The Fate of Chiral Gauge Theories

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Based on

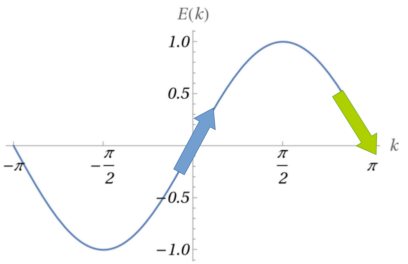
HHL, A. Pastor-Gutieruz, S.Vatani, L.-X. Xu JHEP 12 (2025) 020

HHL, A. Pastor-Gutieruz, S.Vatani, arXiv:2603.19355

Why Chiral Gauge Theories

- Appealing UV completion
  Asymptotic free, scales generated dynamically, conjectured rich IR dynamics,
  -- Original SU(5) GUT
  [Georgi,Glashow'74 Raby,Dimopoulos,Susskind '79 Bars,Yankielowitz'81]
  -- Composite Higgs models
  [Cacciapaglia,Sannino 1508.00016, Cacciapaglia, Sannino, Wagner 2605.08294]

- Chiral fermions are wired
  Nielsen-Ninomiya no-go theorem,
  fermion doubling problem when putting on lattice
  [Nielsen,Ninomiya'80'81]



- SM is a chiral gauge theory, no non-perturbative formulation yet

- Confinement w/o dSB → Symmetric Mass generation → Chiral Gauge Theory on Lattice
  [D. Tong 2104.03997, Wang, You 2204.14271]

- Crucial to study the phase of CGTs:
  Conformal? Confinement? Dynamical Symmetry Breaking?

Which model

- Benchmark models $SU(N_c)$ with two kinds of left-handed fermions

$$S = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \psi^\dagger \bar{\sigma}^\mu D_\mu \psi + \chi^\dagger \bar{\sigma}^\mu D_\mu \chi$$

- Gauge anomaly free restricts the flavor

Georgi-Glashow

	$SU(N_c)$	Flavor
ψ	$\overline{\Box}$	$N_c - 4$
χ	$\begin{array}{c}\Box\\[-4pt]\Box\end{array}$	1

$\times N_f$

Bars-Yankielowicz

	$SU(N_c)$	Flavor
ψ	$\overline{\Box}$	$N_c + 4$
χ	$\Box\Box$	1

Global Symmetry

$SU(N_c - 4) \times U(1)$ JHEP 12 (2025) 020

This talk

$SU(N_c + 4) \times U(1)$ 2603.19355

State of Art Analysis

- Most Attractive Channel → $<\chi\psi>$ condensate for $N_c = 5$ and $N_c \geq 7$

  [Raby,Dimopoulos,Susskind ’79, Dimopoulos,Raby,Susskind ’80, Eichten,Feinberg ‘82]

- Large Nc analysis → If confinement, no dSB

  [Eichten,Peccei,Preskill,Zeppenfeld’86 Karasik,Önder,Tong 2208.07842]]

- Anomaly mediated SUSY breaking → conformal for $N_c \leq 13$

  [Csáki,Murayama,Telem 2105.03444]

- Anomaly matching of higher forms symmetry → $<\chi\chi>, <\chi\psi>$ color-flavor locked phase

  [Bolognesi,Konishi,Luzio[2101.02601]]

State of Art Analysis

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- Large Nc analysis → If confinement, no dSB

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- Anomaly

  [Csáki,Murayan

- Anomaly or locked phase

  [Bolognesi,Kon

In conclusion:
No good understanding of the
IR dynamics of chiral gauge theory

The tool: Functional renormalization group method

- Functional Renormalization Group:
  a non-perturbative method widely used in QCD

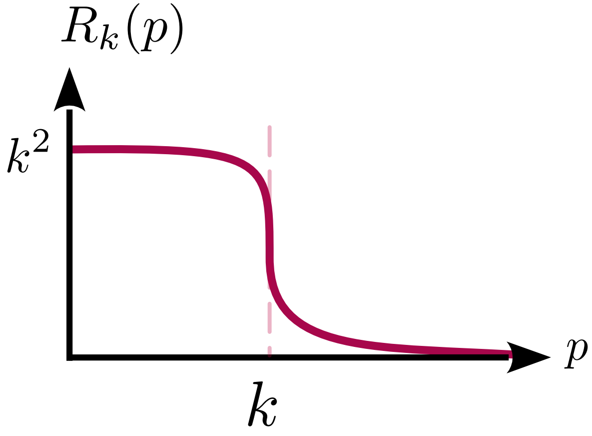
See Dupuis,et.al [2006.04853] for a reivew

- Basic idea: effective average action: $\Gamma_k[\phi]$

$$\int [\mathcal{D}\phi]_{p>k} = \int \mathcal{D}\phi \, \exp\left(-\Delta S_k[\phi]\right) \qquad \Delta S_k[\phi] = \int_p \phi(p) R_k \phi(-p)$$

$$\Gamma_k[\phi] = \int_x J(x)\phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi]$$

IR cutoff scale: $k \sim T \sim \langle p \rangle$



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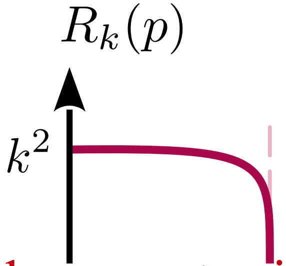


Chiral symmetry intact by fermion regulator

$$R_{\psi/\chi,\,k}(p^2) = \mathrm{i}\, Z_{\psi/\chi}\, \bar{\sigma}_\mu p_\mu \, r_{\psi/\chi}(x)$$

$$r_{\psi/\chi}(x) = \left(1/\sqrt{x} - 1\right) \theta\left(1 - x\right)$$

The tool: Functional renormalization group method

Master formula:

Wetterich '93

$$\boxed{\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k}\partial_t R_k\right]}$$

$\partial_t \equiv k\partial_k$

$$\partial_t \Gamma^{(1)}{}_k = -\frac{1}{2}\mathrm{STr}\ \Gamma^{(3)}{}_k \ \frac{\partial_t R_k}{\Gamma^{(2)} + R_k}$$

Both UV and IR finite

$$\partial_t \Gamma^{(2)}{}_k = -\frac{1}{2}\mathrm{STr}\ \left[\Gamma^{(4)}{}_k - 2\Gamma^{(3)}{}_k\Gamma^{(2)}{}_k\Gamma^{(3)}{}_k\right]\ \frac{\partial_t R_k}{\Gamma^{(2)} + R_k}$$

n-pt depend on m ≤ n+2-pt vertex functions

Infinite tower of flow equation, needs to truncate $\Gamma_k^{(i>n_{\max})} = 0$

We choose $n_{\max} = 4$, enough to capture the fermion condensates

The tool: Functional renormalization group method

Truncated effactive action:

$$\Gamma_k = \int \mathcal{L}_{kin} + \mathcal{L}_{gh} + \mathcal{L}_{gf} + Z_\psi^2 \sum_{i=1}^{2} \lambda_i \mathcal{O}_i^\psi + Z_\chi^2 \sum_{i=4}^{5} \lambda_i \mathcal{O}_i^\chi + Z_\psi Z_\chi \sum_{i=6}^{7} \lambda_i \mathcal{O}_i^{\psi\chi}$$

$$\lambda_i = \lambda_i(p_m)|_{p_m \to 0}$$

$$\begin{cases} \mathcal{O}_1^\psi = (\psi^\dagger \bar{\sigma}^\mu \psi)(\psi^\dagger \bar{\sigma}^\mu \psi) \\ \mathcal{O}_2^\psi = (\psi^{\dagger f_1} \bar{\sigma}^\mu \psi_{f_2})(\psi^{\dagger f_2} \bar{\sigma}^\mu \psi_{f_1}) \end{cases}$$

