Physics Observables for Small x

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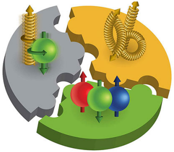
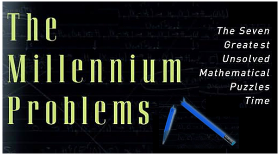
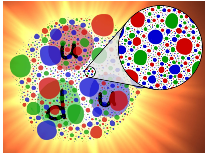
The Chinese University of Hong Kong, Shenzhen

July 6-9, 2026

Emergent Phenomena in QCD



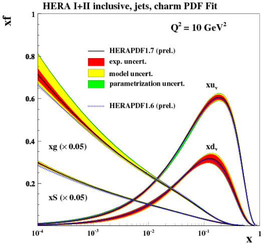


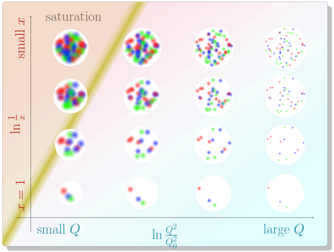


- How does the spin of proton arise from quarks and gluons? (Spin puzzle)
- What are the emergent properties of dense gluon system?
- How does proton mass arise? Mass gap: million dollar question.
- How does gluon bind quarks and gluons inside proton?
- We need the 3D information of the quark and gluon inside the proton!

Saturation Physics (Color Glass Condensate)

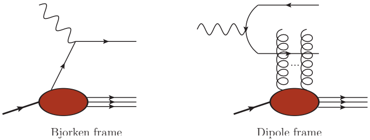
Describe Emergent Phenomenon of the ultra-dense QCD cold matter.





- Gluon density grows rapidly as x gets small. BFKL evolution!
- Resummation of the $\alpha_s \ln \frac{1}{x} \Rightarrow$ BFKL equation.
- Many gluons with fixed size packed in a confined hadron, gluons overlap and recombine $\Rightarrow$ Non-linear QCD dynamics (BK/JIMWLK) $\Rightarrow$ ultra-dense gluonic matter
- Saturation = Multiple Scattering (MV model) + Small-x (high energy) evolution

Dual Descriptions of Deep Inelastic Scattering



Bjorken frame

$$F_2(x,Q^2) = \sum_q e_q^2 x \left[f_q(x,Q^2) + f_{\bar{q}}(x,Q^2)\right].$$

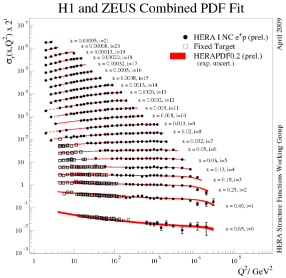
Dipole frame [A. Mueller, 01; Parton Saturation-An Overview]

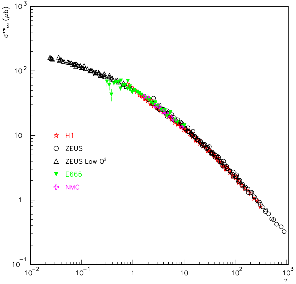
$$F_2(x,Q^2) = \sum_f e_f^2 \frac{Q^2}{4\pi^2\alpha_{\rm em}} S_\perp \int_0^1 \mathrm{d}z \int \mathrm{d}^2 r_\perp \left|\psi\left(z, r_\perp, Q\right)\right|^2 \left[1 - S^{(2)}\left(Q_s r_\perp\right)\right]$$

- Bjorken: partonic picture is manifest. Saturation shows up as limit of number density.
- Dipole: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for scattering. Convenient to resum the multiple gluon interactions.

Geometrical Scaling in DIS

[Golec-Biernat, Stasto, Kwiecinski; 01, Munier, Peschanski, 03]

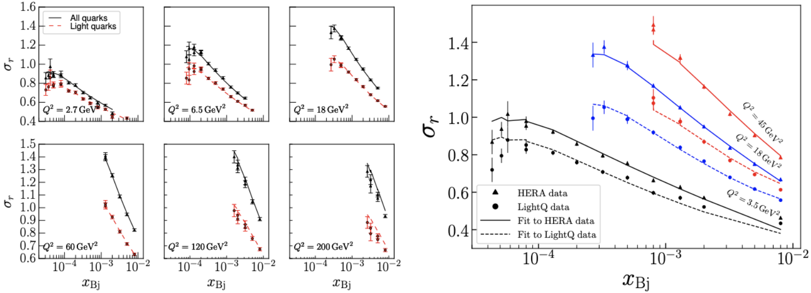




- Define $Q_s^2(x) = (x_0/x)^\lambda \text{GeV}^2$ with $x_0 = 3.04 \times 10^{-3}$ and $\lambda = 0.288$. All low-x data with $x \leq 0.01$ and $Q^2 \leq 450GeV^2$ is function of a single variable $\tau = Q^2/Q_s^2$.

