

Resummed azimuthal decorrelation and transverse momentum imbalance of dijets at the LHC

-- resolving Non-global logarithms with the Winner-Take-All Jet Axis



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Azimuthal decorrelation $\delta\phi$ and transverse momentum imbalance q_T



Back-to-Back Dynamics

In dijet production, momentum conservation in the transverse plane enforces the back-to-back alignment at the Born level.



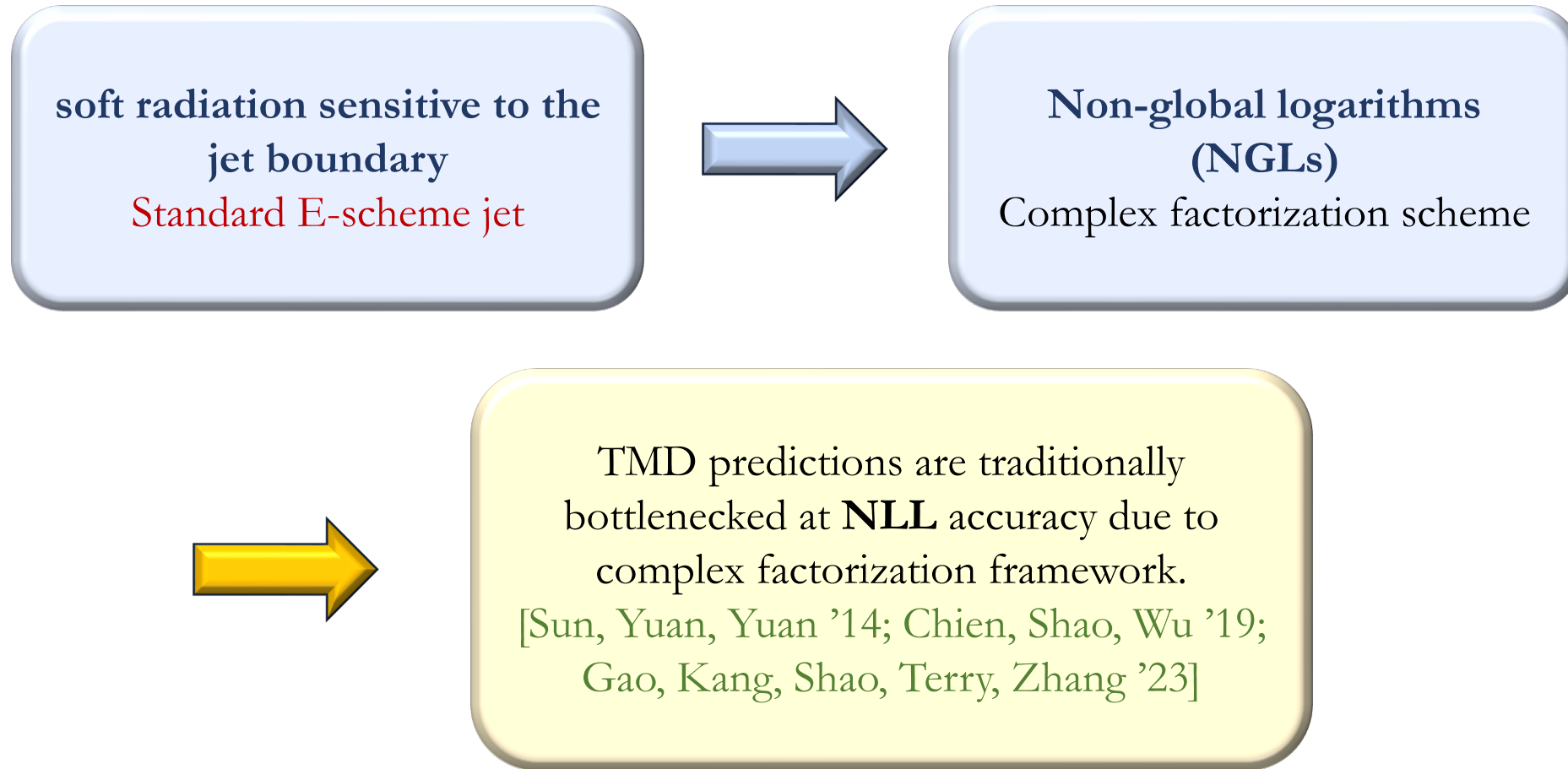
Radiation Imbalance

Soft and collinear radiation induces deviations from this topology, generating imbalances in the transverse momentum q_T or azimuthal angle $\delta\phi$.

$$q_T = |\vec{p}_{1,T} + \vec{p}_{2,T}|$$
$$\delta\phi = |\pi - |\phi_1 - \phi_2|| \approx \frac{q_y}{p_{1,T}}$$

All-order resummation is required in the back-to-back limit!

Non-global logarithms (NGLs)



Main obstacle: Non-global logarithms

Winner-take-all recombination scheme

- **Standard E-scheme** simply sums the four-momenta of all particles within the jet.
- **Winner-take-all (WTA) scheme:** [Bertolini, Chan, Thaler '13]
Clustering step: for the closest emission pair i and j :

$$p_{T,r} = p_{T,i} + p_{T,j}, \quad \hat{n}_r = \begin{cases} \hat{n}_i, & p_{T,i} \geq p_{T,j} \\ \hat{n}_j, & p_{T,i} < p_{T,j} \end{cases}$$

- The WTA scheme is *recoil-free*.
- $\delta\phi$: No NGL anymore. [Chien, Rahn, Velzen, Shao, Waalewijn, Wu '20; Chien, Rahn, Shao, Waalewijn, Wu '22]
- q_T : The standard NGLs of q_T disappear and the NGLs of jet radius R in the small R limit is isolated in the soft sector. [RJF, Rahn, Shao, Waalewijn, Wu '26]

WTA Transverse Momentum

- Consider a multijet scattering process at LHC: $p(P_a) + p(P_b) \rightarrow \sum_{i=1}^N J(p_i) + X$.
- The total transverse momentum of the final jets is defined as:

$$\vec{q}_T \equiv \sum_{i=1}^N \vec{p}_{i,T}^{\text{WTA}} = \sum_{i=1}^N \left(\sum_{k \in \text{jet-}i} p_{k,T} \right) \vec{n}_{W_i,T}.$$

$$\vec{q}_T = \sum_{i=1}^N \sum_{k \in \text{jet-}i} (p_{k,T} \vec{n}_{W_i,T} - \vec{p}_{k,T}) - \vec{p}_{a,T} - \vec{p}_{b,T} - \vec{p}_{s,T}$$

$$= - \sum_{i=1}^N \sum_{k \in \text{jet-}i} \vec{p}_{k,\perp}^{(i)} - \vec{p}_{a,T} - \vec{p}_{b,T} - \vec{p}_{s,T},$$

with $\vec{p}_{k,\perp}^{(i)} \equiv \vec{p}_{k,T} - p_{k,T} \vec{n}_{W_i,T}, \quad k \in \text{jet-}i$.

$\vec{p}_{k,\perp}^{(i)} \cdot \vec{n}_{W_i,T} = 0 + \mathcal{O}\left(p_{k,T} (\phi_k - \phi_i^{\text{WTA}})^2\right)$  In-cone contribution $\vec{p}_{k,\perp}^{(i)}$ is perpendicular to the jet and the beam axis at leading power.

