Measurements of Z-boson pair entanglement in decays of Higgs bosons at the ATLAS experiment

arXiv:2603.26463 (CERN-EP-2026-061)

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Mingyi Liu (刘明依)[1]

1. University of Science and Technology of China

mingyi.liu@cern.ch

Outline

Analysis based on Run2 + partial Run3 (~306 fb^{-1}) ATLAS pp collision data.

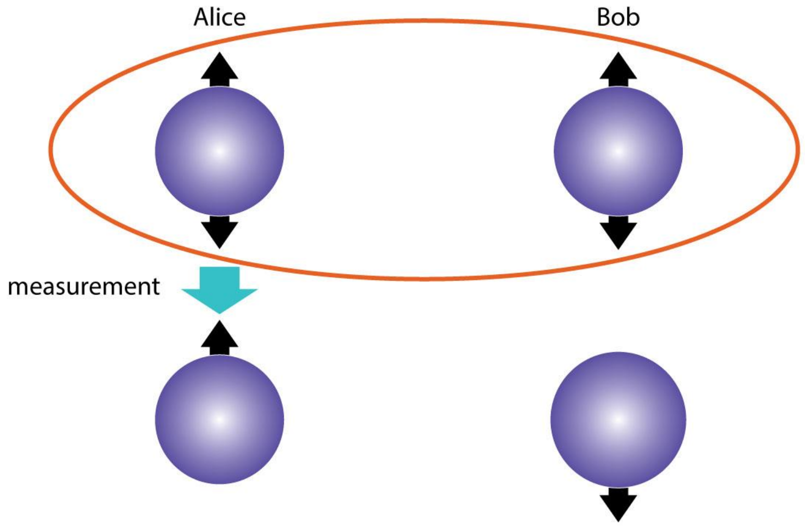
- **Introduction**
  - ✓ Quantum entanglement (QE) test on LHC.
  - ✓ Motivation for entanglement test in $H \rightarrow ZZ^* \rightarrow 4l$.
- **Methodologies, analysis workflow, results and interpretation**
  - ✓ Direct measurement based on quantum tomography.
  - ✓ Hypothesis test against the separable (non-entangled) system.
- **Summary**

Introduction

Motivation:

- Entanglement is a critical phenomenon in QM:

For particles entangled, their individual quantum states cannot be described independently; only the overall state of the system can be characterized.



- Validated by atomic and solid-state experiments @ low energy. Recent years, QE probing @ high energy physics becomes an interesting topic.

… $\mathbf{GeV}\sim\mathcal{O}(\mathbf{M}_{s,\tau,c,b})$ … $\mathbf{100\ GeV}\sim\mathcal{O}(\mathbf{M}_t, \mathbf{M}_{Z/W})$ **Energy Scale**

- KLOE-2: JHEP 04 (2022) 059
- BESIII: Nature Commun. 16 (2025) 4948
- STAR: Nature volume 650, 65–71 (2026)
- BELLE: Phys. Rev. Lett. 99, 131802
- …

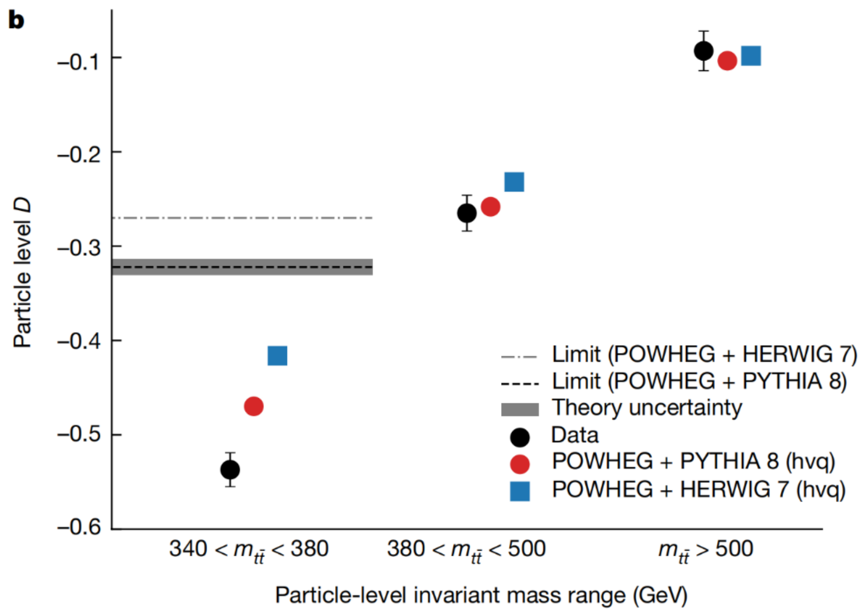
- ATLAS: Nature 633 (2024) 542
- CMS: ROPP 87 (2024) 117801
- CMS: PRD 110 (2024) 112016
- **ATLAS: arXiv:2603.26463**
- More is on the way …

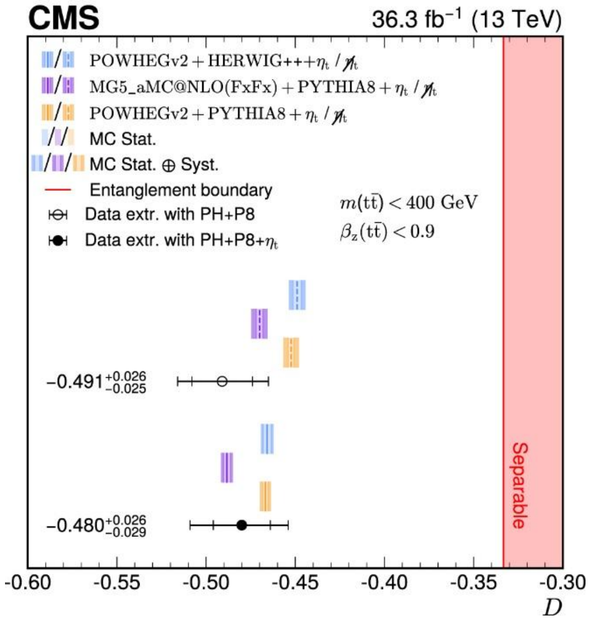
Measurements

Entanglement on the LHC

A new topic for the LHC physics:

- Entanglement can be tested via studying quantum state correlations (e.g. spin states).
- LHC is an ideal platform for QE test at the EW scale (the highest energy frontier).
- The ATLAS and CMS experiments firstly observed entanglement in fermion (top-quark) pairs produced near threshold. Which also innovated the toponium hunting.



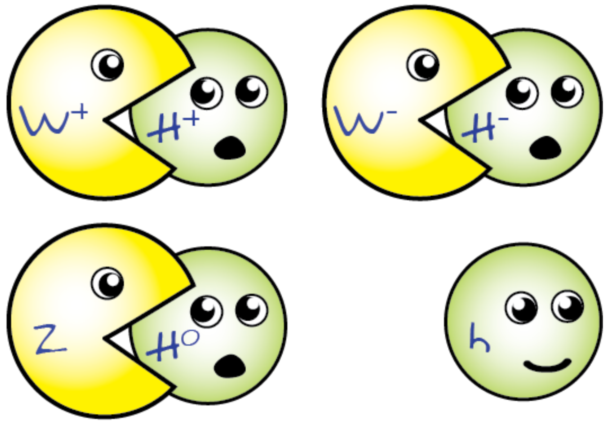


Nature 633 (2024) 542

ROPP 87 (2024) 117801

Entanglement on the LHC

What about bosons?



[Quantum Diaries]

- **Massive vector boson** obtains its longitudinally polarization state via absorbing one Goldstone freedom.
- Qutrit system made up by spin states: $|V\rangle \subset \{|+\rangle, |0\rangle, |-\rangle\}$.
- VV system - Corresponding to 9×9 SDM: $\rho = |V_1 V_2\rangle\langle V_1 V_2|$.

- Study based on spin states of massive vector bosons is the first ever attempt to probe entanglement between fundamental † qutrits, at the Electroweak (EW) scale.

†There were low energy atomic experiments studying QE between qutrits (of energy excitation states), e.g.: Phys. Rev. Applied 17, 024062.

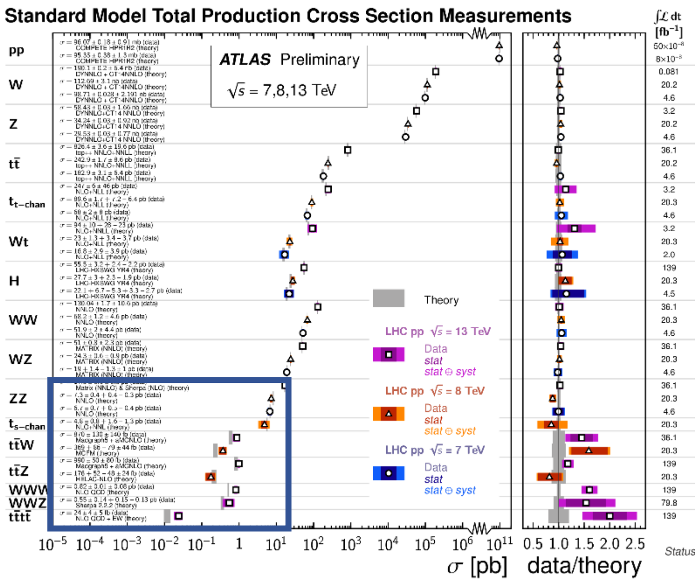
Motivation for Choosing $\mathrm{H} \to ZZ^*$

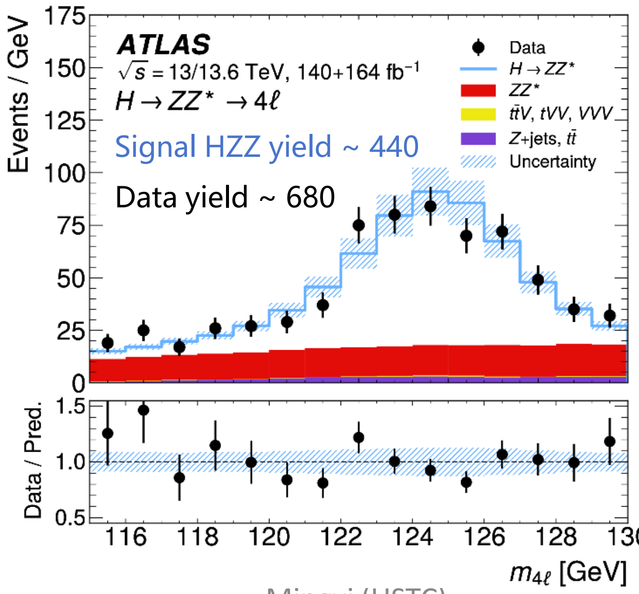
For a bi-qutrit system:

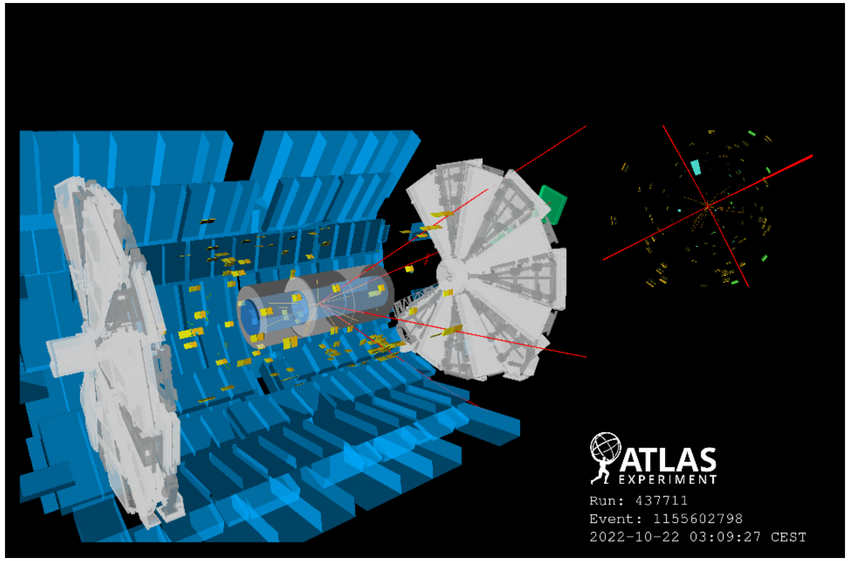
- Maximally entangled pure state: $|VV\rangle_{AB} = \frac{1}{\sqrt{3}}(|+,-\rangle - |0,0\rangle + |-,+\rangle)$.
- The total spin is constrained to be 0 $\leftrightarrow$ Vector bosons decayed from a scalar, i.e., the Higgs.
- $\mathrm{H} \to ZZ^* \to 4l$ process has fully detectable final states (unlike neutrinos decayed from $W^{\pm}$)

Besides, 4 lepton final state is a golden channel: high signal purity.

