Probing the Color Glass Condensate at the EIC

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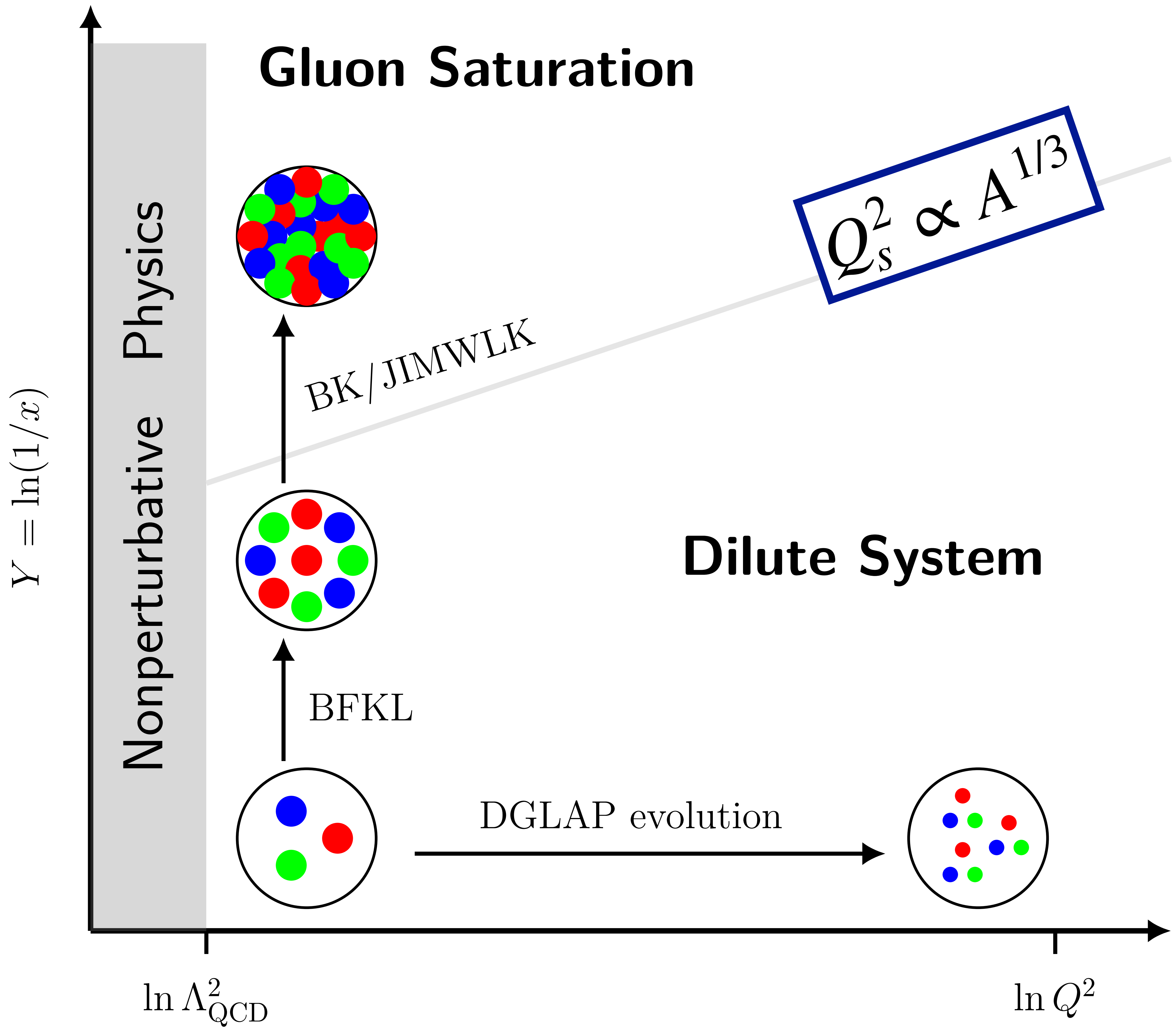
Contents

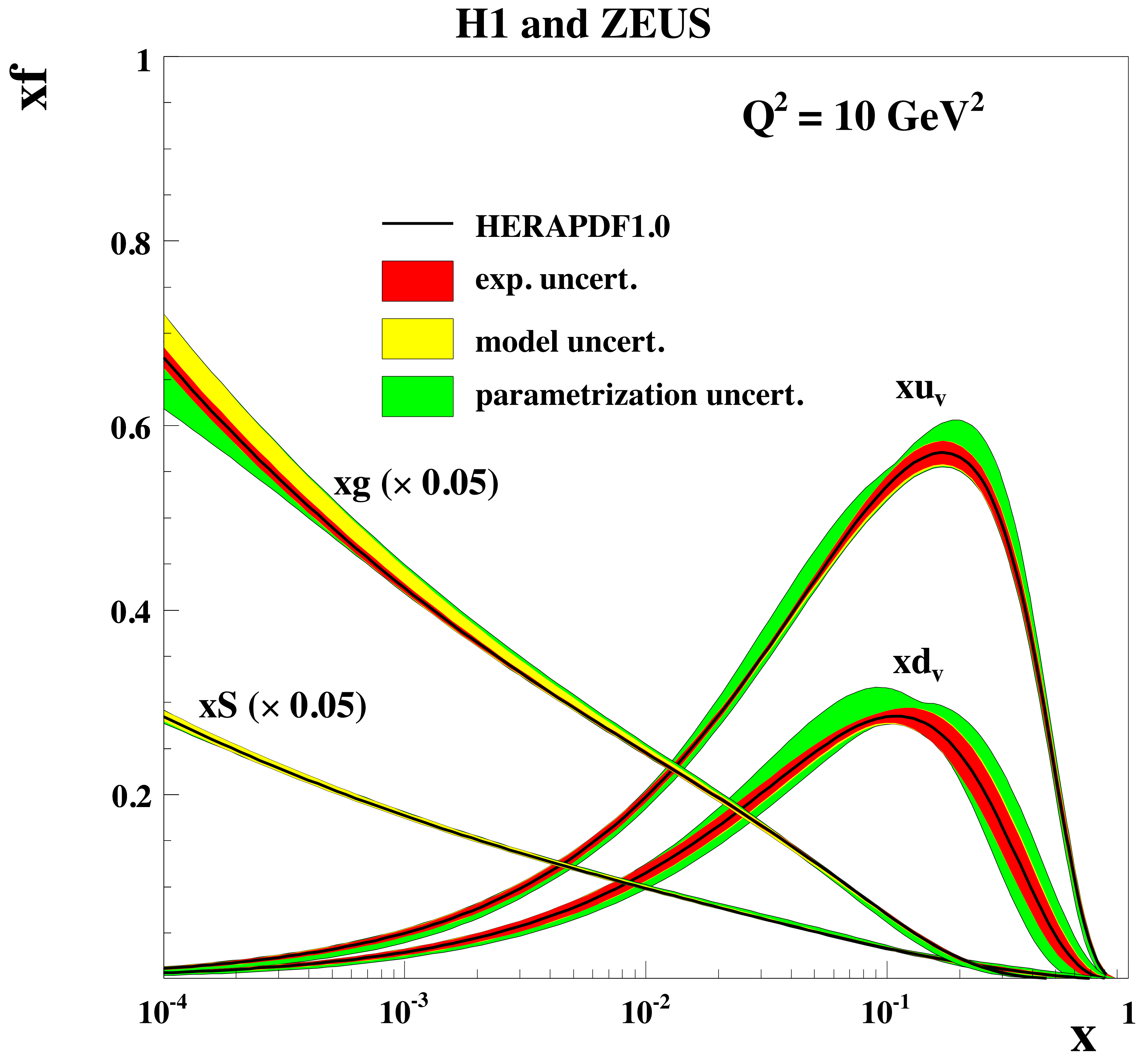
- Structure Function
- Dihadron Correlation
- Diffractive Dijet Production

See also Qiping & Dingyu's talk

Introduction

Gluon density rapidly increases at small-x





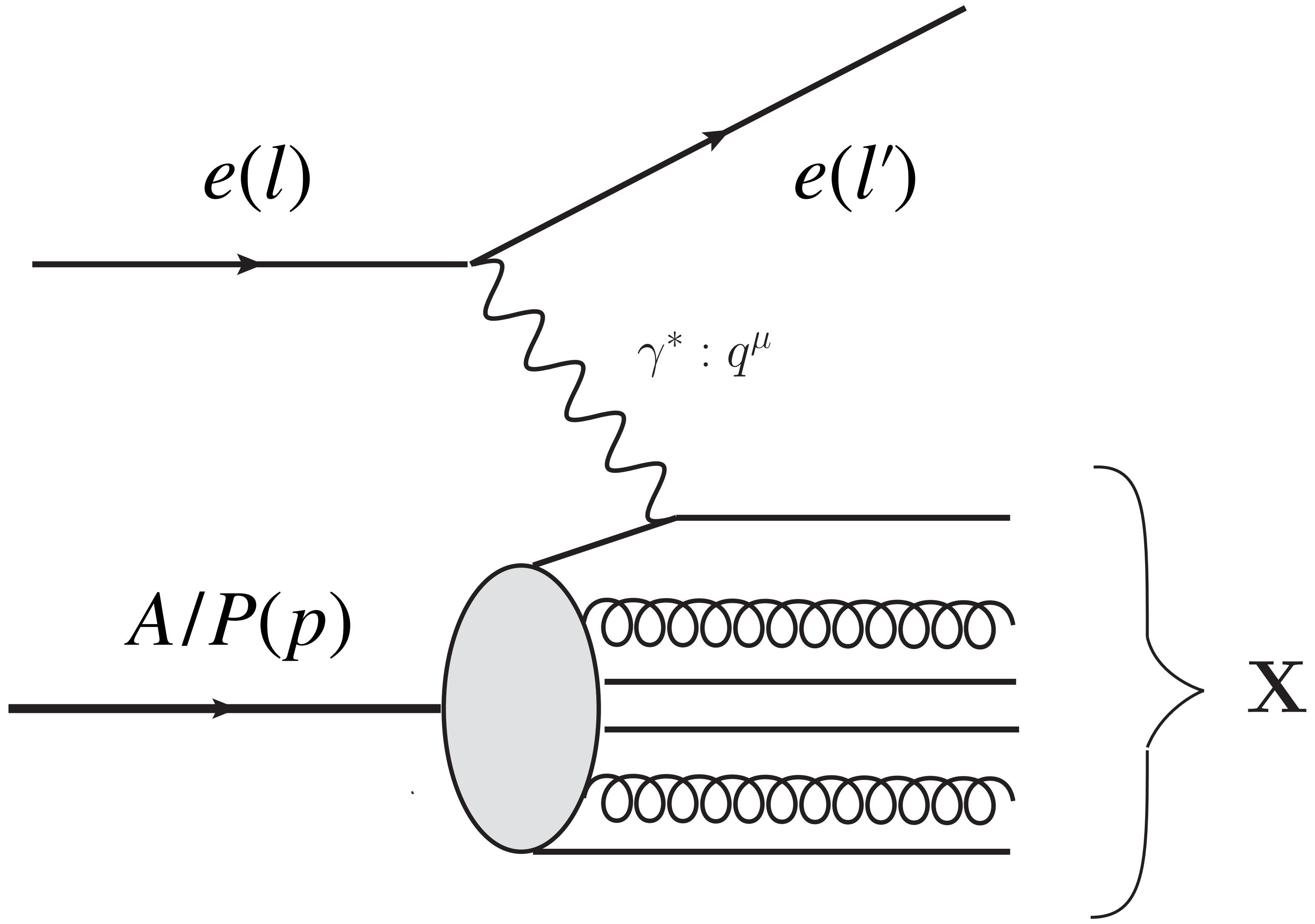
Non-linear evolution effect becomes important.

Keywords: Small-x

Introduction

Deep inelastic scattering

$$e + p/A \to e + X$$



$$q^2 = -Q^2$$

$$x_{\mathrm{Bj}} = \frac{Q^2}{2p \cdot q}$$

$$y = \frac{q \cdot p}{l \cdot p}$$

$$d\sigma \propto L^{\mu\nu} W_{\mu\nu}$$

$$L^{\mu\nu} = \frac{1}{2} \mathrm{Tr}\left[\not{l}' \gamma^\mu \ \not{l} \gamma^\nu \right]$$

$$W^{\mu\nu} = \left(-g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2} \right) F_1 + \frac{1}{p \cdot q} \left(p^\mu + \frac{p \cdot q}{Q^2} q^\mu \right) \left(p^\nu + \frac{p \cdot q}{Q^2} q^\nu \right) F_2$$

$$\frac{d\sigma}{dx_{\mathrm{Bj}}\, dy} = \frac{2\pi \alpha_{\mathrm{em}}^2}{x_{\mathrm{Bj}}\, y\, Q^2} \left[[1 + (1-y)^2]\ F_2(x_{\mathrm{Bj}}, Q^2) - y^2\ F_L(x_{\mathrm{Bj}}, Q^2) \right]$$

$$= \frac{2\pi \alpha_{\mathrm{em}}^2}{x_{\mathrm{Bj}}\, y\, Q^2} [1 + (1-y)^2] \left[F_2(x_{\mathrm{Bj}}, Q^2) - \frac{y^2}{1 + (1-y)^2}\ F_L(x_{\mathrm{Bj}}, Q^2) \right]$$

$$F_2 = 2x_{\mathrm{Bj}}\, F_1 + F_L$$

Reduced dimensionless cross section $\sigma_r(x_{\mathrm{Bj}}, y, Q^2)$

Introduction

Geometric scaling

$$\sigma_{tot}^{\gamma^* p} = \sigma_T^{\gamma^* p} + \sigma_L^{\gamma^* p} = \frac{4\pi^2 \alpha_{em}}{Q^2} F_2(x_{Bj}, Q^2)$$

Stasto, Golec-Biernat, Kwiecinski, arXiv:hep-ph/0007192

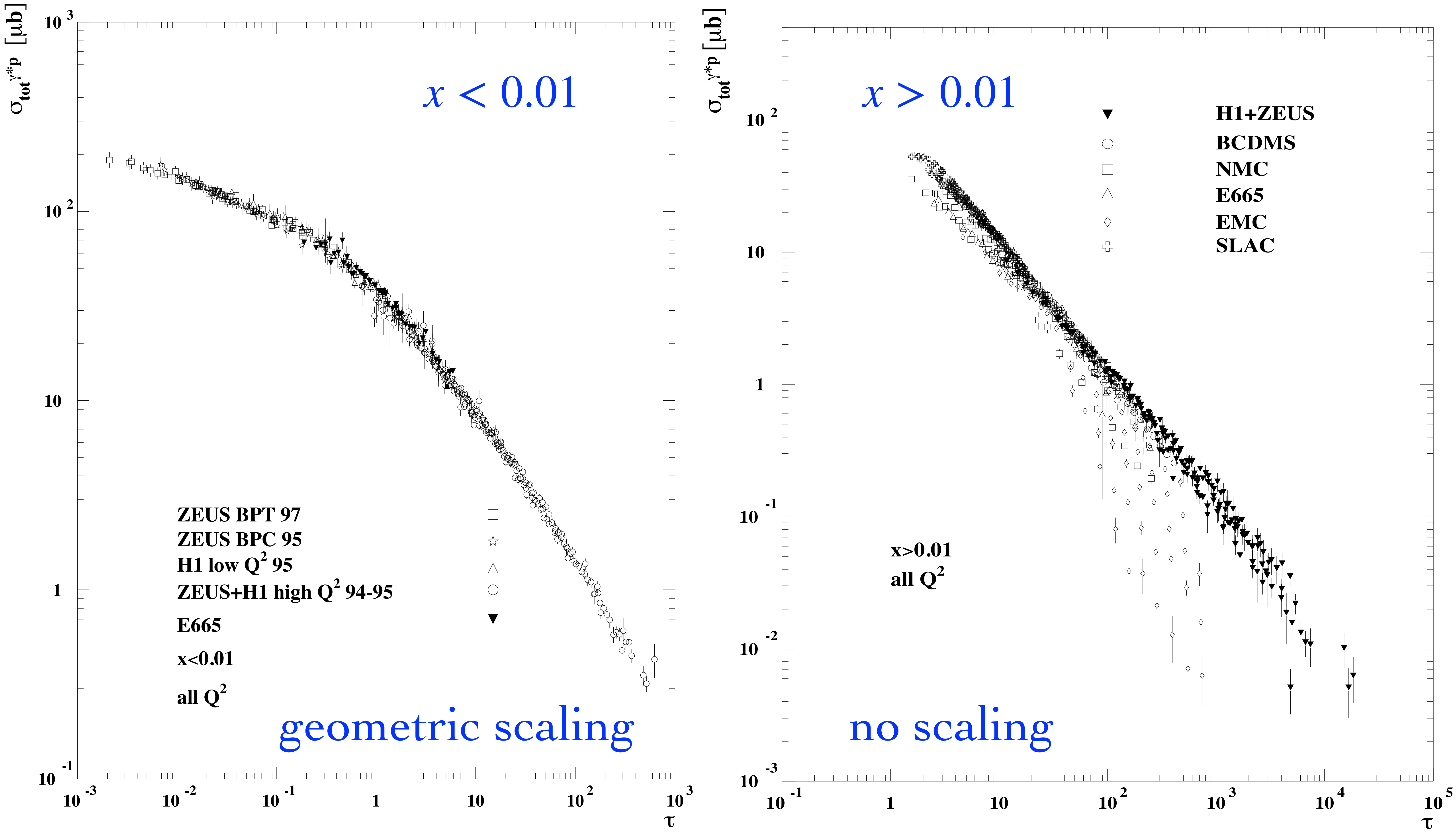
$$\tau = Q^2/Q_s^2(x_{Bj})$$

$$Q_s^2(x_{Bj}) = Q_0^2\,(x_0/x_{Bj})^{\lambda}$$

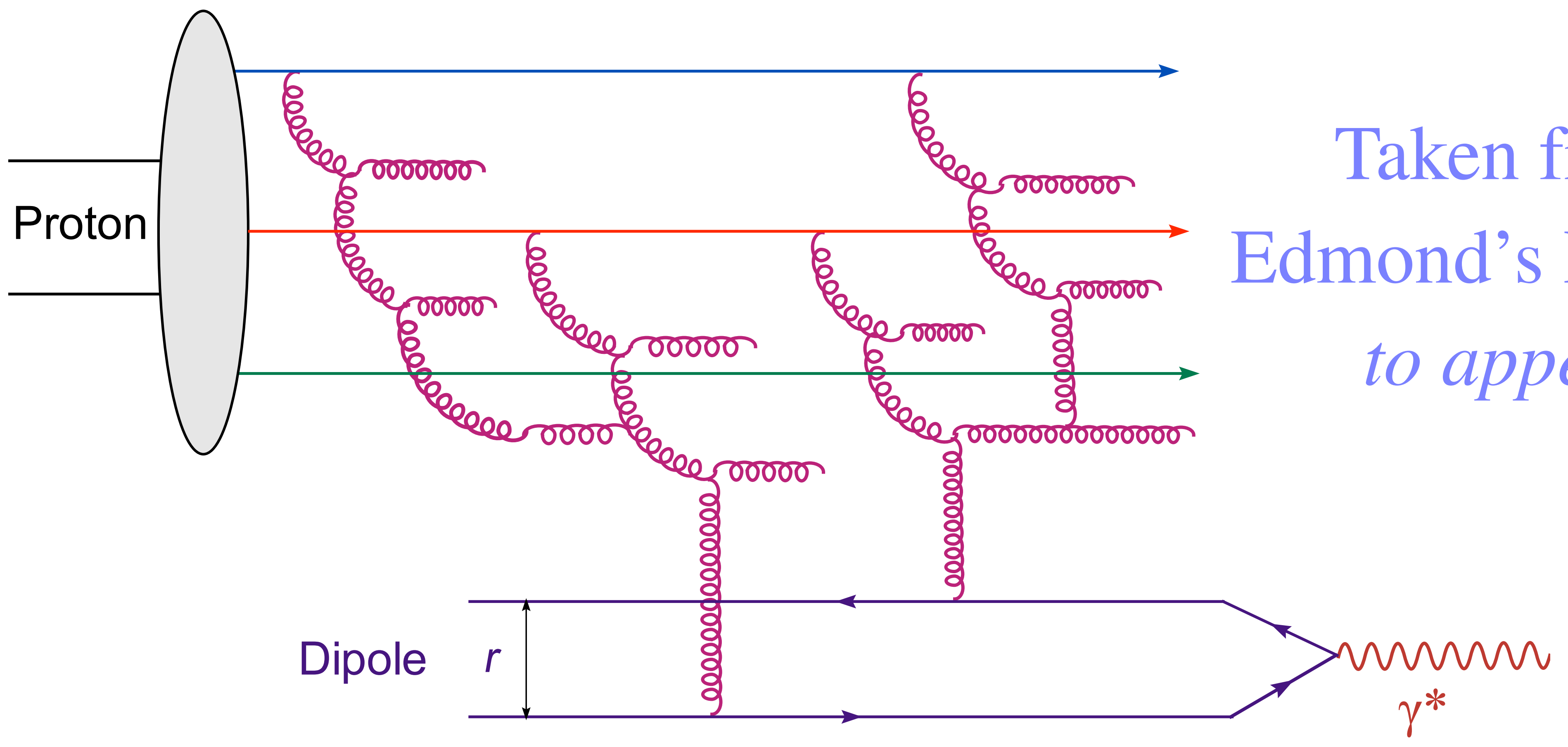
Hint of gluon saturation:

$$\sigma_{tot}^{\gamma^* p}(x_{Bj}, Q^2) \;\rightarrow\; \sigma_{tot}^{\gamma^* p}(\tau = Q^2/Q_s^2(x_{Bj}))$$

for $x_{Bj} \lesssim 0.01$



Dipole picture



Taken from Edmond's Review *to appear*

$$\sigma_{T,L}^{\gamma^* p}(x_{\rm Bj}, Q^2) = \sum_q \int_0^1 dz \int \frac{d^2 r}{4\pi} \left| \psi_{T,L}^{\gamma^* \to q\bar{q}}(z, \boldsymbol{r}, Q^2) \right|^2 \sigma_{\rm dipole}(x_{\rm Bj}, \boldsymbol{r})$$

photon splitting $\times$ X of $q\bar{q}$ dipole with A

$$|\psi_T|^2 = \frac{2N_c \alpha_{\rm em} e_q^2}{\pi} \left[z^2 + (1-z)^2\right] \bar{Q}^2 K_1^2(\bar{Q} r_\perp)$$

$$|\psi_L|^2 = \frac{2N_c \alpha_{\rm em} e_q^2}{\pi} 4z(1-z)\, \bar{Q}^2 K_0^2(\bar{Q} r_\perp)$$

$$\bar{Q}^2 = z(1-z)\, Q^2$$

+ mass effect

dipole scattering amplitude

$$\sigma_{\rm dipole}(x_{\rm Bj}, \boldsymbol{r}) = 2 \int d^2\boldsymbol{b}\; \mathcal{N}(x_{\rm Bj}, \boldsymbol{r}, \boldsymbol{b})$$

$$\approx 2 \int d^2\boldsymbol{b}\; T(\boldsymbol{b})\; N(x_{\rm Bj}, r_\perp) = 2\pi R_p^2\; N(x_{\rm Bj}, r_\perp)$$

