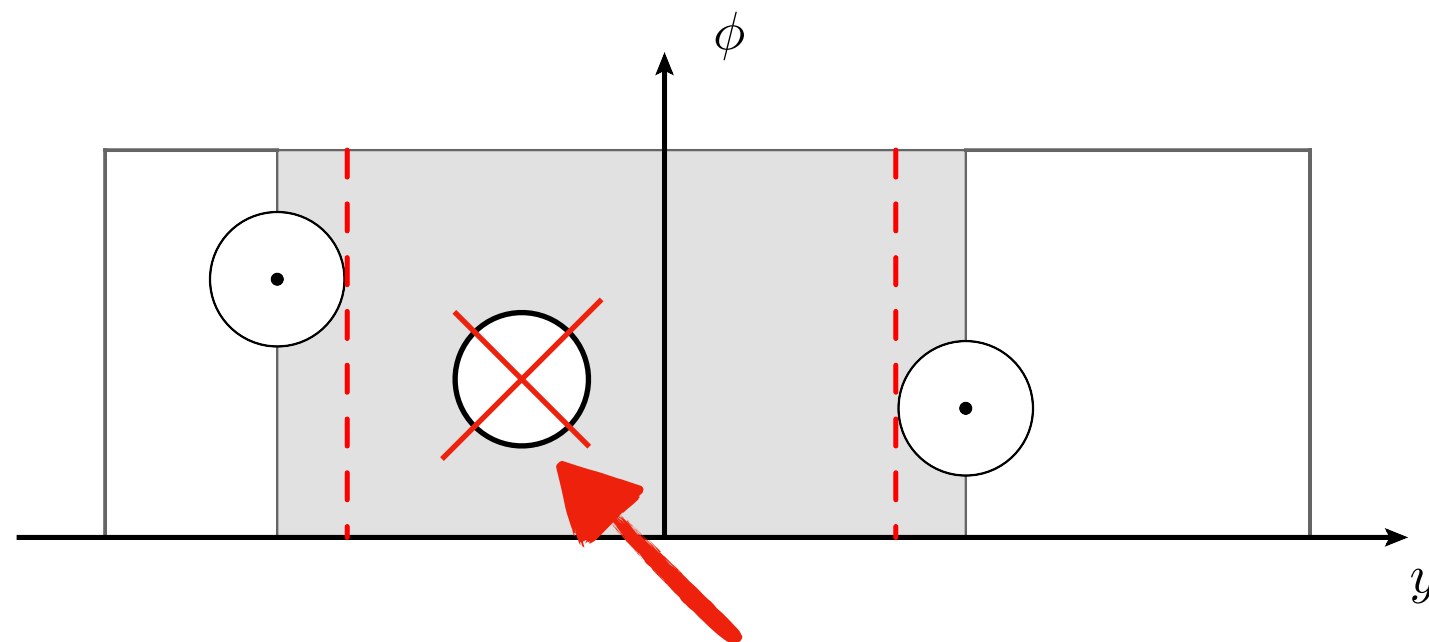
The SCETCalc framework:

- > Being built on FeynCalc [SMO20] development version that supports light-cone vectors.
- > QGRAF [Nog93] to build the SCET “model”
- > Automate all SCET Feynman rules
- > Combine with Tapir [GHL23] for filtering cut diagrams
- > In-house software in Mathematica for accessing and playing with cut edges.



Gap fraction and Glauber modes

Gap fraction at the LHC



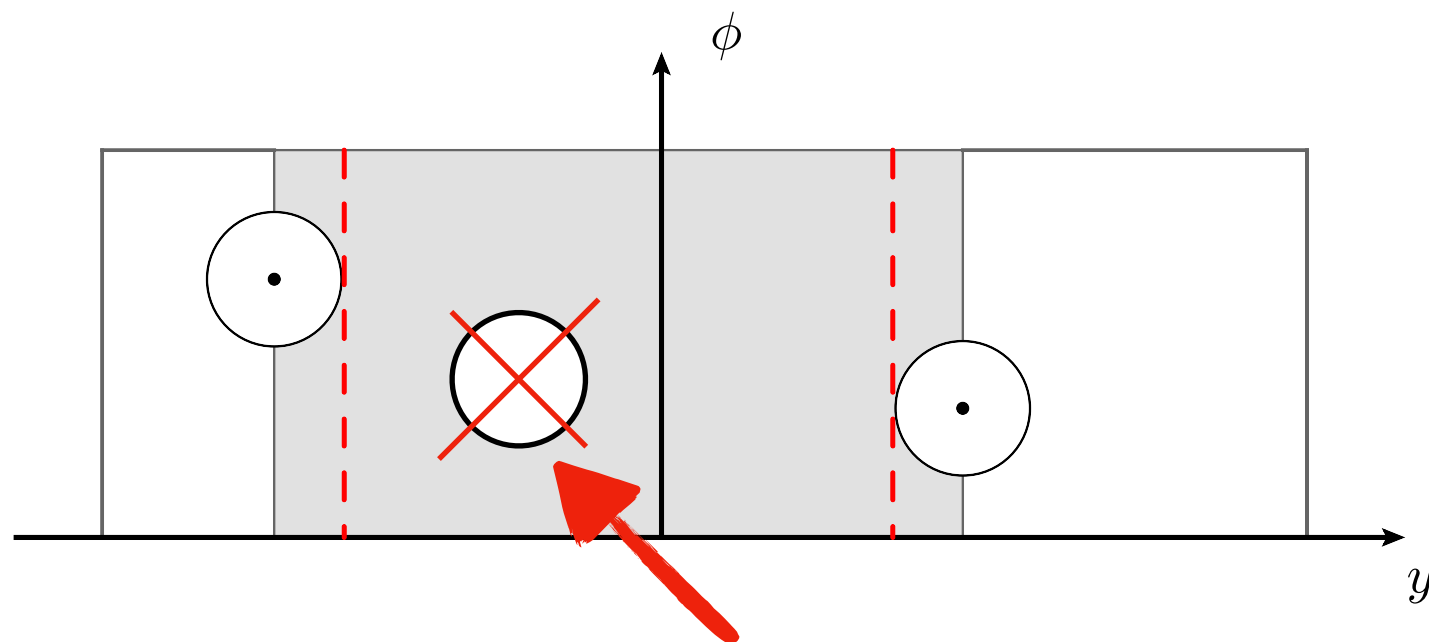
$$R(\bar{p}_T, Q_0) = \frac{\sigma_{2\text{-jet}}(\bar{p}_T, Q_0)}{\sigma_{2\text{-jet}}(\bar{p}_T, Q_0 = \bar{p}_T)}$$

$$\bar{p}_T = \frac{1}{2} (p_{T,\text{jet1}} + p_{T,\text{jet2}})$$

veto events with $p_{T,\text{jet3}} > Q_0$

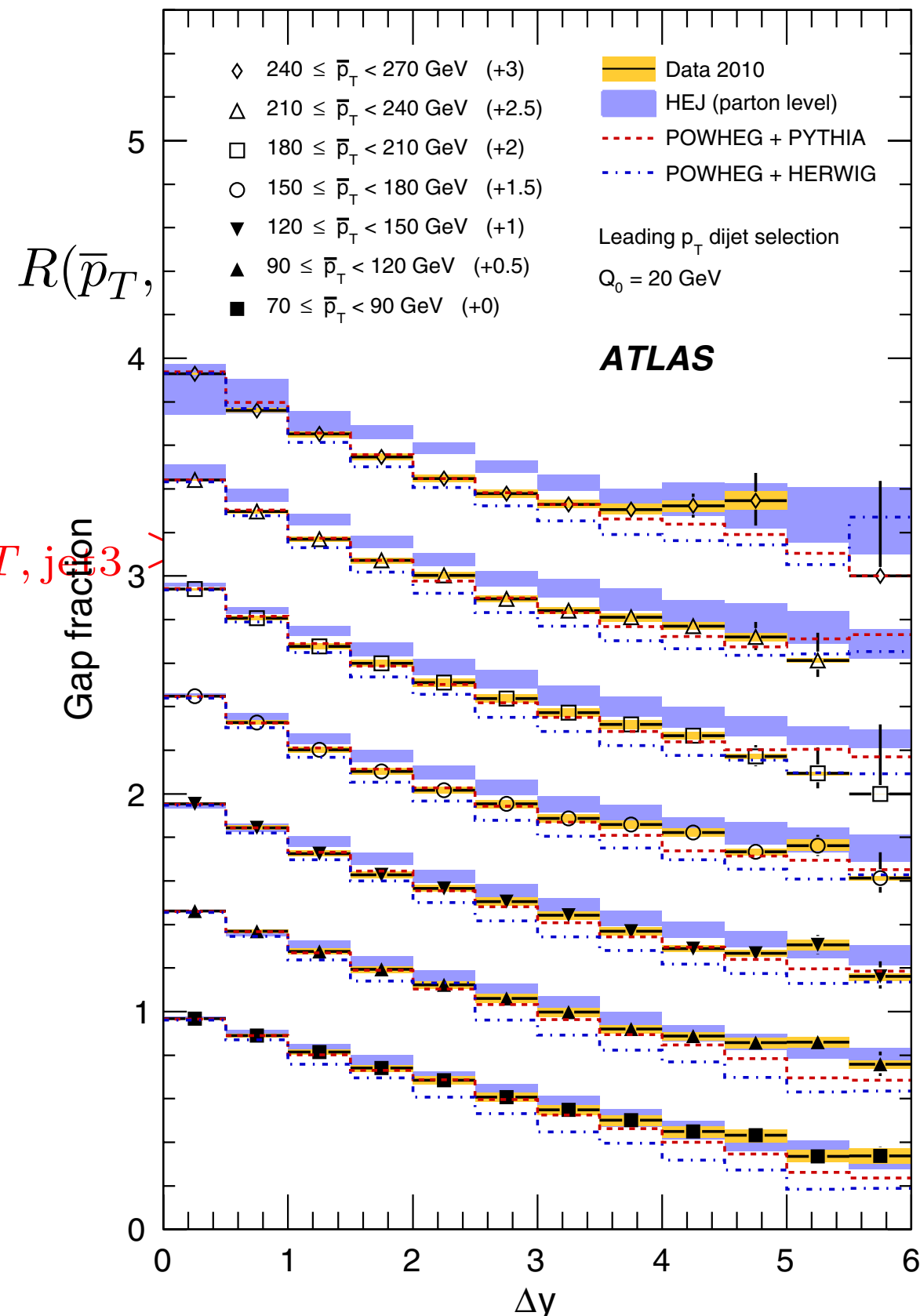
- Gap fraction originally suggested on the basis of color flow considerations in QCD **Bjorken '93**
- BFKL evolution at large gap
- soft wide angle gluon resummation at small Q_0
- **ATLAS '11 '14** observes data and Monte Carlo predictions is not totally satisfactory
 - fixed-order calculations matched with parton shower tend to be below data at large Δy
 - HEJ (BFKL logs) overestimates data at large p_T

Gap fraction at the LHC



veto events with $p_{T, \text{jet } 3}$

- Gap fraction originally suggested on the basis of color flow considerations in QCD Bjorken '93
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Resummation

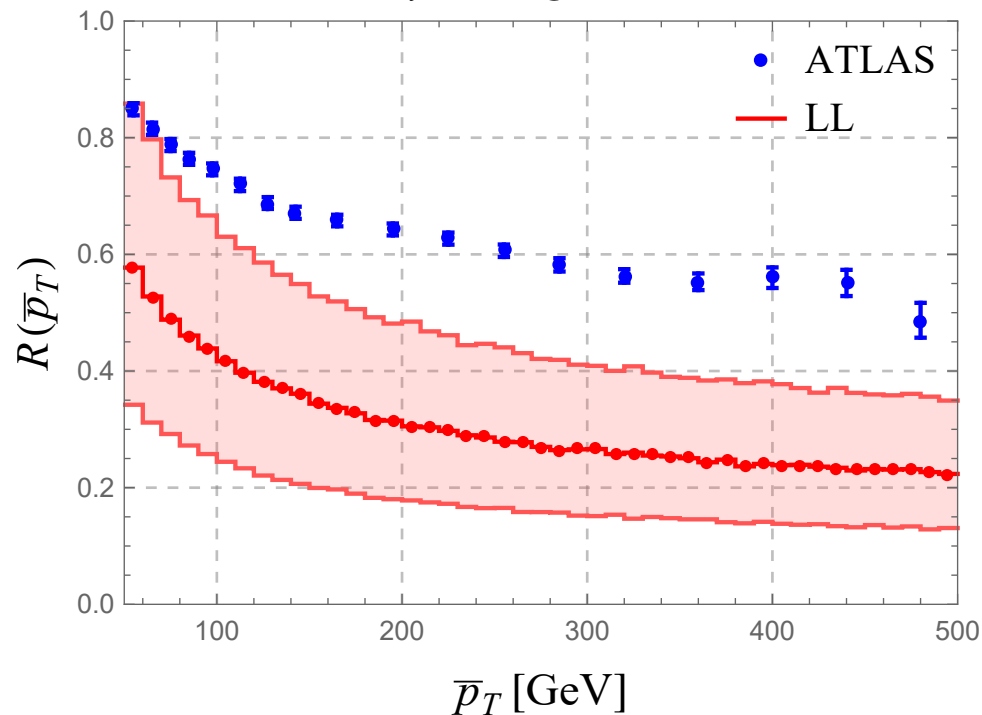
Balsiger, Becher, DYS '18

- Factorization formula to all order is unknown due to glauher gluons
- In LL and large N_c limit

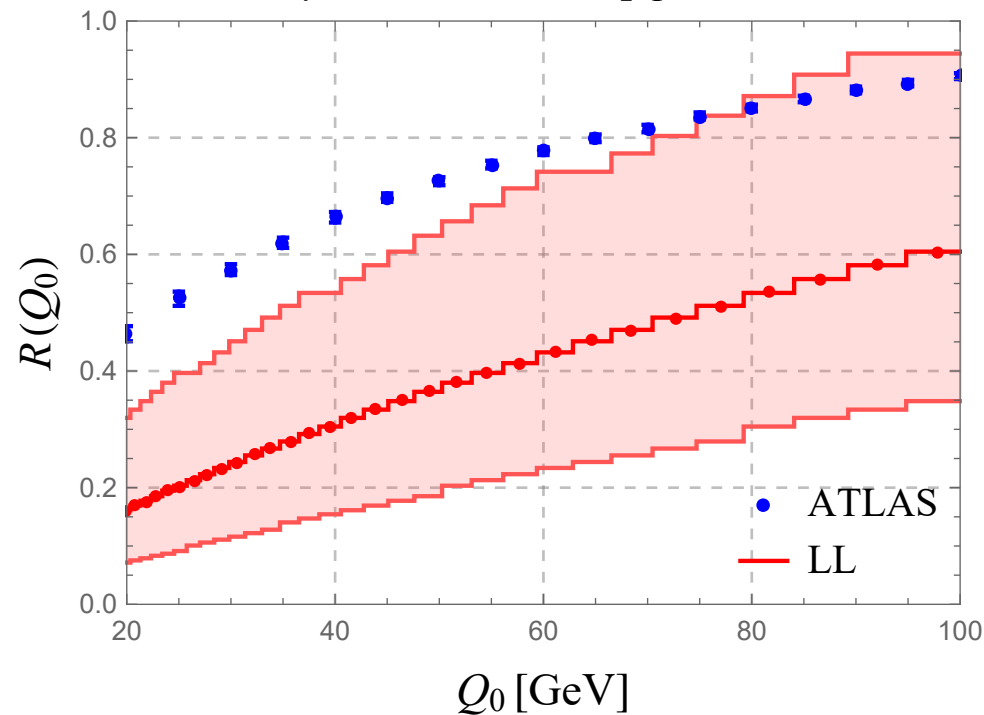
$$\frac{d\sigma(Q_0)}{d\Delta y d\bar{p}_T} = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 f_a(x_1, \mu_f) f_b(x_2, \mu_f) H_2^{ab}(\hat{s}, \Delta y, \bar{p}_T, \mu_h) \langle U_{2m}(\mu_s, \mu_h) \hat{\otimes} 1 \rangle$$

- scale setting $\mu_f = \mu_h = \bar{p}_T$ and $\mu_s = Q_0$
- focus on central jets, small gap & no collinear logs

$1 < \Delta y < 2, Q_0 = 20 \text{ GeV}$



$2 < \Delta y < 3, 210 \text{ GeV} < \bar{p}_T < 240 \text{ GeV}$

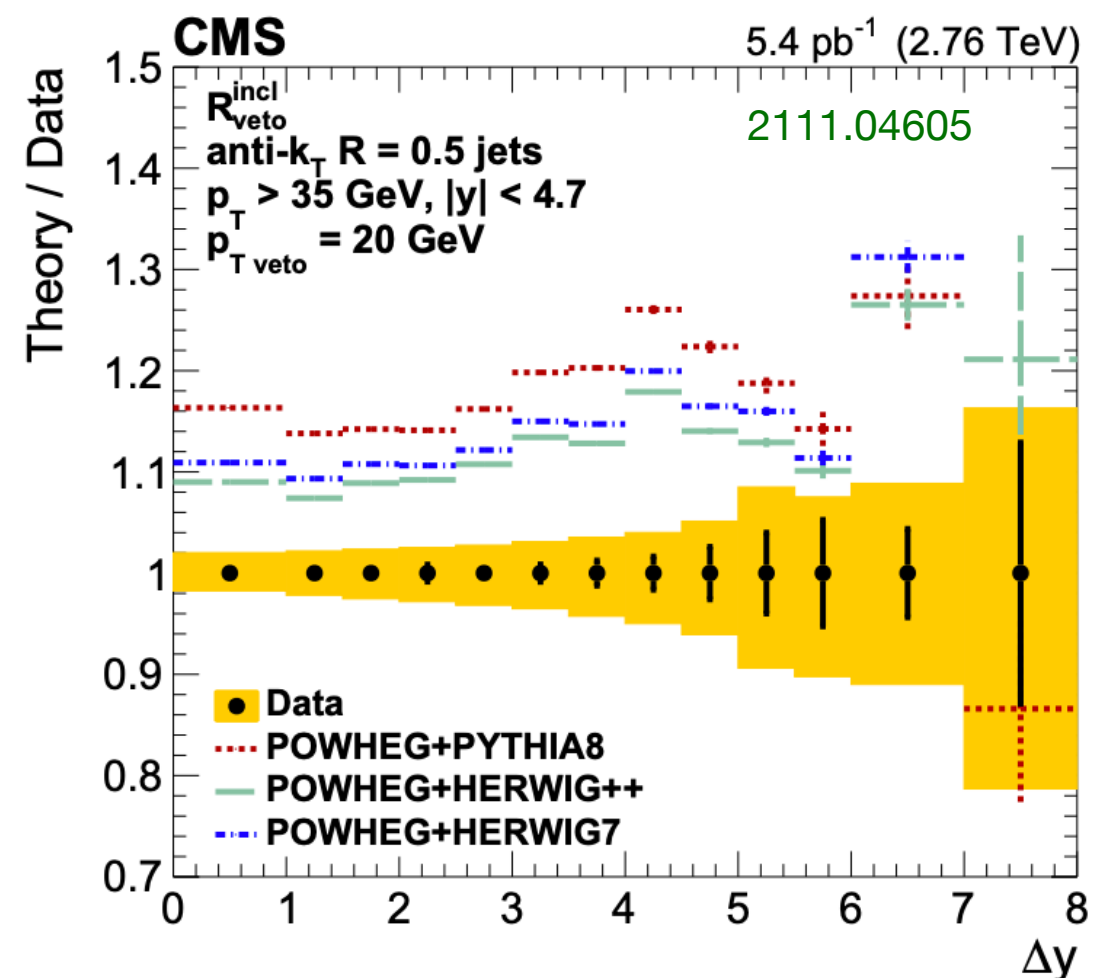
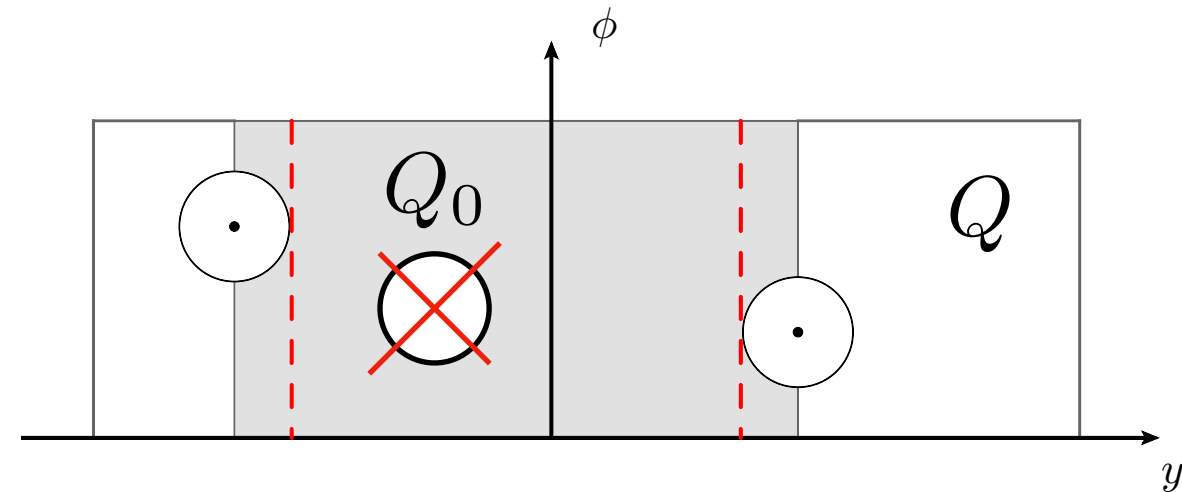


$$\mu_f, \mu_h \in [\bar{p}_T/2, 2\bar{p}_T]$$

$$\mu_s \in [Q_0/2, 2Q_0]$$

Gap fraction and super-leading logs

- Gap fraction originally suggested on the basis of color flow considerations in QCD Bjorken '93
- Forshaw, Kyrieleis, Seymour '06 have analyzed the effect of Glauber phases in the exclusive jet cross section directly in pQCD
- **Collinear logarithms** starting at 4 loops:
Super-leading logs
- Even 15 years after this effect was discovered, leading order resummation is unknown, process dependence is unknown, ...
- At forth order there are 1,746,272 diagrams !!!
- We apply renormalization-group approach and obtain the all-order results of leading SLLs
Becher, Neubert, DYS '21 PRL

