中国物理学会高能物理分会
HIGH ENERGY PHYSICS BRANCH OF CPS

華南師範大學

Towards Quantum Simulations of Sphaleron Dynamics at Colliders

*Based on the work PRD **113** (2026) 056021*
in collaboration with Min Huang, Ying-Ying Li, and Yandong Liu

Hao Zhang

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For “中国物理学会高能物理分会第十二届全国会员代表大会暨学术年会”, Jul 15th, 2026, Guangzhou

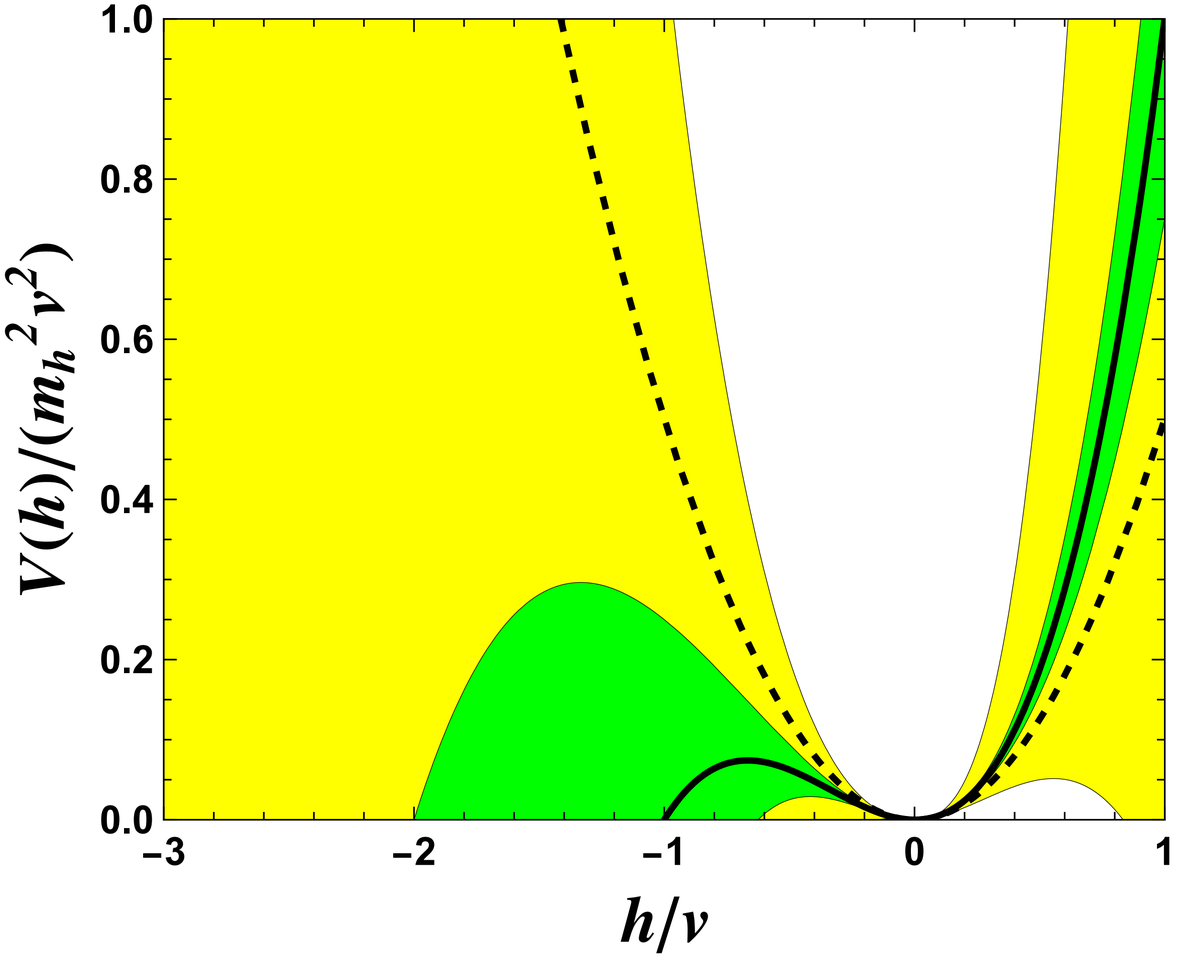
The Eternal Theme of the HL-LHC and future *pp*-collider

The Potential

- Allowed region of the cubic potential by current Higgs pair data.

$$V(h) = \frac{1}{2} m_h^2 \left(h^2 + \frac{\kappa_\lambda}{v} h^3 \right)$$

$$\kappa_\lambda \in [-1.2,\ 7.02]$$



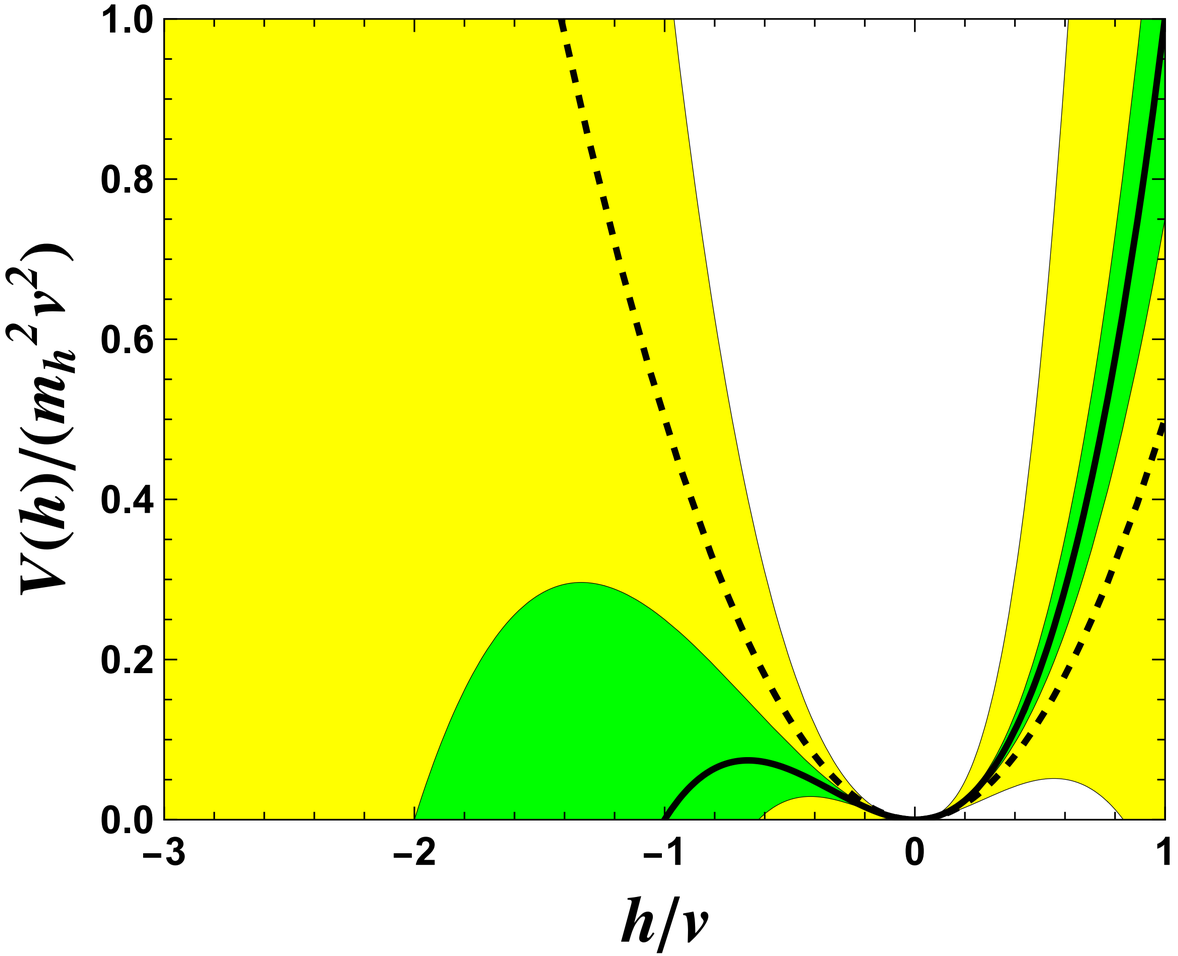
ATLAS Collaboration, Phys. Rev. Lett. 133 (2024) 101801;
CMS Collaboration, CMS PAS HIG-20-011.

The Potential

- (Expected) Allowed region of the cubic potential by HL-LHC.

$$V(h) = \frac{1}{2} m_h^2 \left(h^2 + \frac{\kappa_\lambda}{v} h^3 \right)$$

$$\kappa_\lambda \in [0.5,\ 1.6]$$



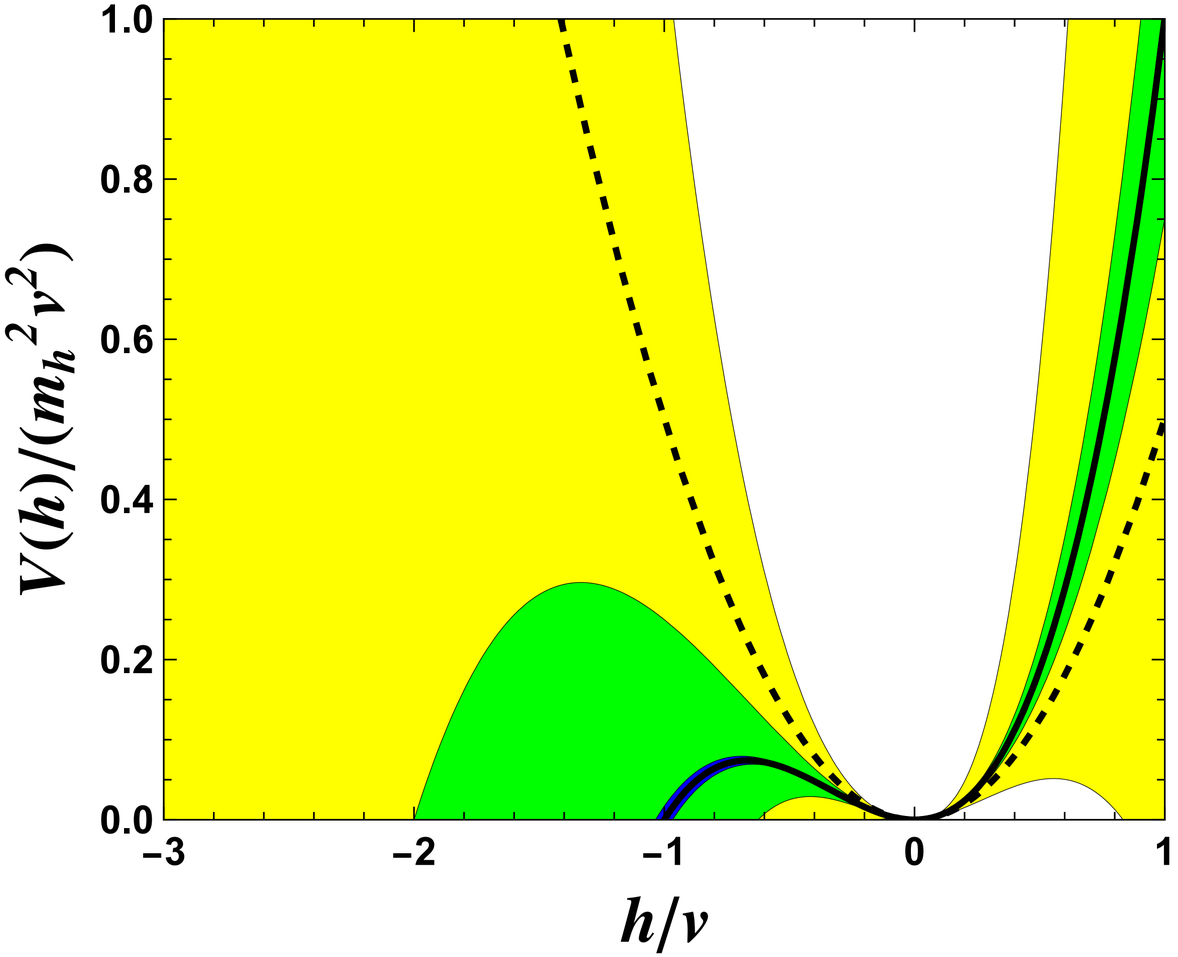
M. Mlynarikova, on behalf of the ATLAS and CMS collaborations, arXiv:2307.07772[hep-ex].

