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Complex τ Dipole Moment From GeV-Scale New Physics

Probing CP violation in τ sector[1]

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TeV 物理和超出标准模型新物理

[1] based on *Chin.Phys.Lett. 43 (2026) 030201,2606*,and *arXiv. 2606.01178*

Dipole moment: a brief review

- The anomalous magnetic dipole moment (MDM), $a_f \equiv (g-2)_f/2$,

$$\mathcal{L}_{\text{eff}} \supset \frac{iea_f}{4m_f}\bar{f}\sigma^{\mu\nu}fF_{\mu\nu} \sim \mu_a\hat{S}\cdot B, \quad \mu_a = \frac{e}{m}a_f$$

- electric dipole moment (EDM): d_f,

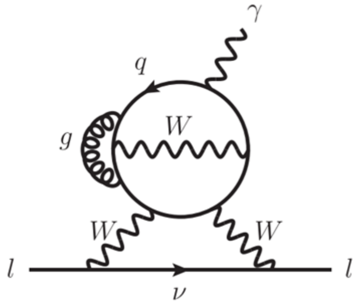
$$\mathcal{L}_{\text{eff}} \supset \frac{d_f}{2}\bar{f}\sigma^{\mu\nu}\gamma^5 fF_{\mu\nu} \sim d\hat{S}\cdot E, \quad \textcolor{red}{C\text{ even}, P\text{ odd}}$$

- At tree level, $g = 2$ and $d_f = 0$.
- they are sensitive to the new particle and new interaction. ($\propto (m_l/\Lambda)^2$.)
- An EDM is a direct probe of CP violation.

The SM prediction

- In SM, the CP-violating invariant: $\bar{\delta} = \mathrm{Im}[V_{us}V_{cs}^{*}V_{cb}V_{ub}]$.

Suppressed by Glashow–Iliopoulos–Maiani (GIM) mechanism, EDM $\propto \left(m_b^2 m_c^2 m_s^2\right)/m_t^6$.



2006.00281

Atoms 7 (2019) 1, 28:

$$a_e \approx 1.15\,965\,218\,059 \times 10^{-3},$$

Lepton EDM in SM:

$$|d_e^{\mathrm{SM}}| \sim 5.8 \times 10^{-40} e\mathrm{cm}$$

$$|d_\mu^{\mathrm{SM}}| \sim 1.4 \times 10^{-38} e\mathrm{cm}$$

$$|d_\tau^{\mathrm{SM}}| \sim 7.3 \times 10^{-38} e\mathrm{cm}$$

2505.21476:

$$a_\mu \approx 1.16592033 \times 10^{-3}.$$

hep-ph/0701260:

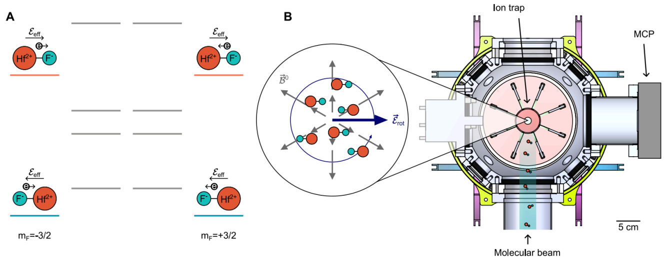
$$a_\tau \approx 1.1721 \times 10^{-3}$$

Lepton sector

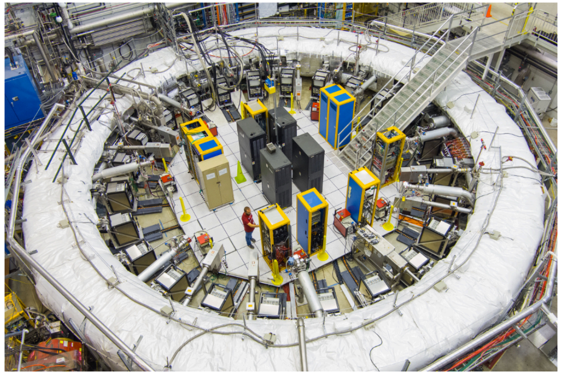
- $a_e^{\text{exp}} = 1\,159\,652\,180\,59(13) \times 10^{-14}$

arXiv:2209.13084

- $d_e^{\text{exp}} < 4.1 \times 10^{-30} ecm$ 90% C.L. (Ion trap, arXiv:2212.11841)



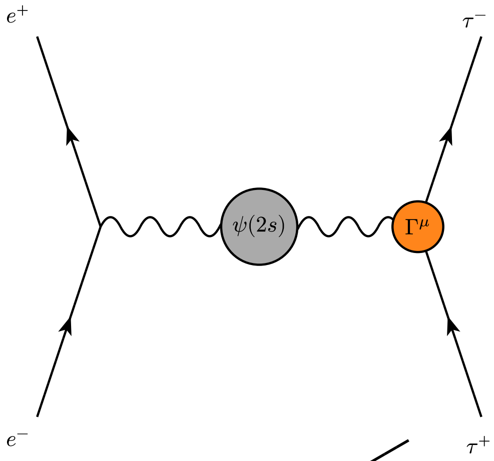
- $a_\mu^{\text{exp}} = 116\,592\;\;0715(145) \times 10^{-12}$ 2506.03069
- $d_\mu^{\text{exp}} < 1.8 \times 10^{-19} ecm$ 95% C.L. arXiv: 0811.1207

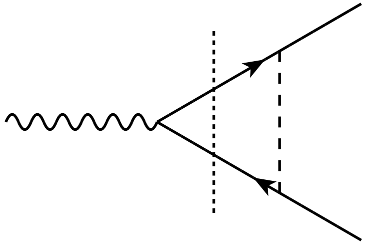


- the a_τ and d_τ remain poorly constrained due to τ **short life** $(t_\tau \sim 10^{-13} s)$.
  - Suggest to study at collider.

EM form factor

$$\Gamma^\mu = \bar{u}\left(\gamma^\mu F_V + \overbrace{\frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau}}^{(g-2)_\tau} H_\sigma + \gamma^\mu\gamma^5 F_A + \underbrace{\sigma^{\mu\nu}\gamma^5 q_\nu H_T}_{CP\text{ violating } d_\tau}\right) v$$





- In τ pair production, $F_i, H_i \to F_i(q^2), H_i(q^2)$.

  $$d_\tau(q^2) \equiv eH_T(q^2),\ a_\tau(q^2) \equiv \frac{(g-2)_\tau}{2} = H_\sigma(q^2)$$

- Intermediate particles are on shell:
  - form factor develop **imaginary** part.