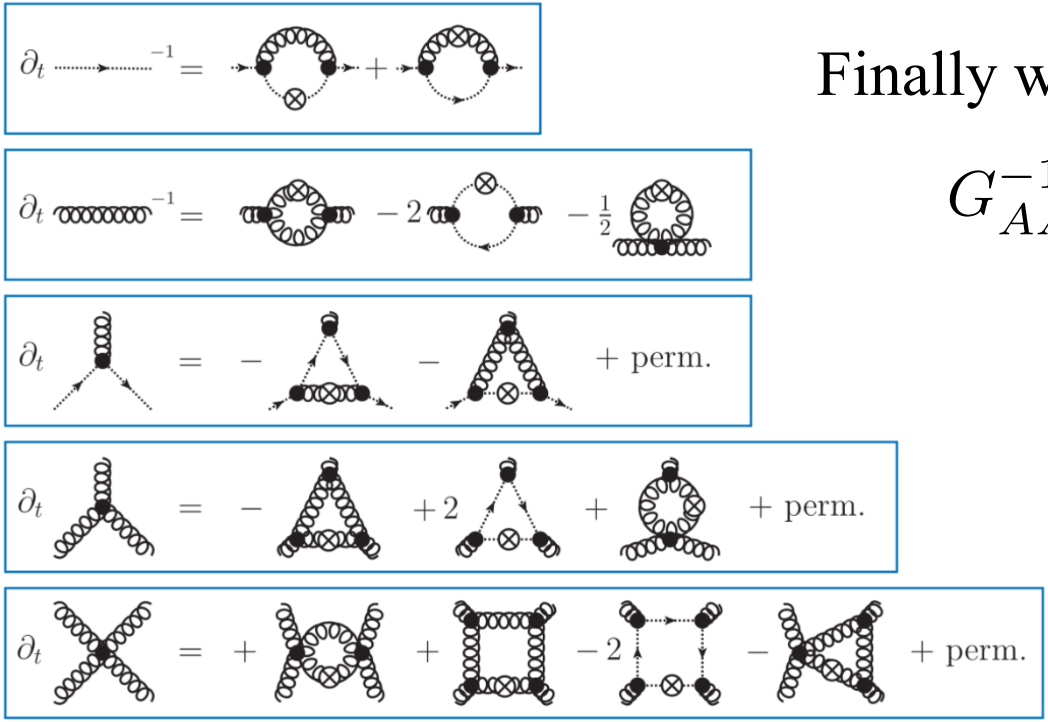
$$\begin{cases} \mathcal{O}_4^\chi = (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi) \\ \mathcal{O}_5^\chi = (\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi) \end{cases}$$

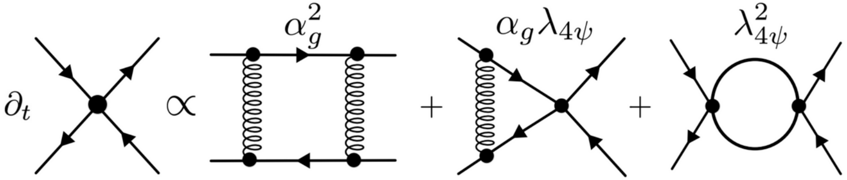
$$\begin{cases} \mathcal{O}_6^{\chi\psi} = (\psi^\dagger \bar{\sigma}^\mu \psi)(\chi^\dagger \bar{\sigma}^\mu \chi) \\ \mathcal{O}_7^{\chi\psi} = (\psi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \psi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi) \end{cases}$$

Using Young Tensor Method
[**HLL**,Pastor-Gutierrez,Vatani,Xu 2507.21208]
And prove it's valid for any N_c

[**HLL**,Pastor-Gutierrez,Vatani,Xu 2507.21208]

The tool: Functional renormalization group method





Finally we solve a coupled differential equations

$$G^{-1}_{AA,k}(p) = \tilde{Z}_{A,k}\left(p^2 + m^2_{\mathrm{gap},k}\right)\Pi^{\perp}_{\mu\nu}(p)$$

$$\begin{array}{ll} \{\bar{m}_{\mathrm{gap}}, \tilde{Z}_A, Z_c, Z_\psi, Z_\chi & \text{2-pt} \\ g_{A^3}, g_{A^4}, g_{A\bar{c}c}, g_{A\chi^\dagger\chi}, g_{A\psi^\dagger\psi} & \text{3-pt} \\ \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_4, \bar{\lambda}_5, \bar{\lambda}_6, \bar{\lambda}_7, \}_k & \text{4-pt} \end{array}$$

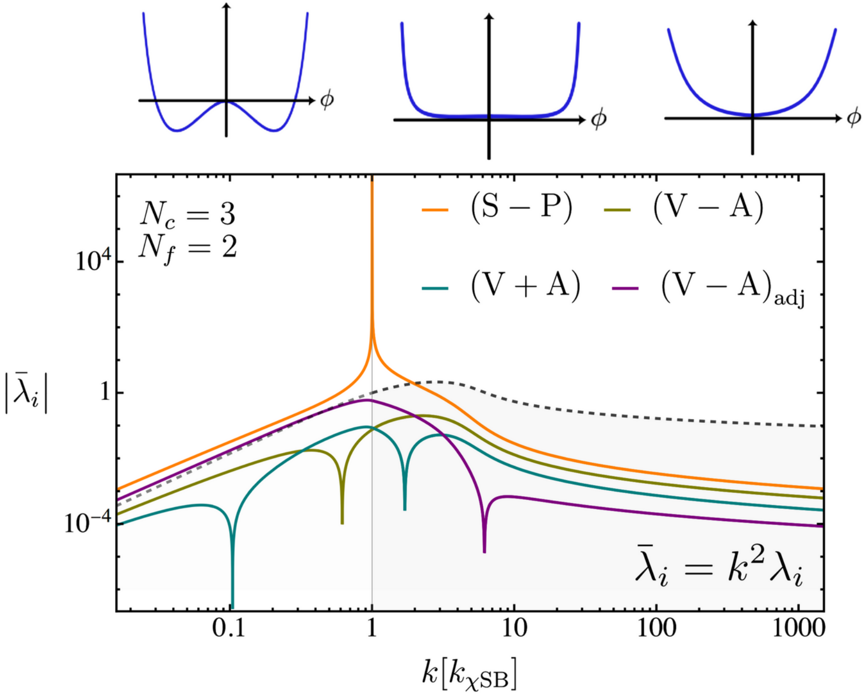
Physics are in the solutions of correlation functions. Confinement, Dynamical Symmetry breaking

The tool: Functional renormalization group method

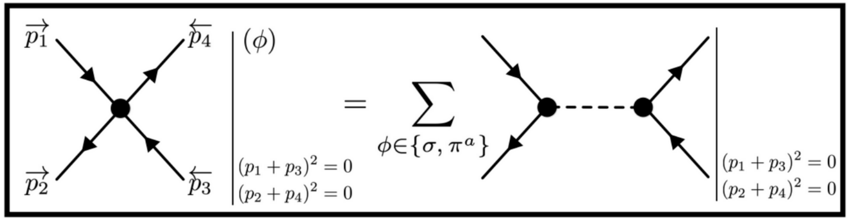
- **How to diagnose Symmetry breaking?**

Divergent 4-fermi flow
$(\psi_1\psi_2)(\psi_1\psi_2)$

Fermion condensate
$\langle\psi_1\psi_2\rangle \neq 0$



Exact field redefinition Stratonovich'57 Hubbard'59



$\lambda_i \propto 1/m^2_{\phi_i}$ $\quad m^2_{\phi_i} = \partial_{\phi_i^2} V(\phi_1)$

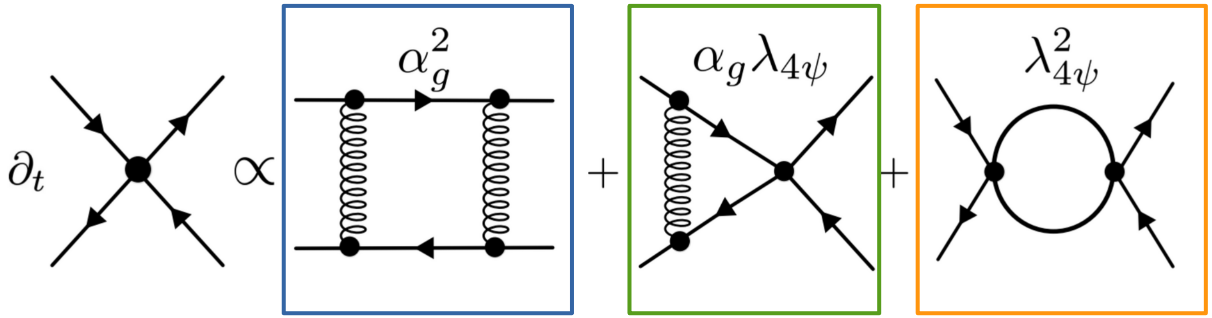
$\lambda_i \to \infty \quad m^2_{\phi_i} \to 0 \quad \xi \to \infty$