The Potential

- (Expected) Allowed region of the cubic potential by FCC-hh.

$$V(h) = \frac{1}{2} m_h^2 \left(h^2 + \frac{\kappa_\lambda}{v} h^3 \right)$$

$$\kappa_\lambda \in [0.966,\ 1.034]$$



A. Abada, et. al., FCC-hh: The Hadron Collider, Eur. Phys. J. Special Topics 228 (2019) 755.

The Potential

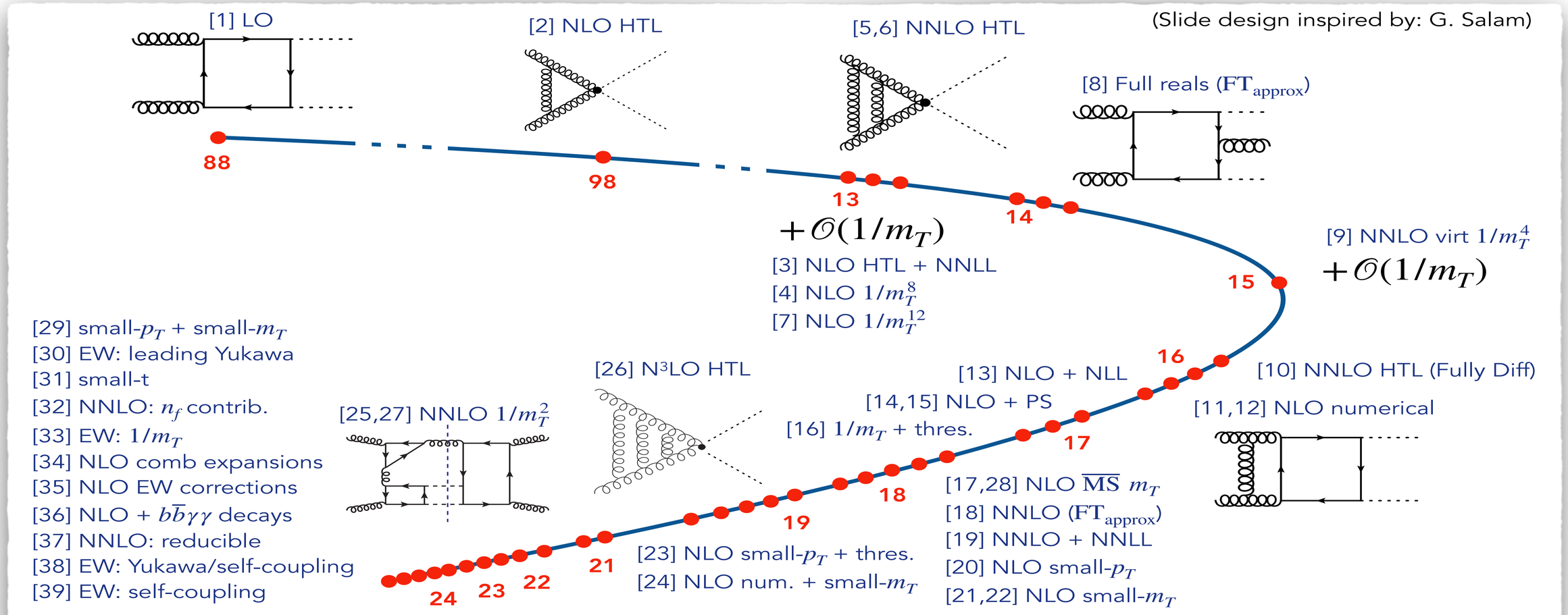
- (Expected) Allowed region of the cubic potential by FCC-hh.

When?

The tentative timeline is:

- **2025**: Completion of the FCC Feasibility Study
- **2027–2028**: Decision by the CERN Member States and international partners
- **2030s**: Start of construction
- **Mid-2040s**: **FCC-ee** begins operation and runs for approximately 15 years
- **2070s**: **FCC-hh** begins operation and runs for approximately 25 years

https://home.cern/science/accelerators/future-circular-collider



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23 [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner [39] Li, Si, Wang, Zhang, Zhao 24

From S. Jones, “Higgs production: A theory overview”, Higgs Hunting 2024.

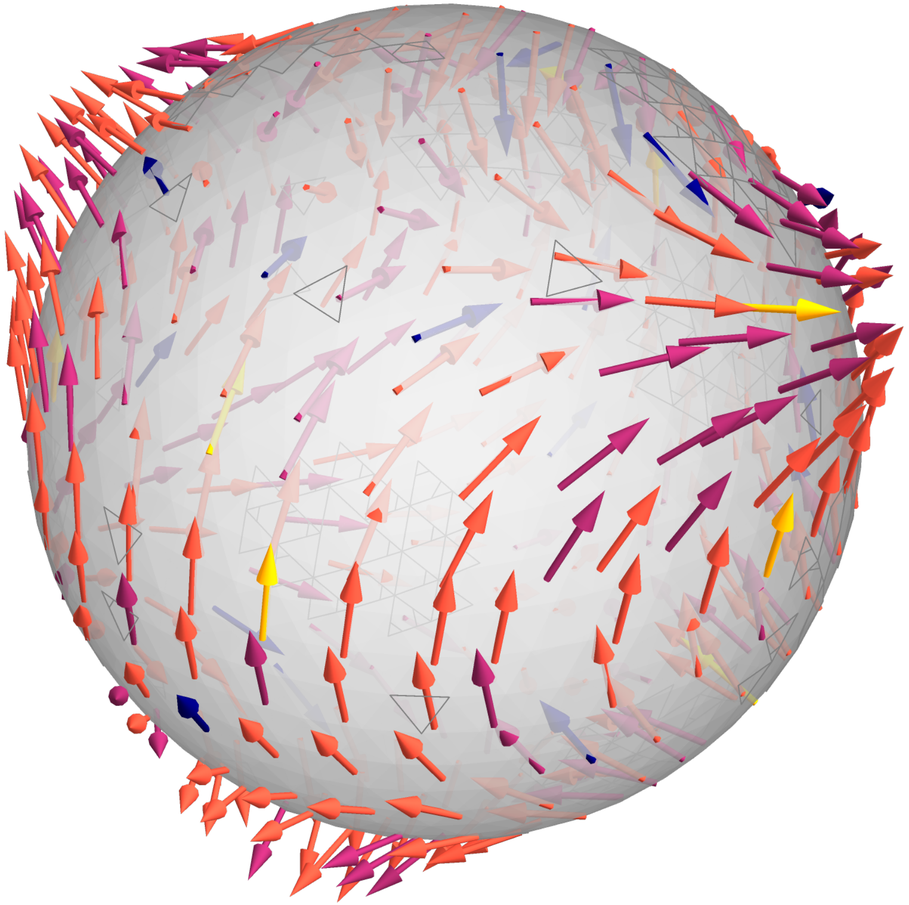
The Sphaleron Configuration

- Manton's insight

N. S. Manton, Topology in the Weinberg-Salam theory, Phys. Rev. D 28 (1983) 2019.

The Sphaleron Configuration

- A "*σφαλερος*" (means "ready to fall") with some ansatz



F. R. Klinkhamer, N. S. Manton, A saddle-point solution in the Weinberg-Salam theory, Phys. Rev. D 30 (1984) 2212.

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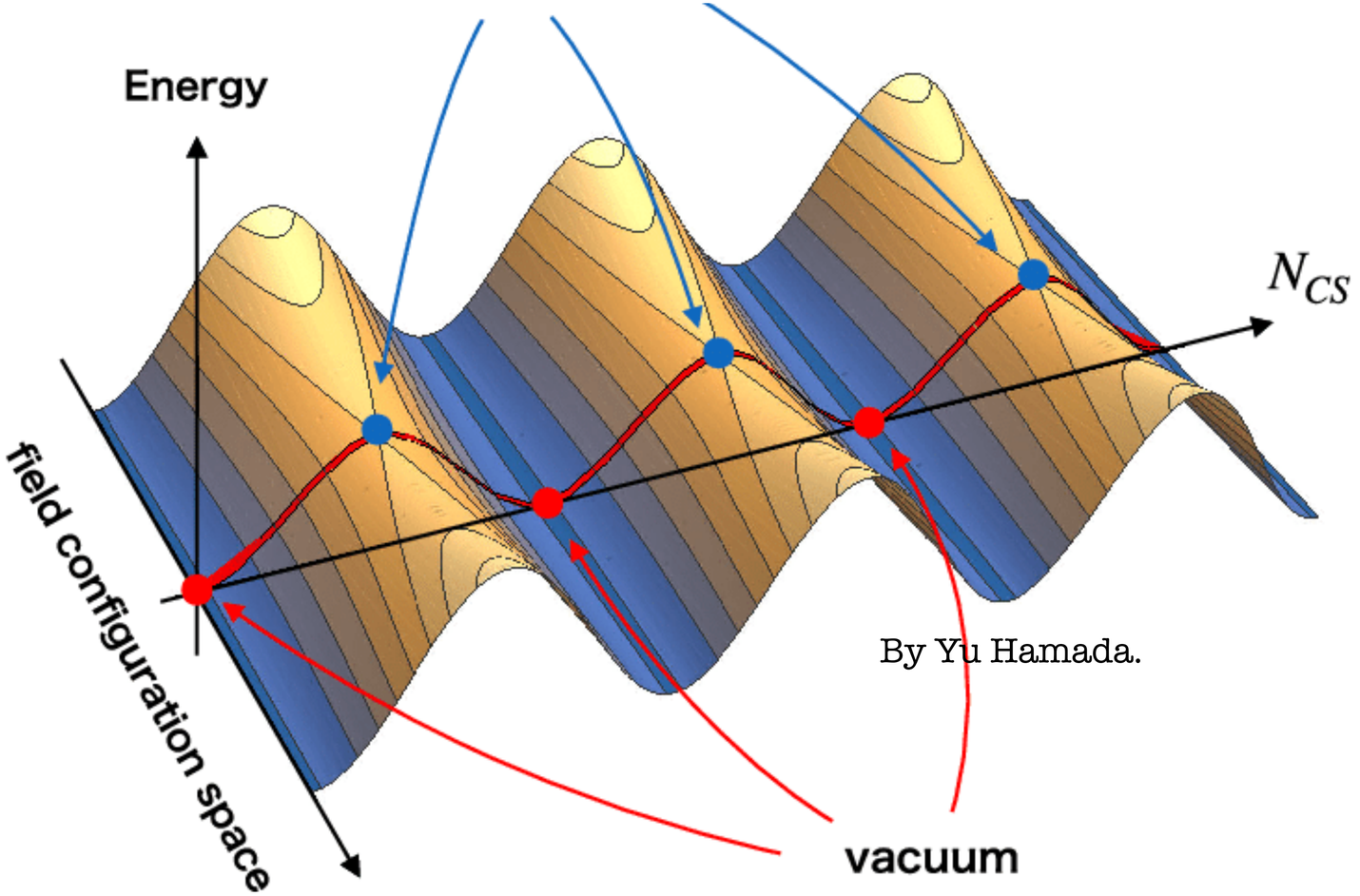

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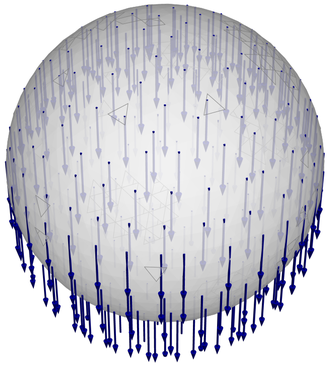