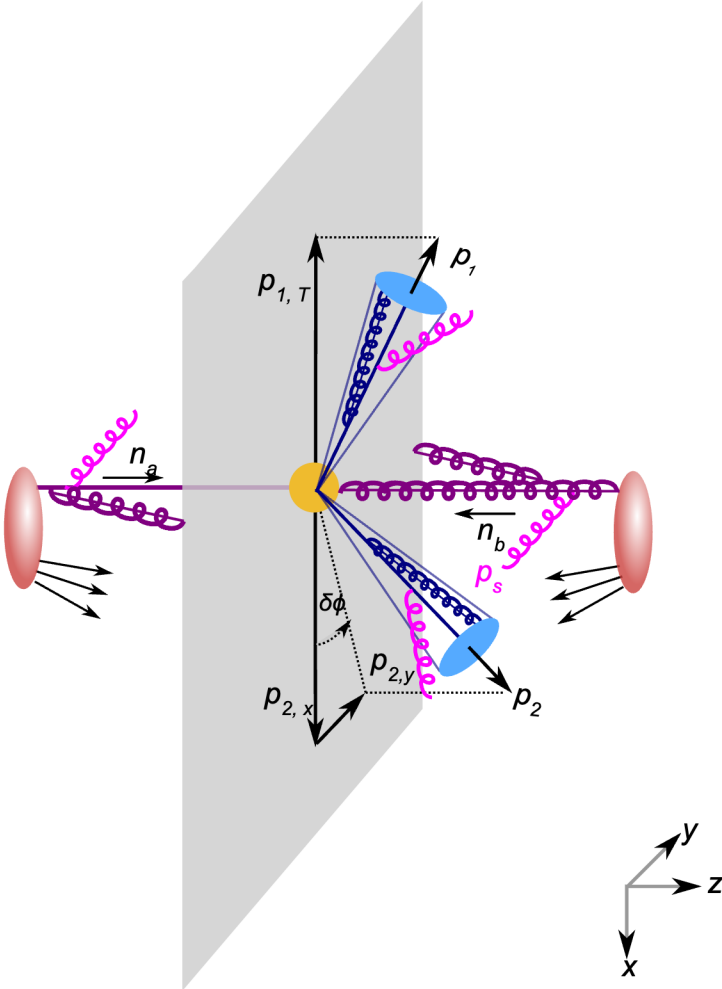
Azimuthal decorrelation $\delta\phi \equiv |\pi - |\phi_1^{\text{WTA}} - \phi_2^{\text{WTA}}||$.

- In the back-to-back limit ($\delta\phi \ll 1$) $\delta\phi \approx \frac{q_y}{p_T}$

- **hard:** $p_h^\mu \sim p_T(1, 1, 1),$
- $n_{a,b}$ -**collinear:** $p_c^\mu \sim p_T(\delta\phi^2, 1, \delta\phi)_{n_i \bar{n}_i},$
- **soft:** $p_s^\mu \sim p_T(\delta\phi, \delta\phi, \delta\phi),$
- $n_{1,2}$ -**collinear:** $p_c^\mu \sim p_T(\delta\phi^2, 1, \delta\phi)_{n_i \bar{n}_i}.$

- SCET-II Factorization formula:

$$\begin{aligned} \frac{d^4\sigma}{d\eta_1 d\eta_2 dp_T dq_y} &= \sum_{ijkl} \frac{x_a x_b}{16\pi \hat{s}^2} \frac{2p_T}{1 + \delta_{k\ell}} \mathcal{H}_{ij \rightarrow k\ell, IJ}(p_T, \eta_1 - \eta_2, \mu) \\ &\times \int_{-\infty}^{\infty} \frac{db_y}{2\pi} e^{ib_y q_y} \mathcal{S}_{ijkl, IJ}(b_y, \eta_1, \eta_2, \mu, \nu) \mathcal{J}_k(b_y, \omega_1, \mu, \nu) \mathcal{J}_\ell(b_y, \omega_2, \mu, \nu) \\ &\times B_{i/p}(x_a, b_y, \omega_a, \mu, \nu) B_{j/p}(x_b, b_y, \omega_b, \mu, \nu) \end{aligned}$$



Resummation formula for $\delta\phi$



Rapidity **R**enormalization **G**roup [Chiu, Jain, Neill, Rothstein '11]

$$\begin{aligned}
 \frac{d^4\sigma}{d\eta_1 d\eta_2 dp_T dq_y} &= \sum_{ijkl} \frac{x_a x_b}{16\pi \hat{s}^2} \frac{2p_T}{1 + \delta_{kl}} \int_0^\infty \frac{db_y}{\pi} \cos(b_y q_y) \prod_{i=a,b,1,2} \left(\frac{\nu_s}{\nu_i} \right)^{\Gamma_\nu^i(b_y, \mu_f)} \\
 &\times \sum_{KK'} \exp \left\{ \int_{\mu_h}^{\mu_f} \frac{d\mu}{\mu} \left[\gamma_{\text{cusp}}(\alpha_s) \left(C_H \ln \frac{\hat{s}}{\mu^2} + \lambda_K + \lambda_{K'}^* \right) + 2\gamma_H(\alpha_s) \right] \right\} \\
 &\times \mathcal{H}_{ij \rightarrow kl, KK'}(p_T, \eta_1 - \eta_2, \mu_h) \mathcal{S}_{ijkl, K'K}(b_y, \eta_1, \eta_2, \mu_f, \nu_s) \\
 &\times B_{i/p}(x_a, \omega_a, b_y, \mu_f, \nu_a) B_{j/p}(x_b, \omega_b, b_y, \mu_f, \nu_b) \\
 &\times \mathcal{J}_k(b_y, \omega_1, \mu_f, \nu_1) \mathcal{J}_\ell(b_y, \omega_2, \mu_f, \nu_2) \\
 &\times \exp \left[-S_{\text{NP}}^i(b_y, Q_0, \omega_a) - S_{\text{NP}}^j(b_y, Q_0, \omega_b) \right].
 \end{aligned}$$

Transverse momentum imbalance q_T

$$\vec{q}_T \equiv \sum_{i=1}^N \vec{p}_{i,T}^{\text{WTA}} = - \sum_{i=1}^N \sum_{k \in \text{jet}-i} \vec{p}_{k,\perp}^{(i)} - \vec{p}_{a,T} - \vec{p}_{b,T} - \vec{p}_{s,T}$$

- SCET Factorization formula:

$$\begin{aligned} \frac{d^4\sigma}{d\eta_1 d\eta_2 dp_T dq_T} &= \sum_{ijkl} \frac{x_a x_b}{16\pi \hat{s}^2} \frac{2p_T}{1 + \delta_{k\ell}} \mathcal{H}_{ij \rightarrow k\ell, II}(p_T, \eta_1 - \eta_2, \mu) q_T \int_0^{2\pi} d\phi_q \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \\ &\quad \times \mathcal{S}_{ijkl, IJ}(\vec{b}_T, \eta_1, \eta_2, R, \mu, \nu) \mathcal{J}_k(b_y, \omega_1, \mu, \nu) \mathcal{J}_\ell(b_y, \omega_2, \mu, \nu) \\ &\quad \times B_{i/p}(x_a, \omega_a, b_T, \mu, \nu) B_{j/p}(x_b, \omega_b, b_T, \mu, \nu). \end{aligned}$$

- Only soft function is new: Outside the jet ($\vec{q}_T$), inside the jet (q_y).

Small- R refactorization of q_T -soft function ($q_T/p_T \ll R \ll 1$)