$\langle\phi\rangle \sim \langle\bar{\psi}\psi\rangle \neq 0$

See [2006.04853] for detailed discussion

The tool: Functional renormalization group method

4-fermion flows:



$$\bar{\lambda} = k^2 \lambda$$

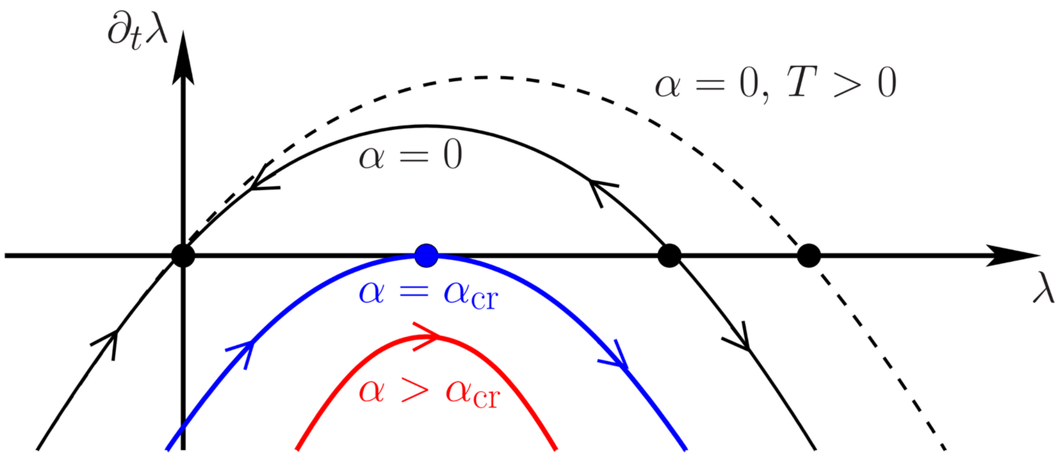
$$\partial_t \bar{\lambda}_i \propto 2\,\bar{\lambda}_i + \boldsymbol{c}_{\mathrm{A},i} \cdot \alpha_g^2 + \boldsymbol{c}_{\mathrm{B},ij} \cdot \alpha_g\,\bar{\lambda}_j + \boldsymbol{c}_{\mathrm{C},ijk} \cdot \bar{\lambda}_j \bar{\lambda}_k + \ldots$$

Canonical scaling

The tool: Functional renormalization group method

Focus on one $\bar{\lambda}_i$, the flow is a quadratic function of $\bar{\lambda}_i$:

$$\partial_t \bar{\lambda}_i \propto 2\,\bar{\lambda}_i + \boldsymbol{c}_{\mathrm{A},i} \cdot \alpha_g^2 + \boldsymbol{c}_{\mathrm{B},ij} \cdot \alpha_g\,\bar{\lambda}_j + \boldsymbol{c}_{\mathrm{C},ijk} \cdot \bar{\lambda}_j \bar{\lambda}_k + \ldots$$



Necessary condition for dSB:

1. reosnant structure (naive)

$$\frac{\boldsymbol{c}_{A,i}}{\boldsymbol{c}_{C,iii}} > 0$$

2. $\alpha_{\mathrm{SB}}^{\mathrm{crit}}$ is reached

two scenarios can prevents $\alpha_{\mathrm{SB}}^{\mathrm{crit}}$ being reached:
confinement, conformal

The tool: Functional renormalization group method

- **How to diagnose Confinement?**

Features of confinement:

- No Asymptotic colored states
- Massive spectrum of bound states (glueballs)
- Existence of gluon mass gap:

  - Wilson area law

Constraints on correlation functions

- Gluon mass gap in 2-pt functions

$$\Gamma_k^{AA}(p) = Z_{A,k}(p)(p^2 + m^2_{\mathrm{gap},k}) = \hat{Z}_{A,k}(p)p^2$$

- Kugo-Ojima relation: colorless aymptotic states

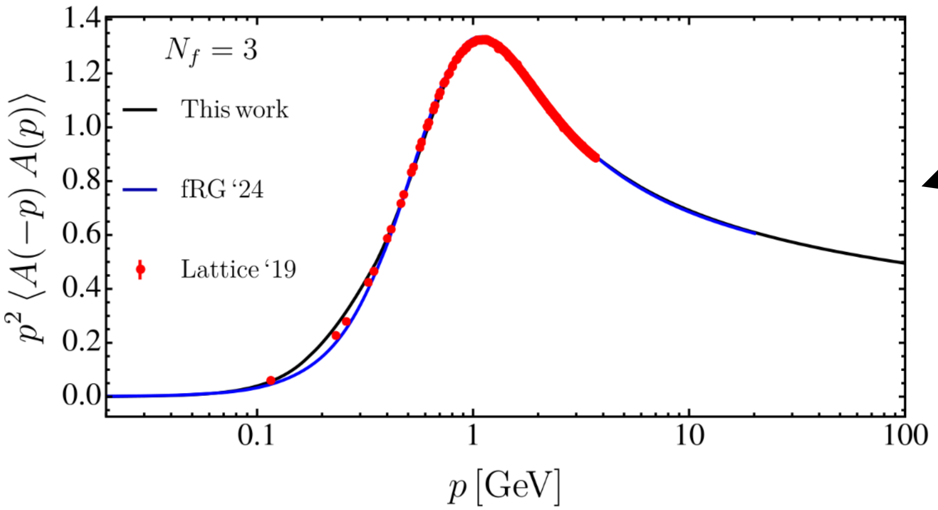
[Kugo, Ojima'79 Nakanishi,Ojima'90 Kugo hep-th/9511033]

$$\lim_{p\to 0} Z_c(p^2) \sim (p^2)^\kappa \quad \lim_{p\to 0} Z_A(p^2) \sim (p^2)^{-2\kappa}$$

The tool: Functional renormalization group method

➢ **How to diagnose Confinement?**

Features of confinement:



Constraints on correlation functions in fRG

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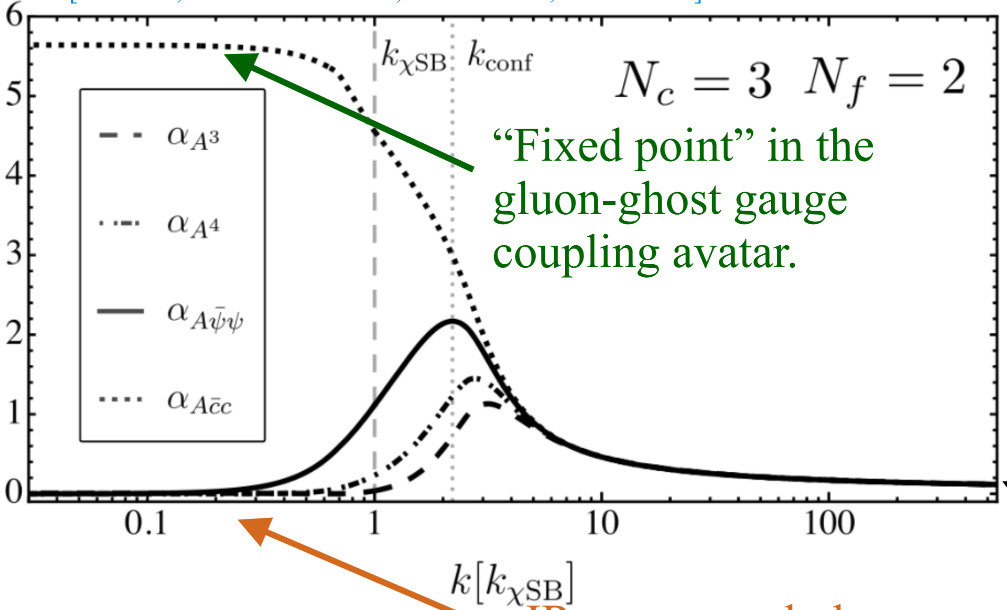
The tool: Functional renormalization group method

- **How to diagnose Confinement?**

$$Z_{A,\bar{k}}(p^2)p^2 \approx Z_{A,k}(k^2 + m^2_{\mathrm{gap},k})\big|_{k\approx p}$$

[F. Goertz, A. Pastor-Gutiérrez, J. Pawlowski, 2602.11265]



"Fixed point" in the gluon-ghost gauge coupling avatar.