Image credit







Entanglement Criterions

Peres: Phys. Rev. Lett. **77**, 1413
Horodecki: Phys. Lett. A **232**, 5

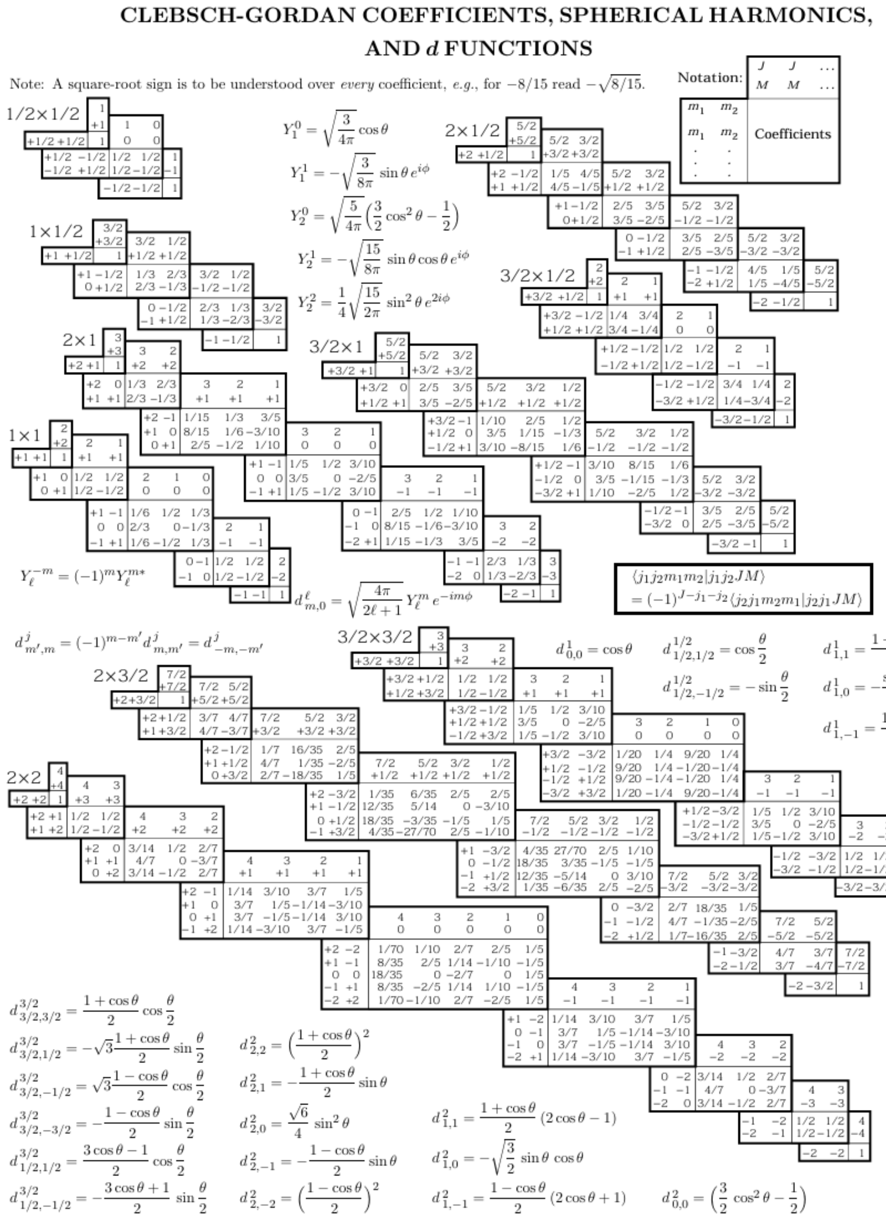
- **Peres-Horodecki criterion:**
  - Density matrix ρ: Positive definite; $\mathrm{Tr}(\rho) = 1$; $\rho^\dagger = \rho$.
  - The system is entangled if above features violated for partial transposed $\rho^{\mathrm{T}_{A/B}}$.
  - A sufficient condition from P-H criterion:

If ρ^{T_A} (or ρ^{T_B}) has negative eigenvalues → Non-separable (entangled)

- Other criterions:
  - CGLMP inequality (A qutrit version of the Bell-inequality) Phys. Rev. Lett. 88, 040404 (2002)
  - Concurrence $(\mathcal{C}(\rho))^2 \geq 2\max(\mathrm{Tr}\,\rho^2 - \mathrm{Tr}\,\rho_a^2, \mathrm{Tr}\,\rho^2 - \mathrm{Tr}\,\rho_b^2)$, $(\mathcal{C}(\rho))^2 \leq 2\min(1 - \mathrm{Tr}\,\rho_a^2, 1 - \mathrm{Tr}\,\rho_b^2)$ JHEP09(2025)013
  - …
- Entanglement witness is designed based on the P-H criterion because of sensitivity.

Analysis Methodology 1: Tomography

- The QE criterions apply to the Spin Density Matrix (SDM); However, impossible to measure the spin states of Z/W bosons, top quark, etc. directly on the LHC.
- **Tomography** technique is developed to reconstruct the SDM, based on the decay angles of the final state particles in the frame of the to-be-tested particles.



[Eur. Phys. J. Plus 136, 907 (2021)](#)
[PhysRevD.107.016012](#)
[JHEP05(2023)020](#)

The Spin Density Matrix

- **Reconstruct the ZZ-SDM based on the leptonic final states decayed from the bosons.**

- SDM can be decomposed into basis of **3D representatives of $\mathrm{SU(2)_{Spin}}$** (**J.A.A.S, el.**) :

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + \overset{\times 8}{\boxed{A^1_{LM}}}\, T^L_M \otimes \mathbb{1}_3 + \overset{\times 8}{\boxed{A^2_{LM}}}\, \mathbb{1}_3 \otimes T^L_M + \overset{\times 64}{\boxed{C_{L_1M_1L_2M_2}}}\, T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

$M \in \{1,2\}$

$L \in \{-M, -M+1, \dots, M-1, M\}$

- With the fully differential distribution:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + \boxed{A^1_{LM}} B_L Y^M_L(\theta_1,\varphi_1) + \boxed{A^2_{LM}} B_L Y^M_L(\theta_2,\varphi_2) + \boxed{C_{L_1M_1L_2M_2}} B_{L_1} B_{L_2} Y^{M_1}_{L_1}(\theta_1,\varphi_1) Y^{M_2}_{L_2}(\theta_2,\varphi_2)\Big],$$

$$B_1 = -\sqrt{2\pi}\,\eta_\ell\,, \quad B_2 = \sqrt{\frac{2\pi}{5}}\,. \qquad \eta_\ell = \frac{1-4s_W^2}{1-4s_W^2+8s_W^4}$$

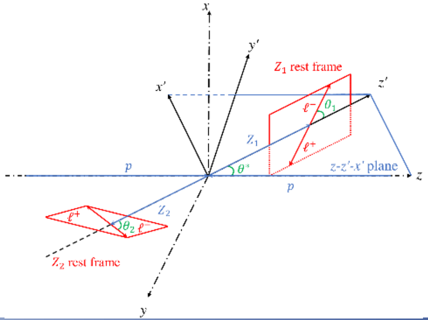
$Y^M_L(\theta_i,\varphi_i)$: Spherical harmonic functions

- Thus, the coefficients A_{LM}, $C_{L_1M_1L_2M_2}$, can be extracted from angular variables, by likelihood fit (with efficiency considered); or by integrating the partial waves (over full phase space):

$$\begin{aligned}
\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^M_L(\Omega_j)\, d\Omega_j &= \frac{B_L}{4\pi} A^j_{LM}, \qquad j = 1,2\,. \\
\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2)\, d\Omega_1 d\Omega_2 &= \frac{B_{L_1}B_{L_2}}{(4\pi)^2} C_{L_1M_1L_2M_2}\,,
\end{aligned}$$

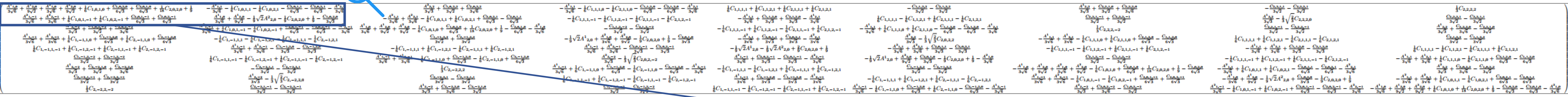
θ, φ are lepton decay angles in V rest-frames

With V in H rest-frame



Simplify?

- For a general ZZ system (w/o constraints from Higgs), the SDM comes in a quite complex form:



$$\frac{A^1{}_{1,0}}{3\sqrt{6}} + \frac{A^1{}_{2,0}}{9\sqrt{2}} + \frac{A^2{}_{1,0}}{3\sqrt{6}} + \frac{A^2{}_{2,0}}{9\sqrt{2}} + \frac{1}{6}C_{1,0,1,0} + \frac{C_{1,0,2,0}}{6\sqrt{3}} + \frac{C_{2,0,1,0}}{6\sqrt{3}} + \frac{1}{18}C_{2,0,2,0} + \frac{1}{9} \qquad -\frac{A^2{}_{1,1}}{3\sqrt{6}} - \frac{1}{6}C_{1,0,1,1} - \frac{1}{6}C_{1,0,2,1} - \frac{C_{2,0,1,1}}{6\sqrt{3}} - \frac{C_{2,0,2,1}}{6\sqrt{3}} - \frac{A^2{}_{2,1}}{3\sqrt{6}}$$

$$\frac{A^2{}_{1,-1}}{3\sqrt{6}} + \frac{A^2{}_{2,-1}}{3\sqrt{6}} + \frac{1}{6}C_{1,0,1,-1} + \frac{1}{6}C_{1,0,2,-1} + \frac{C_{2,0,1,-1}}{6\sqrt{3}} + \frac{C_{2,0,2,-1}}{6\sqrt{3}} \qquad \frac{A^1{}_{1,0}}{3\sqrt{6}} + \frac{A^1{}_{2,0}}{9\sqrt{2}} - \frac{1}{9}\sqrt{2}A^2{}_{2,0} - \frac{1}{9}C_{2,0,2,0} + \frac{1}{9} - \frac{C_{1,0,2,0}}{3\sqrt{3}}$$

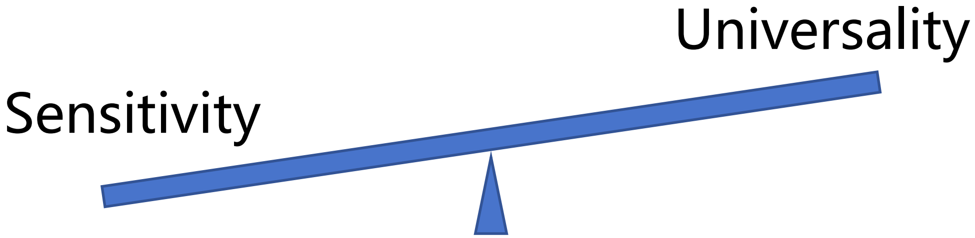
- The statistical fluctuation **blows up** if we keep using the general form of ZZ SDM. If pre-request the Z boson pair to be decayed from a Higgs, i.e., $|ZZ\rangle_{Spin} = \frac{1}{\sqrt{2+\beta^2}}(|+,-\rangle - \beta|0,0\rangle + |-,+\rangle)$:

$$\rho = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & \boxed{\frac{1}{3}C_{2,1,2,-1}} & 0 & \boxed{\frac{1}{3}C_{2,2,2,-2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}\left(1-\sqrt{2}A^1_{2,0}\right) & 0 & \boxed{\frac{1}{3}C_{2,1,2,-1}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

9 element form

The Peres-Horodecki simplified as:

$C_{222-2} \neq 0$ **or** $C_{212-1} \neq 0$ $\leftrightarrow$ ZZ Entangled



Caveat

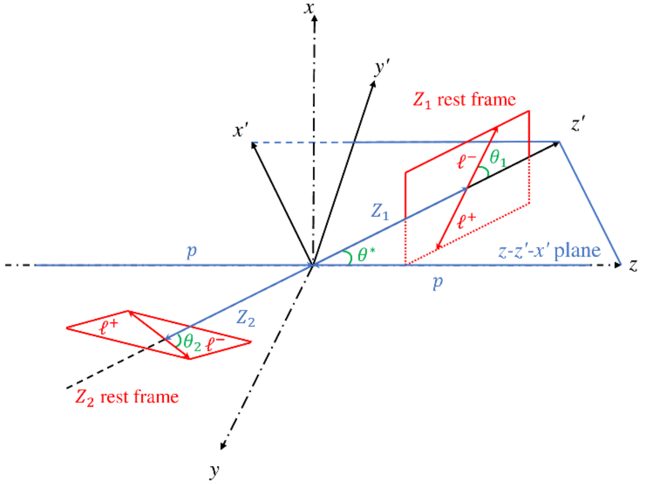
The rationality and solidness of directly using the criterion based on $\boldsymbol{C_{222-2}}$ and $\boldsymbol{C_{212-1}}$, are ensured by the following assumptions:

- Assume SM predicted angular distributions of the leptons from each helicity state of $Z^{(*)}$. → The foundation of tomography.
- Assume $|ZZ^*\rangle$ from Higgs decay form a scalar, CP-even state, consistent with measurements. → Ensuring the SDM of the (H)ZZ system is 9-element-formed.
- Assume the angular distributions of the four-lepton background processes are well-described by the Standard Model. → All backgrounds, including non-Higgs ZZ process (simulated via MC), should be subtracted from data (Otherwise, 9-element form of SDM violated).