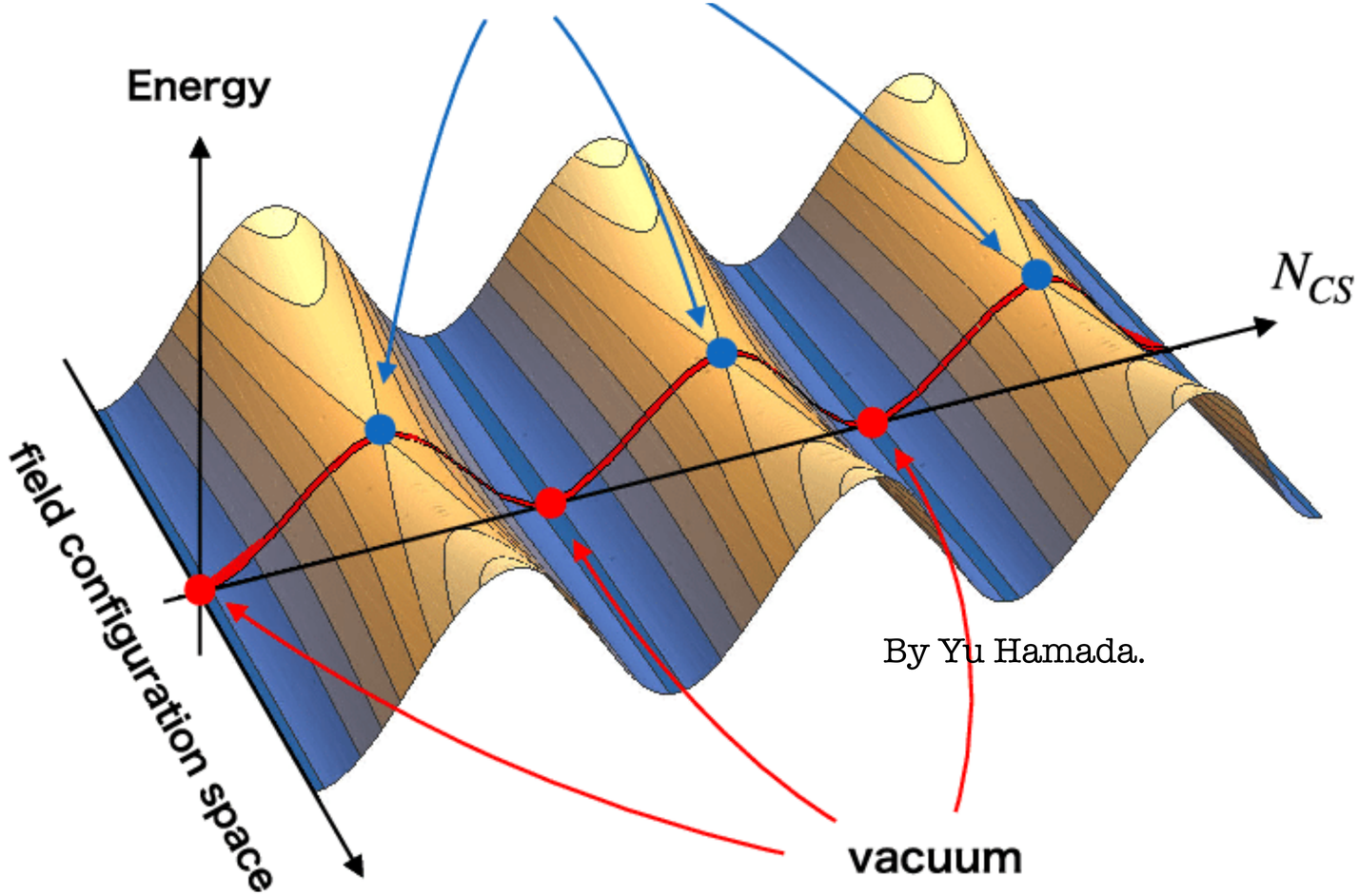

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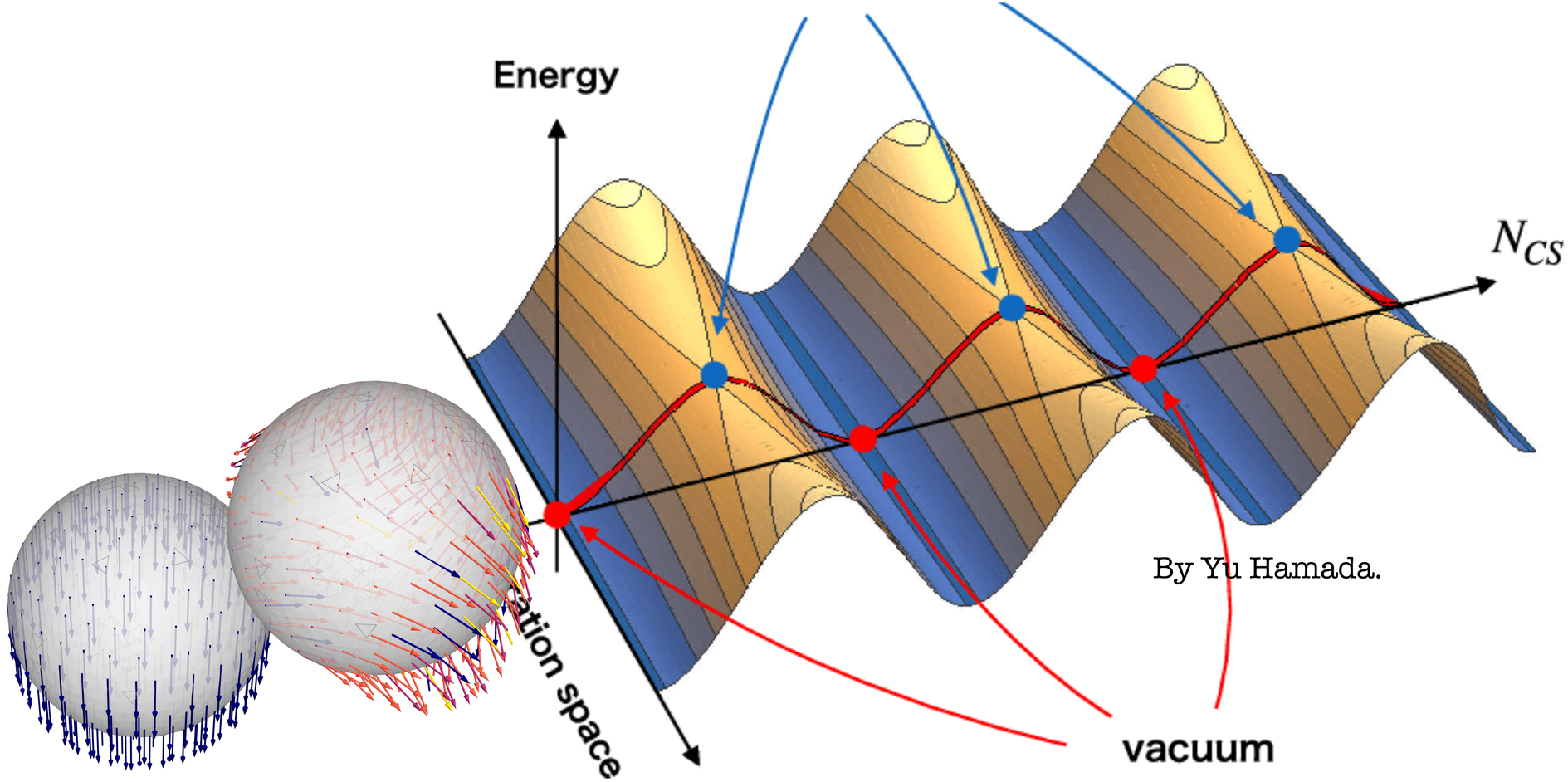

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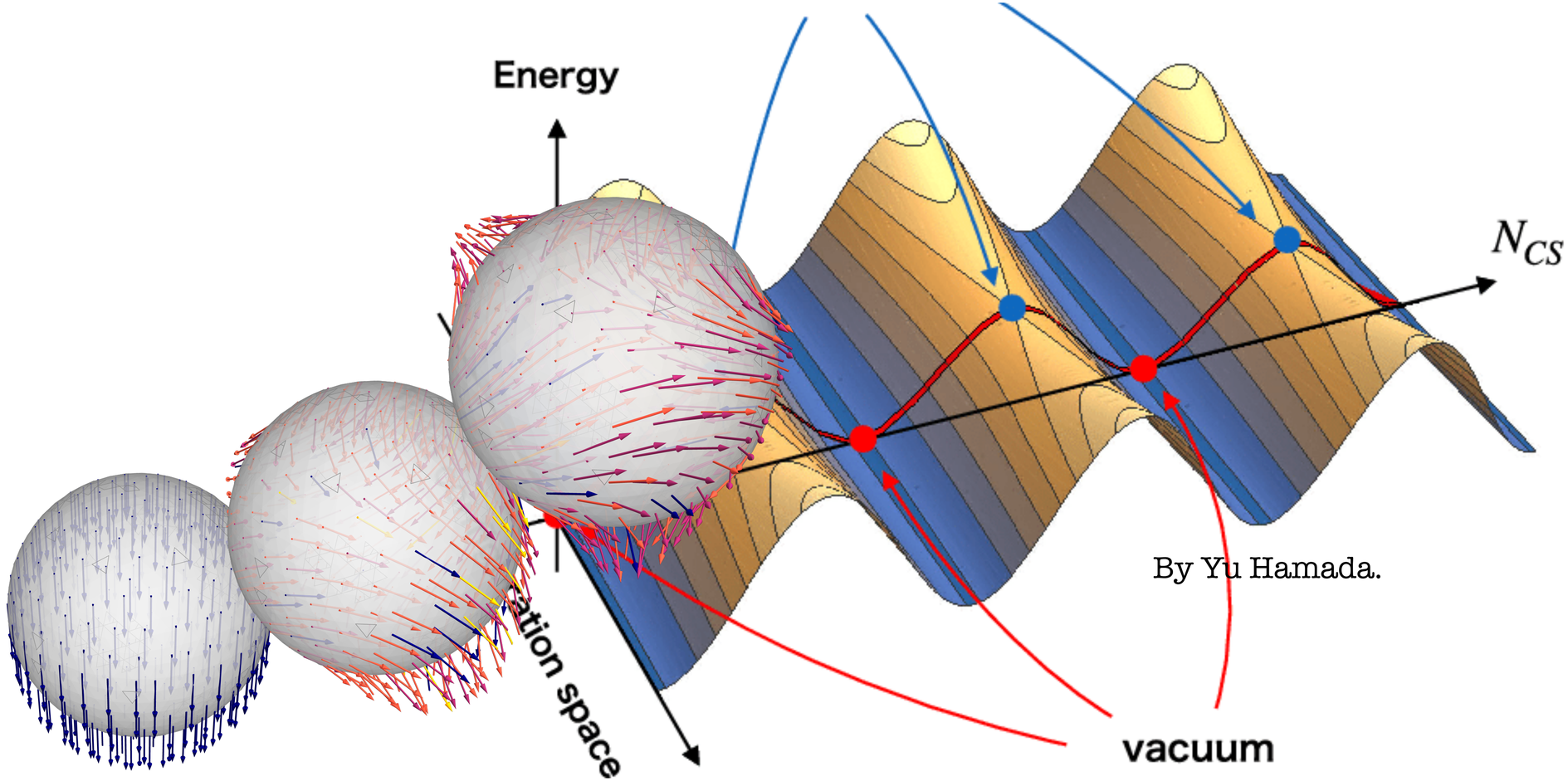