NLO CGC meets HERA data

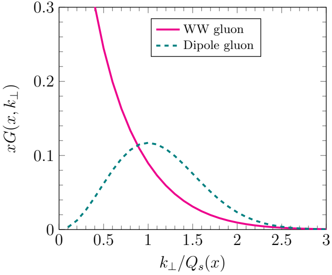
[Beuf, Hänninen, T. Lappi, and H. Mäntysaari, 20]



- Dipole-amplitude fits to HERA inclusive data using the full NLO impact factor combined with an improved BK evolution. Robust predictions for future EIC data.
- The needs for extension to heavy quark case at NLO. [Beuf, Lappi, Paatelainen, 22]

A Tale of Two Gluon Distributions[1]

In small-x physics, there are two gluon distributions:[Kharzeev, Kovchegov, Tuchin; 03]
I. Weizsäcker Williams gluon distribution([Kovchegov, Mueller, 98] and MV model)
II. Color Dipole gluon distributions: (known for many years)



- In MV model, these two gluon distributions are different.
- Same perturbative tail when $k_\perp \gg Q_s$.
  "A Tale of Two Gluon Distributions" $\Rightarrow$
  "A Tail of Two Gluon Distributions" [B. Zajc]
- Integrated gluon numbers are the same $(AxG_N(x))$
  indicating number conservation, thus no saturation.
- Which gluon distribution to use in a given process?
- Why are there exactly two gluon distributions?
  Generalized Universality

[1] From Y. Kovchegov and C. Dickens.

A Tale of Two Gluon Distributions

In terms of operators (TMD def. [Bomhof, Mulders and Pijlman, 06]), two gauge invariant gluon definitions: [Dominguez, Marquet, Xiao and Yuan, 11]

I. Weizsäcker Williams gluon distribution: conventional gluon distributions

$$xG_{\text{WW}}(x, k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \text{Tr}\langle P|F^{+i}(\xi^-, \xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distributions:

$$xG_{\text{DP}}(x, k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \text{Tr}\langle P|F^{+i}(\xi^-, \xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

- Modified Universality for Gluon Distributions: $\times \Rightarrow$ Do Not Appear. $\surd \Rightarrow$ Apppear.

	Inclusive	Single Inc	DIS dijet	γ +jet	dijet in pA
xG_{WW}	$\times$	$\times$	$\surd$	$\times$	$\surd$
xG_{DP}	$\surd$	$\surd$	$\times$	$\surd$	$\surd$

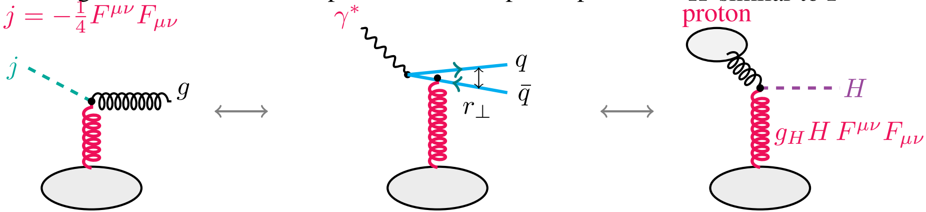
- From CGC perspective, Complementary physics missions in pA and eA collisions.

How to Probe Gluons in the Color Glass Condensate?

Gedanken probe
color-singlet current
$j = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$

DIS realization
virtual photon $\Rightarrow$ a compact dipole

Higgs-like probe
H similar to F^2



CGC target
current strikes
a gluon out

CGC target
keep $q\bar{q}$
close together

CGC target
Higgs gets k_T
from gluons

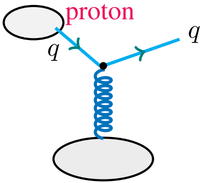
- A scalar current $j \sim F^{\mu\nu} F_{\mu\nu}$ is an idealized direct probe of the gluon field.
- In DIS, γ^* first splits into a compact $q\bar{q}$ dipole as a probe to CGC gluon field.
- Higgs production is analogous due to the effective Higgs-gluon coupling $H F^{\mu\nu} F_{\mu\nu}$.

How to Probe Gluons in the Color Glass Condensate?