Measurement workflow

- Measure $\boldsymbol{C_{222-2}}$ and $\boldsymbol{C_{212-1}}$ by calculating the mean values of $\boldsymbol{c_{2,M,2,-M}}$, in $4e$, 4μ, $2e2\mu$ channels separately:

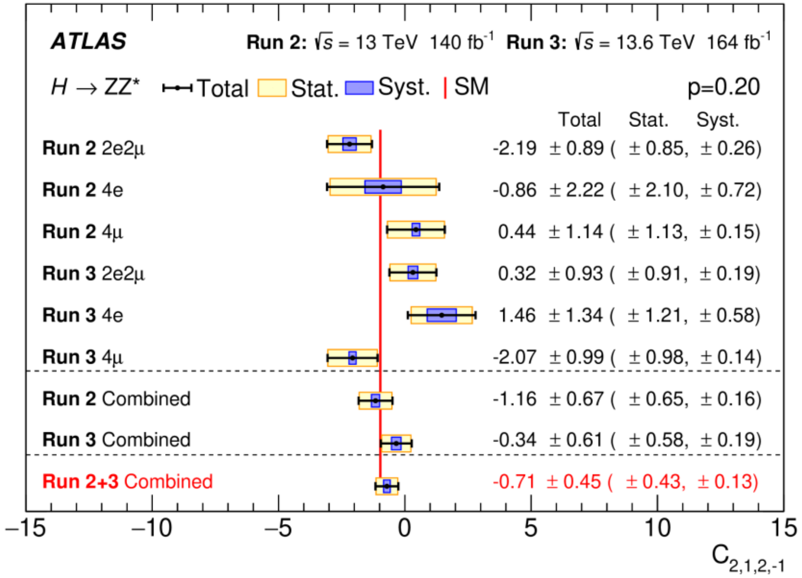
$$\boldsymbol{C_{2,M,2,-M} = \langle c_{2,M,2,-M} \rangle \equiv \frac{(4\pi)^2}{B_2 B_2} \left\langle Y_2^M(\Omega_1)\, Y_2^{-M}(\Omega_2) \right\rangle}$$

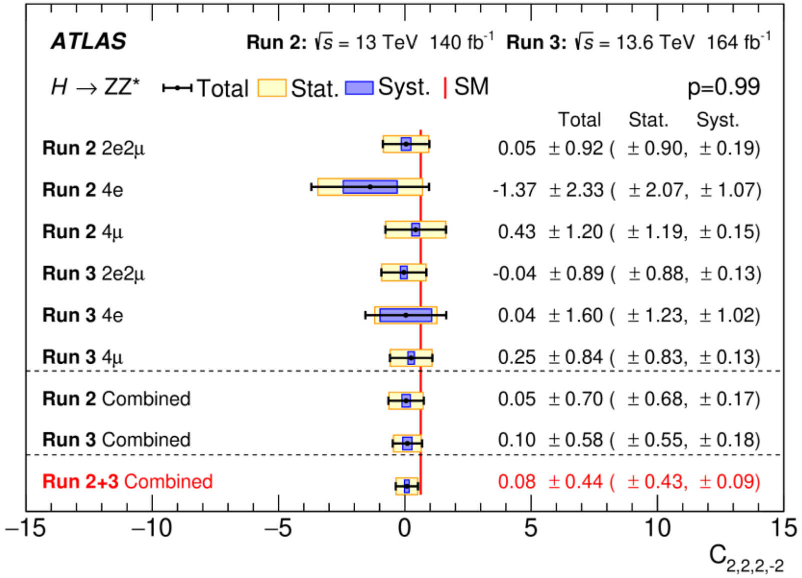


- Workflow: from data to full phase space at the Truth level
  - Calculate data average $\quad C^{Data} = \frac{1}{N_{Data}} \sum_i^{N_{Data}} c_i \quad \oplus$ Data stat. unc.
  - Background subtraction $\quad C_{Reco}^{HZZ} = \frac{N_{Data} C^{Data} - N_{BKG} C^{BKG}}{N_{Data} - N_{BKG}} \quad \oplus$ BKG exp., theo. unc.
  - Reco-Truth Extrapolation $\quad C_{Truth,\, no\ bias\ corr.}^{HZZ} = k C_{Reco}^{HZZ} + b \quad \oplus$ Extrapolation function unc.
  - Intf./EW NLO corr. $\quad C_{Truth}^{HZZ} = C_{Truth,\, no\ bias\ corr.}^{HZZ} + \Delta C_{intf.}^{4e/4\mu} \quad \oplus$ Intf., EW NLO unc.

- Follow the BLUE procedure for channel combination: $\{2e2\mu, 4e, 4\mu\} \times \{Run2, Run3\}$.

Method 1: C_{212-1} and C_{222-2}





Coefficient	$C_{2,1,2,-1}$			$C_{2,2,2,-2}$		
Channel	$2e2\mu$	$4e+4\mu$	Combined	$2e2\mu$	$4e+4\mu$	Combined
Observed	-0.98 ± 0.64	-0.44 ± 0.63	-0.71 ± 0.45	0.00 ± 0.64	0.15 ± 0.61	0.08 ± 0.44
SM expected	-0.97			0.64		
Leading uncertainty components						
Fake-lepton bkg.	±0.16	±0.16	±0.12	±0.11	±0.18	±0.12
Irreducible bkg.	±0.05	±0.04	±0.04	±0.01	±0.01	±0.01
Interference	—	±0.04	±0.02	—	±0.03	±0.02
EW NLO effects	±0.02	±0.01	±0.01	<0.01	<0.01	<0.01
Migration	±0.01	±0.02	±0.01	±0.01	±0.02	±0.01
Statistical	±0.62	±0.61	±0.43	±0.63	±0.58	±0.43

$$C_{2,1,2,-1} = -0.71 \pm 0.43\ (Data\ Stat.) \pm 0.13\ (Syst.) = -0.71 \pm 0.45$$

$$C_{2,2,2,-2} = 0.08 \pm 0.43\ (Data\ Stat.) \pm 0.12\ (Syst.) = 0.08 \pm 0.44$$

Agree with SM prediction, however, sensitivity constrained fiercely by statistical uncertainty:

$$|C_{2,1,2,-1} - 0| = 1.6\sigma_{C_{2,1,2,-1}},\ |C_{2,2,2,-2} - 0| = 0.2\sigma_{C_{2,2,2,-2}}$$

Analysis methodology 2: Hypothesis test

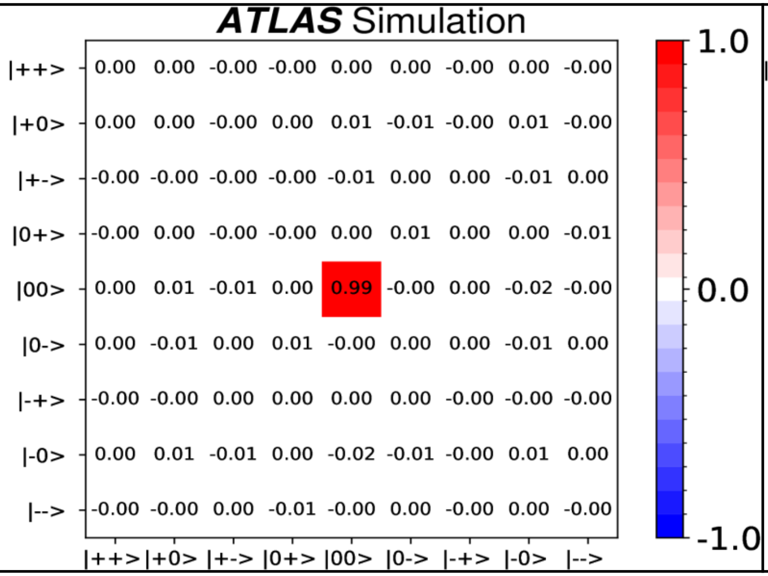
- To directly use QE criteria requires high statistics: $5\sigma \leftrightarrow \sim 3000\ fb^{-1}$ data.
- Another valid approach: hypothesis test rejecting the non-QE template.
  - The state of the $\mathbf{H}\boldsymbol{V_L V_L}$ (longitudinally polarized) process is separable: $|\mathbf{00}\rangle_{\mathbf{AB}} = |\mathbf{0}\rangle_{\mathbf{A}} \otimes |\mathbf{0}\rangle_{\mathbf{B}}$
  - For events from Higgs process (conclusion from arxiv:2209.14033 by J.A.A.S):

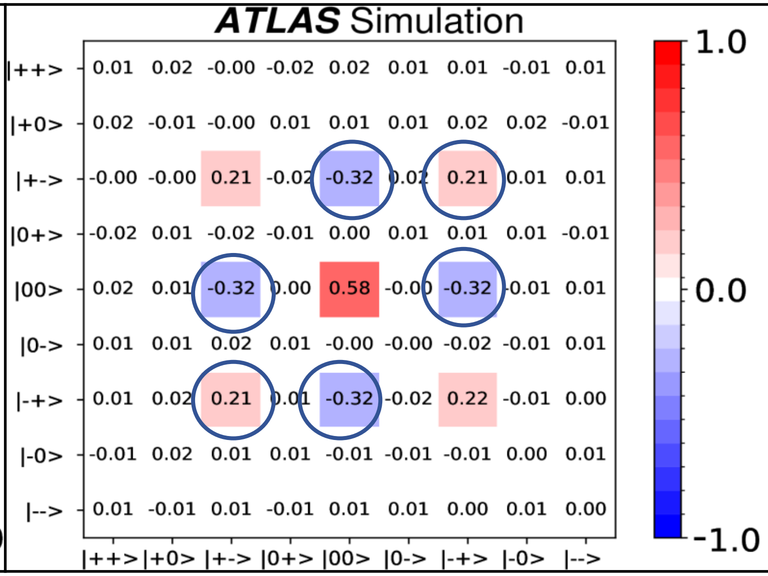
HVV system not in pure $|00\rangle$ state ⇔ Non-separable (Entangled)

SDM for HZ_LZ_L:

$C_{2,1,2,-1} = 0$
$C_{2,2,2,-2} = 0$

→Non-QE





SDM for HZZ:

$C_{2,1,2,-1} \neq 0$
$C_{2,2,2,-2} \neq 0$

→Entangled

Hypothesis test

- Fitting discriminants: c_{222-2} and c_{212-1} (same observables in method 1).
- Based on Two templates made up by:
  - HZZ (QE) Signal + ZZ background ($isQE = 1$)
  - HZ_LZ_L (non-QE) Signal + ZZ background ($isQE = 0$)

$$\mathcal{L}(data|\boxed{isQE},\mu,\vec{\theta}) = \prod_{i}^{N_{bins}} Poisson\left(N_i|\mu S_i^{(isQE)}(\vec{\theta}) + B_i(\vec{\theta})\right) \times Gauss(\vec{\theta})$$

Post-fit entangled template

Post-fit separable template

- Apply post-fit template μ
- randomize NPs (including μ)
- randomize Poisson

QE Toy

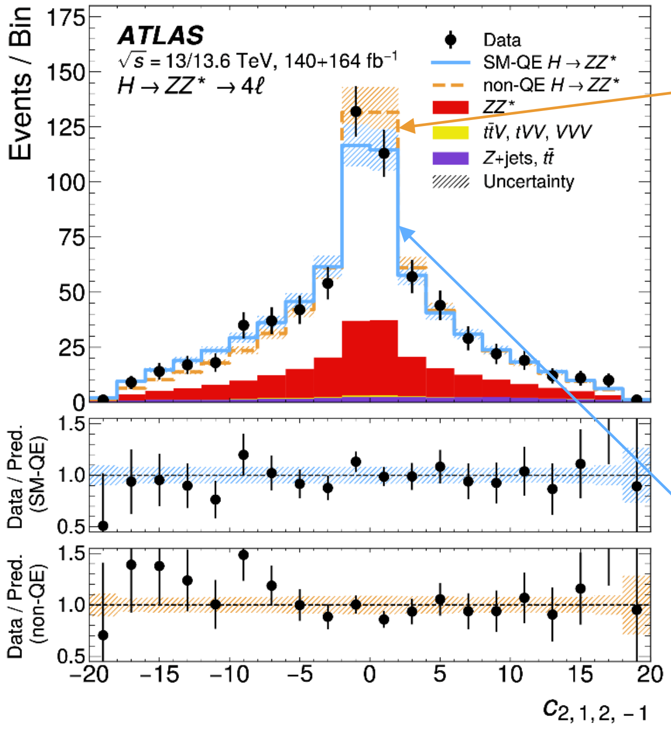
non-QE Toy

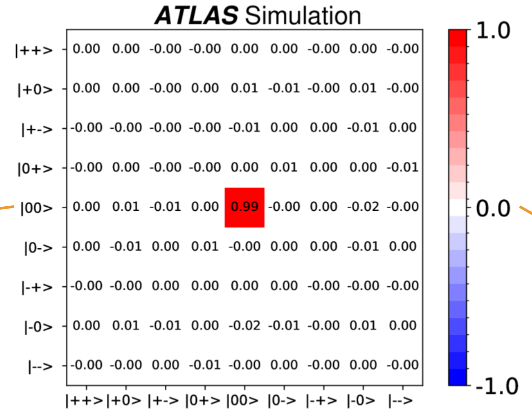
profile both to each toy data

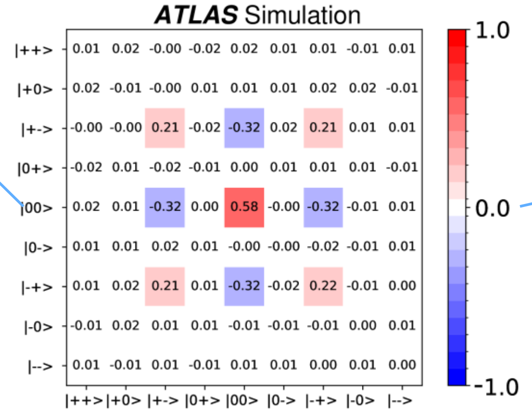
$$\tilde{q} = log\frac{\mathcal{L}(data|isQE = 1, \mu_{QE}, \hat{\hat{\theta}}_{isQE=1})}{\mathcal{L}(data|isQE = 0, \mu_{nonQE}, \hat{\hat{\theta}}_{isQE=0})}.$$

- Compare $\tilde{q}_{Data}$ with the two distributions made up by $\tilde{q}_{QE\,TOY}$ and $\tilde{q}_{non-QE\,TOY}$.