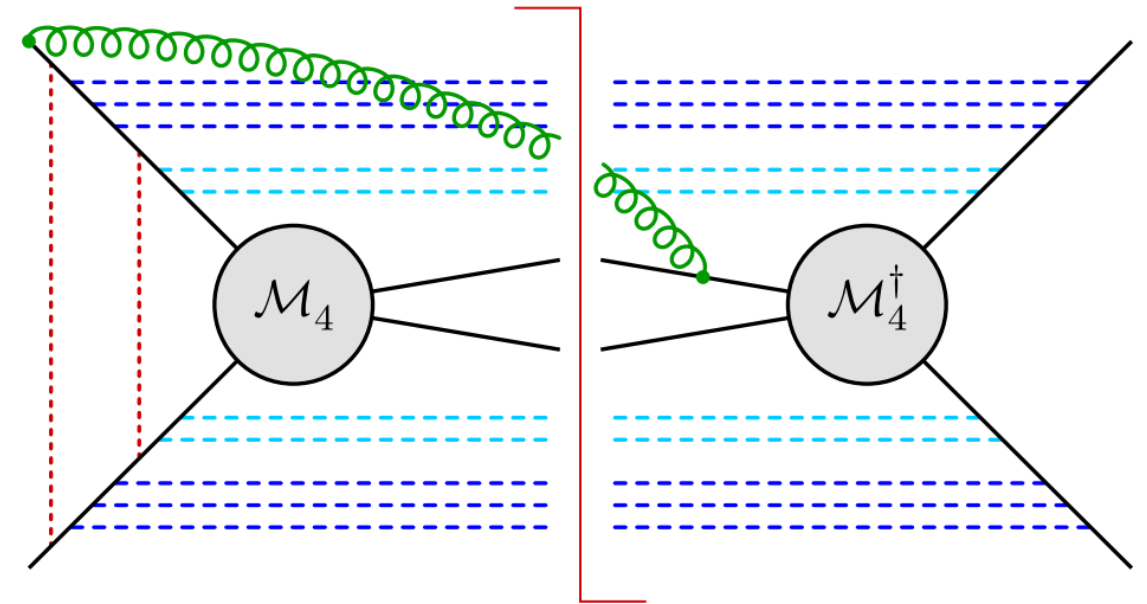


All-order evolution of leading Super-Leading Logs

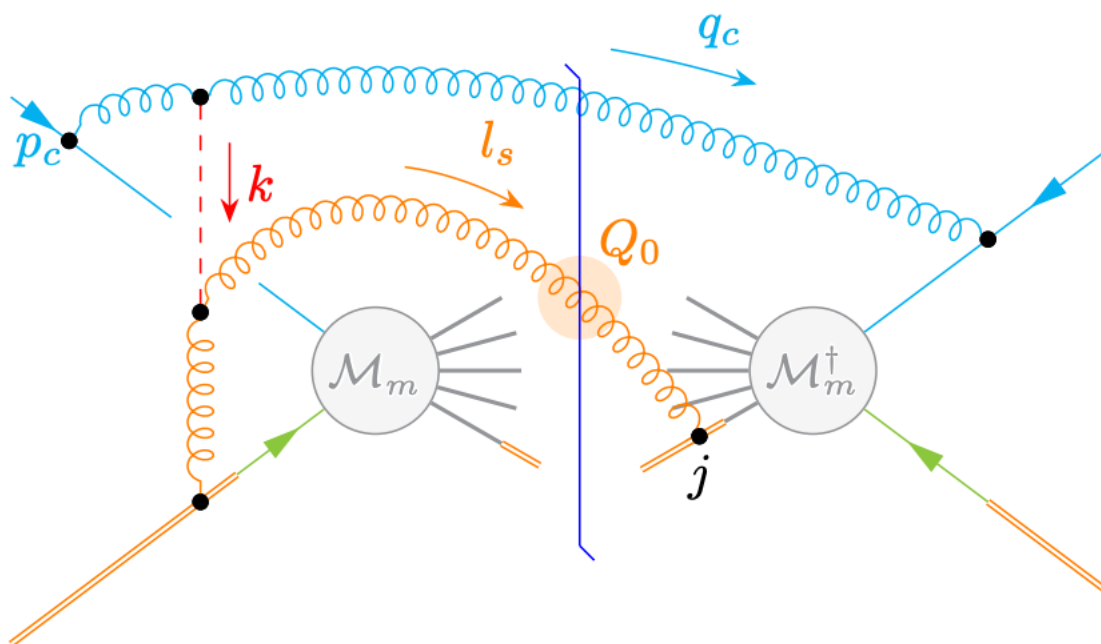
Becher, Neubert, DYS '21 PRL + Stillger'23 JHEP

All-order Super-leading logs: Kampe de Fariet function

$$\begin{aligned} \text{Global logs} &\longrightarrow e^{-\omega} \\ \text{Superleading logs} &\xrightarrow{\omega \rightarrow \infty} \frac{1}{\omega} \end{aligned}$$



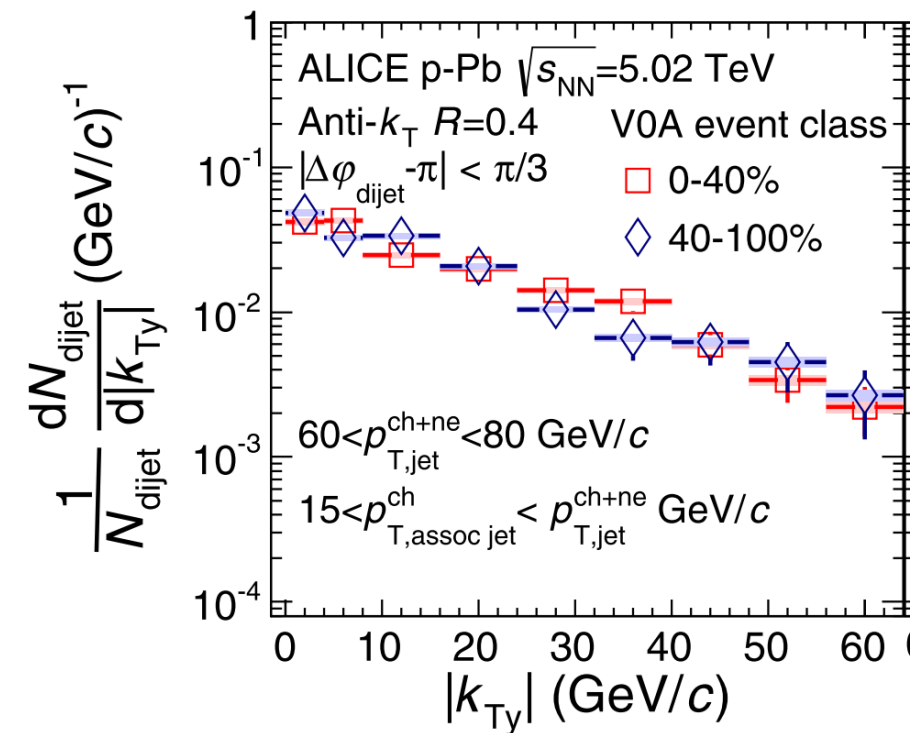
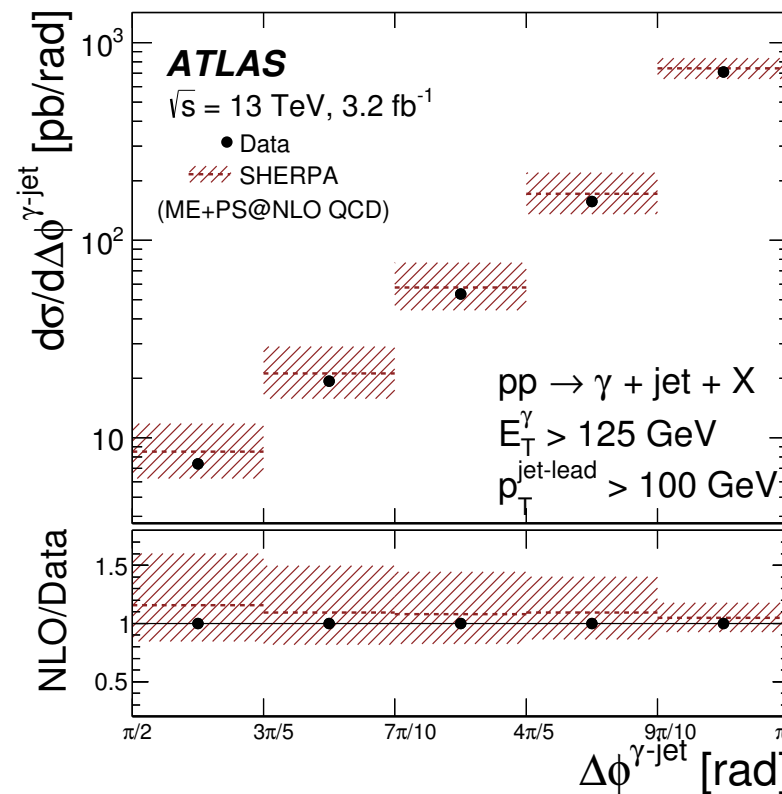
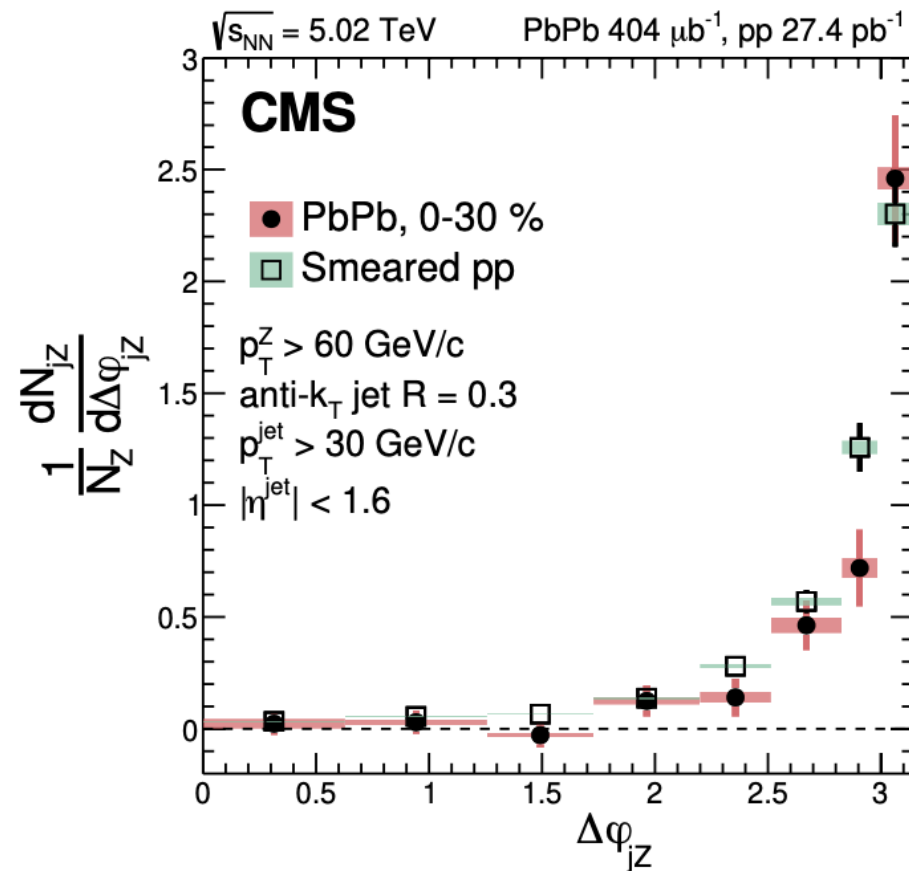
Consistent factorization needs Glauber Becher, Hager, Jaskiewicz, Neubert, Schwienbacher '24



$$\begin{aligned} \mathcal{W}_m^{\text{bare}} &\ni \frac{i\alpha_s^3}{12\pi^2 \varepsilon^3} f^{abc} f^{ade} \sum_{j>2} J_j \\ &\times \left[\mathbf{T}_{1L}^d \mathbf{T}_{1R}^e \mathbf{T}_{2L}^b \mathbf{T}_{jR}^c \left(-\frac{1}{2\eta} - \ln \frac{\nu}{p_c^-} \right) \right. \\ &\quad \left. + \mathbf{T}_{2L}^d \mathbf{T}_{2R}^e \mathbf{T}_{1L}^b \mathbf{T}_{jR}^c \left(\frac{1}{2\eta} + \ln \frac{\nu \bar{p}_c^+}{Q_0^2} \right) \right] \\ &- (L \leftrightarrow R), \end{aligned}$$

Correlation observables

Azimuthal correlation in the back-to-back limit



- jet calibration at pp
- strong coupling measurement
- spin asymmetry
- TMDs, Nuclear TMDs
- QGP at HIC
- naive factorization violation
- ...

In the back-to-back limit,
 one needs all-order results
 ($\log(\pi - \Delta\phi)$ resummation)

Azimuthal decorrelations of jets with the standard jet axis

- All-order resummation of azimuthal decorrelation of QCD jets was first studied by (Banfi, Dasgupta & Delenda '08)

$$q_T = \left| \sum_{i \notin \text{jets}} \vec{k}_{T,i} \right| + \mathcal{O}(k_T^2)$$

- CSS framework
 - dijet (Sun, Yuan & Yuan '14 & '15) jet + V (Sun, Yuan & Yuan '18; Chen, Qin, Wang, Wei, Xiao, Zhang '18)
lepton + jet (Liu, Yuan & Felix '19) jet + top (Cao, Sun, Yan, Yuan & Yuan '18 & '19)

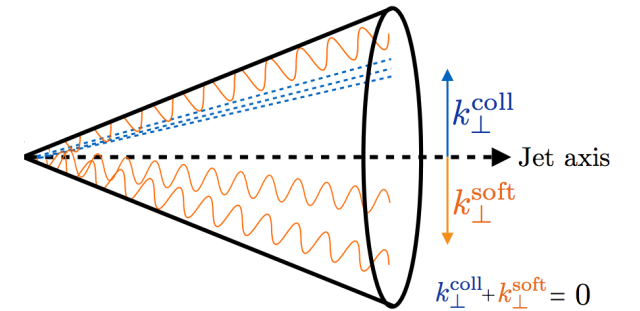
Resummation formula:
$$\frac{d\sigma}{d\Delta\phi} = x_a f_a(x_a, \mu_b) x_b f_b(x_b, \mu_b) \frac{1}{\pi} \frac{d\sigma_{ab \rightarrow cd}}{d\hat{t}} b J_0(|\vec{q}_\perp|b) e^{-S(Q,b)}$$

Perturbative Sudakov factor:
$$S_P(Q, b) = \sum_{q,g} \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln \frac{Q^2}{\mu^2} + B + D \ln \frac{1}{R^2} \right]$$

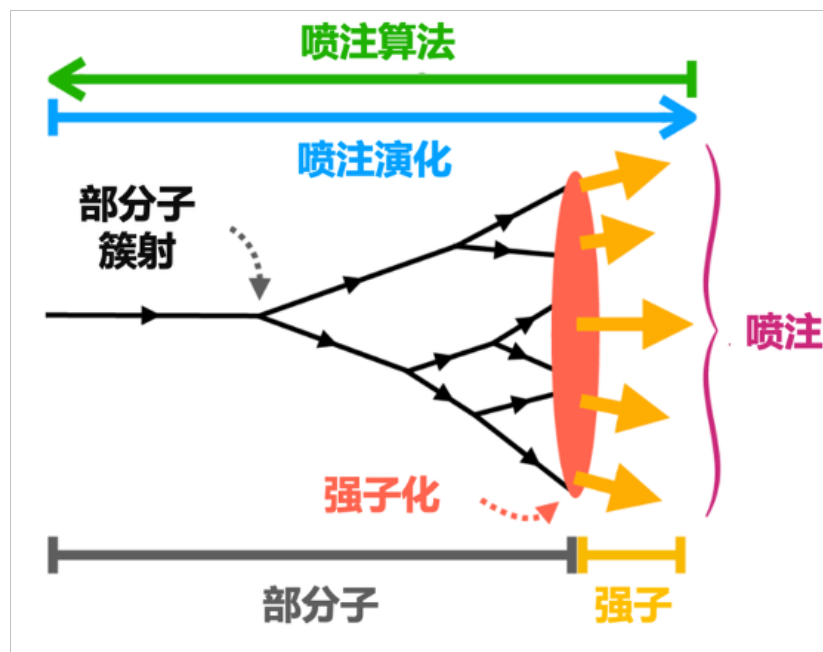
Jet TMDs and all-order structure

- Large logarithms in jet TMDs

$$q_T = \left| \sum_{i \notin \text{jets}} \vec{k}_{T,i} \right| + \mathcal{O}(k_T^2) \ll Q$$



- sum over all soft and collinear partons not combined with hard jets
 - deviation from $q_T = 0$ are only caused by particle flow outside the jet regions
 - non-global observables (Dasgupta & Salam '01)
 - Recoil absent for the p_T weighted recombination scheme (Banfi, Dasgupta & Delenda '08)
- $n \rightarrow \infty$ Winner-take-all scheme (Salam; Bertolini, Chan, Thaler '13)



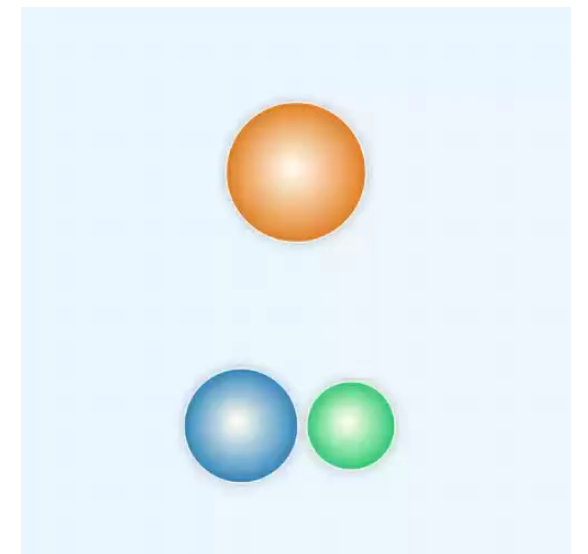
“赢者通吃” 动量重组方案

$$p_{t,r} = p_{t,i} + p_{t,j}$$

$$\phi_r = (w_i \phi_i + w_j \phi_j) / (w_i + w_j)$$

$$y_r = (w_i y_i + w_j y_j) / (w_i + w_j)$$

$$w_i = (p_{t,i})^n \quad n \rightarrow \infty$$

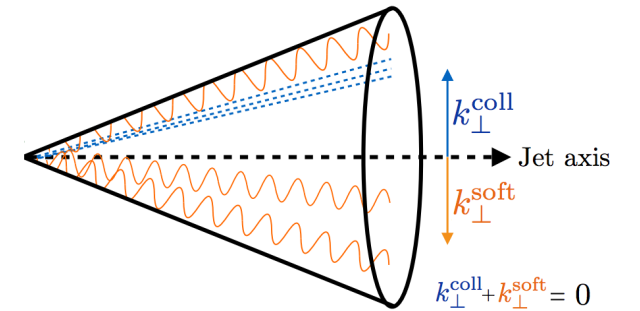


- N³LL resummation for jet q_T @ ee and ep (Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi '18 '19)

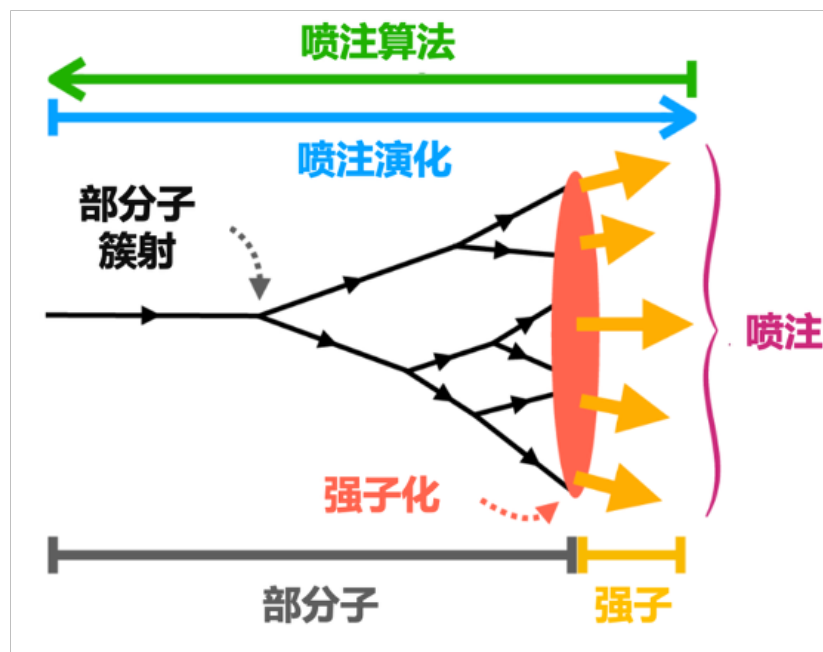
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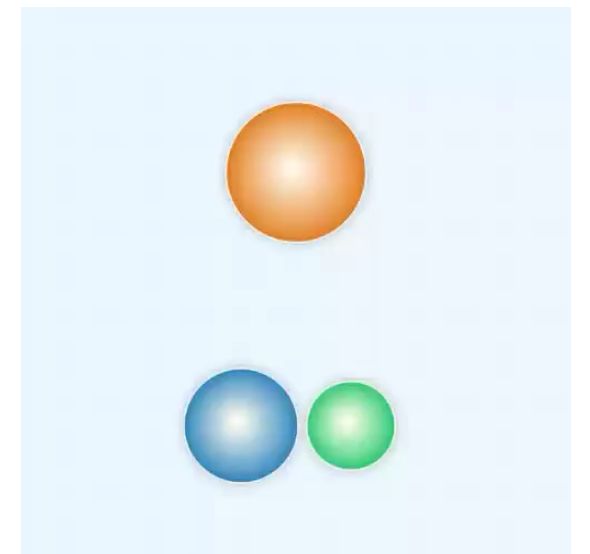
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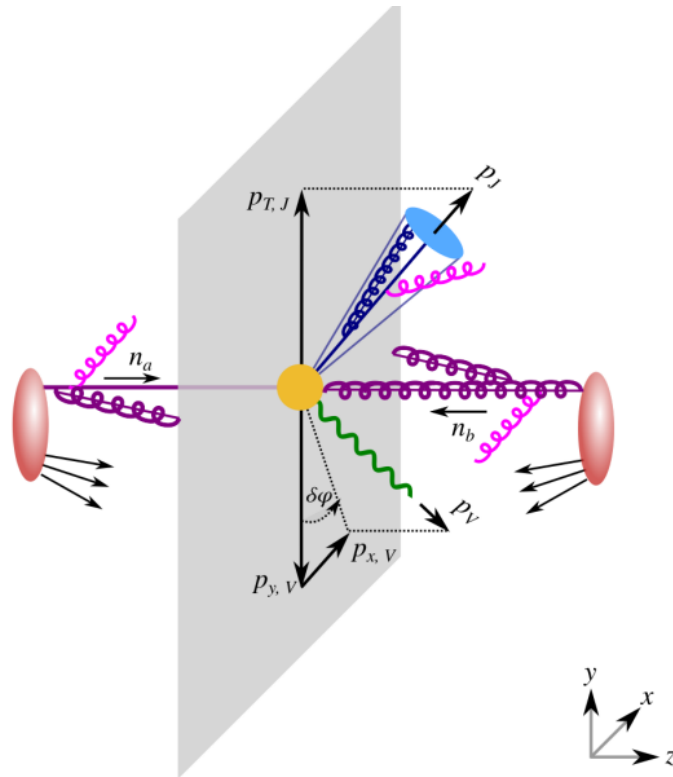
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WTA jet TMDs and all-order structure

- **NNLL resummation of V+jet for $\delta\phi$ @ pp** (Chien, Rahn, DYS, Waalewijn & Wu '22 JHEP + Schrignder '21 PLB)
- **NNLL resummation for $\delta\phi$ @ ep & eA** (Fang, Ke, DYS, Terry '23 JHEP)
- **NNLL resummation for $\delta\phi$ @ ep** (Fang, Gao, Li, DYS, Terry '24 JHEP)
- **NNLO WTA q_T slicing for jets** (Fu, Rahn, DYS, Waalewijn & Wu '25 PRL)
- **NNLL resummation of dijets for $\delta\phi$ and q_T @ pp** (Fu, Rahn, DYS, Waalewijn & Wu '26 JHEP)
- **NNLO WTA q_T slicing for hadron + jets @ ep** (Dong, Fang, Gao, Li, DYS, Zhu '26 submitted to PRL)
- **NNNLO two cut off (WTA q_T + hadron q_T) SIDIS @ ep** (Dong, Fang, Gao, Li, DYS, Zhu, Zhu '26 submitted to PRL)
- **WTA q_T slicing for jets and hadron @ UPC $0nXn$** (Chen, Chen, Fang, DYS, Xing in progress)

