Spin Entanglement of Fermion Pair Near Threshold

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Content

- History of entanglement
- Theoretical calculation
- Spin entanglement of toponium
- Spin entanglement of tau pair
- Summary

- **1935,EPR Paradox**, Einstein, Podolsky & Rosen, challenge of quantum mechanics' completeness (locality and realism);

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

- **same year, Entanglement** Named. Schrödinger coined "Verschränkung" (entanglement) to respond for EPR paradox

1949 年，吴健雄和 Shaknov 基于正负电子湮没所产生的纠缠光子，通过测量散射后的角关联，首次实现了 EPR 关联

- **1964, Bell Theorem**, John Bell proved local hidden-variable theories and classical mechanics obey Bell inequalities, which quant

local hidden variable λ

$$1 + P(\vec{b}, \vec{c}) \geq |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})|$$

$$P(a,b) \equiv \int_\Gamma A(a,\lambda)B(b,\lambda)\rho(\lambda)d\lambda$$

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(Received 4 November 1964)

- **1969,CHSH Inequality**,John F. Clauser & Michael A. Horne & Abner Shimony & Richard A. Holt derivated the inequality that classical biqubits systems satisfy, which can be violated by quantum entanglement state(famous Alice & Bob game in two-qubit system)

PROPOSED EXPERIMENT TO TEST LOCAL HIDDEN-VARIABLE THEORIES*

John F. Clauser†
Department of Physics, Columbia University, New York, New York 10027

and

Michael A. Horne
Department of Physics, Boston University, Boston, Massachusetts 02215

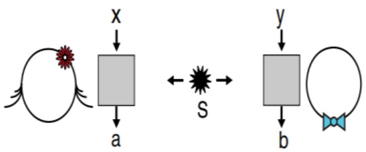
and

Abner Shimony
Departments of Philosophy and Physics, Boston University, Boston, Massachusetts 02215

and

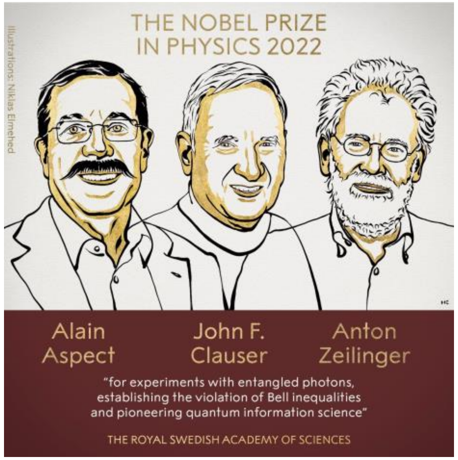
Richard A. Holt
Department of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 4 August 1969)

$$|P(a,b)-P(a,b')|+P(a',b)+P(a',b')\leq 2$$



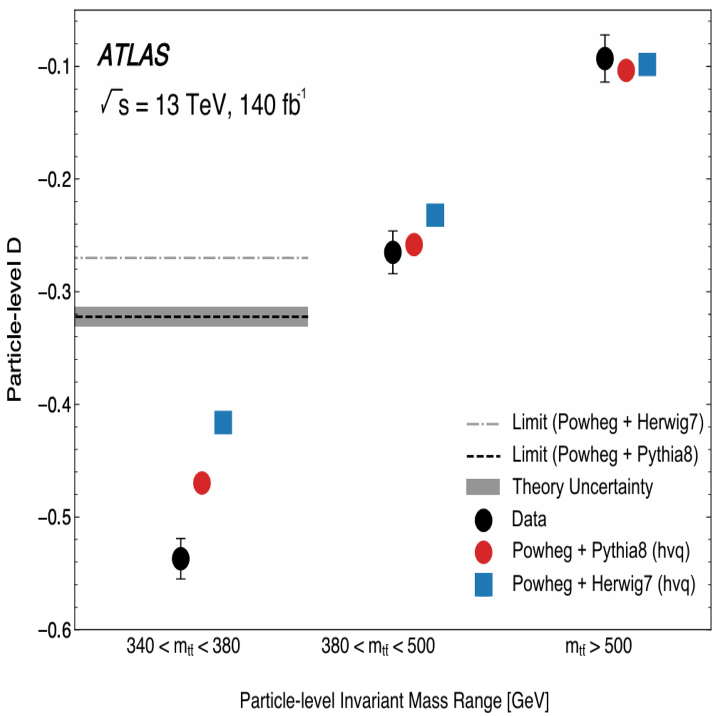
- **1970s-1990s, Bell Inequality violation Experiments**. Several Bell inequality violation measured experiments falsify the local realism of quantum mechanic