Building block in saturation physics

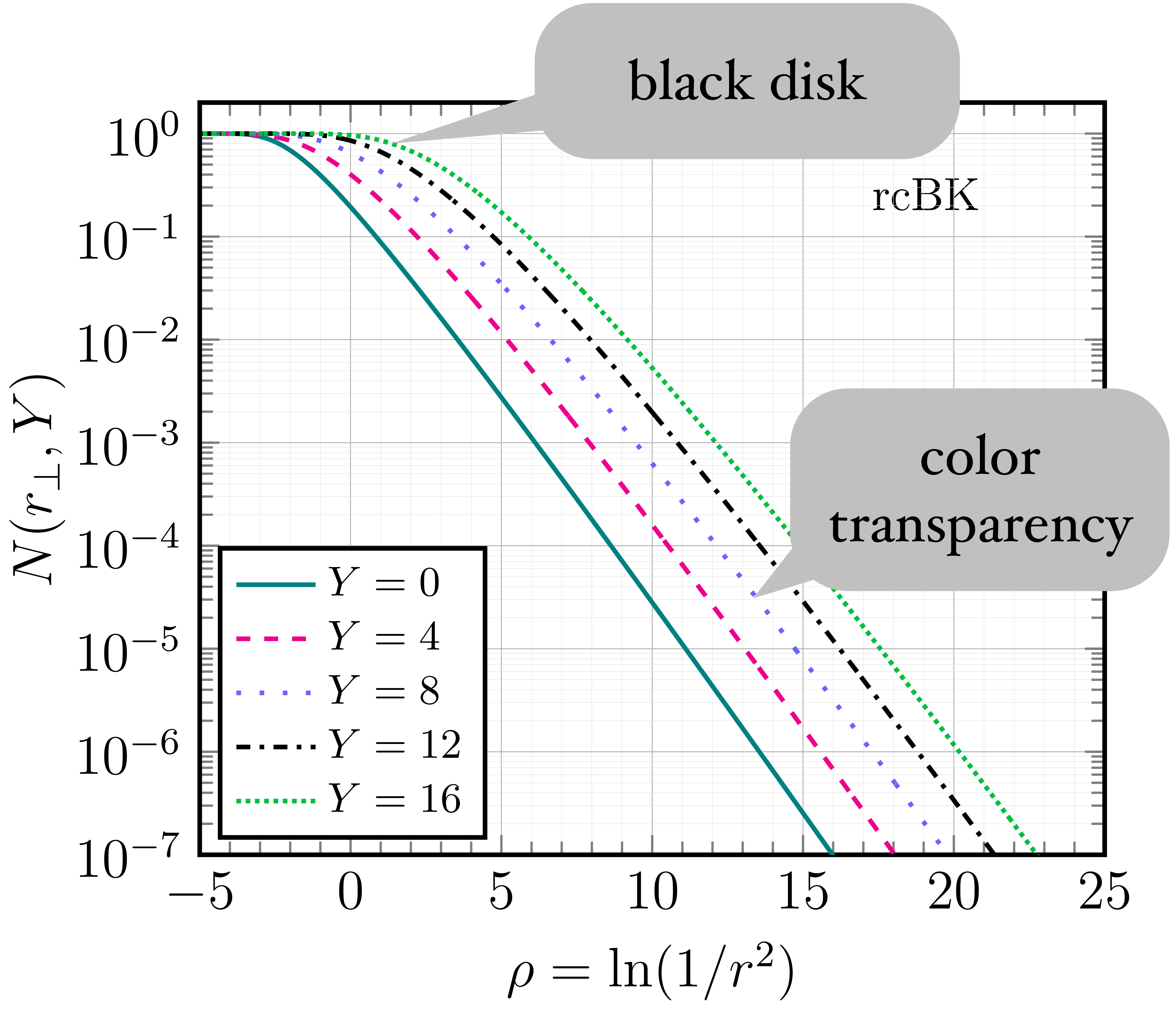
Structure Function

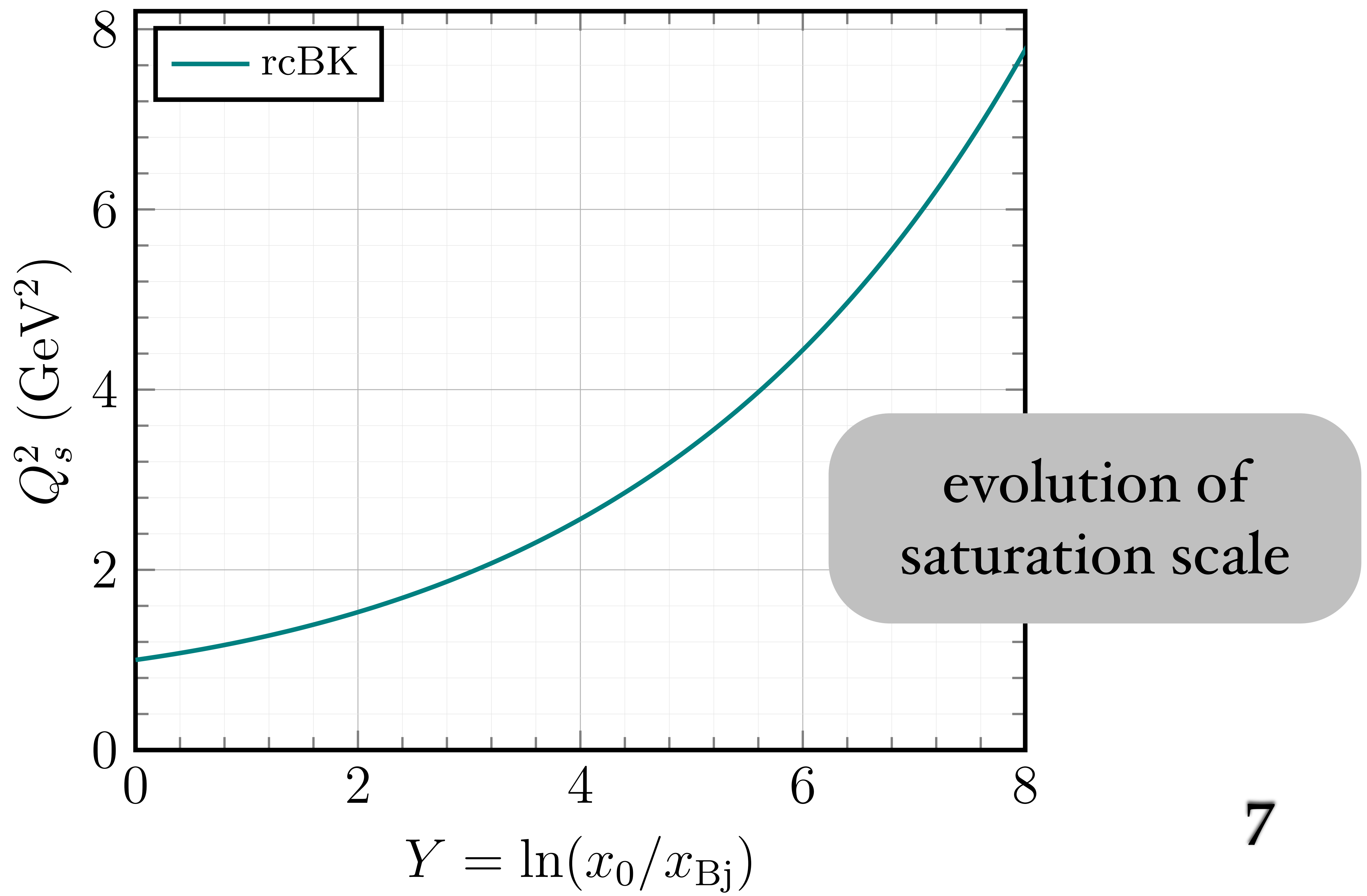
$Y = \ln(x_0/x)$

BK/JIMWLK evolution

$$\frac{\partial N(Y,\boldsymbol{r})}{\partial Y} = \int d^2\boldsymbol{r}_1 \, \frac{\bar{\alpha}_s}{2\pi} \, \frac{\boldsymbol{r}^2}{\boldsymbol{r}_1^2 \boldsymbol{r}_2^2} \left[N(Y,\boldsymbol{r}_1) + N(Y,\boldsymbol{r}_2) - N(Y,\boldsymbol{r}) - N(Y,\boldsymbol{r}_1)N(Y,\boldsymbol{r}_2)\right]$$

MV model $\quad N_0(r_\perp) = 1 - \exp\left[-\frac{1}{4} Q_A^2 r_\perp^2 \ln\left(e + \frac{2}{\Lambda r_\perp}\right)\right]$





Pretend proton is a large nucleus

Structure Function

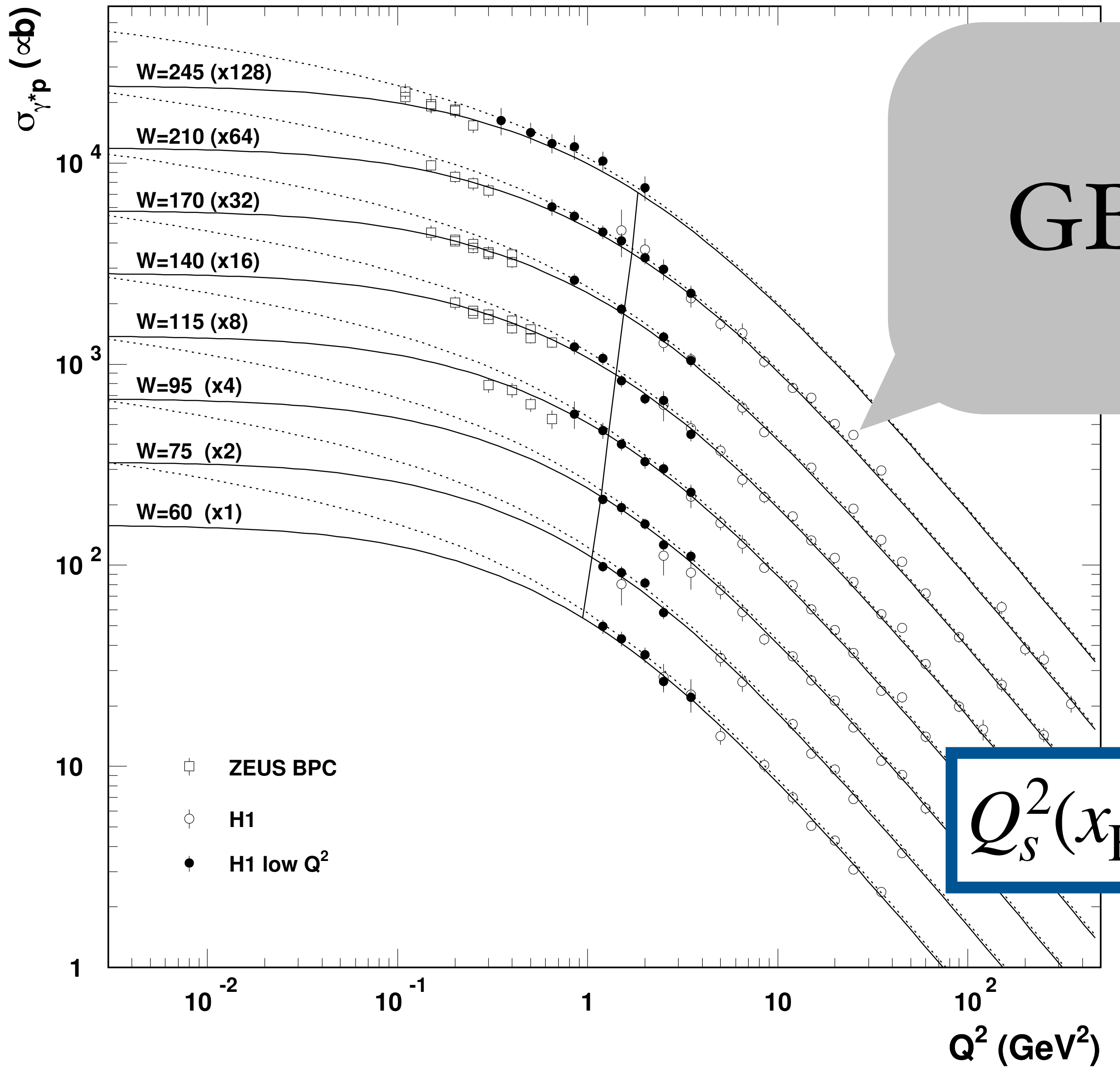
$$\text{MV-}\gamma\text{: } N(r_\perp) = 1 - \exp\left[-\frac{1}{4}\left(Q_{s0}^2 r_\perp^2\right)^{\gamma} \ln\left(e + \frac{2}{\Lambda r_\perp}\right)\right]$$

Extracting the dipole scattering amplitude

Pre-history: Golec-Biernat, Wusthoff, 9807513 GBW model

Iancu, Itakura, Munier, 0301338 approximate solution

Modern Era: AAMQS 2010, 1012.4408



GBW model

$$Q_s^2(x_{\rm Bj}) = Q_0^2\,(x_0/x_{\rm Bj})^{\lambda}$$

	fit	$\frac{\chi^2}{d.o.f}$	Q_{s0}^2	σ_0	γ	C	m_l^2
	GBW						
a	$\alpha_{fr} = 0.7$	1.226	0.241	32.357	0.971	2.46	fixed
a'	$\alpha_{fr} = 0.7$ (Λ_{m_τ})	1.235	0.240	32.569	0.959	2.507	fixed
b	$\alpha_{fr} = 0.7$	1.264	0.2633	30.325	0.968	2.246	1.74E-2
c	$\alpha_{fr} = 1$	1.279	0.254	31.906	0.981	2.378	fixed
c'	$\alpha_{fr} = 1$ (Λ_{m_τ})	1.244	0.2329	33.608	0.9612	2.451	fixed
d	$\alpha_{fr} = 1$	1.248	0.239	33.761	0.980	2.656	2.212E-2
	MV						
e	$\alpha_{fr} = 0.7$	1.171	0.165	32.895	1.135	2.52	fixed
f	$\alpha_{fr} = 0.7$	1.161	0.164	32.324	1.123	2.48	1.823E-2
g	$\alpha_{fr} = 1$	1.140	0.1557	33.696	1.113	2.56	fixed
h	$\alpha_{fr} = 1$	1.117	0.1597	33.105	1.118	2.47	1.845E-2
h'	$\alpha_{fr} = 1$ (Λ_{m_τ})	1.104	0.168	30.265	1.119	1.715	1.463E-2

	fit	$\frac{\chi^2}{d.o.f}$	Q_{s0}^2	σ_0	γ	Q_{s0c}^2	σ_{0c}	γ_c	C	m_l^2
	GBW									
a	$\alpha_{fr}=0.7$	1.269	0.2294	36.953	1.259	0.2289	18.962	0.881	4.363	fixed
a'	$\alpha_{fr}=0.7$ (Λ_{m_τ})	1.302	0.2341	36.362	1.241	0.2249	20.380	0.919	7.858	fixed
b	$\alpha_{fr}=0.7$	1.231	0.2386	35.465	1.263	0.2329	18.430	0.883	3.902	1.458E-2
c	$\alpha_{fr}=1$	1.356	0.2373	35.861	1.270	0.2360	13.717	0.789	2.442	fixed
d	$\alpha_{fr}=1$	1.221	0.2295	35.037	1.195	0.2274	20.262	0.924	3.725	1.351E-2
	MV									
e	$\alpha_{fr}=0.7$	1.395	0.1673	36.032	1.355	0.1650	18.740	1.099	3.813	fixed
f	$\alpha_{fr}=0.7$	1.244	0.1687	35.449	1.369	0.1417	19.066	1.035	4.079	1.445E-2
g	$\alpha_{fr}=1$	1.325	0.1481	40.216	1.362	0.1378	13.577	0.914	4.850	fixed
h	$\alpha_{fr}=1$	1.298	0.156	37.003	1.319	0.147	19.774	1.074	4.355	1.692E-2

Pretend proton is a large nucleus light-flavor only

light-flavor + heavy flavor

Structure Function

$$\text{MV-}\gamma\text{: } N(r_\perp) = 1 - \exp\left[-\frac{1}{4}\left(Q_{s0}^2 r_\perp^2\right)^\gamma \ln\left(e + \frac{2}{\Lambda r_\perp}\right)\right]$$