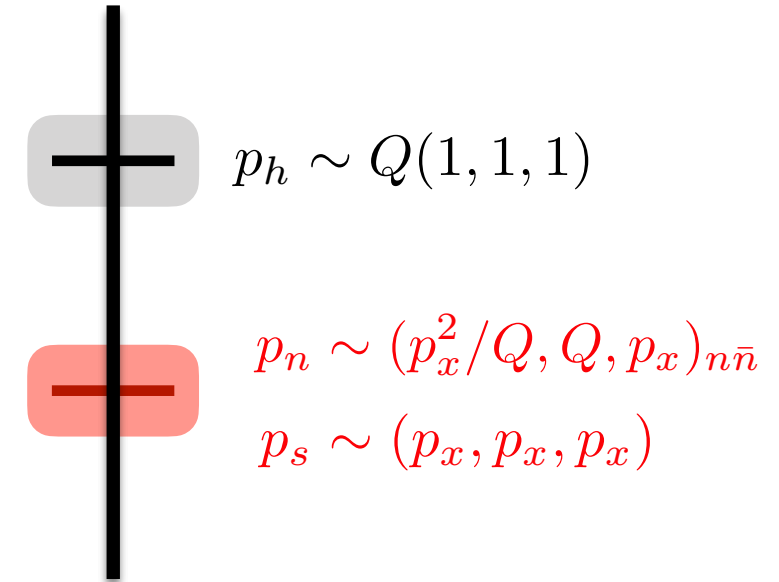
Recoil-free azimuthal angle for boson-jet correlation

(Chien, Rahn, DYS, Waalewijn & Wu '23 JHEP + Schrignder '21 PLB)



$$\pi - \Delta\phi \equiv \delta\phi \approx \sin(\delta\phi) = |p_{x,V}|/p_{T,V}$$

Standard SCET2 (CSS ...) $\delta\phi \ll \mathcal{O}(1)$



Effect of soft radiation in jet algorithm is power suppressed

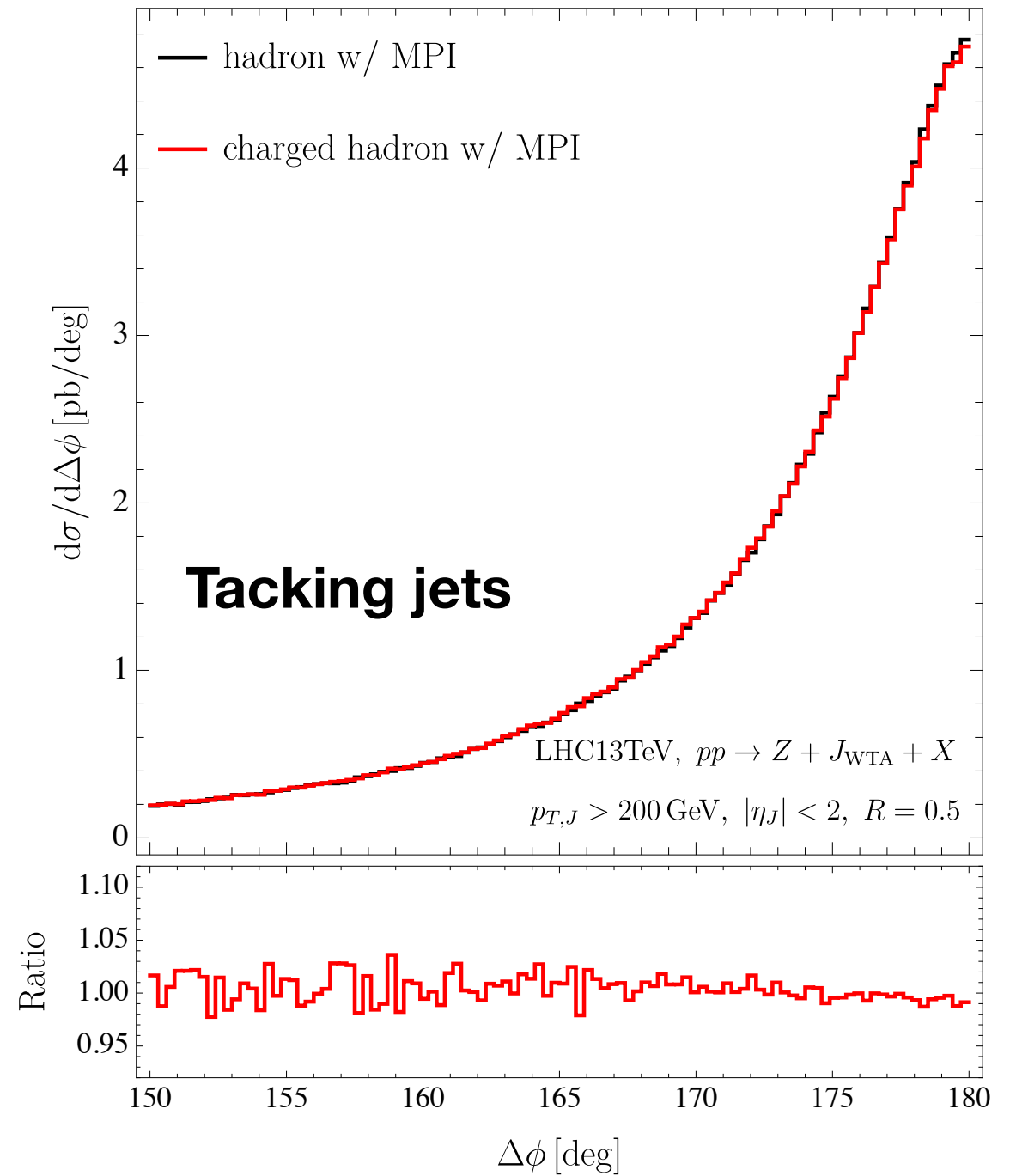
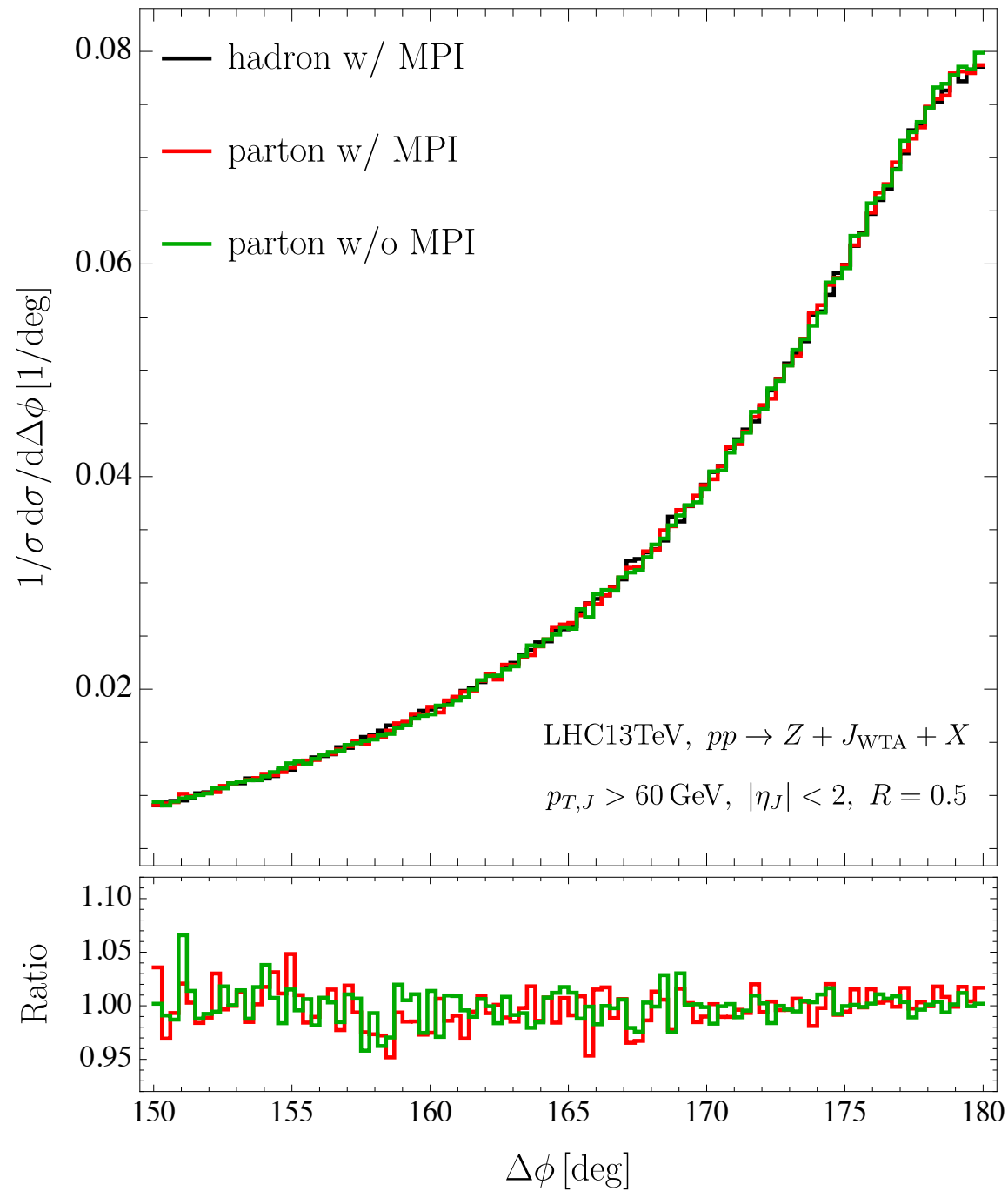
Following the standard steps in SCET2 we obtain the following factorization formula

$$\frac{d\sigma}{dp_{x,V} dp_{T,J} dy_V d\eta_J} = \int \frac{db_x}{2\pi} e^{ip_{x,V} b_x} \sum_{i,j,k} B_i(x_a, b_x) B_j(x_b, b_x) S_{ijk}(b_x, \eta_J) H_{ij \rightarrow V k}(p_{T,V}, y_V - \eta_J) J_k(b_x)$$

Fourier transformation in 1-dim

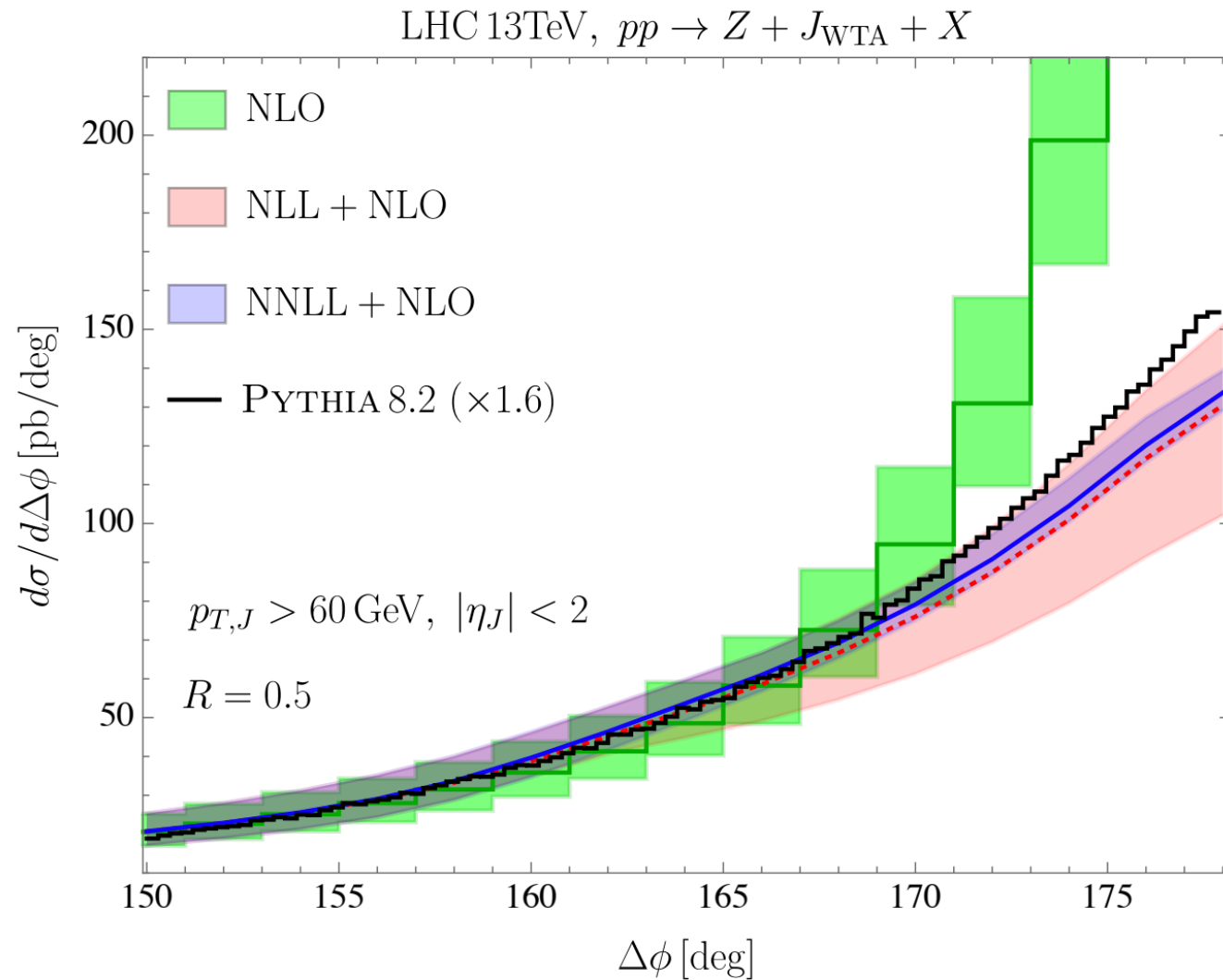
Soft function can be obtained by boosted invariance

Pythia simulation results

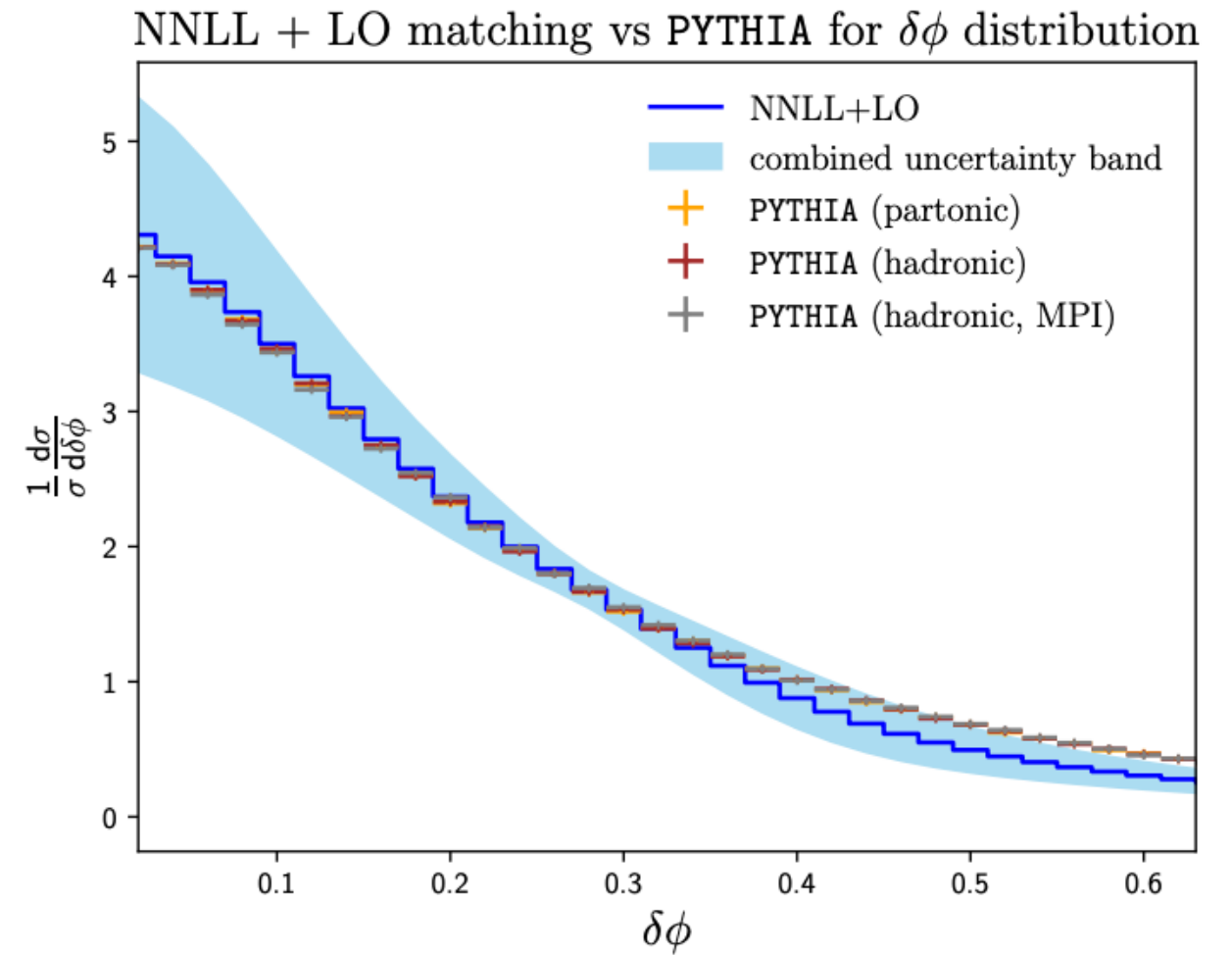


- Non-perturbative effects (hadronization and MPI) are mild

Numerical results



(Chien, Rahn, DYS, Waalewijn & Wu '23 JHEP)

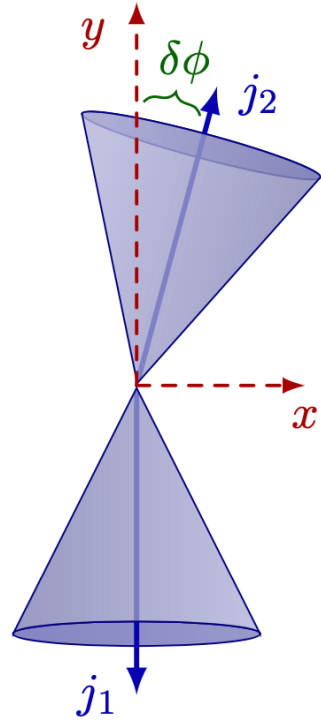


Fu, Rahn, DYS, Waalewijn & Wu '26 JHEP

- first N²LL resummation including jet dynamics
- good perturbative convergence
- N³LL+O(α_s^2) resummation at the LHC in on progress
- interesting to perform the same measurement at the LHC

QCD resummation of the azimuthal decorrelation of dijets in pp and pA

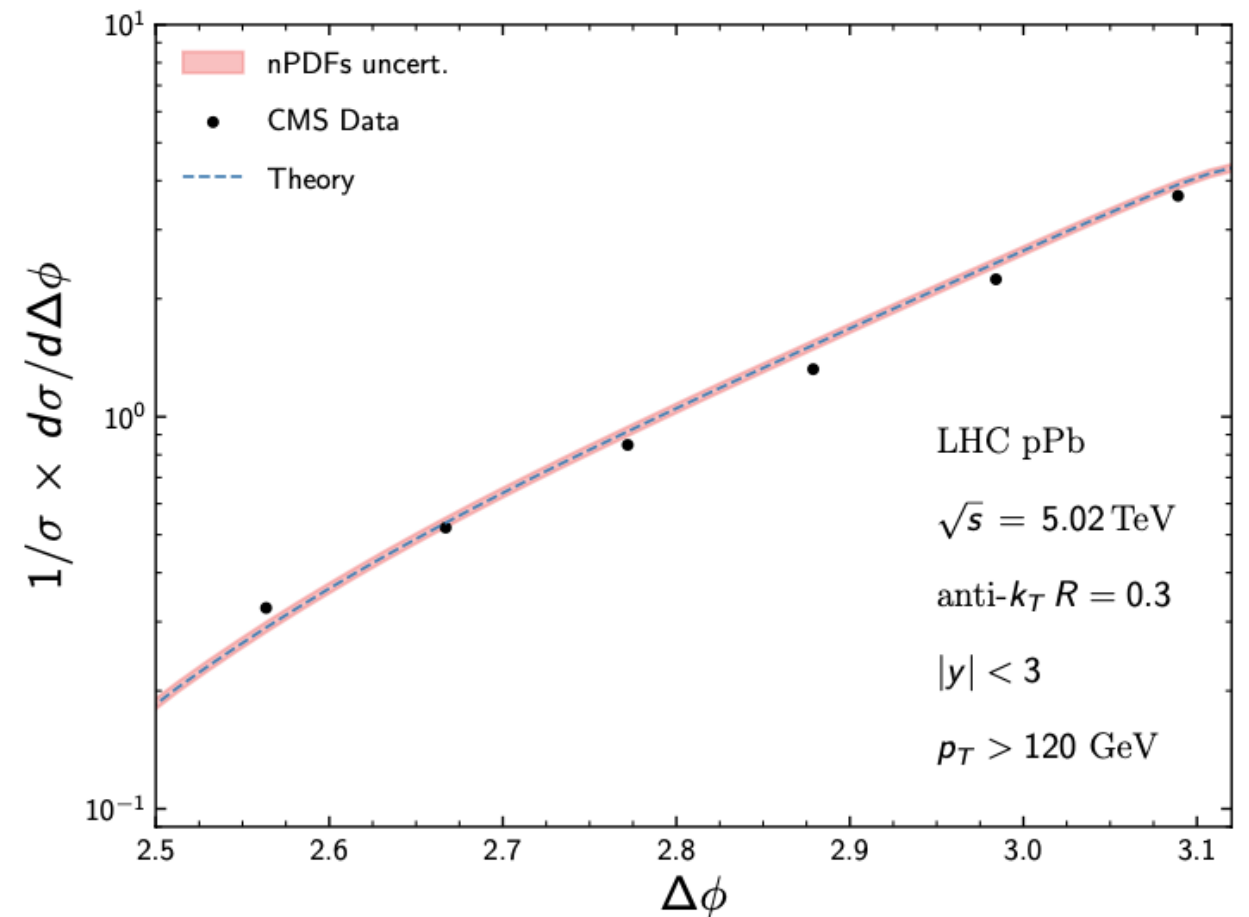
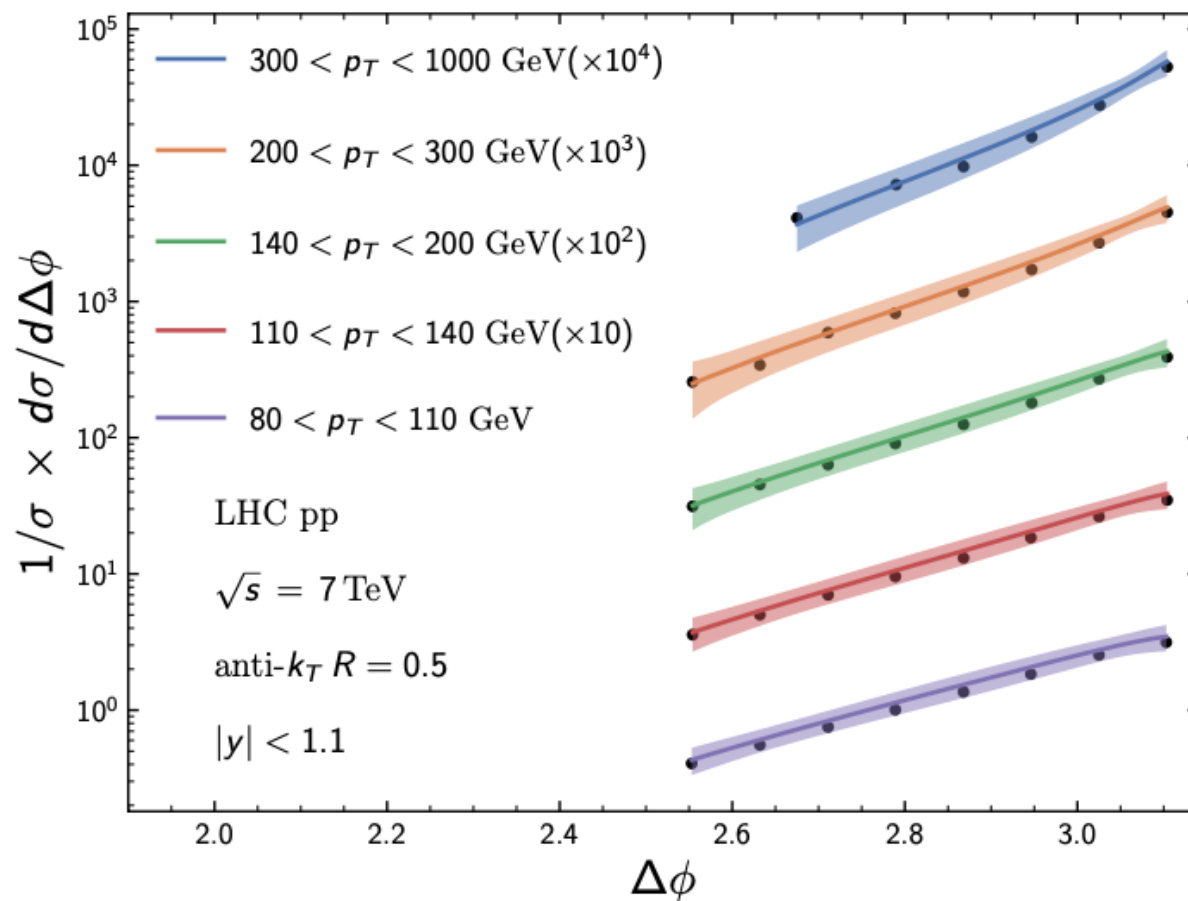
Gao, Kang, DYS, Terry, Zhang '23 JHEP