Quark production

$q + p/A_{\mathrm{CGC}} \to q + X$

single forward hadron



CGC target

dipole gluon

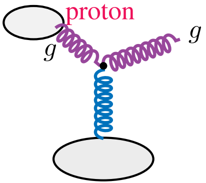
xG_{DP}

⟷

Gluon production

$g + p/A_{\mathrm{CGC}} \to g + X$

single forward hadron



CGC target

adjoint dipole

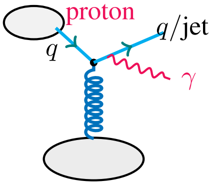
gluon field

⟷

Quark + photon production

$q + A_{\mathrm{CGC}} \to q + \gamma + X$

forward γ + jet



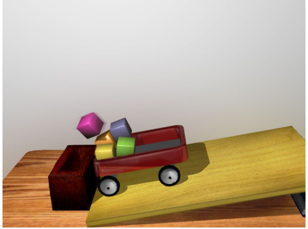
CGC target

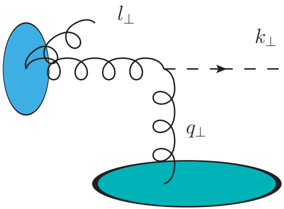
dipole gluon

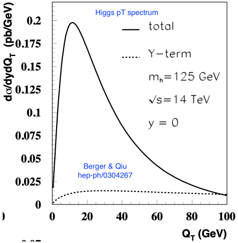
in γ + jet

- Forward Probes (q/g) of the Dipole Gluon Distribution
- The dense target is described by dipole Wilson correlators; probes dipole gluon.
- Either prompt photon or photon + jet correlation. [Jalilian-Marian, Gelis, 02; Jalilian-Marian, Rezaeian, 12; Stasto, Xiao, Zaslavsky, 12]

Sudakov effects (parton showers in MC generators) 101





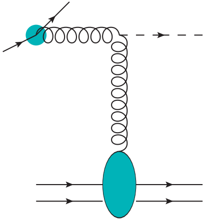


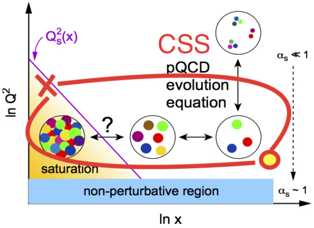
- When an energetic gluon is annihilated to create the color neutral and heavy Higgs particle, all other gluons in its wave function due to inertia are forced to be produced.
- Sudakov factor $S \equiv e^{-\mathcal{S}_{\text{sud}}} \simeq 1 - \mathcal{S}_{\text{sud}}$ is the probability of less than $k_\perp$.
- Use prob conservation $S + T = 1$ with T the prob of radiation $l_\perp > k_\perp$

$$T = \frac{\alpha_s N_c}{\pi} \int_{k_\perp^2}^{M^2} \frac{dl_\perp^2}{l_\perp^2} \int_{l_\perp^2/M^2}^{1} \frac{d\xi}{\xi} = \frac{\alpha_s N_c}{2\pi} \ln^2 \frac{M^2}{k_\perp^2} \simeq \mathcal{S}_{\text{sud}}$$

Sudakov resummation in cold nuclear medium

[A. Mueller, BX and F. Yuan, 13] Multiple scales problem. $k_\perp^2 \ll Q^2 \sim M^2 \ll s$.



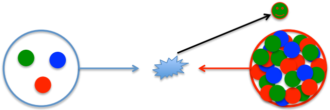


- The CSS equation resums Sudakov logarithms $\left[\frac{\alpha_s C_R}{2\pi} \ln^2 \frac{Q^2}{k_\perp^2}\right]^n + \cdots$ with $Q^2 \gg k_\perp^2$
- Small-x evolution resums $\left[\frac{\alpha_s N_c}{2\pi} \ln \frac{1}{x}\right]^n$, with $x = \frac{Q^2}{s} \ll 1$
- Joint resummation of Small-x $\left[\frac{\alpha_s N_c}{2\pi} \ln \frac{1}{x}\right]^n$ and Sudakov factor $\left[\frac{\alpha_s C_R}{2\pi} \ln^2 \frac{Q^2}{k_\perp^2}\right]^n$ logs.