Negativity Problem

Giraud, Peschanski, 1604.01932

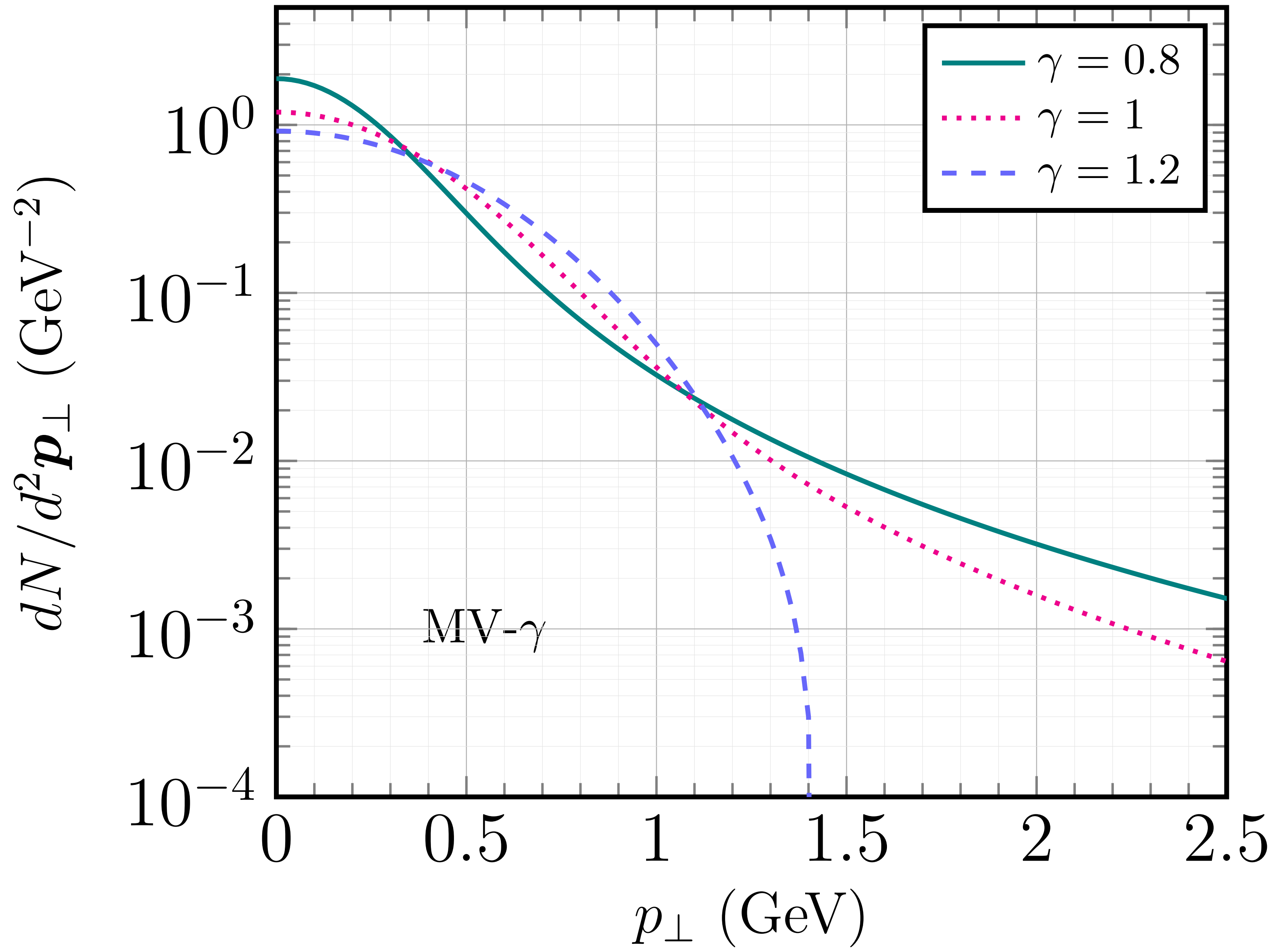
- Broadening of quark transverse momentum

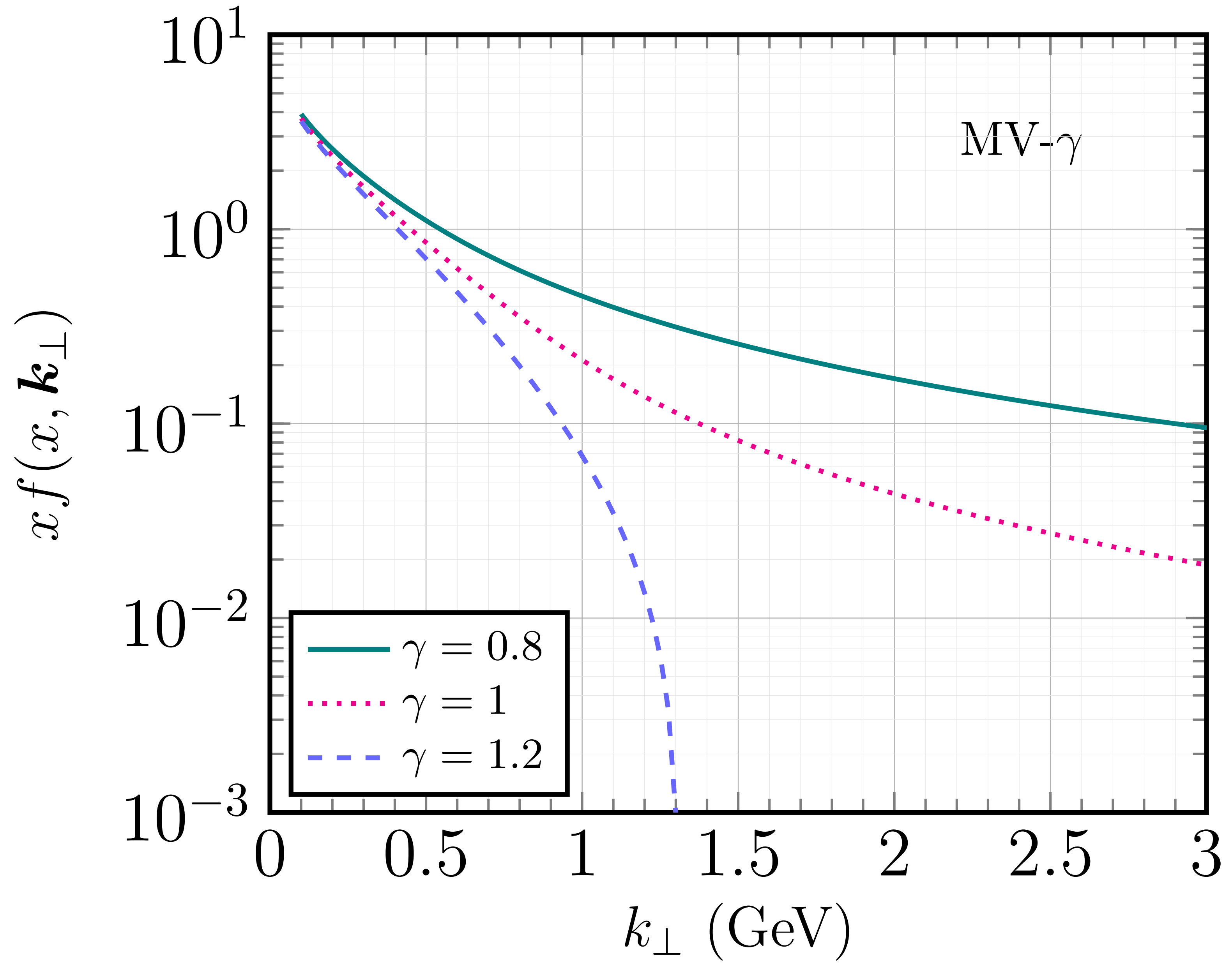
$$\frac{dN}{d^2\boldsymbol{p}_\perp} = \int \frac{d^2\boldsymbol{r}}{(2\pi)^2} e^{-i\boldsymbol{p}_\perp\cdot\boldsymbol{r}}\, S(r_\perp) = \int \frac{d^2\boldsymbol{r}}{(2\pi)^2} e^{-i\boldsymbol{p}_\perp\cdot\boldsymbol{r}}\, [1 - N(r_\perp)]$$

- Weizsacker-Williams gluon distribution

$$xf_g(x, \boldsymbol{k}_\perp) \propto \int d^2\boldsymbol{r}_\perp e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} \frac{1}{r_\perp^2}\, \tilde{N}(r_\perp)$$

The Fourier transform turns into negative, if $\gamma > 1$





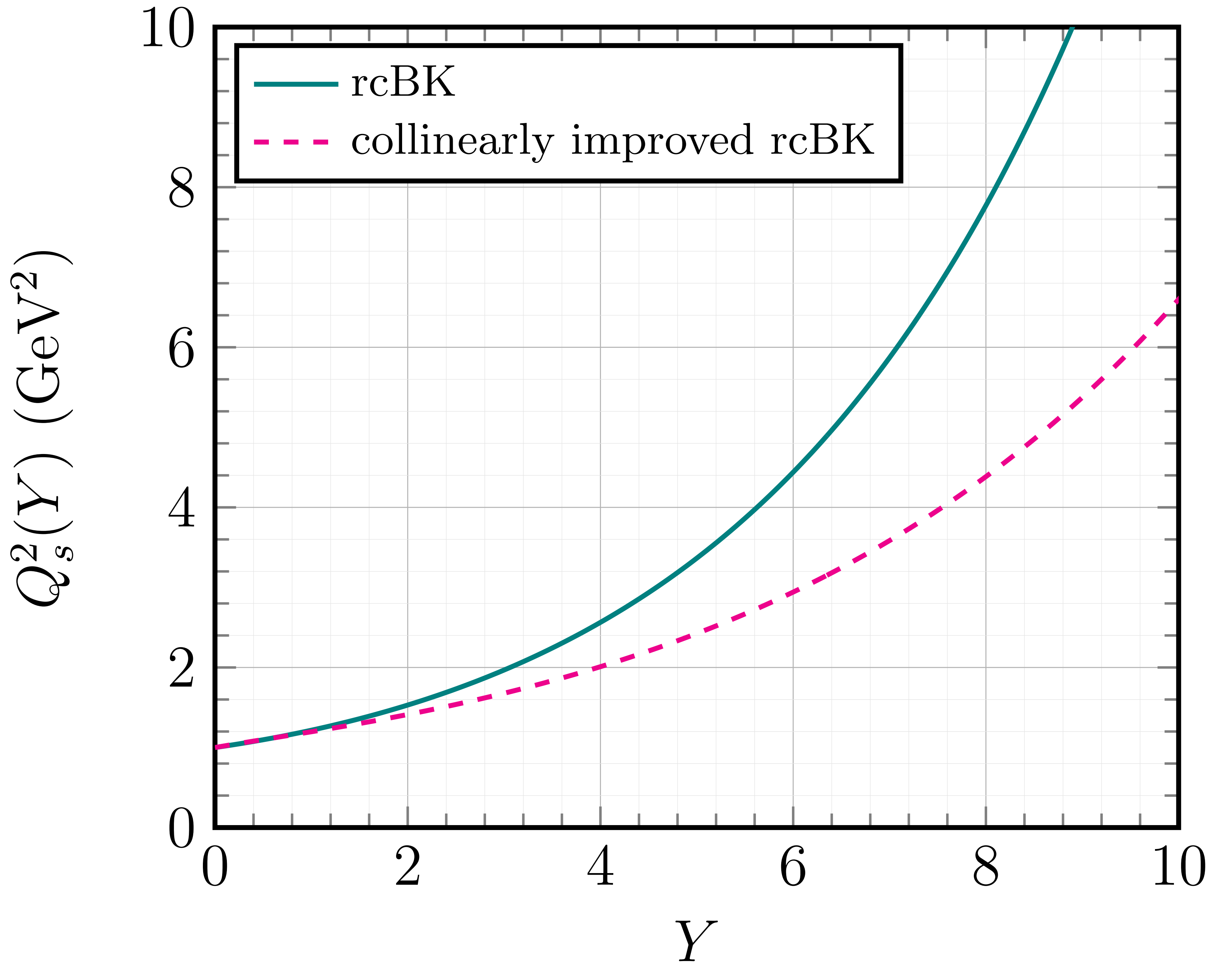
Structure Function

collinearly-improved rcBK

Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos, 1507.03651
Ducloué, Iancu, Soyez, Triantafyllopoulos, 1912.09196

$$\frac{\partial S_{\boldsymbol{xy}}(\eta)}{\partial \eta} = \int \frac{\mathrm{d}^2\boldsymbol{z}}{2\pi}\,\bar{\alpha}_{\mathrm{BLM}}\,\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\left[\frac{r^2}{\bar{z}^2}\right]^{\pm A_1}\left[S_{\boldsymbol{xz}}(\eta-\delta_{\boldsymbol{xz};r})S_{\boldsymbol{zy}}(\eta-\delta_{\boldsymbol{zy};r}) - S_{\boldsymbol{xy}}(\eta)\right]$$

non-local in rapidity



Pretend proton is a large nucleus

Full NLO BK is still challenging to implement

- Large logarithms to be resummed.
- Consumes enormous computing resources.

We observed the well-known preference of HERA small-x data for a steep initial dipole amplitude with a large anomalous dimension γ. This, in turn, leads to a large effective running coupling scale. While DIS cross section calculations do not forbid large anomalous dimensions, $\gamma > 1$ does not yield a positive definite unintegrated gluon distribution (UGD) [59]. Functional forms that force positive UGD were also explored in this work, but those did not result in good agreement with the HERA data.

2007.01645; 2506.00487; 2604.22332, et cetera

Machine Learning Techniques

$$N(r, x_0) = 1 - \exp\left[-\left(\frac{r^2 Q_{s0}^2}{4}\right)^{\gamma} \ln\left(\frac{1}{\Lambda_{\rm QCD}\, r} + e\, e_c\right)\right]$$

M. Gao, Z.B. Kang, J. Penttala, D.Y. Shao, 2510.26779

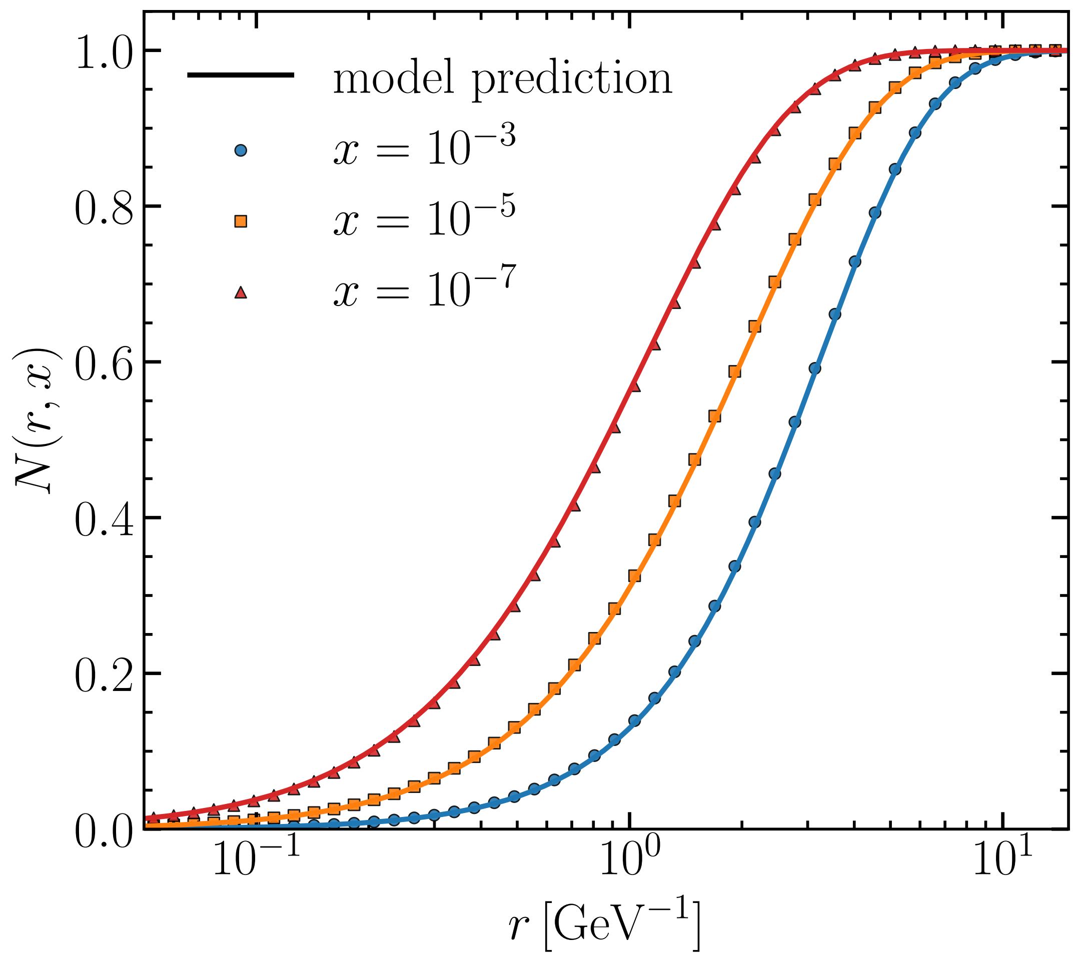
Transformer

Initial Condition
MV: Q_{s0}, γ, e_c

LO rcBK
C^2

Solve numerically to train the NN

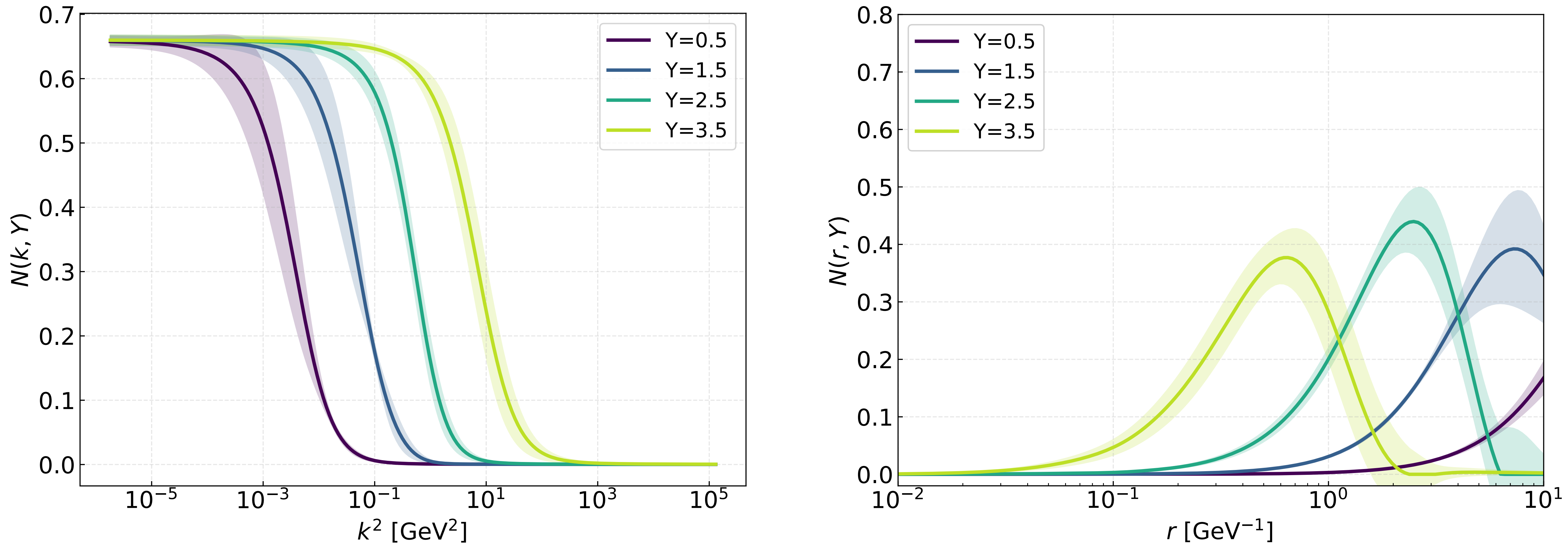
$N(r, Y; Q_{s0}^2, \gamma, e_c, C^2)$



W. Kou, X. Chen, 2601.16391

Momentum space

Diffusion approximation of BK equation.