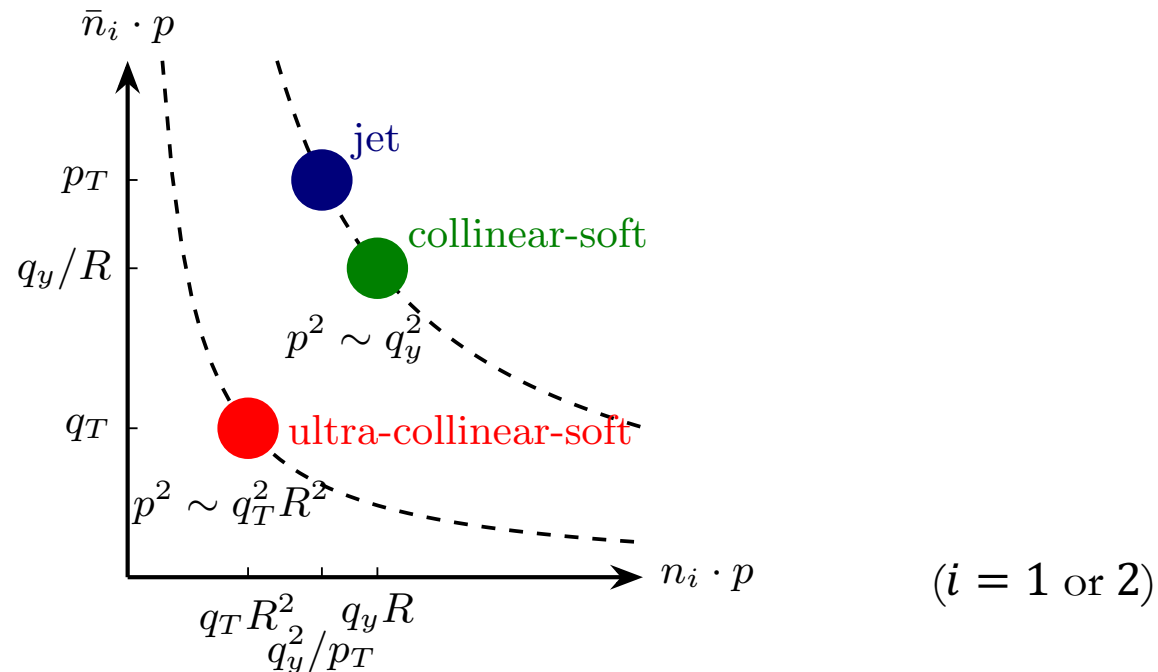
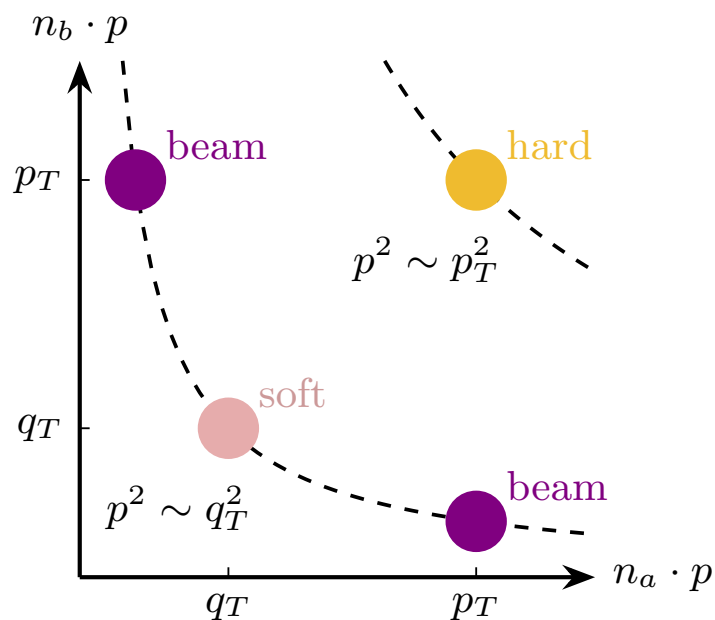
- In small- R limit, the q_T -soft function refactorizes into global and “collinear-soft” contributions, up to power corrections in the jet radius.,

$$\mathcal{S}_{ijkl}(\vec{b}_T, R, \eta_1, \eta_2, \mu, \nu) = \mathcal{S}_{ijkl}^{\text{global}}(\vec{b}_T, \eta_1, \eta_2, \mu, \nu) \boxed{S_k(\vec{b}_T, \eta_1, R, \mu, \nu)} S_\ell(\vec{b}_T, \eta_2, R, \mu, \nu) + \mathcal{O}(R^{2n}).$$

Encoding the collinear-soft and ultra-collinear-soft modes

- **Non-Global Logarithms:** $S_k(S_\ell)$ contains NGLs of the jet radius.
- These can be computed using the **multiplicity-interacting EFT framework**. [Becher, Neubert, Rothen, Shao '15, '16; Becher, Pecjak, Shao '16; Larkoski, Moult, Neill '15]

SCET modes for q_T distribution



- **hard:** $p_h^\mu \sim p_T(1, 1, 1),$
- **$n_{a,b}$ -collinear:** $p_{c_i}^\mu \sim (q_T^2/p_T, p_T, q_T)_{n_i \bar{n}_i},$
- **$n_{1,2}$ -collinear:** $p_{c_i}^\mu \sim (q_y^2/p_T, p_T, q_y)_{n_i \bar{n}_i}.$
- **soft:** $p_s^\mu \sim q_T(1, 1, 1),$
- **$n_{1,2}$ -collinear-soft:** $p_{cs}^\mu \sim |q_y|/R(R^2, 1, R)_{n_i \bar{n}_i},$
- **$n_{1,2}$ -ultra-collinear-soft:** $p_{ucs}^\mu \sim q_T(R^2, 1, R)_{n_i \bar{n}_i}.$

Refactorization of S_i

- Applying the factorization framework for non-global observables in the small R limit:
[Becher, Neubert, Rothen, Shao '15, '16; Becher, Pecjak, Shao '16]

$$S_i(\vec{b}_T, \eta_i, R, \mu, \nu) = \sum_{m_i=0}^{\infty} \prod_{j=1}^{m_i} \int \frac{d\Omega(\vec{u}_j)}{4\pi} \frac{1}{d_i} \text{Tr} [\mathcal{S}_{m_i}^{\text{CS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_y, R, \mu, \nu) \mathcal{S}_{m_i}^{\text{UCS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_x, R, \mu)]$$

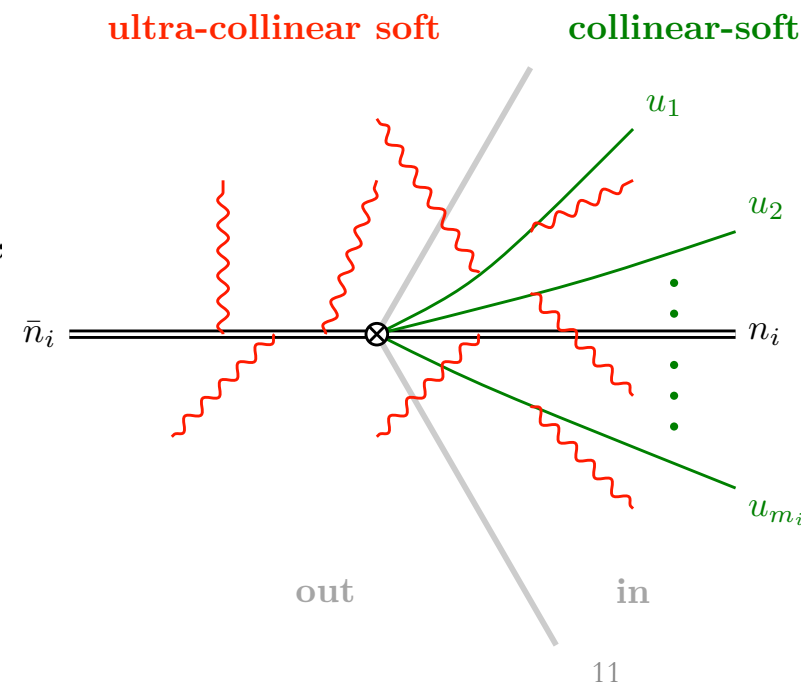
$$:= \sum_{m_i=0}^{\infty} \langle \mathcal{S}_{m_i}^{\text{UCS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_x, R, \mu) \otimes \mathcal{S}_{m_i}^{\text{CS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_y, R, \mu, \nu) \rangle.$$

$$\mathcal{S}_{m_i}^{\text{CS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_y, R, \mu, \nu) =$$