Factorization and resummation formula in SCET

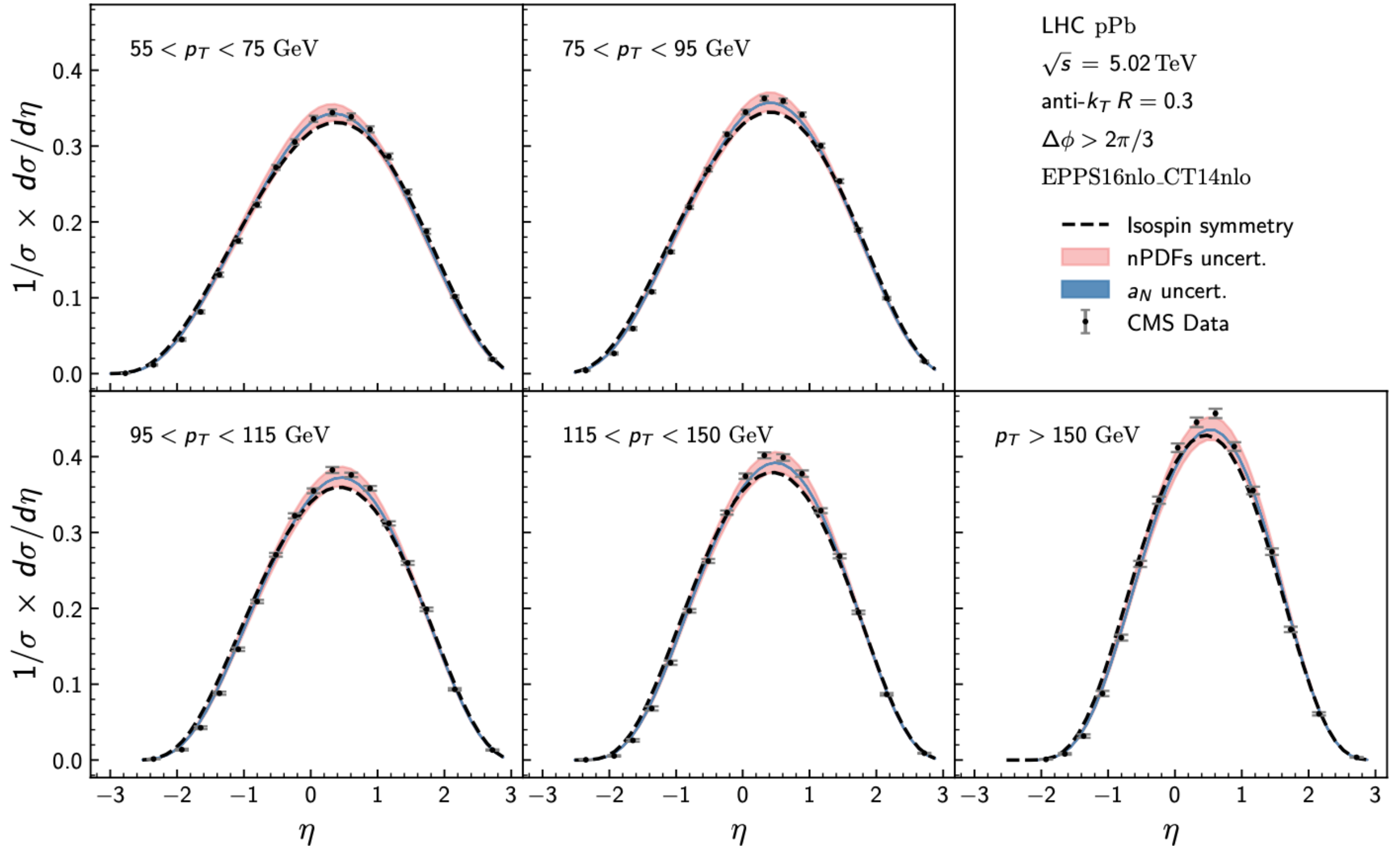
$$\begin{aligned} \frac{d^4\sigma}{dy_c dy_d dp_T^2 d\delta\phi} &= \sum_{abcd} \frac{p_T}{16\pi\hat{s}^2} \frac{1}{1+\delta_{cd}} \int \frac{db}{2\pi} e^{ibp_T\delta\phi} x_a \tilde{f}_{a/p}^{\text{unsub}}(x_a, b, \mu, \zeta_a/\nu^2) x_b \tilde{f}_{b/p}^{\text{unsub}}(x_b, b, \mu, \zeta_b/\nu^2) \\ &\times \text{Tr} \left[\mathbf{H}_{ab \rightarrow cd}(\hat{s}, \hat{t}, \mu) \tilde{\mathbf{S}}_{ab \rightarrow cd}^{\text{unsub}}(b, \mu, \nu) \right] J_c(p_T R, \mu) \tilde{S}_c^{\text{cs}}(b, R, \mu, \nu) \\ &\times J_d(p_T R, \mu) \tilde{S}_d^{\text{cs}}(b, R, \mu, \nu), \end{aligned}$$

Nuclear modified TMD PDFs (Alrashed, Anderle, Kang, Terry & Xing, '22)



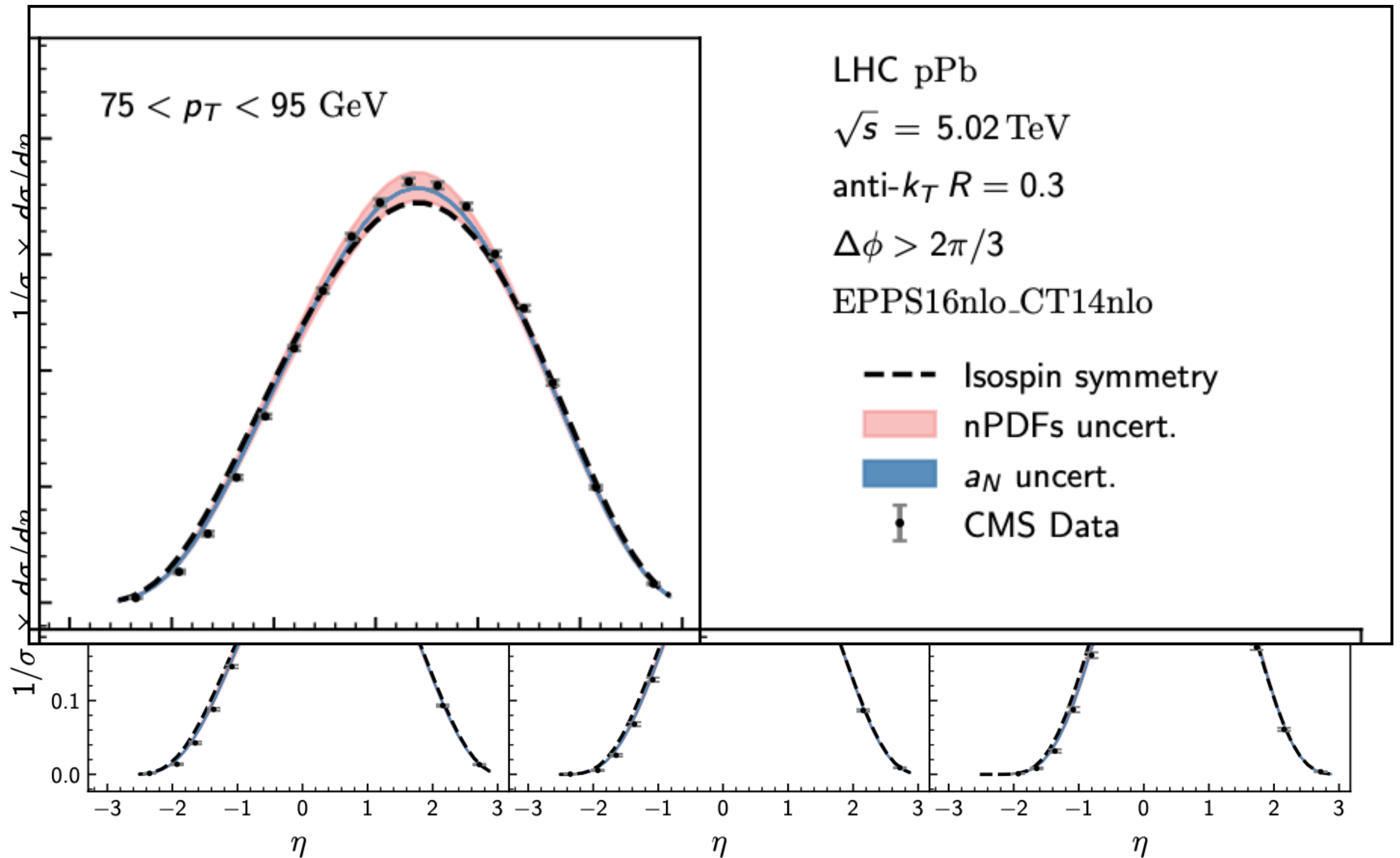
Numerical results in pA

Gao, Kang, DYS, Terry, Zhang '23 JHEP



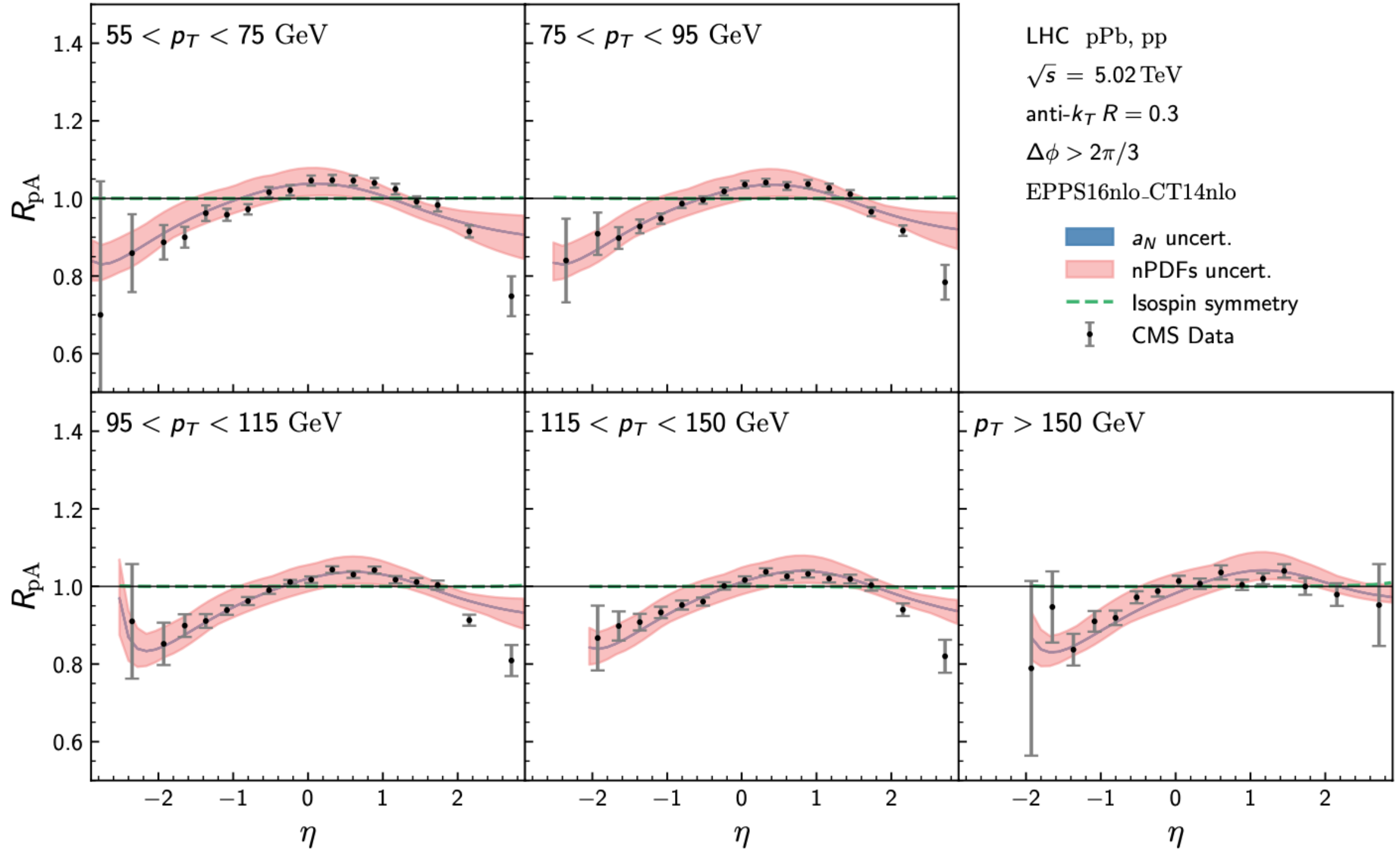
Numerical results in pA

Gao, Kang, DYS, Terry, Zhang '23 JHEP

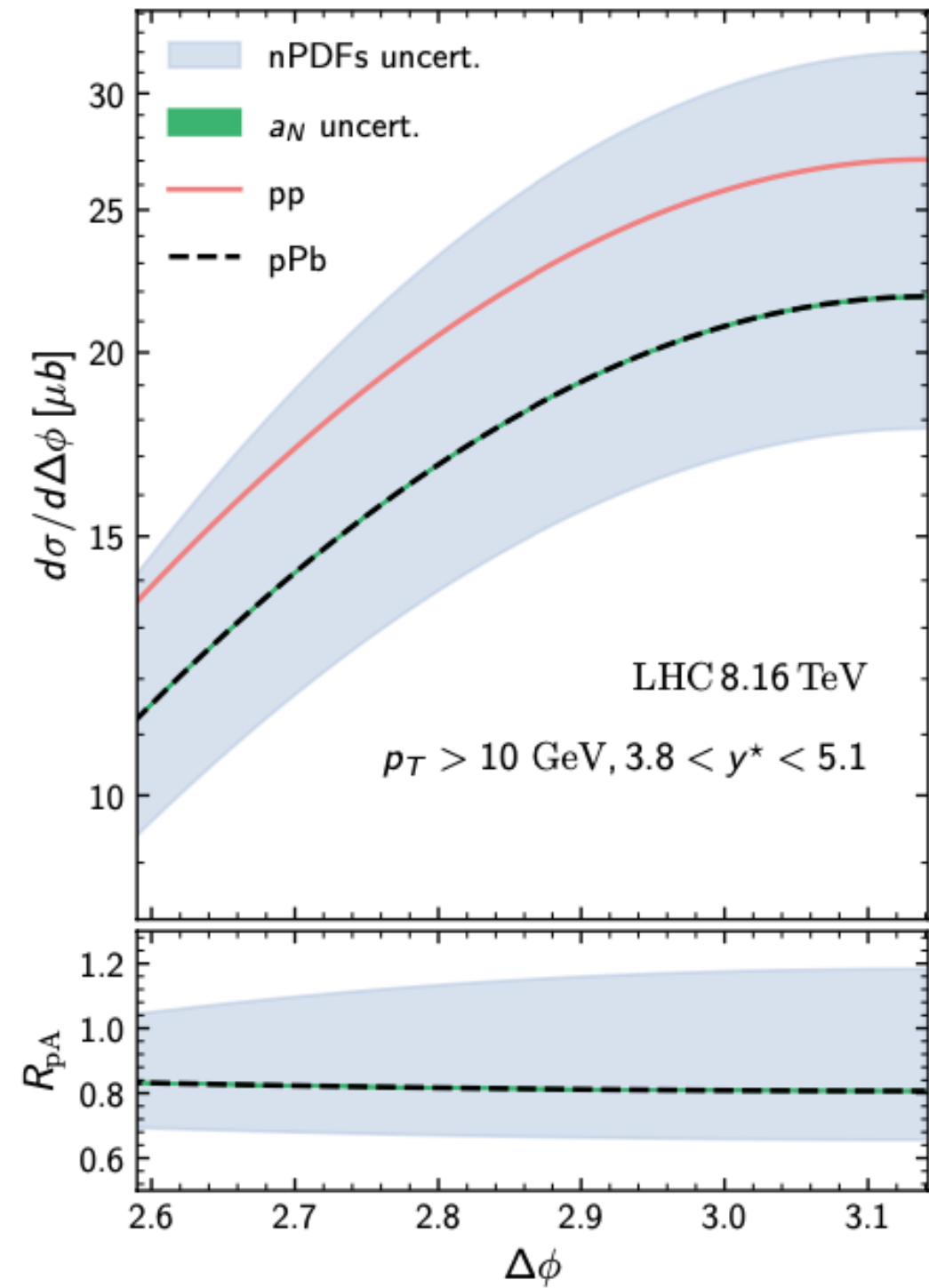
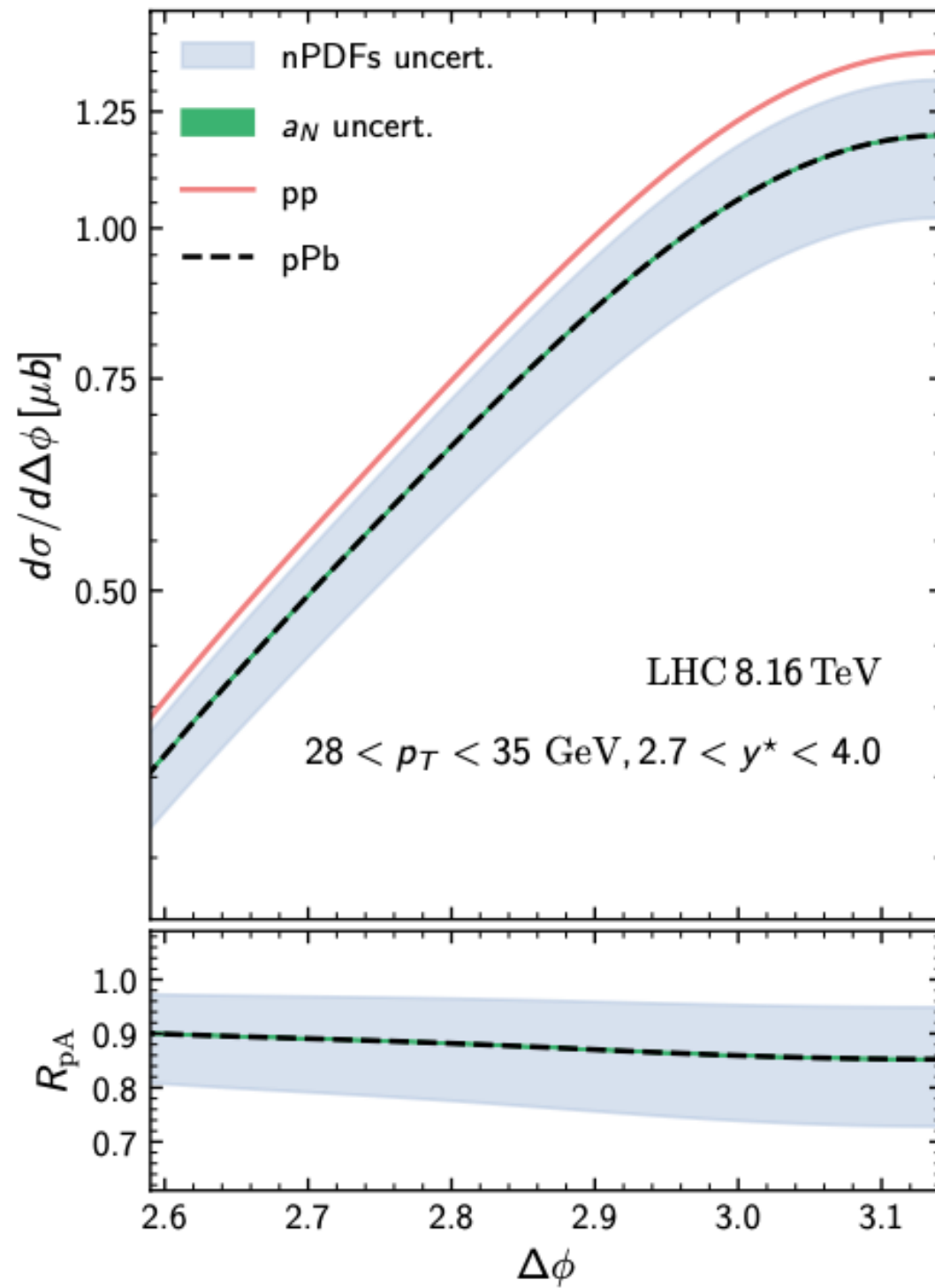


Numerical results in pA

Gao, Kang, DYS, Terry, Zhang '23 JHEP



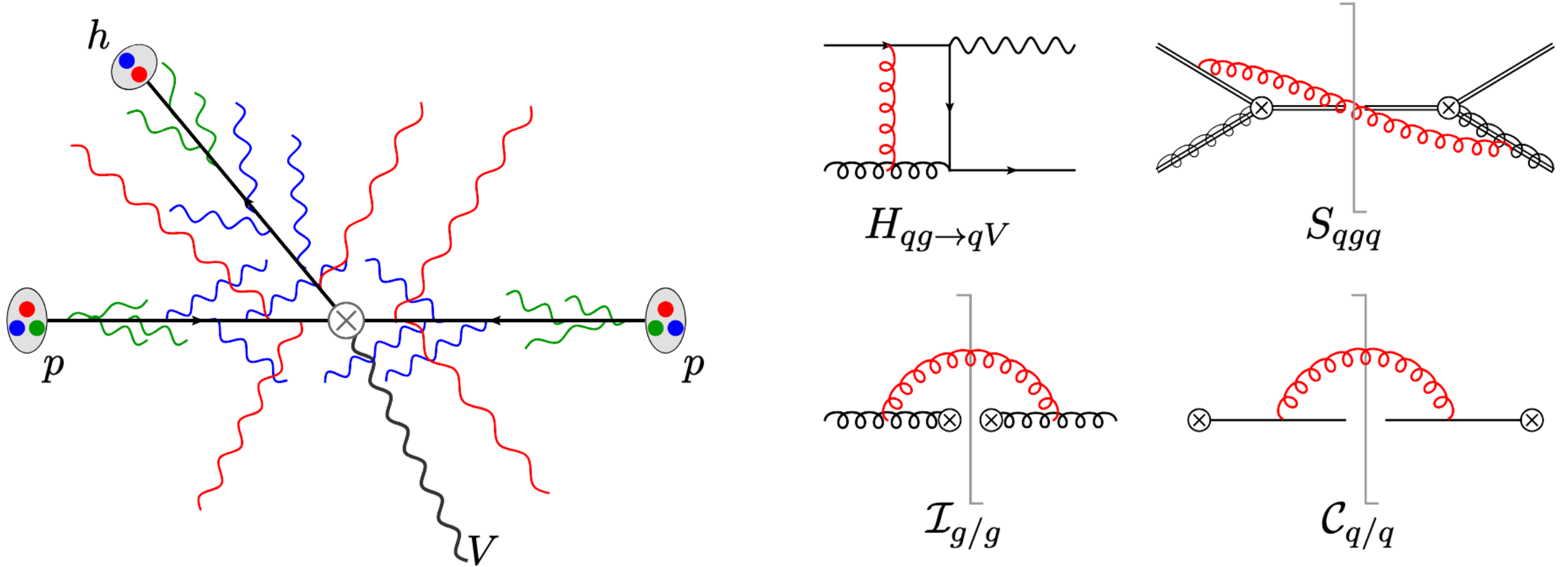
Numerical results in pA



shadowing effect in the small- x region

TMD resummation for the vector boson and hadron angular correlation at hadron colliders

