Entanglement in Electroweak Theory

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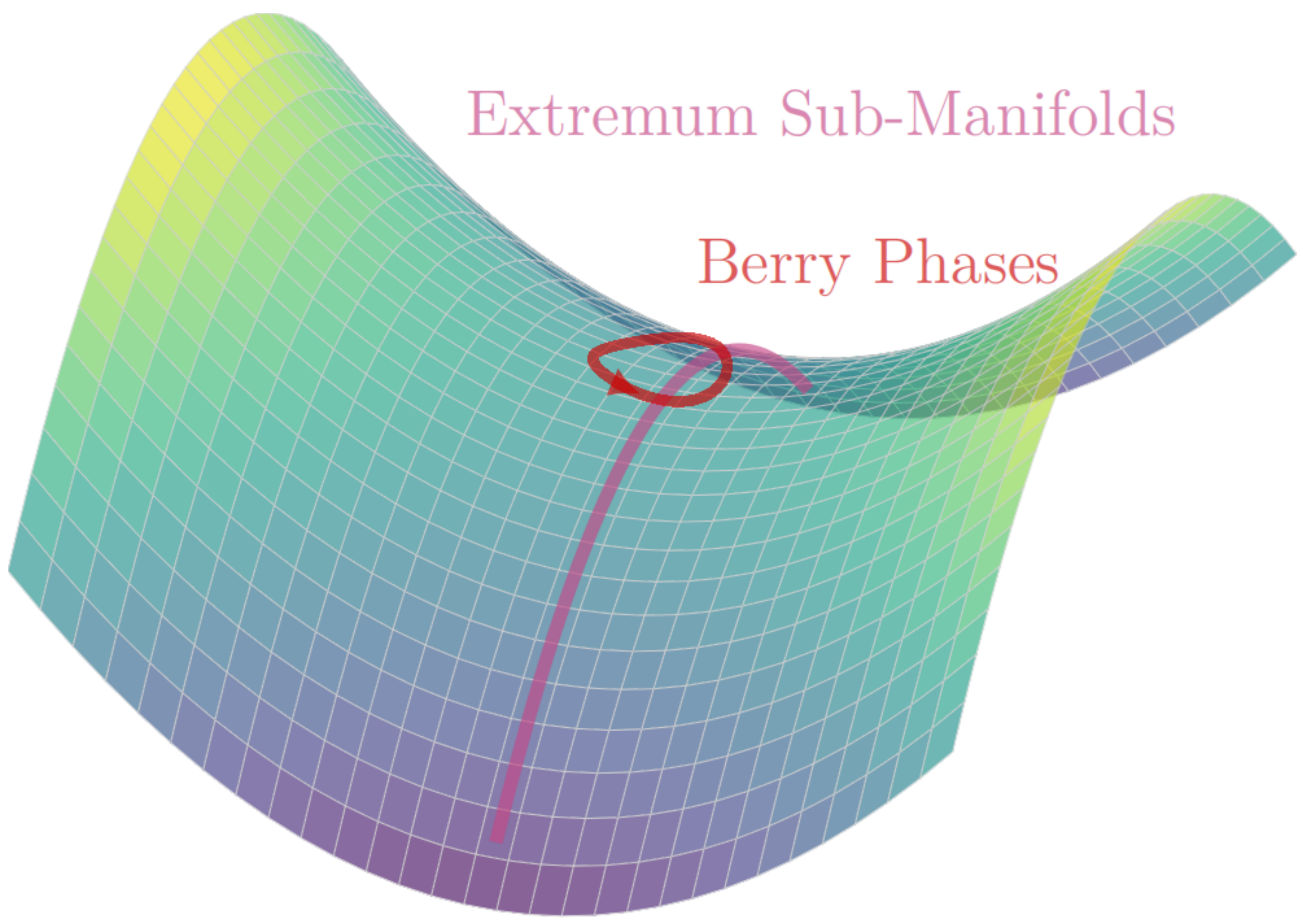
Based on arXiv:2605.22070 and 2607.xxxxx

"It From Qubits" : What does Entanglement Probe ?

$$\mathscr{L}_I(G) = \sum_n \frac{c_n}{\Lambda^n} \hat{\mathcal{O}}_n(G)$$

- Operator forms are bounded by Symmetry G: $\hat{\mathcal{O}} \to g^{-1}\hat{\mathcal{O}}g, g \in G$
- Wilson Coefficients are matched by Experiments $c_n\left(\mu\right)$



Parameter Space

Ab initio Parameter Constraints ?