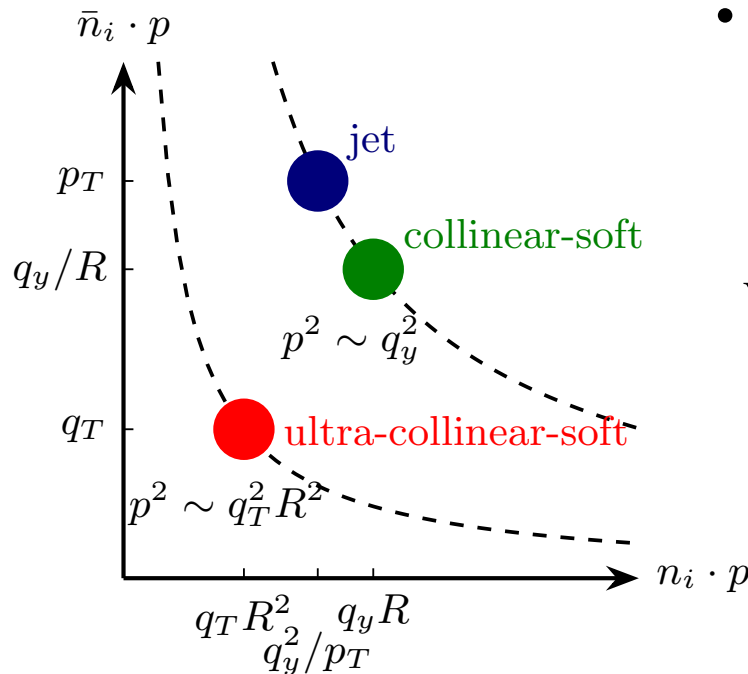
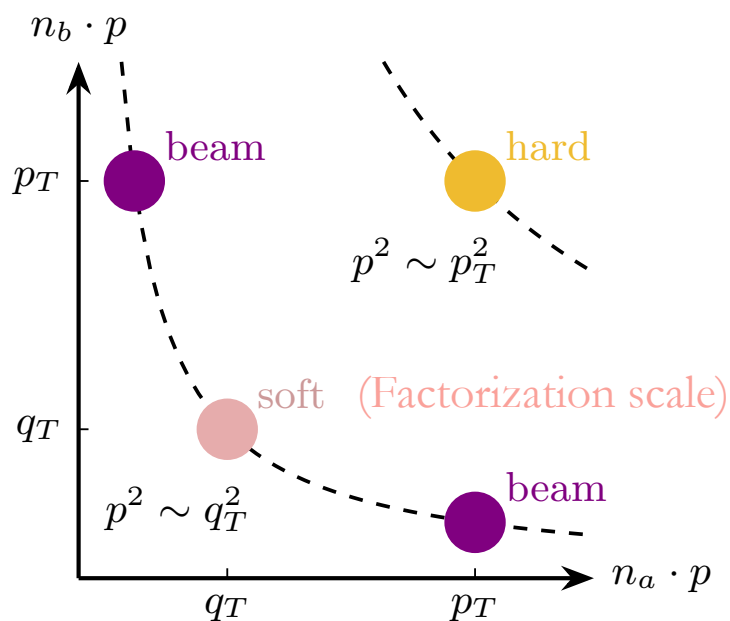
$$\prod_{j=1}^{m_i} \int \frac{dE_j E_j^{d-3}}{(2\pi)^{d-2}} |\mathcal{M}_{m_i}^{\text{CS}}(n_i, \bar{n}_i, \{p\})\rangle \langle \mathcal{M}_{m_i}^{\text{CS}}(n_i, \bar{n}_i, \{p\})| \exp\left(ib_y \sum_{k=1}^{m_i} p_{k,y}\right) \Theta_{\text{in}}^i(\{p\}).$$

$$\mathcal{S}_{m_i}^{\text{UCS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_x, R, \mu) = \oint_{X_{\text{ucs}}} \langle 0 | U_{m_i}^\dagger(u_{m_i}) \cdots U_1^\dagger(u_1) U_b^\dagger(\bar{n}_i) U_a^\dagger(n_i) | X_{\text{ucs}} \rangle$$

$$\langle X_{\text{ucs}} | U_a(n_i) U_b(\bar{n}_i) U_1(u_1) \cdots U_{m_i}(u_{m_i}) | 0 \rangle \exp\left(-\frac{i}{2} b^+ \sum_{j \notin \text{jet}-i} k_j^-\right)$$



RRG and RG evolutions



- Two sets of RRG evolution:

$$\prod_{i=ab} \left(\frac{\nu_s}{\nu_i} \right)^{\Gamma_{\nu}^i(b_T, \mu_f)} \prod_{i=12} \left(\frac{\nu_{cs,i}}{\nu_i} \right)^{\Gamma_{\nu}^i(b_y, \mu_{cs})}$$

with the rapidity scales

$$\nu_i \sim \omega_i \quad (i = a, b, 1, 2), \quad \nu_s \sim b_0/b_T,$$

$$\nu_{cs,i} \sim b_0/(|b_y|R_i) \quad (i = 1, 2).$$

- The evolution matrix for the collinear-soft and ultra-collinear-soft functions:

$$\prod_{i=1,2} \exp \left[\int_{\mu_{cs}}^{\mu_f} \frac{d\mu}{\mu} \Gamma_i^{cs} + \int_{\mu_{ucs}}^{\mu_f} \frac{d\mu}{\mu} \Gamma_i^{ucs} \right] U_{NG}^i(\mu_{ucs}, \mu_{cs}).$$

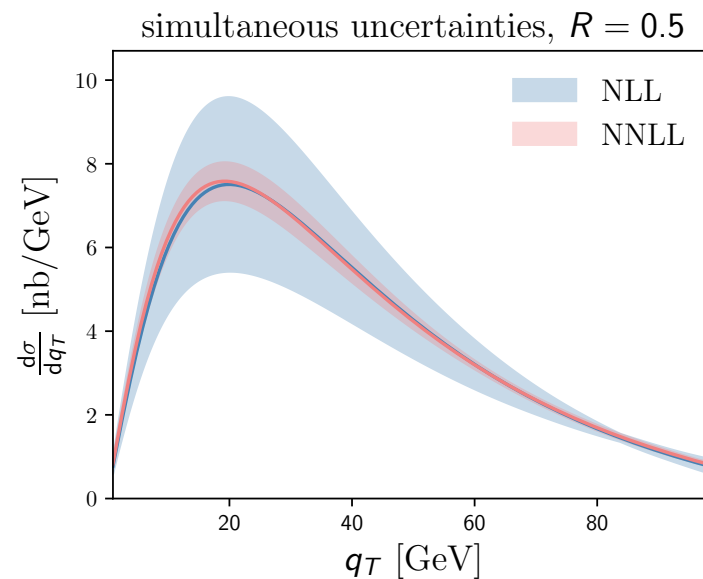
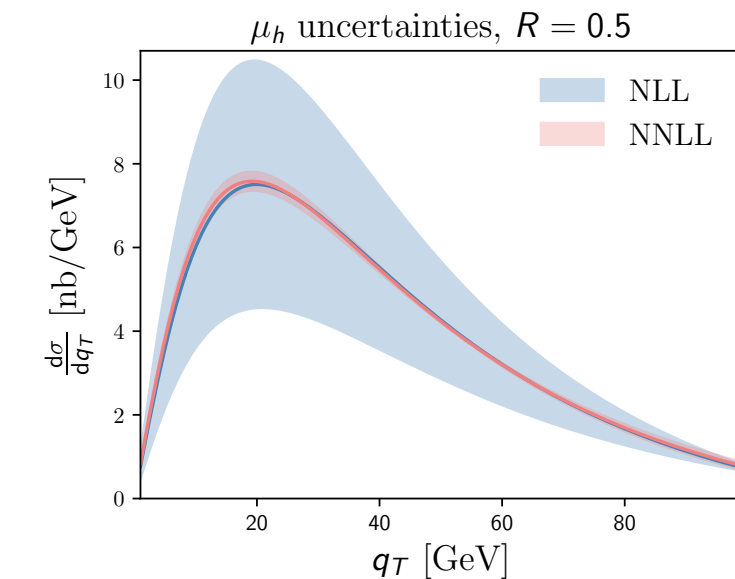
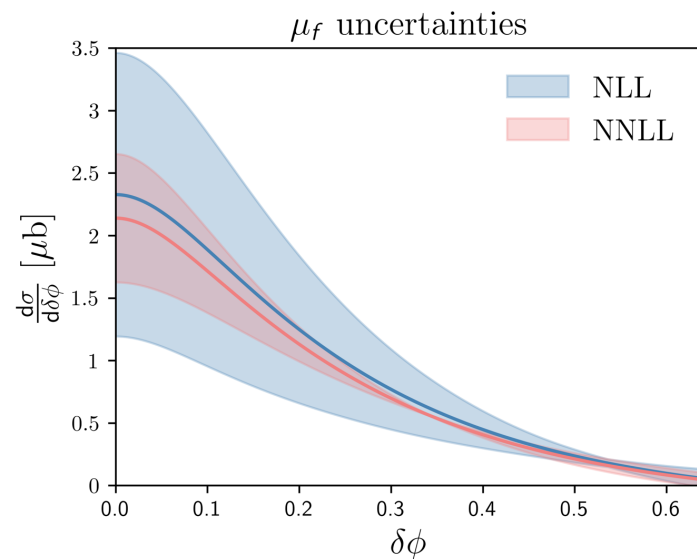
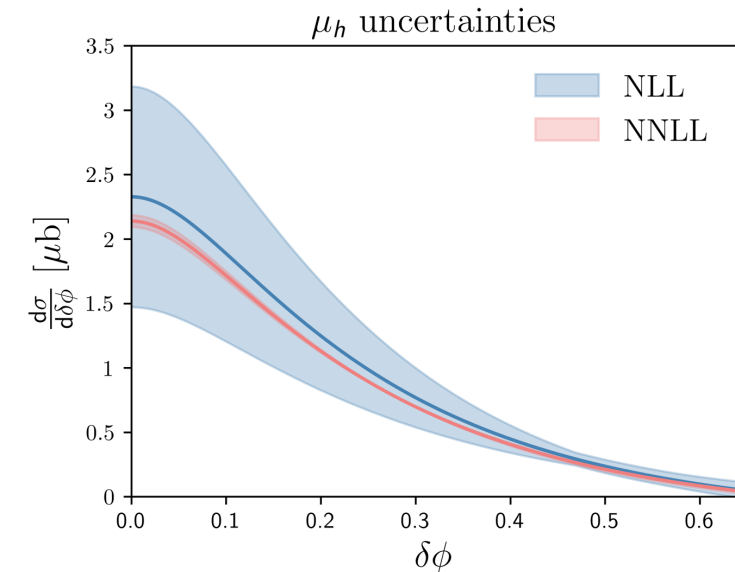
- Dasgupta–Salam parametrization: $U_{NG}^i(\mu_{ucs}, \mu_{cs}) \approx \exp \left(-C_A C_i \frac{\pi^2}{3} u^2 \frac{1 + (au)^2}{1 + (bu)^c} \right), \quad u = \frac{1}{\beta_0} \ln \frac{\alpha_s(\mu_{ucs})}{\alpha_s(\mu_{cs})},$