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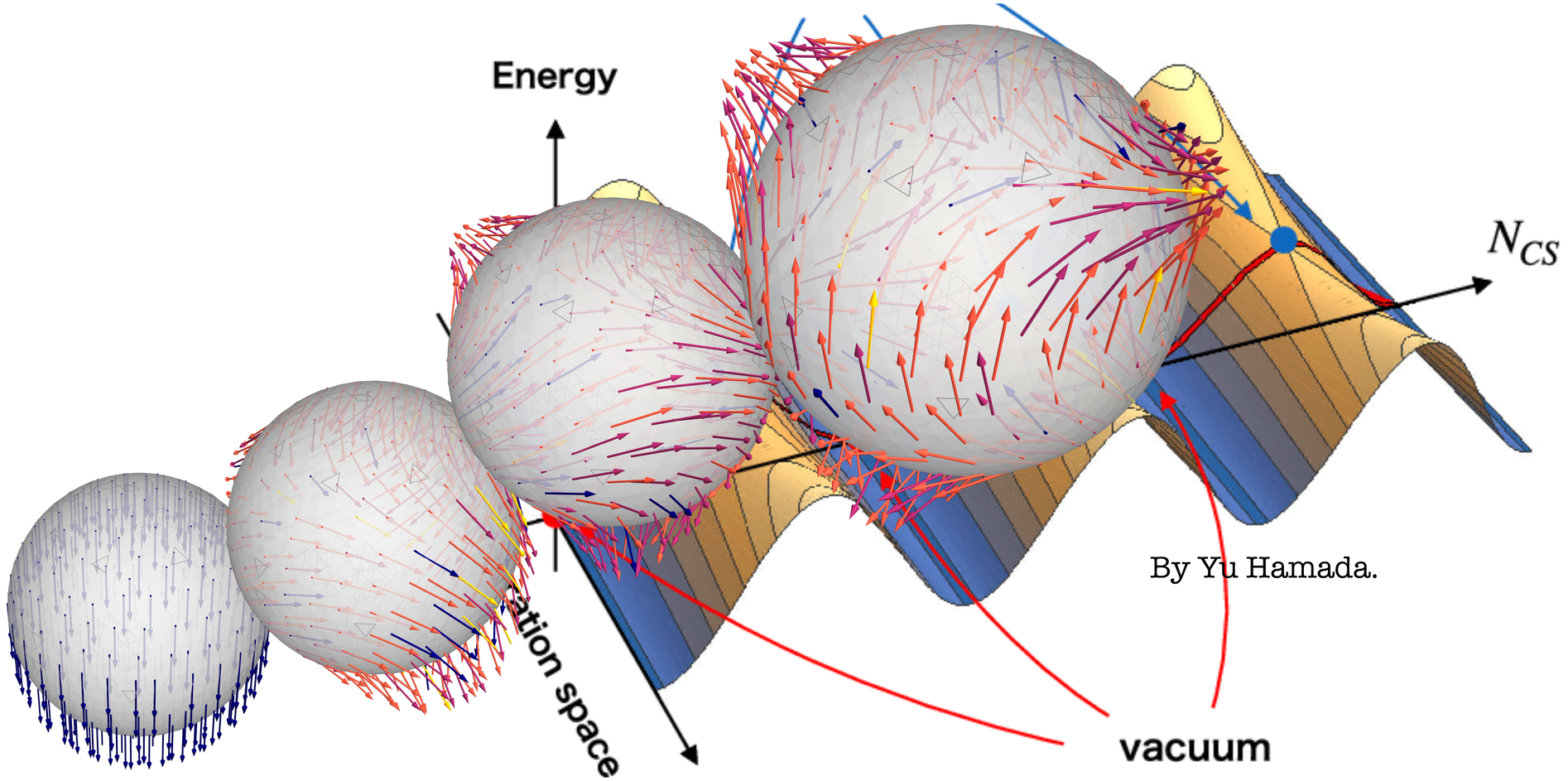

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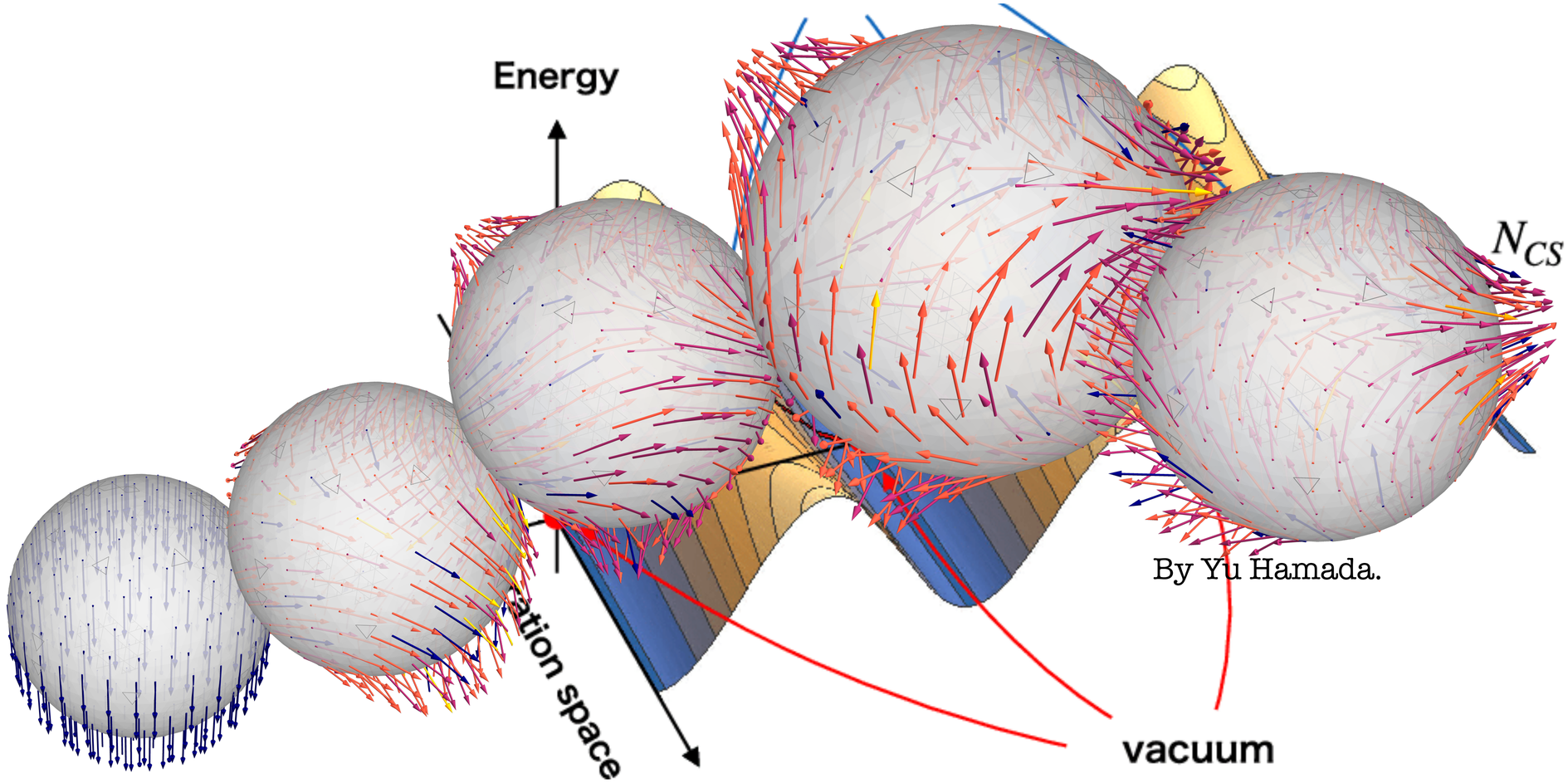

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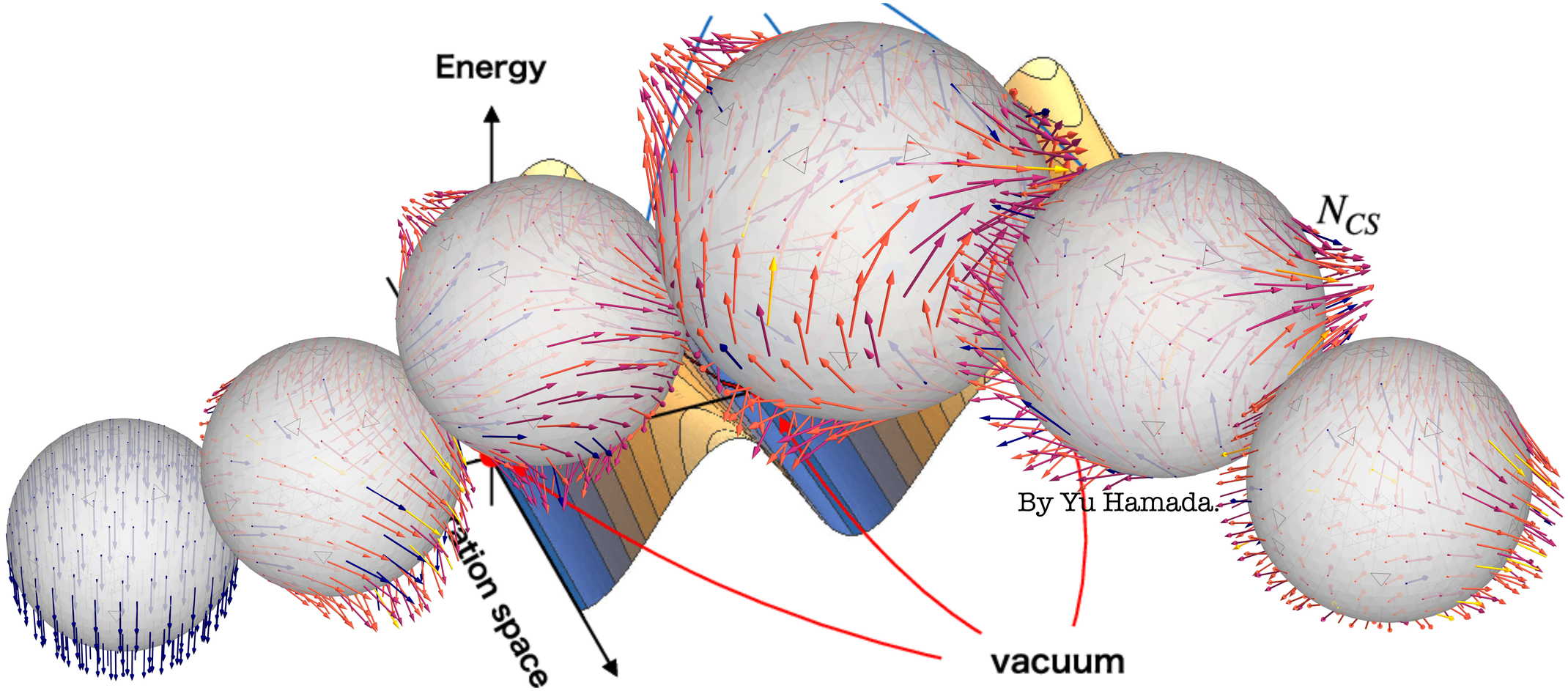

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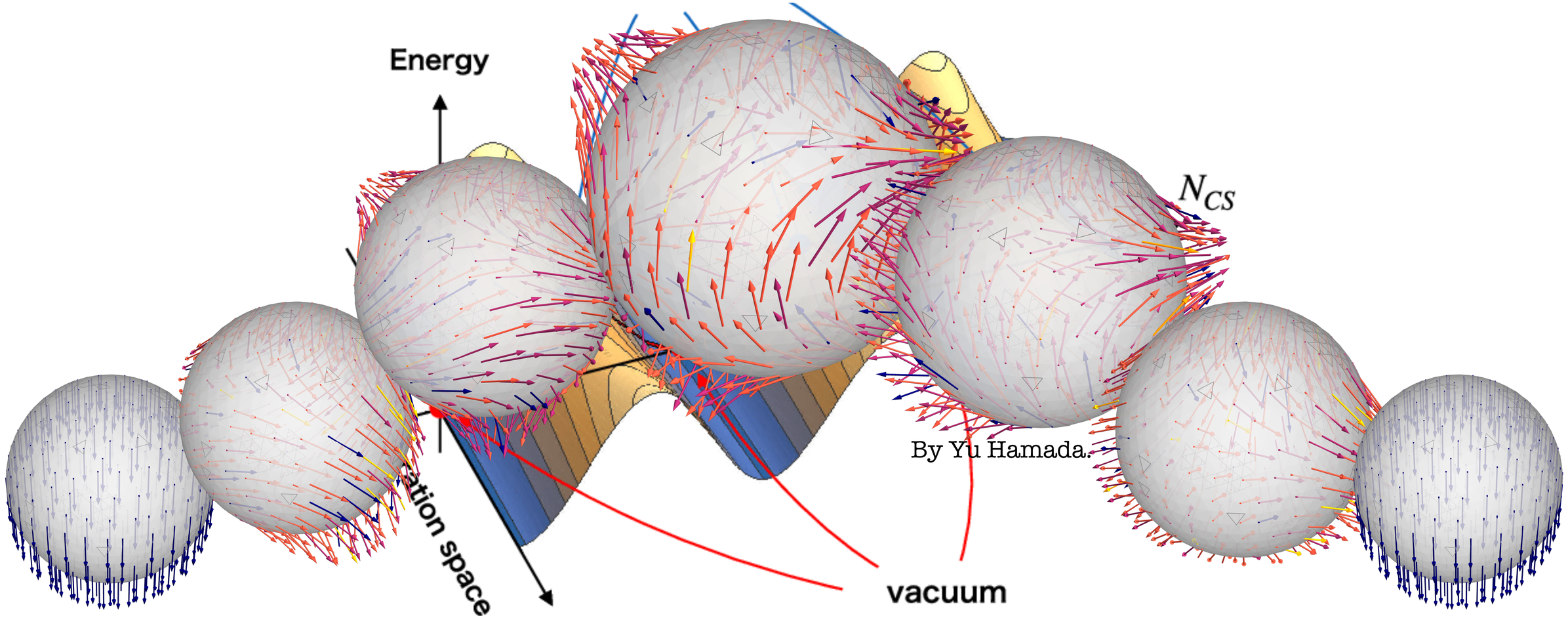