Entanglement Entropy (EE) in Parameter Space

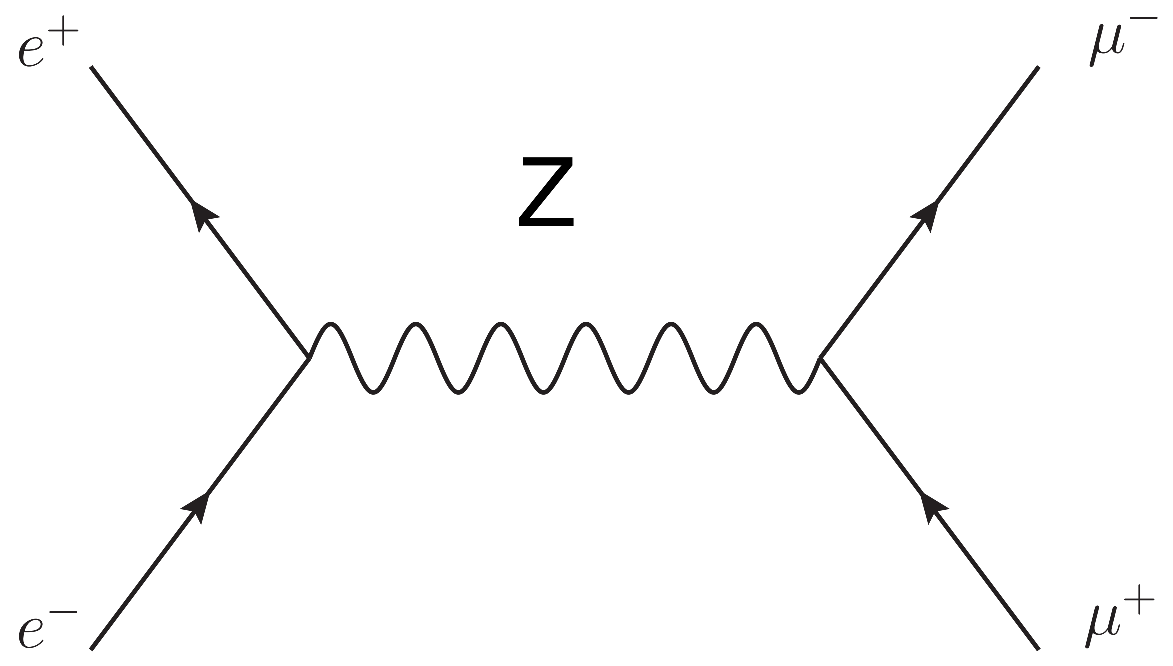
$$\frac{\delta S\{c_n\}}{\delta c_n} = 0 \qquad S\{c_n\} \geqslant 0 \qquad \ldots\ldots$$

[J. Thaler et al., arXiv:2410.23343]
[Q. Liu et al., arXiv:2509.12851]
……

[Q.-H Cao et al., arXiv:2211.08065]
[R. Aoude et al., arXiv:2402.16956]
……

A puzzle to us

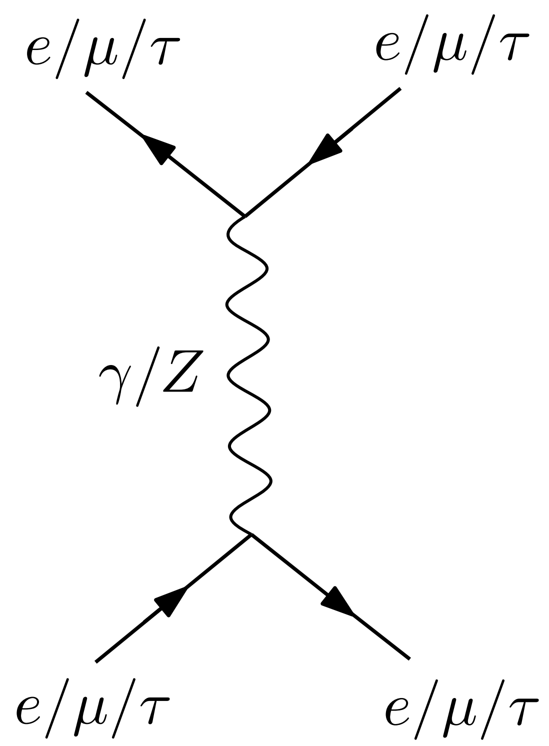
Cervera-Lierta, Latorre, Rojo, Rottoli,1703.02989