Structure Function

$$\mathcal{R}_{\text{ciBK}}(r,Y) = \frac{\partial \mathcal{N}_\theta(r,Y)}{\partial Y} - \int d^2\mathbf{r}_1\, K_{\text{ciBK}}(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2)\, \mathcal{F}[\mathcal{N}_\theta]$$

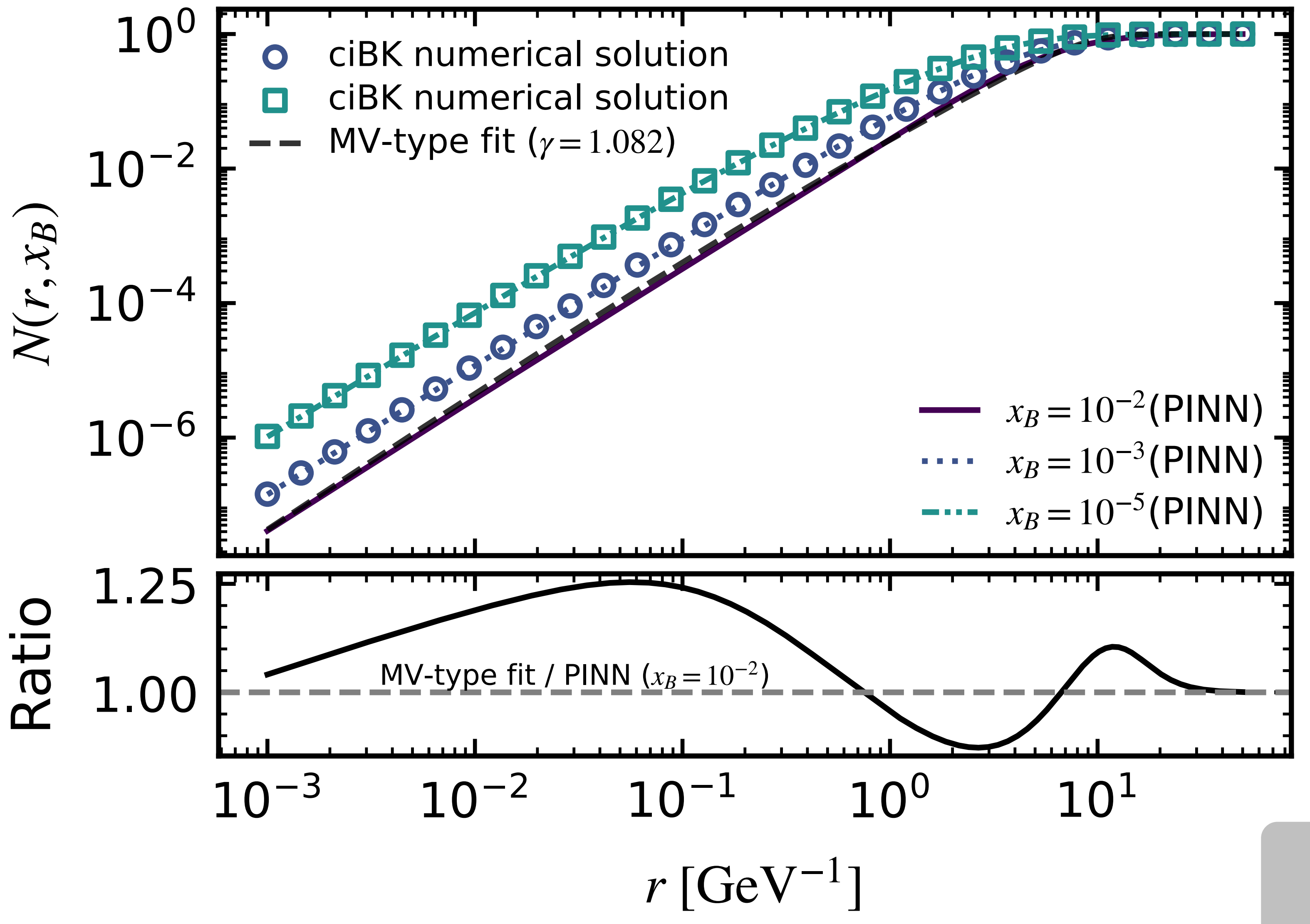
Machine Learning Techniques

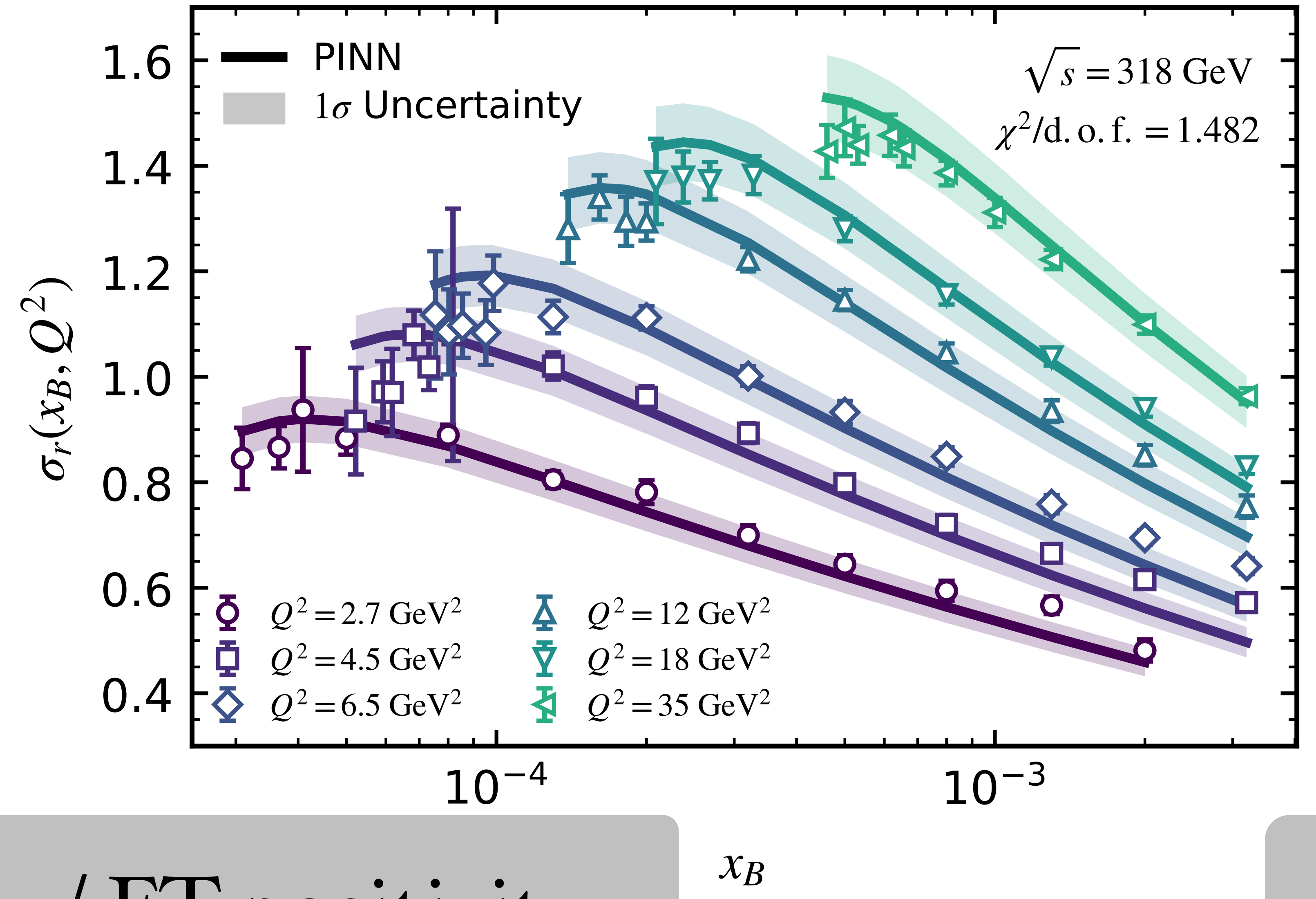
S.W. Dai, F.P. Li,
L.G. Pang, G.Y. Qin,
S.Y. Wei, H.Z. Zhang,
W. Zhao, 2603.08008

w/o a priori MV-type IC

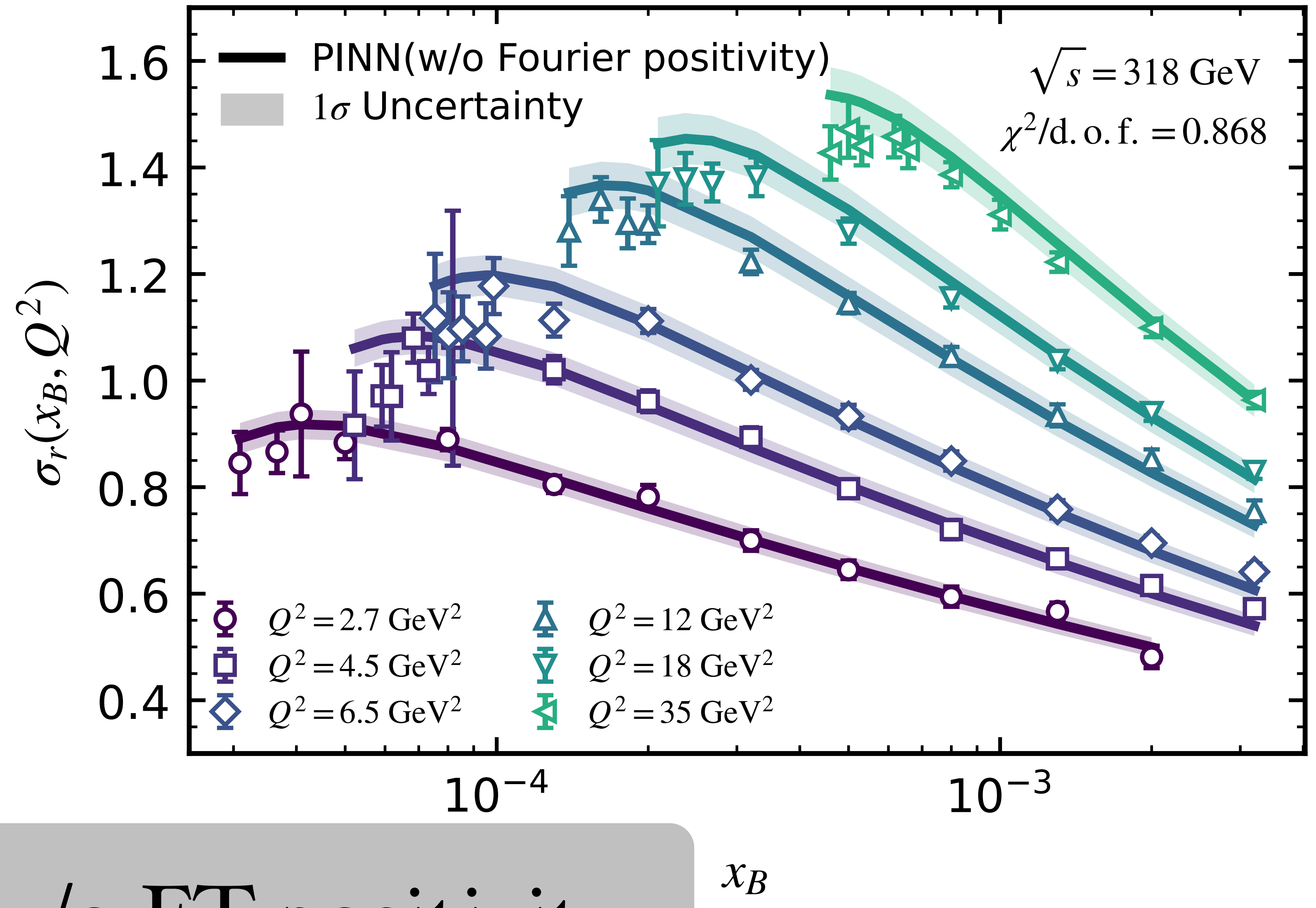
smooth surrogate $\mathcal{N}_\theta\ (r, Y)$

- Black disk limit at $r \gg 1/Q_s$
- Color Transparency at $r \ll 1/Q_s$
- Obey ciBK evolution equation
- Fourier Positivity
- Can simultaneously describe σ_r and $\sigma_r^{c\bar{c}}$





w/ FT positivity



w/o FT positivity

Do not pretend proton is a large nucleus

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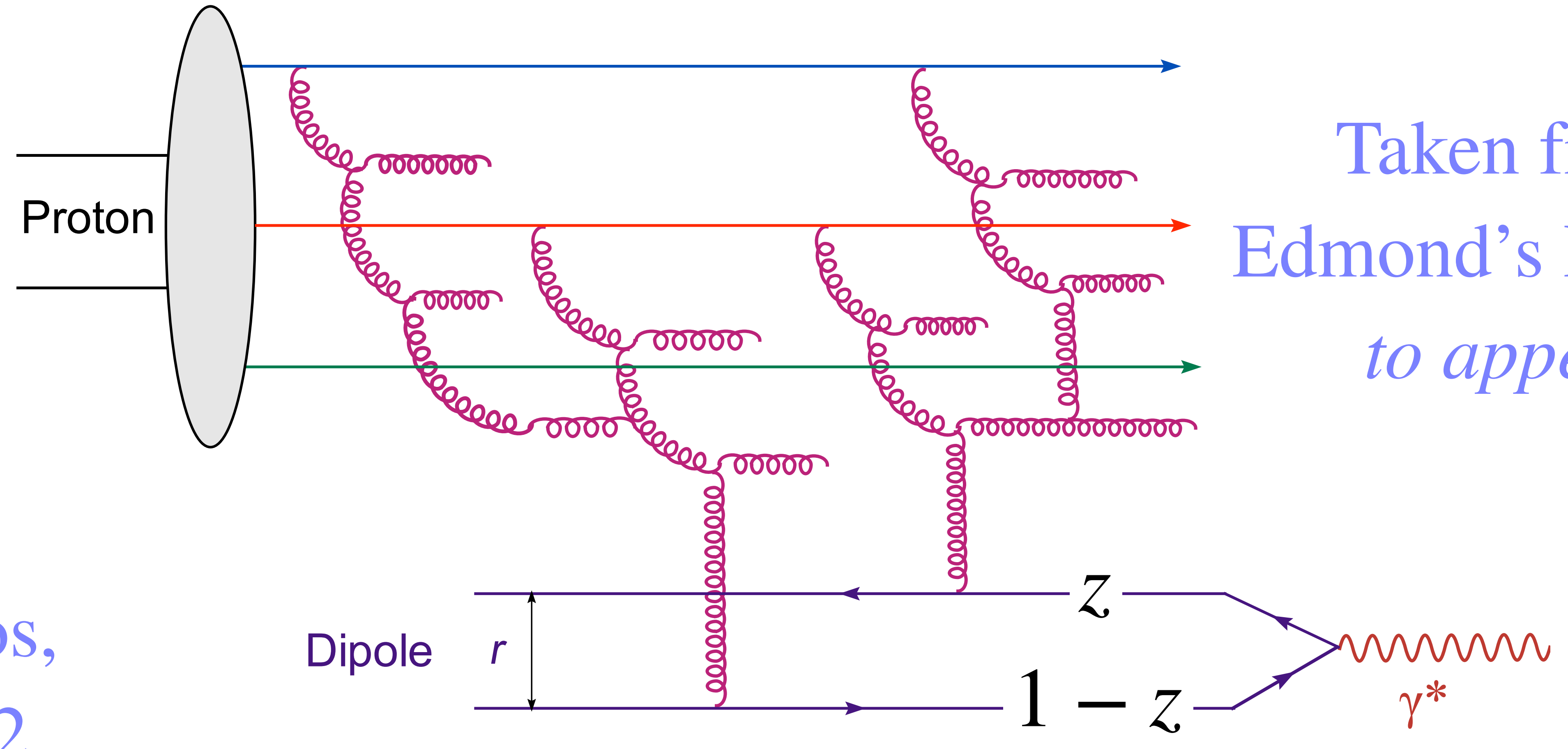
Dihadron Correlation in *ep/eA*

From inclusive to semi-inclusive

$\bar{Q}^2 = z(1-z)Q^2$



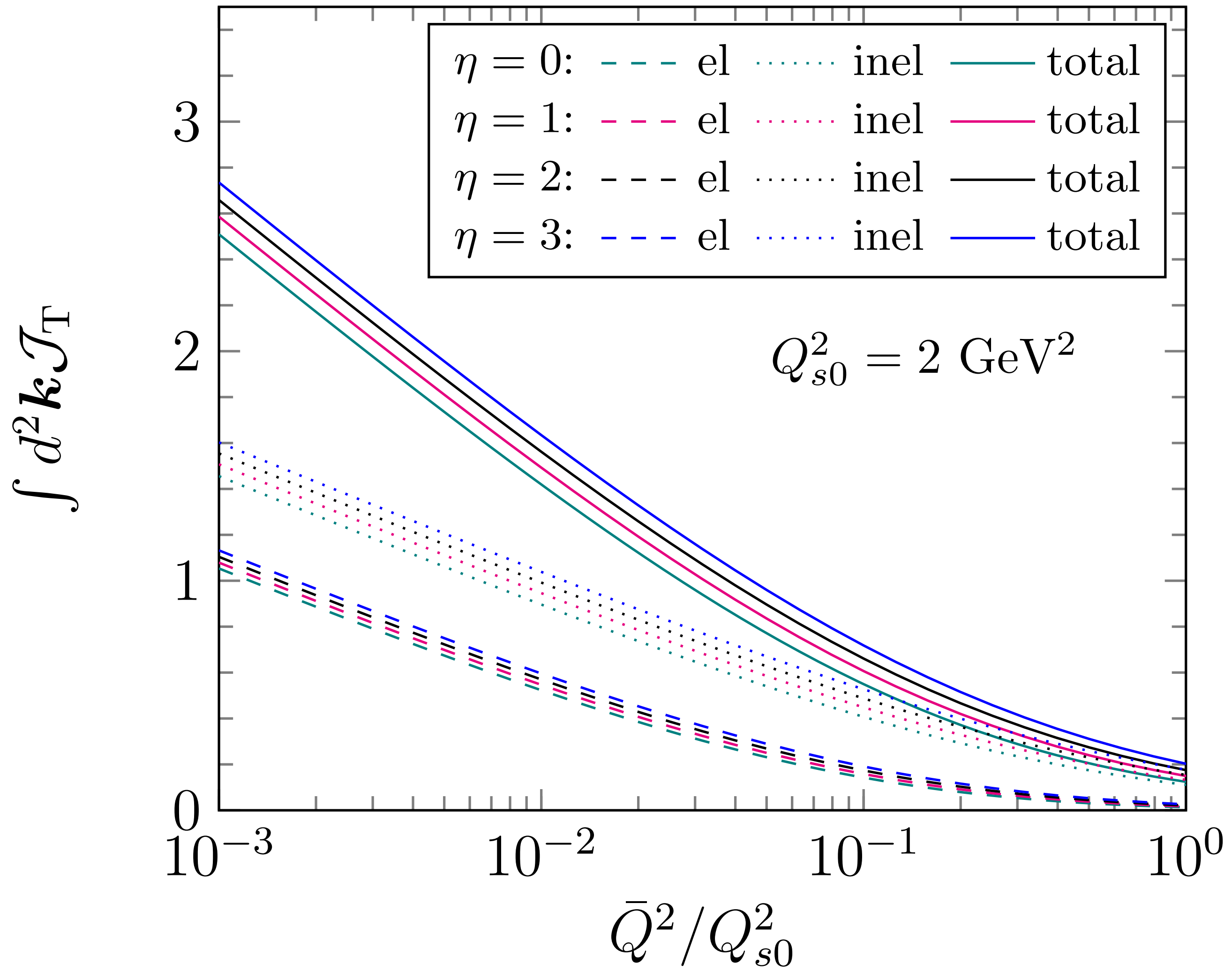
Taken from Edmond's Review *to appear*

$$\int d^2\boldsymbol{k}_T \mathcal{J}_{T,\mathrm{el}} = \bar{Q}^2 \int d^2\boldsymbol{r} K_1^2(\bar{Q}r)\,|N(r)|^2$$