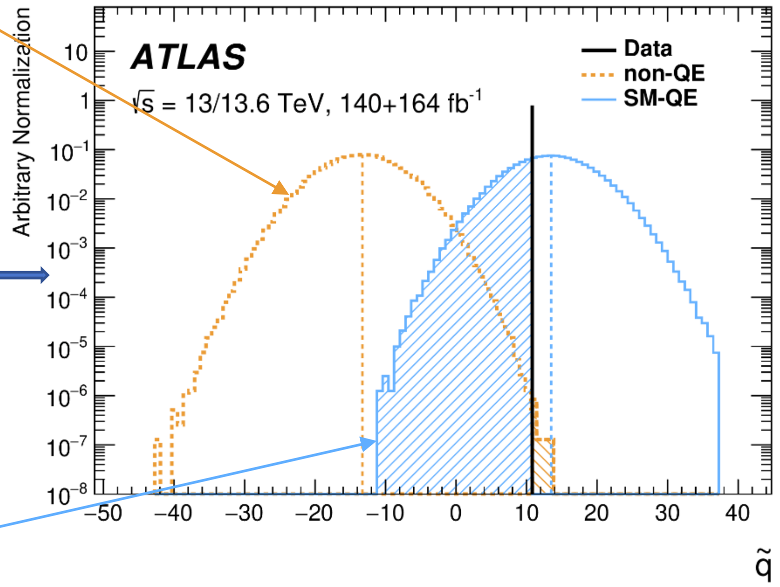
Hypothesis test







$$\tilde{q} = log \frac{\mathcal{L}(data|isQE=1, \mu_{QE}, \hat{\hat{\theta}}_{isQE=1})}{\mathcal{L}(data|isQE=0, \mu_{nonQE}, \hat{\hat{\theta}}_{isQE=0})}.$$

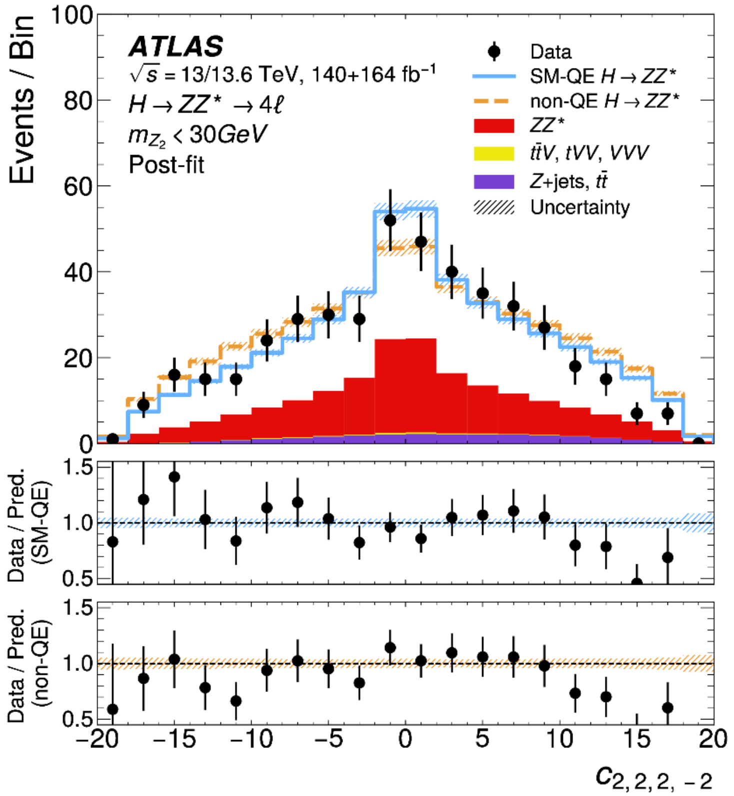


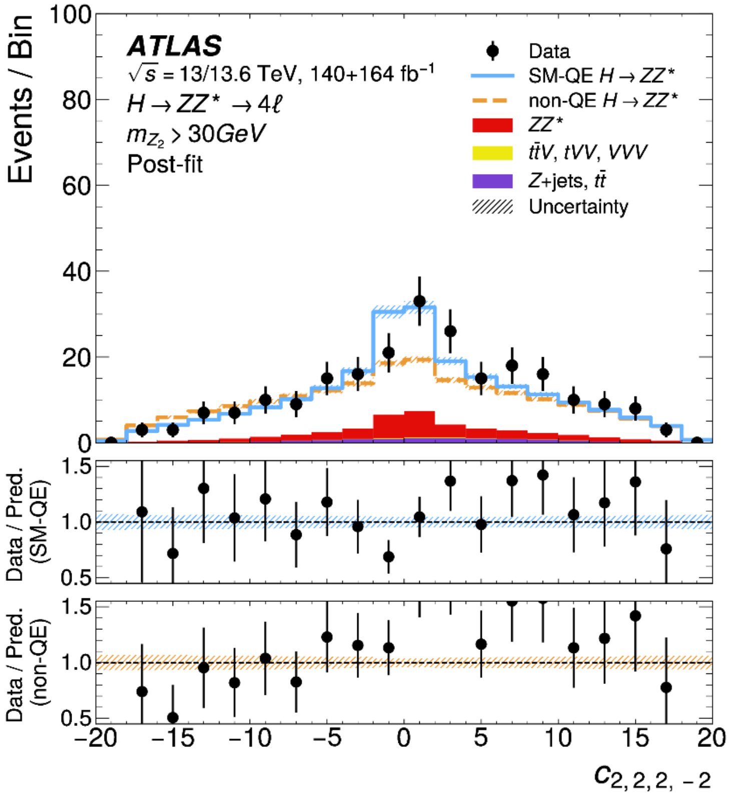
$$CL_s(nonQE) = \frac{\int_{\tilde{q}^{data}}^{+\infty} d\tilde{q} f^{non-QE}(\tilde{q})}{1 - \int_{-\infty}^{\tilde{q}^{data}} d\tilde{q} f^{QE}(\tilde{q})}$$

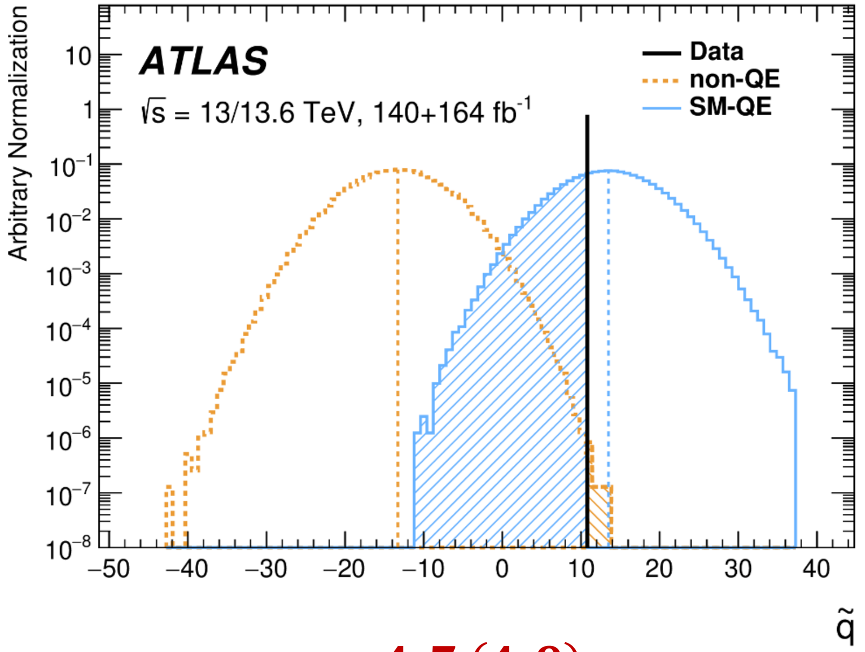
- Make full use of the line shape, instead of merely the mean value → Higher sensitivity, but more model dependent.

Method 2: Hypothesis Test Result

- The SR further split into SR1 ($m_{Z_2} < 30\mathrm{GeV}$) and SR2 ($m_{Z_2} > 30\mathrm{GeV}$), then combine SR1/SR2 for hypothesis test to enhance the significance.
- The longitudinally polarized HZZ process dense more at the low m_{Z_2} region.







$4.7\ (4.9)\ \sigma$

Summary

- We implemented the first measurement of Z-boson pair entanglement in decays of Higgs bosons.
- The observed (expected) values for the Peres-Horodecki witnesses:

$$C_{2,1,2,-1} = -0.71 \pm 0.45\ (-0.97 \pm 0.47)$$

$$C_{2,2,2,-2} = 0.08 \pm 0.44\ (0.63 \pm 0.46)$$

- A complementary hypothesis test disfavors the separable-state hypothesis at a significance of 4.7 standard deviations (expected 4.9σ) relative to the entangled Standard Model hypothesis.

Thanks!

Backup

Tensor basis based on $\mathrm{SU}(2)_{\mathrm{Spin}}$

A convenient way to parametrize the 9×9 spin density-operator of the two vector bosons is to use the basis of irreducible tensor operators $\{T^{L_1}_{M_1}\otimes T^{L_2}_{M_2}\}$ [18], where

$$T^{L_1}_{M_1}, T^{L_2}_{M_2} \in \{\mathbb{1}_3; T^1_1, T^1_0, T^1_{-1}; T^2_2, T^2_1, T^2_0, T^2_{-1}, T^2_{-2}\}. \quad (13)$$

Here T^L_M are normalized such that $\mathrm{Tr}\{T^L_M(T^L_M)^\dagger\} = 3$, where $(T^L_M)^\dagger = (-1)^M T^L_{-M}$. More precisely, defining J_x, J_y and J_z as the spin-1 component operators, we have $T^1_{\pm1} = \mp\sqrt{3}/2(J_x \pm iJ_y)$ and $T^1_0 = \sqrt{\frac{3}{2}}J_z$, i.e.

$$T^1_1 = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad T^1_0 = \sqrt{\frac{3}{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$T^1_{-1} = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \quad (14)$$

On the other hand,

$$\begin{aligned} T^2_{\pm2} &= \frac{2}{\sqrt{3}}(T^1_{\pm1})^2, \\ T^2_{\pm1} &= \sqrt{\frac{2}{3}}[T^1_{\pm1}T^1_0 + T^1_0 T^1_{\pm1}], \\ T^2_0 &= \frac{\sqrt{2}}{3}[T^1_1 T^1_{-1} + T^1_{-1}T^1_1 + 2(T^1_0)^2]. \end{aligned} \quad (15)$$

$$T^2_2 = \sqrt{3}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad T^2_{-2} = \sqrt{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$T^2_1 = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad T^2_{-1} = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix},$$

$$T^2_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (16)$$

CGLMP inequality

CGLMP inequality,

$$
\begin{aligned}
\mathcal{I}_3 = &\left[P(a_1 = b_1) + P(b_1 = a_2 + 1) + P(a_2 = b_2) + P(b_2 = a_1)\right] \\
&- \left[P(a_1 = b_1 - 1) + P(b_1 + a_2) + P(a_2 = b_2 - 1) + P(b_2 = a_1 - 1)\right]
\end{aligned}
$$

$$I_3 = \mathrm{Tr}\{\rho \mathcal{O}_{\mathrm{Bell}}\} > 2.$$

$$
\begin{aligned}
\mathcal{O}_{\mathrm{Bell}} = &\left(\frac{2}{3\sqrt{3}}(T_1^1 \otimes T_1^1 - T_0^1 \otimes T_0^1 + T_1^1 \otimes T_{-1}^1) + \frac{1}{12}(T_2^2 \otimes T_2^2 + T_2^2 \otimes T_{-2}^2)\right. \\
&\left. + \frac{1}{2\sqrt{6}}(T_2^2 \otimes T_0^2 + T_0^2 \otimes T_2^2) - \frac{1}{3}(T_1^2 \otimes T_1^2 + T_1^2 \otimes T_{-1}^2) + \frac{1}{4}T_0^2 \otimes T_0^2\right) + \mathrm{H.c.} \qquad (41)
\end{aligned}
$$