Forward hadron production in pA collisions

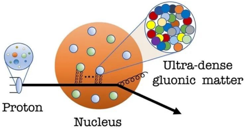
[Dumitru, Jalilian-Marian, 02] Dilute-dense factorization at forward rapidity

$$\frac{d\sigma_{\mathrm{LO}}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[x_1 q_f(x_1,\mu)\mathcal{F}_{x_2}(k_\perp)D_{h/q}(z,\mu) + x_1 g(x_1,\mu)\tilde{\mathcal{F}}_{x_2}(k_\perp)D_{h/g}(z,\mu)\right].$$



projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}}e^{+y} \sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}}e^{-y} \ll 1$ gluon

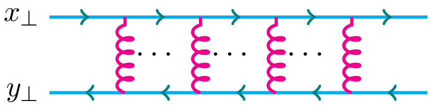


- Use proton (deuteron) to probe the dense gluon in the heavy nuclei target.
- Proton: Collinear PDFs and FFs (Strongly depends on μ^2).; Nucleus: Small-x gluon!
- Full NLO + Resummation CGC: [Chirilli, BX and Yuan, 12; Stasto, Xiao, Zaslavsky, 14; Shi, Wang, Wei, Xiao, 22]

Wilson Lines in Color Glass Condensate Formalism

The Wilson loop (color singlet dipole) in McLerran-Venugopalan (MV) model

$$\frac{1}{N_c}\left\langle \mathrm{Tr}U(x_\perp)U^\dagger(y_\perp)\right\rangle = e^{-\frac{Q_s^2(x_\perp - y_\perp)^2}{4}}$$



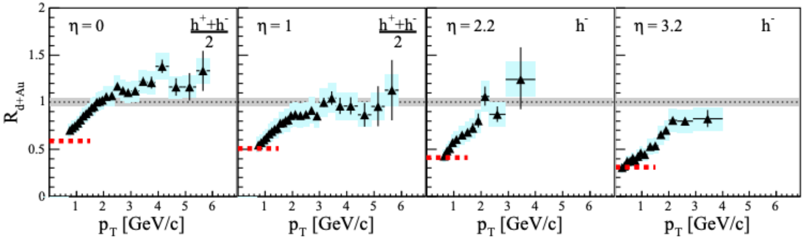
- Dipole amplitude $S^{(2)}$ produces quark k_T spectrum $\frac{dN}{d^2k_\perp}$ via Fourier transform

$$\mathcal{F}(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} \frac{1}{N_c}\left\langle \mathrm{Tr}U(x_\perp)U^\dagger(y_\perp)\right\rangle \propto \frac{1}{\pi Q_s^2} e^{-k_\perp^2/Q_s^2}.$$

- Higher point correlators such as quadrupole, etc.

d+Au collisions at RHIC

$$R_{d+Au} = \frac{1}{\langle N_{\rm coll}\rangle} \frac{d^2N_{d+Au}/d^2p_T d\eta}{d^2N_{pp}/d^2p_T d\eta}$$



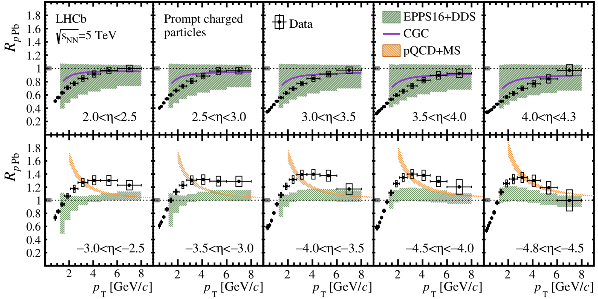
BRAHMS

- Cronin effect at middle rapidity ($\Downarrow$ in low p_T and $\Uparrow$ at high p_T)
- Rapidity evolution of the nuclear modification factors R_{d+Au}
- Promising evidence for gluon saturation effects

New LHCb Results

[R. Aaet al. (LHCb Collaboration), Phys. Rev. Lett. 128 (2022) 142004]

$$R_{pPb} = \frac{1}{\langle N_{\text{coll}} \rangle} \frac{d^2N_{p+Pb}/d^2p_T d\eta}{d^2N_{pp}/d^2p_T d\eta}$$

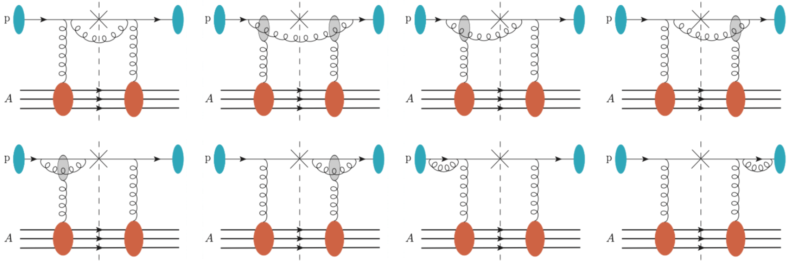


- Forward rapidity: Nuclear effects Small-x suppression
- Backward rapidity: Cronin enhancement