- The **imaginary** d_τ is more sensitive to some NP model.

Current Experiment Status (From PDG (2026))

d_τ @ BELLE ($\sqrt{q^2} = 10.58$ GeV) (2108.11543):

$$\mathrm{Re}[d_\tau] = (6.2 \pm 6.3) \times 10^{-18} e\mathrm{cm},$$
$$\mathrm{Im}[d_\tau] = (4.0 \pm 3.2) \times 10^{-18} e\mathrm{cm}.$$

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a_τ @ ATLAS EXPERIMENT, $pp \to \tau^+\tau^-$ (2503.19836):

$$a_\tau = -0.5^{+2.6}_{-0.9} \times 10^{-3},$$

And a_τ @ CMS, $\gamma\gamma \to \tau^+\tau^-$ (2406.03975):

$$a_\tau = 0.9^{+3.2}_{-3.1} \times 10^{-3},$$

- The data is analyzed in the EFT framework.

$$\mathcal{L}_{\mathrm{LEFT}} = \frac{v}{\sqrt{2}\Lambda^2} c_{\tau\gamma} \bar{\tau}_L \sigma^{\mu\nu} \tau_R F_{\mu\nu} + \mathrm{h.c.}$$

$$\delta a_\tau = \frac{2\sqrt{2} m_\tau v}{e} \frac{\mathrm{Re}(c_{\tau\gamma})}{\Lambda^2}, \quad \delta d_\tau = \sqrt{2} v \frac{\mathrm{Im}(c_{\tau\gamma})}{\Lambda^2}.$$

Both a_τ measurement assume *no new physical* at low energy.

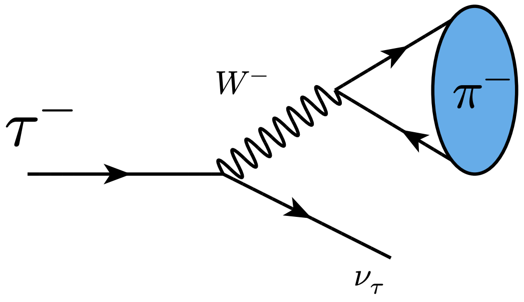
No $H_\sigma(q^2)$ or $a_\tau(q^2)$ measured yet.

The measurement

$$\Gamma^{\mu} = \bar{u}\left(\gamma^{\mu} F_V + \frac{i\sigma^{\mu\nu} q_\nu}{2m_\tau} H_\sigma + \gamma^{\mu}\gamma^{5} F_A + \sigma^{\mu\nu}\gamma^{5} q_\nu H_T\right) v$$

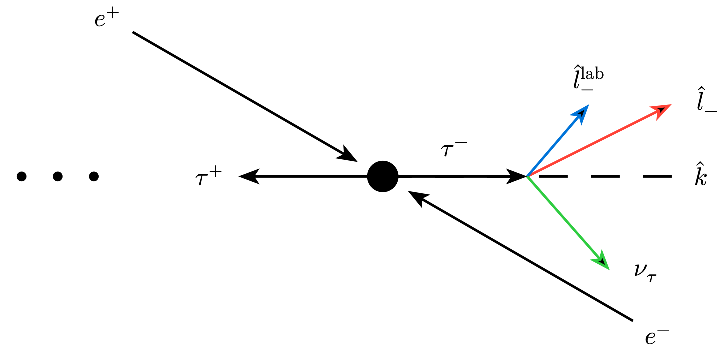
$$\left|i\mathcal{M}\right|^2 \sim |\epsilon_\mu \Gamma^\mu|^2 \sim 1 + b_+ \cdot \hat{s}_+ + b_- \cdot \hat{s}_- - \hat{s}_+ \cdot \overleftrightarrow{c} \cdot \hat{s}_-,$$

- $\hat{s}_\pm$: spin polarization of $\tau^\pm$, analyzed by τ decay
- The CP violation is included in the asymmetry of $\hat{s}_\pm$.[1]
- $\mathrm{d}\Gamma/\,\mathrm{d}\cos\theta_\pm \sim \frac{1}{2}\left(1 \pm \alpha_\pi \hat{l}_{\pi^\pm} \cdot \hat{s}_\pm\right)$



[1] See *JHEP 04, 110 (2022)*, *Phys. Rev. D* **110**, *no.7, 076019 (2024)*, *JHEP 04 (2025) 001*, *Phys. Rev. D* **112**, *no.7, 075039 (2025)*, *Phys.Rev.D 112 (2025) 7,075039* ...

JHEP 04 (2025) 001, *arXiv. 2606.01178*



$$\mathrm{Im}[d_\tau] = -\frac{3}{4}\frac{e\left(s+2m_\tau^2\right)}{m_\tau\sqrt{s}\sqrt{s-4m_\tau^2}}\left(\frac{\langle \hat{l}_-\cdot\hat{k}\rangle}{\alpha_h}+\frac{\langle \hat{l}_+\cdot\hat{k}\rangle}{\bar{\alpha}_h}\right), \quad 7$$

$$\mathrm{Re}[d_\tau] = e\frac{9}{4}\frac{s+2m_\tau^2}{\alpha_h\bar{\alpha}_{h'}m_\tau\sqrt{s^2-4sm_\tau^2}}\langle\left(\hat{l}_-\times\hat{l}_+\right)\cdot\hat{k}\rangle.$$

$$\mathrm{Re}[a_\tau] = \frac{2\left(s+2m_\tau^2\right)\left(s+m_\tau^2\right)}{s\left(s-4m_\tau^2\right)} - \frac{5\left(s+2m_\tau^2\right)^2}{s\left(s-4m_\tau^2\right)}\langle\left(\hat{p}\cdot\hat{k}\right)^2\rangle,$$

$$\mathrm{Im}[a_\tau] = -\frac{64m_\tau\left(2m_\tau^2+s\right)}{\pi\alpha_{h_{+-}}\sqrt{s}\left(s-4m_\tau^2\right)}\left\langle\frac{\left(\hat{p}\cdot\hat{k}\right)\left[l_\mp\cdot\left(\hat{p}\times\hat{k}\right)\right]}{\left|\hat{p}\times\hat{k}\right|}\right\rangle.$$

	$\sqrt{q^2}$(GeV)	$\delta_{d_\tau}(10^{-18}e\mathrm{cm})$	$\delta_{a_\tau}(10^{-3})$
STCF	6.3	$4.0+0.7i$	/
STCF	$m_{\psi(2s)}$	$83+1.8i$	$2.2+4.9i$
Belle II	$m_{\Upsilon(4s)}$	$0.7+0.5i$	$0.06+0.05i$

68% C.L.