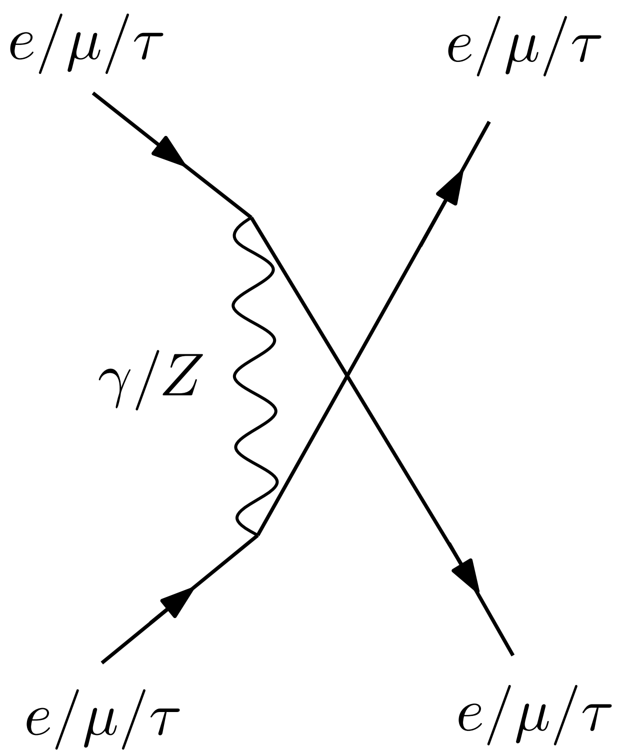


Maximal entanglement
(cocurrentce)
occurs at $g_V = 0$

$$\sin^2\theta_W = \frac{1}{4}$$

Liu, Low, Yin, 2509.18251





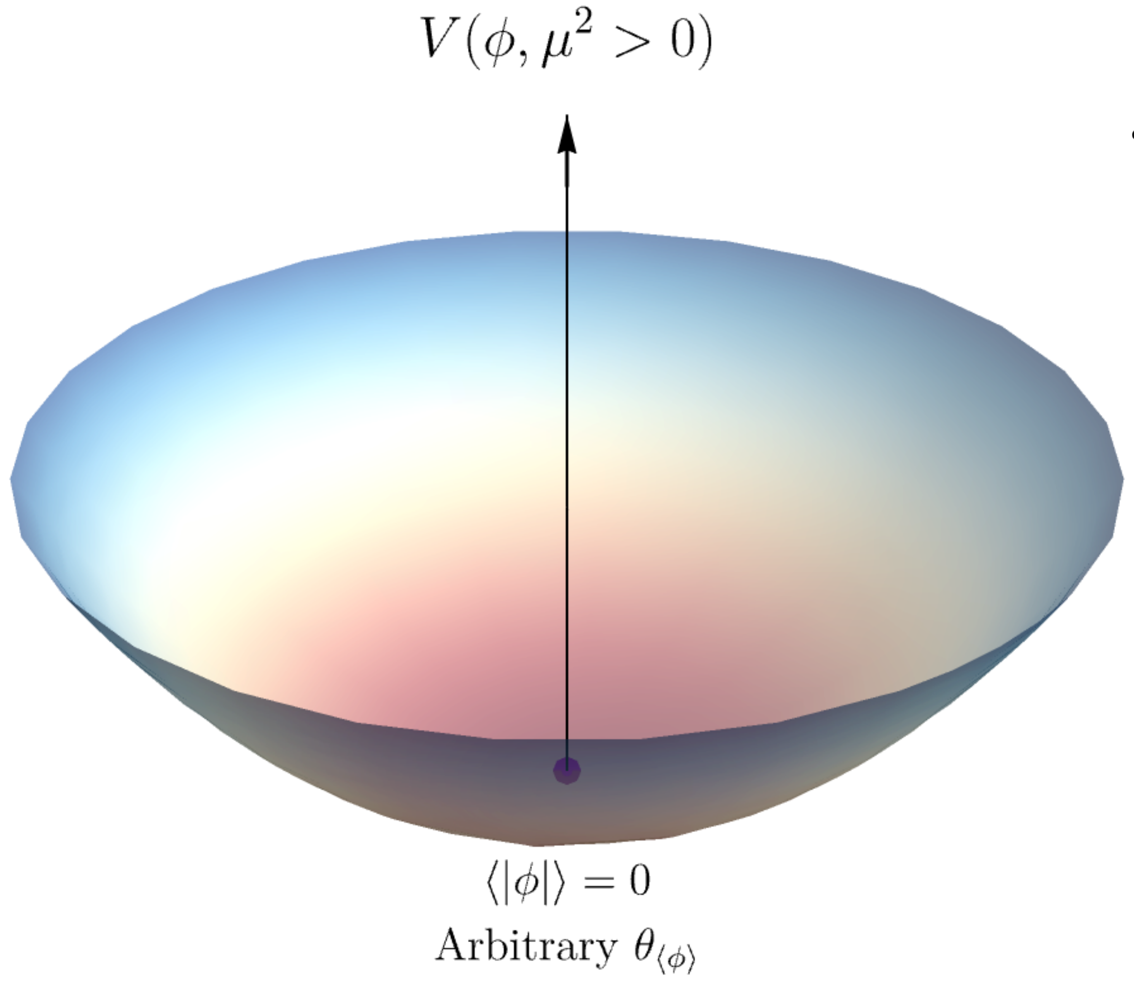
SRE (magic) minimization
in Møller scattering