- Rapidity evolution of the nuclear modification factors R_{pPb} similar to RHIC

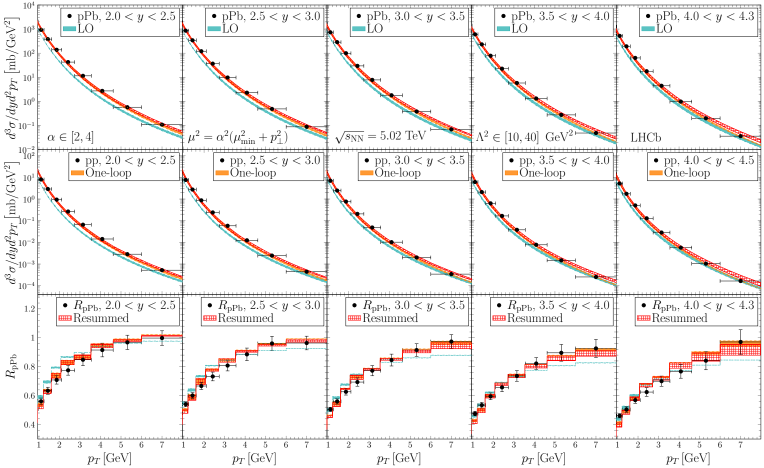
NLO diagrams in the $q \to q$ channel

[Chirilli, BX and Yuan, 12; Shi, Wang, Wei, Xiao, 21]



- Take into account real (top) and virtual (bottom) diagrams together!
- Non-linear multiple interactions inside the grey blobs!
- Integrate over gluon phase space ⇒Rapidity and Collinear Divergences!.

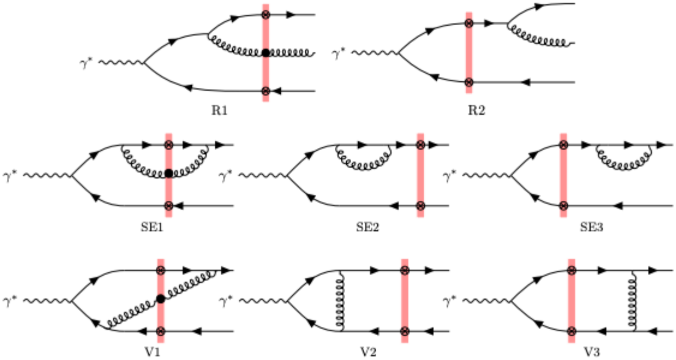
Comparison with the new LHCb data

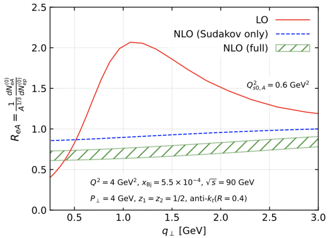


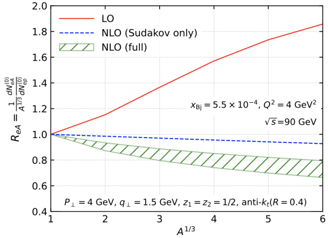
- LHCb data: 2108.13115
- LO and NLO Predictions from CGC. [Shi, Wang, Wei, Xiao, 21]
- Consistent rapidity evolution of the suppression R_{pPb}.

Reliable NLO DIS dijets

[Caucal, Salazar, Schenke, Stebel and Venugopalan, 2304.03304; 2308.00022]; [Taels, Altinoluk, Beuf and Marquet, 2204.11650]

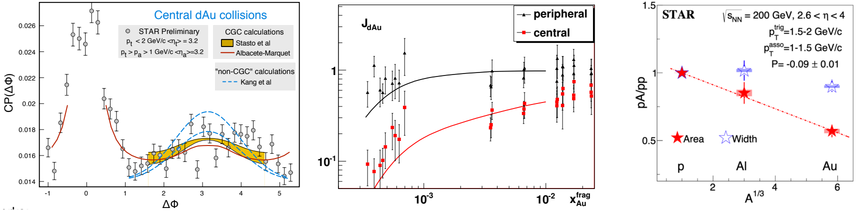






Dihadron correlations in dAu collisions

$$C(\Delta\phi) = \frac{\int_{|p_{1\perp}|,|p_{2\perp}|} \frac{d\sigma^{pA\to h_1 h_2}}{dy_1 dy_2 d^2p_{1\perp} d^2p_{2\perp}}}{\int_{|p_{1\perp}|} \frac{d\sigma^{pA\to h_1}}{dy_1 d^2p_{1\perp}}} \qquad J_{dA} = \frac{1}{\langle N_{\text{coll}}\rangle} \frac{\sigma_{dA}^{\text{pair}}/\sigma_{dA}}{\sigma_{pp}^{\text{pair}}/\sigma_{pp}}$$