(Fang, Gao, Kang, DYS in progress)

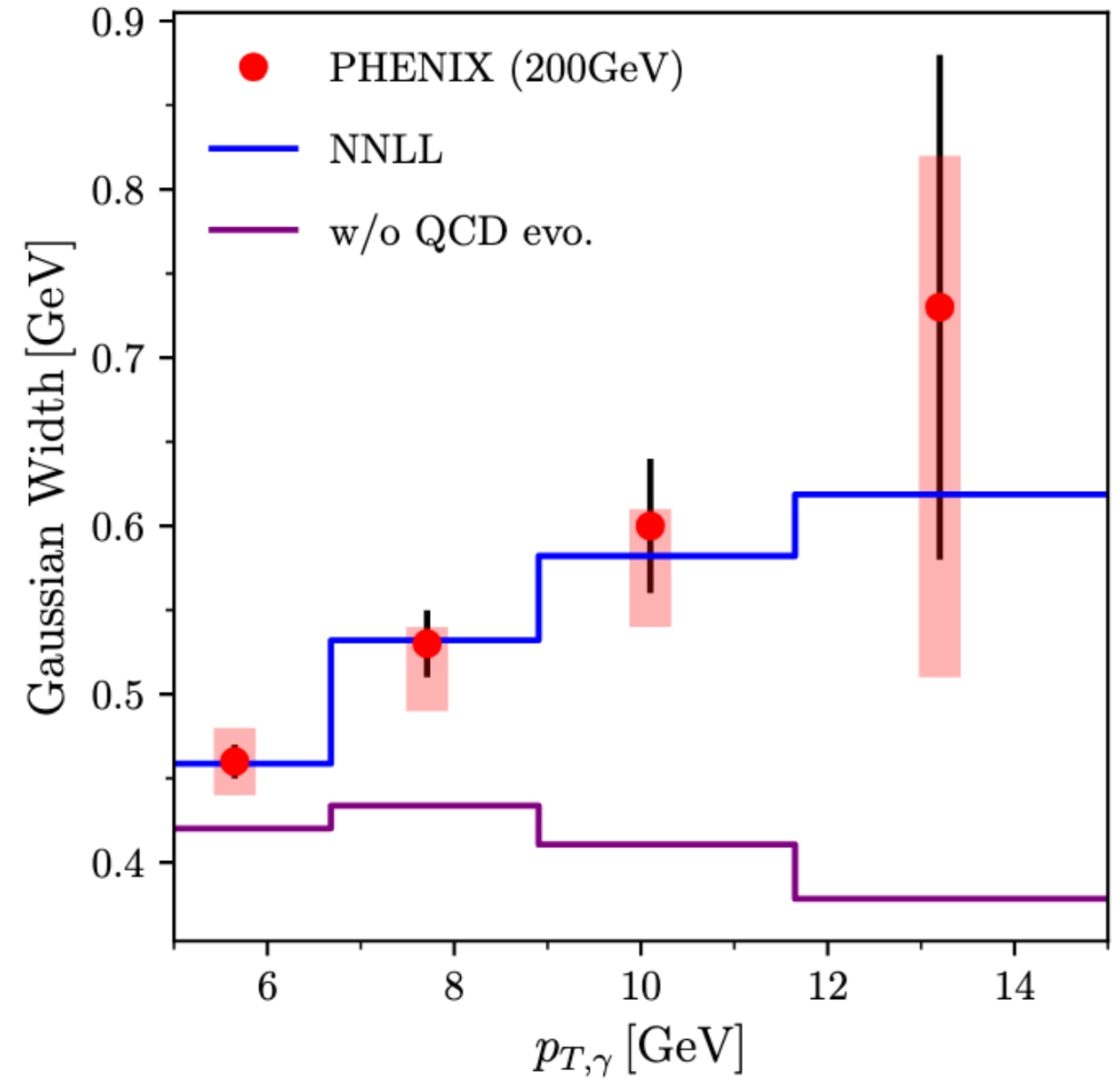
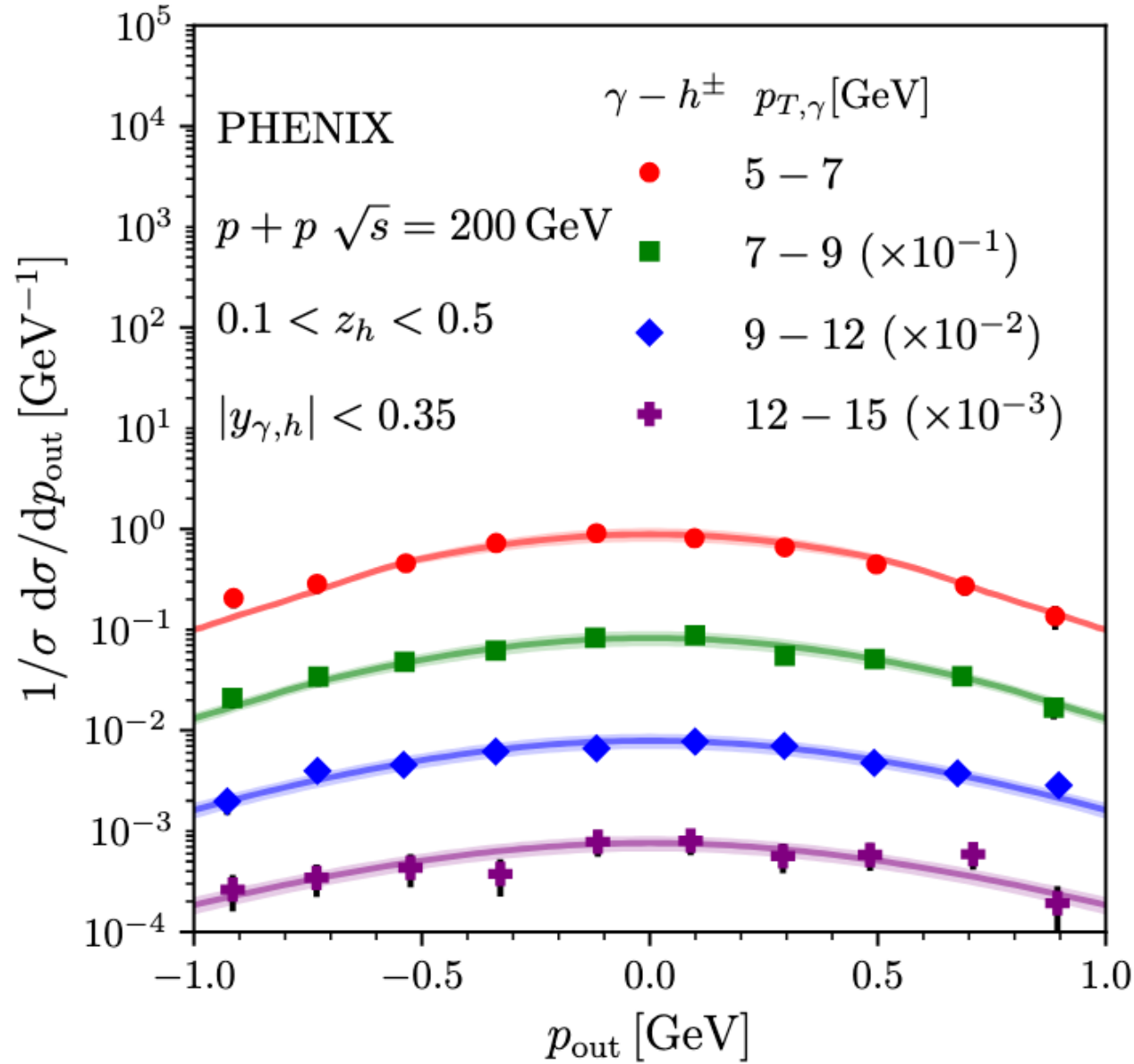


$$\frac{d\sigma^{\text{Resum}}}{dp_{\text{out}} d\mathcal{PS}} = \sum_{ijk} \int_0^\infty \frac{db}{\pi} e^{ibp_{\text{out}}/z_h} H_{ij \rightarrow kV}(Q, \mu_h) e^{-\int_{\mu_b}^{\mu_h} \frac{d\mu}{\mu} \Gamma_{H_{ij \rightarrow kV}}^\mu(\mu)}$$

$$\times f_{i/p}(x_1, b, \mu_b, \zeta_s) f_{j/p}(x_2, b, \mu_b, \zeta_s) D_{h/k}(z_h, b, \mu_b, \zeta_s) \prod_{a=ijk} \left(\frac{\zeta_a}{\zeta_s} \right)^{\Gamma_{Fa}^\zeta(b, \mu_b)/2}.$$

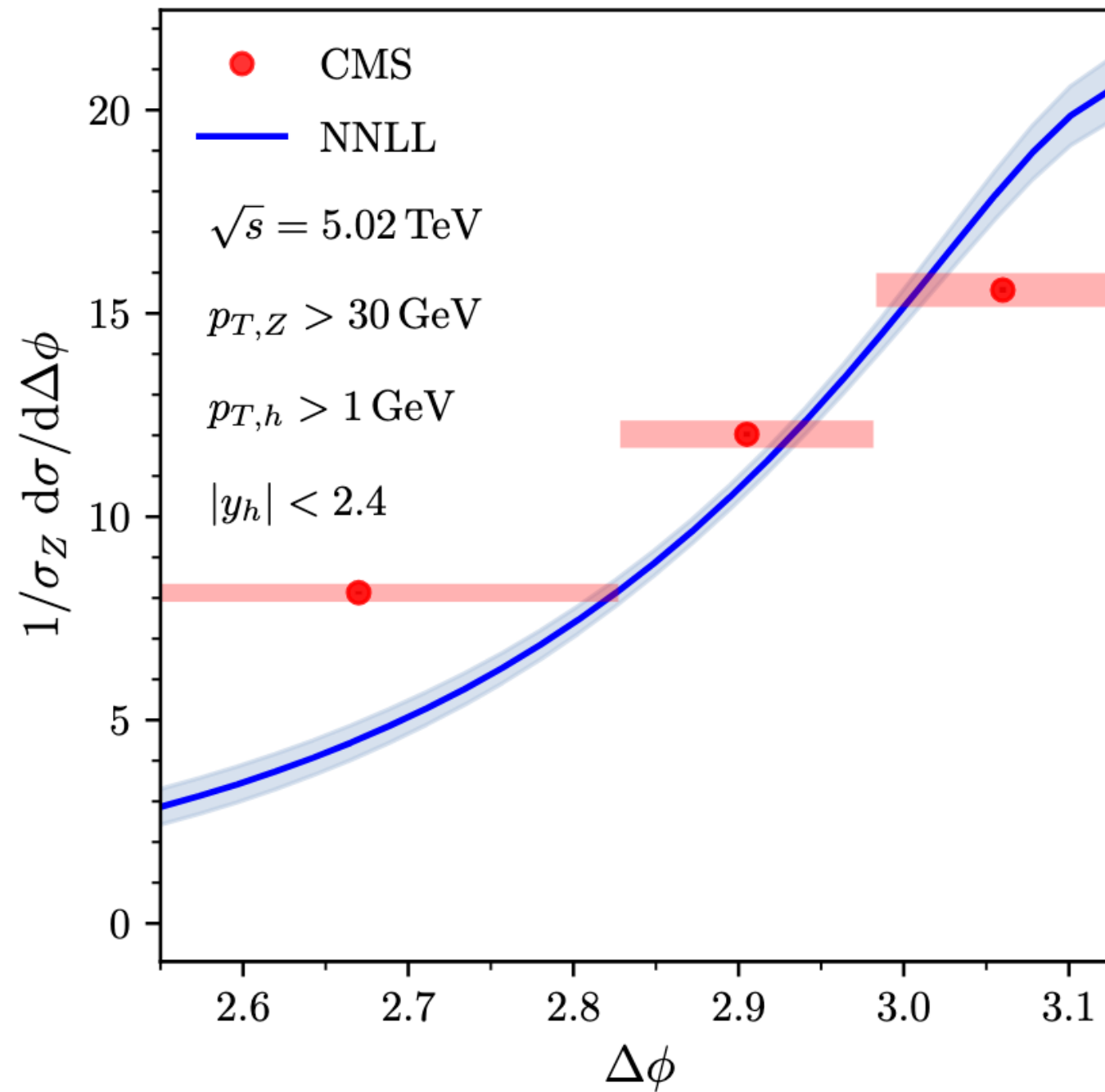
TMD resummation for the vector boson and hadron angular correlation at hadron colliders

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TMD resummation for the vector boson and hadron angular correlation at hadron colliders

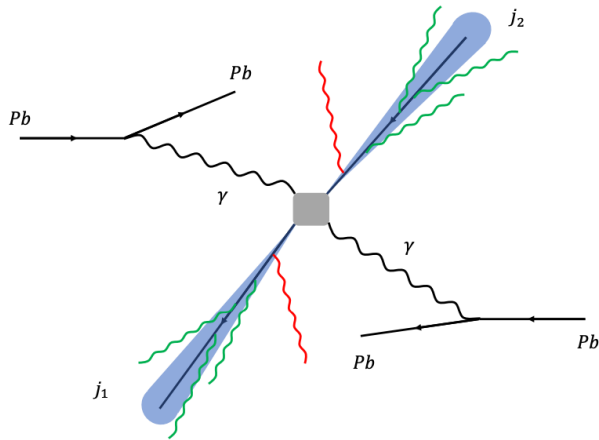
(Fang, Gao, Kang, DYS in progress)



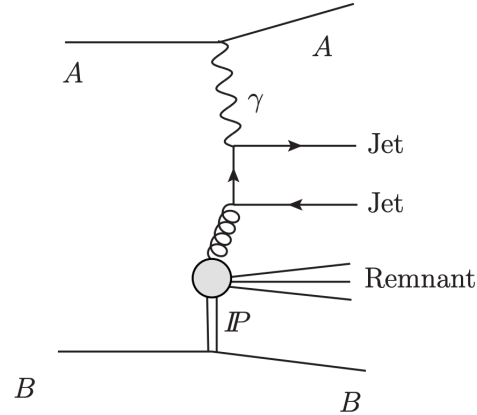
Azimuthal decorrelation of QCD jets in ultra-peripheral collisions

(Zhang, Dai, DYS, '23 JHEP)

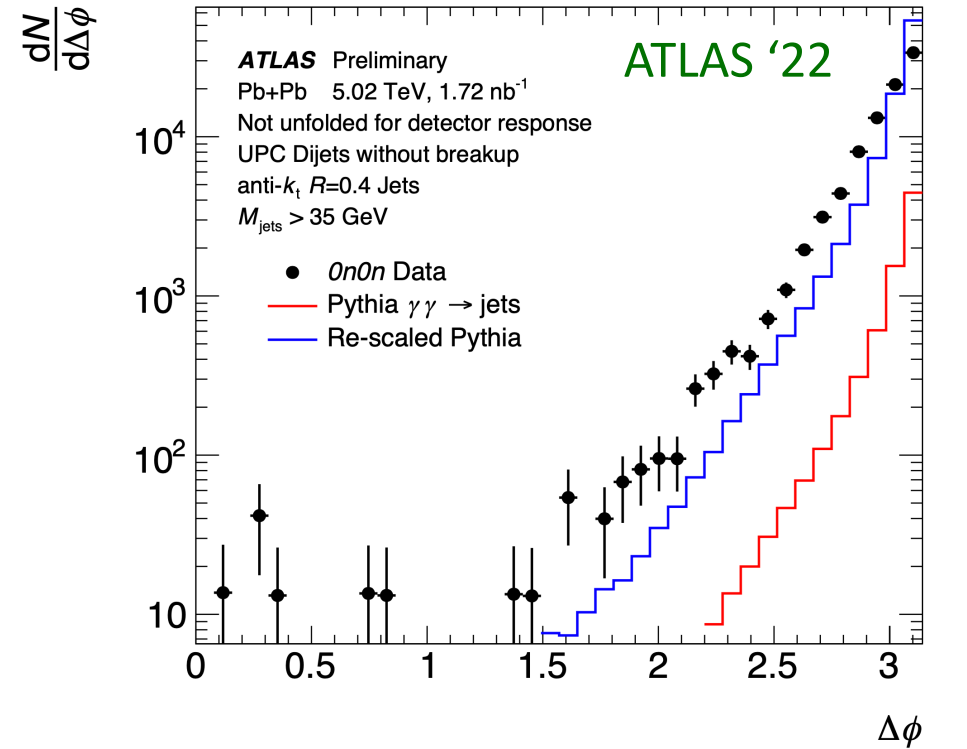
Dijet production with no nuclear breakup



Photon-photon fusion



diffractive photo-production

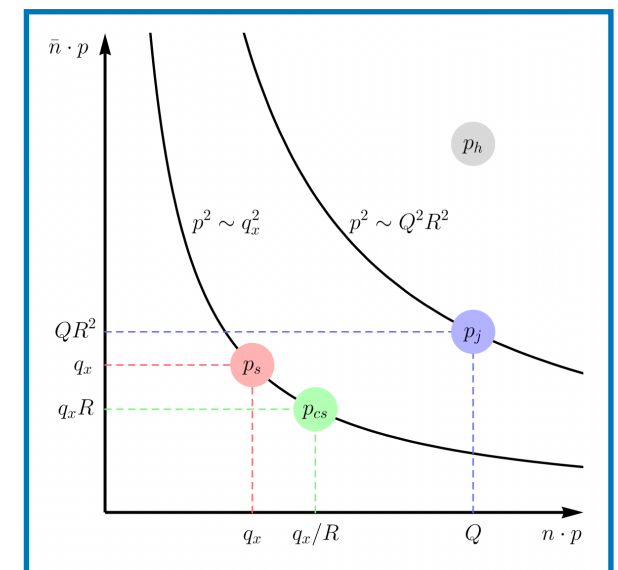


We apply **equivalent photon approximation** (Fermi 1924; Weizsacker 1934; Williams 1935) + **SCET**

$$\frac{d^4\sigma}{dq_x dp_T dy_1 dy_2} = \int_{-\infty}^{+\infty} \frac{db_x}{2\pi} e^{iq_x b_x} \tilde{B}(b_x, p_T, y_1, y_2) H(p_T, \Delta y, \mu) \tilde{S}(b_x, y_1, y_2, \mu, \nu) \tilde{U}_1(b_x, R, y_1, \mu, \nu) J_1(p_T, R, \mu) \tilde{U}_2(b_x, R, y_2, \mu, \nu) J_2(p_T, R, \mu)$$

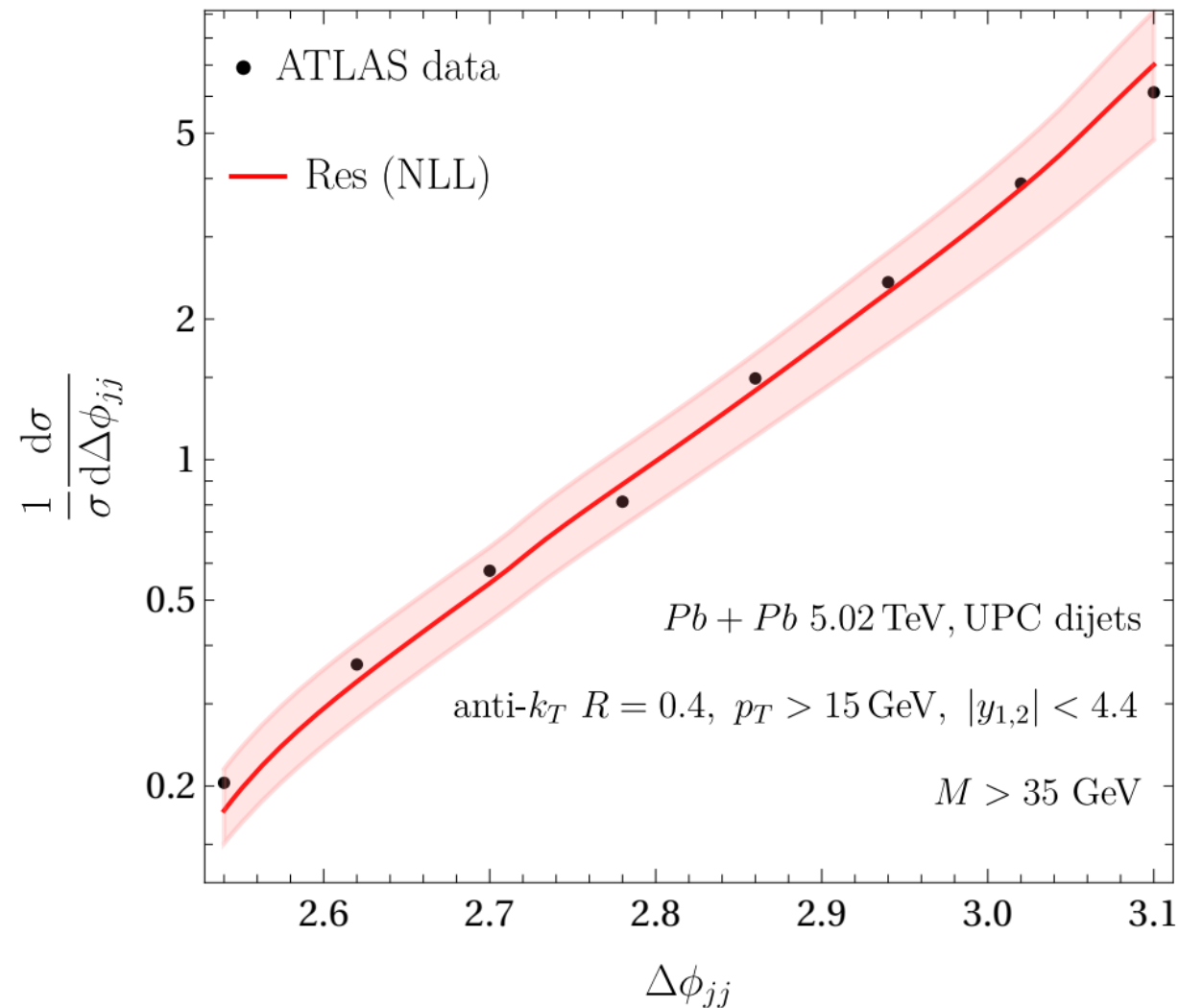
Photon Wigner distribution: (Klein, Mueller, Xiao, Yuan, '20, also see Ji, '03; Belitsky, Ji, Yuan, '04, ...)

$$x f_\gamma(x, k_T; b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} \int \frac{d\xi^- d^2r_\perp}{(2\pi)^3} e^{ixP^+\xi^- - ik_T \cdot r_\perp} \times \left\langle A, -\frac{\Delta_\perp}{2} \left| F^{+\perp} \left(0, \frac{r_\perp}{2} \right) F^{+\perp} \left(\xi^-, -\frac{r_\perp}{2} \right) \right| A, \frac{\Delta_\perp}{2} \right\rangle$$



Numerical results

(Zhang, Dai, DYS, '23 JHEP)