[Dasgupta, Salam, '01]

NNLL+LL_R Resummation formula for q_T

$$\begin{aligned}
& \frac{d^5\sigma}{d\eta_1 d\eta_2 dp_T dq_T d\phi_q} \\
&= \sum_{ijkl} \frac{x_a x_b}{16\pi \hat{s}^2} \frac{2p_T}{1 + \delta_{k\ell}} q_T \int_0^{2\pi} d\phi_b \int_0^\infty \frac{b_T db_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \\
&\quad \times \sum_{KK'} \exp \left\{ \int_{\mu_h}^{\mu_f} \frac{d\mu}{\mu} \left[\gamma_{\text{cusp}}(\alpha_s) \left(C_H \ln \frac{\hat{s}}{\mu^2} + \lambda_K + \lambda_{K'}^* \right) + 2\gamma_H(\alpha_s) \right] \right\} \\
&\quad \times \prod_{p=ab} \left(\frac{\nu_s}{\nu_p} \right)^{\Gamma_\nu^p(b_T, \mu_f)} \prod_{p=12} \left(\frac{\nu_{cs,p}}{\nu_p} \right)^{\Gamma_\nu^p(b_y, \mu_{cs})} \\
&\quad \times \prod_{p=12} \exp \left[\int_{\mu_{cs}}^{\mu_f} \frac{d\mu}{\mu} (\Gamma^{J_p} + \Gamma_p^{\text{cs}}) + \int_{\mu_{cs}}^{\mu_f} \frac{d\mu}{\mu} \Gamma_p^{\text{ucs}} \right] U_{\text{NG}}^p(\mu_{ucs}, \mu_{cs}) \\
&\quad \times \mathcal{H}_{ij \rightarrow k\ell, KK'}(p_T, \eta_1 - \eta_2, \mu_h) \mathcal{S}_{ijkl, K'K}^{\text{global}}(\vec{b}_T, \eta_1, \eta_2, \mu_f, \nu_s) \\
&\quad \times \prod_{p=12} \sum_{m_p=0}^{\infty} \left\langle \mathcal{S}_{m_p}^{\text{ucs}}(\{n_p, \bar{n}_p, \underline{u}\}, \vec{b}_T, R, \mu_{ucs}) \otimes \mathcal{S}_{m_p}^{\text{cs}}(\{n_p, \bar{n}_p, \underline{u}\}, b_y, R, \mu_{cs}, \nu_{cs,p}) \right\rangle \\
&\quad \times B_{i/p}(x_a, b_T, \omega_a, \mu_f, \nu_a) B_{j/p}(x_b, b_T, \omega_b, \mu_f, \nu_b) \mathcal{J}_k(b_y, \omega_1, \mu_j, \nu_1) \mathcal{J}_\ell(b_y, \omega_2, \mu_j, \nu_2) \\
&\quad \times \exp \left[-S_{\text{NP}}^i(b_T, \omega_a, \nu_a) - S_{\text{NP}}^j(b_T, \omega_b, \nu_b) \right].
\end{aligned}$$

NLL and NNLL Resummation results



- b_* -prescription:

$$b_{T,*} = \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \quad b_{y,*} = \frac{|b_y|}{\sqrt{1 + b_y^2/b_{\max}^2}}, \quad b_{x,*} = \frac{|b_x|}{\sqrt{1 + b_x^2/b_{\max}^2}}$$

- Scales in $\delta\phi$ resummation:

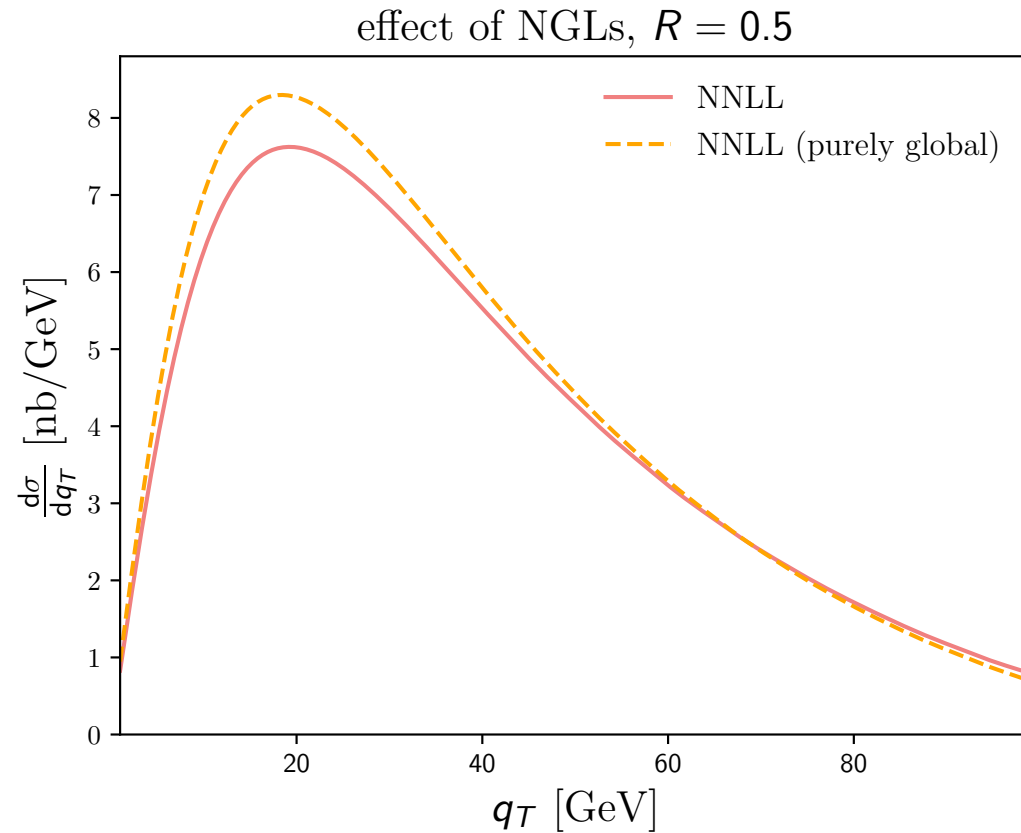
$$\mu_h = v_h \cdot 2p_T \quad \mu_f = \min\{v_f \cdot \mu_{b_{y,*}}, \mu_h\}$$

- Scales in q_T resummation:

$$\mu_s = \mu_b = \mu_f = \min\left\{v_f \frac{b_0}{b_{T,*}}, \mu_h\right\}, \quad \mu_j = \mu_{cs} = v_f \frac{b_0}{b_{y,*}},$$

$$\mu_h = v_h \cdot 2p_T \quad \mu_{ucs} = \min\left\{v_f \frac{Rb_0}{2b_{x,*}}, \mu_{cs}\right\}.$$