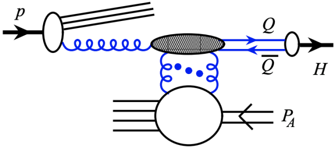


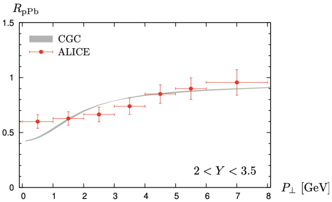
Comparing to STAR and PHENIX data

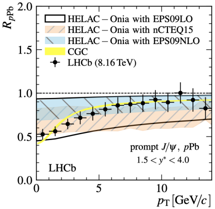
- CGC calculation [Marquet, 09; Marquet, Albacete, 10; Stasto, BX, Yuan, 11]
- Di-hadron correlations in pp and pA collisions at STAR. ([Xiaoxuan Chu])

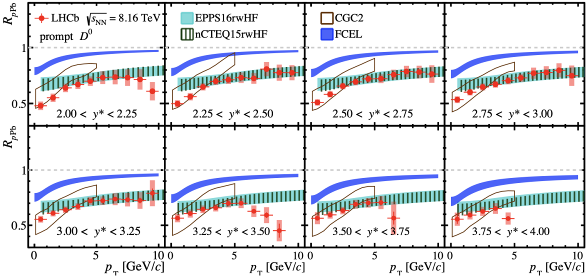
Forward Heavy Meson Results from LHC

- [LHCb Collaboration, 1706.07122; 2205.03936] Forward rapidity: Small-x suppression; Rapidity evolution similar to RHIC;
- [Qiu, Sun, Xiao, Yuan, 13; Ducloue, Lappi, and Mansysaari, 16]

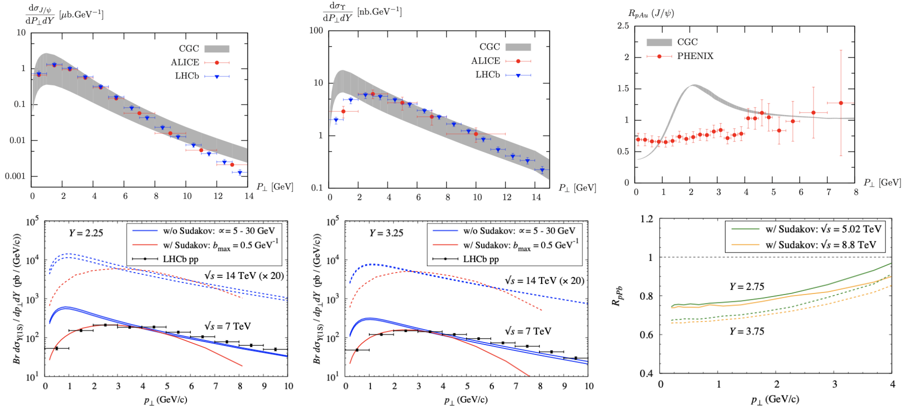








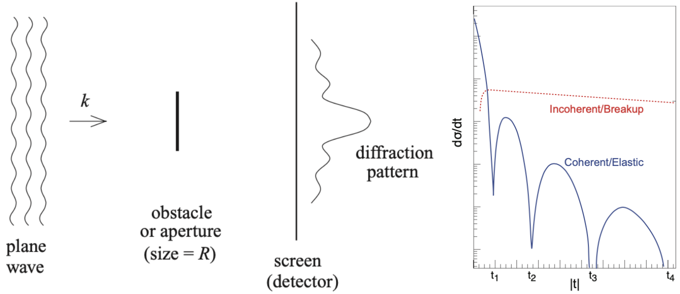
Comparing J/Ψ and Υ spectrum



- [Ducloue, Lappi, and Mansysaari, 15] Υ has higher p_T than J/Ψ. Need Sudakov!
- [Watanabe, Xiao, 15] Sudakov explains spectra and R_{pA}. Similar to NLO dijet.