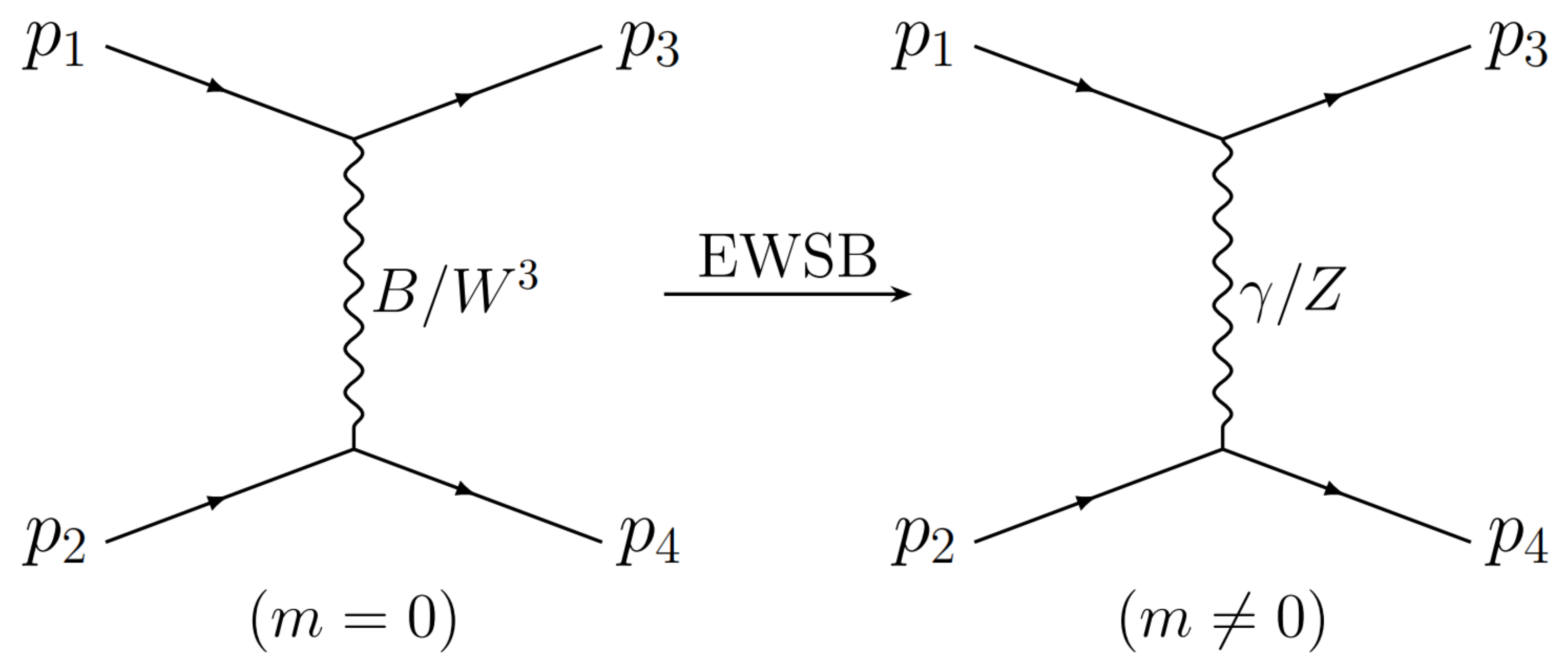
- Two independent QI probes, different bases, different initial states
- Different physical quantities (entanglement vs. magic)