- **2022,Nobel Prize in Physics**, Alain Aspect & John F. Clauser & Anton Zeilinger, "For experiments with entangled photons, establishing Bell inequality violations, and pioneering quantum information science



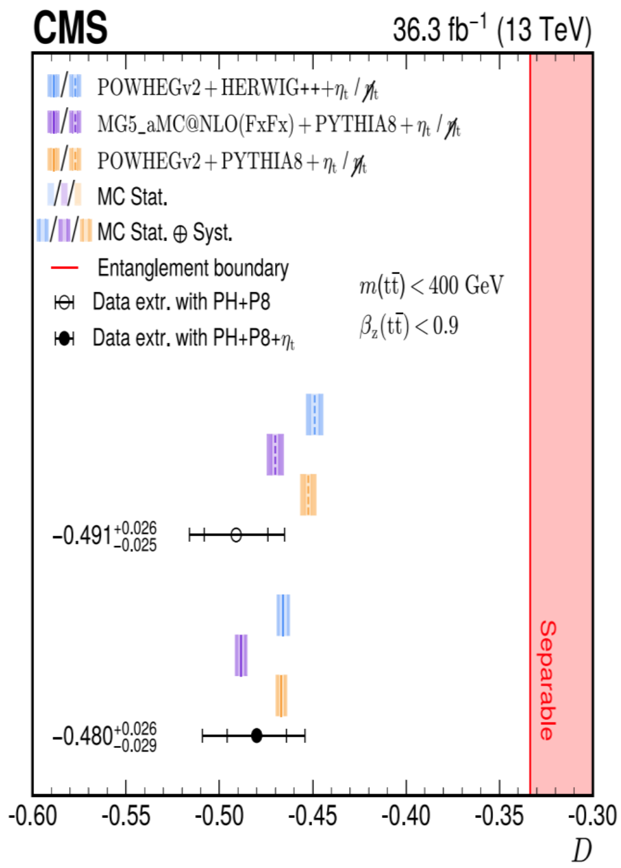
Measurement

ATLAS,2311.07288 2 Oct 2024



$$D = -0.537 \pm 0.002\ (\text{stat.}) \pm 0.019\ (\text{syst.})\ \text{for}\ 340 < m_{t\bar{t}} < 380\ \text{GeV}$$

CMS,2406.0397 29 Oct 2024



The density matrix of the spin correlation of the biqubit (spin 1/2 particles A and B) system

$$\rho = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

$$= \frac{1}{4}\left(\mathbf{I}_4 + \sum_i B_i^A \sigma_i \otimes \mathbf{I}_2 + \sum_i B_i^B \mathbf{I}_2 \otimes \sigma_i + \sum_{ij} C_{ij} \sigma_i \otimes \sigma_j\right)$$

the spin correlation matrix between subsystems A and B

net polarizations of spin A and spin B

concurrence

$$\mathcal{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

entanglement : $\mathcal{C} > 0$ (maximum 1)

Phys. Rev. Lett. 80, 2245

λ_i are the eigenvalues of operator $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$

($\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$) with $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ in order

非纠缠纯态 $|\psi\rangle = \frac{1}{2}(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$ $\quad \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \{0,0,0,0\}$

$$|\psi\rangle = \left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}\right)_A \otimes \left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}\right)_B \qquad C(\psi) = |\langle\psi|\tilde{\psi}\rangle| = 0$$

纠缠纯态 $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ $\quad \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \{1,0,0,0\}$