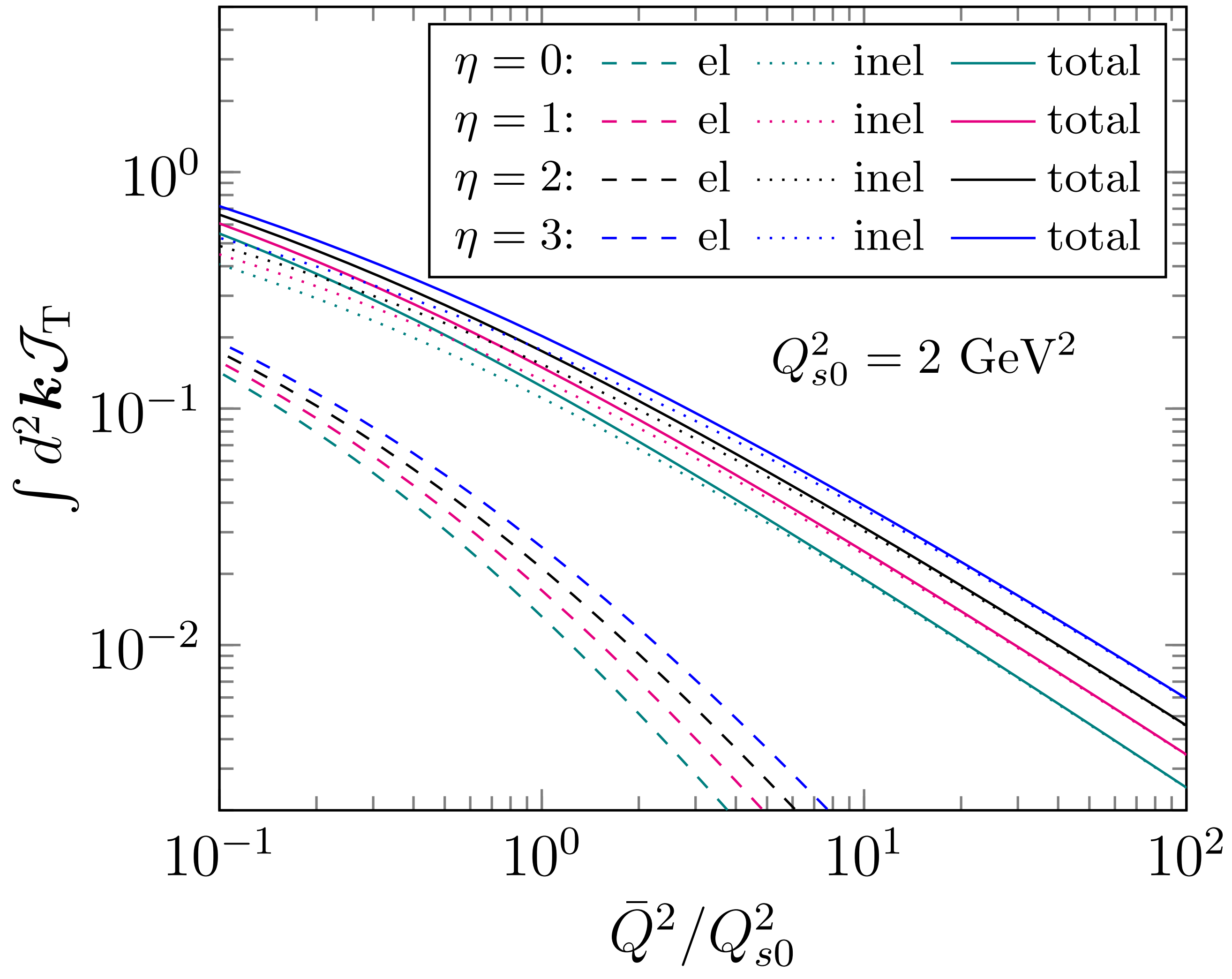
$$\int d^2\boldsymbol{k}_T \mathcal{J}_{T,\mathrm{inel}} = \bar{Q}^2 \int d^2\boldsymbol{r} K_1^2(\bar{Q}r)\left[2N(r) - |N(r)|^2\right]$$

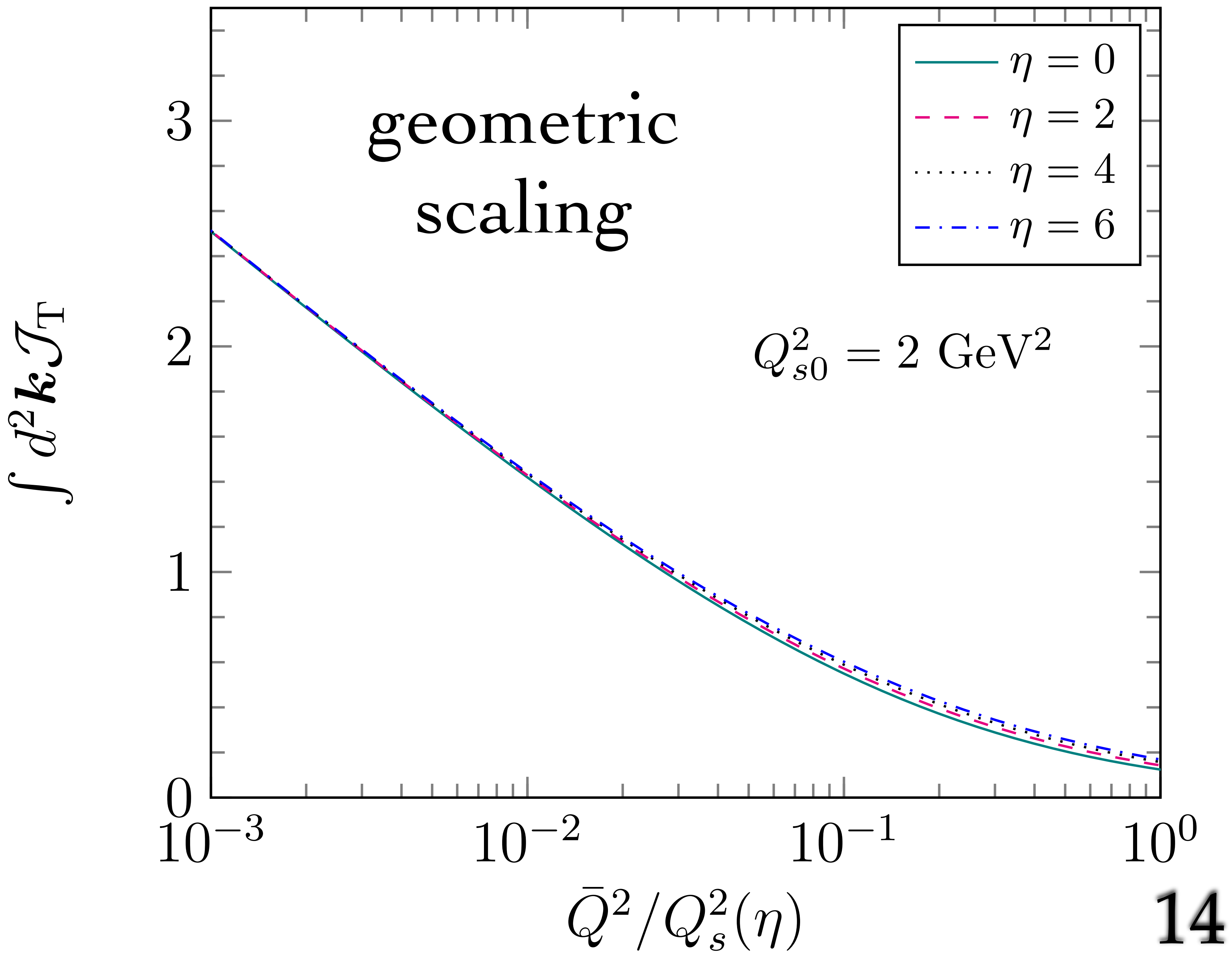
Iancu, Mueller, Triantafyllopoulos, Wei, [2012.08562](https://arxiv.org/abs/2012.08562)

$\bar{Q}^2 \ll Q_s^2$: black disk



$\bar{Q}^2 \gg Q_s^2$: dilute limit





Dihadron Correlation in *ep/eA*

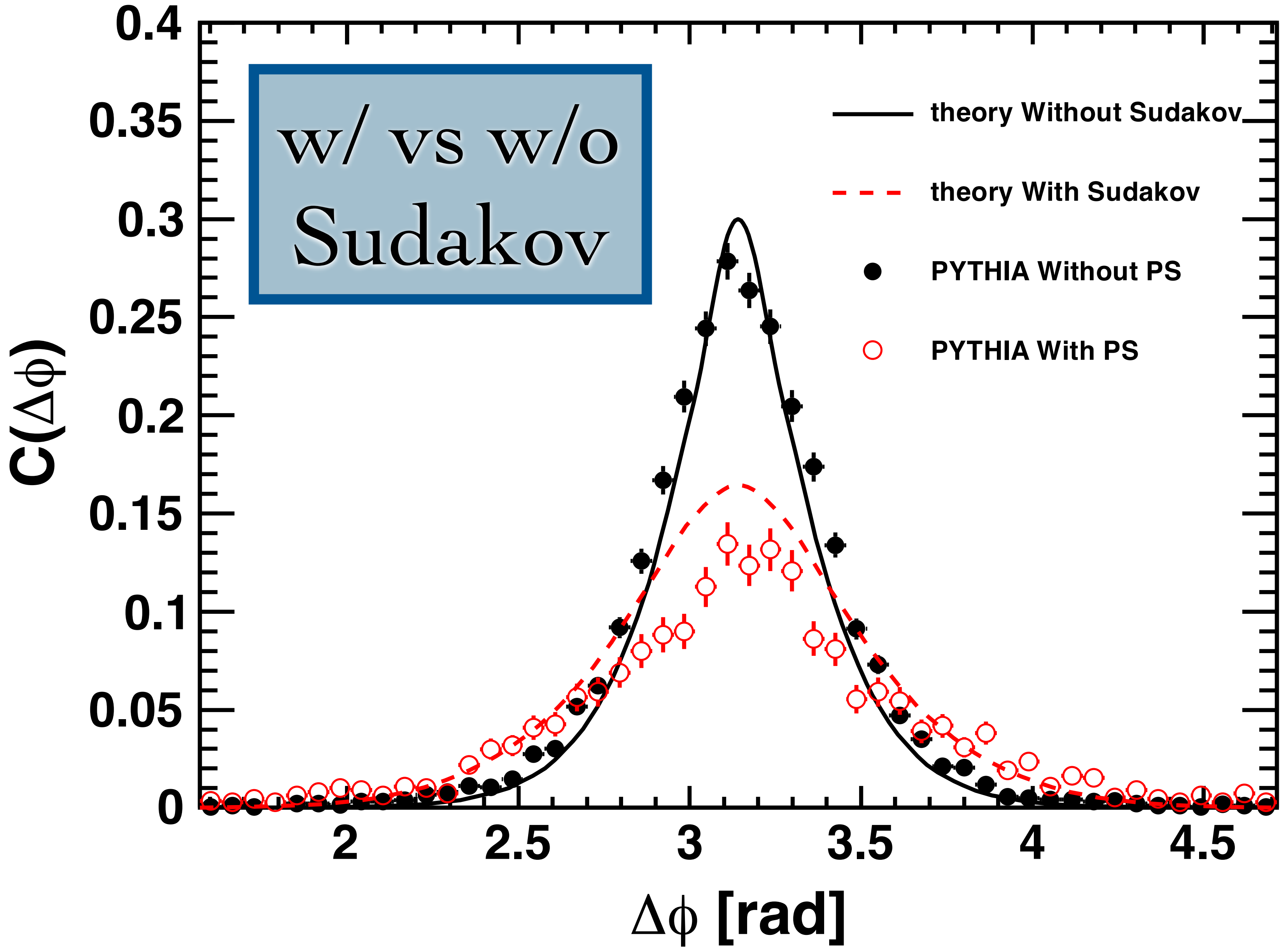
TMD factorisation

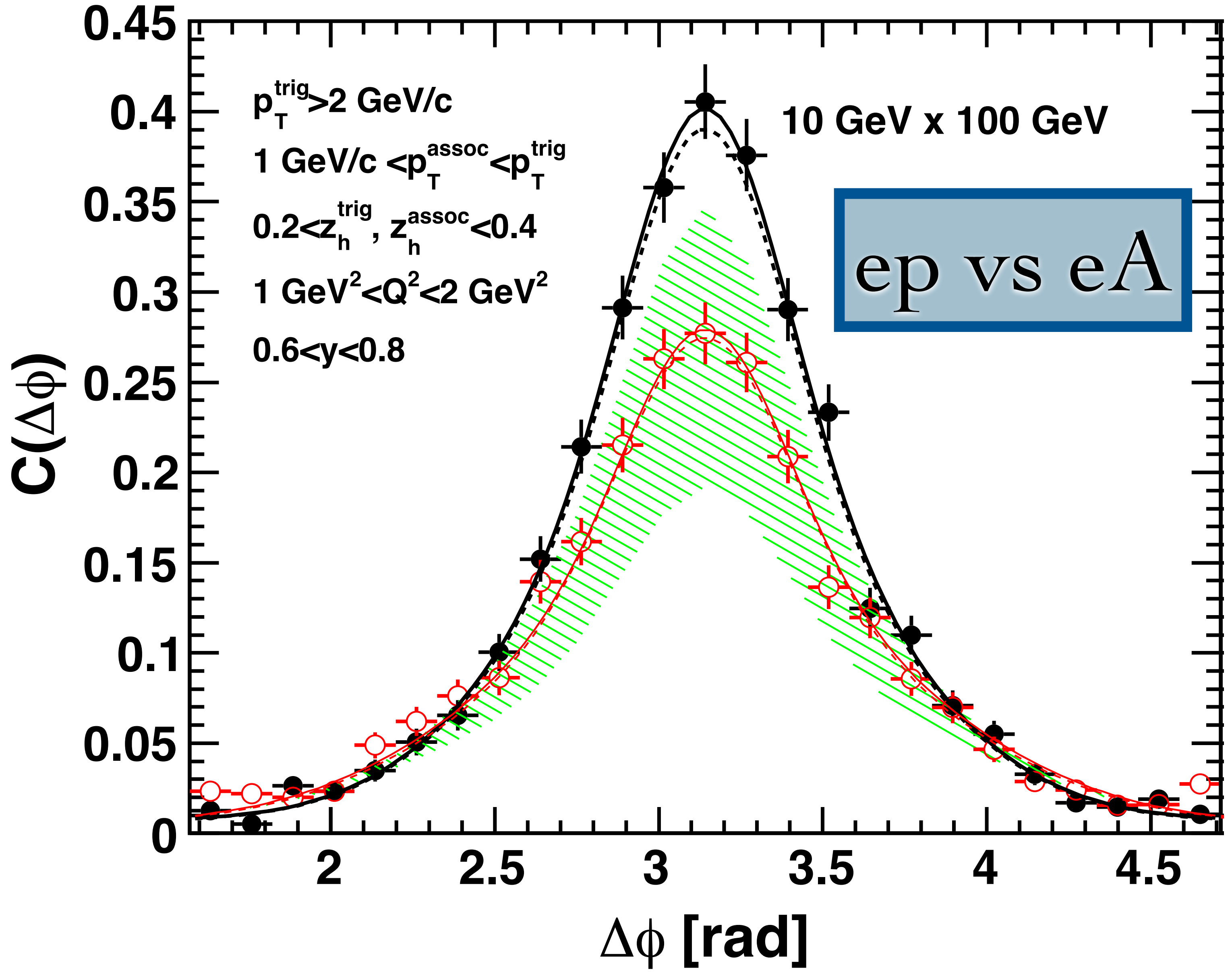
F. Dominguez, C. Marquet, B.W. Xiao, Feng Yuan, 1101.0715

$$\frac{d\sigma^{\gamma^* A\to q\bar{q}+X}}{dPS} \propto xG^{(1)}(x, q_T)\; H_{\gamma^* g\to q\bar{q}} \qquad xG^{(1)}(x, q_T) \propto \int d^2 r e^{-i q_T\cdot r}\, \frac{\tilde{N}(x, r)}{r^2}$$

+ Sudakov/parton shower

L. Zheng, E.C. Aschenauer, J.H. Lee, B.W. Xiao, F. Yuan, 1403.2413

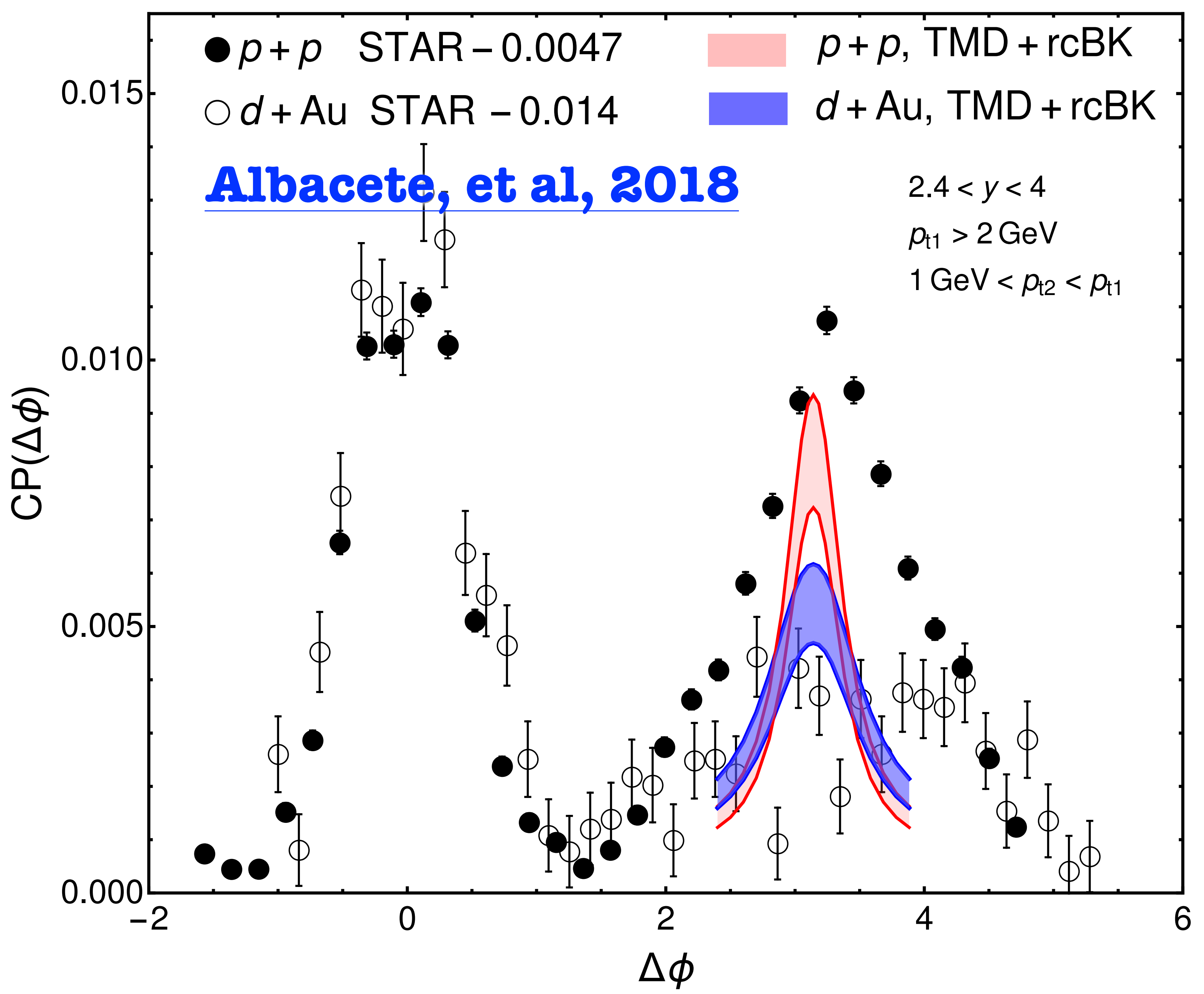




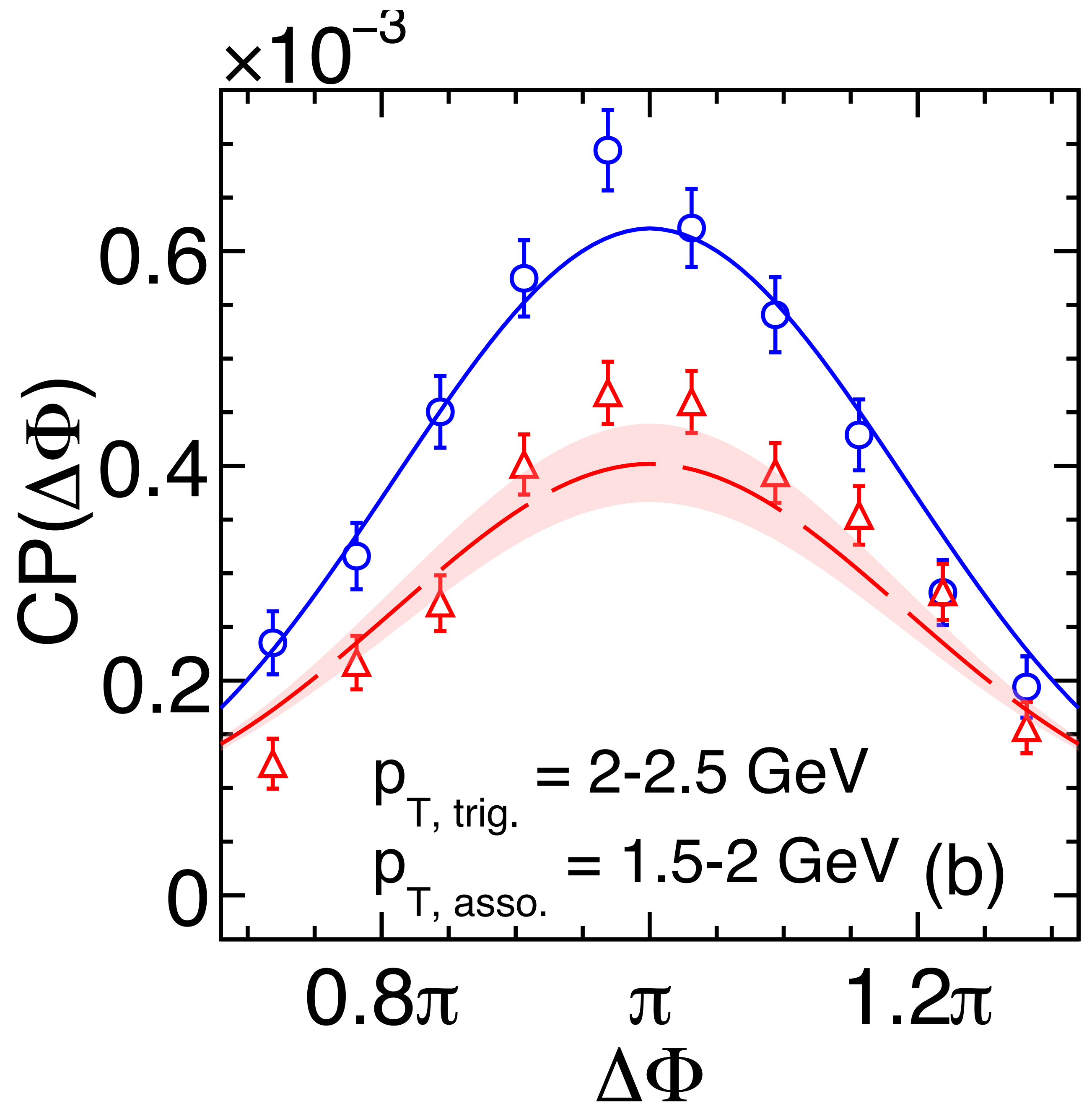
- p_T-broadening
- Suppression of per trigger yield