IR suppressed gluon-fermion, gluon-self gauge coupling avatars

Constraints on correlation functions in fRG

Gluon mass gap in 2-pt functions

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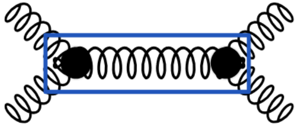
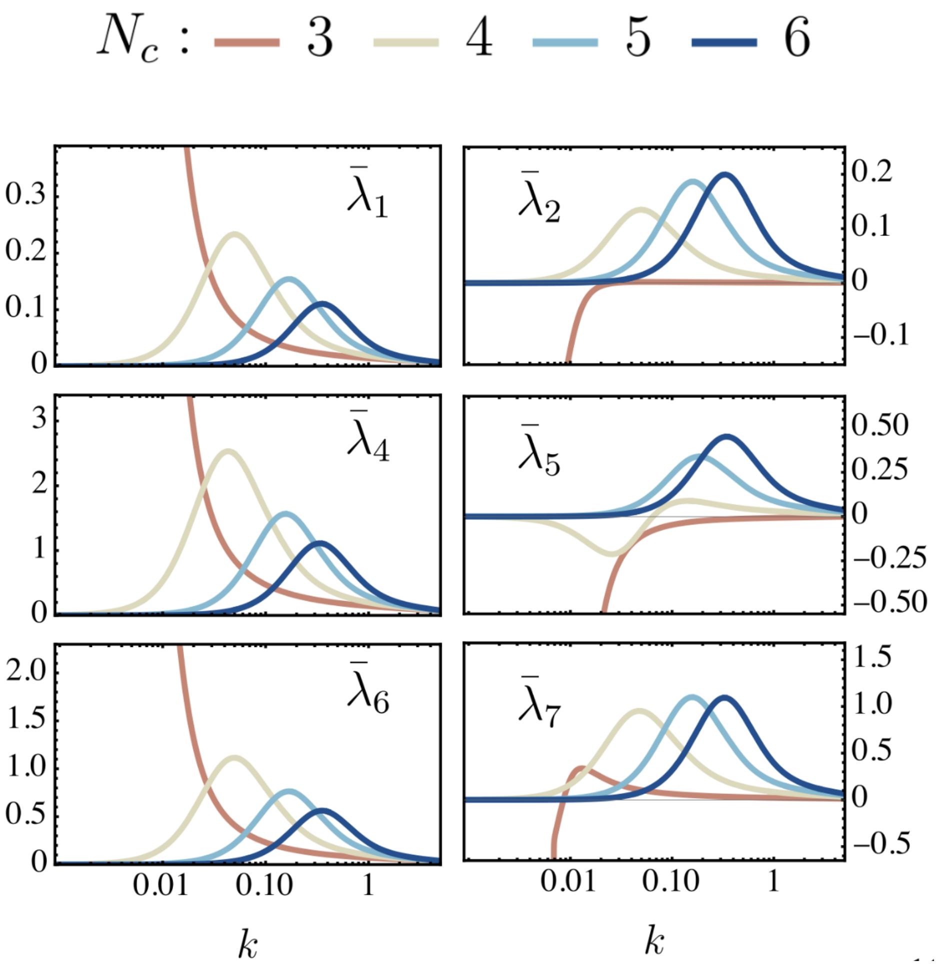
$$\alpha_{A^3} = \left[\tilde{\Gamma}_k^{(A^3)}\right]^2 / (4\pi\, Z_A^3) =$$



Results of BY model



N_c=3

donminant divergent operators:

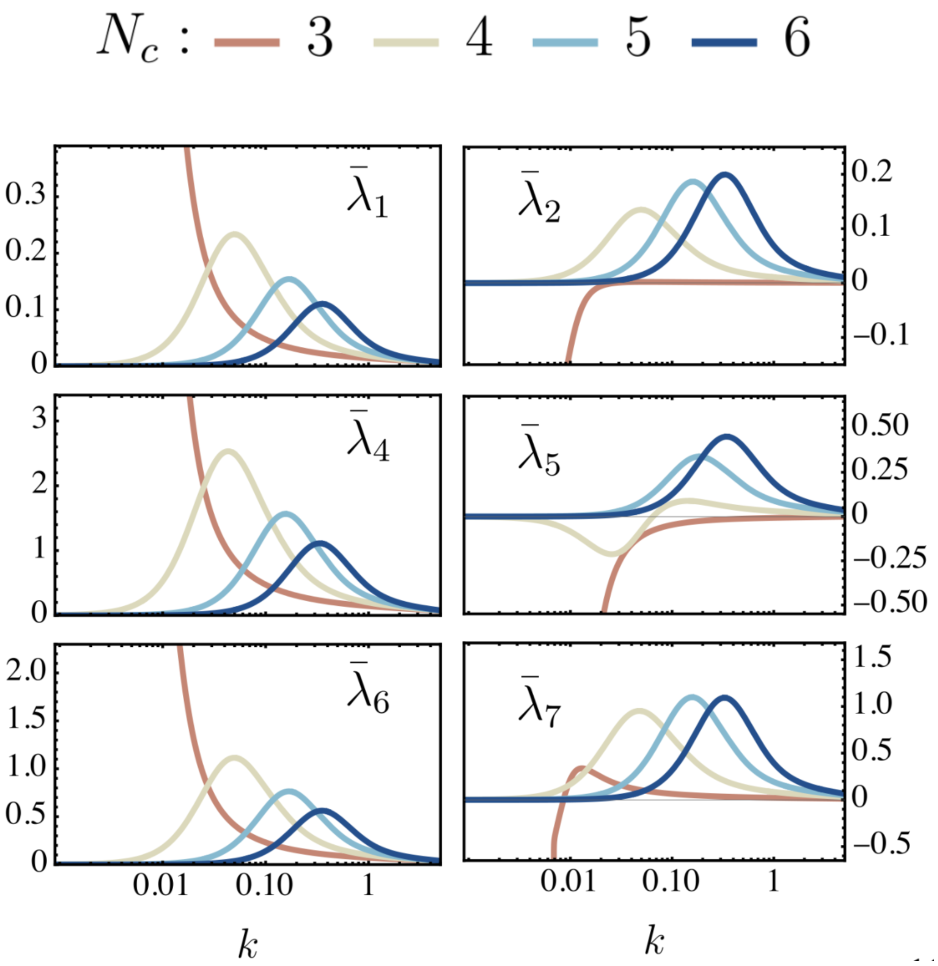
$$\mathcal{O}_4^\chi = (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi)$$

$$\mathcal{O}_5^\chi = (\chi^\dagger \bar{\sigma}^\mu T_{\mathrm{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\mathrm{sym}} \chi)$$