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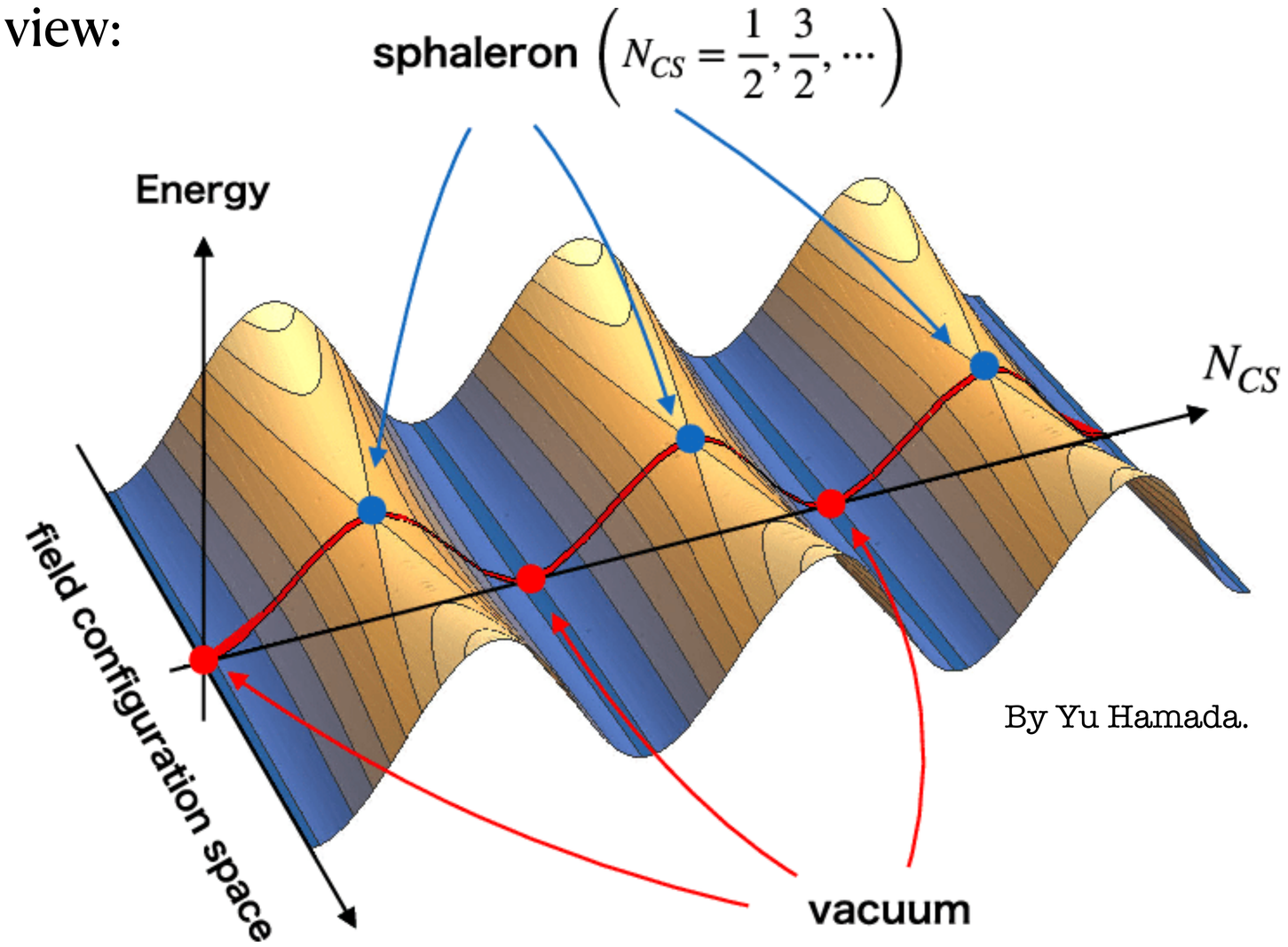
The Sphaleron, a climber

- From the fermion's point of view:



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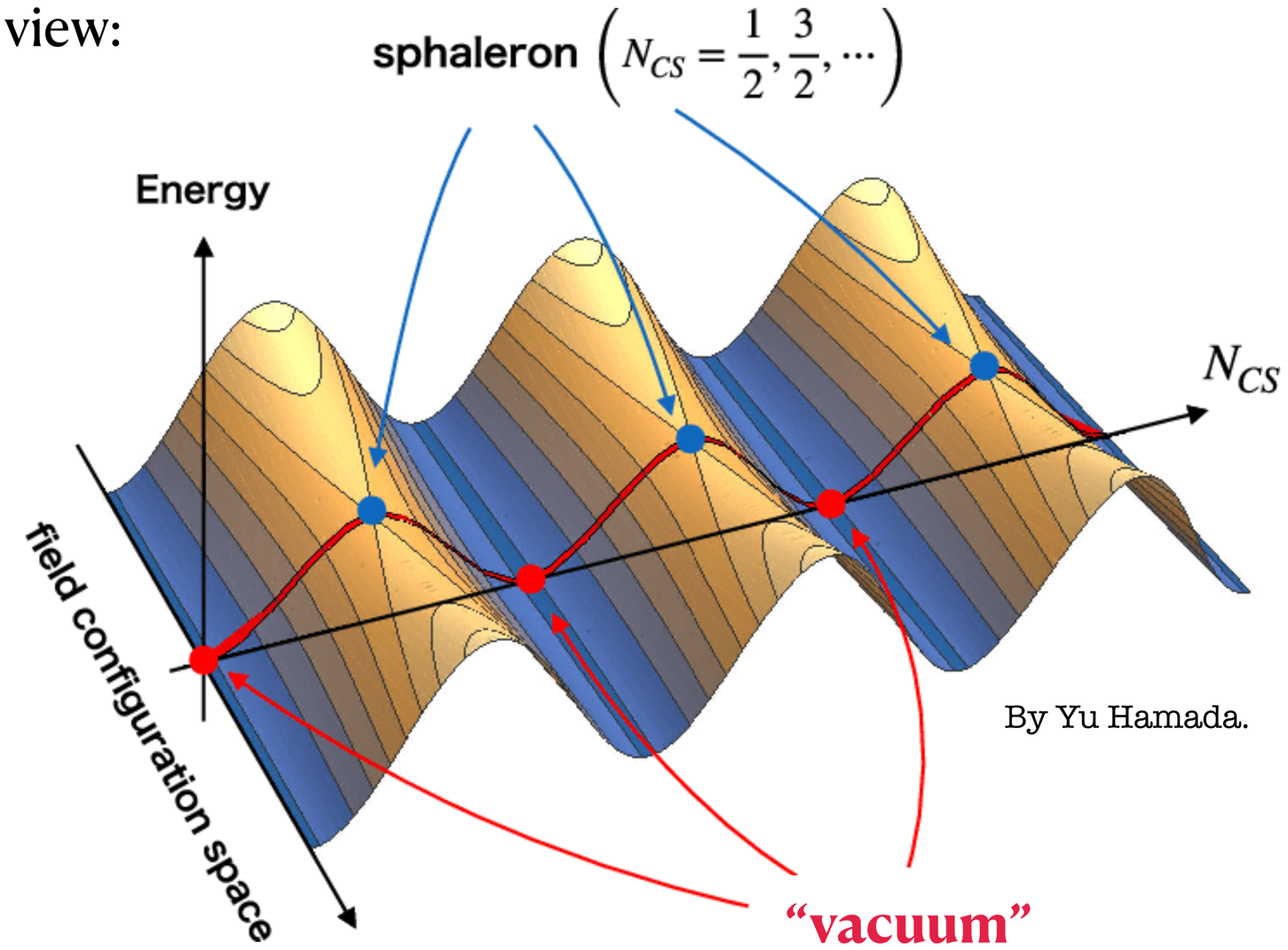

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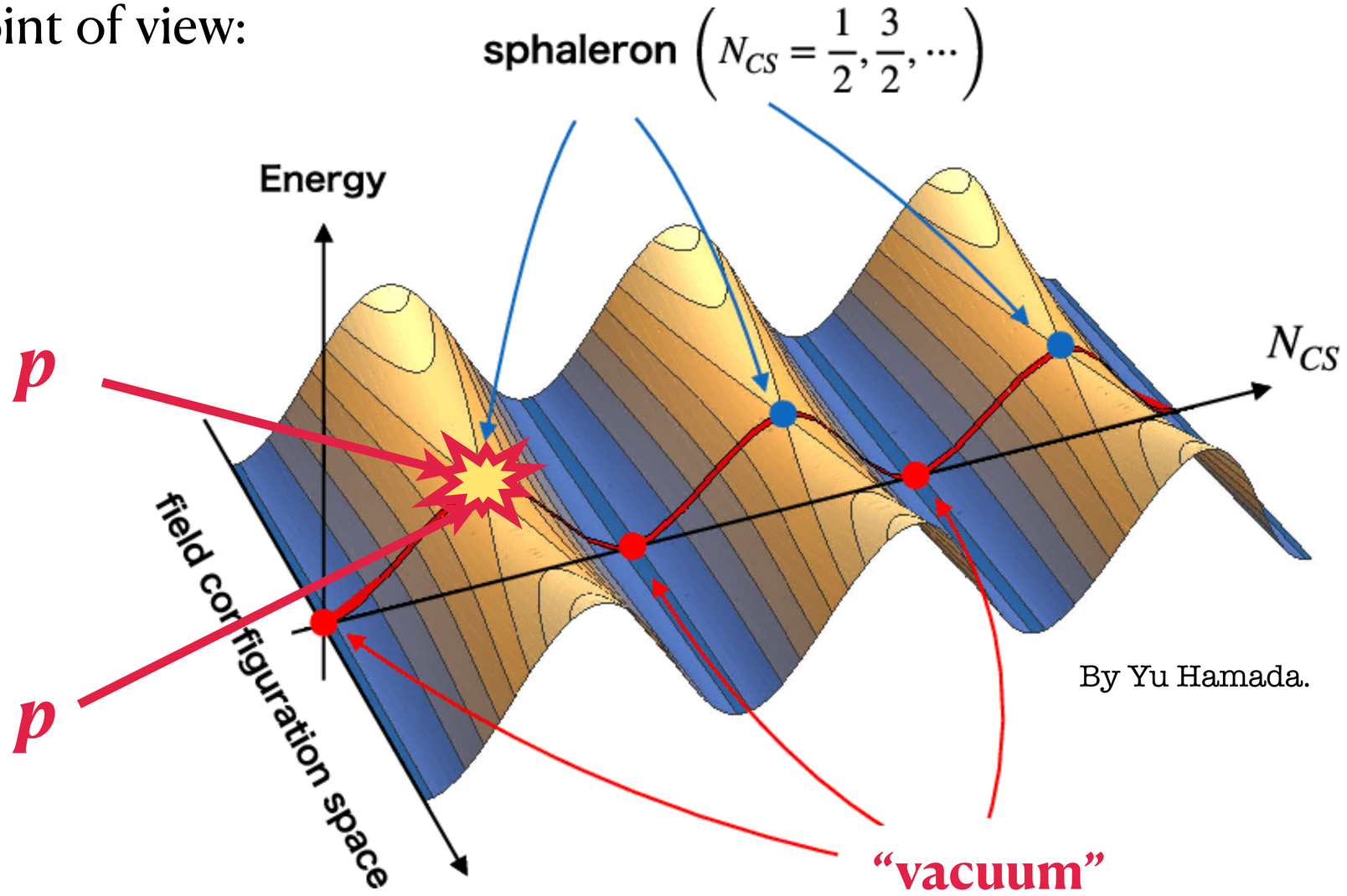
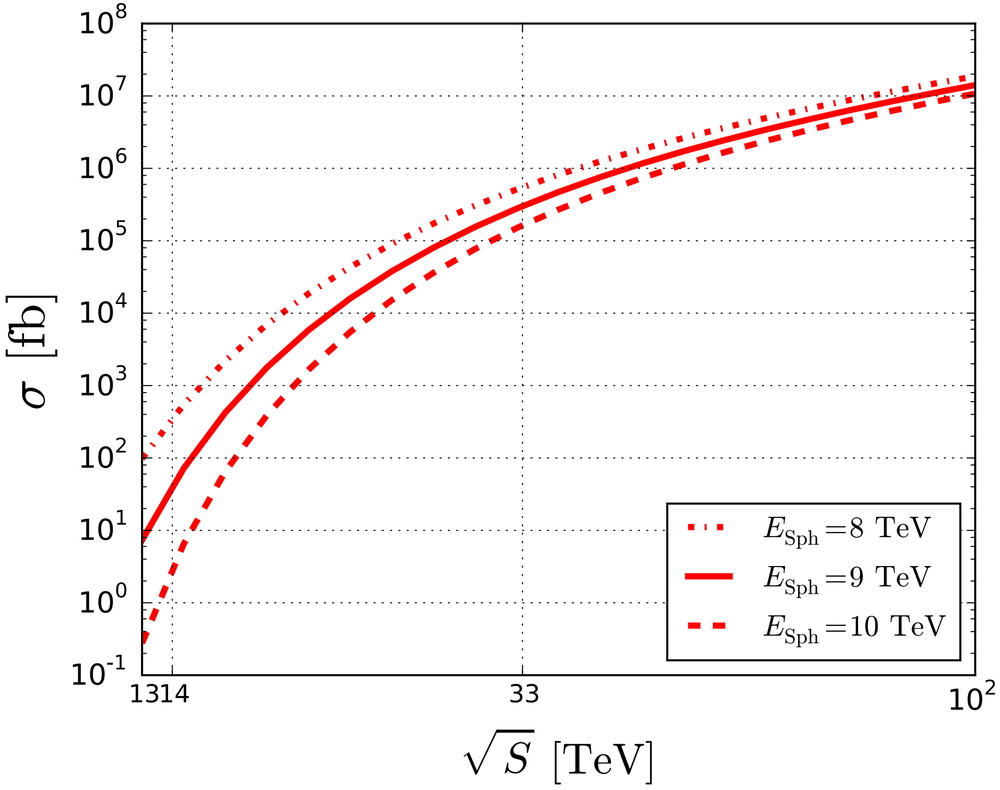