Dihadron Correlation in *pp*/*pA*

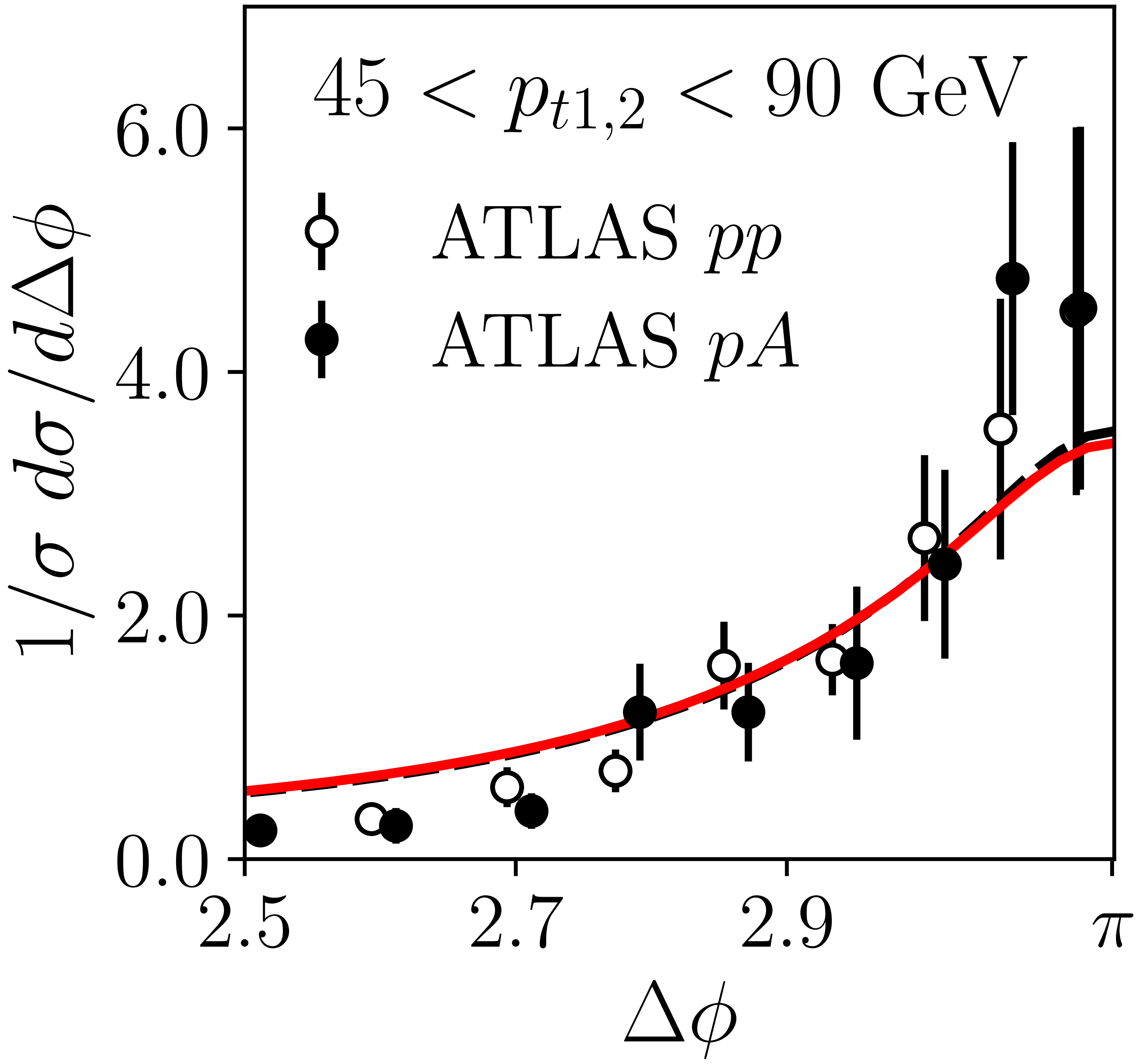
Sudakov effect dominates at large Q



w/o Sudakov



rcBK + Sudakov



high p_T: no visible broadening

GBW+ Sudakov: A. Stasto, S.Y. Wei, B.W. Xiao, F. Yuan, 1805.05712

rcBK + Sudakov: Caucal, Kang, Korcyl, Salazar, Schenke, Stebel, Venugopalan, Zhao, 2512.21466

See also: G. Giacalone, C. Marquet, M. Matas, S.Y. Wei, Never Appeared

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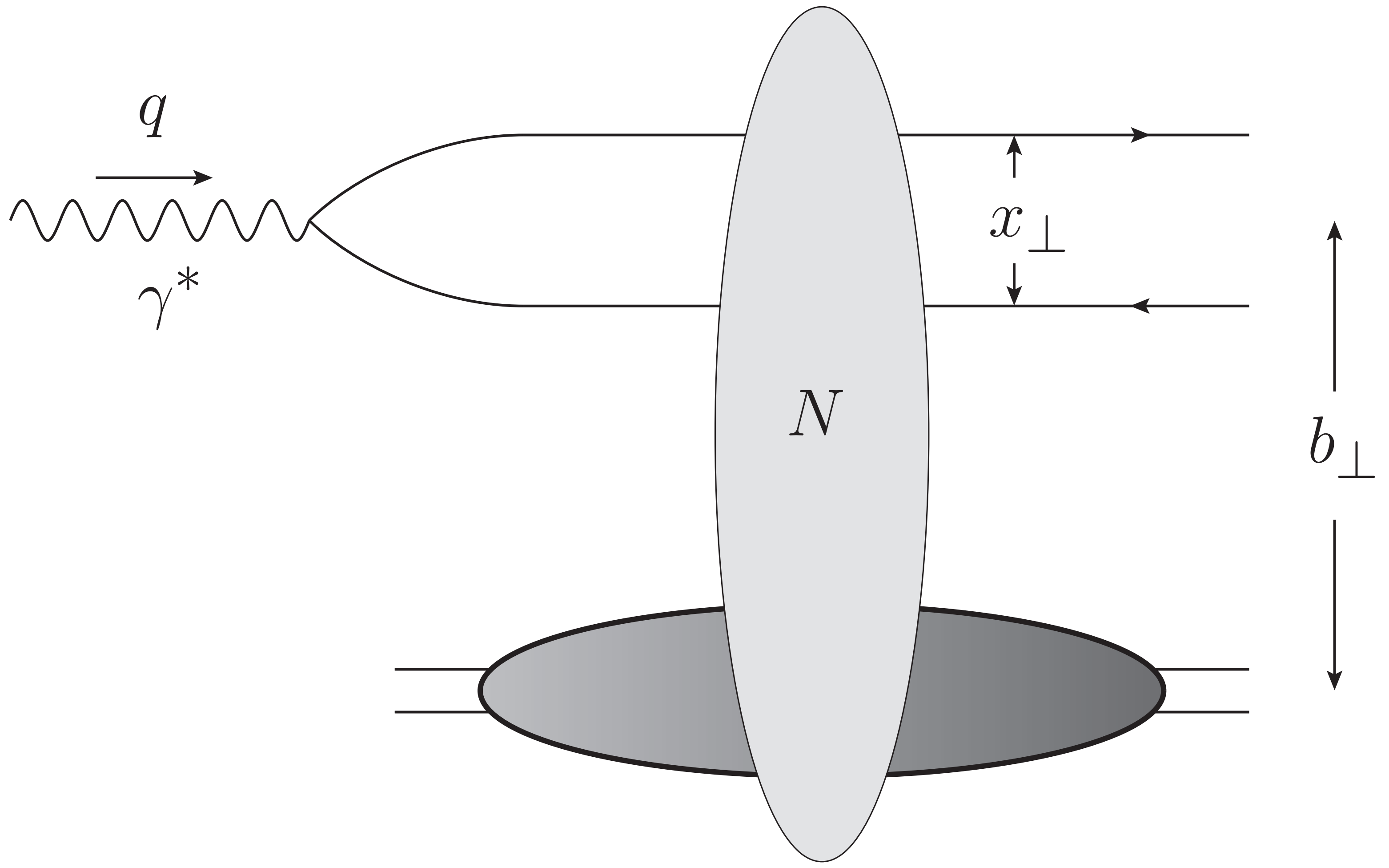
Diffractive Dijet

Elastic vs. Inelastic

Total Cross Section:	$\sigma_{\text{tot}}^{q\bar{q}A} = 2S_\perp N(x,r)$
Diffractive Process:	$\sigma_{\text{el}}^{q\bar{q}A} = S_\perp N^2(x,r)$



Exclusive diffractive dijet production at large $\boldsymbol{P}_\perp$

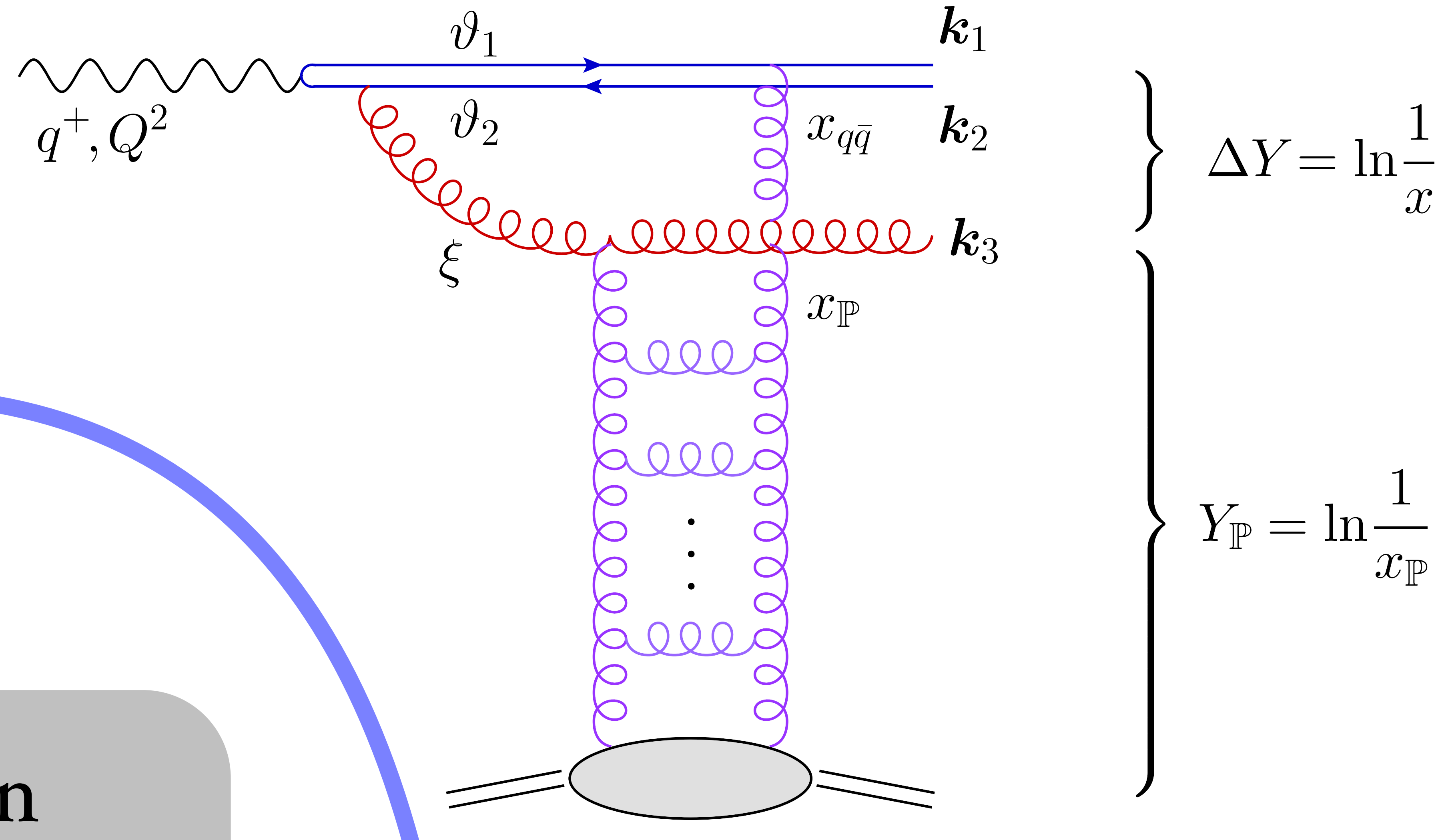
$$\frac{\mathrm{d}\sigma_{\text{el}}^{\gamma_T^* A\to q\bar{q}A}}{\mathrm{d}\vartheta_1 \mathrm{d}\vartheta_2 \mathrm{d}^2\boldsymbol{P}} = \frac{S_\perp \alpha_{em} N_c}{2\pi^2}\left(\sum e_f^2\right)\left(\vartheta_1^2+\vartheta_2^2\right)\delta(1-\vartheta_1-\vartheta_2)\left|\mathcal{A}_{\text{el}}^l(\boldsymbol{P},\bar{Q})\right|^2$$

$$\left|\mathcal{A}_{\text{el}}^l(P_\perp,\bar{Q})\right|^2 = (2\pi)^4 \frac{\bar{Q}^4 P_\perp^2}{\left(P_\perp^2+\bar{Q}^2\right)^6}\left[\frac{\alpha_s}{N_c}\frac{xG\left(x,P_\perp^2\right)}{S_\perp}\right]^2$$

$\sim 1/P_\perp^6$

(Hauksson), Iancu, Mueller, Triantafyllopoulos, Wei, 2207.06268, 2402.14748

2+1 configuration dominates at large $P_\perp \gg K_\perp$



$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma^*_{T,L}A\to q\bar{q}gA}}{\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}\,\mathrm{d}Y_{\mathbb{P}}} = H_{T,L}\left(\vartheta_1,\vartheta_2,Q^2,P_\perp^2\right)\frac{\mathrm{d}xG_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}$$

$\gamma^* + g \to q + \bar{q}$ scattering $\sim 1/P_\perp^4$

TMD gluon distribution of the "Pomeron"

The photon interact with a gluon inside of the Pomeron.

Gluon diffractive TMD $$\mathcal{F}_g^{(0)}(x,x_{\mathbb{P}},K_\perp^2) = \frac{S_\perp N_g}{4\pi^3}\frac{[\mathcal{G}_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)]^2}{2\pi(1-x)}$$

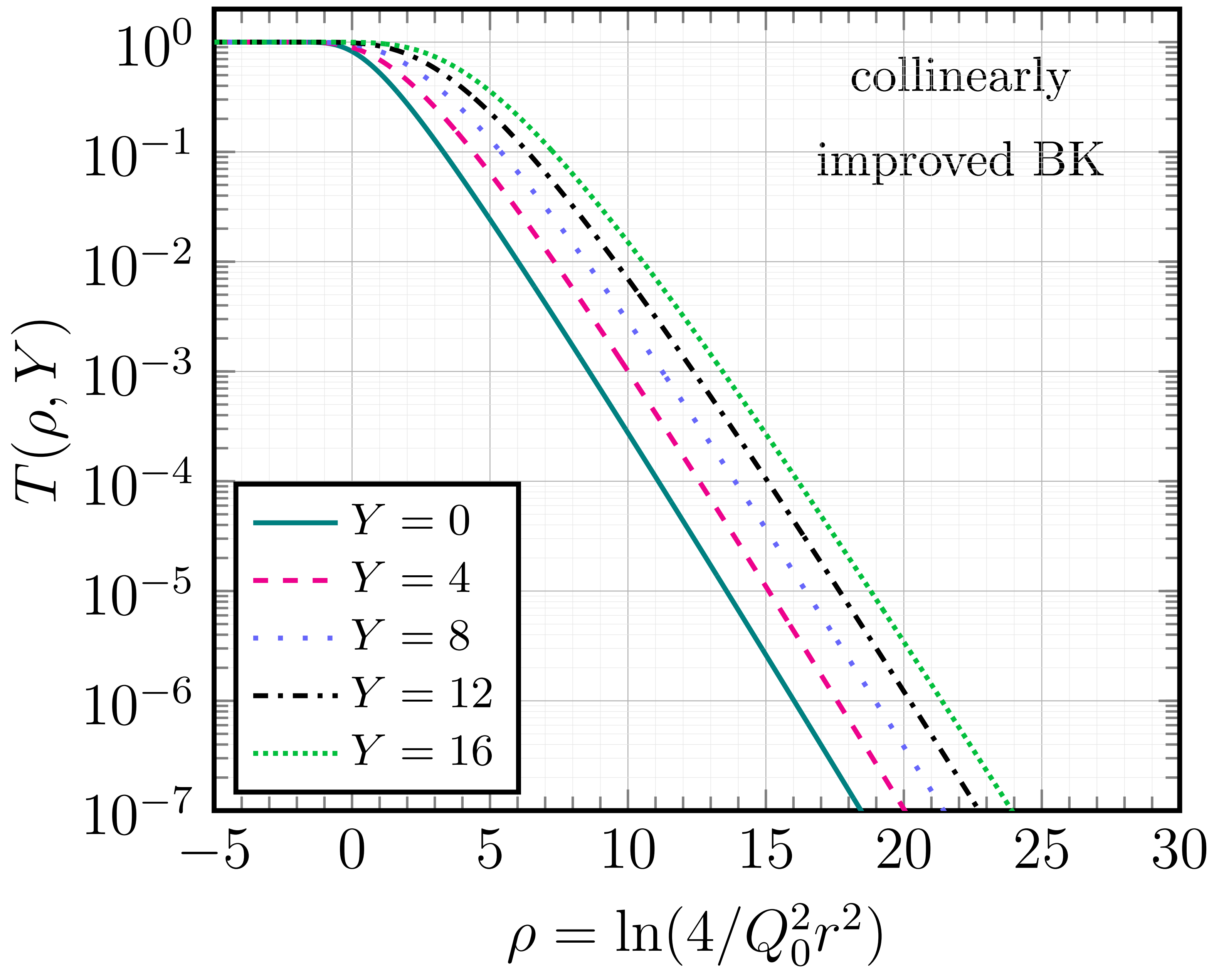
Gluon diffractive PDF $$xG^{(0)}(x,x_{\mathbb{P}},Q^2) \equiv \pi\int_0^{Q^2}\mathrm{d}K_\perp^2\,\mathcal{F}_g^{(0)}(x,x_{\mathbb{P}},K_\perp^2)$$

$$\mathcal{G}_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2) = \mathcal{M}^2\int dRR\,J_2(K_\perp R)\,K_2(\mathcal{M}R)\,\tilde{N}(R,x_{\mathbb{P}})$$