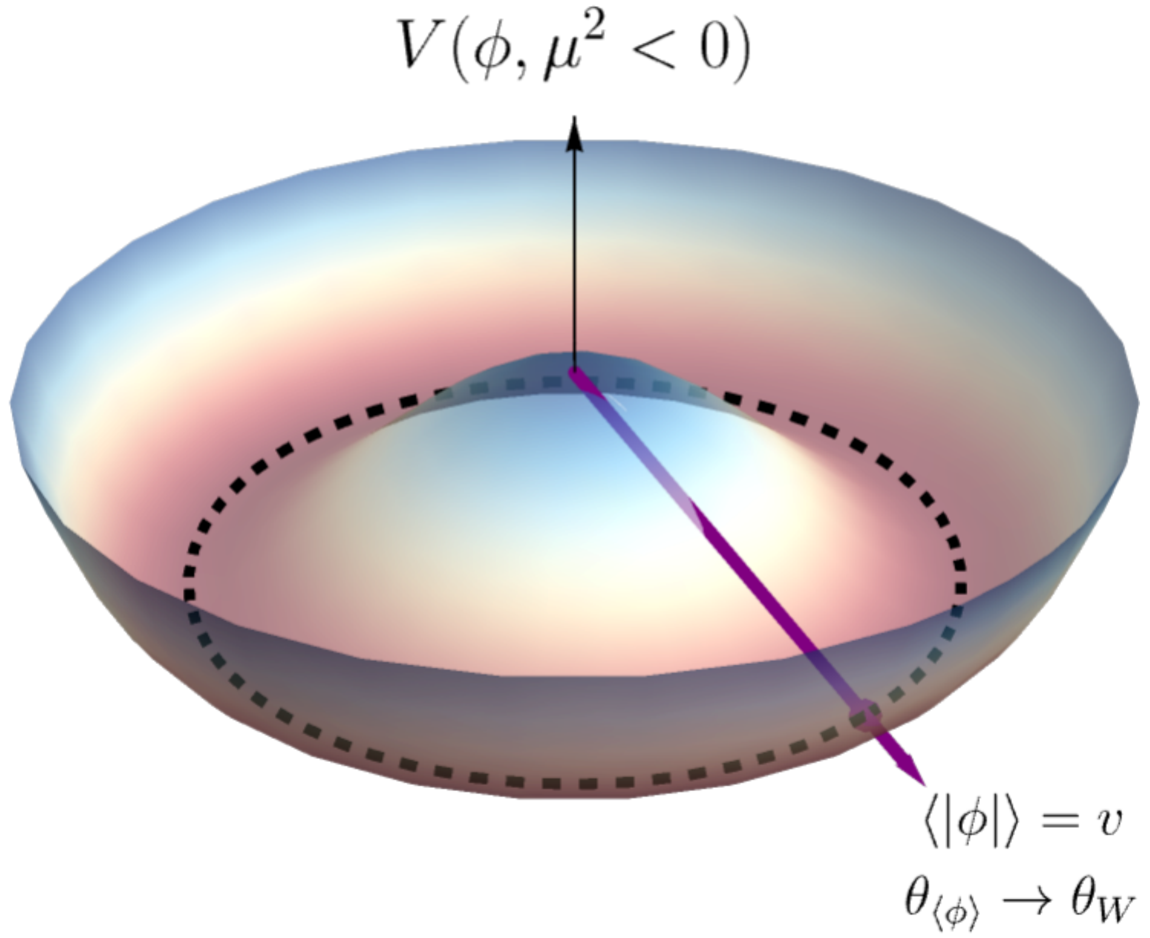
Why do they point to the same number?

Revisit Electroweak Symmetry Breaking From EE

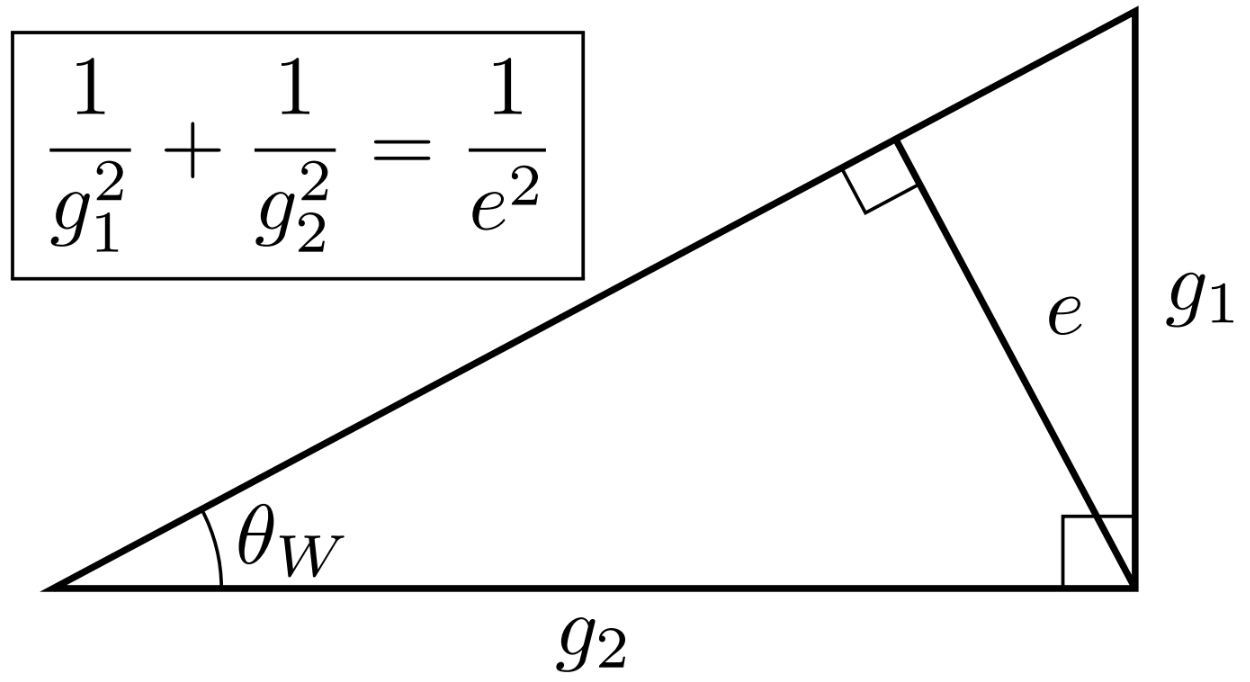
$$\underbrace{SU(2)_L}_{g_2} \times \underbrace{U(1)_Y}_{g_1} \xrightarrow{\text{EWSB}} \underbrace{U(1)_{\text{em}}}_{e}$$