$$C(\psi^-) = |\langle\psi^-|\tilde{\psi}^-\rangle| = 1$$

Spin Entanglement Analysis on $t\bar{t}$ Production Progress

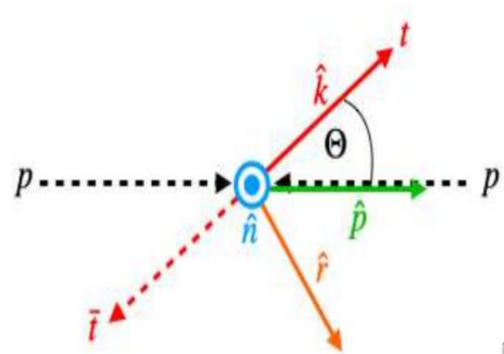
t lifetime $\sim 10^{-25}$ s

QCD hadronization time scale $1/\Lambda_{QCD} \approx 10^{-24}$ s

Spin decorrelation time scale $m_t/\Lambda_{QCD}^2 \approx 10^{-21}$s

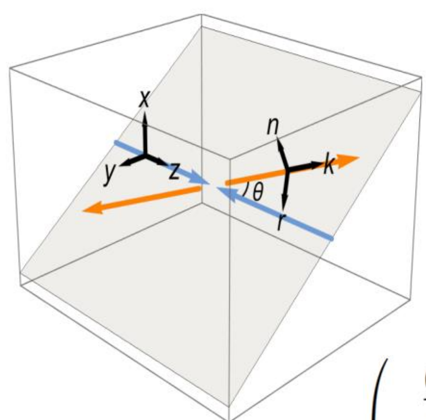
Two orthogonal bases

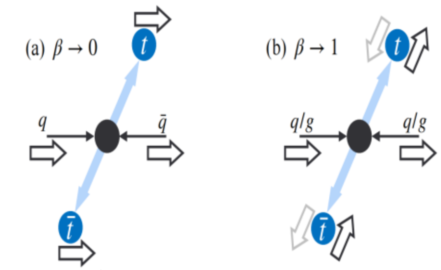
Helicity basis



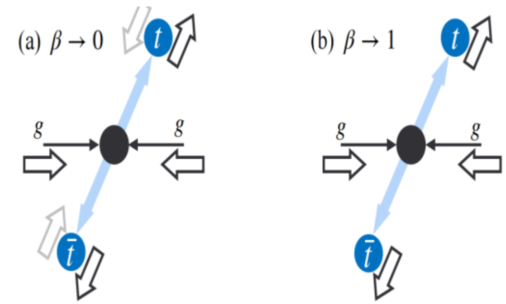
Transformation relation:

$$\hat{r} = \frac{1}{\sin\theta}(\hat{z} - \cos\theta \hat{k}),$$
$$\hat{n} = \hat{r} \times \hat{k},$$
$$\cos\theta = \hat{k} \cdot \hat{z}.$$