-- condensate in IR, dSB

$$\langle \chi\chi \rangle$$

Results of BY model

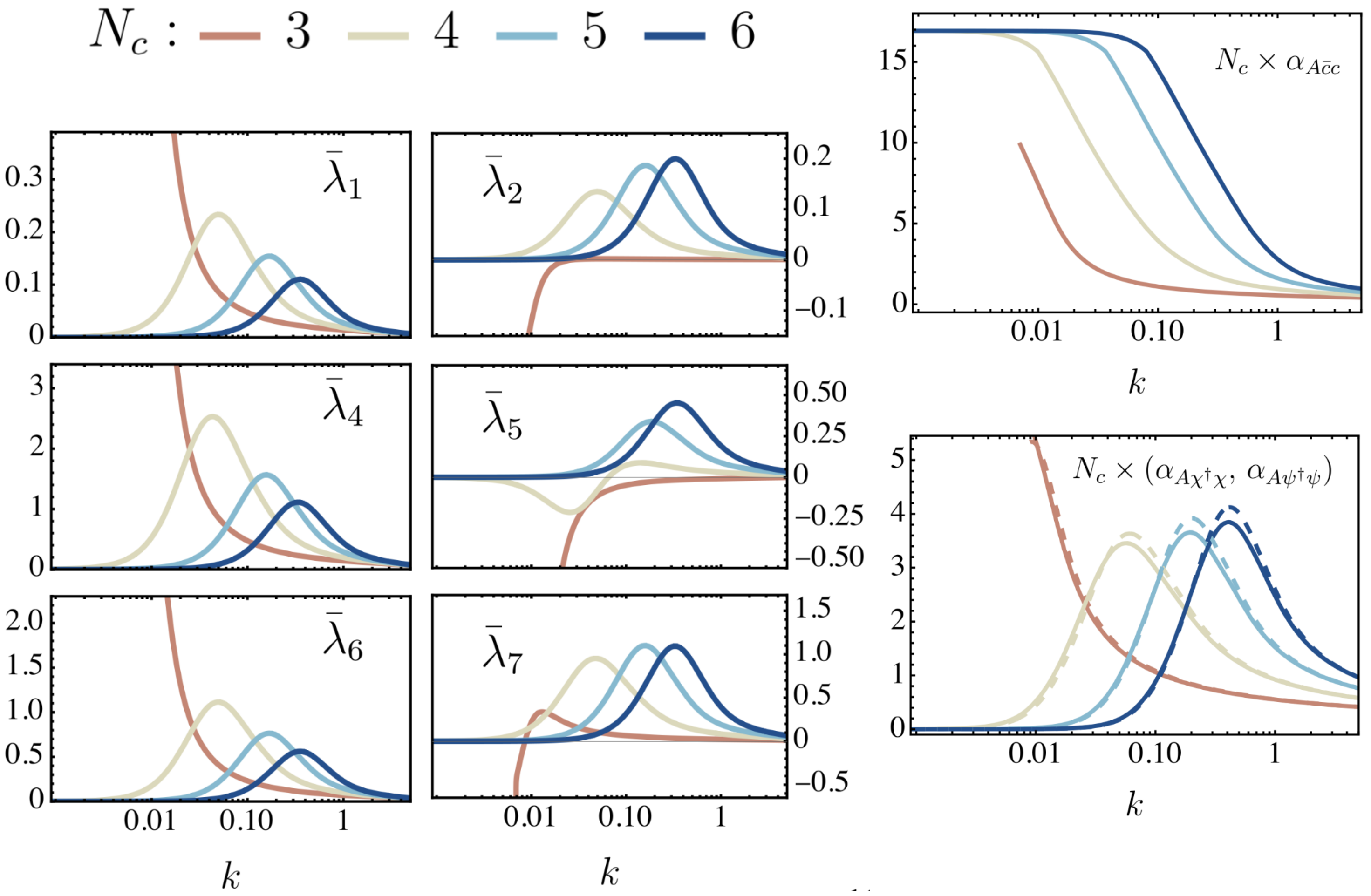


N_c=4,5,6

- No divergent 4F operators

-- no dynamical SB

Results of BY model



N_c=4,5,6

- No divergent 4F operators

  -- no dynamical χSB

- Found Scaling solution satisfies the KO condition

  -- confinement in Landau gauge

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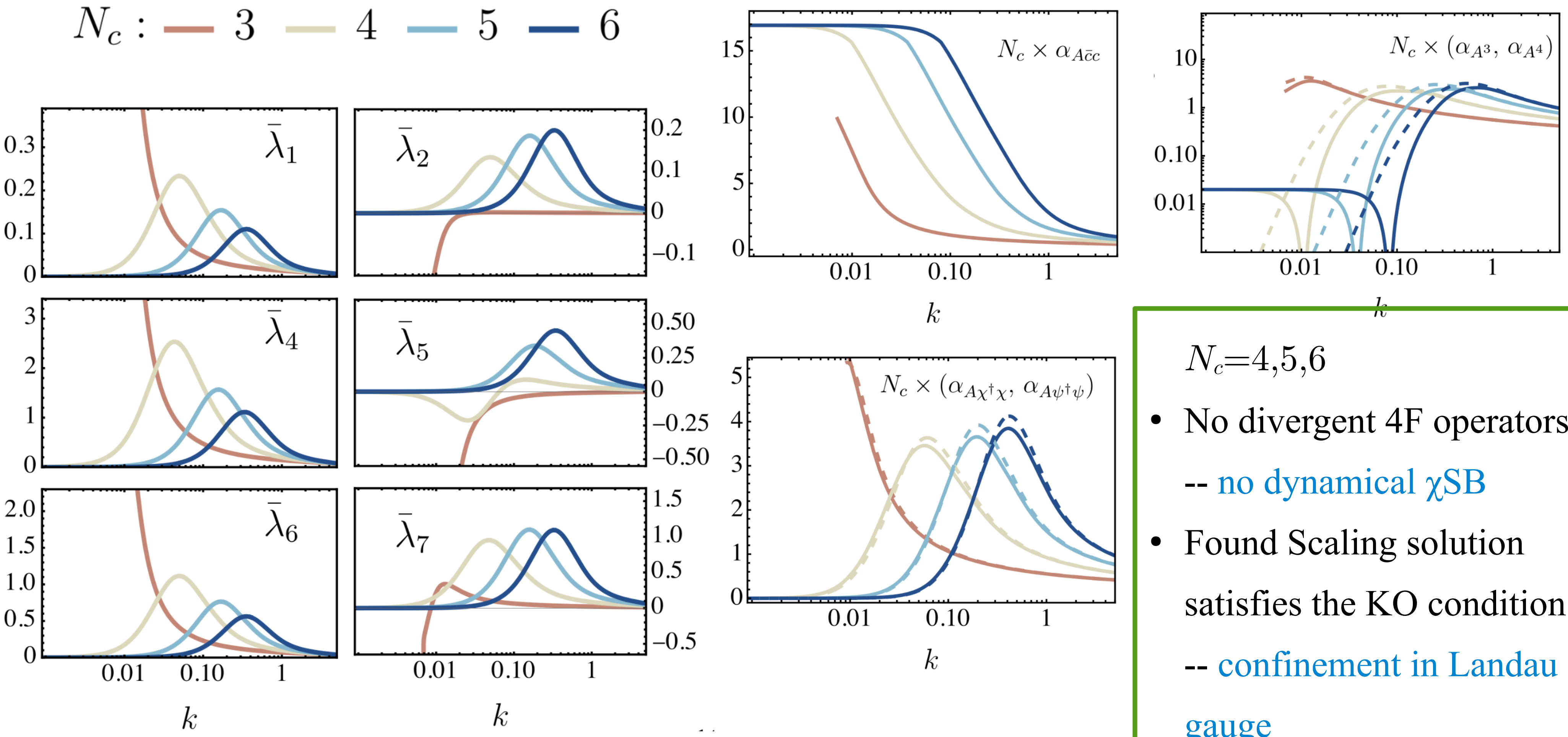

N_c=4,5,6

- No divergent 4F operators
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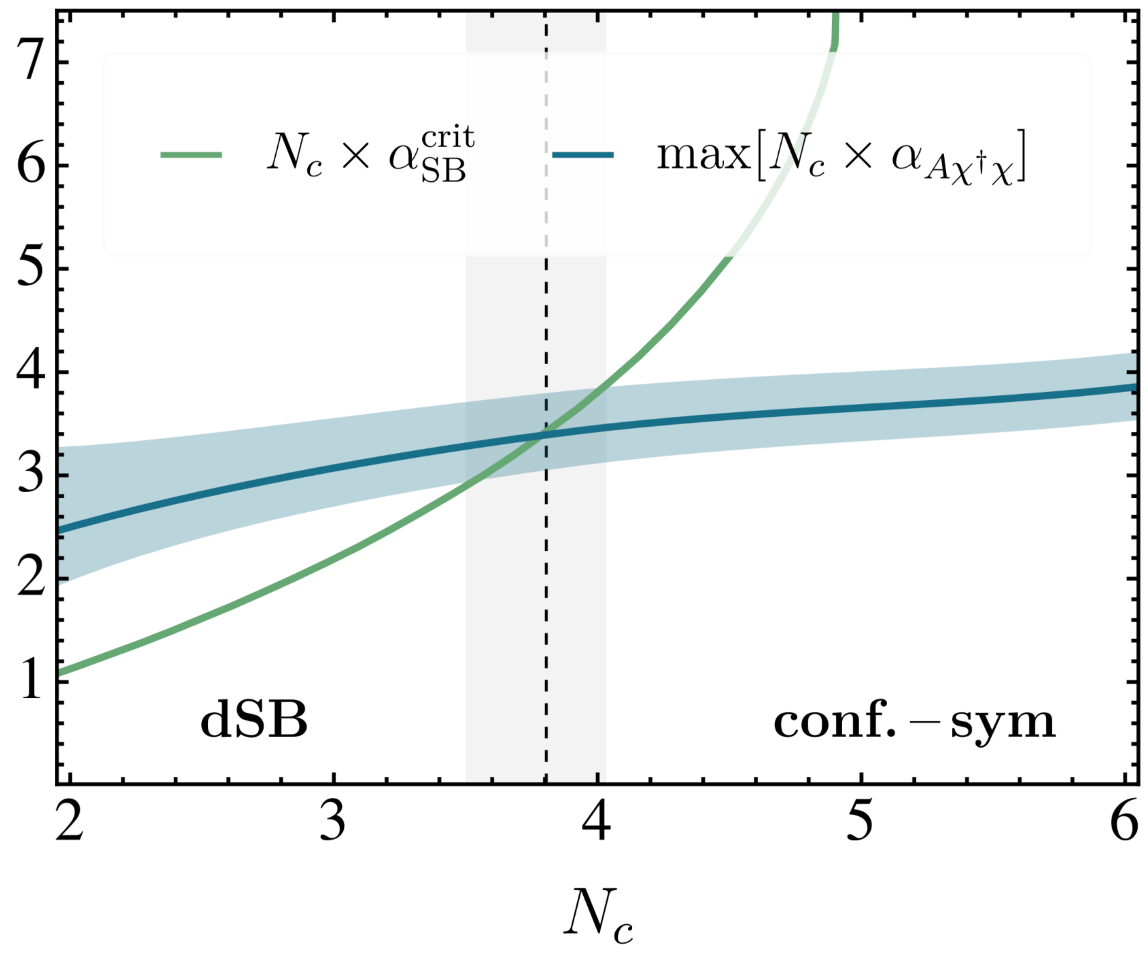
Evidence for Confinement w/o dynamical symmetry breaking!