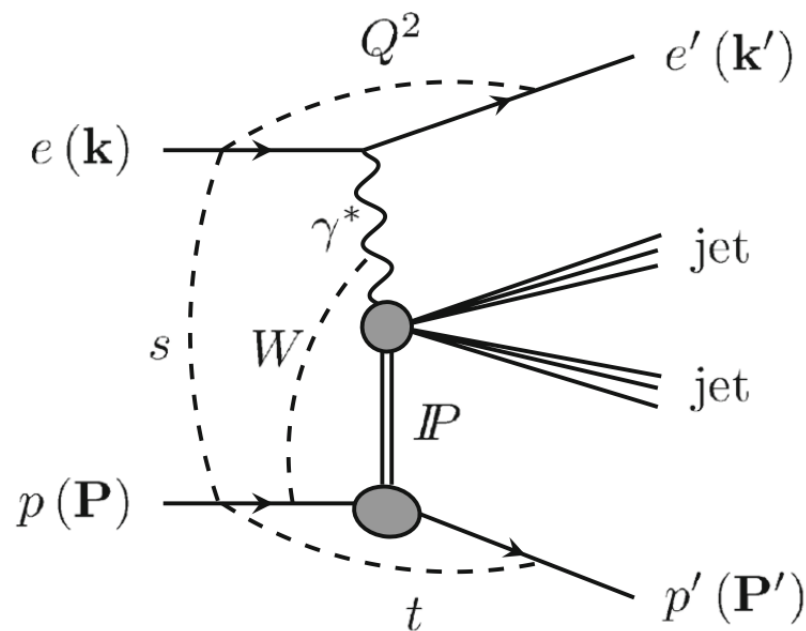


- A good agreement with the ATLAS data in the nearly back-to-back region
- Photo-productions may enhance the dijet production rate, but should barely change the shape

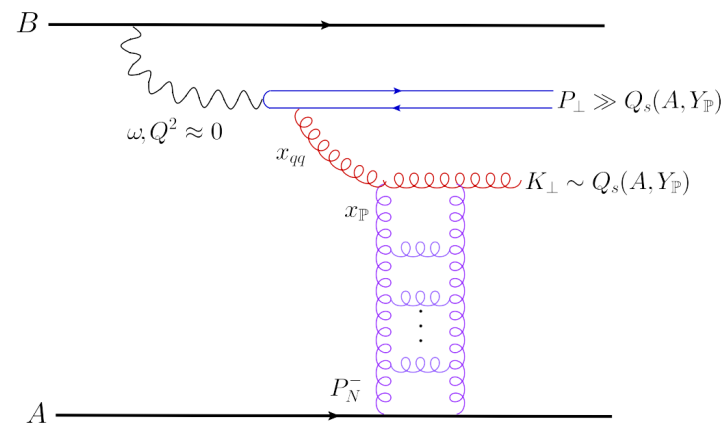
Diffractive dijets photo-production

DYS, Y. Shi, C. Zhang, J. Zhou, Y. Zhou '24 JHEP

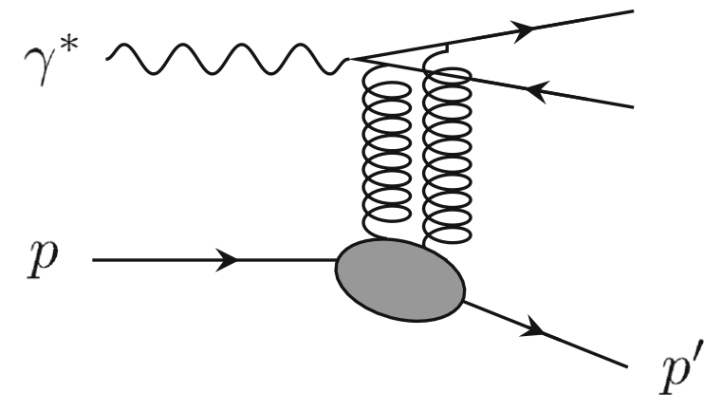
- Diffractive di-jet production provide rich information on nucleon internal structure.



diffractive production of (2+1) jets



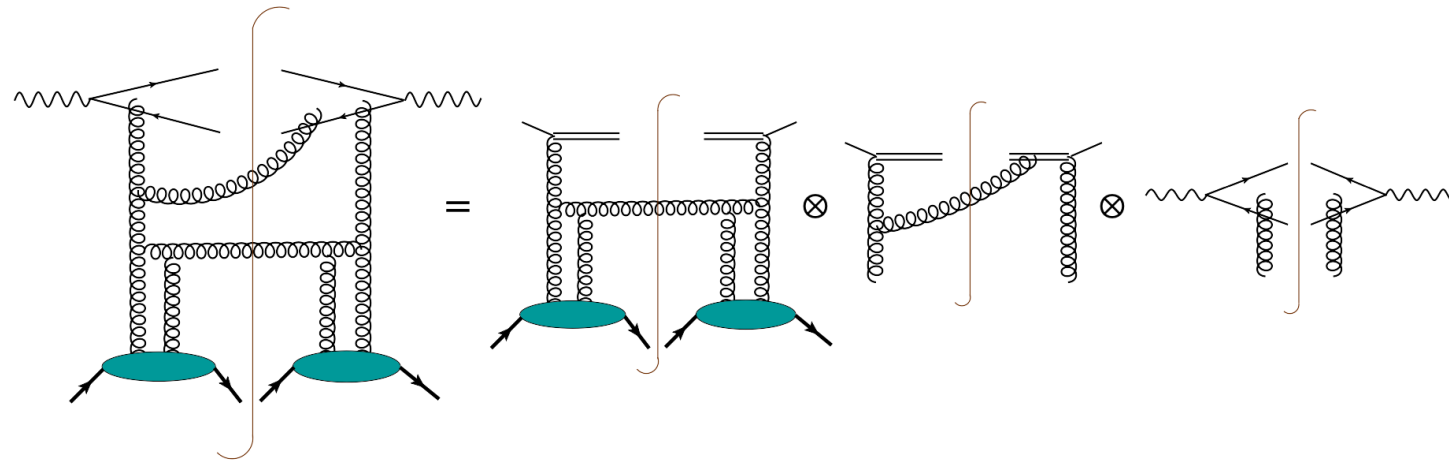
diffractive production of exclusive dijets



- In cases of diffractive tri-jet production, where a semi-hard gluon is emitted towards the target direction and remains undetected, the experimental signature of this process becomes indistinguishable from that of exclusive di-jet production.
- Recent studies have shown that the cross section for coherent tri-jet photo-production significantly surpasses that of exclusive di-jet production [Iancu, Mueller & Triantafyllopoulos '21](#)
- The production of color octet hard quark-anti-quark dijets enables the emission of soft gluons from the initial state. This mechanism significantly influences the total transverse momentum $q_\perp$ distribution of the dijet.

Factorization and resummation

- By treating the gluon DTMD as if it were an ordinary TMD, we assume that the standard TMD factorization framework can be used in the back-to-back region
Hatta, Xiao & Yuan '22



$$\frac{d\sigma}{dy_1 dy_2 d^2\mathbf{P}_\perp d^2\mathbf{q}_\perp} = \sigma_0 x_\gamma f_\gamma(x_\gamma) H_{\gamma^*g}(P_\perp, R, \mu) \int d^2\mathbf{k}_\perp d^2\mathbf{\lambda}_\perp \delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_\perp - \mathbf{\lambda}_\perp) \\ \times S(\mathbf{\lambda}_\perp, R, \mu) \int \frac{dx_\mathbb{P}}{x_\mathbb{P}} x_g G_\mathbb{P}^{\text{unsub}}(x_g, x_\mathbb{P}, k_\perp, \mu).$$

- We refactorize the gluon DTMD as the matching coefficients and the integrated pomeron gluon function

DGLAP evolution of the pomeron gluon DPDF ?

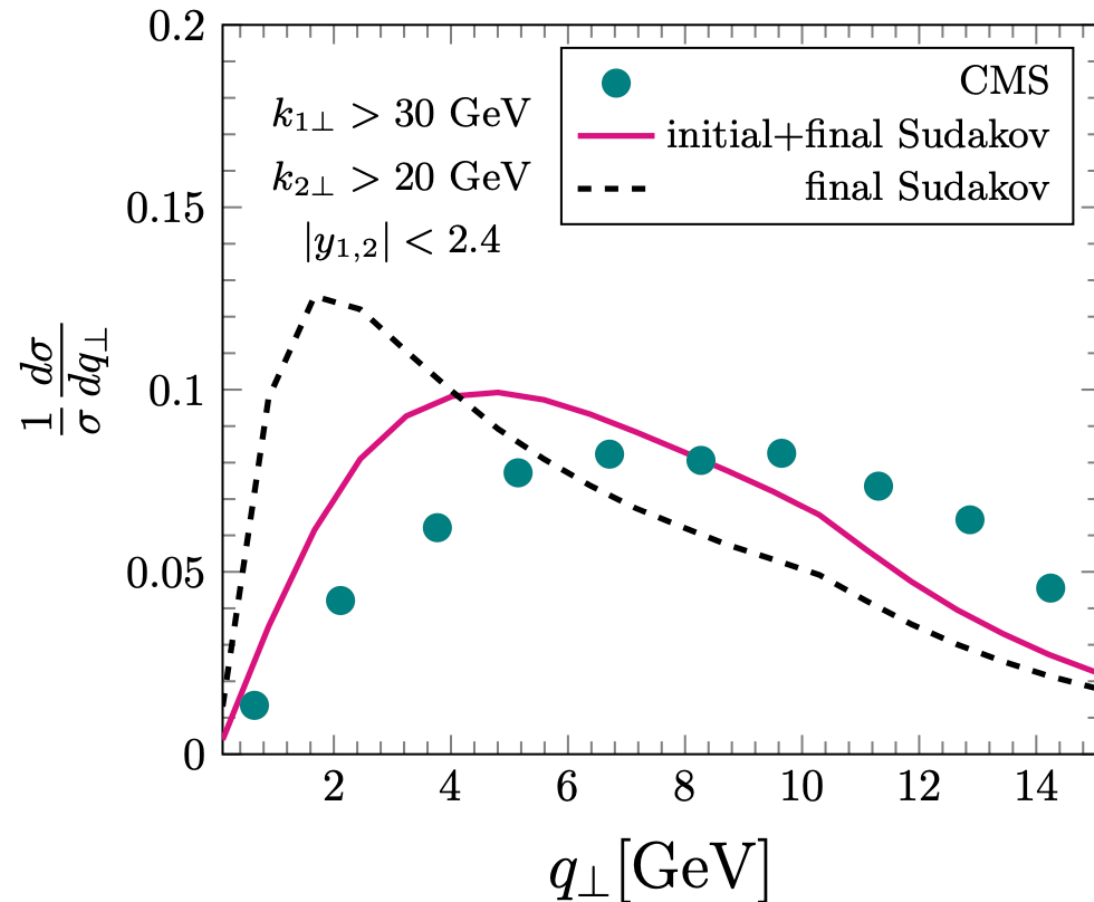
Glauber SCET Rothstein, Stewart, '16

$$G_\mathbb{P}(x_g, x_\mathbb{P}, k_\perp, \mu, \zeta) = \int_{x_g}^1 \frac{dz}{z} I_{g \leftarrow g}(z, k_\perp, \mu, \zeta) G_\mathbb{P}(x_g/z, x_\mathbb{P}, \mu) + G_\mathbb{P}(x_g, x_\mathbb{P}, k_\perp)$$

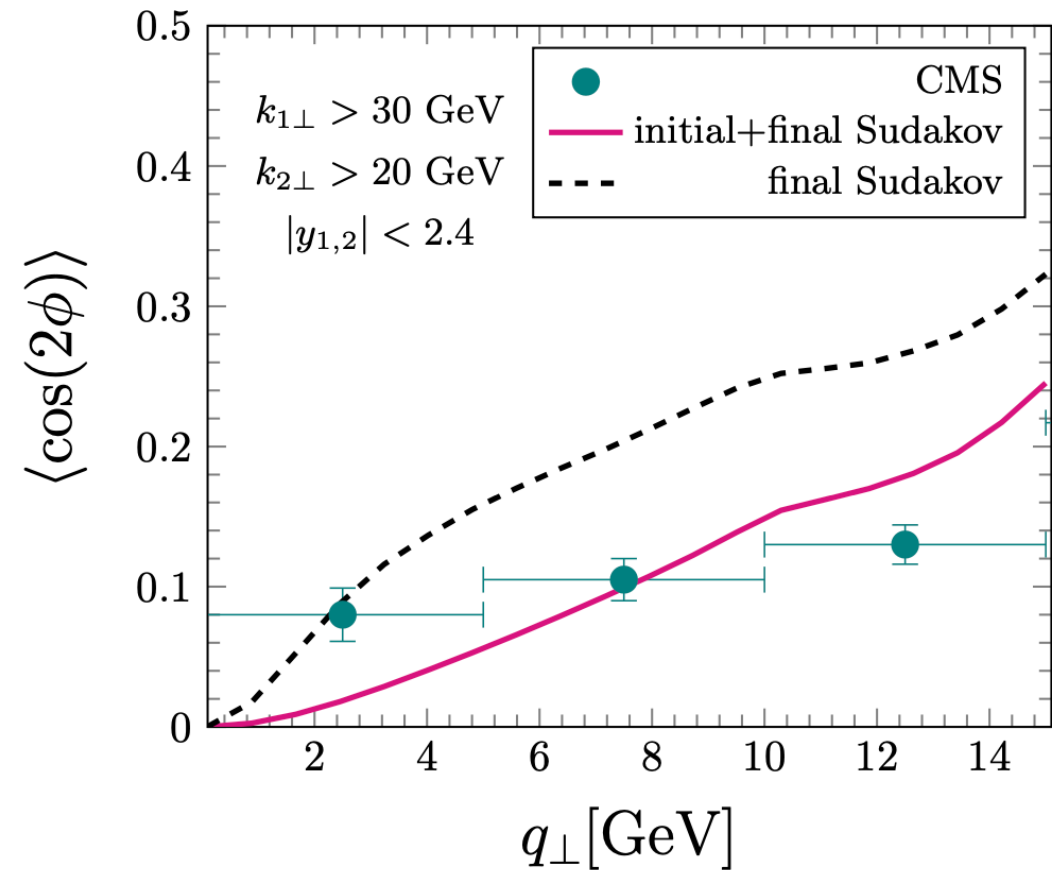
additional static source term in the modified DGLAP equation
Iancu, Mueller, Triantafyllopoulos, & Wei '23

Numerical results and measurements in UPCs

DYS, Y. Shi, C. Zhang, J. Zhou, Y. Zhou '24 JHEP



- Incorporating the initial state gluon radiation offers a more accurate representation of the CMS data
- Difference remains.



$$\langle \cos(2\phi) \rangle \equiv \frac{\int d\mathcal{P}.S. \cos(2\phi) \frac{d\sigma}{dy_1 dy_2 d^2 \mathbf{P}_{\perp} d^2 \mathbf{q}_{\perp}}}{\int d\mathcal{P}.S. \frac{d\sigma}{dy_1 dy_2 d^2 \mathbf{P}_{\perp} d^2 \mathbf{q}_{\perp}}}$$

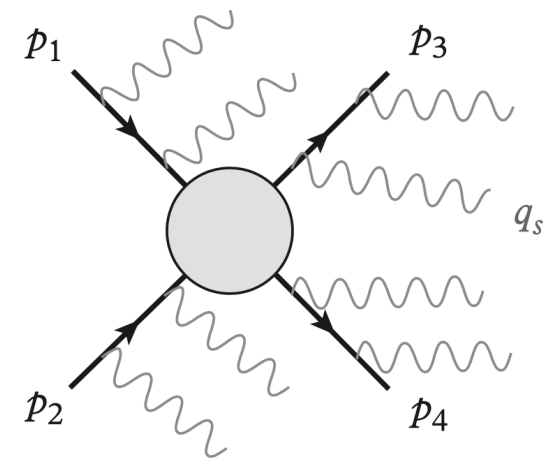
The azimuthal asymmetry: Our result underestimates the asymmetry at low $q_{\perp}$ and overshoots it at high

UPC and heavy quarks (fermion)

Soft effective theory in QED

- When we talk about electron–electron scattering, we really measure the inclusive process

$$e^-(p_1) + e^-(p_2) \rightarrow e^-(p_3) + e^-(p_4) + X_s(q_s)$$



- It will be sufficient to assume that the total energy fulfills $E_s \ll m_e$
- We will now analyse the above process up to terms suppressed by powers of the expansion parameter $\lambda = E_\gamma / m_e$
- The effective Lagrangian $\mathcal{L}_{\text{eff}}^\gamma = \mathcal{L}_4^\gamma + \frac{1}{m_e^2} \mathcal{L}_6^\gamma + \frac{1}{m_e^4} \mathcal{L}_8^\gamma$

Soft Photons in Electron–Electron Scattering

- The field h_ν cannot describe other fermion lines which have different velocities. To account for all four fermion lines, we need to include four auxiliary fermion fields

$$\mathcal{L}_{\text{eff}} = \sum_{i=1}^4 \bar{h}_{v_i}(x) i v_i \cdot D h_{v_i}(x) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \Delta\mathcal{L}_{\text{int}}, \quad v_i^\mu = p_i^\mu / m_e$$

- We need different fields to represent the electrons along the different directions in the effective theory, while all of these were described by a single field in QED

Soft Photons in Electron–Electron Scattering

- The interaction terms

$$\Delta\mathcal{L}_{\text{int}} = \sum_i C_i(v_1, v_2, v_3, v_4, m_e) \bar{h}_{v_3}(x) \Gamma_i h_{v_1}(x) \bar{h}_{v_4}(x) \Gamma_i h_{v_2}(x),$$

- In principle we could also write down interaction terms involving only two fields such as

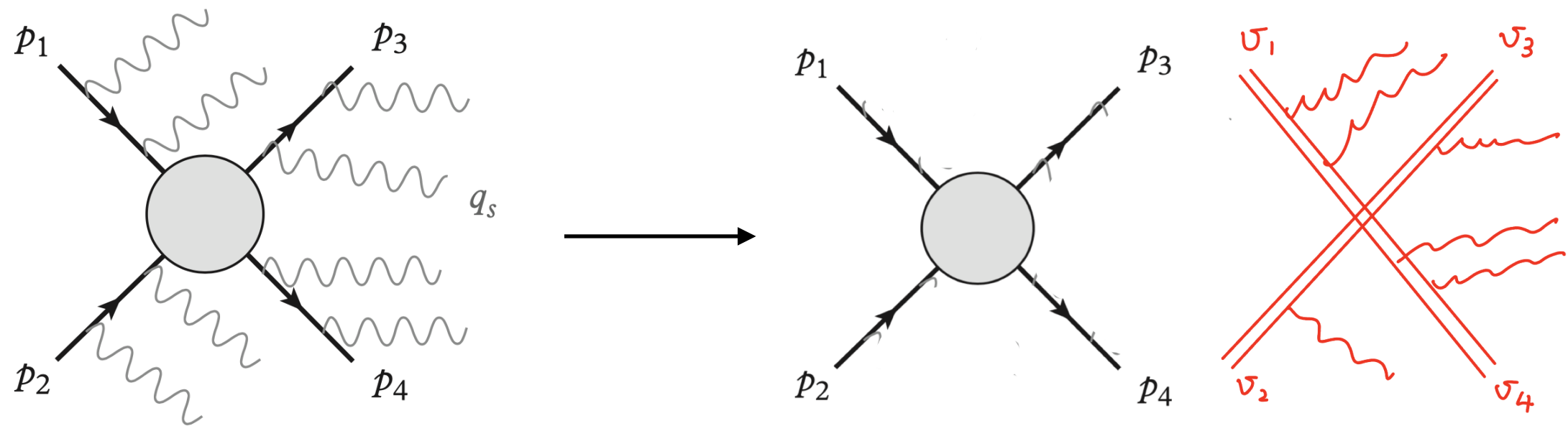
$$\Delta\mathcal{L}'_{\text{int}} = C_{\alpha\beta}(v_1, v_3) h_{v_1}^{\alpha}(x) \bar{h}_{v_3}^{\beta}(x),$$

- Their Wilson coefficients are zero if the velocities are different, since the corresponding operator would describe a process in which a fermion spontaneously changes its velocity, which violates momentum conservation.
- We could also write interactions terms with covariant derivatives or more fields, but these are higher-dimensional operators, whose contributions are suppressed by powers of the electron mass

Effective Lagrangian

- We end up with Wilson lines along the directions of all particles in the scattering process.

$$\Delta\mathcal{L}_{\text{int}} = \sum_i C_i(v_1, v_2, v_3, v_4) \bar{h}_{v_3}^{(0)} \bar{S}_3^\dagger \Gamma_i S_1 h_{v_1}^{(0)} \bar{h}_{v_4}^{(0)} \bar{S}_4^\dagger \Gamma_i S_2 h_{v_2}^{(0)}$$



Azimuthal asymmetries of muon pair production in UPCs

DYS, Zhang, Zhou, Zhou '23

- We consider

$$\gamma(x_1 P + k_{1\perp}) + \gamma(x_2 \bar{P} + k_{2\perp}) \rightarrow l^+(p_1) + l^-(p_2)$$

- The joint impact parameter and transverse momentum dependent cross section

$$\frac{d\sigma_0}{d^2 q_\perp d^2 P_\perp dy_1 dy_2 d^2 b_\perp} = A_0 + A_2 \cos 2\phi + A_4 \cos 4\phi$$

$$q_\perp \equiv p_{1\perp} + p_{2\perp}$$

$$P_\perp \equiv (p_{1\perp} - p_{2\perp})/2$$

- The resummed cross section reads

$$\frac{d\sigma(q_\perp)}{d\mathcal{P}.S.} = \int \frac{d^2 r_\perp}{(2\pi)^2} \left[1 - \frac{2\alpha_e c_2}{\pi} \cos 2\phi_r + \frac{\alpha_e c_4}{\pi} \cos 4\phi_r \right] e^{ir_\perp \cdot q_\perp} e^{-\text{Sud}(r_\perp)} \int d^2 q'_\perp e^{ir_\perp \cdot q'_\perp} \frac{d\sigma_0(q'_\perp)}{d\mathcal{P}.S.}$$

- Sudakov factor in the large mass approximation

$$\text{Sud}(r_\perp) = \frac{\alpha_e}{\pi} \ln \frac{M^2}{m^2} \ln \frac{P_\perp^2}{\mu_r^2}$$

Hatta, Xiao, Yuan & Zhou '21 PRL & PRD

Azimuthal asymmetries of muon pair production in UPCs

DYS, Zhang, Zhou, Zhou '23

- However, at RHIC energy, where the muon mass is roughly the same order of pT or M , the soft factor receives the sizable finite lepton mass correction

$$q_{\perp} \ll m \sim M$$

- The soft factor reads Li, Li, DYS, Yang, Zhu '13, Catani, Grazzini, & Torre '14

$$S(l_{\perp}, m, M) = \sum_{i,j} \int \frac{d^d k}{(2\pi)^{d-1}} \delta(k^2) \theta(k^0) \frac{e^2 v_i \cdot v_j \Pi_{ij}}{v_i \cdot k v_j \cdot k} \delta^{(2)}(l_{\perp} - k_{\perp}) \quad p^{\mu} = m v^{\mu} + k^{\mu}$$

- The resumed formula

$$e^{-\text{Sud}(r_{\perp})} \left[1 + \frac{\alpha_e}{4\pi} (s_{11} + s_{22} + 2s_{12}) \right]$$

$$\text{Sud}(r_{\perp}) = \frac{\alpha_e}{\pi} \ln \frac{P_{\perp}^2}{\mu_r^2} \left(-1 - \frac{1 + \beta^2}{2\beta} \ln \frac{1 - \beta}{1 + \beta} \right)$$

$$\beta = \sqrt{1 - 4m^2/M^2}$$

$$s_{11} = s_{22} = \frac{4c_r}{\sqrt{c_r^2 + 1}} \ln \left(\sqrt{c_r^2 + 1} + c_r \right),$$

$$s_{12} = -\frac{1 + \beta^2}{2\beta} \text{sign}(c_r) [L_{\zeta}[\zeta(c_r, \alpha_r), \alpha_r] - L_{\zeta}[\zeta(-c_r, \alpha_r), \alpha_r]],$$

with

$$c_r = \cos \phi_r P_{\perp} / m, \quad \beta = \sqrt{1 - 4m^2/M^2},$$

$$\alpha_r = \frac{2P_{\perp}^2 \cos^2 \phi_r}{-m^2 + P_{\perp}^2 + (m^2 + P_{\perp}^2) \cosh(y_1 - y_2)},$$

$$\zeta(a, b) = (a + \sqrt{1 + a^2})(a + \sqrt{a^2 + b}),$$

$$L_{\zeta}(a, b) = 2 \left[-\text{Li}_2 \left(\frac{a+b}{b-1} \right) + \text{Li}_2(-a) + \ln(a+b) \ln(1-b) \right] - \ln^2 \left(\frac{a}{a+b} \right) + \frac{1}{2} \ln^2 \left[\frac{a(a+1)}{a+b} \right].$$

