



山东大学
SHANDONG UNIVERSITY

Approximate N^2LO and N^3LO QCD Predictions for tW Production

Phys. Rev. D 114, 013002

丁家乐
山东大学

In collaboration with Hai Tao Li and Jian Wang

中国物理学会高能物理分会第十二届全国会员代表大会暨学术年会

2026年7月13日-7月19日，广州

Introduction

- Among the various production modes of a single top quark, the associated production of a top quark with a W boson has the second-largest cross section at Large Hadron Collider (LHC).
- Inclusive and differential cross sections for tW production have been measured extensively.
- The cross section is directly proportional to the square of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{tb} .

Progress toward complete NNLO QCD correction for tW production:

- ✓ Two-loop hard function. [\[2106.12093\]](#)[\[2111.14172\]](#)[\[2211.13713\]](#)[\[2204.13500\]](#)[\[2208.08786\]](#)[\[2212.07190\]](#)
- ✓ Two-loop N -jettiness soft function. [\[1611.02749\]](#)[\[1804.06358\]](#)
- ✓ Two-loop threshold soft function. [\[2502.18648\]](#)

Introduction

The inclusive associated production of top quark and W boson at the LHC is

$$p(P_1) + p(P_2) \rightarrow t/\bar{t}(p_3) + W^-/W^+(p_4) + X(p_X).$$

The LO partonic scattering channel for this process is

$$b/\bar{b}(p_1) + g(p_2) \rightarrow t/\bar{t}(p_3) + W^-/W^+(p_4).$$

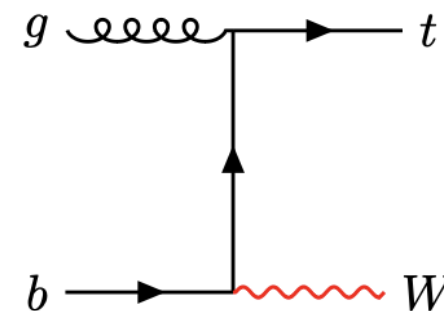
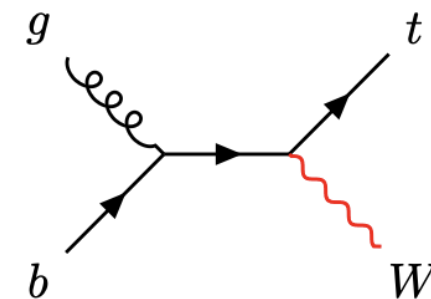
$$s = (P_1 + P_2)^2 \quad \text{hadronic energy of the collider}$$

$$\hat{s} = (p_1 + p_2)^2 \quad \text{partonic invariant mass}$$

$$Q^2 = (p_3 + p_4)^2 \quad tW \text{ invariant mass}$$

$$\tau = Q^2/s \quad \text{hadronic threshold variable}$$

$$z = Q^2/\hat{s} \quad \text{partonic threshold variable}$$

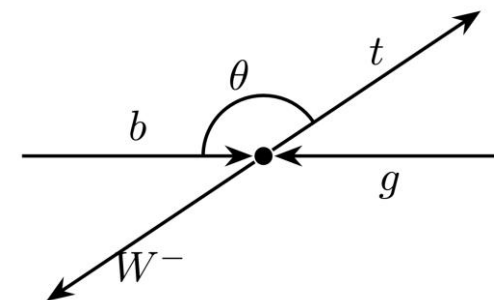


Introduction

In the hadronic threshold limit, the cross section takes a factorization form:

$$\frac{d\sigma}{dQ^2 d\Phi_2} = \frac{1}{s} \int_{\tau}^1 \frac{dz}{z} \mathcal{L}\left(\frac{\tau}{z}, \mu\right) \frac{1}{2Q^2} H(\beta_t, y, \mu) \mathcal{S}(1-z, \beta_t, y, \mu)$$

$$\mathcal{L}\left(\frac{\tau}{z}, \mu\right) = \int_{\tau/z}^1 \frac{dx}{x} \left[f_b(x, \mu) f_g\left(\frac{\tau}{xz}, \mu\right) + f_g(x, \mu) f_b\left(\frac{\tau}{xz}, \mu\right) \right].$$