The **QED** oneloop prediction (in unit of 10^{-3}):

$a_\tau\left(m_\psi(2s)\right) \sim 1.1+6.4i, \quad a_\tau\left(m_{\Upsilon(4s)}\right) \sim 0.24+0.22i.$

- q^2-dependence matters.

also see *Phys.Rev.D 112 (2025) 7, 075039, JHEP 07 (2025) 172,...*

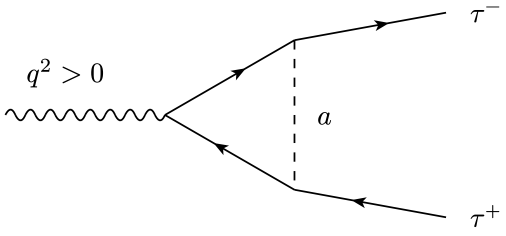
Light new physics generate CP violation

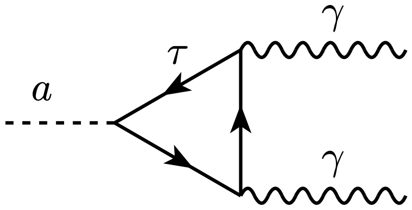
$$\mathcal{L}_{\text{int}} = a\bar{\tau}(\tilde{g}_\tau + i g_\tau \gamma^5)\tau$$

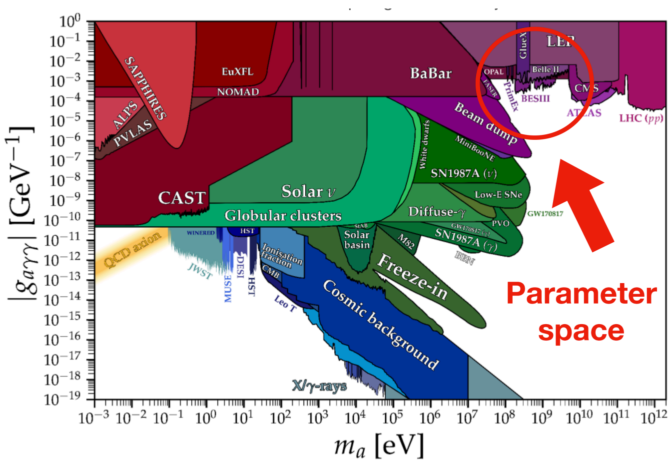
Scenario: A new light scalar (GeV) couple to τ with both parities.

- May arise from CPV *axion-like particle*(*ALP*) model, multi-Higgs doublets model(e.g. *2HDM*), ...

- The tightest constraint $\gamma^* \to a\gamma$[1],

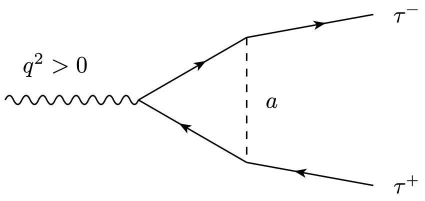


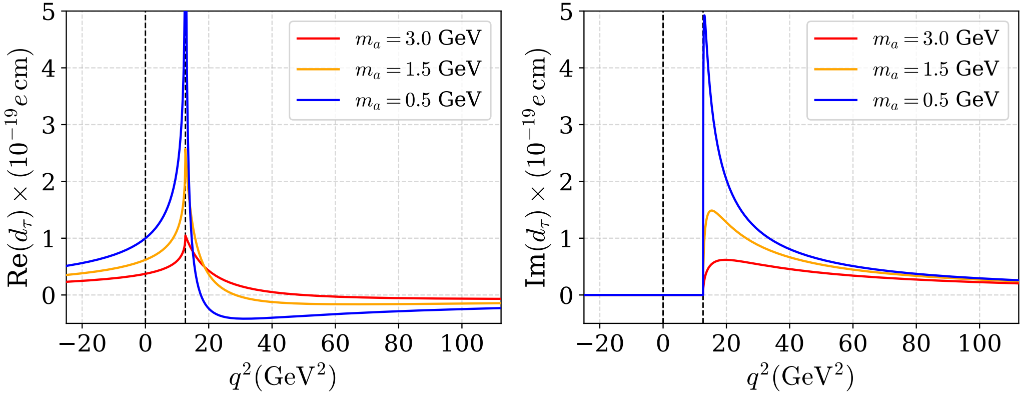




[1] https://github.com/cajohare/AxionLimits

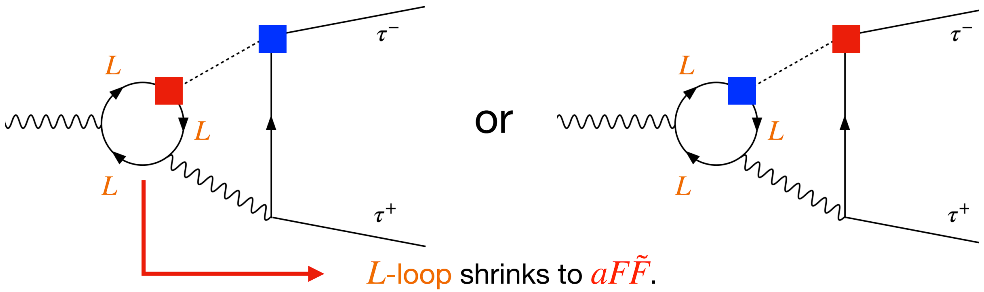
EDM result





- At one loop, $d_\tau \sim 10^{-19}e\mathrm{cm}$ at $\psi(2s)$.
- $d_\tau \sim 10^{-20}e\mathrm{cm}$ at $\Upsilon(4s)$.

benchmark: $g_\tau \sim \tilde{g}_\tau \sim 0.2$,



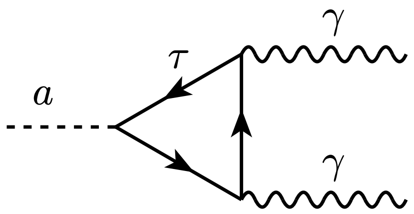
$\longleftarrow$Bar-Zee type diagram,[1]