$$\boxed{\frac{1}{g_1^2} + \frac{1}{g_2^2} = \frac{1}{e^2}}$$



- Weak mixing angle s_W^2 determines EWSB direction
- Scattering across EWSB depends on s_W^2

How does Entanglement vary across EWSB ?

Rényi Mutual Information (RMI) as a probe of EWSB

[Q.-H Cao et al., arXiv:2605.22070]

Rényi Entropy of order α

$$S_\alpha(\rho) = (1-\alpha)^{-1} \ln \mathrm{Tr}(\rho^\alpha)$$

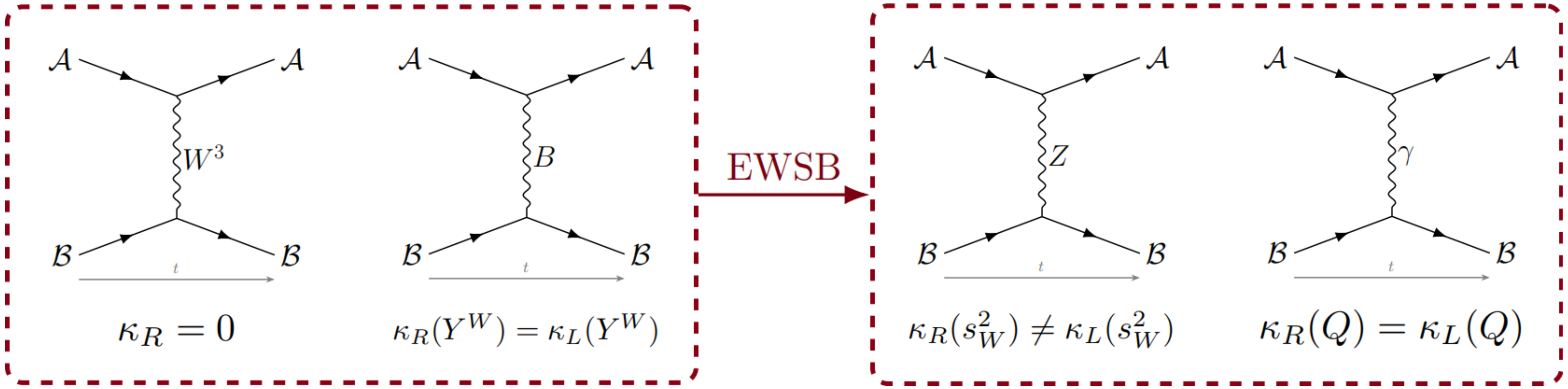
Bi-partition via Particles

$$\mathcal{H} = \mathcal{H}_\mathcal{A} \otimes \mathcal{H}_\mathcal{B}$$

Rényi Mutual Information

$$I_\alpha(\mathcal{A}:\mathcal{B}) = S_\alpha(\rho_\mathcal{A}) + S_\alpha(\rho_\mathcal{B}) - S_\alpha(\rho)$$

$\rho^{(\mathrm{S})}$ $(m_{W^3} = 0, m_B = 0, \boldsymbol{m_f = 0})$ $\rho^{(\mathrm{B})}$ $(m_Z \neq 0, m_A = 0, \boldsymbol{m_f \neq 0})$



- s_W^2 is encoded in the chiral couplings in $\bar{f}fV$
- RMI is extracted from $2 \to 2$ elastic scattering

$$\Delta_I(\rho^{(\mathrm{S})}, \rho^{(\mathrm{B})}) = \left| I^{(\mathrm{S})} - I^{(\mathrm{B})} \right| \xrightarrow{\sqrt{s}\to\infty} \left(\frac{m_f^2}{s}\right) \Delta_\mathcal{I}(\rho^{(\mathrm{S})}, \rho^{(\mathrm{B})})$$

Δ_I Capture entanglement variation across EWSB

Global Resources: Stabilizer Rényi Entropy (SRE)

Quantum resources of spin systems is characterized by **Non-Stabilizerness**

Clifford Group as normalizer of Pauli Group

$$\mathcal{C}_n := \{C : CPC^\dagger \in \mathcal{P}_n, \forall P \in \mathcal{P}_n\}$$