(Hauksson), Iancu, Mueller, Triantafyllopoulos, Wei, 2207.06268, 2402.14748

Non-linear evolution of dipole scattering amplitude

$$\frac{\partial N(Y,\boldsymbol{r})}{\partial Y} = \int d^2\boldsymbol{r}_1 \,\frac{\bar{\alpha}_s}{2\pi}\,\frac{\boldsymbol{r}^2}{\boldsymbol{r}_1^2\boldsymbol{r}_2^2}\,[N(Y,\boldsymbol{r}_1)+N(Y,\boldsymbol{r}_2)-N(Y,\boldsymbol{r})-N(Y,\boldsymbol{r}_1)N(Y,\boldsymbol{r}_2)]$$



$x_{\mathbb{P}} \ll 1$: non-linear evolution

DGLAP Evolution

$$\frac{\partial xG(x,x_{\mathbb{P}},Q^2)}{\partial \ln Q^2} = \pi Q^2 \mathcal{F}_g^{(0)}(x,x_{\mathbb{P}},Q^2) + \Theta(Q,\mu_0)\,\frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \mathrm{d}\xi\, P_{gg}(\xi)\,\frac{x}{\xi}G\left(\frac{x}{\xi},x_{\mathbb{P}},Q^2\right)$$

$x \sim 1$: DGLAP Evolution

(Hauksson), Iancu, Mueller, Triantafyllopoulos, Wei, 2207.06268, 2402.14748

Kinematics: $x \sim \mathcal{O}(1)$, $x_{\mathbb{P}} \sim \mathcal{O}(0)$

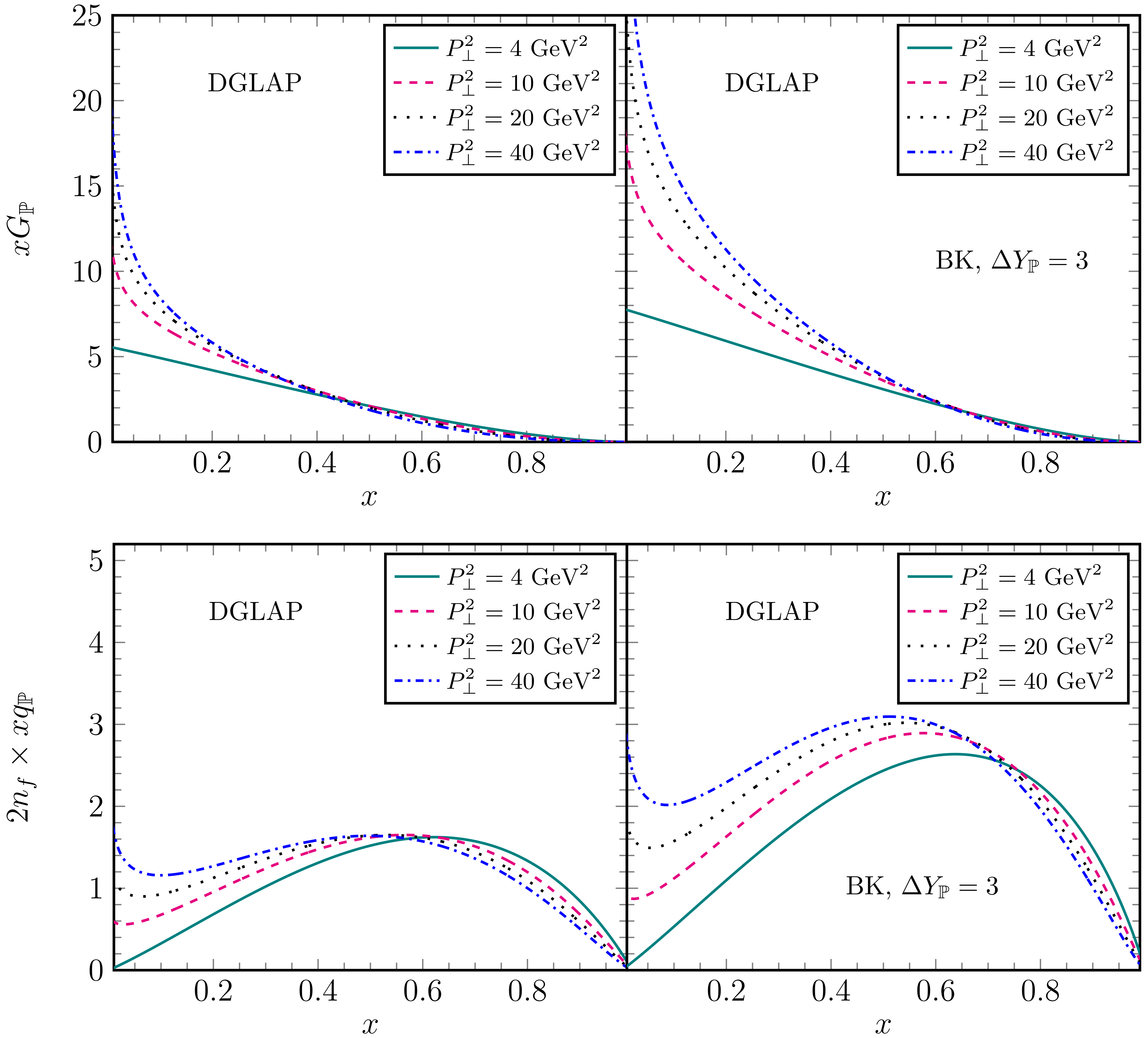
BK + DGLAP evolution

The diffractive parton distribution functions are somehow more predictable

The only inputs:

Dipole scattering amplitude
Saturation Scale

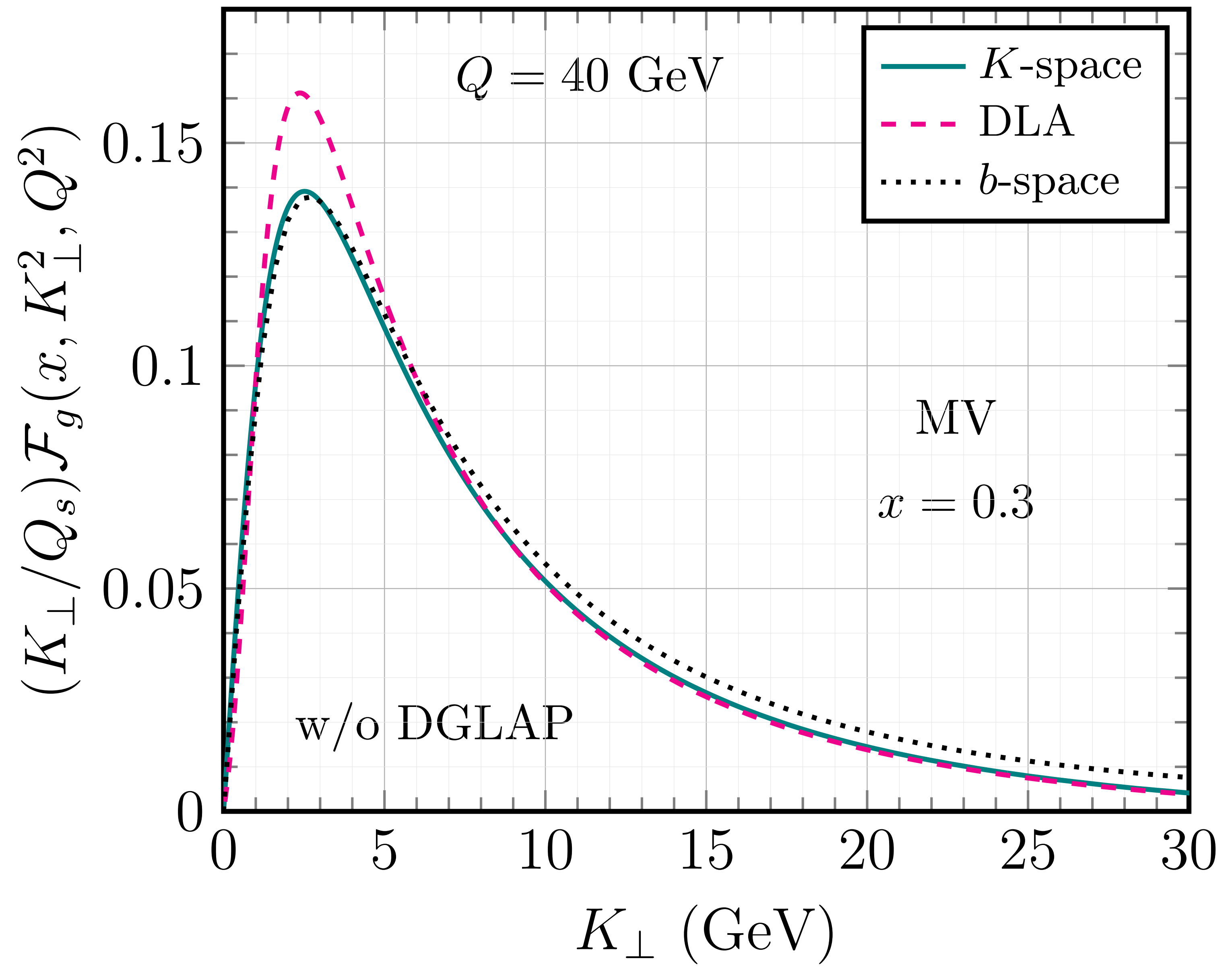
(Hauksson), Iancu, Mueller, Triantafyllopoulos, Wei, 2207.06268, 2402.14748

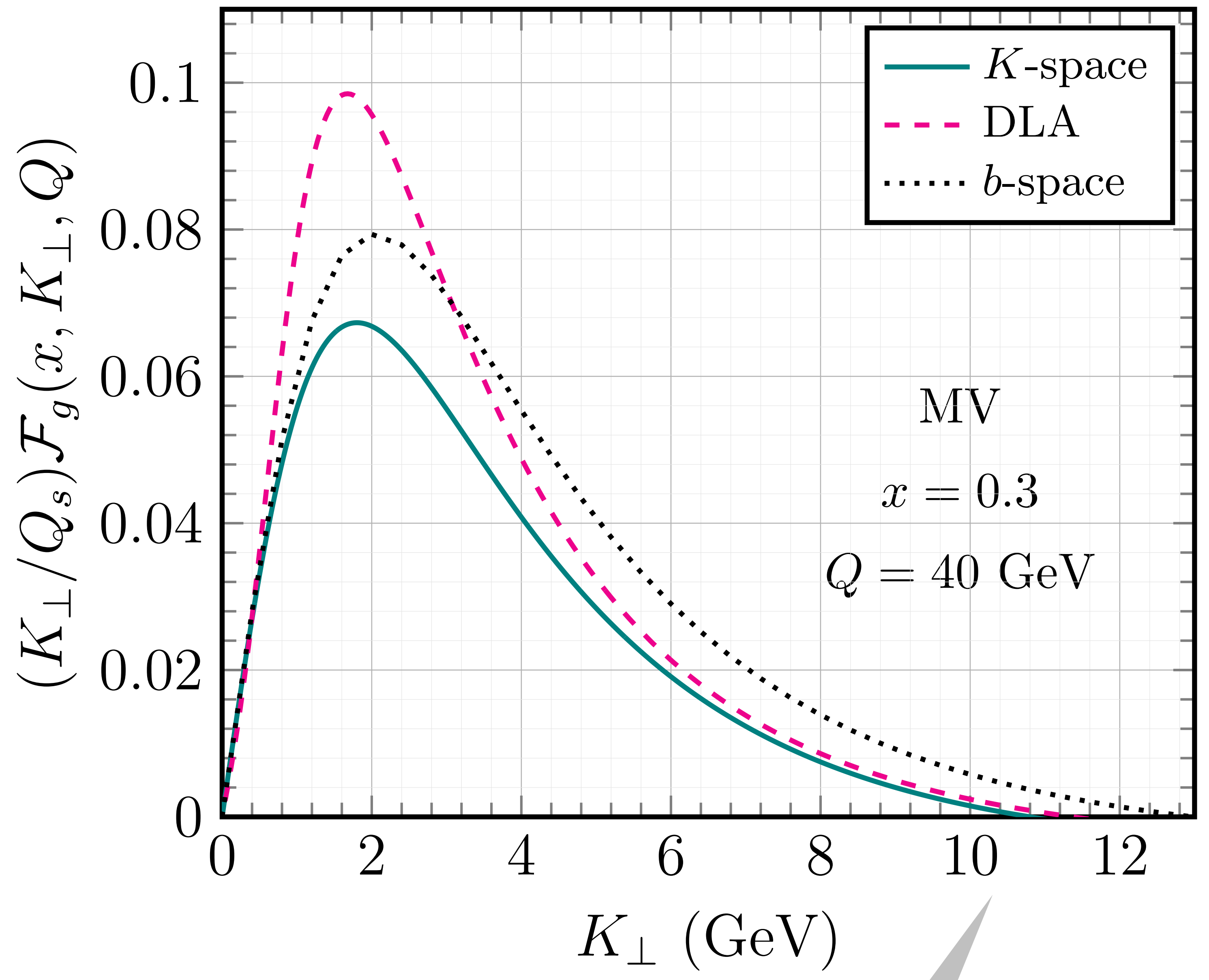


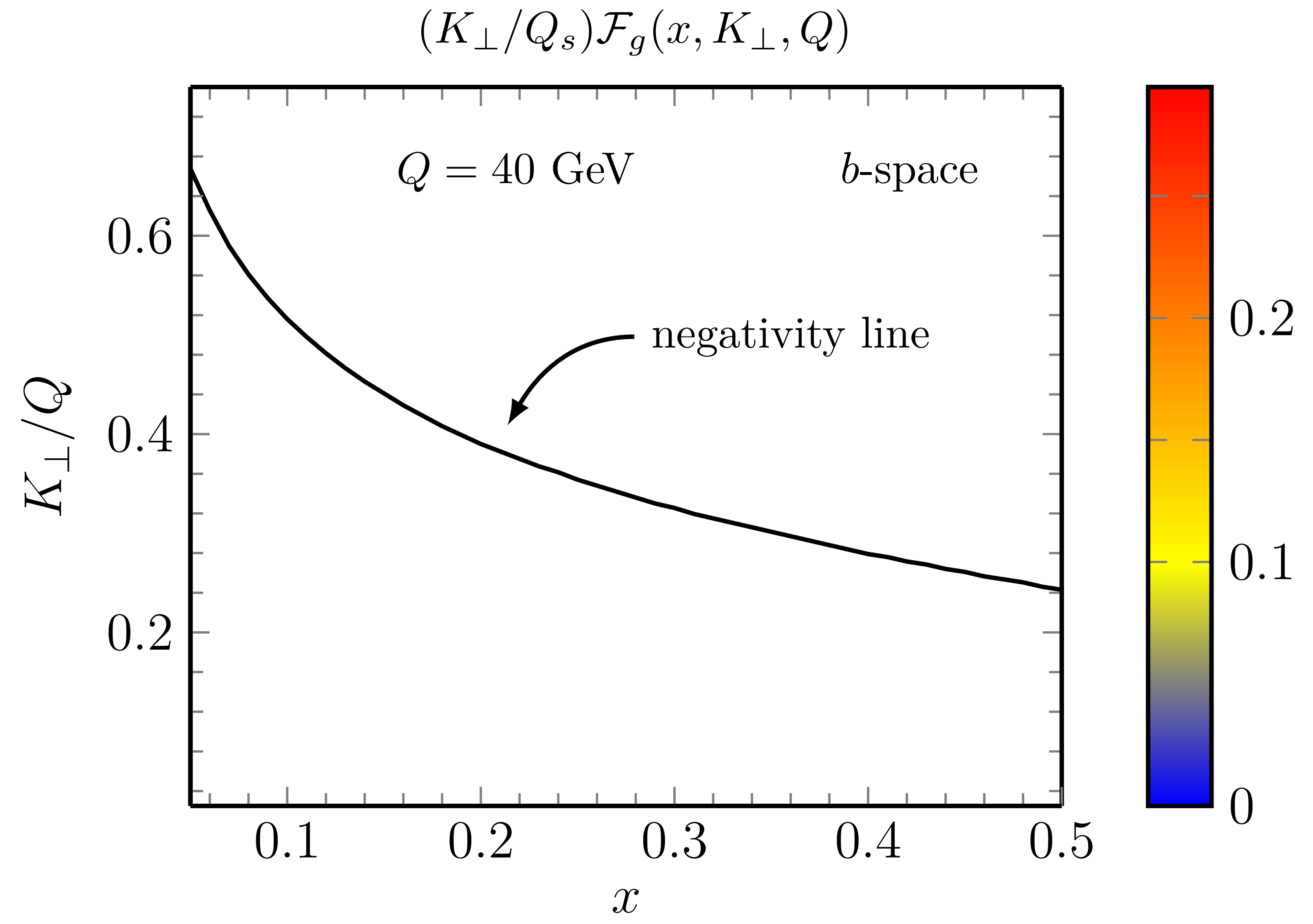
Diffractive Dijet

Collins-Soper Evolution

$$\mathcal{F}_g(x, K_\perp^2, Q^2) = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{K}\cdot\boldsymbol{b}} \frac{xG_{\mathbb{P}}(x,\mu_b^2)}{xG_{\mathbb{P}}^{(0)}(x,\mu_b^2)} \tilde{\mathcal{F}}_g(x, b_\perp) \exp\left[-S_{\text{pert}}(\mu_b^2, Q^2)\right]$$







Distribution becomes negative

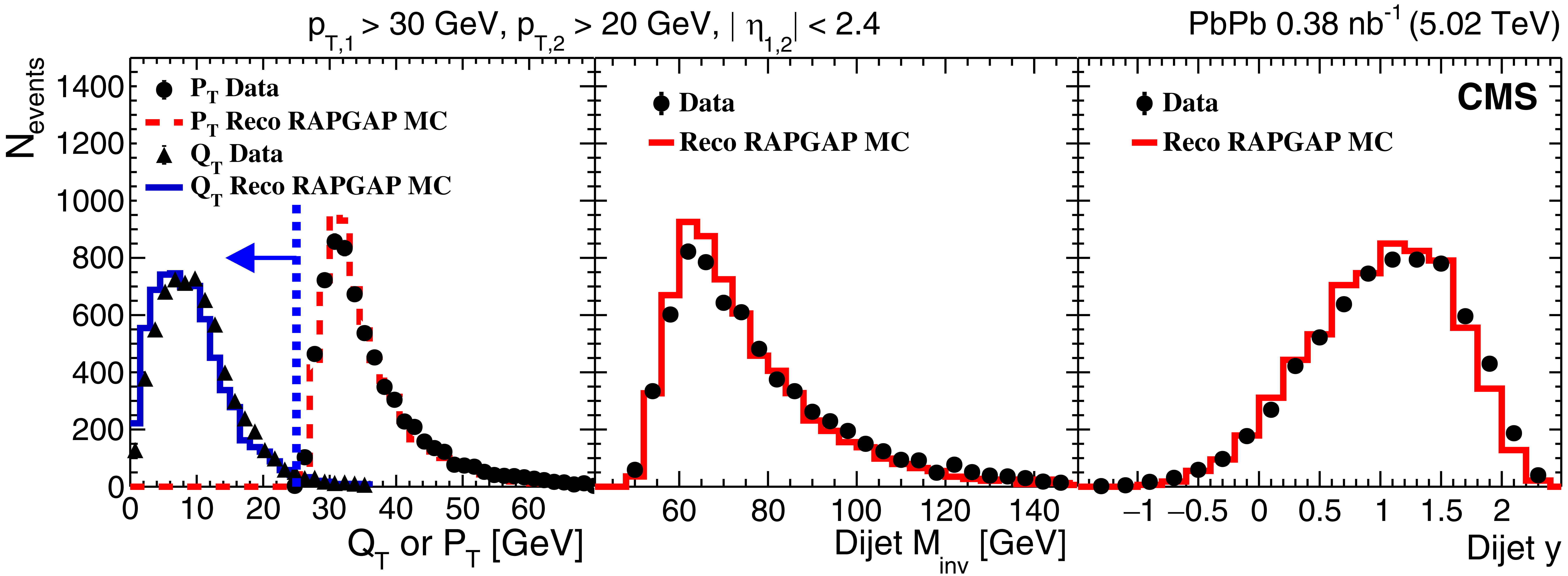
The most simple process with the most complicated QCD dynamics