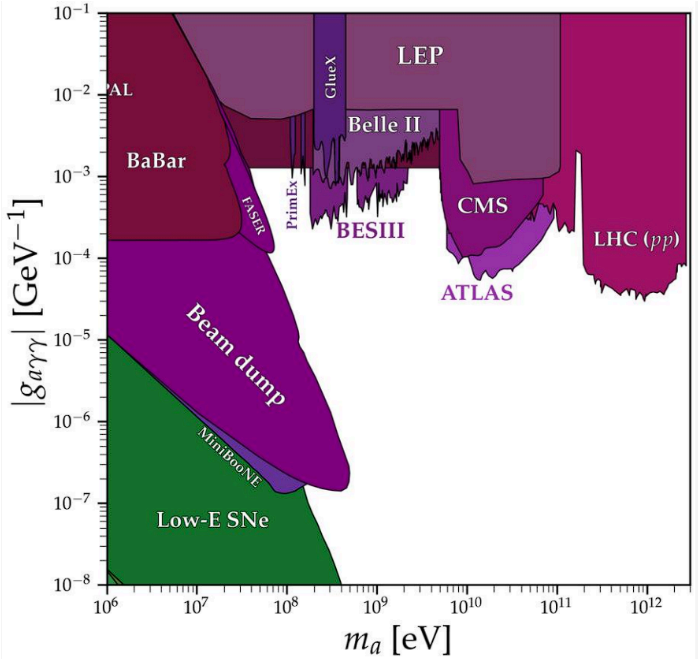
- no chiral enhancement.
- Numerically, two orders smaller than experiment sensitivity.

[1] *Rev. Lett. 65, 21 (1990)*

Parameter space

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{4} g F\tilde{F} + \frac{1}{4}\tilde{g} F F$$