Azimuthal asymmetries of muon pair production in UPCs

DYS, Zhang, Zhou, Zhou '23

- However, at RHIC energy, where the muon mass is roughly the same order of pT or M , the soft factor receives the sizable finite lepton mass correction

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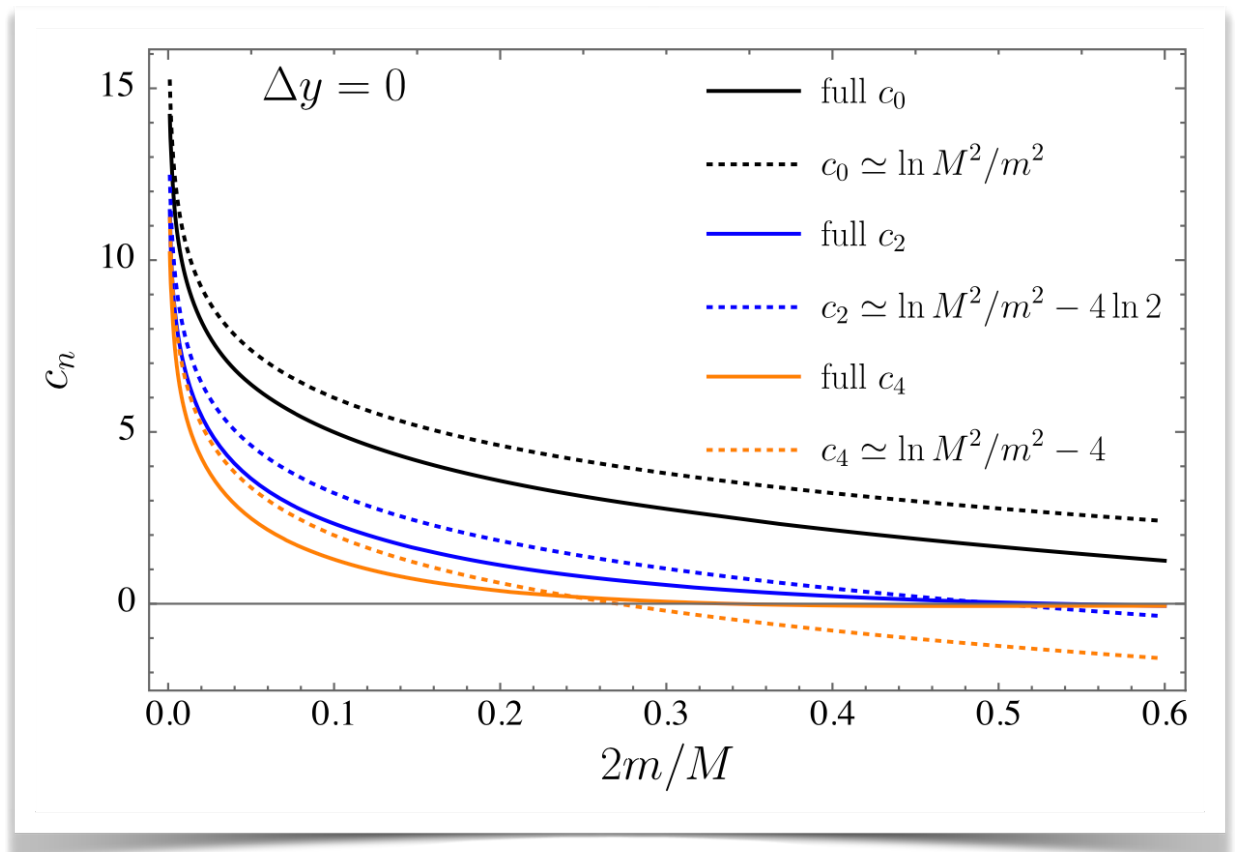
$$S(l_{\perp}, m, M) = \sum_{i,j} \int \frac{d^d k}{(2\pi)^{d-1}} \delta(k^2) \theta(k^0) \frac{e^2 v_i \cdot v_j \Pi_{ij}}{v_i \cdot k v_j \cdot k} \delta^{(2)}(l_{\perp} - k_{\perp}) \quad p^{\mu} = m v^{\mu} + k^{\mu}$$

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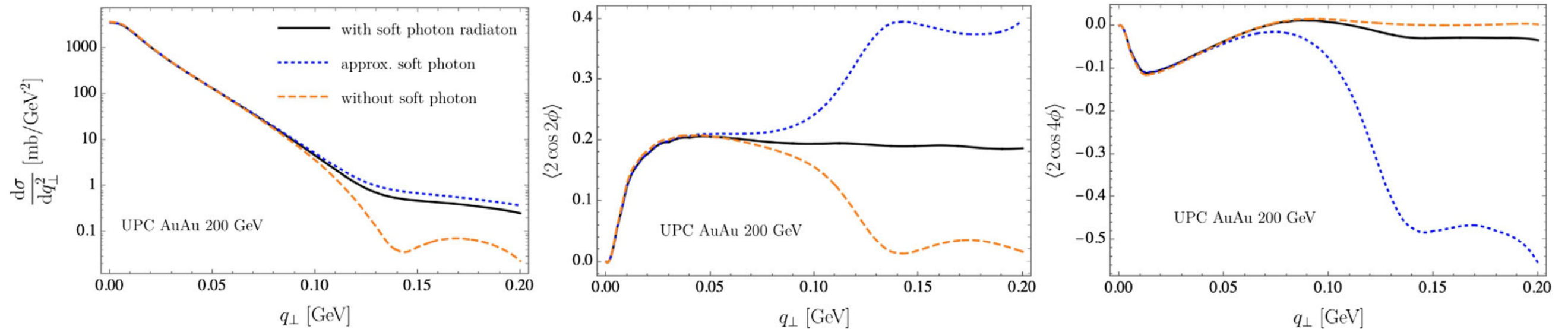
$$\text{Sud}(r_{\perp}) = \frac{\alpha_e}{\pi} \ln \frac{P_{\perp}^2}{\mu_r^2} \left(-1 - \frac{1 + \beta^2}{2\beta} \ln \frac{1 - \beta}{1 + \beta} \right)$$

$$\beta = \sqrt{1 - 4m^2/M^2}$$



Azimuthal asymmetries of muon pair at RHIC

DYS, Zhang, Zhou, Zhou '23



$$\langle \cos(n\phi) \rangle = \frac{\int \frac{d\sigma}{d\mathcal{P}.S.} \cos(n\phi) d\mathcal{P}.S.}{\int \frac{d\sigma}{d\mathcal{P}.S.} d\mathcal{P}.S.}$$

$$400 \text{ MeV} < M < 640 \text{ MeV}$$

$$|y_{1,2}| < 0.8$$

- Compared to the previous study, our resummation formula takes into account the full finite lepton-mass correction.
- Its correction to the unpolarized cross section is tiny
- The contribution from the muon mass effect to the asymmetries is quite **sizeable**

More EFTs

$$q_{\perp} \ll m \sim M$$

$$q_{\perp} \ll m \ll M$$

$$q_{\perp} \sim m \ll M$$

Boost is dynamical

$n(p^2)$

k^2

$\hat{k}_i (i=1,2,3), \hat{J}_i$

Lorentz group

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Xiangdong Ji

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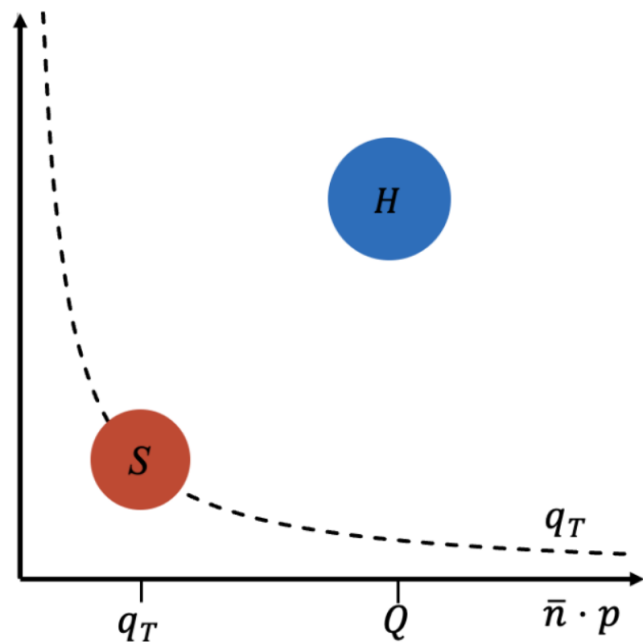
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“Boost in dynamical” - Xiangdong Ji

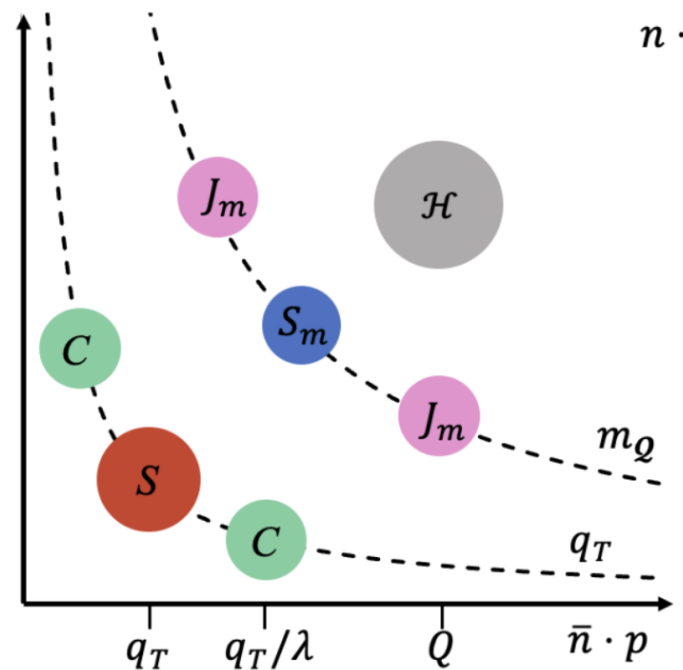
More EFTs

$$q_{\perp} \ll m \sim M$$



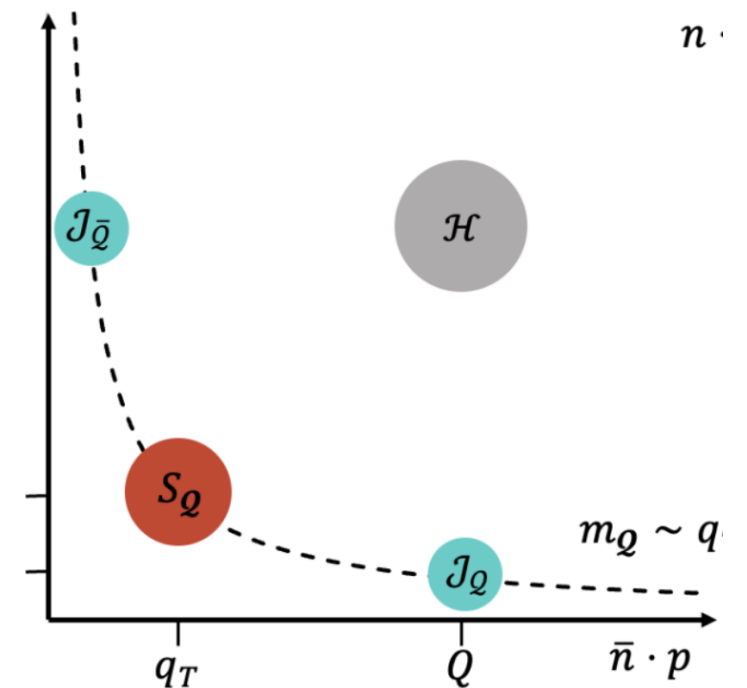
HQET

$$q_{\perp} \ll m \ll M$$



b-HQET

$$q_{\perp} \sim m \ll M$$



SCET-M

“Boost in dynamical” - Xiangdong Ji

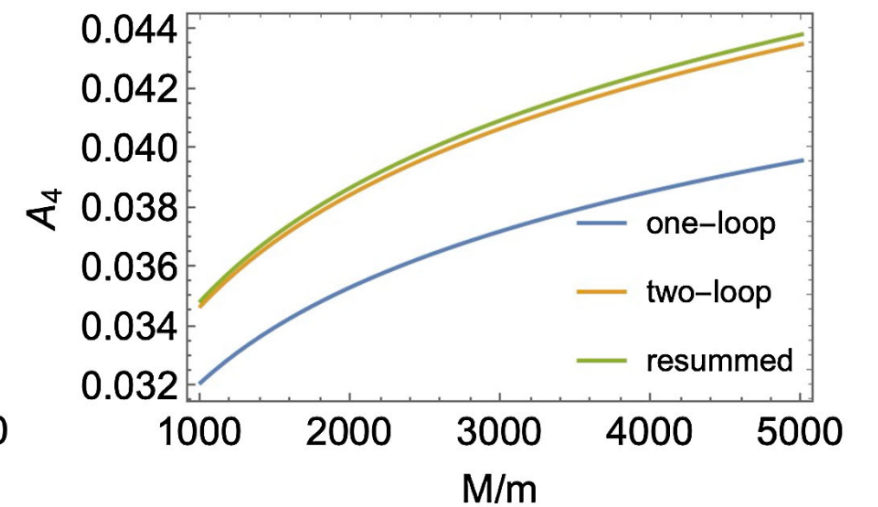
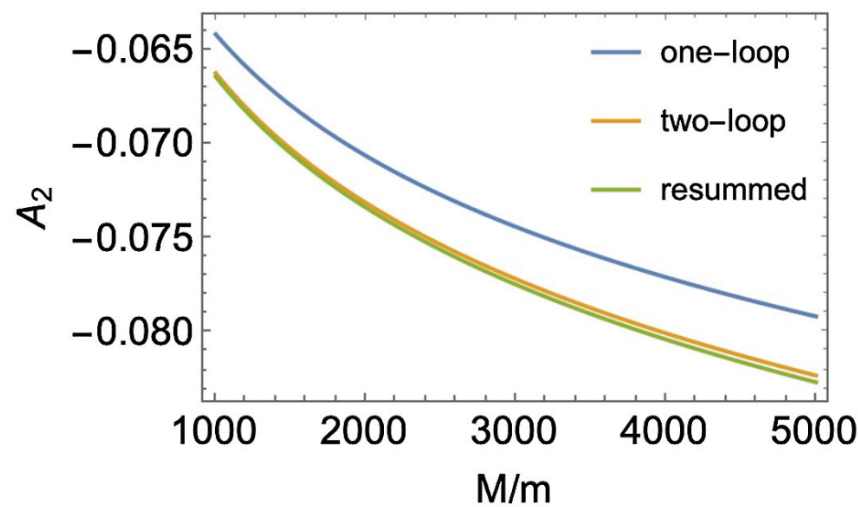
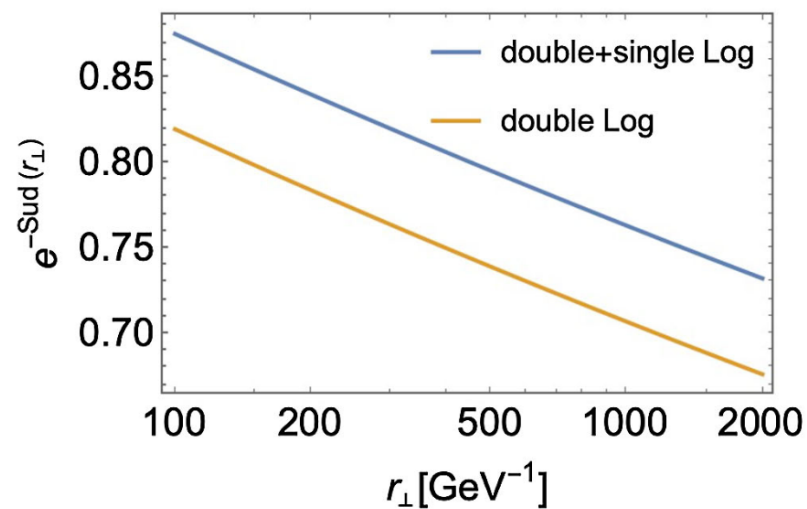
Lepton pair production in UPCs

Toward a precision test of the resummation formalism

DYS, Zhang, Zhou, Zhou '23

Sudakov factor in the limit $q_\perp \ll m \ll M$

$$\text{Sud}(r_\perp) = \int_{\mu_r}^M \frac{d\mu}{\mu} \Gamma_H + 2 \int_{\mu_r}^m \frac{d\mu}{\mu} \Gamma_J + \int_{\mu_r}^{\mu_r m/(2P_\perp)} \frac{d\mu}{\mu} \Gamma_{C_1} + \int_{\mu_r}^{\mu_r m/(2P_\perp)} \frac{d\mu}{\mu} \Gamma_{C_2}$$



$$A_2 \equiv \int_0^{2\pi} d\phi_r \frac{\cos 2\phi_r}{\pi} \exp \left(-\frac{\alpha_e}{\pi} \ln \frac{M^2}{m^2} \ln 4 \cos^2 \phi_r \right)$$

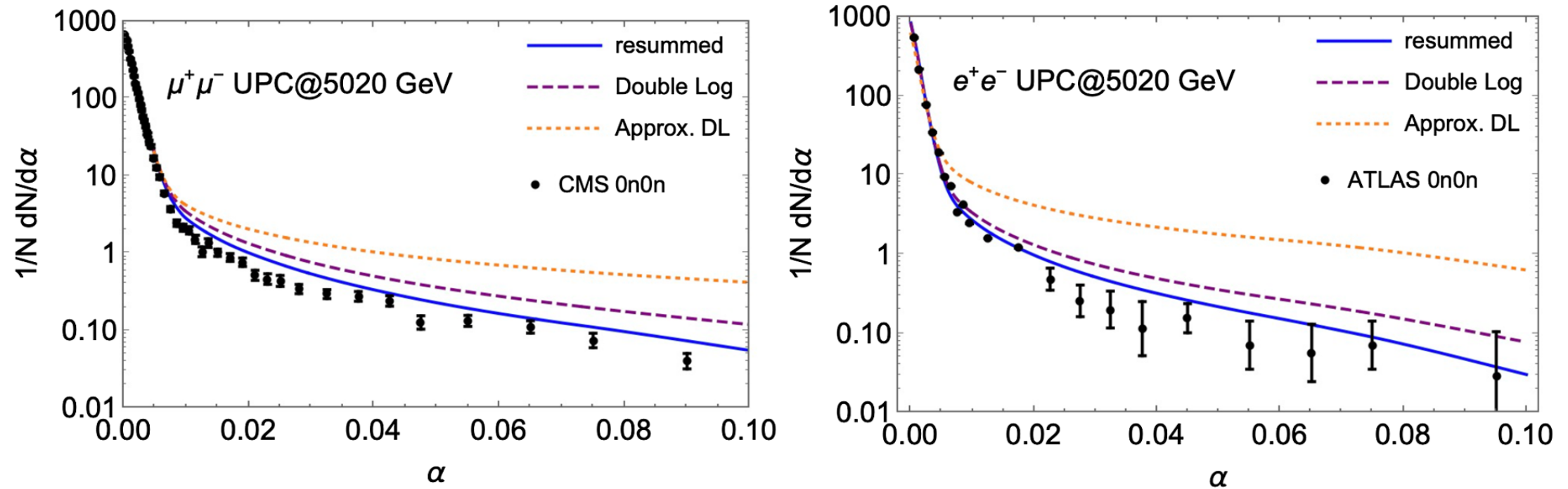
$$A_4 \equiv \int_0^{2\pi} d\phi_r \frac{\cos 4\phi_r}{\pi} \exp \left(-\frac{\alpha_e}{\pi} \ln \frac{M^2}{m^2} \ln 4 \cos^2 \phi_r \right)$$

We expand the above expression at one loop and find they reproduce the coefficients c_2 and c_4

Lepton pair production in UPCs

Toward a precision test of the resummation formalism

DYS, Zhang, Zhou, Zhou '23

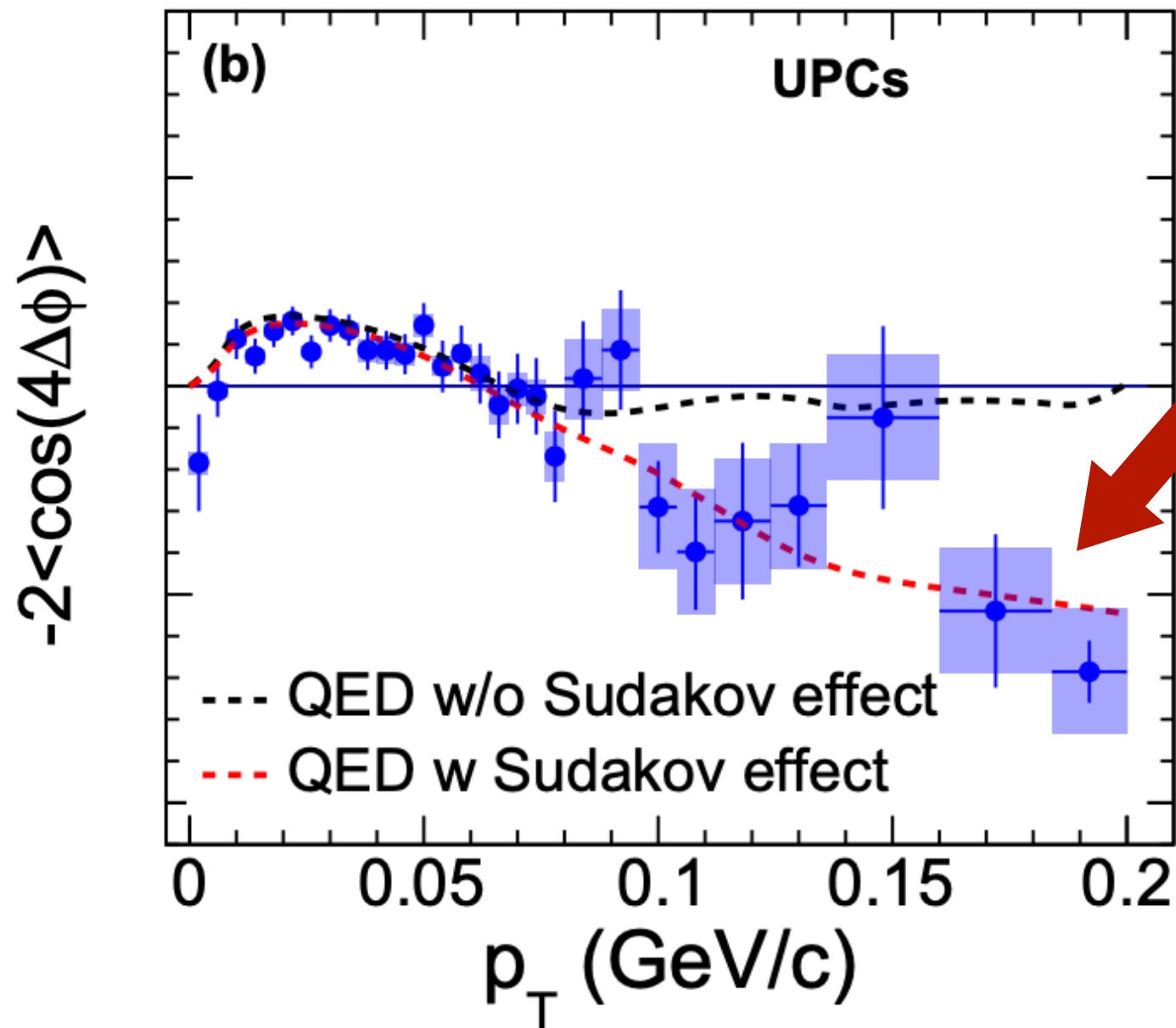


- We improve the accuracy of the previous calculations beyond the double leading logarithm approximation.
- Notably, the single logs arising from the collinear region are greatly enhanced by the small mass of the leptons.
- Our findings demonstrate the accessibility of these subleading resummation effects through the analysis of angular correlations in lepton pairs produced in UPCs.