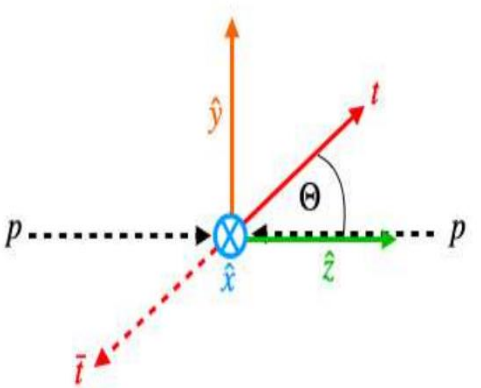


spin-triplet



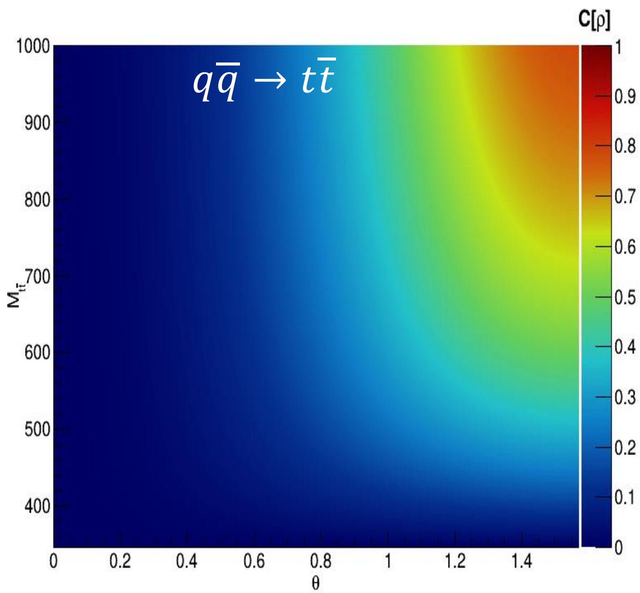
spin-singlet

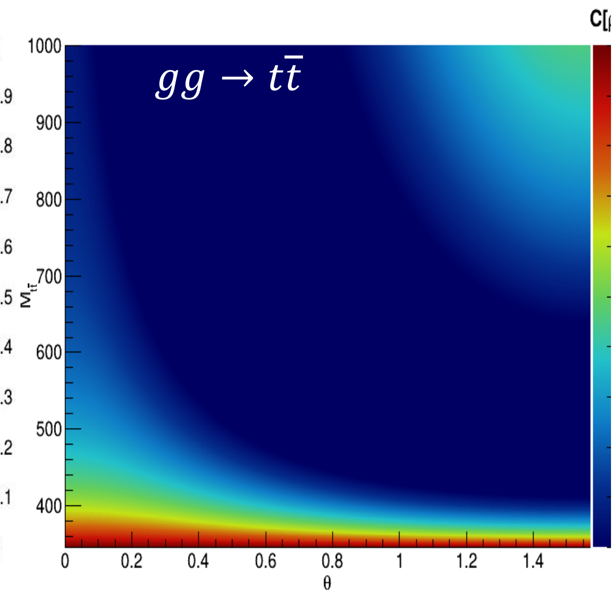
Beamline basis



$$C_{ij}^{q\bar{q}/g_L g_R} = \begin{pmatrix} \frac{(2-\beta^2)s_\theta^2}{2-\beta^2 s_\theta^2} & 0 & -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} \\ 0 & \frac{-\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} & 0 \\ -\frac{2c_\theta s_\theta\sqrt{1-\beta^2}}{2-\beta^2 s_\theta^2} & 0 & \frac{2c_\theta^2+\beta^2 s_\theta^2}{2-\beta^2 s_\theta^2} \end{pmatrix}
\qquad
C_{ij}^{g_L g_L/g_R g_R} = \begin{pmatrix} \frac{\beta^2-1}{\beta^2+1} & 0 & 0 \\ 0 & \frac{\beta^2-1}{\beta^2+1} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Spin Entanglement Analysis on $t\bar{t}$ Production Progress





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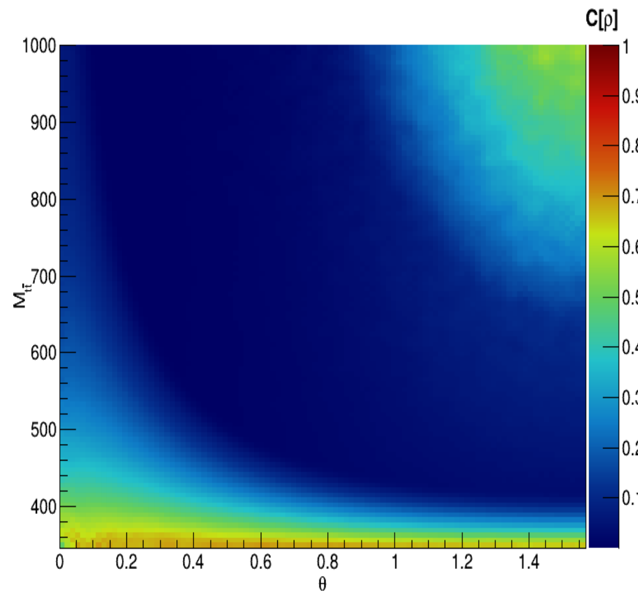
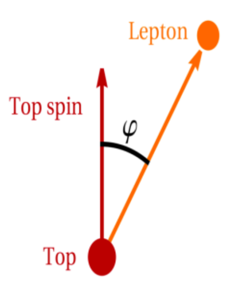
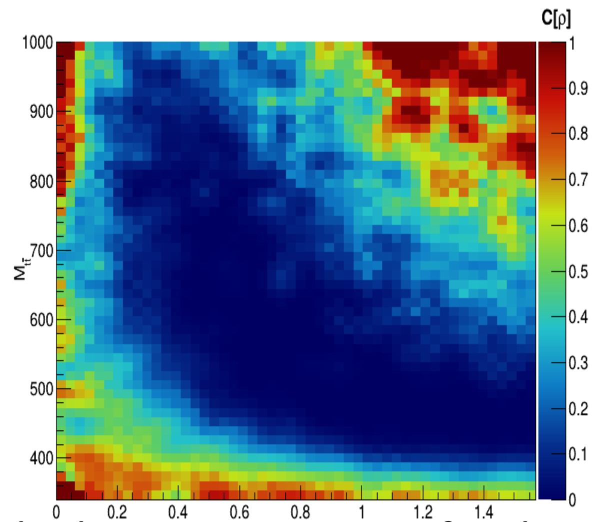






$$\rho^{t\bar{t}} = \omega^{q\bar{q}} \rho^{q\bar{q} \to t\bar{t}} + \omega^{gg} \rho^{gg \to t\bar{t}}.$$