The Signal at Colliders

- It is believed that the typical signal of a sphaleron (if we can produce it) would be *B*+*L* violation final state with multi-Higgs bosons and multi-weak gauge bosons.

$$\partial_\mu J^\mu_B = \frac{1}{32\pi^2}\frac{N_B}{3}\left(g^2\mathbf{F}\wedge\mathbf{F} + g'^2 F\wedge F\right)$$

$$q_L + q_L \to (e^- ddd)(\mu^- ccc)(\tau^- bbb)uu + X$$



J. Ellis, K. Sakurai, Search for sphalerons in proton-proton collisions, JHEP 04 (2016) 086.

How many?

- An classically estimation of the “mass” of the sphaleron is ~ 9TeV (depends on the weak angle, the ansatz, ...).
- Though it seems to be too heavy to be produced at the LHC (even with the parton distribution enhancement), a 28TeV HE-LHC or 100TeV FCC-hh is energetic enough to produce it.
- It is not a easy job to predict the production and decay rate of the sphaleron precisely. Such a prediction (formula) is necessary for reading out the information of the Higgs potential hidden in the sphaleron processes.
- The effort to improve it would probably help us understand the non-perturbative properties of the quantum field theory.
- We would have 70 years to improve the prediction before our experimentalist colleagues have chance to discover it?

New Methods?

- Quantum computing and quantum simulation.

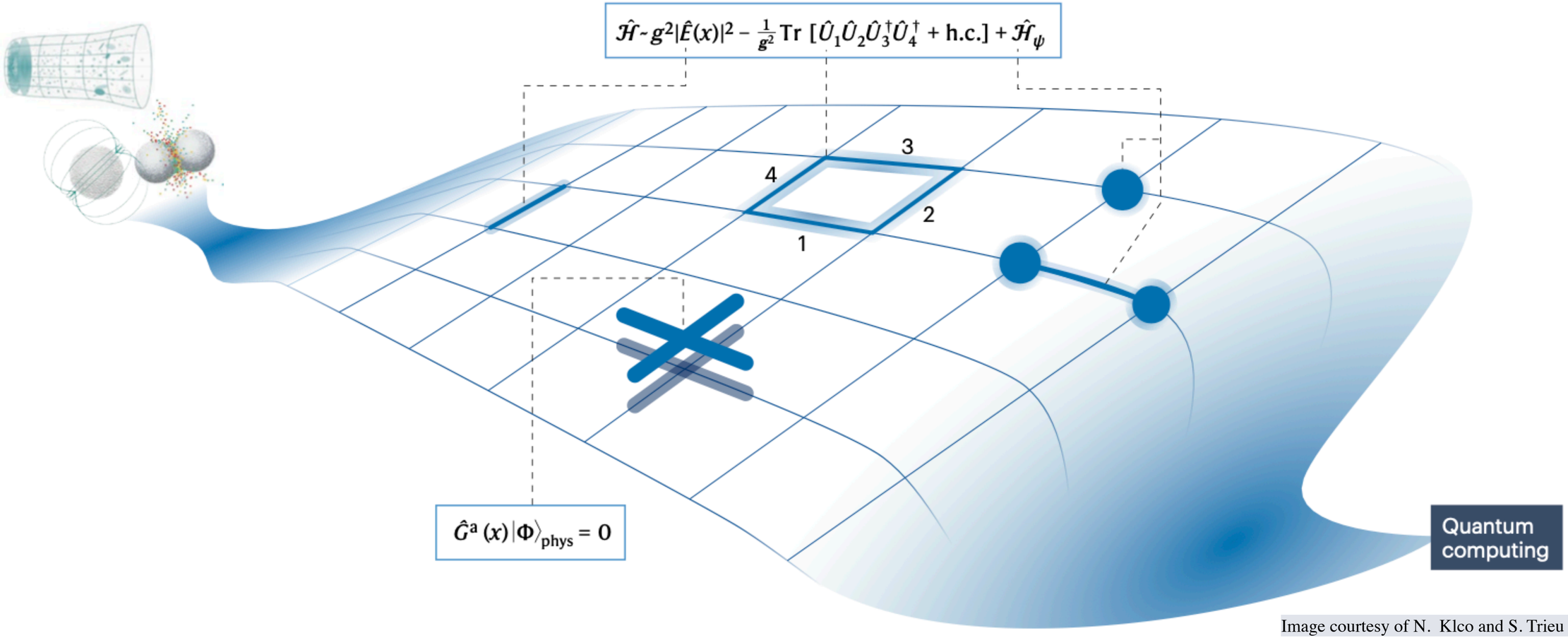


Image courtesy of N. Klco and S. Trieu

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- 9TeV energy in ~10^{-18}m with quite short time scale! Should there be significant quantum effect?
- The configuration is a saddle point which is also (at least a little) subtle in path integral formalism.
- The configuration does not correspond to any eigenstate of the Hamiltonian.
- What is a quantum sphaleron? Or what is (is there) a quantum analog of the real classical sphaleron configuration, and what is the role of it in the quantum evolution?

Towards to it

- The highest program of the plan: computing (or simulating) a real-time evolution with sphaleron configuration in the SM.
- But

Towards to it

- The highest program of the plan: computing (or simulating) a real-time evolution with sphaleron configuration in the SM.
- The 3+1D SU(2)×U(1) gauge theory with spontaneously symmetry breaking from a Higgs doublet and three generation chiral fermions is too complicate for a baby quantum computing.
- In this first attempt, we investigate the saddle-point configuration in a much simpler quantum field model — the 1+1D O(3) non-linear σ-model.

E. Mottola, A. Wipf, Unsuppressed fermion-number violation at high temperature: An O(3) model, Phys. Rev. D 39 (1989) 588.

The 1+1D O(3) Model

- The classical sphaleron configuration

$$S = \frac{1}{g^2}\int \mathrm{d}t\,\mathrm{d}x\left[\frac{1}{2}\partial_\mu \mathbf{n}\cdot\partial^\mu \mathbf{n} - \omega^2(1+\mathbf{n}\cdot\mathbf{e}_z)\right] \qquad \mathbf{n}(x)\cdot\mathbf{n}(x)\equiv 1$$

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$$\mathbf{n}(\xi,\eta) = \begin{pmatrix} \sin\eta\sin\xi \\ \sin\eta\cos\eta(\cos\xi-1) \\ -\sin^2\eta\cos\xi - \cos^2\eta \end{pmatrix}$$

$$H = \frac{1}{g^2}\int \mathrm{d}x\left[\frac{1}{2}(1-\cos\xi)^2\left(\frac{\partial\eta}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\eta}{\partial x}\sin\xi\cos\eta + \frac{\partial\xi}{\partial x}\sin\eta\right)^2 + \omega^2\sin^2\eta(1-\cos\xi)\right].$$

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$$\text{fix } \eta = \pi/2, \Rightarrow \qquad E = \frac{\sin^2\eta}{g^2}\int \mathrm{d}x\left[\frac{1}{2}\left(\frac{\partial\xi}{\partial x}\right)^2 + \omega^2(1-\cos\xi)\right]$$

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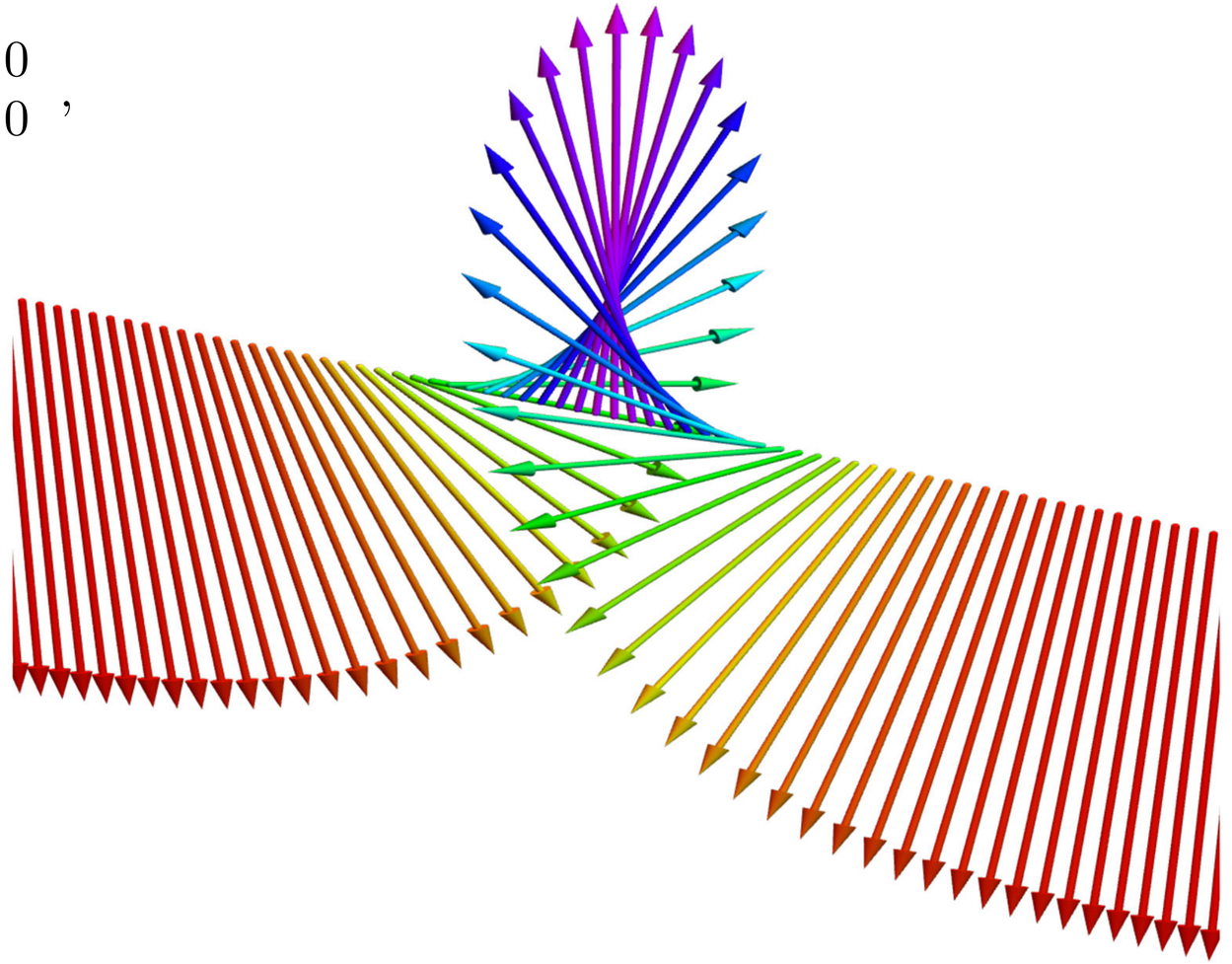
A chain of infinite coupled pendulums.