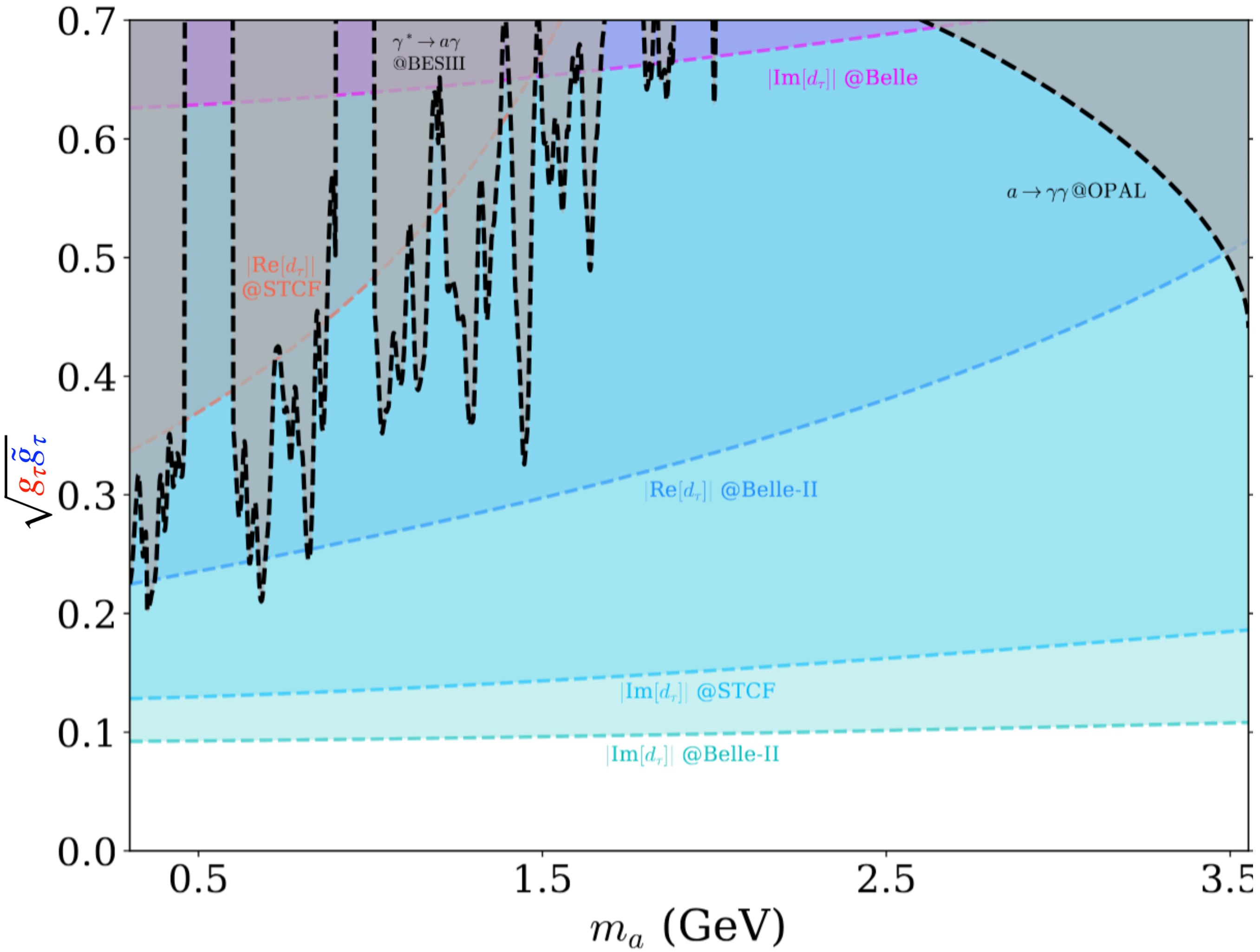


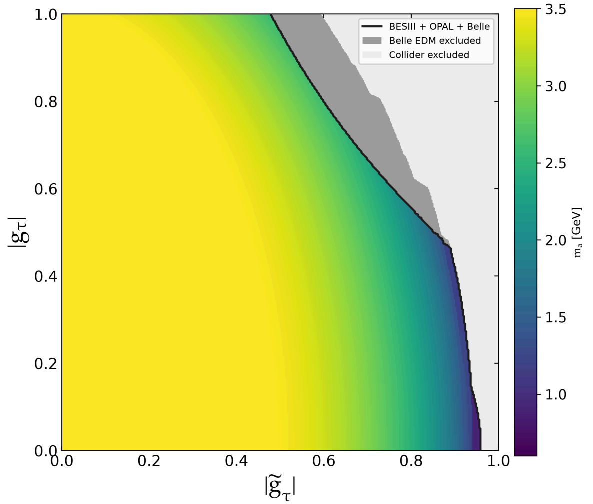


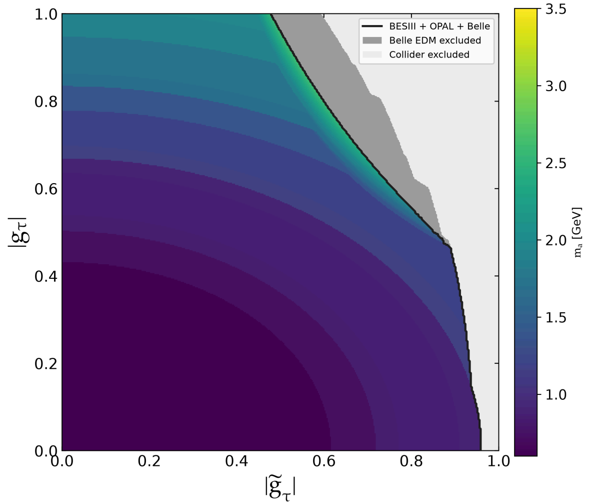
collider searching: @BESIII (*2211.12699*) + @OPAL(*hep-ex/0210016*)

$$\frac{\mathcal{B}(\gamma^* \to \gamma a)}{\mathcal{B}(\gamma^* \to e^+e^-)} = \frac{q^2(g^2+\tilde{g}^2)}{32\pi\alpha_{\text{em}}}\left(1-\frac{m_a^2}{q^2}\right)^3,$$

$$g \propto g_\tau, \quad \tilde{g} \propto \tilde{g}_\tau.$$


$\gamma^* \to a\gamma$
@BESIII
$|\mathrm{Im}[d_\tau]|$ @Belle
$a \to \gamma\gamma$@OPAL
$|\mathrm{Re}[d_\tau]|$
@STCF
$|\mathrm{Re}[d_\tau]|$ @Belle-II
$|\mathrm{Im}[d_\tau]|$ @STCF
$|\mathrm{Im}[d_\tau]|$ @Belle-II
$\sqrt{g_\tau \tilde{g}_\tau}$
m_a (GeV)


BESIII + OPAL + Belle
Belle EDM excluded
Collider excluded
$|g_\tau|$
$|\tilde{g}_\tau|$
m_a [GeV]


BESIII + OPAL + Belle
Belle EDM excluded
Collider excluded
$|g_\tau|$
$|\tilde{g}_\tau|$
m_a [GeV]

Conclusion

1. Spin-dependent τ decay correlations provide access to both real and imaginary part of d_τ and a_τ.
2. Timelike τ dipole moments open a new window for probing new physics at future e^+e^- colliders.
3. $\mathrm{Im}[d_\tau]$ is particularly sensitive to CP violation involving **light new scalar**.
4. Meanwhile, a precise measurement of a_τ is still needed to test the **QED prediction**.

Thank you for your attention!

Back up

Two Higgs Doublet(2HDM) Realization

- Two-Higgs-doublet models are conservative extensions of the Higgs sector.
- They add a second $\mathrm{SU}(2)_L$ doublet and produce a scalar spectrum containing two CP-even neutral states, one CP-odd neutral state, and charged Higgs bosons in the CP-conserving limit.

$$
\begin{aligned}
V(\Phi_1,\Phi_2) = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right) \\
& + \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\
& + \left[\frac{1}{2}\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \left(\lambda_6\Phi_1^\dagger\Phi_1 + \lambda_7\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + \mathrm{c.c.}\right]
\end{aligned}
$$

Higgs flavor symmetry: $\mathrm{SU}(2)\times\mathrm{U}(1)$ on (Φ_1,Φ_2),

$$
\Phi_a \to e^{i\theta_a}\Phi_a \quad , \quad \Phi_a \to U_{ab}\Phi_b
$$

In the Higgs basis, $\Phi_1 = \left(G^+, \frac{v+h+iG^0}{\sqrt{2}}\right)^T$, $\Phi_2 = \left(H^+, \frac{H+iA}{\sqrt{2}}\right)^T$.

The mass matrix (h, H, A) is

$$\frac{M^2}{v^2} = \begin{pmatrix} \lambda_1 & \mathrm{Re}[\lambda_6] & -\mathrm{Im}[\lambda_6] \\ \mathrm{Re}[\lambda_6] & m_{22}^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \mathrm{Re}[\lambda_5])v^2 & -\frac{1}{2}\,\mathrm{Im}[\lambda_5] \\ -\mathrm{Im}[\lambda_6] & -\frac{1}{2}\,\mathrm{Im}[\lambda_5] & m_{22}^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \mathrm{Re}[\lambda_5])v^2 \end{pmatrix}$$

- Decoupling: $m_H, m_A \gg v$.

☺ Alignment limit: $\lambda_6 = 0$. (allow light states)

- corresponds to suppressing the mixing between the SM-like direction and the extra neutral states.

- Consider the τ sector (**no flavor changing**)

$$\mathcal{L}_Y \supset -\left(\frac{1}{\sqrt{2}}\right)\overline{\tau}[Y_1(v+h) + Y_2(H + iA\gamma_5)]\tau$$

There are two invariant CP sources: $\mathrm{Im}\left[\lambda_5^* Y_2^2\right]$, $\mathrm{Im}\left[\lambda_5^* \lambda_7^2\right]$ in alignment limit.

A nonzero $\mathrm{Im}[\lambda_5]$ induces mixing between the CP-even and CP-odd neutral scalars.