Entanglement signals appear in threshold region and high-p_T region

Correlations between the leptons as a proxy for those between the top quark spins

Spin Entanglement of $t\bar{t}$ Bound State

Toponium is the theoretical bound state of a top and anti-top quark .
A peak slightly less than 2
Dominated by $gg \to S_0^{[1]}$($S_0^{[1]}$ is color-singlet state) progress.

$$\left[-\frac{\nabla^2}{m_t} + V_s(r) - E\right] G^S(\vec{r}, \vec{r}\,'; E) = \delta^3(\vec{r} - \vec{r}\,')$$
$$E = \sqrt{s} - 2m_t$$

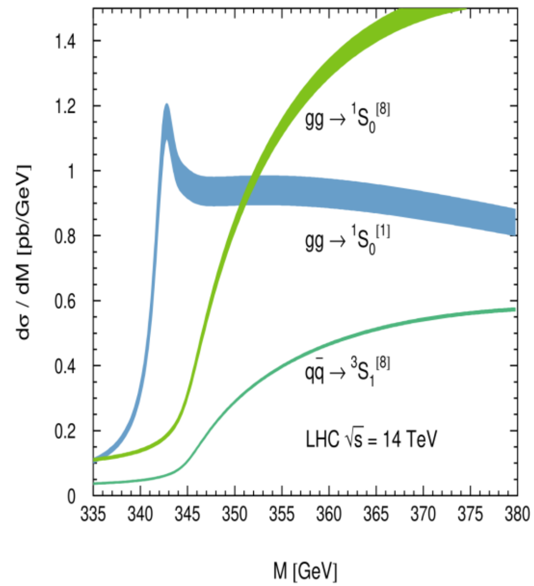
Elusive romance of top-quark pairs observed at the LHC

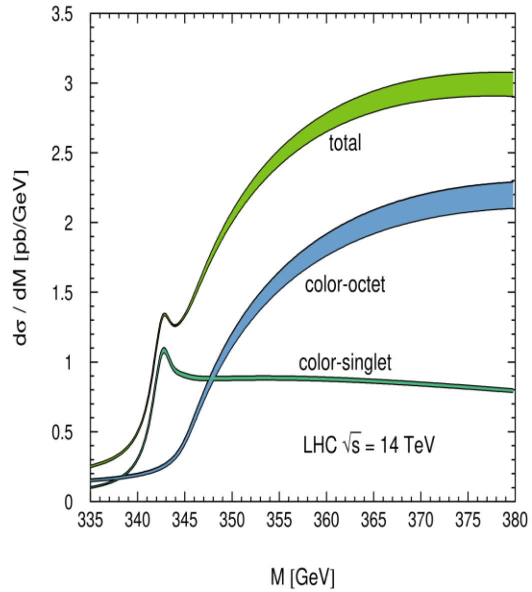
The CMS and ATLAS experiments at CERN's Large Hadron Collider have observed an unforeseen feature in the behaviour of top quarks that suggests that these heaviest of all elementary particles form a fleeting union