In general, the state of the ZZ system is given by the density operator ρ shown in Eq. (26) and the corresponding prediction for $I_3 = \mathrm{Tr}\{\rho \mathcal{O}_{\mathrm{Bell}}\}$ reads

$$I_3 = \frac{1}{36}(18 + 16\sqrt{3} - \sqrt{2}(9 - 8\sqrt{3})A_{2,0}^1 - 8(3 + 2\sqrt{3})C_{2,1,2,-1} + 6C_{2,2,2,-2}). \qquad (42)$$

Concurrence

https://arxiv.org/pdf/2504.03841v2

We can thus rewrite the lower ($\mathcal{C}_{LB}$) and upper ($\mathcal{C}_{UB}$) bounds on the squared concurrence as following:

$$
\begin{aligned}
(\mathcal{C}(\rho))^2 \geq \frac{2}{9}\max\Bigg[&\Big(-2-2\sum_{L,M}|A^a_{L,M}|^2+\sum_{L,M}|A^b_{L,M}|^2+\sum_{L_a,L_b,M_a,M_b}|C_{L_a,M_a,L_b,M_b}|^2\Big),\\
&\Big(-2+\sum_{L,M}|A^a_{L,M}|^2-2\sum_{L,M}|A^b_{L,M}|^2+\sum_{L_a,L_b,M_a,M_b}|C_{L_a,M_a,L_b,M_b}|^2\Big)\Bigg], \qquad (2.48)\\
(\mathcal{C}(\rho))^2 \leq \frac{2}{3}\min\Bigg[&\Big(2-\sum_{L,M}|A^a_{L,M}|^2\Big),\Big(2-\sum_{L,M}|A^b_{L,M}|^2\Big)\Bigg]. \qquad (2.49)
\end{aligned}
$$

While these quantities by themselves do not provide a precise measure of entanglement, they can be used to establish the presence of entanglement: $\mathcal{C}_{LB} > 0$ implies a non vanishing concurrence and consequently an entangled state, on the other hand a null $\mathcal{C}_{UB}$ implies a separable state.

η_l expression

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_L^M(\Omega_j) d\Omega_j = \frac{B_L}{4\pi} A_{LM}^j, \qquad j = 1, 2\,.$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_{L_1}^{M_1}(\Omega_1) Y_{L_2}^{M_2}(\Omega_2) d\Omega_1 d\Omega_2 = \frac{B_{L_1} B_{L_2}}{(4\pi)^2} C_{L_1 M_1 L_2 M_2}\,,$$

$$B_1 = -\sqrt{2\pi}\eta_\ell\,, \quad B_2 = \sqrt{\frac{2\pi}{5}}\,.$$

$$\eta_l = \begin{cases} \dfrac{1-4\sin^2\theta_W}{1-4\sin^2\theta_W+8\sin^4\theta_W} & \text{for } Z \\ -1 & \text{for } W^+ \\ +1 & \text{for } W^- \end{cases}$$

Entanglement Judgement

- A system is not entangled $\leftrightarrow$ It is <u>separable</u>!

$$|VV\rangle_{AB} = |V\rangle_A \otimes |V\rangle_B \quad \text{or} \quad \rho = \sum_i p_i\, \rho_i^A \otimes \rho_i^B$$

- For a pure bi-qutrit system, Von Neumann entropy $S(\rho_A)$, where $\rho_A \equiv \mathrm{Tr}_B \rho$:

$$S(\rho_A) \equiv -\mathrm{Tr}(\rho_A log_3 \rho_A) = 0 \leftrightarrow Separable$$

- However, on the LHC, the vector bosons are produced in mixed states, because of different kinematical configurations, i.e.:

$$\rho = \int d\beta\, \mathcal{P}(\beta) \rho_\beta$$

- Thus, entanglement witnesses are needed to quantify the separability.
- Easier to judge whether ρ (density matrix) is non-separable $\leftrightarrow$ Entangled.

Object and Event Selection

- Well developed experiences from previous HZZ analyses, e.g. PLB 843(2023)137880
- Two same flavored opposite charged (SFOC) lepton pairs, with (Signal Region):

$$m_{4l} \in (115,130)\ \text{GeV},\ m_{12} \in (50,106)\ \text{GeV},\ m_{34} \in (12,115)\ \text{GeV}$$

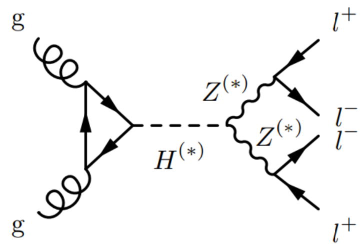
Physics Objects
LEPTONS
Loose Likelihood quality electrons with hit in innermost layer, $E_T > 7$ GeV and $\lvert\eta\rvert < 2.47$
MUONS
Loose identification with $p_T > 5$ GeV and $\lvert\eta\rvert < 2.7$ (2.5) for Run2 (Run3)
Combined, stand-alone (with ID hits if available) and segment-tagged muons with $p_T > 5$ GeV
VERTEX
At least one collision vertex with at least two associated track
PRIMARY VERTEX
Vertex with the largest p_T sum

Reject	Against	Overlap Criteria
electron	electron	shared track, $p_T^1 < p_T^2$, the electron with larger p_T kept
calo muon	electron	shared ID track
electron	muon	shared ID track

Event Selection	
QUADRUPLET SELECTION	- Require at least one quadruplet of leptons consisting of two pairs of same-flavor opposite-charge leptons fulfilling the following requirements: - p_T thresholds for three leading leptons in the quadruplet: 20, 15 and 10 GeV - Leading di-lepton mass requirement: $50 < m_{12} < 106$ GeV - Sub-leading di-lepton mass requirement: $m_{\text{threshold}} < m_{34} < 115$ GeV - $\Delta R(\ell, \ell') > 0.10$ for all leptons in the quadruplet - Remove quadruplet if alternative same-flavour opposite-charge di-lepton gives $m_{\ell\ell} < 5$ GeV - Keep all quadruplets passing the above selection
ISOLATION	- Contribution from the other leptons of the quadruplet is subtracted - FixedCutPFlowLoose WP for Run2 leptons - Loose_VarRad (PFlowLoose_VarRad) WP for Run3 elec (muon)
IMPACT PARAMETER SIGNIFICANCE	- Apply impact parameter significance cut to all leptons of the quadruplet - For electrons: $d_0/\sigma_{d_0} < 5$, $\lvert z_0 \times \sin\theta\rvert < 0.5$ mm - For muons: $d_0/\sigma_{d_0} < 3$, $\lvert z_0 \times \sin\theta\rvert < 0.5$ mm
BEST QUADRUPLET	- If more than one quadruplet has been selected, choose the quadruplet with the highest Higgs decay ME according to channel: 4μ, $2e2\mu$, $2\mu 2e$ and $4e$
VERTEX SELECTION	- Require a common vertex for the leptons: - χ^2/ndof < 5 for 4μ and < 9 for others decay channels

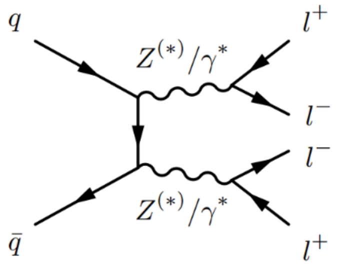
MC Modeling

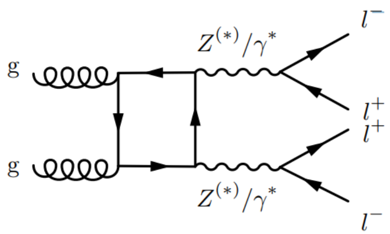
- Signal: SM $\mathrm{H} \to ZZ^* \to 4l$ with Z polarized (For hypothesis test)/unpolarized



Signal	Mode	Generator	Precision
ggF, VBF, VH	ZZ	Powheg+Hto4l+Pythia8	EW NLO, QCD NNLO
	$Z_L Z_L$	MadGraph5+Pythia8	EW LO, QCD NNLO*

- Irreducible Backgrounds (Other SM processes with $4l$ final state)



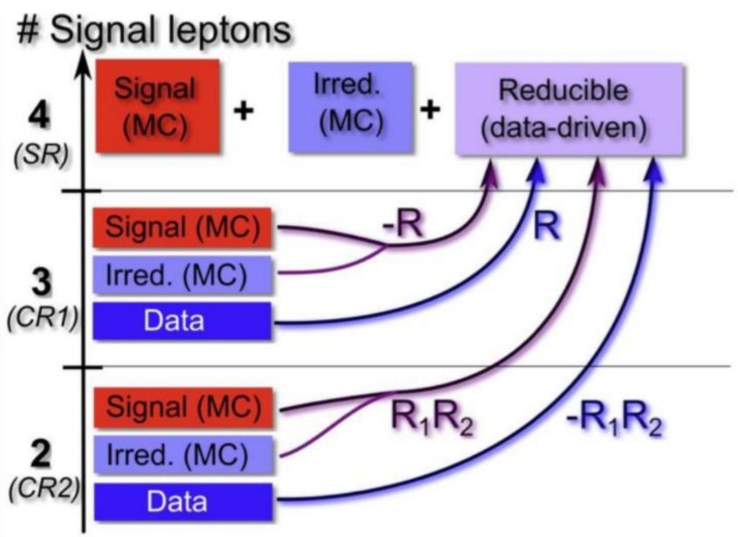


Non-Higgs ZZ	$qq \to ZZ$	Sherpa
	$gg \to ZZ$	
tri-Boson (VVV)	WWZ, WZZ, ZZZ	
$ttll$	ttZ	

*The MadGraph5 samples are simulated at QCD LO, but tuned to QCD NNLO by reweighting Higgs (P_T, η) w.r.t. to the Powheg sample.

Reducible Background

- Reducible/Fake Backgrounds (SM processes with different final states)
  - Recognized as $4l$ events by mistake, due to the detector mis-identification.
  - Poor MC modeling, a data-driven **fake factor method** used.
- Fake Factor, a function of (p_T, η) for electrons and muons separately, representing the mis-identification ratio for leptons
  - Obtained in Fake process Enriched Regions.
  - Applied to SR-nearby, but fake dominant regions (defined by loosing the lepton quality requirement), to obtain the fake prediction in SR.
- Systematics: Statistical uncertainties, choice of Fake enriched region for FF calculation.
- Small contribution to SR yield: ~5% of total SR event yield



Uncertainties

Statistical Uncertainty from data (dominant for both methods)

- For direct measurement, also depends on the intrinsic width of the c_{2M2-M} distribution:

$$\sigma_{C_{2M2-M}}^{\text{Data Stat.}} = \frac{\mathrm{s.t.d.}\left(c_{2M2-M}^{Data}\right)}{\sqrt{N_{Data}}} \times \frac{N_{Data}}{N_{\text{HZZ}}}$$

Experimental Uncertainties

- Detecting uncertainties for electrons and muons (Identification, energy resolution, …)
- Track-to-Vertex Association (TTVA)
- Trigger, Pileup, luminosity uncertainties

Theoretical Uncertainties

- PDF + α_S nncertainty
- QCD renormalization scale uncertainty
- Parton showering uncertainty

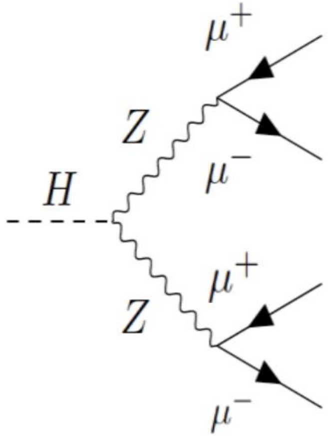
Scale	Value
μ_R	0.5, 0.5, 1.0, 1.0, 2.0, 2.0
μ_F	0.5, 1.0, 0.5, 2.0, 1.0, 2.0

RMS $$\sigma_{\hat{O}}^{\text{PDF}} = \sqrt{\frac{1}{100}\sum_{i=303201}^{303300}\left(\hat{O}^{\text{PDF}_i} - \hat{O}^{\text{Nominal}}\right)^2}$$