Effects of NGLs

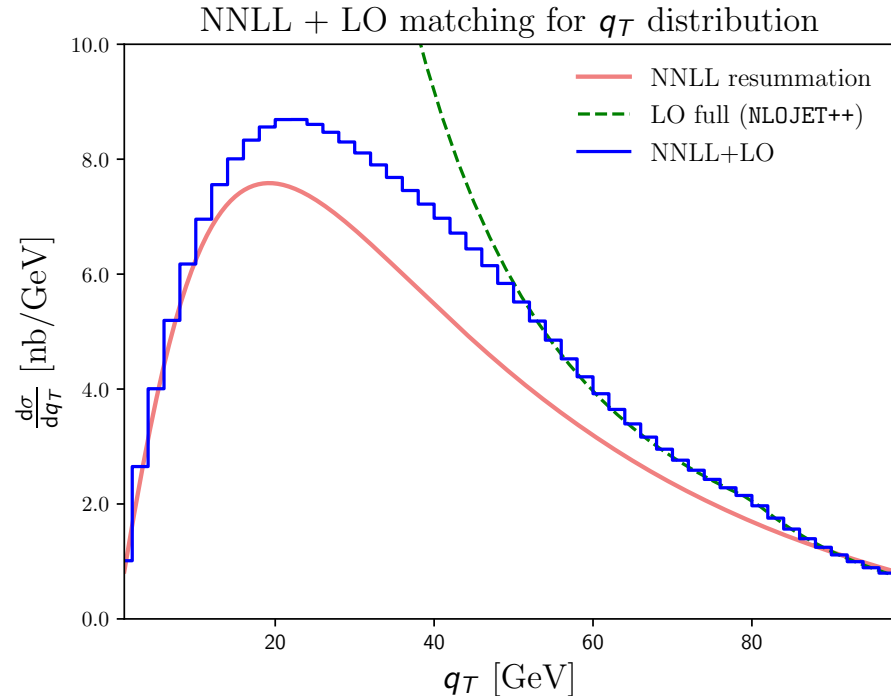
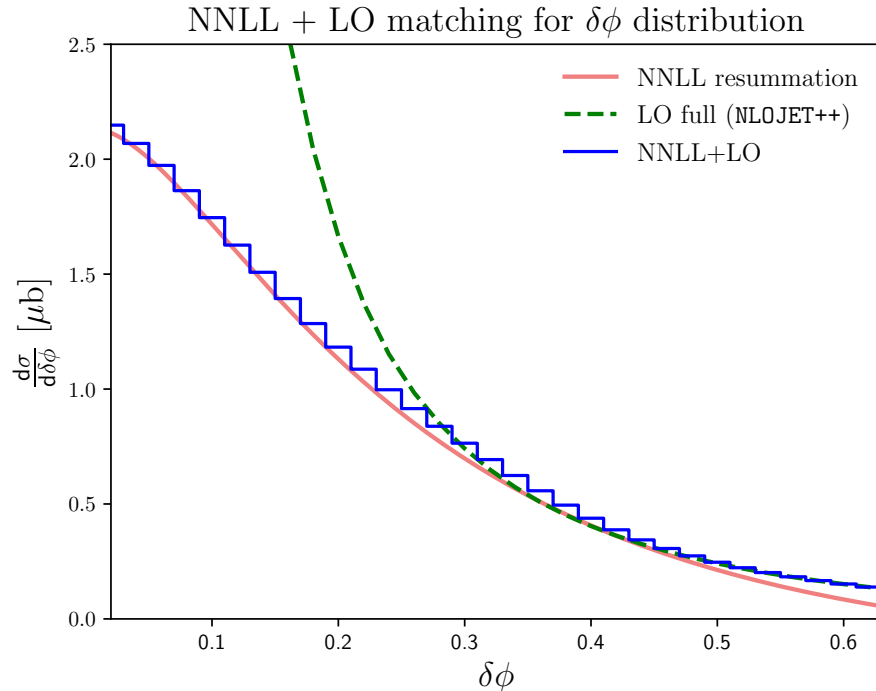


- The comparison reveals that the inclusion of the resummation of NGLs leads to a noticeable **suppression** of the cross-section in the peak region.

NNLL+LO Matching

$$d\sigma_{\text{match}}(\text{NNLL} + \text{LO}) = (1 - t(\mathcal{O})) \left(d\sigma(\text{NNLL}) + U_H^{\text{eff}}(\mathcal{O}) d\sigma(\text{LO non-singular}) \right) + t(\mathcal{O}) d\sigma(\text{LO full}).$$

- The effective evolution matrix, $U_H^{\text{eff}}(\mathcal{O}) = \exp \left[\int_{2p_T}^{\mu_{\mathcal{O}}} \frac{d\mu}{\mu} \left(\frac{\alpha_s(2p_T)}{\pi} 4C_A \ln \frac{4p_T^2}{\mu^2} \right) \right]$,
is used to suppress the divergent non-singular terms in the infrared regime.
- The transition function $t(\mathcal{O})$ acts as a smooth switch, transitioning from 0 in the resummation-dominated region to 1 in the fixed-order region.



Conclusions

- WTA scheme eliminates standard non-global logarithms (NGLs), simplifying the factorization;
- Small- R Limit: Exploiting $R \ll 1$ allows the soft function to be **refactorized** into global, collinear-soft, and ultra-collinear-soft modes, enabling the calculation of NGLs of R ;
- Joint resummation involving NNLL accuracy for global logarithms and LL accuracy for the small- R NGLs.

THANK YOU!

Motivation

3D Tomography

Single-spin asymmetries in dijet production at polarized proton-proton colliders probe 3D proton structure and TMD PDFs. [Boer, Vogelsang '04; Bomhof, Mulders, Vogelsang Yuan '07; Qiu, Vogelsang, Yuan '07 ...]

Factorization Violation

Transverse momentum dependent (TMD) factorization is violated in dijet production. [Collins, Qiu '07; Collins '07, Vogelsang, Yuan '07; Rogers, Mulders '10 ...]

q_T –slicing for multiple jet

By defining the slicing variable with WTA jet momentum, we can generalize q_T -slicing to colored multi-jet systems. [RJF, Rahn, Shao, Waalewijn, Wu '24]

Backup: Combination of the collinear-soft and ultra-collinear-soft functions

- The perturbative expansion of S_i is organized as

$$S_i(b_\perp, b^+) = 1 + \frac{Z_\alpha \alpha_s(\mu)}{4\pi} S_i^{(1)}(b_\perp, b^+) + \left(\frac{Z_\alpha \alpha_s(\mu)}{4\pi} \right)^2 S_i^{(2)}(b_\perp, b^+) + \mathcal{O}(\alpha_s^3)$$

- NLO S_i : $S_i^{(1)}(b_\perp, b^+) = \langle \mathbf{S}_0^{\text{ucs}(1)} \mathbf{S}_0^{\text{cs}(0)} \rangle + \langle \mathbf{S}_0^{\text{ucs}(0)} \mathbf{S}_0^{\text{cs}(0)} \rangle + \langle \mathbf{S}_1^{\text{ucs}(0)} \otimes \mathbf{S}_1^{\text{cs}(1)} \rangle = \langle \mathbf{S}_0^{\text{ucs}(1)} \rangle + \langle \mathbf{1} \otimes \mathbf{S}_1^{\text{cs}(1)} \rangle$

with: $\langle \mathbf{1} \otimes \mathbf{S}_1^{\text{cs}(1)}(b_\perp) \rangle = C_i \left(\frac{\mu |b_\perp|}{b_0} \right)^{2\epsilon} \left(\frac{\nu |b_\perp| R_i}{b_0} \right)^\eta h_{\text{in}}, \quad \langle \mathbf{S}_0^{\text{ucs}(1)}(b^+) \rangle = C_i \left(\frac{i b^+ \mu}{b_0 R_i} \right)^{2\epsilon} s_{\text{out}},$

$$h_{\text{in}} = \frac{2}{\epsilon^2} - \frac{4}{\eta\epsilon} - \frac{\pi^2}{6} + \left(-\frac{\eta}{\epsilon^3} + \frac{\pi^2\eta}{12\epsilon} - \frac{\zeta_3}{3}\eta - \frac{\pi^2\epsilon}{3\eta} - \frac{4\zeta_3}{3}\epsilon - \frac{17\pi^4}{1440}\eta\epsilon - \frac{4\zeta_3\epsilon^2}{3\eta} - \frac{3\pi^4}{80}\epsilon^2 - \frac{\pi^4\epsilon^3}{40\eta} \right)$$

$$s_{\text{out}} = -\frac{2}{\epsilon^2} - \frac{\pi^2}{2} + \left(-\frac{14\zeta_3}{3}\epsilon - \frac{7\pi^4}{48}\epsilon^2 \right)$$