H : hard function

β_t : the velocity of the top quark

$\mathcal{S}$: soft function

θ : the polar angle of the momentum of the top quark

f_i : parton distribution functions

$y = \cos\theta$

Fixed-order

Performing the Laplace transformation, which converts the cross section to a product of the PDFs, hard function and soft function:

$$\frac{2Q^2 s d\tilde{\sigma}}{dQ^2 d\Phi_2} = \tilde{\mathcal{L}}(t, \mu_f) H(\mu_f, \beta_t, \gamma) \tilde{\mathcal{S}}(t, \mu_f, \beta_t, \gamma).$$

The approximate N³LO results for hard and soft functions are obtained by solving the Renormalization Group (RG) evolution equations:

$$\frac{d\tilde{\mathcal{S}}}{d\ln\mu} = \gamma_s \tilde{\mathcal{S}}, \quad \frac{dH}{d\ln\mu} = \gamma_h H.$$

The RG evolution of the PDFs in the threshold limit is diagonal

$$\frac{d\tilde{f}_i(t, \mu)}{d\ln\mu} = (2\mathbf{T}_i^2 \gamma_{\text{cusp}} \ln t + 2\gamma_f^i) \tilde{f}_i(t, \mu).$$

Fixed-order



The soft anomalous dimension is

$$\begin{aligned}\gamma_s = & -(\mathbf{T}_1^2 + \mathbf{T}_2^2) \gamma_{\text{cusp}} \ln \left(\frac{\hat{s} t^2}{\mu^2} \right) + \mathbf{T}_1 \cdot \mathbf{T}_3 \gamma_{\text{cusp}} \ln \frac{(1 - \beta_t y)^2}{1 - \beta_t^2} + \mathbf{T}_2 \cdot \mathbf{T}_3 \gamma_{\text{cusp}} \ln \frac{(1 + \beta_t y)^2}{1 - \beta_t^2} \\ & - 2\gamma^q - 2\gamma^g - 2\gamma^Q - 2\gamma_f^q - 2\gamma_f^g - 2\mathcal{T}_{1233} \left(\frac{\alpha_s}{4\pi} \right)^3 \mathcal{F}_{h2} \left(\frac{1 - \beta_t^2}{1 - \beta_t^2 y^2} \right).\end{aligned}$$

$$\mathcal{T}_{1233} = \frac{1}{2} f^{ade} f^{bce} \mathbf{T}_1^a \mathbf{T}_2^b (\mathbf{T}_3^c \mathbf{T}_3^d + \mathbf{T}_3^d \mathbf{T}_3^c)$$

The total physical cross section must be independent of the factorization scale μ :

$$\frac{d \ln H}{d \ln \mu} + \frac{d \ln \tilde{f}_q}{d \ln \mu} + \frac{d \ln \tilde{f}_g}{d \ln \mu} + \frac{d \ln \tilde{S}}{d \ln \mu} = 0,$$

and hard anomalous dimension can be driven as

$$\begin{aligned}\gamma_h = & (\mathbf{T}_1^2 + \mathbf{T}_2^2) \gamma_{\text{cusp}} \ln \frac{\hat{s}}{\mu^2} - \mathbf{T}_1 \cdot \mathbf{T}_3 \gamma_{\text{cusp}} \ln \frac{(1 - \beta_t y)^2}{1 - \beta_t^2} - \mathbf{T}_2 \cdot \mathbf{T}_3 \gamma_{\text{cusp}} \ln \frac{(1 + \beta_t y)^2}{1 - \beta_t^2} \\ & + 2\gamma^q + 2\gamma^g + 2\gamma^Q + 2\mathcal{T}_{1233} \left(\frac{\alpha_s}{4\pi} \right)^3 \mathcal{F}_{h2} \left(\frac{1 - \beta_t^2}{1 - \beta_t^2 y^2} \right).\end{aligned}$$

Fixed-order



The cross section in momentum space can be written as

$$\frac{d\sigma}{dQ^2 d\Phi_2} = \frac{1}{s} \int_{\tau}^1 \frac{dz}{z} \mathcal{L} \left(\frac{\tau}{z}, \mu \right) \frac{1}{2Q^2} H(\mu, \beta_t, y) \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \\ \times \tilde{\mathcal{S}} \left(\ln \frac{Q^2(1-z)^2}{z\mu^2} + \partial_{\eta}, \beta_t, y \right) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \Big|_{\eta=0}$$