After $U(1)$ rephasing: $H_2 \to H_2 e^{-\frac{1}{2}\arg\lambda_5} \Rightarrow (H, A) \to H', A'$

the CP eigenstates (H, A) become the mass eigenstates (H', A').

$$\mathcal{L}_Y = \overline{\tau}(a_H + ib_H\gamma_5)\tau H' + \overline{\tau}(a_A + ib_A\gamma_5)\tau A'$$

- For simplicity, set $\mathrm{Im}[\lambda_5] = 0$,

$$a_H = \mathrm{Re}\frac{[Y_2]}{\sqrt{2}}, \quad b_H = \mathrm{Im}\frac{[Y_2]}{\sqrt{2}}. \quad a_A = -\,\mathrm{Im}\frac{[Y_2]}{\sqrt{2}}, \quad b_A = \mathrm{Re}\frac{[Y_2]}{\sqrt{2}}$$

Undetermined

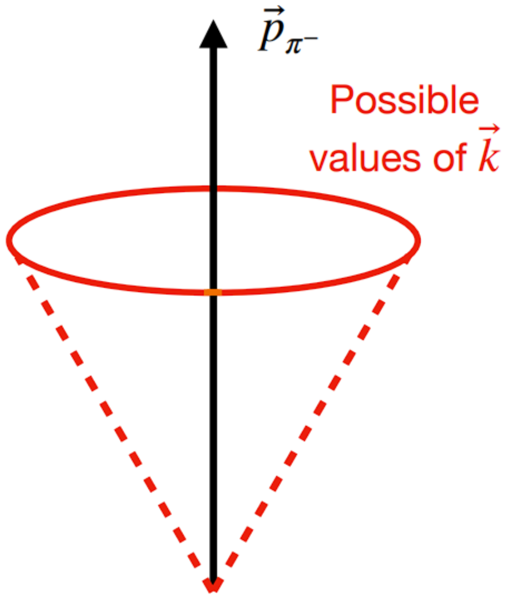
- The momenta **cannot** be fully reconstructed.

$$\mathrm{Im}\left(d_\tau\right) = -\frac{3}{4}\frac{e\left(s+2m_\tau^2\right)}{m_\tau\sqrt{s}\sqrt{s-4m_\tau^2}}\left(\left\langle \hat{p}_{\pi^-}\cdot\hat{k}\right\rangle + \left\langle \hat{p}_{\pi^+}\cdot\hat{k}\right\rangle\right)$$

- Fortunately, we can use $(k^\mu - p^\mu_{\pi^-})^2 = m_\nu^2$ to reconstruct $\hat{p}_{\pi^-}\cdot\hat{k}$.

$$\hat{p}_{\pi^\pm}\cdot\hat{k} = \pm\frac{4E_{\pi^\pm}m_\tau^2 - m_h^2\sqrt{s} - m_\tau^2\sqrt{s}}{\left(m_\tau^2 - m_h^2\right)\sqrt{s-4m_\tau^2}}$$

- With E_π , we can determine $\hat{p}_{\pi^\pm}\cdot\hat{k}$ and $\hat{k}$ up to a circle.



$\sqrt{s}$	$m_{\psi(2S)}$	4.2 GeV	4.9 GeV	5.6 GeV	6.3 GeV	7 GeV
δ_{Im}	1.8	0.9	0.7	0.7	0.7	0.7

Table. Precision at STCF in units of $10^{-18}e$cm, an order better than current data.

Interpretation

The analysis of a_τ for experiment depends on the EFT framework:

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{\Lambda^2}\left[C_{eB}^{33}(\bar{L}_\tau \sigma^{\mu\nu}\tau_R)HB_{\mu\nu} + C_{eW}^{33}(\bar{L}_\tau \sigma^{\mu\nu}\tau_R)\tau^I H W_{\mu\nu}^I\right] + \text{h.c.}$$

$$\longrightarrow \mathcal{L}_{\text{LEFT}} = \frac{v}{\sqrt{2}\Lambda^2} c_{\tau\gamma}\bar{\tau}_L \sigma^{\mu\nu}\tau_R F_{\mu\nu} + \text{h.c.} \quad , \quad c_{\tau\gamma} = c_W C_{eB}^{33} - s_W C_{eW}^{33},$$

$$\delta a_\tau = \frac{2\sqrt{2}m_\tau v}{e}\frac{\text{Re}\left(c_{\tau\gamma}\right)}{\Lambda^2}, \quad \delta d_\tau = \sqrt{2}v\frac{\text{Im}\left(c_{\tau\gamma}\right)}{\Lambda^2}.$$

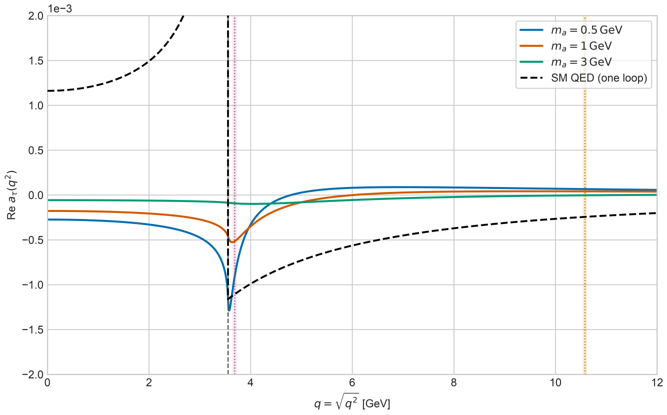
The @ ATLAS EXPERIMENT experiment actually means to $\text{Re}\left(c_{\tau\gamma}\right) \sim -0.40^{+0.63}_{-0.24}\left(\frac{\Lambda}{\text{TeV}}\right)^2$,

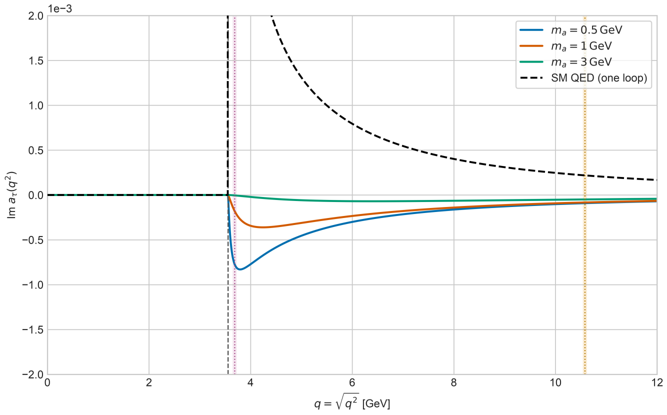
and @ CMS means $\text{Re}\left(c_{\tau\gamma}\right) \sim -0.22^{+0.78}_{-0.76}\left(\frac{\Lambda}{\text{TeV}}\right)^2$.

Both assume *no new physical* at low energy. No $H_\sigma(q^2)$ or $a_\tau(q^2)$ measured yet.