8 JULY, 2025

Bound to be discovered? ATLAS explores top-quark interactions near threshold

7 July 2025 | By ATLAS Collaboration

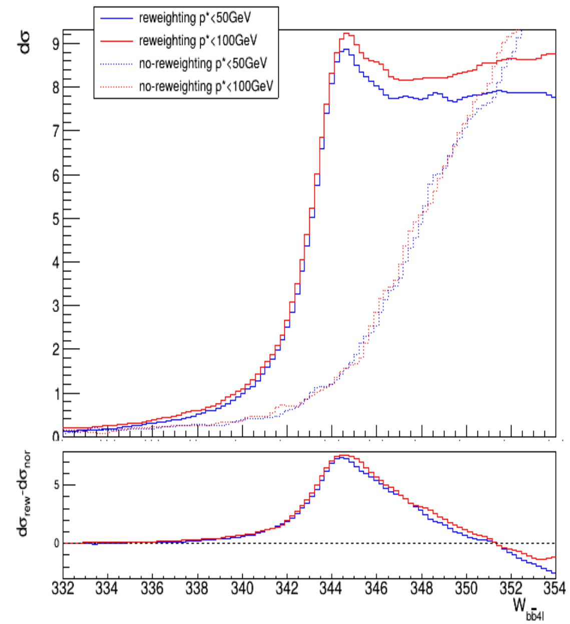


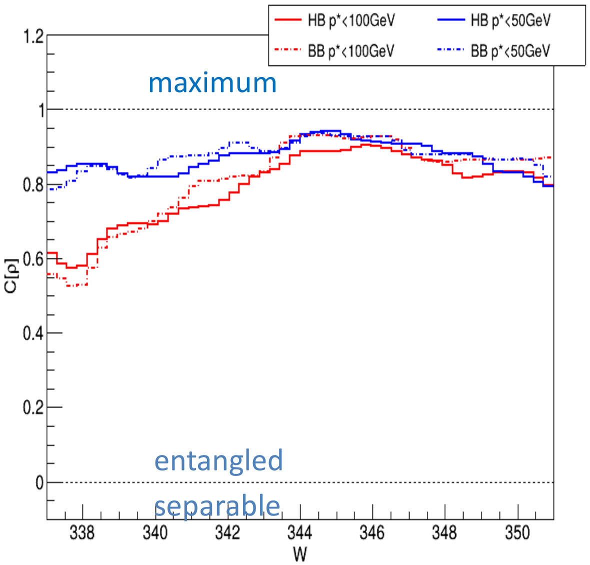


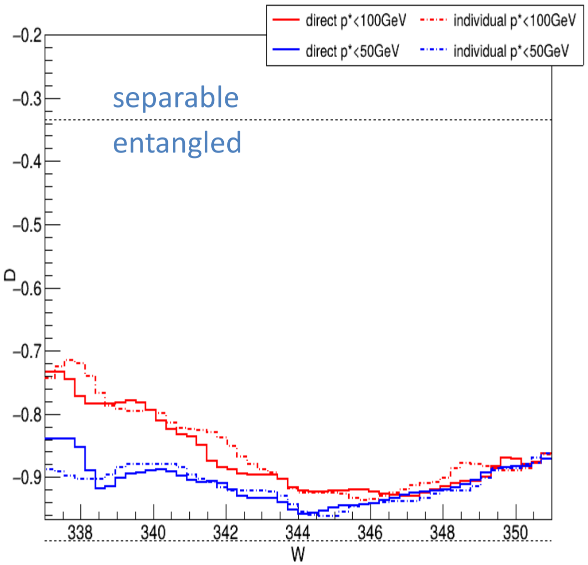
Spin Entanglement of $t\bar{t}$ Bound State

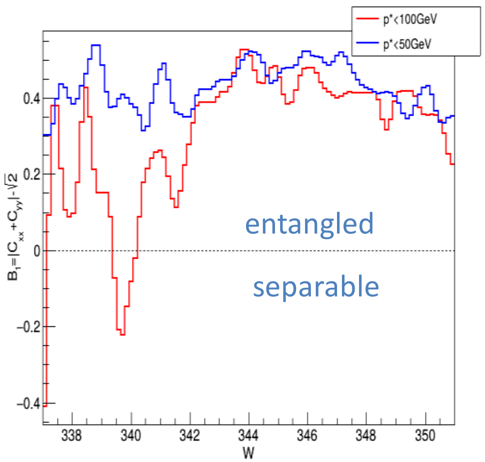
The toponium can be simulated by utilizing Green's function

Benjamin Fuks, Kaoru Hagiwara, Kai Ma, and Ya-Juan Zheng. Simulating toponium formation signals at the LHC. The European Physical Journal C, 85(2), February 2025.









- Toponium region exists strongly correlated signals.