Precision physics at Heavy ion collisions

Recent UPC measurements at RHIC

Phys.Rev.C 111 (2025) 1, 014909



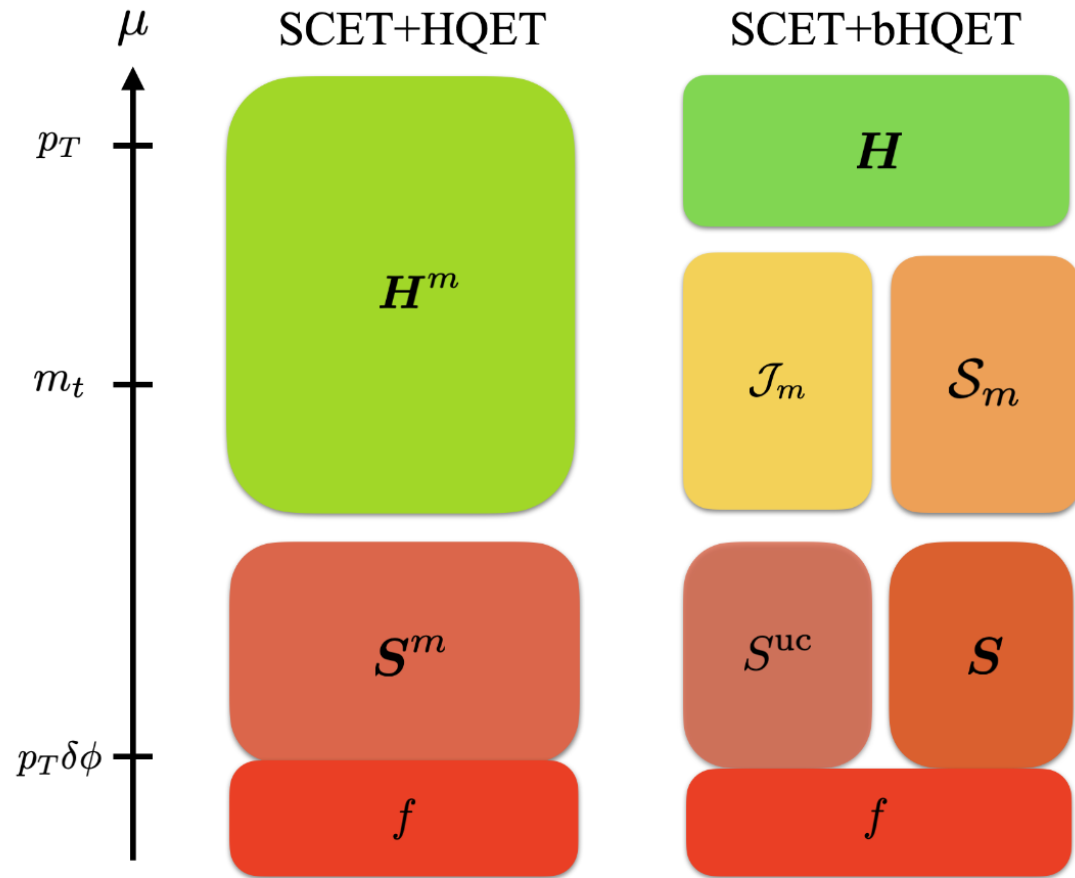
$$\text{Sud}(r_{\perp}) = \int_{\mu_r}^M \frac{d\mu}{\mu} \Gamma_H + 2 \int_{\mu_r}^m \frac{d\mu}{\mu} \Gamma_J + \int_{\mu_r}^{\mu_r m / (2P_{\perp})} \frac{d\mu}{\mu} \Gamma_{C_1} + \int_{\mu_r}^{\mu_r m / (2P_{\perp})} \frac{d\mu}{\mu} \Gamma_{C_2}$$

- At higher p_T , UPC measurements are consistent with QED when the Sudakov effects are included
- RHIC measurements show the importance of including the radiation of final-state soft photons in UPCs.

NNLL' resummation of azimuthal decorrelation for boosted heavy quark pair production at the LHC

Dai, Liu, DYS 26 JHEP

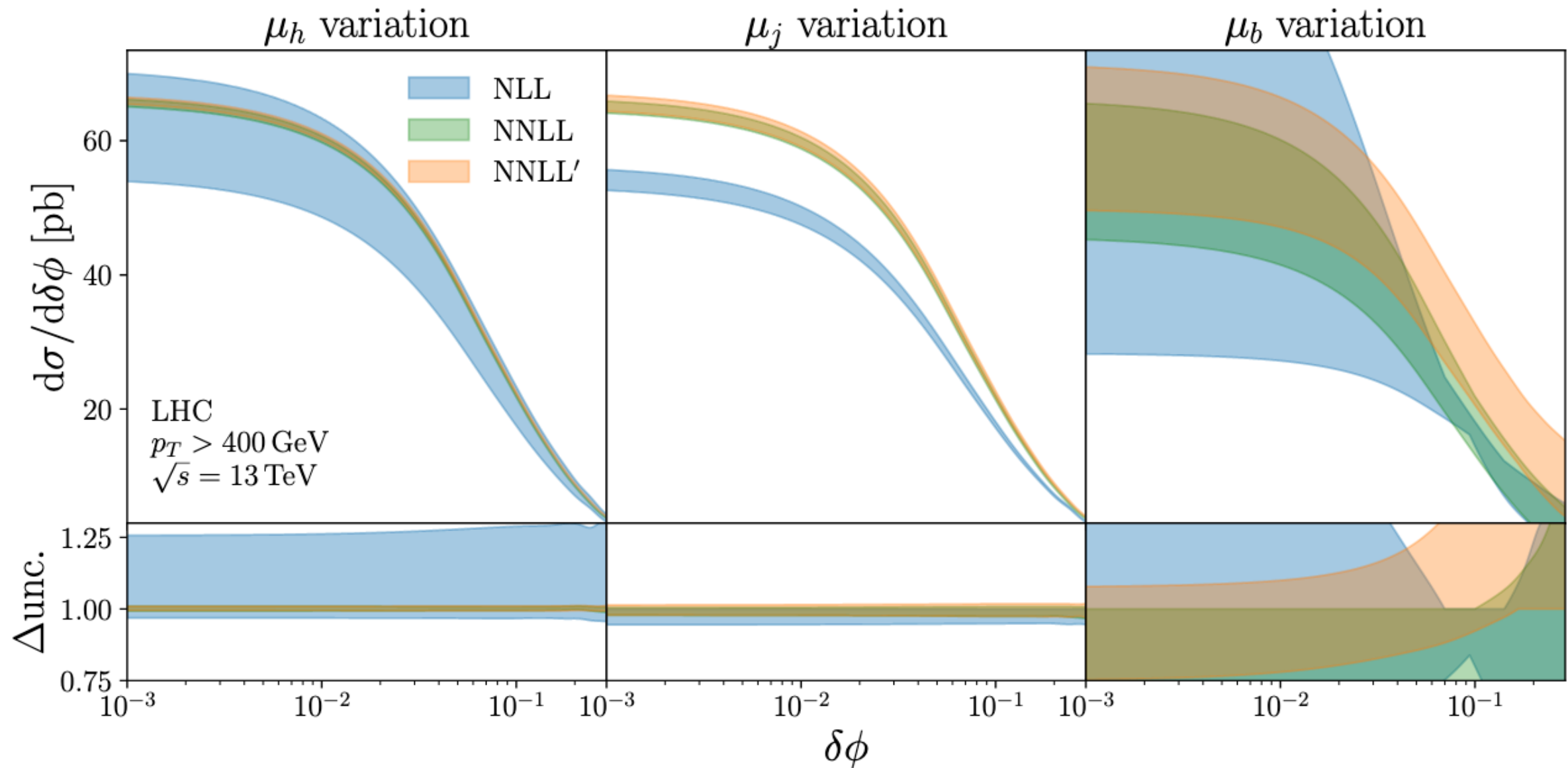
$$N(P_a) + N(P_b) \rightarrow Q(p_c) + \bar{Q}(p_d) + X(p_X)$$



$$\begin{aligned} \frac{d^4\sigma}{dy_c dy_d dp_T^2 d\delta\phi} &= \sum_{ab} \frac{x_a x_b p_T}{16\pi\hat{s}^2} \int_0^\infty \frac{2 db_x}{\pi} \cos(b_x p_T \delta\phi) \\ &\times \tilde{f}_{a/p}(x_a, b_*, \mu_{b_*}, \zeta_{a,i}) \tilde{f}_{b/p}(x_b, b_*, \mu_{b_*}, \zeta_{b,i}) \left(\sqrt{\frac{\zeta_{a,f}}{\zeta_{a,i}}} \right)^{\kappa_a(b_*, \mu_{b_*})} \left(\sqrt{\frac{\zeta_{b,f}}{\zeta_{b,i}}} \right)^{\kappa_b(b_*, \mu_{b_*})} \\ &\times \exp \left\{ - \int_{\mu_j}^{\mu_h} \frac{d\mu}{\mu} \left[\gamma_{\text{cusp}}^{(n_f)}(\mu) C_H \ln \frac{\hat{s}}{\mu^2} + 2\gamma_H^{(n_f)}(\mu) \right] \right\} \\ &\times \exp \left\{ - \int_{\mu_{b_*}}^{\mu_j} \frac{d\mu}{\mu} \left[\gamma_{\text{cusp}}^{(n_l)}(\mu) C_H \ln \frac{\hat{s}}{\mu^2} + 2\gamma_H^{(n_l)}(\mu) \right] \right\} \\ &\times \sum_{KK'} \left\{ \exp \left[- \int_{\mu_j}^{\mu_h} \frac{d\mu}{\mu} \gamma_{\text{cusp}}^{(n_f)}(\mu) (\lambda_K + \lambda_{K'}^*) \right] \right. \\ &\times \exp \left[- \int_{\mu_{b_*}}^{\mu_j} \frac{d\mu}{\mu} \gamma_{\text{cusp}}^{(n_l)}(\mu) (\lambda_K + \lambda_{K'}^*) \right] H_{KK'}(M, y^*, \mu_h) \tilde{S}_{K'K}(b_*, y^*, \mu_{b_*}, \nu_s) \left. \right\} \\ &\times \exp \left[- \int_{\mu_{b_*}}^{\mu_j} \frac{d\mu}{\mu} (\Gamma^{\mathcal{J}_c}(\mu) + \Gamma^{\mathcal{J}_d}(\mu)) \right] \mathcal{J}_c(m_t, \mu_j, \nu_{m,c}) \mathcal{J}_d(m_t, \mu_j, \nu_{m,d}) \\ &\times \exp \left[- \int_{\mu_{b_*}}^{\mu_j} \frac{d\mu}{\mu} \Gamma^{\mathcal{S}_m}(\mu) \right] \mathcal{S}_m(m_t, \mu_j, \nu_m) \left(\frac{\nu_m^2}{\nu_{m,c} \nu_{m,d}} \right)^{\Gamma_{\nu}^{\mathcal{S}_m}(\mu_{b_*})} \\ &\times S_c^{\text{uc}}(b_*, m_t, \mu_{b_*}, \nu_c) S_d^{\text{uc}}(b_*, m_t, \mu_{b_*}, \nu_d) \left(\frac{\nu_s^2}{\nu_c \nu_d} \right)^{\Gamma_{\nu}^{\text{uc}}(\mu_{b_*})} \\ &\times \exp \left[-S_{\text{NP}}^a(b_x, Q_0, \sqrt{\hat{s}}) - S_{\text{NP}}^b(b_x, Q_0, \sqrt{\hat{s}}) \right]. \end{aligned}$$

NNLL' resummation of azimuthal decorrelation for boosted heavy quark pair production at the LHC

Dai, Liu, DYS 26 JHEP

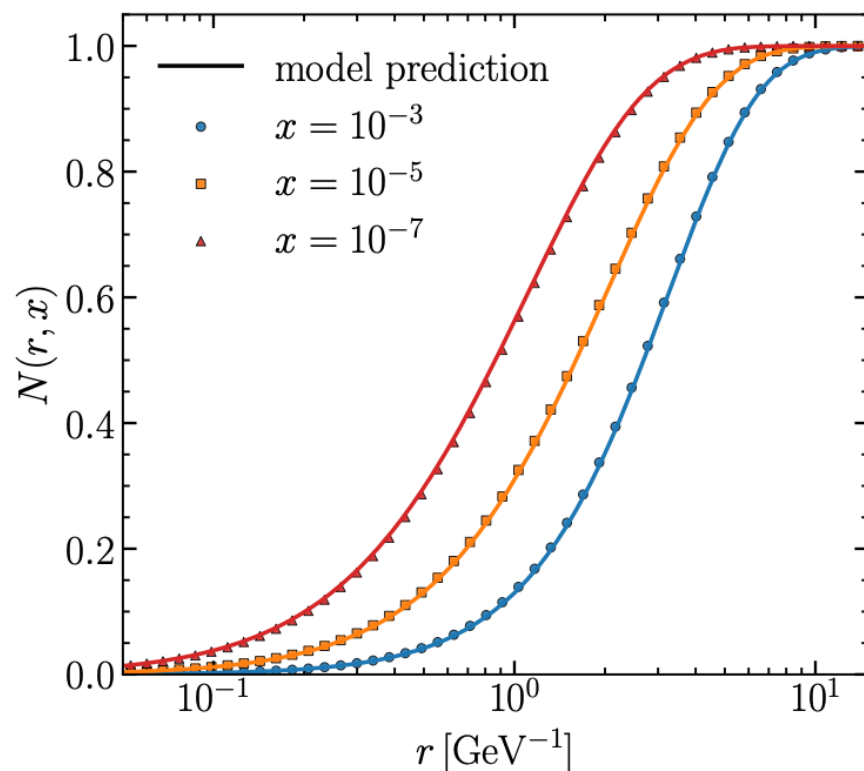


Most precise resummation results for heavy quark azimuthal decorrelation in pp collisions

Small x and AI

Fast BK solution

- GPU accelerated solution to BK equation with resummation corrections from kinematical constraint and DGALP terms Kutak, Li, Stasto, Straka '25
- Physics-Informed Global Extraction of the Universal Small-x Dipole Amplitude Dai, Li, Pang, Qin, Wei, Zhang, Zhao '26
- Fast BK solution with Transformer Gao, Kang, Penttala, DYS '25 JHEP
 - We train a transformer model to learn the energy dependence of the dipole amplitude, replacing repeated time-consuming numerical BK evolutions during the fit by fast emulator predictions.

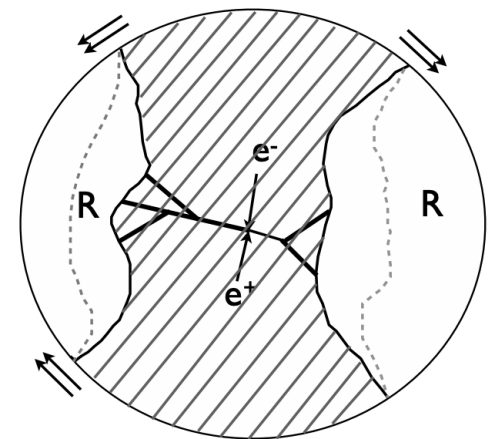


Parameter	Prior range	4-param	5-param
$Q_{s,0}^2$ [GeV ²]	[0.01, 0.11]	$0.063^{+0.001}_{-0.004}$	$0.068^{+0.024}_{-0.015}$
e_c	[0.5, 70.0]	$29.0^{+8.7}_{-3.1}$	$18.7^{+20.1}_{-9.6}$
C^2	[2.0, 20.0]	$4.45^{+0.99}_{-0.59}$	$4.82^{+1.56}_{-2.63}$
$\sigma_0/2$ [mb]	[12.0, 20.0]	$14.5^{+0.6}_{-0.4}$	$14.8^{+0.9}_{-2.4}$
γ	[0.9, 1.3]	1.00 (fixed)	$1.006^{+0.037}_{-0.019}$
χ^2/dof	—	0.854	0.857

Fast simulation: BMS v.s. BK

- NGL resummation is governed by the non-linear BMS equation, physically interpreted as a dipole shower. Banfi, Marchesini, Smye '02

$$Q \frac{\partial}{\partial Q} G_{ab}(Q) = \int \frac{d^2 \Omega_k}{4\pi} \bar{\alpha}_s W_{ab}(k) [u(k) G_{ak}(Q) G_{kb}(Q) - G_{ab}(Q)]$$



- This non-linear NGL evolution connects to the BK equations via conformal duality Caron-Huot '18

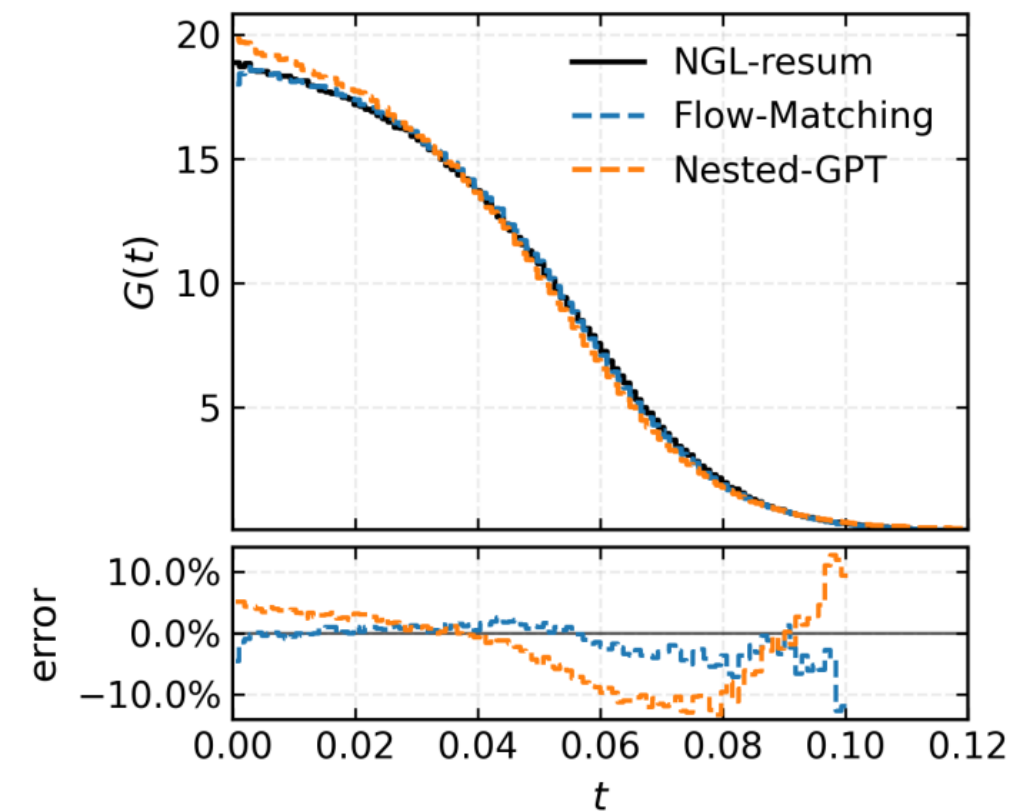
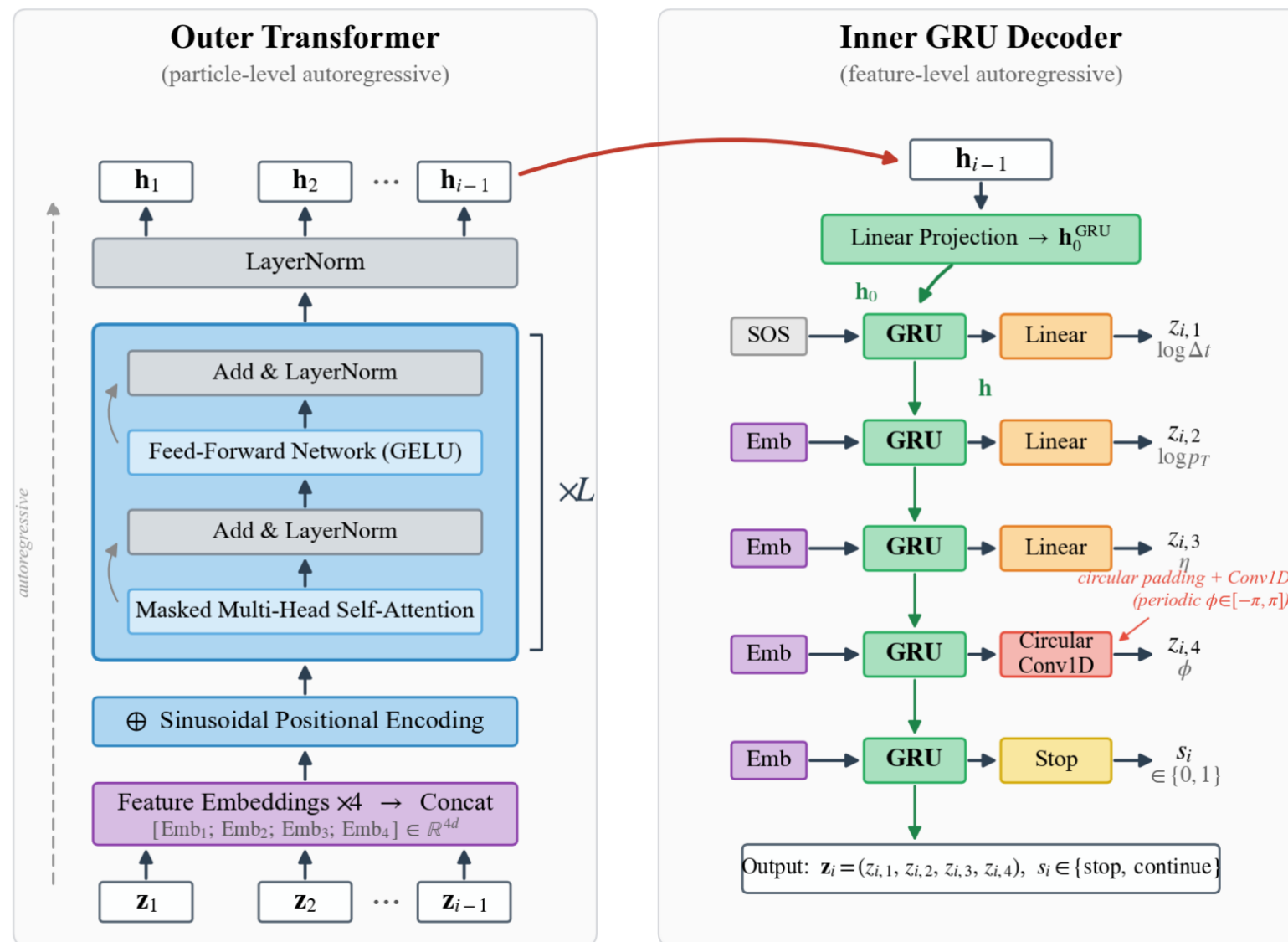
$$\alpha_{ij} \equiv -\frac{\beta_i \cdot \beta_j}{2} \rightarrow (x_i - x_j)_\perp^2, \quad \int \frac{d^2 \Omega_i}{4\pi} \rightarrow \frac{d^2 x_\perp}{\pi}.$$

- BMS equation can be solved via Monte Carlo dipole shower. Dasgupta, Salam '01, Balsiger, Becher, DYS '18

Nested-GPT for variable-multiplicity parton showers: A case study in the resummation of non-global logarithms

Li, DYS, Shi, Sun '26

- We introduce Nested-GPT, a hierarchical autoregressive Transformer architecture for simulating the variable-multiplicity parton-shower histories.



Application in small- x shower? Shi, Wei, Zhou '23

Summary

- **Gap fraction**
 - **Superseding logs from Glauber regions**
- **Correlations**
 - **Precision calculation**
 - **TMD background**
 - **nPDF, nuclear shadowing**
- **Heavy quark**
 - **Fermion mass effects**
- **Small x and AI**
 - **Fast global fit**
 - **Fast simulation**

Thank you