MDM result

with the same parameter setup: $g_\tau \sim \tilde{g}_\tau \sim 0.2$,





- The oneloop QED dominated.

but with $g_\tau \sim 0.2, \tilde{g} \sim 0.8$, it can achieves the QED level.

- The precision test of QED.

$a_\tau(q^2)$ effect is hard to isolate in physical observable. But numerically possible.

- may suffers other spin-mode as background. (no symmetry protect)
  - e.g. QED box diagram which can be ignored (~ $O(m_e^2)$), see *JHEP 07 (2025) 172*.
- as long as the observable is symmetric under Mandelstam variables exchange $t \leftrightarrow u$,

$(\cos\theta \leftrightarrow -\cos\theta)$.

$$\mathrm{Re}[a_\tau] = \frac{2(s+2m_\tau^2)(s+m_\tau^2)}{s(s-4m_\tau^2)} - \frac{5(s+2m_\tau^2)^2}{s(s-4m_\tau^2)}\langle(\hat{p}\cdot\hat{k})^2\rangle,$$

$$\mathrm{Im}[a_\tau] = -\frac{64m_\tau(2m_\tau^2+s)}{\pi\alpha_{h_+-}\sqrt{s}(s-4m_\tau^2)}\left\langle\frac{(\hat{p}\cdot\hat{k})[l_\mp\cdot(\hat{p}\times\hat{k})]}{|\hat{p}\times\hat{k}|}\right\rangle.$$

The measurement of $(g-2)_\tau$

$$\mathrm{Re}\,(a_\tau) = \frac{2(s+2m_\tau^2)(s+m_\tau^2)}{s(s-4m_\tau^2)} - \frac{5(s+2m_\tau^2)^2}{s(s-4m_\tau^2)}\left\langle(\hat{p}\cdot\hat{k})^2\right\rangle,$$

$$\mathrm{Im}\,(a_\tau) = -\frac{64m_\tau(2m_\tau^2+s)}{\pi\alpha_{h^\pm}\sqrt{s}(s-4m_\tau^2)}\left\langle\frac{(\hat{p}\cdot\hat{k})\left[\hat{l}_\mp\cdot(\hat{p}\times\hat{k})\right]}{|\hat{p}\times\hat{k}|}\right\rangle.$$

$$\Delta\hat{O}_i = \sqrt{\frac{\langle\hat{O}_i^2\rangle-\langle\hat{O}_i\rangle^2}{N}}, \quad i = 1,2,\ldots,6.$$

$$\Delta\,\mathrm{Re}\,(a_\tau) = \frac{s+2m_\tau^2}{s(s-4m_\tau^2)}\sqrt{\frac{32m_\tau^4+64m_\tau^2 s+17s^2}{14N_{\mathrm{eff}}^1}},$$

$$\Delta\,\mathrm{Im}\,(a_\tau) = \frac{64m_\tau}{\pi(s-4m_\tau^2)\sqrt{s}}\sqrt{\frac{4m_\tau^4+6m_\tau^2 s+2s^2}{30N_{\mathrm{eff}}^2}}.$$

Table II. Projected sensitivities to $g-2$ at $\sqrt{s} = m_{\psi(2s)}$ and $\sqrt{s} = m_{\Upsilon(4S)}$. The one-loop QED prediction are also shown in this table. Here we consider both π and ρ decay channel, and take the spatial resolution $D = 30\mu$m and 15μm for STCF and Belle II, respectively, as an illustration. At STCF, the luminosity is expected to reach $1\ ab^{-1}$ per year. Over ten years of data collection, the total number of $\psi(2S)$ events is anticipated to be about 2×10^{10} [67]. At Belle II, the events number $N_{\tau^+\tau^-} \approx 4.5 \times 10^{10}$ [65, 66].

$\sqrt{q^2}$ [GeV]	$\lvert\mathrm{Re}(a_\tau^{QED})\rvert$	$\Delta\mathrm{Re}\,(a_\tau)$	$\lvert\mathrm{Im}(a_\tau^{QED})\rvert$	$\Delta\mathrm{Im}\,(a_\tau)$
$m_{\psi(2S)}$(3.686GeV)	1.11×10^{-3}	2.17×10^{-3}	6.39×10^{-3}	4.90×10^{-3}
$m_{\Upsilon(4S)}$(10.58GeV)	2.44×10^{-4}	6.29×10^{-5}	2.18×10^{-4}	5.02×10^{-5}

The SMEFT for $(g-2)_\tau$ and d_τ

$$\mathcal{L}_{\text{eff}} \supset \frac{C_{SS}^{\tau\tau}}{\Lambda^2}(\overline{\tau}\tau)^2 + \frac{C_{PP}^{\tau\tau}}{\Lambda^2}(\overline{\tau}i\gamma_5\tau)^2 + \frac{C_{SP}^{\tau\tau}}{\Lambda^2}(\overline{\tau}i\gamma_5\tau)(\overline{\tau}\tau) - \frac{ea_\tau^0}{4m_\tau}F^{\mu\nu}\overline{\tau}\sigma_{\mu\nu}\tau - d_\tau^0\frac{i}{2}F^{\mu\nu}\overline{\tau}\sigma_{\mu\nu}\gamma_5\tau.$$

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{C_{(eH)^2}^{33}}{\Lambda^4}\left[\left(\overline{L}_\tau H\right)\tau_R\right]^2 + \frac{C_{eB}^{33}}{\Lambda^2}\left(\overline{L}_\tau\sigma^{\mu\nu}\tau_R\right)HB_{\mu\nu} + \frac{C_{eW}^{33}}{\Lambda^2}\left(\overline{L}_\tau\sigma_{\mu\nu}\tau_R\right)\sigma_a HW_a^{\mu\nu} + \text{h.c.}\,.$$

$$C_{SS}^{\tau\tau} = -C_{PP}^{\tau\tau} = \frac{v^2}{4\Lambda^2}\,\text{Re}\left[C_{(eH)^2}^{33}\right],\quad C_{SP}^{\tau\tau} = \frac{v^2}{2\Lambda^2}\,\text{Im}\left[C_{(eH)^2}^{33}\right],$$

$$a_\tau^0 = -\frac{2\sqrt{2}vm_\tau}{e\Lambda^2}\,\text{Re}\left[c_W C_{eB}^{33} - s_W C_{eW}^{33}\right],\quad d_\tau^0 = -\frac{\sqrt{2}v}{\Lambda^2}\,\text{Im}\left[c_W C_{eB}^{33} - s_W C_{eW}^{33}\right].$$