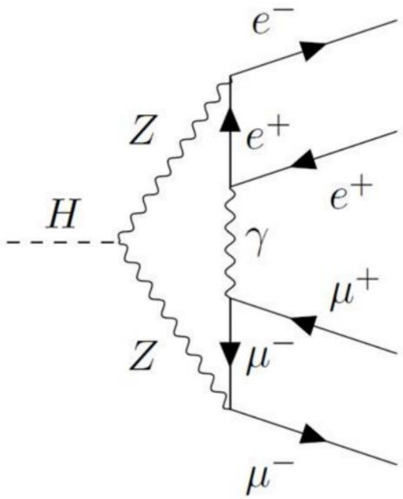
Envelope $$\sigma_{\hat{O}}^{QCD} = \max\left|\hat{O}^{(\mu_F,\mu_R)} - \hat{O}^{\text{Nominal}}\right|$$

Identical Particle and EW NLO Effects

- **Identical particle/mis-pairing effects in same-flavor ($4e$ and 4μ) channels:**
  - $|M(ZZ \to e_1^- e_2^+ e_3^- e_4^+)|^2 = |M_{1,2} + M_{1,4}|^2 = |M_{1,2}|^2 + |M_{1,4}|^2 + 2Re(M_{1,2}M_{1,4}^*)$
  - <u>Patch</u>: Isolate and correct the interfering contribution
    - A correction of $\Delta C_{intf.} = C_{2e2\mu} - C_{4e/4\mu}$ (obtained via MC simulation) applied to $4e$ and 4μ channels
    - Appending a theory uncertainty of $50\% \times \Delta C_{intf.}$

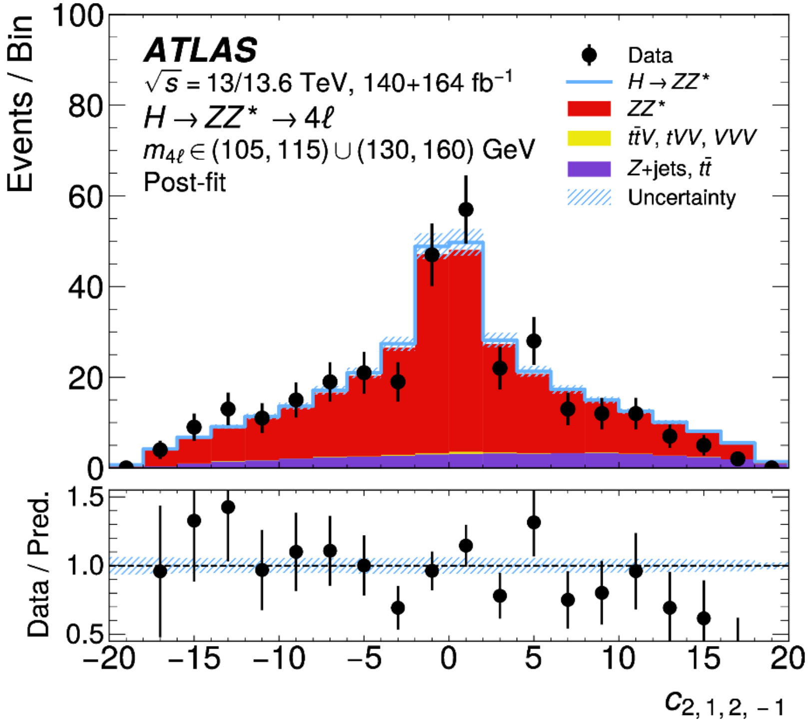


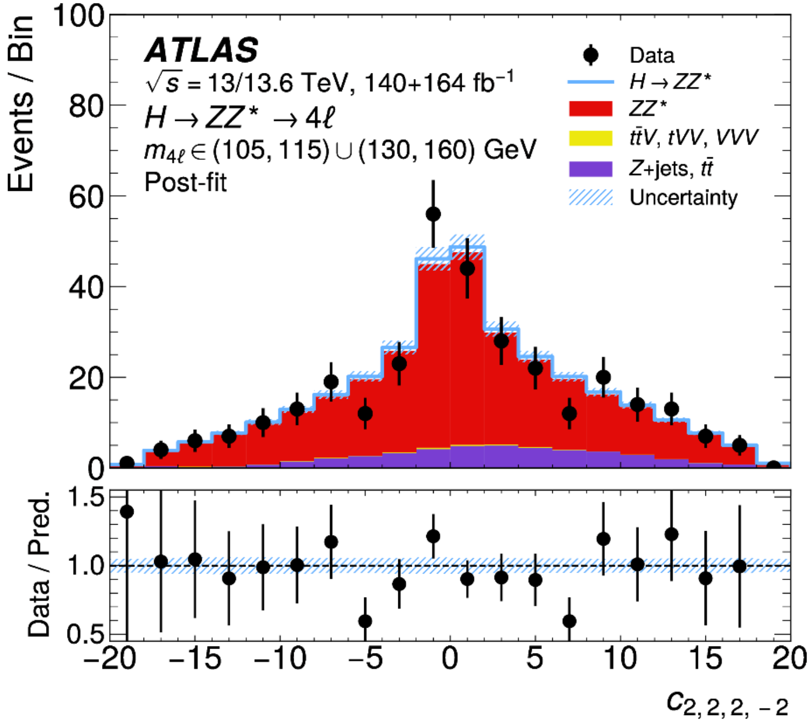
- **EW NLO corrections corrupt the qutrit-qutrit picture:**
  - Virtual contributions with no ZZ states
  - <u>Patch</u>: Considered as a theory uncertainty
    - On $\boldsymbol{C_{222-2}}$ and $\boldsymbol{C_{212-1}}$, the (EW NLO-LO) correction is [tiny]
    - $\Delta C_{2,1,2,-1}^{EW\,NLO} = 1.1\%$, $\Delta C_{2,2,2,-2}^{EW\,NLO} = 1.4\%$



Validation: Background Modeling

- SR: $m_{4l} \in (115{,}130)$ GeV
- Two validation regions (VR) designed for checking background modeling
  - ZZ enriched VR (validating pure ZZ modeling): $m_{4l} \in (170{,}300)$ GeV
  - Sideband VR (more closer to SR): $m_{4l} \in (105{,}115) \cup (130{,}160)$ GeV
- Good agreement between SM prediction and Data

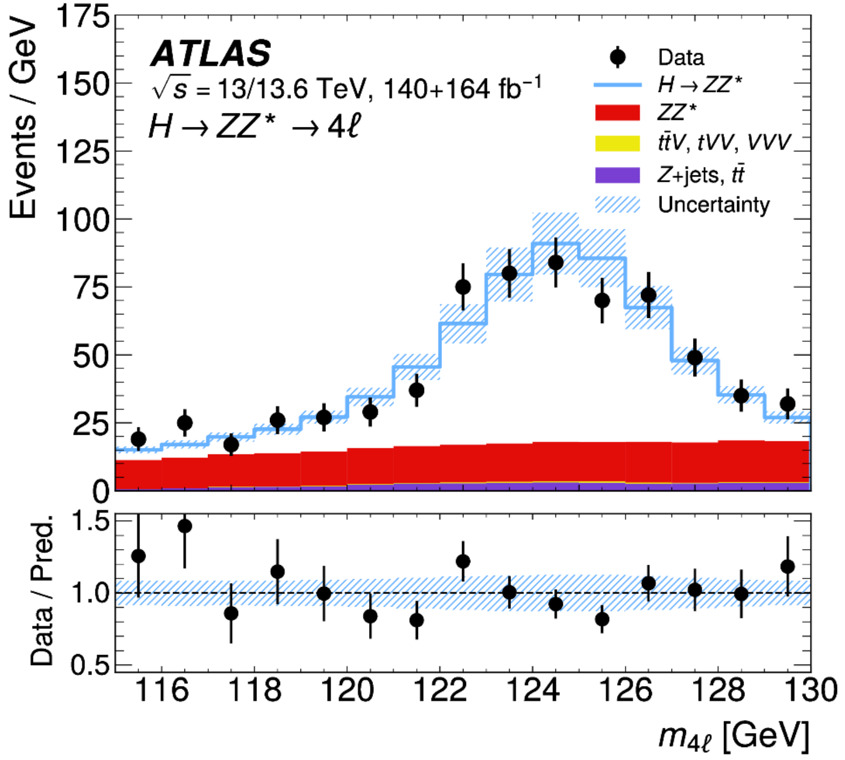




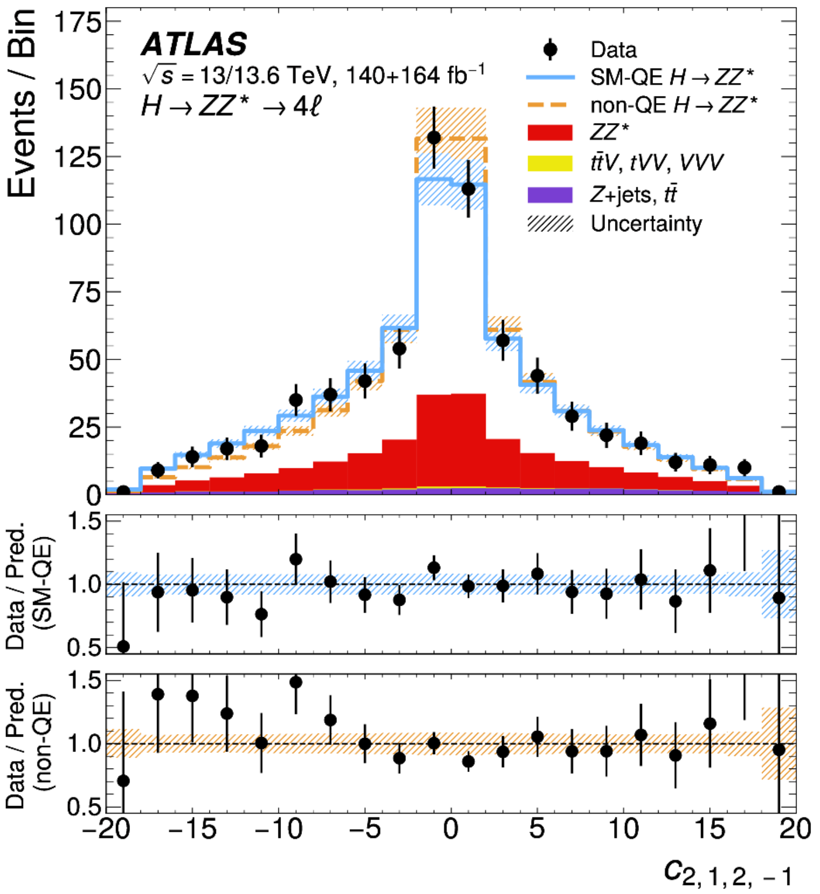
Signal Region Data vs SM

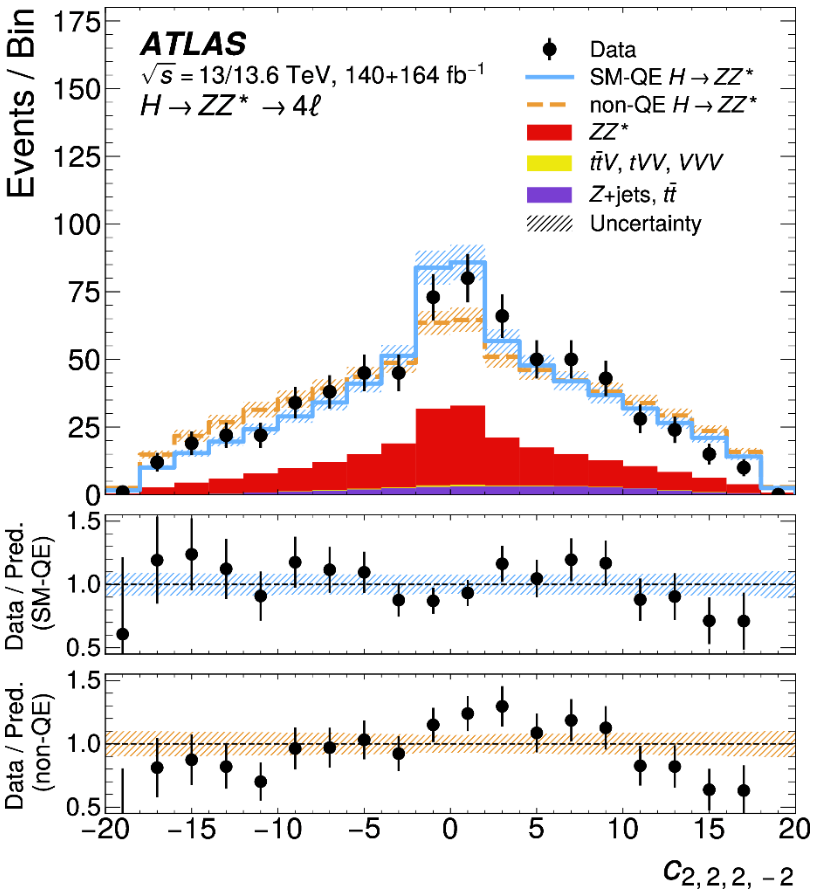
Data yield ~ 680

Signal HZZ yield ~ 440



Also compared Data vs MC (with HZZ signal replaced by HZ_LZ_L)