Spin entanglement of tau pair $\psi(2S) \to \tau^+\tau^-,\ \tau^+ \to \pi^+\bar{\nu}_\tau,\ \tau^- \to \pi^-\nu_\tau$

$$\rho = \frac{1}{4}\left(I_2 \otimes I_2 + \sum_i B_i^+ \sigma_i \otimes I_2 + \sum_j B_j^- I_2 \otimes \sigma_j + \sum_{ij} C_{ij}\, \sigma_i \otimes \sigma_j\right)$$

$$\mathcal{C}(\rho) = \frac{1}{2}(C_{11} + C_{33} - C_{22} - 1)$$

1. Kinematic Method

$$C_{ij} = \frac{1}{2-\beta^2 \sin^2\theta}\begin{pmatrix} (2-\beta^2)\ \sin^2\theta & 0 & \sqrt{1-\beta^2}\sin(2\theta) \\ 0 & -\beta^2\sin^2\theta & 0 \\ \sqrt{1-\beta^2}\sin(2\theta) & 0 & \beta^2 + (2-\beta^2)\cos^2\theta \end{pmatrix}$$

β : The speed of the τ^- lepton in the center-of-mass frame of the e^+e^- collision

θ : τ^- Scattering angle

2. Decay Method

$$C_{ij} = \frac{9\langle \cos\theta_{A,i} \cos\theta_{B,j}\rangle}{\kappa_A \kappa_B}$$

$\theta_{A,i}$: Angle of decay product A with the i-axis

κ_A : Spin analyzing power

Solving for the Four-Momentum of the τ Lepton

In the rest frame of the $\psi(2S)$ particle :

$$E_{\tau^+} = E_{\tau^-} = \frac{m_\psi}{2}$$

$$|\vec{p}_{\tau^+}| = |\vec{p}_{\tau^-}| = \sqrt{(\frac{m_\psi}{2})^2 - m_\tau^2}$$

For the decay $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$,four-momentum conservation gives:

$$p_{\tau^+} = p_{\pi^+} + p_{\bar{\nu}_\tau}$$

Squaring both sides and using the massless neutrino property

$$\vec{p}_{\tau^+} \cdot \vec{p}_{\pi^+} = E_{\tau^+} E_{\pi^+} - \frac{m_\tau^2 + m_\pi^2}{2}$$

$$\cos\theta^+ = \frac{E_{\tau^+} E_{\pi^+} - \frac{m_\tau^2 + m_\pi^2}{2}}{|\vec{p}_{\tau^+}||\vec{p}_{\pi^+}|}$$

A similar derivation for the other tau gives:

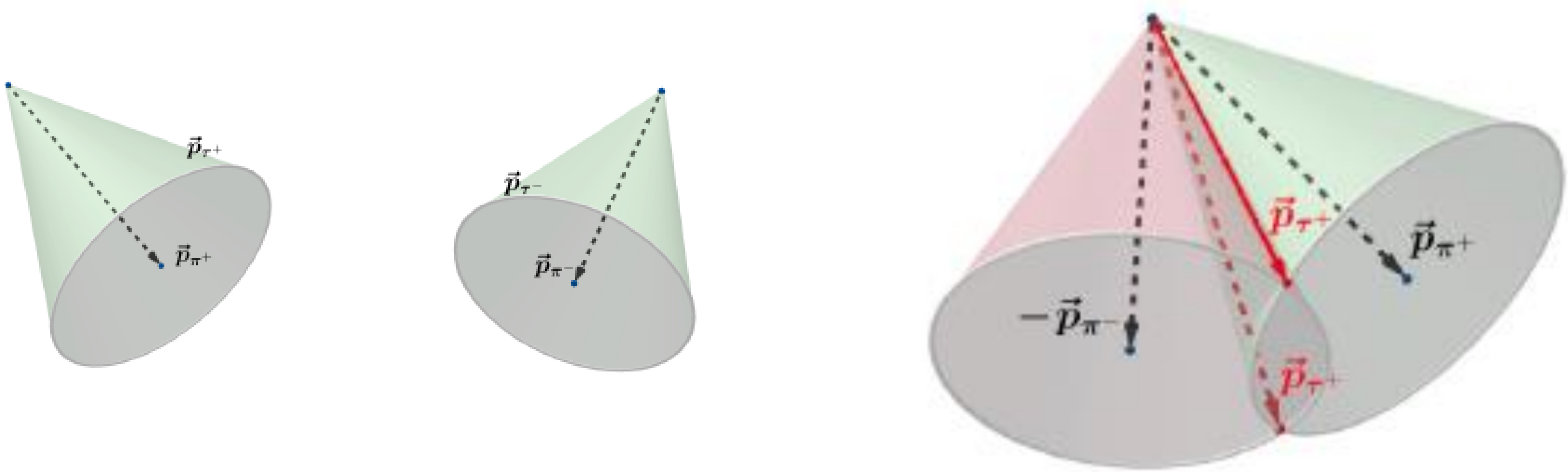
$$\cos\theta^- = \frac{E_{\tau^-} E_{\pi^-} - \frac{m_\tau^2 + m_\pi^2}{2}}{|\vec{p}_{\tau^-}||\vec{p}_{\pi^-}|}$$