Coefficient	fit result (linear)	fit result (linear + quad.)
$c_{lq}^{(3)}$	$-0.002^{+0.018}_{-0.016}$	$-0.002^{+0.018}_{-0.016}$
$c_{lq}^{(1)}$	$0.01^{+0.06}_{-0.07}$	$0.01^{+0.18}_{-0.05}$
c_{lu}	$-0.05^{+0.15}_{-0.20}$	$-0.05^{+0.15}_{-0.20}$
c_{ld}	$-0.02^{+0.21}_{-0.21}$	$-0.02^{+0.21}_{-0.21}$
$c_{q\tau}$	$0.01^{+0.07}_{-0.08}$	$0.01^{+0.07}_{-0.16}$
$c_{\tau u}$	$0.01^{+0.04}_{-0.04}$	$0.01^{+0.04}_{-0.04}$
$c_{\tau d}$	$0.07^{+0.32}_{-0.25}$	$0.07^{+0.32}_{-0.25}$
$c_{ll}^{(1)}$	$-1.1^{+1.1}_{-1.1}$	$-1.0^{+1.1}_{-1.1}$
$c_{Hl}^{(3)}$	$0.7^{+0.8}_{-0.8}$	$0.7^{+0.8}_{-0.8}$
$c_{Hl}^{(1)}$	$-1.7^{+1.6}_{-1.6}$	$-1.5^{+1.6}_{-1.6}$
$c_{H\tau}$	6^{+6}_{-6}	$0.7^{+2.1}_{-1.9}$
$c_{\tau W}$	15^{+15}_{-15}	$0.0^{+0.4}_{-0.5}$
$c_{\tau B}$	-30^{+31}_{-30}	$-0.2^{+1.8}_{-1.5}$
$c_{\tau Z}$	20^{+19}_{-20}	$0.0^{+0.4}_{-0.5}$
$c_{\tau\gamma}$	-17^{+17}_{-16}	$-0.1^{+1.0}_{-0.8}$

$c_{\tau\gamma}$ measurement by ATLAS

Two loop table

$d_\tau \propto g_\tau \tilde{g}_\tau \equiv 1$ in table.

m_s(GeV)		$m_F = m_\tau$	$m_F = 100$ GeV	$m_F = 200$ GeV	$m_F = 500$ GeV	$m_F = 1000$ GeV	$m_F = 2000$ GeV
0.5	**S**	$-66.4 + 48.1i$	$2.69 + 0.83i$	$1.49 + 0.39i$	$0.68 + 0.16i$	$0.38 + 0.09i$	$0.20 + 0.06i$
	P	$-172 + 228i$	$4.46 + 1.22i$	$2.43 + 0.58i$	$1.11 + 0.22i$	$0.61 + 0.11i$	$0.33 + 0.06i$
1	**S**	$-70.9 + 44.3i$	$2.64 + 0.81i$	$1.46 + 0.39i$	$0.67 + 0.15i$	$0.37 + 0.08i$	$0.20 + 0.06i$
	P	$-165 + 239i$	$4.39 + 1.15i$	$2.40 + 0.55i$	$1.08 + 0.21i$	$0.60 + 0.10i$	$0.33 + 0.05i$
2	**S**	$-94.4 + 43.4i$	$2.43 + 0.73i$	$1.34 + 0.35i$	$0.62 + 0.14i$	$0.34 + 0.08i$	$0.18 + 0.06i$
	P	$-133 + 268i$	$4.05 + 0.89i$	$2.21 + 0.42i$	$1.00 + 0.16i$	$0.55 + 0.08i$	$0.30 + 0.04i$
3	**S**	$-122 + 76.4i$	$6.27 + 1.76i$	$3.45 + 0.84i$	$1.58 + 0.34i$	$0.87 + 0.19i$	$0.47 + 0.15i$
	P	$-76.8 + 288i$	$10.2 + 1.66i$	$5.55 + 0.79i$	$2.52 + 0.30i$	$1.38 + 0.15i$	$0.76 + 0.07i$

Table I. The upper bound of the two-loop corrections to d_τ at $\sqrt{s} = 6.3\,\text{GeV}$, in units of $10^{-21}\,e\,\text{cm}$. Here, **S** and **P** denotes new particle couples to F as a scalar and pseudoscalar, respectively.

m_s (GeV)		$m_F = m_\tau$	$m_F = 100$ GeV	$m_F = 200$ GeV	$m_F = 500$ GeV	$m_F = 1000$ GeV	$m_F = 2000$ GeV
0.5	**S**	$-27.9 + 15.2i$	$2.37 + 0.92i$	$1.34 + 0.43i$	$0.627 + 0.17i$	$0.35 + 0.08i$	$0.19 + 0.04i$
	P	$-88.3 + 5.17i$	$3.97 + 1.37i$	$2.20 + 0.65i$	$1.02 + 0.25i$	$0.56 + 0.12i$	$0.31 + 0.06i$
1	**S**	$-27.8 + 14.6i$	$2.32 + 0.90i$	$1.31 + 0.43i$	$0.61 + 0.17i$	$0.34 + 0.08i$	$0.19 + 0.04i$
	P	$-88.3 + 53.8i$	$3.91 + 1.32i$	$2.17 + 0.63i$	$1.00 + 0.24i$	$0.55 + 0.12i$	$3.06 + 0.06i$
2	**S**	$-28.5 + 12.6i$	$2.13 + 0.82i$	$1.20 + 0.39i$	$0.56 + 0.15i$	$3.14 + 0.08i$	$0.17 + 0.04i$
	P	$-86.8 + 61.0i$	$3.60 + 1.14i$	$1.99 + 0.54i$	$0.92 + 0.21i$	$0.51 + 0.10i$	$0.28 + 0.05i$
3	**S**	$-31.0 + 10.5i$	$5.40 + 2.04i$	$3.04 + 0.97i$	$1.42 + 0.38i$	$0.79 + 0.19i$	$0.44 + 0.10i$
	P	$-82.7 + 70.3i$	$9.16 + 2.61i$	$5.06 + 1.24i$	$2.33 + 0.48i$	$1.29 + 0.24i$	$0.71 + 0.12i$

Table II. The legend is the same as in Table I, but for $\sqrt{s} = 10.58\text{GeV}$.