The 1+1D O(3) Model

- The classical sphaleron configuration

$$\xi^{\mathrm{sph}}(x) = \begin{cases} 2\arcsin(\mathrm{sech}(\omega x)), & x < 0 \\ 2\pi - 2\arcsin(\mathrm{sech}(\omega x)), & x \geqslant 0 \end{cases},$$

$$\mathbf{n}(x,t) = \begin{pmatrix} -2\mathrm{sech}(\omega x)\tanh(\omega x) \\ 0 \\ -1 + 2\mathrm{sech}^2(\omega x) \end{pmatrix}.$$



The 1+1D O(3) Model

- Reconstruct the classical sphaleron configuration on a lattice

$$H_{\mathrm{tot}} = \frac{g^2}{2a}\sum_{k=-N}^{N}\mathbf{L}_k^2 + \frac{1}{ag^2}\sum_{k=-N}^{N-1}(1-\mathbf{n}_k\cdot\mathbf{n}_{k+1}) + \frac{a\omega^2}{g^2}\sum_{k=-N}^{N}(1+\mathbf{n}_k\cdot\hat{e}_3)$$

$$\mathbf{L}_k = \frac{a}{g^2}\frac{\mathrm{d}\mathbf{n}_k}{\mathrm{d}t}$$

The kinetic term

The tension term

The external field term

The 1+1D O(3) Model

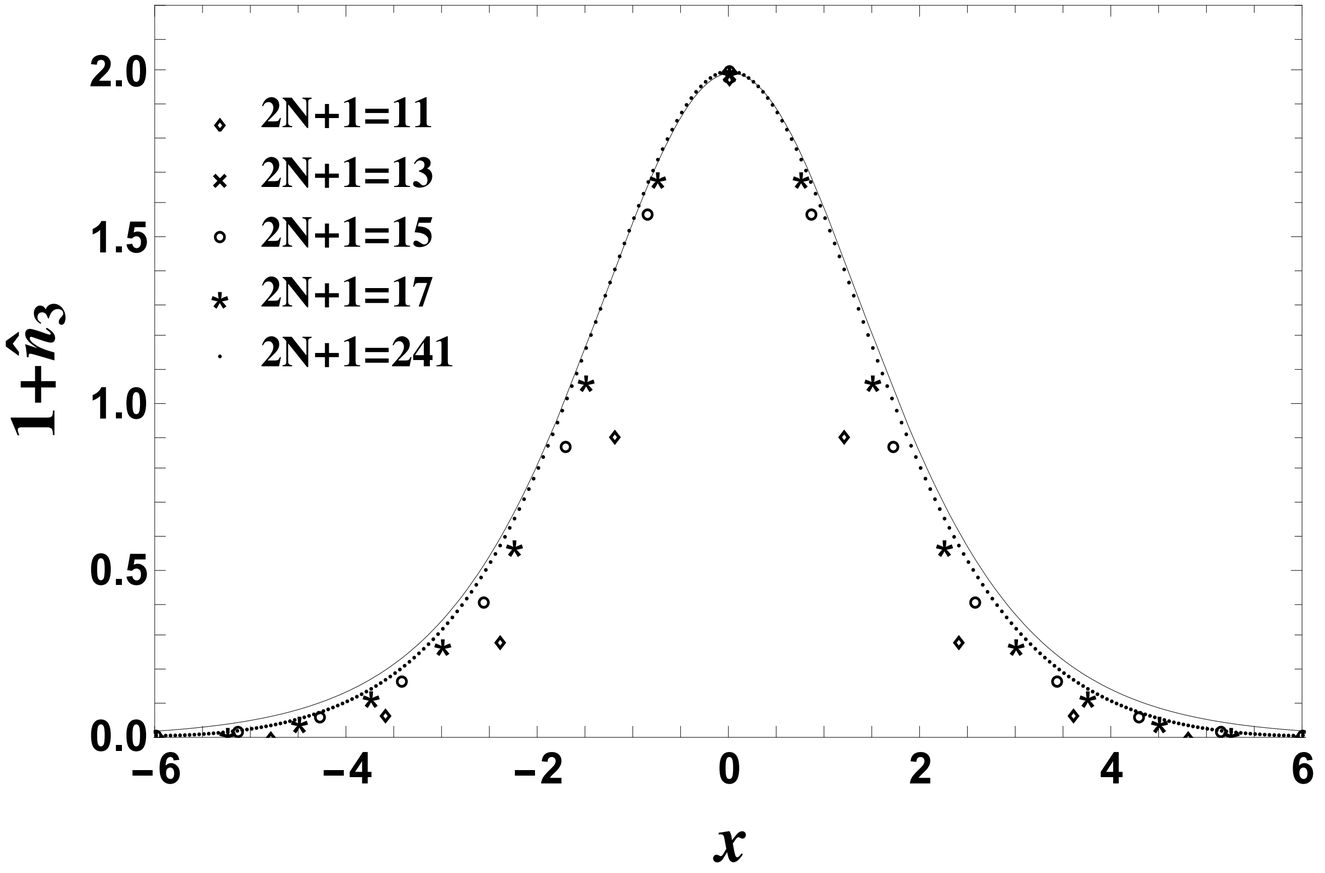
- Reconstruct the classical sphaleron configuration on a lattice

$$
\begin{aligned}
H_{\text{static}} &= \frac{1}{ag^2} \sum_{k=-N}^{N-1} (1 - \mathbf{n}_k \cdot \mathbf{n}_{k+1}) + \frac{a\omega^2}{g^2} \sum_{k=-N}^{N} (1 + \mathbf{n}_k \cdot \hat{e}_3) \\
&= \frac{\sin^2 \eta_0}{g^2} \left(\frac{2N}{L} \right) \left\{ \sum_{k=-N}^{N-1} [1 - \cos(\xi_k - \xi_{k+1})] + \left(\frac{\omega L}{2N} \right)^2 \sum_{k=-N}^{N} (1 - \cos \xi_k) \right\}
\end{aligned}
$$

The 1+1D O(3) Model

- But $\pi_1(\mathbb{Z}_{2N+1}) \neq \pi_1(S^1)$
- Boundary conditions

$$\xi_{-N} = 0, \quad \xi_0 = \pi, \quad \xi_N = 2\pi,$$
$$\text{for } \forall\ k \in [-N, N), \quad \xi_k < \xi_{k+1}.$$



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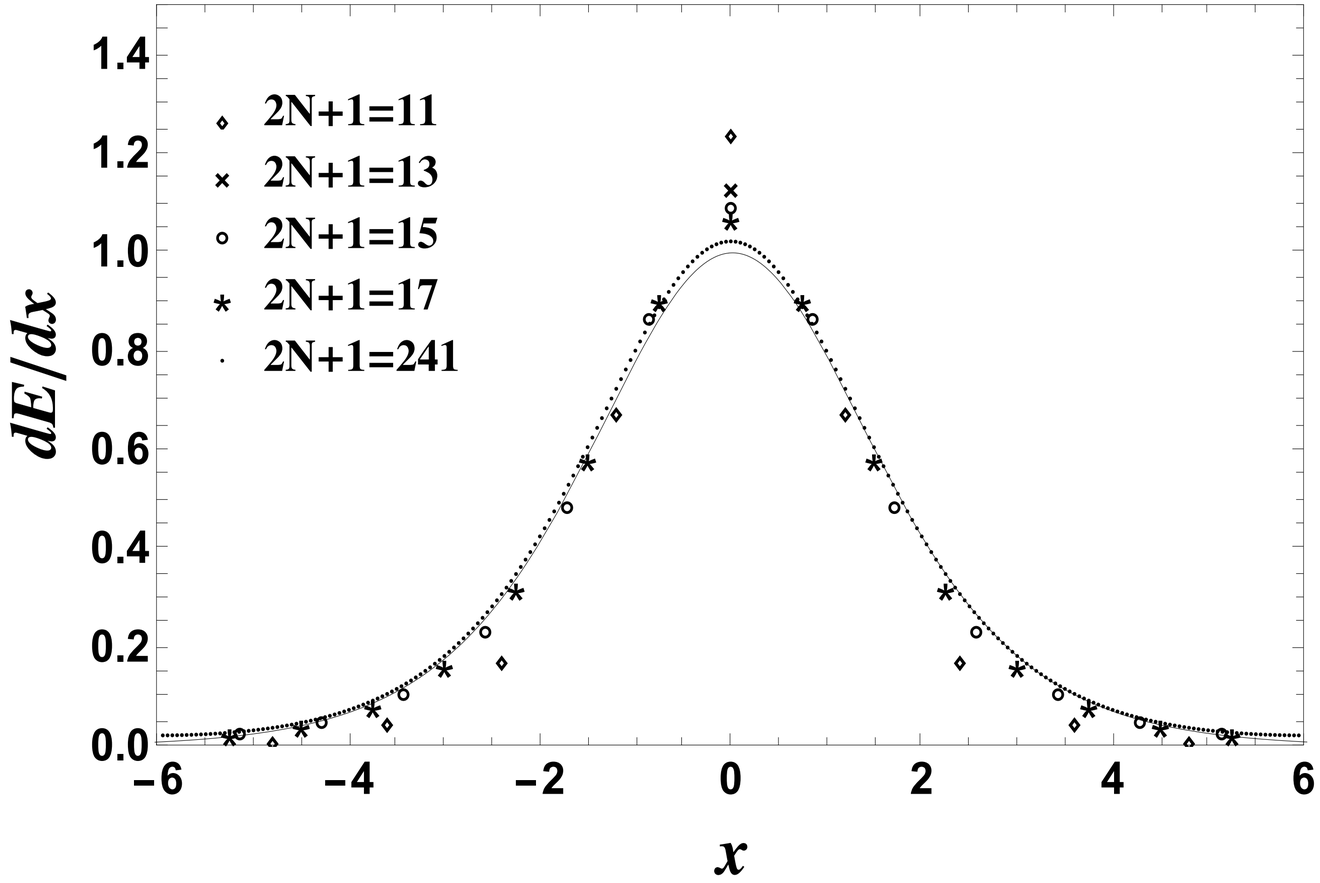