Backup: Combination of the collinear-soft and ultra-collinear-soft functions

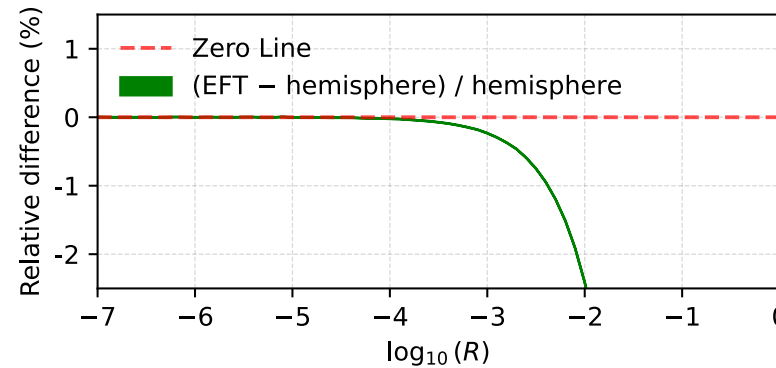
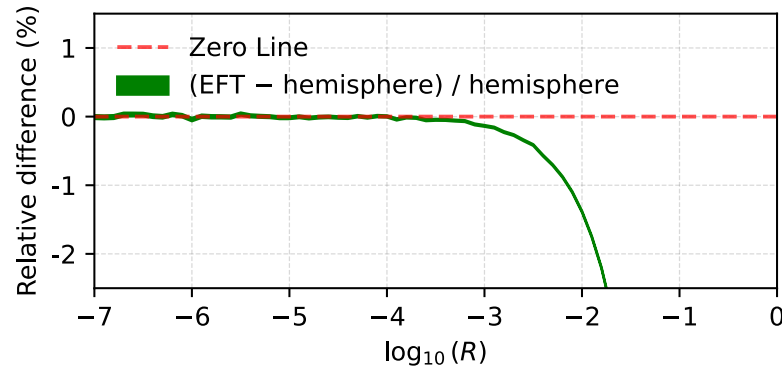
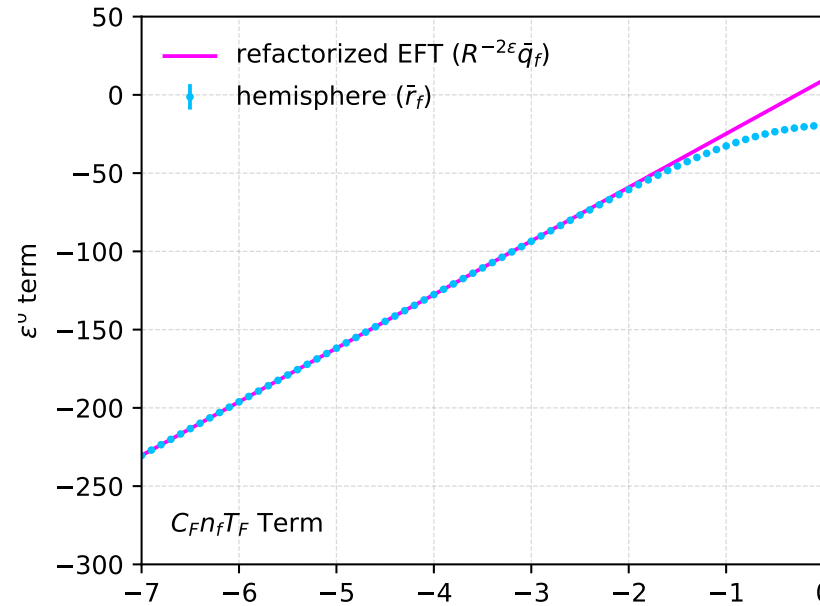
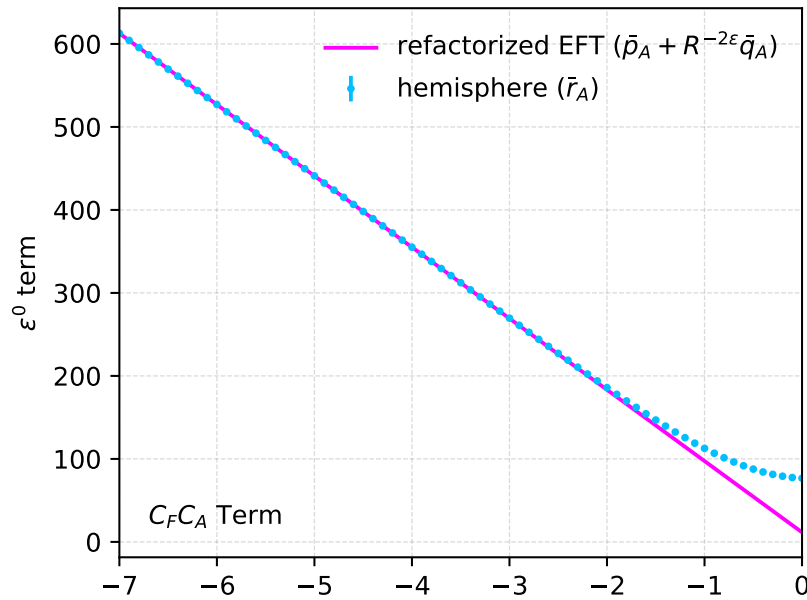
- NNLO S_i :

$$\begin{aligned}
 S_i^{(2)}(b_\perp, b^+) &= \left(\frac{\mu|b_\perp|}{b_0} \right)^{4\epsilon} \left(\frac{\nu|b_\perp|R_i}{b_0} \right)^\eta \omega^2 \left[\omega^2 \left(\frac{\nu|b_\perp|R_i}{b_0} \right)^\eta C_i^2 \frac{h_{\text{in}}^2}{2} + C_i C_A (h_A + v_A^{\text{in}}) + C_i n_f T_F h_f \right] \\
 &\quad + \left(\frac{ib^+ \mu}{b_0 R_i} \right)^{4\epsilon} \left[C_i^2 \frac{s_{\text{out}}^2}{2} + C_i C_A (g_A + v_A^{\text{out}}) + C_i n_f T_F g_f \right] \\
 &\quad + \left(\frac{ib^+ \mu}{b_0 R_i} \right)^{4\epsilon} \left[C_i C_A (s_A - g_A - v_A^{\text{out}}) + C_i n_f T_F (s_f - g_f) \right] \\
 &\quad + \left(\frac{\mu|b_\perp|}{b_0} \right)^{2\epsilon} \left(\frac{ib^+ \mu}{b_0 R_i} \right)^{2\epsilon} \left[\left(\frac{\nu|b_\perp|R_i}{b_0} \right)^\eta \omega^2 C_i^2 h_{\text{in}} s_{\text{out}} + C_i C_A p_A \right].
 \end{aligned}$$

- Double-real ultra-collinear-soft emissions, where one emission is in-cone and unobserved, while the other is emitted outside the jet cone and measured:

$$q_A = s_A - g_A - v_A^{\text{out}} = \left(-\frac{2}{3} + \frac{22\pi^2}{9} - 4\zeta_3 \right) \frac{1}{\epsilon} + \frac{40}{9} - \frac{134\pi^2}{27} + \frac{44\zeta_3}{3} + \frac{8\pi^4}{45}, \quad q_f = s_f - g_f = \left(\frac{4}{3} - \frac{8\pi^2}{9} \right) \frac{1}{\epsilon} - \frac{68}{9} + \frac{64\pi^2}{27} - \frac{16\zeta_3}{3}.$$

Backup: Refactorization of the hemisphere calculation and its power corrections



The comparison demonstrates that the two approaches are consistent in the small radius limit $R \ll 1$.