Stabilizer states can be classically simulated (Gottesman-Knill)

$$|\psi_{\mathrm{stab}}\rangle = C\,|0\rangle^{\otimes n}\Big|_{C \in \mathcal{C}_n}$$

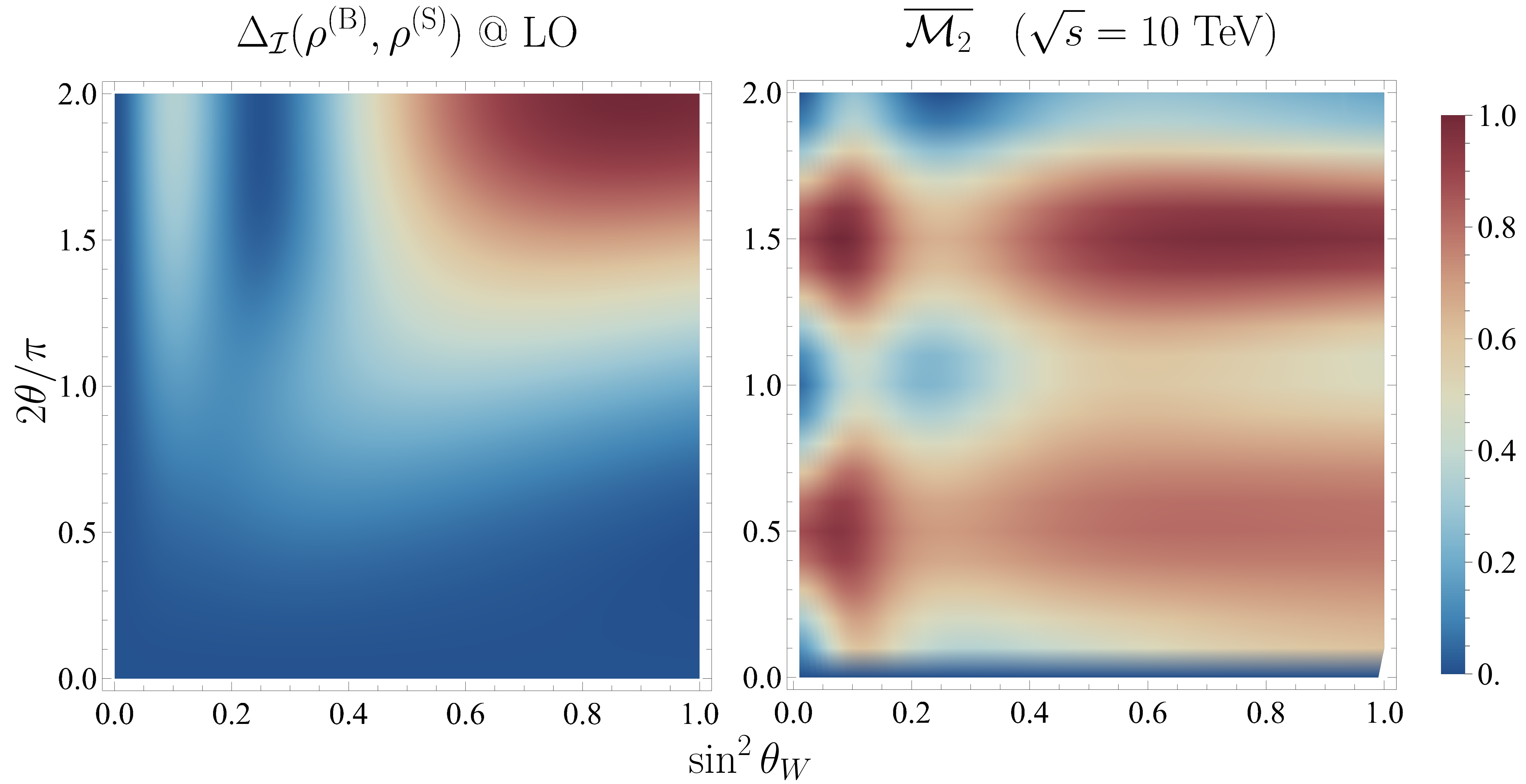
SRE: Rényi Entropy of normalized distribution $\Xi_P = d^{-1}|\langle\psi|P|\psi\rangle|^2$

$$\mathcal{M}_\alpha(|\psi\rangle) = (1-\alpha)^{-1}\log\sum_{P\in\mathcal{P}_n}\Xi_P^\alpha - \log d$$

Non-Stabilizerness of Unitary U (e.g., scattering amplitude)

$$|\psi\rangle = U\,|\psi_{\mathrm{stab}}\rangle \longrightarrow \overline{\mathcal{M}}(U) := \frac{1}{N_{\mathrm{stab}}}\sum_{|\psi_{\mathrm{stab}}\rangle}\mathcal{M}(U\,|\psi_{\mathrm{stab}}\rangle)$$