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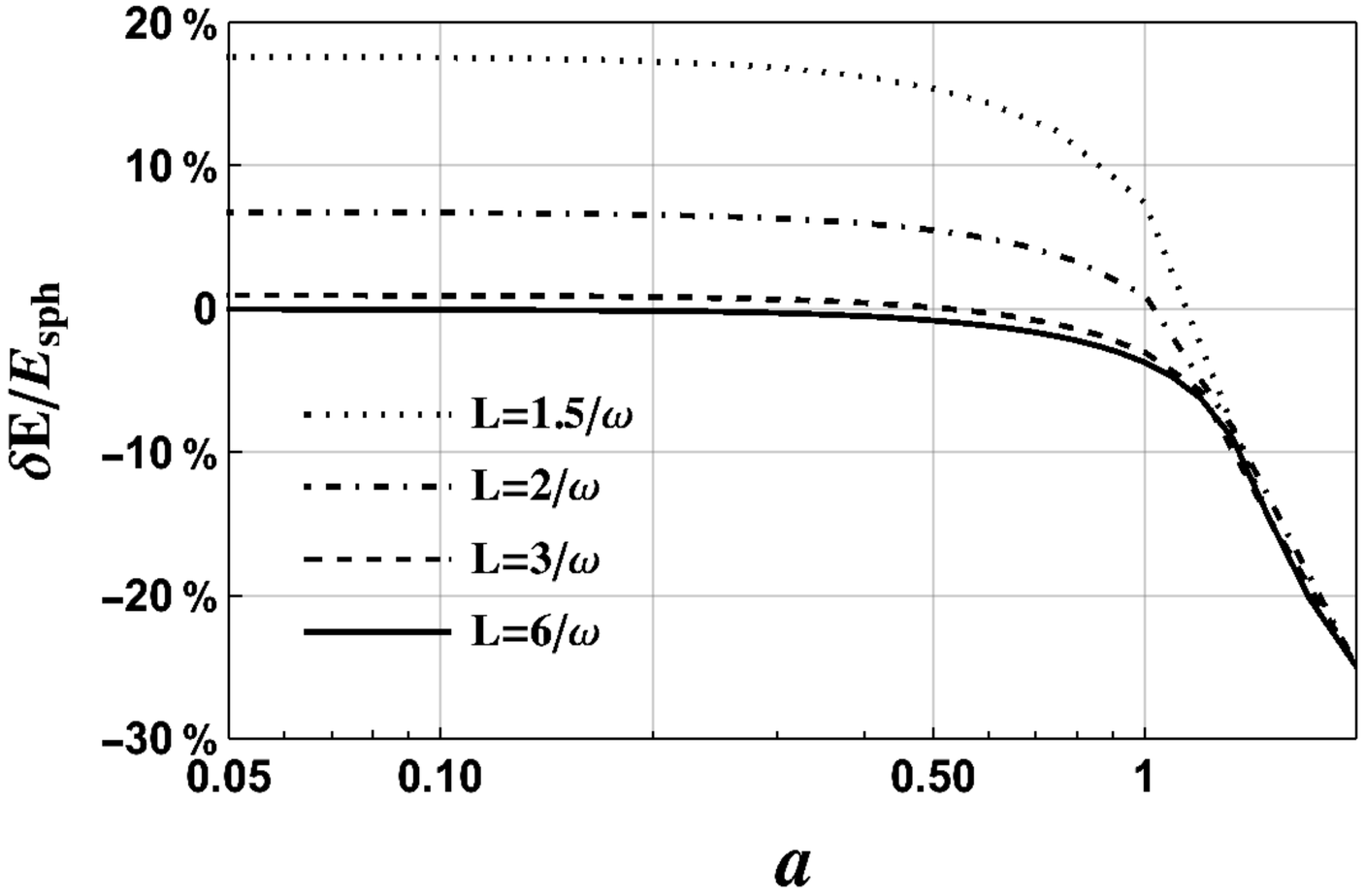
$$\xi_{-N} = 0, \quad \xi_0 = \pi, \quad \xi_N = 2\pi,$$
$$\text{for } \forall\ k \in [-N, N), \quad \xi_k < \xi_{k+1}.$$



The artificial boundary condition is from the fact that the discrete LQFT losing the information of the path connecting the neighbor sites. For a more systematical treatment, see J.-Y. Chen, Instanton Density Operator in Lattice QCD from Higher Category Theory, arXiv:2406.06673[hep-lat].

The 1+1D O(3) Model

- The (discrete) quantum model of the 1+1D O(3) model is a quantum rotor.

$$\hat{H}_{\text{tot}} = \frac{g^2}{2a}\sum_{k=-N}^{N}\hat{\mathbf{L}}_k^2 + \frac{1}{ag^2}\sum_{k=-N}^{N-1}(\mathbb{1} - \hat{\mathbf{n}}_k \cdot \hat{\mathbf{n}}_{k+1}) + \frac{a\omega^2}{g^2}\sum_{k=-N}^{N}(\mathbb{1} + \hat{\mathbf{n}}_k \cdot \mathbf{e}_z),$$

$$[\hat{L}^\alpha, \hat{L}^\beta] = i\epsilon^{\alpha\beta\gamma}\hat{L}^\gamma, \qquad [\hat{L}^\alpha, \hat{n}^\beta] = i\epsilon^{\alpha\beta\gamma}\hat{n}^\gamma$$
$$[\hat{n}^\alpha, \hat{n}^\beta] = 0, \qquad \alpha, \beta, \gamma \in \{x, y, z\},$$

- One may work in the momentum representation where the state of each site is expanded by the eigenstates $|\ell_k m_k\rangle$ of the angular operators.

The 1+1D O(3) Model

- One may work in the momentum representation where the state of each site is expanded by the eigenstates $|\ell_k m_k\rangle$ of the angular operators $\hat{L}_k^2$ and $\hat{L}_k^z$.
- In principle, the momentum could be large. But limited by the computing resource, a truncation $\ell_{\max}$ of ℓ_k is necessary.
- $n_q = [\log_2(\ell_{\max}+1)^2]$ qubits are needed for each site to simulate the evolution with the "UV-cutoff".
- One may show that in the momentum representation, the "coordinate" operators are $\hat{\mathbf{n}}_k \cdot (\mathbf{e}_x \pm i\mathbf{e}_y) = \mp\sqrt{2}\hat{X}_{\pm 1}$ and $\hat{\mathbf{n}}_k \cdot \mathbf{e}_z = \hat{X}_0$, where

$$\langle l,m|\hat{X}_M|l',m'\rangle = (-1)^m\sqrt{(2l+1)(2l'+1)} \times \begin{pmatrix} l & l' & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l & 1 & l' \\ -m & M & m' \end{pmatrix}.$$

Two Strategies

- The first: the Background Field Method.

$$\hat{H}^{\mathrm{bg}}_{\mathrm{tot}} = -\frac{1}{ag^2}\sum_{k=-N}^{N-1}(\hat{\mathbf{n}}_k + \mathbf{n}_k(t))\cdot(\hat{\mathbf{n}}_{k+1} + \mathbf{n}_{k+1}(t))$$
$$+\frac{a\omega^2}{g^2}\sum_{k=-N}^{N}(\hat{\mathbf{n}}_k + \mathbf{n}_k(t))\cdot\mathbf{e}_z + \frac{g^2}{2a}\sum_{k=-N}^{N}\hat{\mathbf{L}}_k^2.$$

The quantum fluctuation

The classical background field

Two Strategies

- The second: mapping the classical configuration to the coherent state of the quantum field.

$$|\psi_{\mathrm{sph}}\rangle \equiv \bigoplus_{k=-N}^{N} \sqrt{\frac{4\pi}{\ell_{\max}+1}} \sum_{\ell=0}^{\ell_{\max}} \left[\sum_{m=-\ell}^{\ell} Y_{\ell m}(\theta_k, \varphi_k)^* \, |\ell m\rangle_k \right]$$

- The coherent state has the property that $\langle \psi_{\mathrm{sph}} | \hat{\mathbf{n}}_k | \psi_{\mathrm{sph}} \rangle = \mathbf{n}_{\mathrm{sph}}(x_k)$, and the kinetic term is certainly divergent due to the Heisenberg uncertainty principle.
- With such prepared initial state, one can simulate the time-evolution with the Trotterization scheme.

Two Strategies

- With such prepared initial state, one can simulate the time-evolution with the Trotterization scheme.

$$U(\Delta t) = \prod_{l=1}^{l=3} U_l(\Delta t) \prod_{l=3}^{l=0} U_l(\Delta t),$$

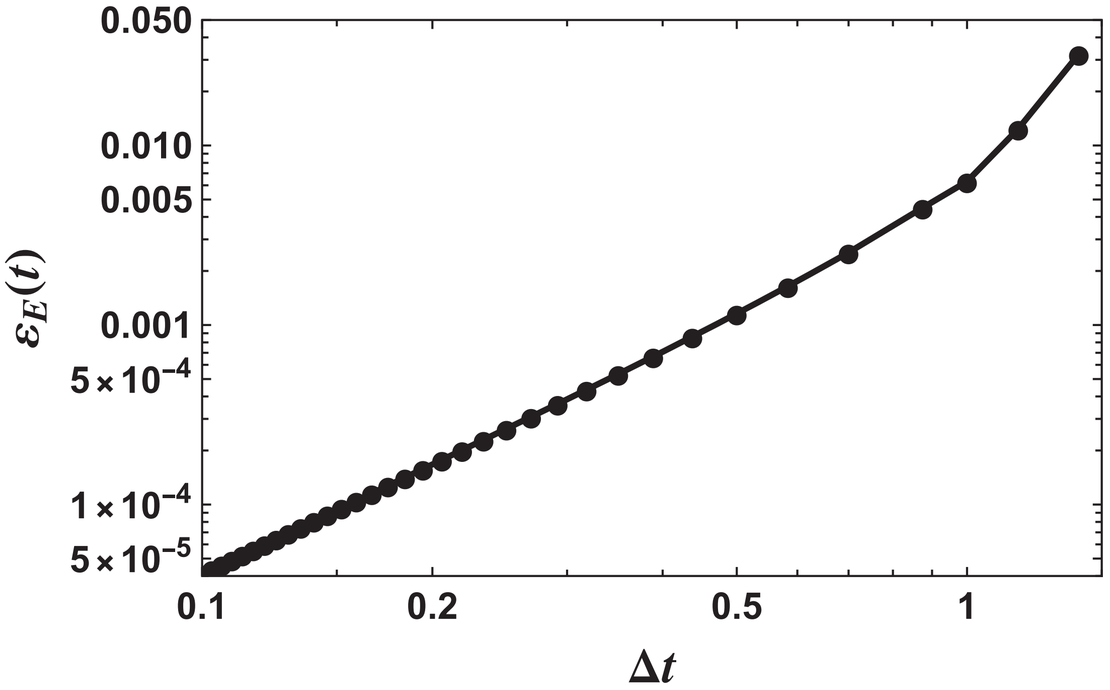
$$\hat{H}_1 = \frac{g^2}{2a} \sum_k \hat{\mathbf{L}}_k^2,$$

$$\hat{H}_2 = \frac{1}{ag^2} \sum_k (\mathbb{1} - \hat{\mathbf{n}}_k \cdot \hat{\mathbf{n}}_{k+1}), \qquad U_2(\Delta t) = e^{i\theta_1 \Delta t} \prod_k U_{k,k+1}(\Delta t),$$

$$\hat{H}_3 = \frac{a\omega^2}{g^2} \sum_k (\mathbb{1} + \hat{\mathbf{n}}_k \cdot \mathbf{e}_z), \qquad U_3(\Delta t) = e^{i\theta_2 \Delta t} \prod_k U_{k,z}(\Delta t).$$

Two Strategies

- It is certainly that 2N+1=5 is the simplest non-trivial one we can simulate.
- With $\ell_{\max} = 1$ ($n_q = 2$), the QISKIT transpiler yields a CNOT circuit depth $\mathcal{O}(3 \times 10^4)$!!!
- With a noiseless simulator, the error just from the Trotterization scheme is



Conclusion and Further Works

- We suggest use quantum computing technology to calculate the sphaleron processes from the first principle.
- A first step attempt of a (overly) simplified model is giving.
- Even for the simplified model, the kinetic terms in the Hamiltonian should be considered and a truncation of the high partial wave modes will be needed.
- To carry out the energy and the topological charge of the sphaleron, fermion modes are introducing in our next step.
- More general sphaleron configurations should be understood as coherent state of the QFT.
- Though 101 years has passed since the first QFT model is proposed, there are still a lot of unsolved mysteries in QFT!

Thank you!