Results of BY model



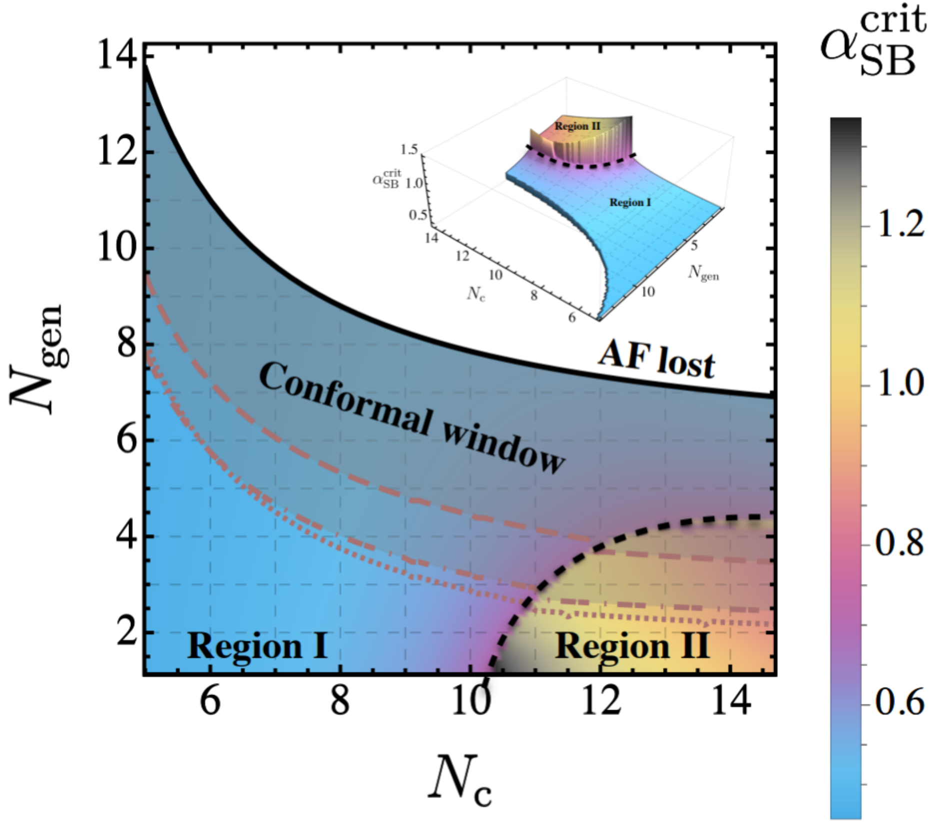
Summary

- We present first fRG study of BY class of chiral gauge theories.
- For $N_c \lesssim 3$ confinement w dSB, $N_c \gtrsim 4$ confinement w/o dSB.

Outlook

- Bosonize the theory to analysis the direction of dSB
- To analysis whether there is massless fermion in IR to match the 't Hooft anomaly
- Can these model help with symmetric mass generation?

Results



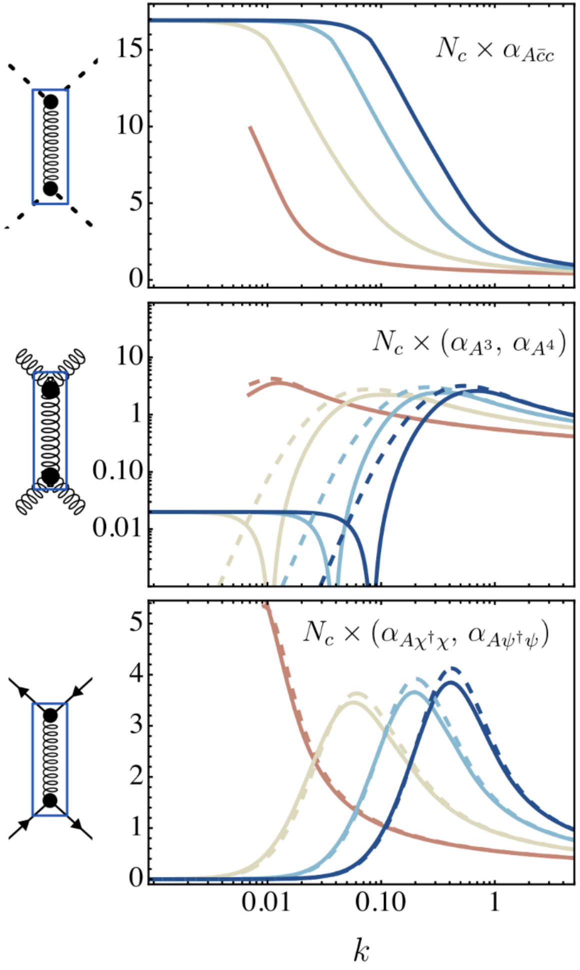
- **Region I:**
  - Weak $\alpha_g^{\rm crit}$: dynamics derivable within perturbation theory
  - Clear dominance of $\mathcal{O}_5 = (\chi^\dagger \bar\sigma^\mu T_{\rm anti}\,\chi)(\chi^\dagger \bar\sigma^\mu T_{\rm anti}\,\chi)$
  - Condensate $\langle\chi\chi\rangle \neq 0$

- **Region II:**
  -strong $\alpha_g^{\rm crit}$ non-perturbative, higer-order effects relevant
  -cannot resolve a clear single resonant channel.

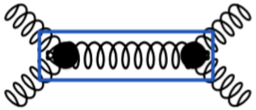
- **Conformal window:**
  $\alpha_g^{\rm crit} = \alpha_g^*$

$$\begin{cases} \alpha_g^*\big|_{\overline{\rm MS}\ 2-{\rm loop}} \\ \alpha_g^*\big|_{\overline{\rm MS}\ 3-{\rm loop}} \\ \alpha_g^*\big|_{\overline{\rm MS}\ 4-{\rm loop}} \end{cases}$$

New!



$$\alpha_{A^3} = \left[\tilde{\Gamma}_k^{(A^3)}\right]^2 / (4\pi\, Z_A^3) \;=$$



, (3)

$$\alpha_{A^4} = \tilde{\Gamma}_k^{(A^4)} / (4\pi\, Z_A^2) \;\text{ and }\; \alpha_{A\bar{c}c} = \left[\tilde{\Gamma}_k^{(A\bar{c}c)}\right]^2 / (4\pi\, Z_c^2 Z_A)\,,$$

$$\alpha_{A\chi^\dagger\chi} = \left[\tilde{\Gamma}_k^{(A\chi^\dagger\chi)}\right]^2 / (4\pi\, Z_\chi^2 Z_A)\,,$$

$$\alpha_{A\psi^\dagger\psi} = \left[\tilde{\Gamma}_k^{(A\psi^\dagger\psi)}\right]^2 / (4\pi\, Z_\psi^2 Z_A)\,.$$

’t Hooft anomaly can be matched by $\mathcal{B} = \chi\psi\psi$

$$Z_{A,\bar{k}}(p^2)\,p^2 \approx Z_{A,k}(k^2 + m^2_{\mathrm{gap},k})\big|_{k\approx p}$$

$$S = \int \bar{\psi} \partial\!\!\!/ \psi - \frac{\lambda}{2}\left(\left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma^5\psi\right)^2 \right)$$

$$Z \propto \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{S} = \mathcal{N} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\phi \;\; e^{\frac{m^2}{2}\phi^2} \;\; e^{S}$$

$$\phi = (\sigma, \pi)$$

$$\sigma \to \sigma + y\frac{\bar{\psi}\psi}{\sqrt{2}m^2} \qquad \pi \to \pi + i\, y\frac{\bar{\psi}\gamma^5\psi}{\sqrt{2}m^2}$$

$$Z \propto \mathcal{N} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\phi e^{S_{BF}}$$

$$S_{FB} = \int \bar{\psi}\partial\psi + \frac{y}{\sqrt{2}}\bar{\psi}\left(\sigma + i\pi\gamma^5\right)\psi + \frac{m^2}{2}\left(\sigma^2 + \pi^2\right) \qquad \lambda \equiv \frac{y^2}{2m^2}$$

$$\sigma = \frac{-y}{\sqrt{2}m^2}\bar{\psi}\psi \qquad \pi = \frac{-iy}{\sqrt{2}m^2}\bar{\psi}\gamma^5\psi$$

$$S_{\mathrm{B}} = \int \mathrm{d}^4x \left\{ m^2\phi^*\phi - \ln\det\left[\mathrm{i}\partial\!\!\!/ + h\left(P_{\mathrm{L}}\phi - P_{\mathrm{R}}\phi^*\right)\right]\right\}$$

$$U(1)_A \qquad \begin{pmatrix}\sigma \\ \pi\end{pmatrix} = \begin{pmatrix}\cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha\end{pmatrix}\begin{pmatrix}\sigma \\ \pi\end{pmatrix}$$