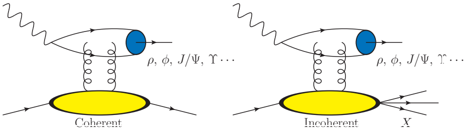
An analogy to Fraunhofer Diffraction in Optics

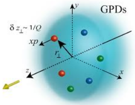
[QCD at high energy, Kovchegov and Levin, 12]

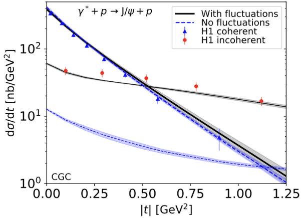


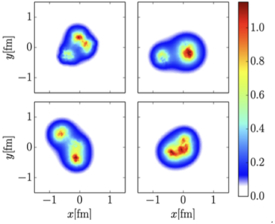
- Treat the hadron target in DIS as a black disk. [Joseph von Fraunhofer, 1821]
- Similar pattern in optics ($\theta_i^{\min} \sim 1/(kR)$) and high energy QCD $t_i \sim \frac{1}{R^2}$.
- Two difference: 1. σ sensitive to gluon distribution; 2. Breakup of the target.
- Use diffractive scattering to study gluon spatial distribution.

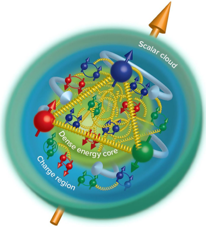
Diffractive vector meson production







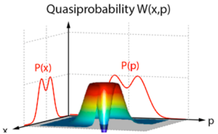


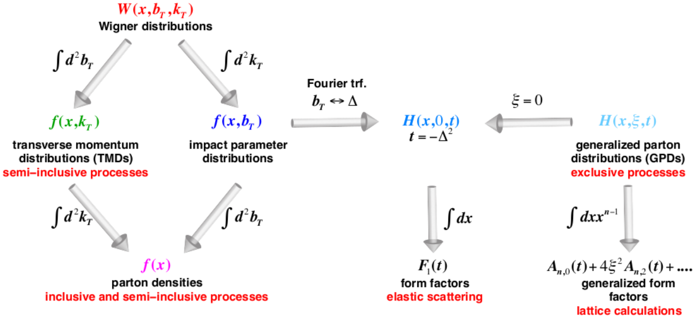


- Inco DVMP $\Rightarrow$ fluctuation. [Mantysaari, Schenke, 16; Mantysaari, Roy, Salazar, Schenke, 20]
- B. Duran, *et al.*, Nature **615** (2023) no.7954, 813-816. A mass radius notably smaller than charge radius?
- Probing gluon mass radius and GFF [Caldwell and Kowalski, 10; Hagiwara, Tong, Xiao, 24]

3D Tomography of Proton

Wigner distributions [Belitsky, Ji, Yuan, 04] ingeniously encode all quantum information of how partons are distributed inside hadrons.

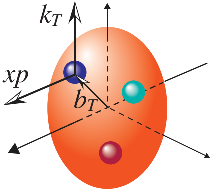


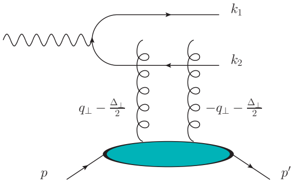


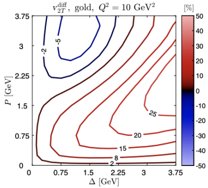
Can we measure Wigner distributions?

- Can we measure Wigner distribution/GTMD? Yes, we can!
- Diffractive back-to-back dijets in ep/eA collisions. [Hatta, Xiao, Yuan, 16] Link
- Further predictions of asymmetries due to correlations.

$$xW_g^T(x,\vec{q}_\perp;\vec{b}_\perp) = x\mathcal{W}_g^T \quad \text{Symmetric part}$$
$$+ \; 2\cos(2\phi)x\mathcal{W}_g^\epsilon + \cdots \quad \text{Anisotropies}$$





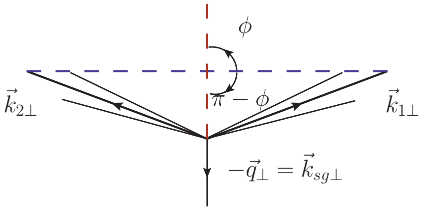


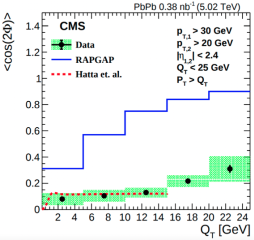
- Study of the elliptic anisotropy.[Mäntysaari, Mueller, Salazar and Schenke, 20] Link

CMS: Dijet photoproduction in UPC (PbPb)