$$\begin{aligned} \bar{r}_A &= \bar{p}_A + R^{-2\epsilon} \bar{q}_A + \mathcal{O}(\epsilon^0 R \ln R, \epsilon^0 R) \\ &= \frac{2\pi^2}{3\epsilon^2} - \frac{2(3 - 11\pi^2 + 6\pi^2 \ln 2 - 18\zeta_3)}{9\epsilon} + \left[\left(\frac{4}{3} - \frac{44\pi^2}{9} + 8\zeta_3 \right) \ln R \right. \\ &\quad \left. + \frac{40}{9} + \frac{7\pi^4}{15} + \frac{\pi^2}{27} (-134 + 36 \ln^2 2) + \frac{44\zeta_3}{3} - 16\zeta_3 \ln 2 \right] + \mathcal{O}(\epsilon^0 R \ln R, \epsilon^0 R), \end{aligned}$$

$$\begin{aligned} \bar{r}_f &= R^{-2\epsilon} \bar{q}_f + \mathcal{O}(\epsilon^0 R \ln R, \epsilon^0 R) \\ &= \left(\frac{4}{3} - \frac{8\pi^2}{9} \right) \frac{1}{\epsilon} + \frac{8}{9} (-3 + 2\pi^2) \ln R + \frac{4}{27} (-51 + 16\pi^2 - 36\zeta_3) + \mathcal{O}(\epsilon^0 R \ln R, \epsilon^0 R). \end{aligned}$$

Back-up: Anomalous dimensions

- The standard form of the RG equations of collinear-soft and ultra-collinear-soft functions can be derived as,

$$\begin{aligned} \frac{d}{d \ln \mu} \mathcal{S}_m^{\text{CS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_y, R, \mu, \nu) &= \sum_{l=0}^m \Gamma_{ml}^{\text{CS}} \mathcal{S}_l^{\text{CS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_y, R, \mu, \nu) \\ &= \sum_{l=0}^m \left(\Gamma^{\text{CS}} \delta_{ml} \mathbf{1} + \hat{\Gamma}_{ml} \right) \mathcal{S}_l^{\text{CS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_y, R, \mu, \nu), \\ \frac{d}{d \ln \mu} \mathcal{S}_l^{\text{UCS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_x, R, \mu) &= - \sum_{m=l}^{\infty} \mathcal{S}_m^{\text{UCS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_x, R, \mu) \hat{\otimes} \Gamma_{ml}^{\text{UCS}} \\ &= \sum_{m=l}^{\infty} \mathcal{S}_m^{\text{UCS}}(\{n_i, \bar{n}_i, \underline{u}\}, b_x, R, \mu) \hat{\otimes} \left(\Gamma^{\text{UCS}} \delta_{ml} \mathbf{1} - \hat{\Gamma}_{ml} \right). \end{aligned}$$

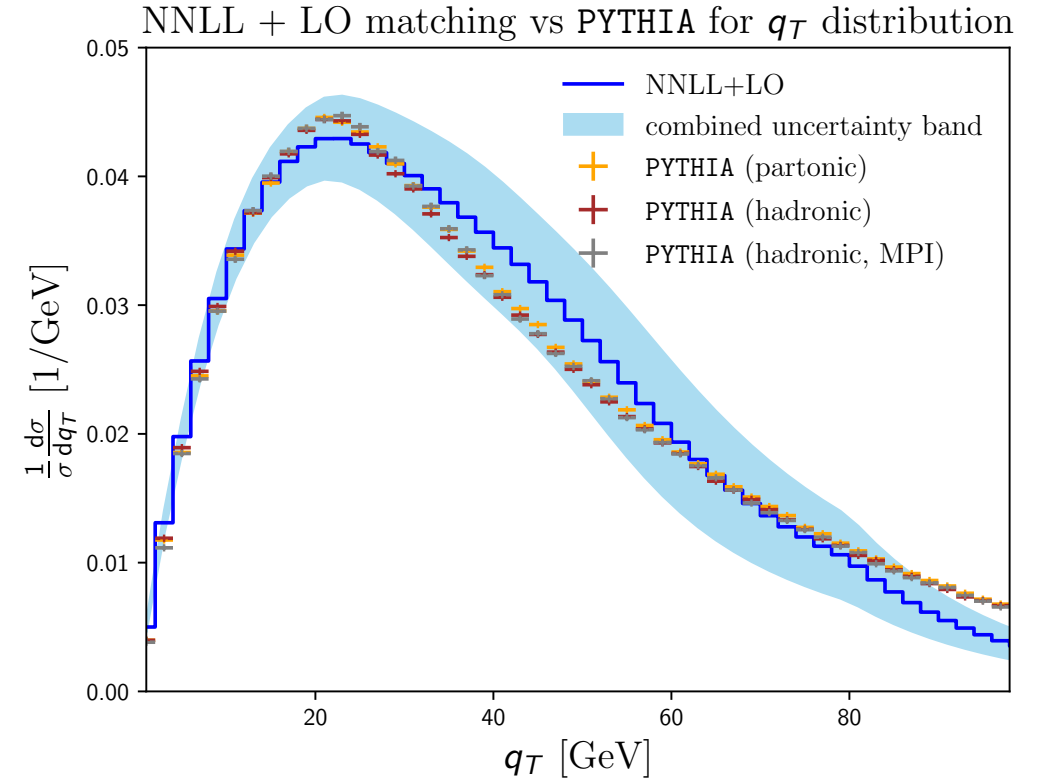
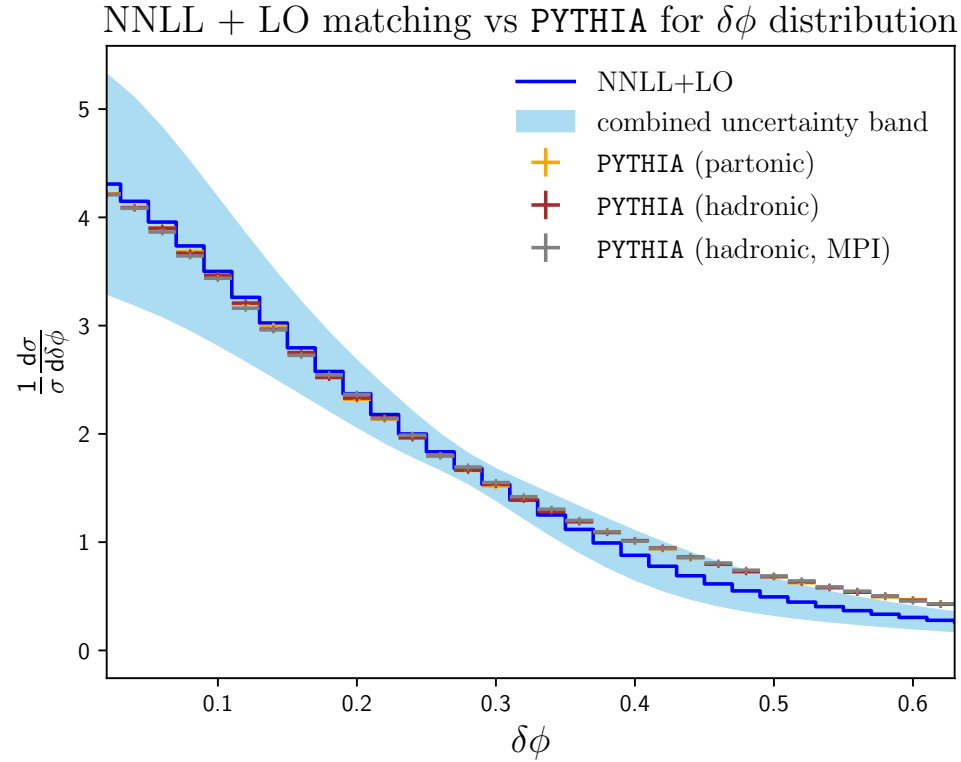
- Their combination S_i , evolves under the RG without any residual multiplicity mixing,

$$\frac{d}{d \ln \mu} S_i(\vec{b}_T, \eta_i, R, \mu, \nu) = (\Gamma^{\text{CS}} + \Gamma^{\text{UCS}}) S_i(\vec{b}_T, \eta_i, R, \mu, \nu).$$

- The two-loop global anomalous dimensions for the collinear-soft and ultra-collinear-soft functions are

$$\begin{aligned} \Gamma^{\text{CS}}(\alpha_s, \{n_i, \bar{n}_i\}, R, \mu, \nu) &= 2C_i \gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu}{\nu R_i}, \\ \Gamma^{\text{UCS}}(\alpha_s, \{n_i, \bar{n}_i\}, b_x, R, \mu) &= -C_i \gamma_{\text{cusp}}(\alpha_s) \left[\ln \frac{4\mu^2 b_x^2}{b_0^2 R^2} - i\pi \text{Sign}(b_x n_{i,x}) \right], \end{aligned}$$

Comparison with PYTHIA 8.3



- Both the azimuthal decorrelation $\delta\phi$ and the transverse momentum imbalance q_T are robust against hadronization and multi-parton interactions (MPI).