$$
\begin{aligned}
\partial_t \Gamma[\bar{\phi}] &= \frac{1}{2} \mathrm{Tr}\, G_k \, \partial_t R_k \,, \\
\partial_t \Gamma^{(1)}[\bar{\phi}] &= -\frac{1}{2} \mathrm{Tr}\, \Gamma_k^{(3)} \left(G_k \, \partial_t R_k \, G_k \right), \\
\partial_t \Gamma^{(2)}[\bar{\phi}] &= -\frac{1}{2} \mathrm{Tr} \left[\Gamma_k^{(4)} - 2\, \Gamma_k^{(3)}\, G_k \, \Gamma_k^{(3)} \right] \left(G_k \, \partial_t R_k \, G_k \right), \\
\partial_t \Gamma^{(3)}[\bar{\phi}] &= -\frac{1}{2} \mathrm{Tr} \left[\Gamma_k^{(5)} - 6\, \Gamma_k^{(4)}\, G_k \, \Gamma_k^{(3)} + 6\, \Gamma_k^{(3)}\, G_k \, \Gamma_k^{(3)}\, G_k \, \Gamma_k^{(3)} \right] \left(G_k \, \partial_t R_k \, G_k \right), \\
\partial_t \Gamma^{(4)}[\bar{\phi}] &= -\frac{1}{2} \mathrm{Tr} \Big[\Gamma_k^{(6)} - 8\, \Gamma_k^{(5)}\, G_k \, \Gamma_k^{(3)} - 6\, \Gamma_k^{(4)}\, G_k \, \Gamma_k^{(4)} + 18\, \Gamma_k^{(4)}\, G_k \, \Gamma_k^{(3)}\, G_k \, \Gamma_k^{(3)} \\
&\qquad + 12\, \Gamma_k^{(3)}\, G_k \, \Gamma_k^{(4)}\, G_k \, \Gamma_k^{(3)} - 24\, G_k \, \Gamma_k^{(3)}\, G_k \, \Gamma_k^{(3)}\, G_k \, \Gamma_k^{(3)} \cdot G_k \, \Gamma_k^{(3)} \Big] \left(G_k \, \partial_t R_k \, G_k \right),
\end{aligned}
$$

$$\Gamma_{\text{gauge},k}[A,c,\bar{c}] = \frac{1}{2}\int_p A^a_\mu(p)\,\left[\tilde{Z}_{A,k}\,\left(p^2+m^2_{\text{gap},k}\right)\Pi^\perp_{\mu\nu}(p) + \frac{1}{\xi}Z^\parallel_{A,k}\,\left(p^2+m^2_{\text{mSTI},k}\right)\frac{p_\mu p_\nu}{p^2}\right] A^a_\nu(-p)$$

$$+\frac{1}{3!}\int_{p_1,p_2}\tilde{Z}^{3/2}_{A,k}\,g_{A^3,k}\left[\mathcal{T}^{(1)}_{A^3}(p_1,p_2)\right]^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}\prod_{i=1}^{3}A^{a_i}_{\mu_i}(p_i) + \frac{1}{4!}\int_{p_1,p_2,p_3}\tilde{Z}^2_{A,k}\,g_{A^4,k}\left[\mathcal{T}^{(1)}_{A^4}(p_1,p_2,p_3)\right]^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}\prod_{i=1}^{4}A^{a_i}_{\mu_i}(p_i)$$

$$+\int_p Z_{c,k}\,\bar{c}^{\,a}(p)\;p^2\delta^{ab}c^b(-p) + \int_{p_1,p_2} Z_{c,k}\tilde{Z}^{1/2}_{A,k}\,g_{c\bar{c}A,k}\left[\mathcal{T}^{(1)}_{A\bar{c}c}(p_1,p_2)\right]^{a_1a_2a_3}_{\mu}\bar{c}^{a_2}(p_2)c^{a_1}(p_1)A^{a_3}_\mu(-p_1-p_2)\,. \tag{13}$$

$$\Gamma_{\text{gauge}-\text{fermion},k}\left[A_\mu,\psi^\dagger,\psi,\chi^\dagger,\chi\right] = \tag{15}$$

$$\int_p Z_{\psi,k}\,\psi^{\dagger\,,i,\,f}(p)\,\bar{\sigma}_\mu\partial_\mu\,\psi^{i,\,f}(-p) - \mathrm{i}\int_{p_1,\,p_2}\tilde{Z}_{\psi,k}\tilde{Z}^{1/2}_{A,k}\,g_{A\psi^\dagger\psi,k}\;\psi^{\dagger\,,i_1f_1}(p_2)\,\left[\mathcal{T}^{(1)}_{A\psi^\dagger\psi}(p_1,p_2)\right]^{a\,i_1i_2f_1f_2}_{\mu} A^a_\mu(-p_1-p_2)\psi^{i_2f_2}(p_1)$$

$$+\int_p Z_{\chi,k}\,\chi^{\dagger\,,\alpha}(p)\,\bar{\sigma}_\mu\partial_\mu\chi^\alpha(-p) - \mathrm{i}\int_{p_1,\,p_2} Z_{\chi,k}\tilde{Z}^{1/2}_{A,k}\,g_{A\chi^\dagger\chi,k}\,\chi^{\dagger\,,\alpha_1}(p_2)\,\left[\mathcal{T}^{(1)}_{A\chi^\dagger\chi}(p_1,p_2)\right]^{a\,\alpha_1\alpha_2}_{\mu} A^a_\mu(-p_1-p_2)\chi^{\alpha_2}(p_1)$$

$$\{g_{A^3},g_{A^4},g_{A\bar{c}c},g_{A\chi^\dagger\chi},g_{A\psi^\dagger\psi},\bar{m}_{\text{gap}},\bar{\lambda}_1,\bar{\lambda}_2,\bar{\lambda}_4,\bar{\lambda}_5,\bar{\lambda}_6,\bar{\lambda}_7,\;\tilde{Z}_A,Z_c,Z_\psi,Z_\chi\}.$$

- Insertion of mass-like regulator: Slavnov-Taylor identities (STIs) for gauge invariance → modified STIs (mSTIs)

$$\int_x \frac{\delta\Gamma}{\delta\mathcal{Q}_i(x)}\frac{\delta\Gamma}{\delta\Phi_i(x)} = 0 \quad \longrightarrow \quad \int_x \frac{\delta\Gamma_k}{\delta\mathcal{Q}_i(x)}\frac{\delta\Gamma_k}{\delta\Phi_i(x)} = \mathrm{Tr}\, R^{ij} G_{jl} \frac{\delta^2\Gamma_k[\Phi,\mathcal{Q}]}{\delta\Phi_l \delta\mathcal{Q}^i}$$

$\Phi_i = \{A_\mu, c, \bar{c}, \ldots\}$

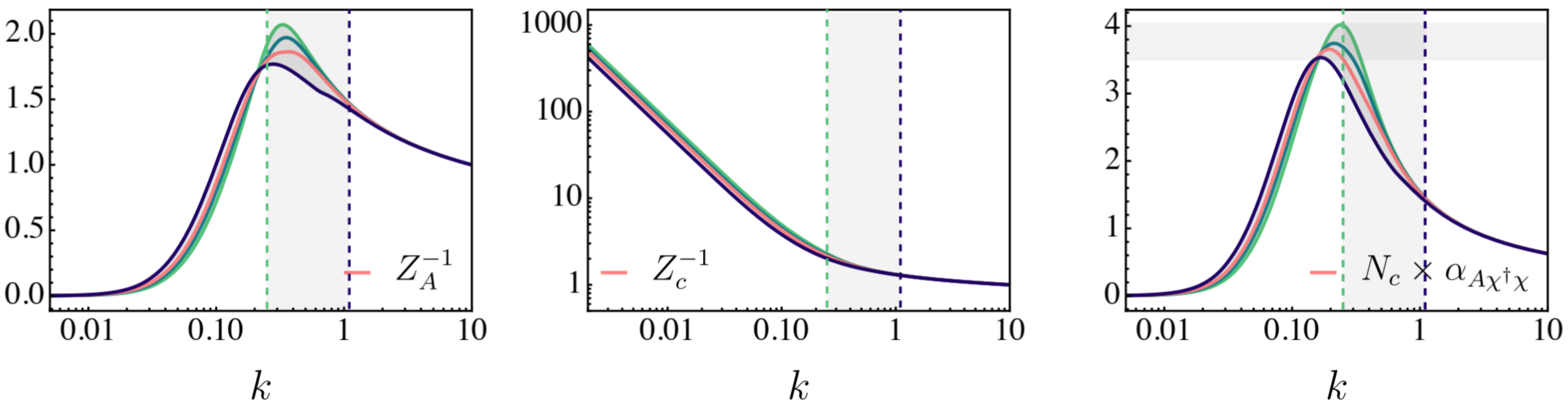
$\mathcal{Q}^i$: BRST sources of the Φ_i fields