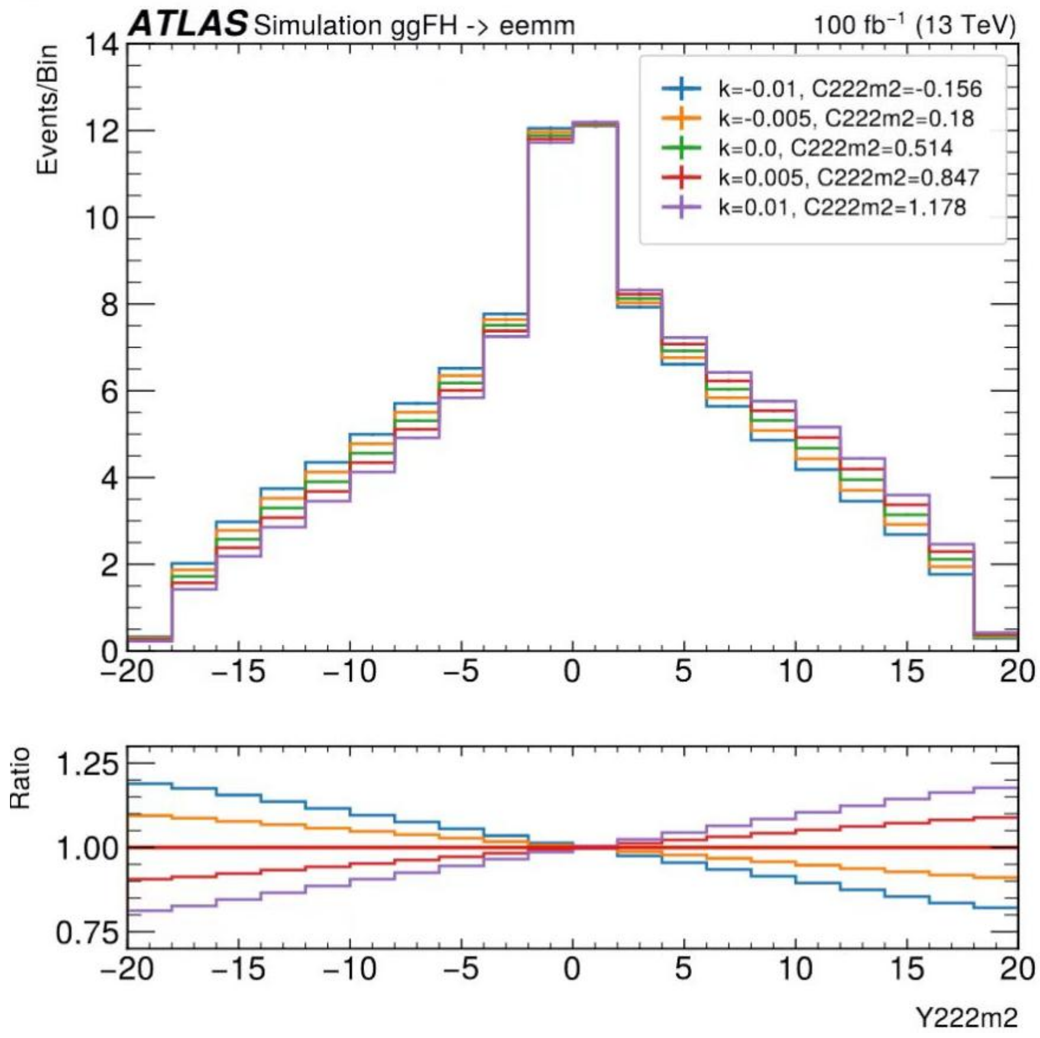
Yield in SR

	Run 2 SR	Run 3 SR
H4l	202.4 ± 0.3	231.2 ± 0.1
ZZ^*	98.1 ± 0.3	97.2 ± 0.2
others	1.8 ± 0.1	2.5 ± 0.1
Non-prompt	15 ± 3	17 ± 4
Predicted	317 ± 3	348 ± 4
Data	311	366

Extrapolation Function

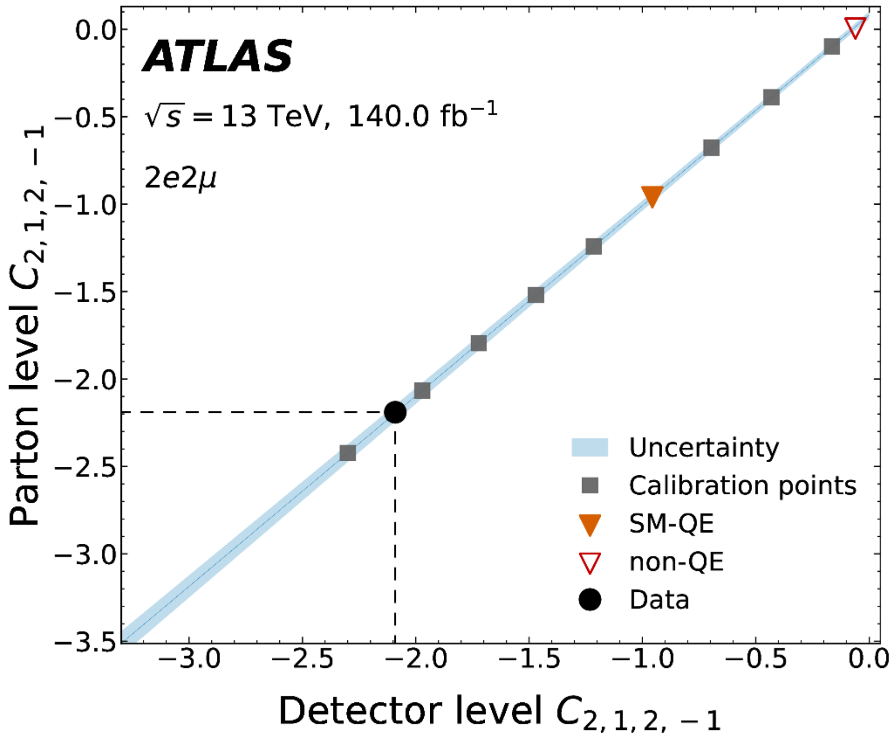
- To obtain the relationship between C_{Truth}^{HZZ} and C_{Reco}^{HZZ}.

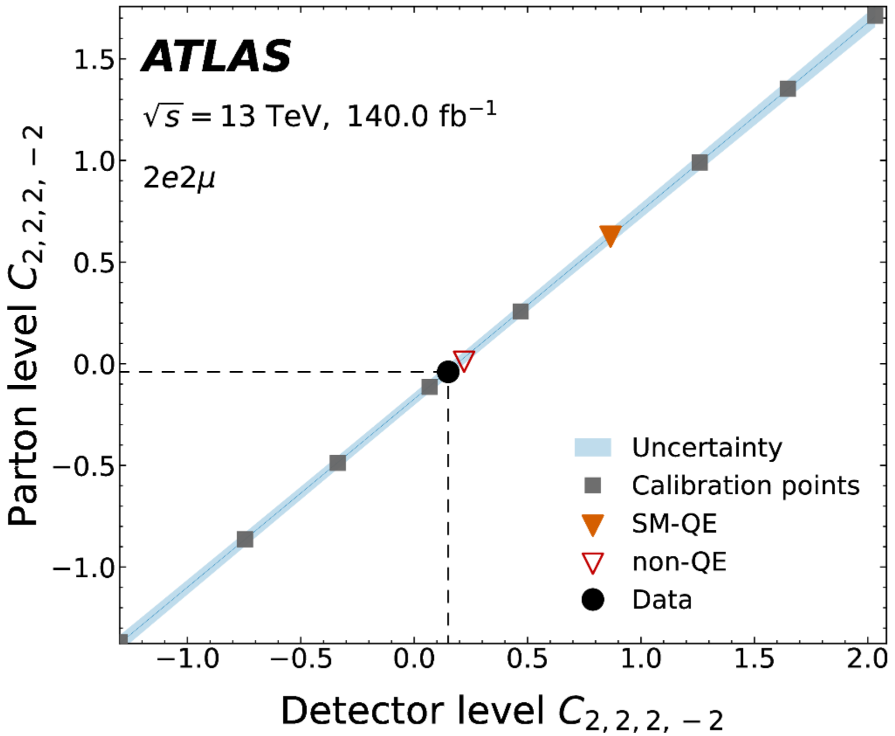
- Reweight the unpolarized HZZ sample to achieve different C_{Truth}^{HZZ} values, see how the corresponding C_{Reco}^{HZZ} evolves. The relation between C_{Truth}^{HZZ} and C_{Reco}^{HZZ} is found to be linear.



Extrapolation Function

- Uncertainties on the extrapolation function
  - Common experimental and theoretical Uncertainties
  - Different choices of the reweighting function, e.g., linear, cubic, quintic
- Alternative longitudinally (non-QE) HZZ sample used for validation





6 sets of extrapolation functions in total for $(4e, 4\mu, 2e2\mu) \times \text{Run } 2/3$

Extrapolation Function: reweighting I

- Tune the $C_{2,2,2,-2}$ and $C_{2,1,2,-1}$ coefficients at Truth level (Full P.S.) → Equivalent to tune the shape of the distributions (whose mean values corresponds to C’s) of:

$$c \equiv \left(\frac{4\pi}{B_2}\right)^2 Y_2^{M_1}(\Omega_1)Y_2^{M_2}(\Omega_2), \mathrm{s.t.} \sim f(c)$$

- Under the full P.S., the C coefficients equal the mean value of c:

$$C = \bar{c} = \frac{\int_{c_{Min}}^{c_{Max}} cf(c)dc}{\int_{c_{Min}}^{c_{Max}} f(c)dc}$$

- Approach: reweight MC events before implementing the integral, with a pre-defined c-dependent reweighting function $\omega(c)$, then $dc \rightarrow \omega(c)dc$:

$$C^{\omega} = \overline{c^{\omega}} = \frac{\int_{c_{Min}}^{c_{Max}} cf(c)\omega(c)dc}{\int_{c_{Min}}^{c_{Max}} f(c)\omega(c)dc}$$

Extrapolation Function: reweighting II

- Implement a simplest linear reweighting function, with a slope of k, with the zero point of the distribution invariant:

$$\omega(c) = kc + 1$$

- Then the mean value after the reweighting equals:

$$C^{\omega} = \overline{c^{\omega}} = \frac{\int_{c_{Min}}^{c_{Max}} cf(c)(kc+1)dc}{\int_{c_{Min}}^{c_{Max}} f(c)(kc+1)dc} = \frac{\bar{c} + k\overline{c^2}}{1 + k\bar{c}}$$

- Where $\overline{c^2}$ corresponds to the mean value of c^2.
- Discussion: obviously $\overline{c^2} \gg \bar{c}$, and when $\bar{c} \gg k$:

$$C^{\omega} \sim \left(\bar{c} + k\overline{c^2}\right)(1 - k\bar{c}) \sim C + \left(\overline{c^2} - \bar{c}^2\right)k$$

$$\Delta C \sim \left(\overline{c^2} - \bar{c}^2\right)k$$

- Which means the shift of C is approximately proportional to k.

Method 2: Hypothesis Test Result

Observed (Expected) significance based on

- $c_{2,1,2,-1}$: $4.0\ (4.0)\ \sigma$
- $\boldsymbol{c_{2,2,2,-2}}$**:** $\mathbf{4.7\ (4.9)}\ \boldsymbol{\sigma}$

A combination of $\boldsymbol{c_{222-2}}$ and $\boldsymbol{c_{212-1}}$ is not realistic under the current statistics.