Solving for the Four-Momentum of the τ Lepton

Geometric constraints $\vec{p}_{\tau^+} + \vec{p}_{\tau^-} = 0$

Green cone： The angle between $\vec{p}_{\pi^+}$ and $\vec{p}_{\tau^+}$

Red cone: The angle between $-\vec{p}_{\pi^-}$ and $\vec{p}_{\tau^+}$



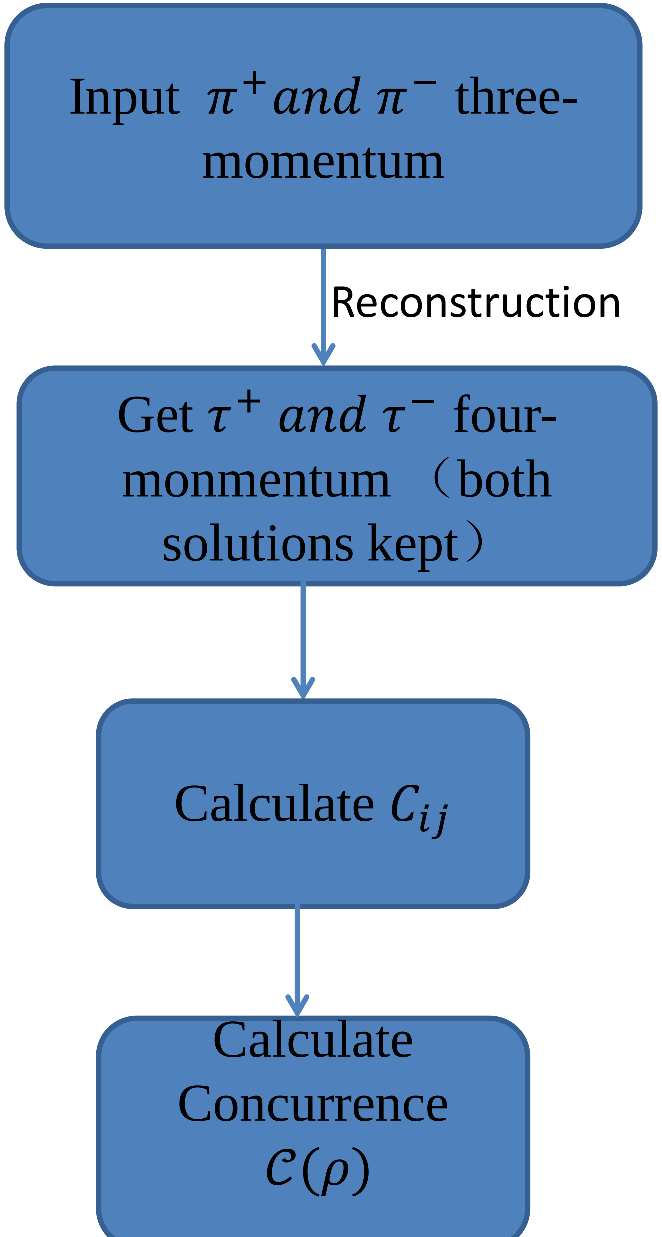
The intersection line of the two cones represents the two directions of τ

$$\alpha^2|v|^2 + 2\alpha(u_0 \cdot v) + (|u_0|^2 - 1) = 0$$

Two fold ambiguity

Possible solutions: 1.New detectors in adopting Decay Method;

2. false solution can give correct concurrence in Kinematic Method



Data Type	$\mathcal{C}(\rho)$
Signal MC	0.023803

Summary

- QFT is required to be local gauge invariant
- Entanglement is nonlocal
- Spin entanglement of toponium, CEPC
- Spin entanglement of tau pair, BESIII

Thank you