[Hatta, Xiao, Yuan, Zhou, 21] ▸ Link [CMS, arXiv:2205.00045] ▸ Link

$$\gamma + \text{Pb} \to \text{Jet} + \text{Jet} + \text{Pb}$$





1. Large asymmetries between $q_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp}$ and difference $\vec{k}_{1\perp} - \vec{k}_{2\perp}$ observed!
2. Asymmtries due to FS gluon radiations [Hatta, Xiao, Yuan, Zhou, 21] ▸ Link

$$g^2 \int \frac{d^3k_g}{(2\pi)^3 2E_{k_g}} \delta^{(2)}(q_\perp + k_{g\perp}) C_F \frac{2k_1 \cdot k_2}{k_1 \cdot k_g k_2 \cdot k_g} = \frac{C_F \alpha_s}{\pi^2 q_\perp^2} \left[c_0^{\text{diff}} + 2\cos(2\phi)\, c_2^{\text{diff}} + \ldots \right].$$

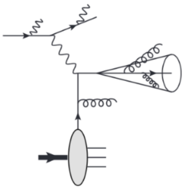
Harmonics of Parton Saturation in Lepton-Jet Correlations at EIC

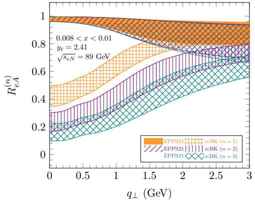
[Tong, Xiao, Zhang, 23] ▸ Link Use Fourier Harmonics to probe saturation

$$\frac{d^5\sigma(\ell P \to \ell' J)}{dy_\ell d^2 P_\perp d^2 q_\perp} = \sigma_0 \int \frac{b_\perp db_\perp}{2\pi} \sum_q e_q^2 x f_q(x, b_\perp) e^{-\mathrm{Sud_P}}$$

$$\left[J_0(q_\perp b_\perp) + \sum_{n=1}^{\infty} 2\cos(n\phi) \alpha_s \frac{C_F c_n(R)}{n\pi} J_n(q_\perp b_\perp) \right],$$

$$R_{eA}^{(n)} = \frac{\langle \cos n\phi \rangle_{eA}}{\langle \cos n\phi \rangle_{ep}}$$

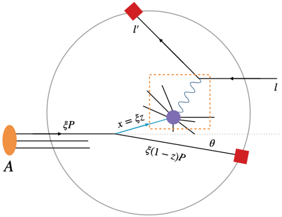


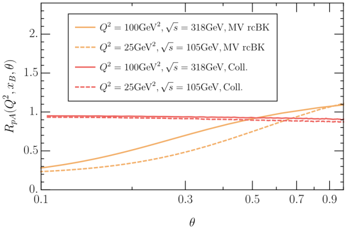


- Significant modification of the asymmetries ⇒ compelling evidence for saturation.
- Potential decisive evidence for saturation. (Nuclear PDF effects cancelled out)

Nucleon Energy Correlator as an Inclusive Probe of Saturation

[H.-Y. Liu, X. Liu, Pan, Yuan, Zhu, 23] ▸ Link Measure forward energy flow in DIS

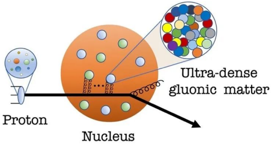


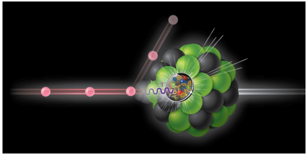


- Weighted DIS observable: $\Sigma(Q^2, x_B, \theta) = \sum_i \int d\sigma \, \frac{E_i}{E_A} \, \delta(\theta^2 - \theta_i^2)$.
- Collinear expectation: $\theta^2\Sigma \simeq$ flat. VS CGC expectation: suppressed for $Q\theta \lesssim Q_s$.
- The turning point gives $Q_s \sim Q\theta_{\text{turn}}$, with stronger suppression for nuclei.

DIS-like inclusive observable sensitive to intrinsic $q_\perp \sim Q_s$.

Summary: From Hints to Precision Tests of Saturation





- Low-x QCD probes an emergent state of dense gluons:

  Gluon growth $\Rightarrow$ Recombination $\Rightarrow$ Saturation.
- Level I: evidence consistent with saturation
  - HERA geometric scaling and small-x DIS fits;
  - RHIC/LHC forward suppression and rapidity evolution of R_{pA};
  - Diffractive vector meson measurements.
- Level II: more discriminating tests at EICs
  - Measure saturation broadening/decorrelation in dijets, lepton-jets, and NEEC;
  - New quantum observables such as Entanglement, etc.
- Goal: establish nonlinear small-x QCD through quantitative, multi-observable tests.