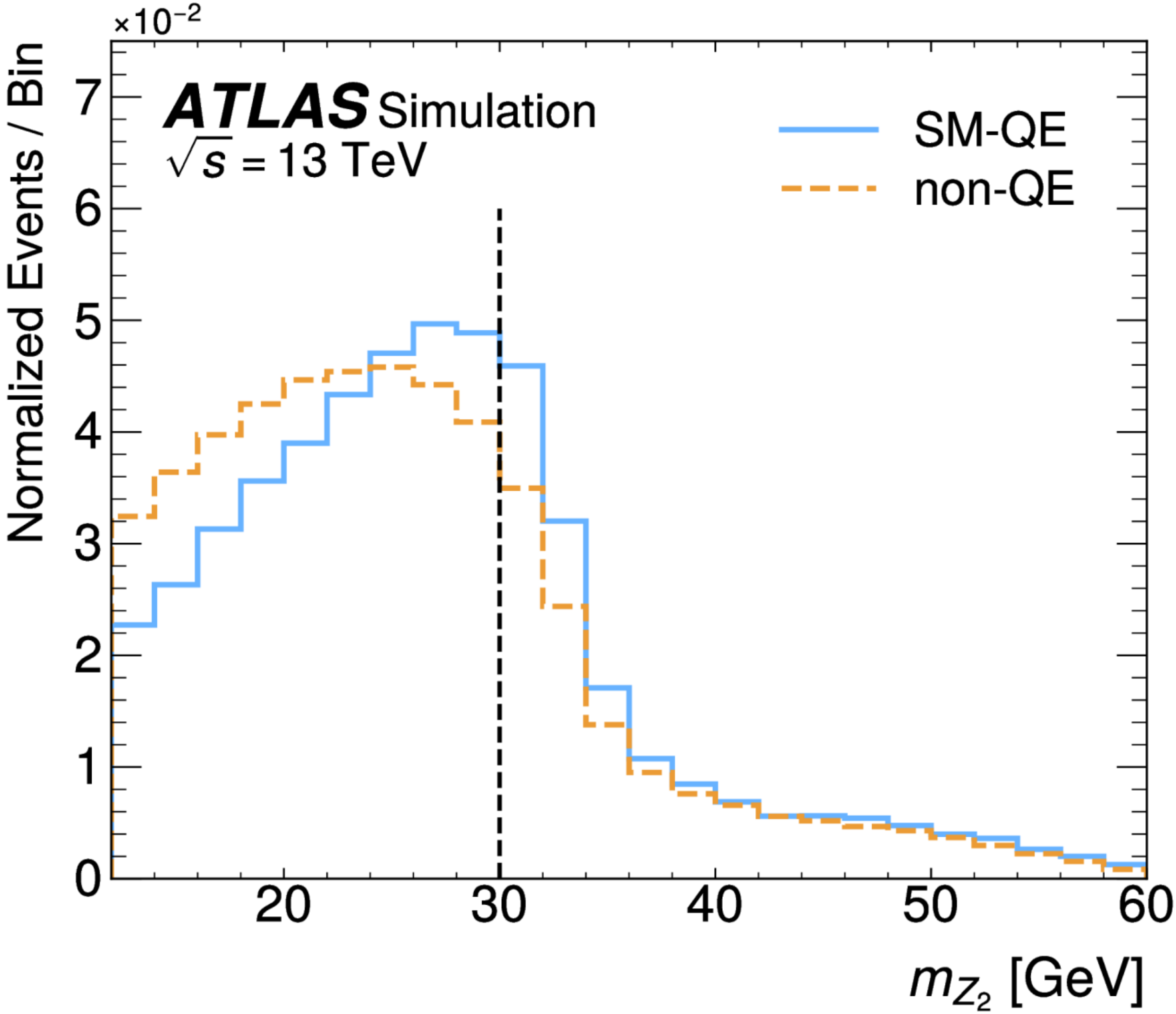
m_{Z_2} dependence

$$|\psi_{ZZ}\rangle = \eta_{\mu\nu} e^{\mu}_{\sigma}(m_{Z_1}, \vec{k}) e^{\nu}_{\lambda}(m_{Z_2}, -\vec{k}) |\vec{k}, \sigma\rangle_A |-\vec{k}, \lambda\rangle_B,$$

where σ, λ represent spin states and

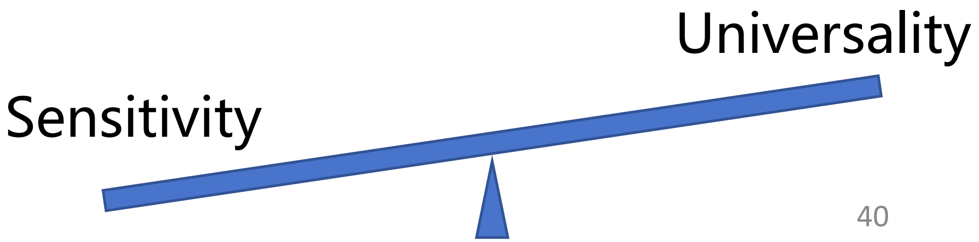
$$e^{\mu}_{\sigma}(m, \vec{k}) = \begin{pmatrix} 0 & \boxed{\frac{|\vec{k}|}{m}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & -\frac{\sqrt{|\vec{k}|^2 + m^2}}{m} & 0 \end{pmatrix}.$$

The polarization vector for the $|0,0\rangle \propto \frac{1}{m}$, thus, its production rate enhance at the low m_{Z_2} region compare with the $|+,-\rangle$, $|-,+\rangle$ state



Synergy between the Two Methods

- Direct measurement based on Peres-Horodecki criterion
  - Makes use of only the mean values of the distributions to apply P-H criterion
  - Attempts to reconstruct the exact ZZ^* spin state via quantum tomography
  - Less modeling dependent
  - Universality: the reconstructed state is interpretable in any theoretical framework
- Hypothesis test
  - Makes use of the shapes of distributions to reject pure $|00\rangle$ state
  - Without reconstructing the state via quantum tomography
  - More dependent on the modeling of c distribution
  - Sensitivity: useful in rejecting specific hypotheses



Foreword on QE interpretation

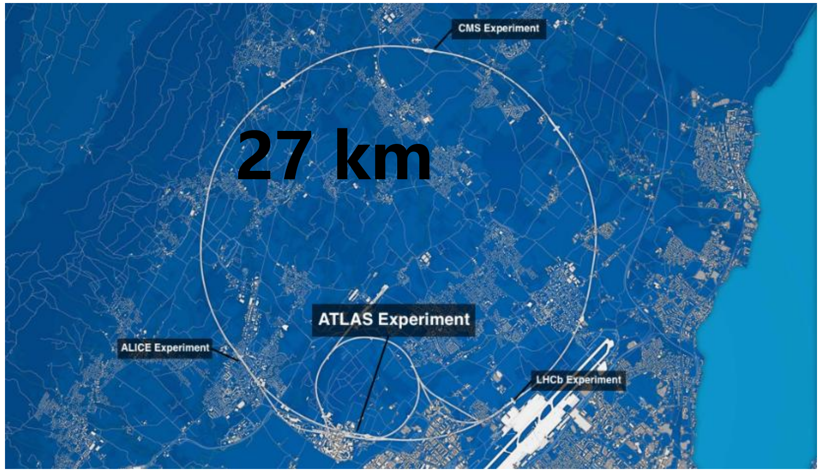
Some critical opinions (e.g. arXiv:2507.15947, arXiv:2507.15949):

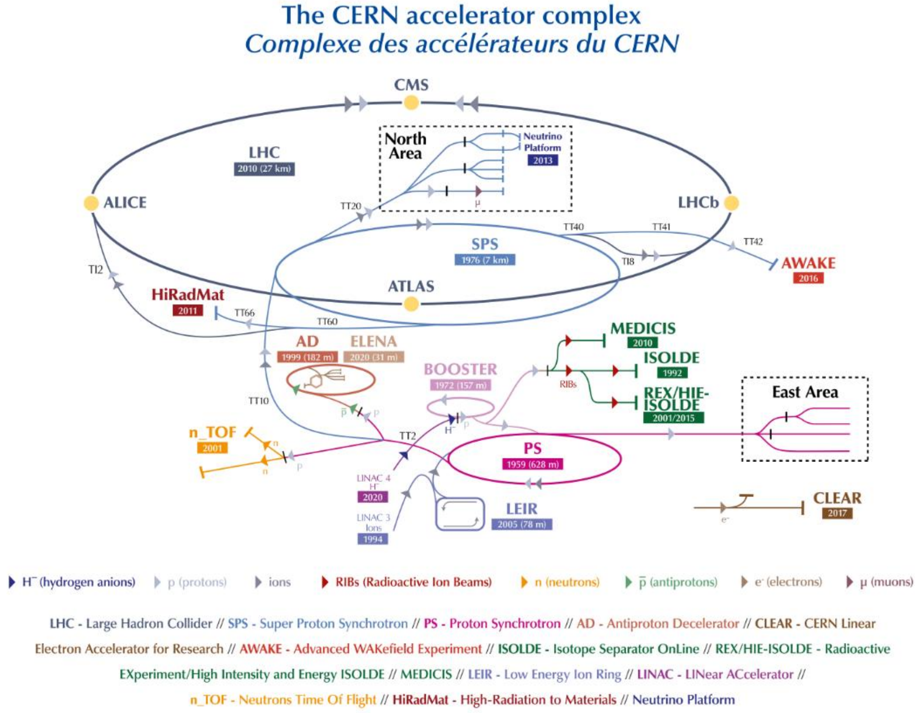
- We cannot measure the spin for Z directly: one have to take the spin analyzing power, or the kinetic distributions as given (these are SM/QM inputs).
- Prove QM by QM, kind of circular proof?

So, some clarification:

- We are analyzing quantum states (under the QM frame) when asking questions of whether separable or non-separable (entanglement).
- There is no circular proof crisis:
    - Not all the quantum states are non-separable.
    - Assuming QM is true $\neq$ Assuming ZZ system is entangled.

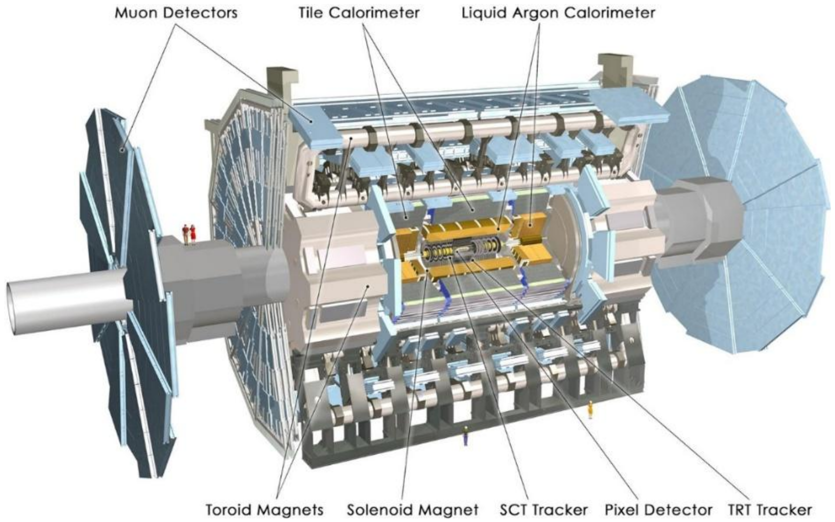
The ATLAS Experiment





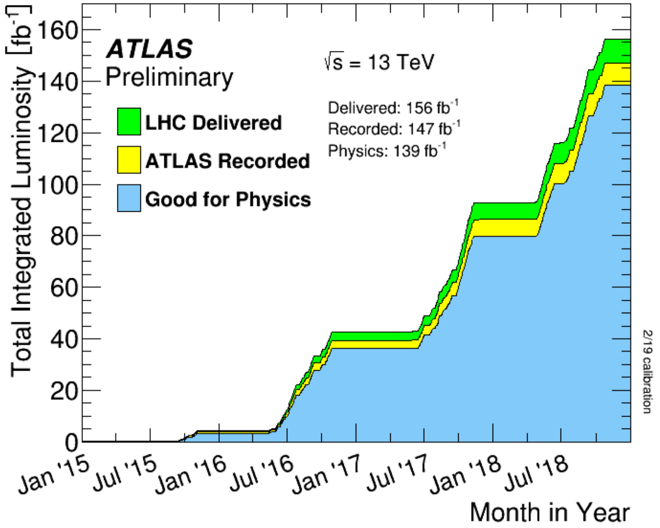
The Large Hadron Collider (High energy frontier ~14 TeV)

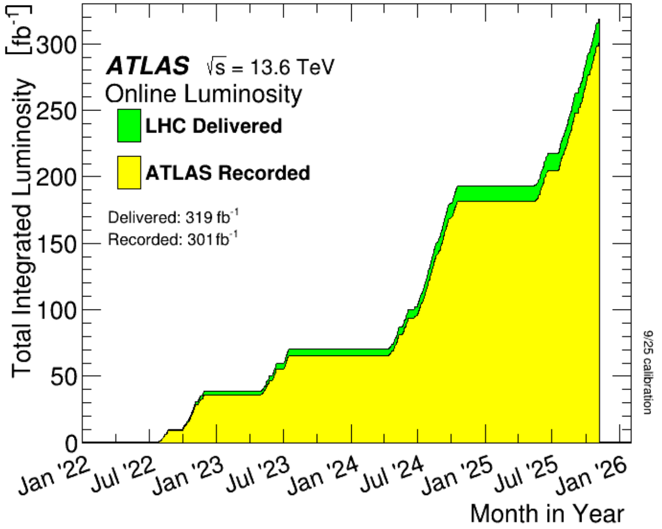
- CERN, Geneva, Switzerland.
- 100 meters underground,
- Colliding ring with circumference of 27 km.
- LHCs: ATLAS, CMS, ALICE, LHCb
- ATLAS is the largest LHC (46m×25m), designed with multi-purpose, covering ~ 4π solid angle.



Data Collection

- This study involves full Run 2 + partial (2022 ~ 2024) Run 3 data.
- Total luminosity: 304 fb^{-1}.





Data taking time line	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2030 -
Run 2	140 fb^{-1}																
Phase 1 upgrade																	
Run3								300+ fb^{-1}									
Phase 2 upgrade																	
Run 4																	