- In the physical limit $k \to 0$ and $R_k \to 0$ and mSTI ☞ STI and no gauge symmetry breaking is present (*)

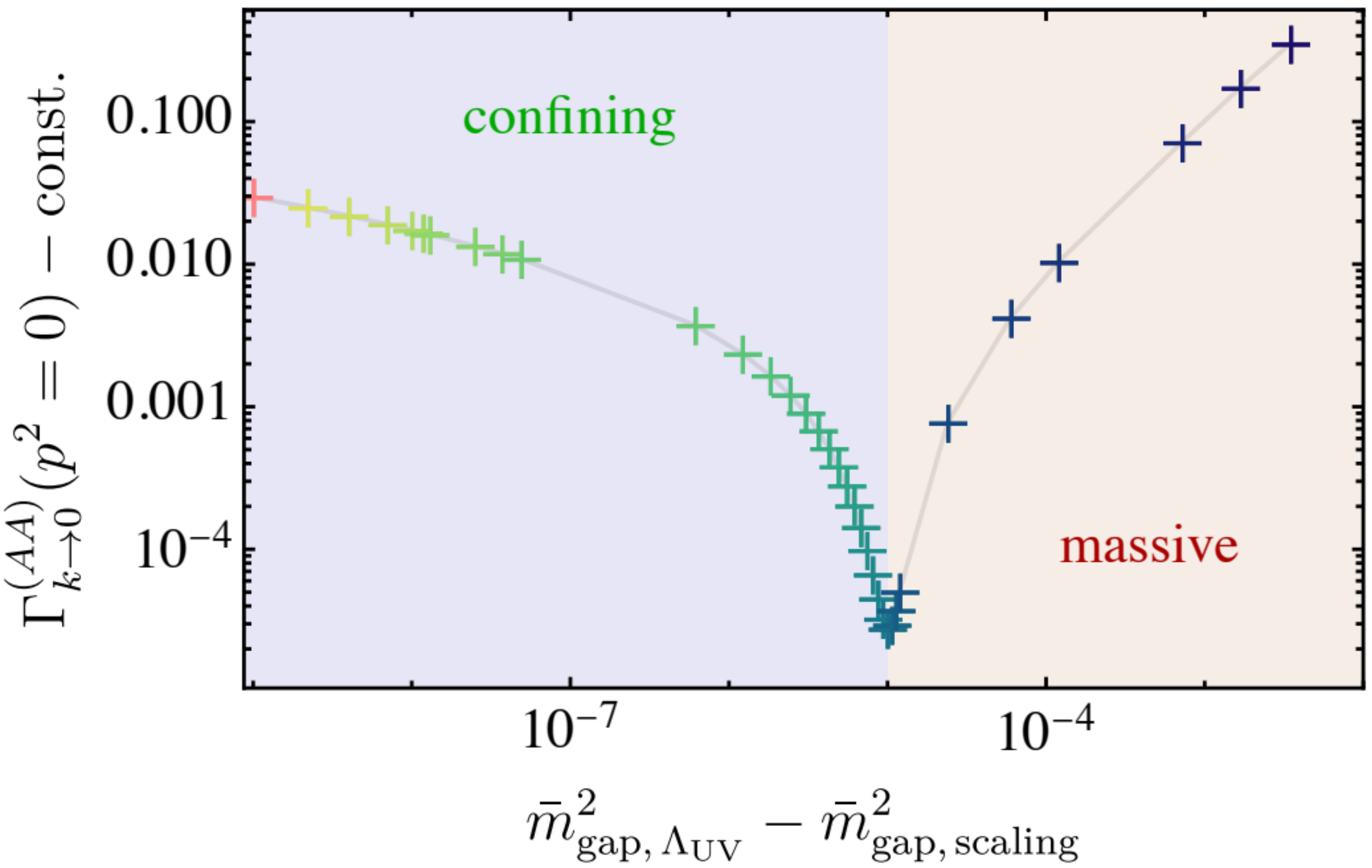
- "easy" approach: physics at k-dependent correlation functions
  - Minimise mSTI and STI deviations for $k \neq 0$
  - Hands-on approach: include problematic power-law corrections to the flow of the gauge mass gap only in the confinement regime

$$\partial_t \bar{m}^2_{\text{gap}} = (-2 + \eta_A)\, \bar{m}^2_{\text{gap}} + \underbrace{\overline{\mathrm{Flow}}_{AA}(p^2 = 0)}$$

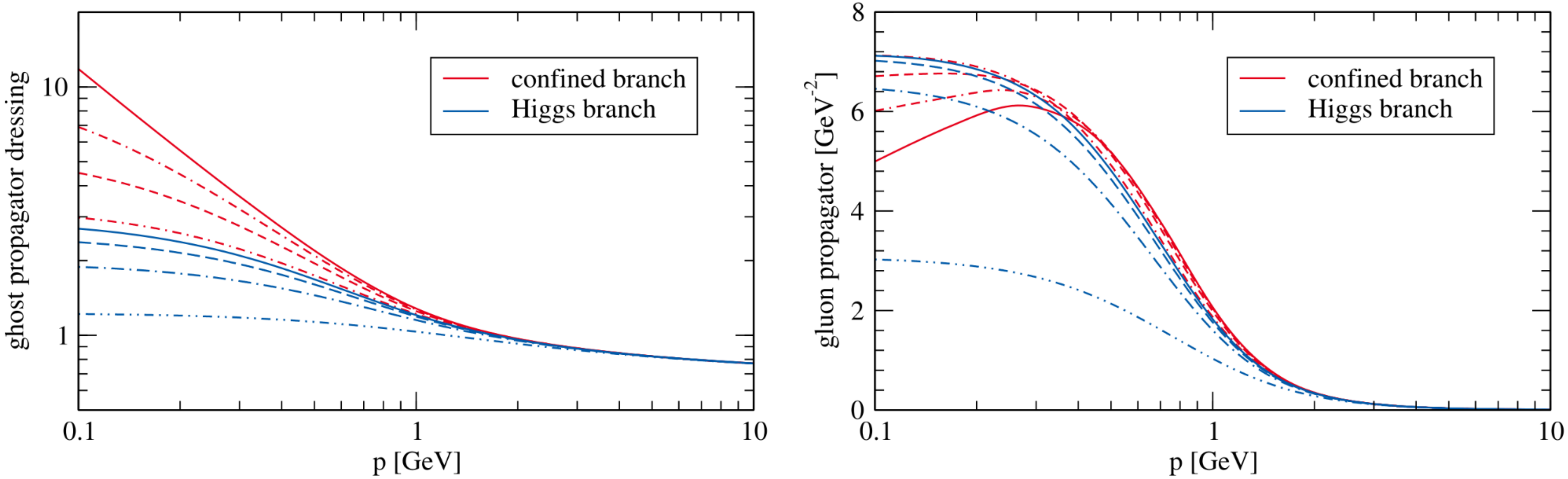
  - Upgrade with full momentum dependencies



2.0
1.5
1.0
0.5
0.0
Z_A^{-1}
0.01
0.10
1
10
k
1000
100
10
1
Z_c^{-1}
0.01
0.10
1
10
k
4
3
2
1
0
$N_c \times \alpha_{A\chi^\dagger\chi}$
0.01
0.10
1
10
k



confining
massive
0.100
0.010
0.001
10^{-4}
10^{-7}
10^{-4}
$\Gamma^{(AA)}_{k\to 0}(p^2=0)$ − const.
$\bar{m}^2_{\mathrm{gap},\,\Lambda_{\mathrm{UV}}} - \bar{m}^2_{\mathrm{gap},\,\mathrm{scaling}}$



ghost propagator dressing
10
1
0.1
1
10
p [GeV]
confined branch
Higgs branch
gluon propagator [GeV^{-2}]
8
6
4
2
0
0.1
1
10
p [GeV]
confined branch
Higgs branch

$$Z_{A,\bar{k}}(p^2)p^2 \approx Z_{A,k}(k^2 + m^2_{\text{gap},k})\big|_{k\approx p}$$

$$G_{A,k}(p) = \frac{1}{Z_{A,k}(p^2)p^2 + R_{A,k}(p)}$$

Gluon propagator dressing: $p^2 \langle A(-p)A(p)\rangle = p^2 \left[\Gamma_k^{(AA)}(p^2=0)\right]^{-1}_{k=c\,p}$

Goertz,APG,Pawlowski[2412.12254]