The fixed-order partonic cross section in the threshold limit contains a series of logarithmically enhanced terms arising from soft and collinear gluon radiation, which can be written as

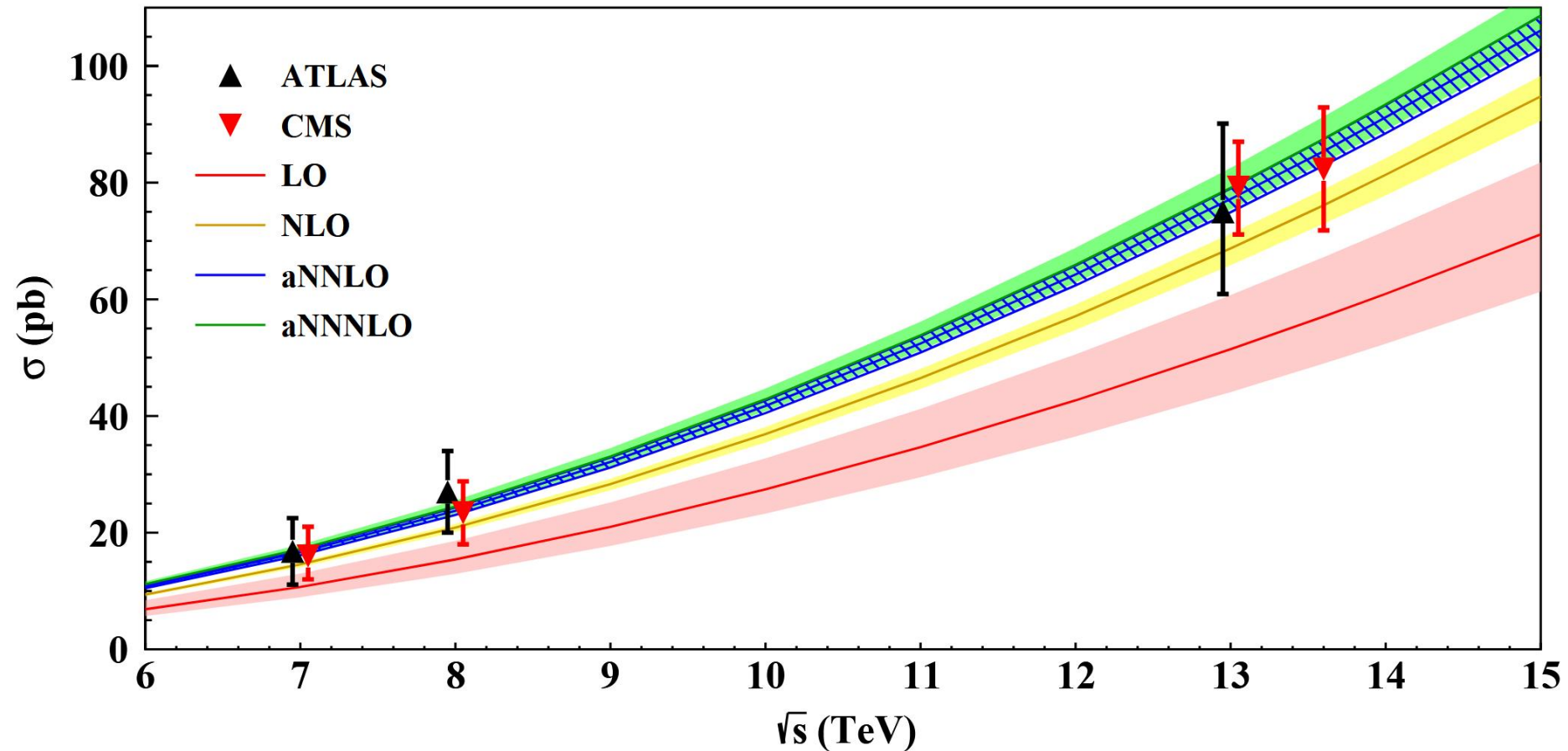
$$\frac{d\hat{\sigma}}{dQ^2 d\Phi_2} = \frac{d\hat{\sigma}_{\text{LO}}}{dQ^2 d\Phi_2} \left\{ \delta(1-z) + \frac{\alpha_s(\mu_r)}{4\pi} \left(C_{1,1}P_1 + C_{1,0}P_0 + C_{1,-1}\delta(1-z) \right) \right. \\ \left. + \left(\frac{\alpha_s(\mu_r)}{4\pi} \right)^2 \left(C_{2,3}P_3 + C_{2,2}P_2 + C_{2,1}P_1 + \dots \right) \right. \\ \left. + \left(\frac{\alpha_s(\mu_r)}{4\pi} \right)^3 \left(C_{3,5}P_5 + C_{3,4}P_4 + C_{3,3}P_3 + \dots \right) + \mathcal{O} \left[\left(\frac{\alpha_s(\mu_r)}{4\pi} \right)^4 \right] \right\} \\ P_n = \left[\frac{1}{1-z} \ln^n \frac{(1-z)^2 Q^2}{z\mu_f^2} \right]_+$$