Shao, Shi, Zhang, Zhou, Zhou, 2402.05465, 2606.02230; Iancu, Triantafyllopoulos, Wei, Yuan, 2512.11730, WIP

Collins-Soper Evolution

Dijet production in Photon-Pb Collions at CMS

Leading jet: $p_{T1} > 30$ GeV; Subleading jet: $p_{T2} > 20$ GeV; Rapidity: $|\eta_{1,2} < 2.4|$



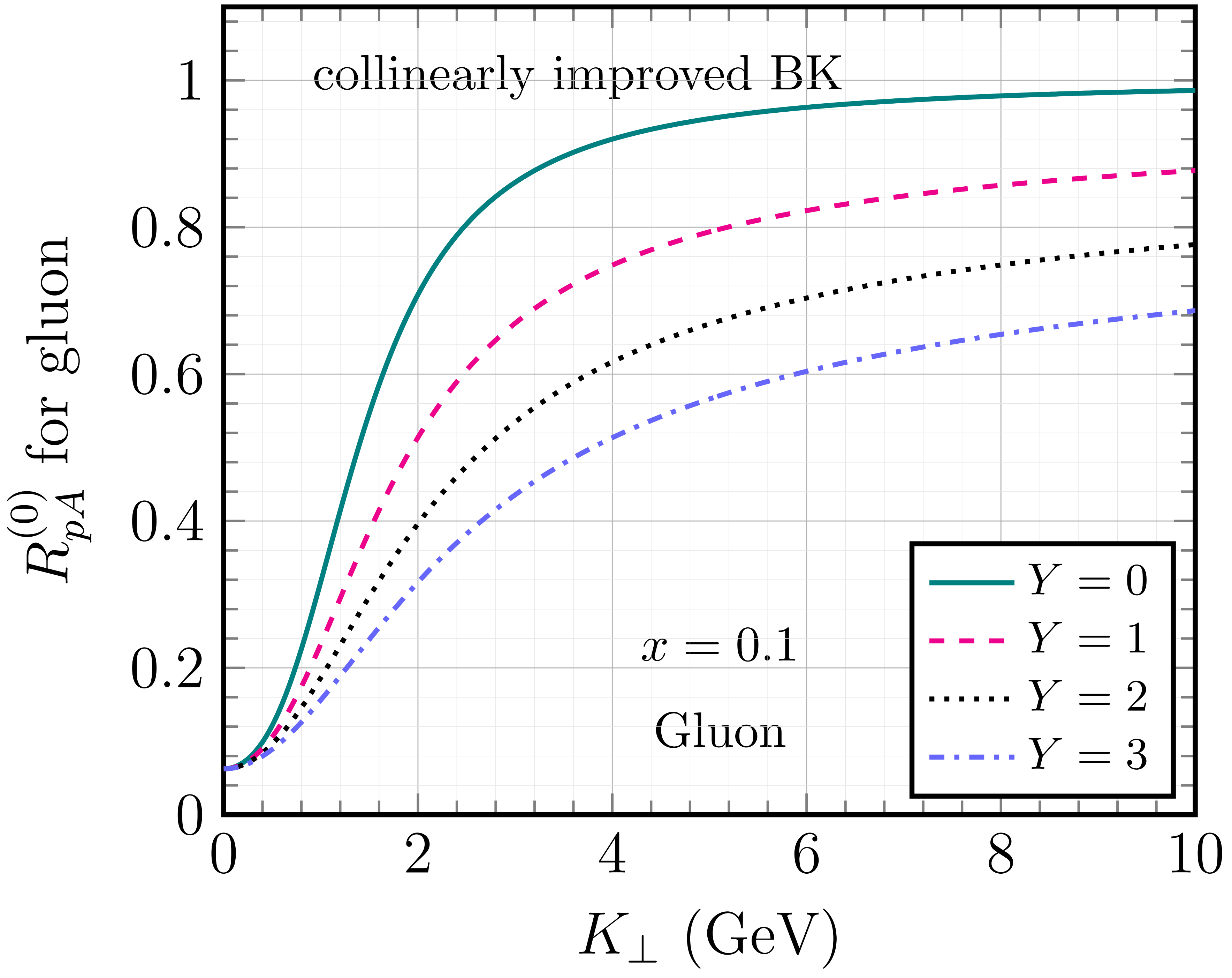
$$\vec{P}_\perp = \frac{1}{2}(\vec{p}_{T1} - \vec{p}_{T2})$$

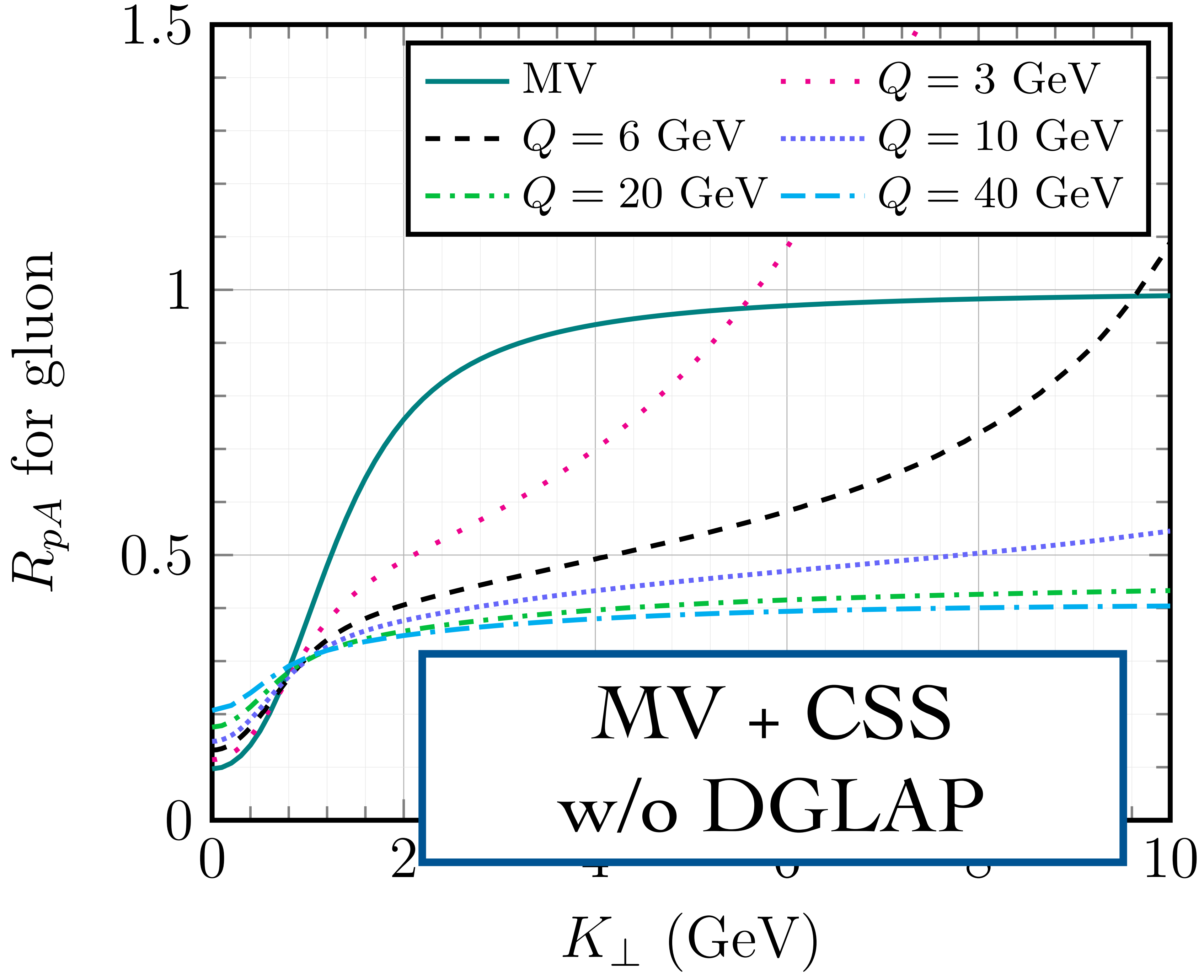
$$\vec{Q}_T = \vec{p}_{T1} + \vec{p}_{T2}$$

CMS, 2205.00045 before unfolding

Nuclear modification factor

$$R_{pA} \equiv \frac{1}{A^{2/3}} \frac{\mathcal{F}^{A}_{g,q}(x, K_\perp^2, Q^2)}{\mathcal{F}^{p}_{g,q}(x, K_\perp^2, Q^2)}$$





MV + CSS + DGLAP

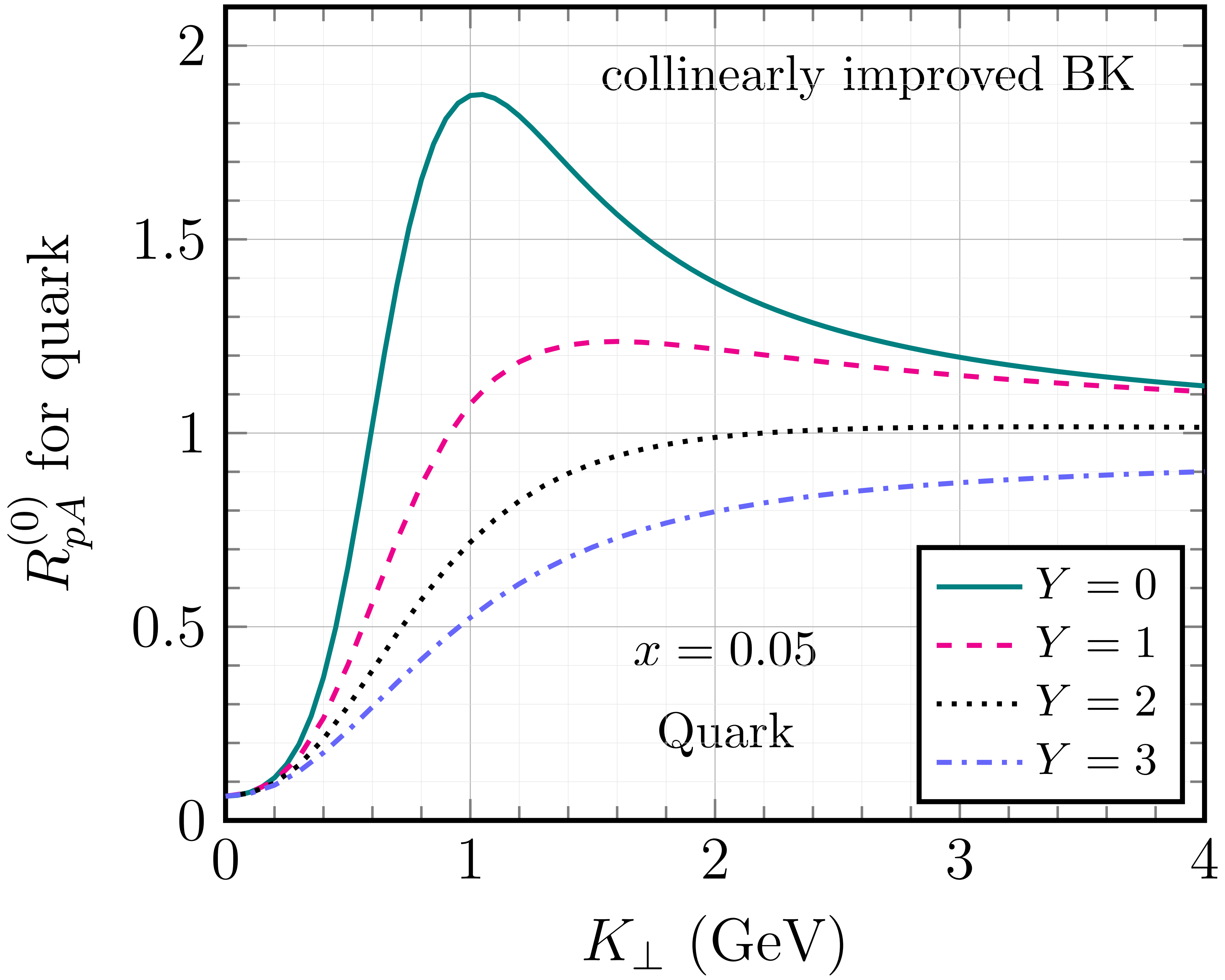
work in progress

Iancu, Triantafyllopoulos, Wei, Yuan, work in progress

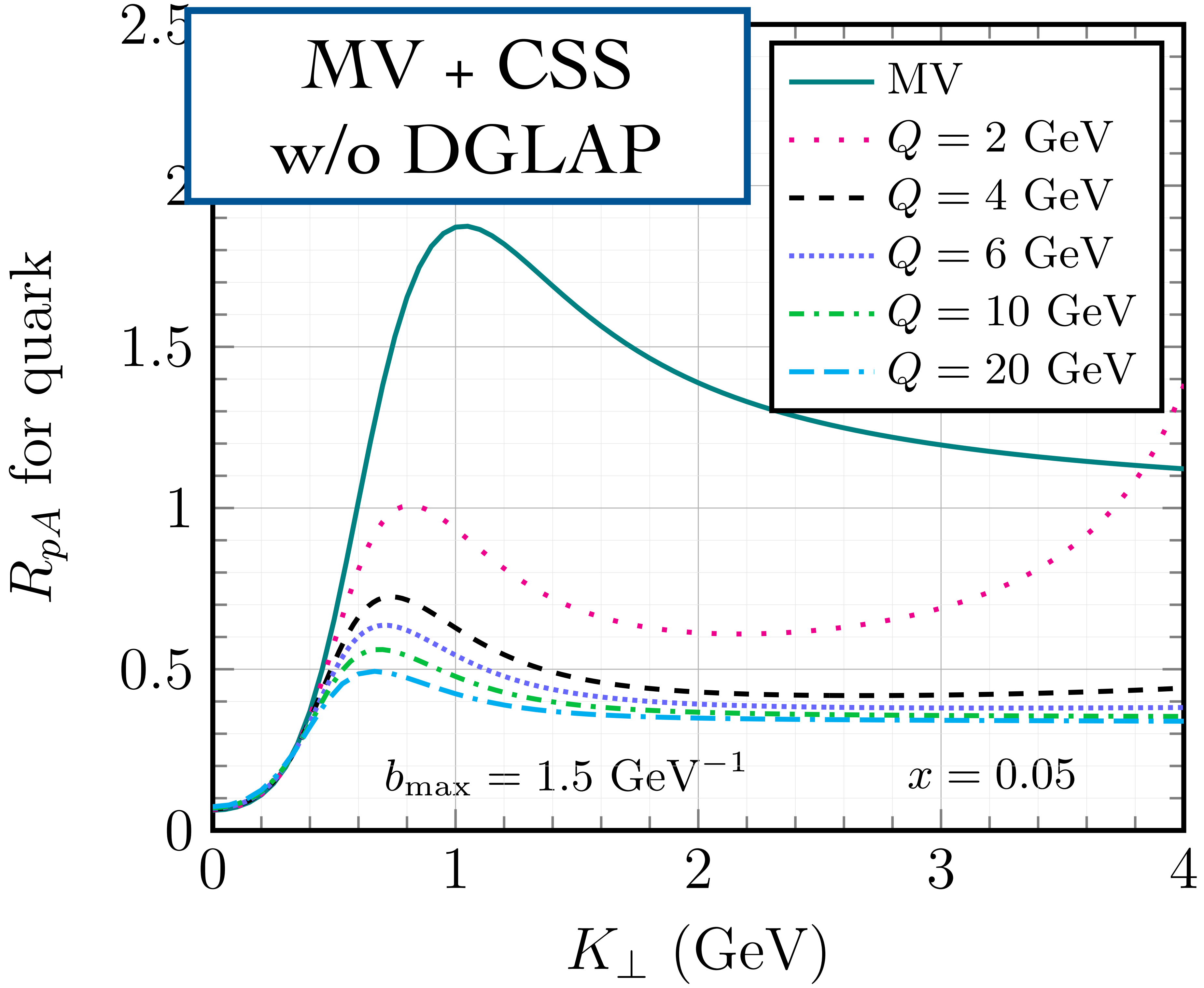
Diffractive Dijet

Nuclear modification factor

$$R_{pA} \equiv \frac{1}{A^{2/3}} \frac{\mathcal{F}^{A}_{g,q}(x, K_\perp^2, Q^2)}{\mathcal{F}^{p}_{g,q}(x, K_\perp^2, Q^2)}$$



Cronin peak disappears
with BK evolution



MV + CSS + DGLAP

work in progress

Iancu, Triantafyllopoulos, Wei, Yuan, work in progress

Conclusion

- Structure function: Dipole scattering amplitude. (**ep and eA**)
- More differential means more information.
- Diffractive dijets production encodes BK+DGLAP+Collins-Soper Evolutions.

Thank you very much for your attention!

Kinematics

Differential Cross Section in SIDIS:

$$\frac{d\sigma}{dx_{\mathrm{Bj}}dy} = \frac{2\pi\alpha_{\mathrm{em}}^2}{x_{\mathrm{Bj}}\,y\,Q^2}\left[(1+(1-y)^2)\;F_{UU}^T(x_{\mathrm{Bj}},Q^2)+2(1-y)\;F_{UU}^L(x_{\mathrm{Bj}},Q^2)\right]$$

for $\gamma^* + q \to q$, $F_{UU}^L = 0$

Replacements: $F_{UU}^T = 2xF_1$, $F_{UU}^L = F_L$, $F_2 = 2xF_1 + F_L$

$$\frac{d\sigma}{dx_{\mathrm{Bj}}dy} = \frac{2\pi\alpha_{\mathrm{em}}^2}{x_{\mathrm{Bj}}\,y\,Q^2}\left[(1+(1-y)^2)\;(F_2-F_L)+2(1-y)\;F_L\right]$$

$$= \frac{2\pi\alpha_{\mathrm{em}}^2}{x_{\mathrm{Bj}}\,y\,Q^2}\left[(1+(1-y)^2)\;F_2+[2-2y-2+2y-y^2]\;F_L\right]$$

$$= \frac{2\pi\alpha_{\mathrm{em}}^2}{x_{\mathrm{Bj}}\,y\,Q^2}\left[(1+(1-y)^2)\;F_2-y^2\;F_L\right]$$

Kinematics

Total DIS

$$\sigma_T^{\gamma^* p}(x_{\rm Bj}, Q^2) = \sum_q \frac{2N_c \alpha_{\rm em} S_\perp e_q^2}{\pi} \int_0^1 dz \int \frac{d^2 r}{4\pi} [z^2 + (1-z)^2]\ \bar{Q}^2\, K_1^2(\bar{Q} r_\perp) \times 2N(x_{\rm Bj}, \boldsymbol{r})$$

Elastic/ diffractive

$$\sigma_{T,\rm el}^{\gamma^* p}(x_{\rm Bj}, Q^2) = \sum_q \frac{2N_c \alpha_{\rm em} S_\perp e_q^2}{\pi} \int_0^1 dz \int \frac{d^2 r}{4\pi} [z^2 + (1-z)^2]\ \bar{Q}^2\, K_1^2(\bar{Q} r_\perp) \times N^2(x_{\rm Bj}, \boldsymbol{r})$$

inelastic

$$\sigma_{T,\rm inel}^{\gamma^* p}(x_{\rm Bj}, Q^2) = \sum_q \frac{2N_c \alpha_{\rm em} S_\perp e_q^2}{\pi} \int_0^1 dz \int \frac{d^2 r}{4\pi} [z^2 + (1-z)^2]\ \bar{Q}^2\, K_1^2(\bar{Q} r_\perp) \times \left[2N(x_{\rm Bj}, \boldsymbol{r}) - N^2(x_{\rm Bj}, \boldsymbol{r})\right]$$