Fixed-order

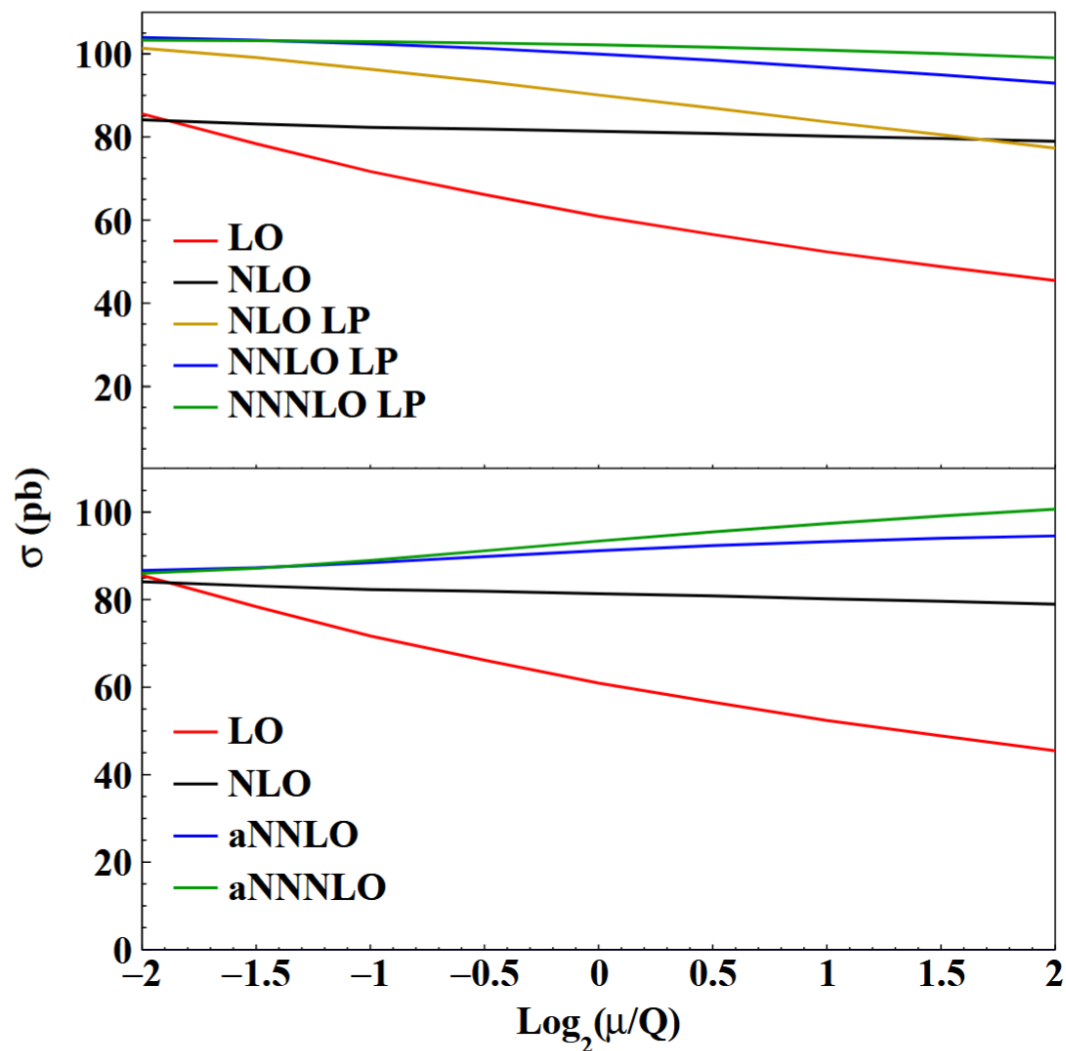


The approximate N^n LO fixed-order cross section are obtained by matching it with full NLO corrections:

$$d\sigma(\text{aN}^n\text{LO}) = d\sigma(\text{N}^n\text{LO}_{\text{LP}}) + d\sigma(\text{NLO}) - d\sigma(\text{NLO}_{\text{LP}}).$$



Fixed-order



$$\mu = \mu_f = \mu_r$$

$$d\sigma(\text{aN}^n\text{LO}) = d\sigma(\text{N}^n\text{LO}_{\text{LP}}) + d\sigma(\text{NLO}) - d\sigma(\text{NLO}_{\text{LP}})$$

$$d\sigma(\text{N}^n\text{LO}_{\text{LP}}) \quad \text{Scale uncertainty} \sim O(\alpha_s^{n+1})$$

$$d\sigma(\text{NLO}_{\text{LP}}) \quad \text{Scale uncertainty} \sim O(\alpha_s^2)$$

$$d\sigma(\text{NLO}) \quad \text{Scale uncertainty} \ll O(\alpha_s^2)$$

$$gb \text{ channel's scale uncertainty} \sim O(\alpha_s^2)$$

cancel with

$$gg \text{ and } d\bar{d} \text{ channels' scale uncertainty} \sim O(\alpha_s^2)$$

V_{tb}

$$|V_{tb}| = \sqrt{\frac{\sigma_{\text{meas.}}}{\sigma_{\text{theo.}}}}$$

$\sigma(\text{pb})$	7 TeV	8 TeV	13 TeV	13.6 TeV
ATLAS	$16.8^{+5.7}_{-5.7}$	27^{+7}_{-7}	$75^{+15.1}_{-14.1}$	-
CMS	16.0^{+5}_{-4}	$23.4^{+5.4}_{-5.4}$	$79.2^{+7.8}_{-8.1}$	$82.3^{+10.6}_{-10.5}$
$\sigma_{\text{theo.}}$	$17.2^{+0.7}_{-0.9}(\text{scale}) \pm 0.5(\text{PDF})$	$24.5^{+1.1}_{-1.2} \pm 0.7$	$79.1^{+3.4}_{-3.7} \pm 1.6$	$87.5^{+3.8}_{-4.1} \pm 1.7$
$ V_{tb} $	7 TeV	8 TeV	13 TeV	13.6 TeV
ATLAS	$0.99^{+0.16}_{-0.18}(\text{meas.})^{+0.03}_{-0.02}(\text{theo.})$	$1.05^{+0.13+0.03}_{-0.15-0.03}$	$0.97^{+0.09+0.03}_{-0.10-0.02}$	-
CMS	$0.96^{+0.14+0.03}_{-0.13-0.02}$	$0.98^{+0.11+0.03}_{-0.12-0.03}$	$1.00^{+0.05+0.03}_{-0.05-0.03}$	$0.97^{+0.06+0.03}_{-0.06-0.02}$

The processes induced by a strange or down quark, $gs(gd) \rightarrow tW$, are highly suppressed due to the tiny values of the CKM matrix elements V_{ts} ($\sim 4 \times 10^{-2}$) and V_{td} ($\sim 9 \times 10^{-3}$).

Consequently, we obtain $|V_{tb}| = 0.99 \pm 0.03(\text{exp.}) \pm 0.03(\text{theo.})$, which agrees with the world average value with comparable accuracy.

Summary and Outlook

- Calculated the approximate NNNLO cross section for tW production at LHC.
- Compared the approximate fixed-order predictions with measured cross sections.
- Analyzed the main source of scale uncertainty in approximate fixed-order predictions.
- Determining the CKM matrix element $|V_{tb}| = 0.99 \pm 0.03(\text{exp.}) \pm 0.03(\text{theo.})$ directly without assuming unitarity.
- Complete $N^2\text{LO}$ calculation will be essential to further reduce theoretical uncertainties.

Thanks for your attention!