RMI vs SRE: Different Kinematics



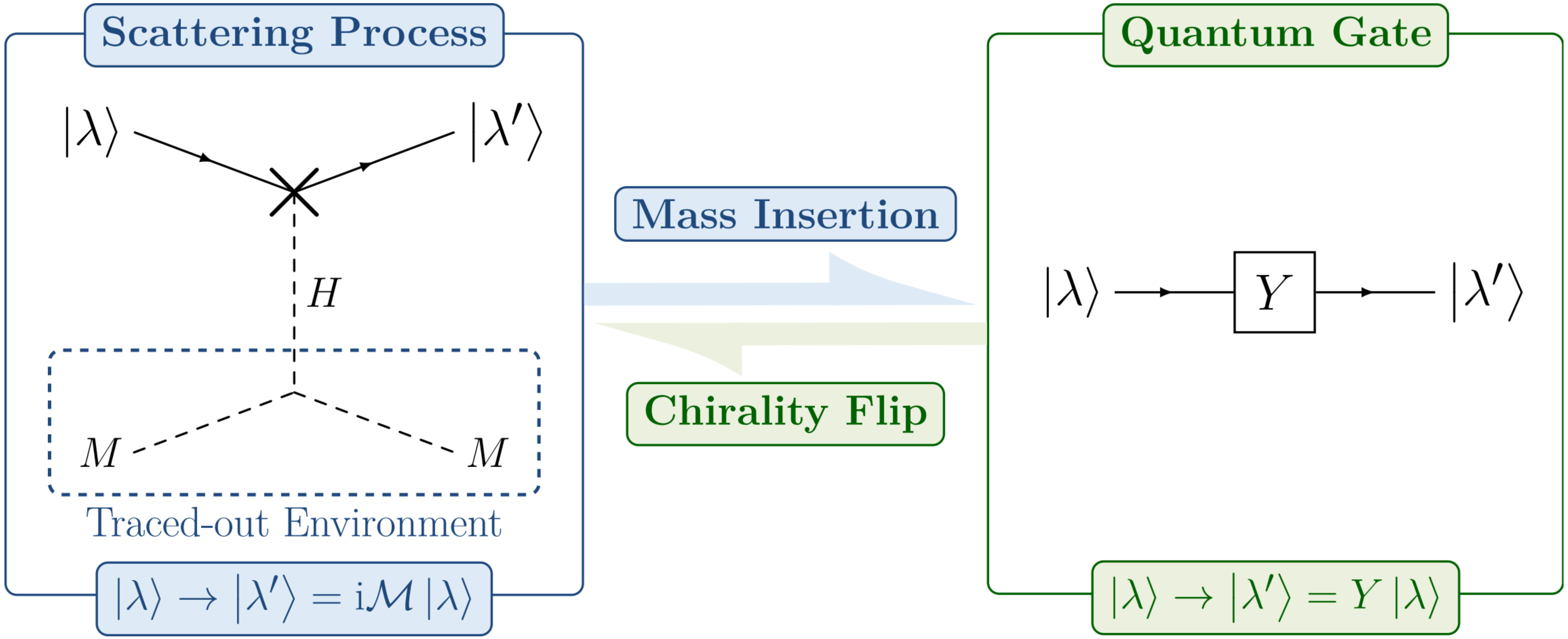
RMI (helicity basis)

- No preferred scattering angle
- Decoherence at $\theta \to 0$
- Initial: $\rho^{(\mathrm{in})} = \mathbf{1}/4$ (maximal mixture)
- Trace-type measure

SRE (fixed beam basis)

- Prefers $\theta = \pi/2$
- Similar to concurrence
- Initial: 60 stabilizer states
- Spherical 2-design

Mass Insertion as Chirality Flip: Gate Correspondence



$$\mathcal{M}_{\lambda'\lambda} \propto \begin{pmatrix} 0 & -\left(1-\frac{M^2}{s}\right)\sin\frac{\theta}{2} \\ \left(1-\frac{M^2}{s}\right)\sin\frac{\theta}{2} & 0 \end{pmatrix} \propto -\mathrm{i}Y$$

Yukawa Interaction $\left(\propto m_f\right)$ is equivalent to $-\mathrm{i}Y$ gate in Scattering

Dominate Gate in EWSB : Reduced SRE

SRE: Rényi Entropy of normalized distribution $\Xi_P = d^{-1}|\langle\psi|P|\psi\rangle|^2$

$$\mathcal{M}_\alpha(|\psi\rangle) = (1-\alpha)^{-1}\log\sum_{P\in\mathcal{P}_n}\Xi_P^\alpha - \log d$$

$\mathscr{P}_n$ is the **Pauli string** operator defined along fixed beam basis

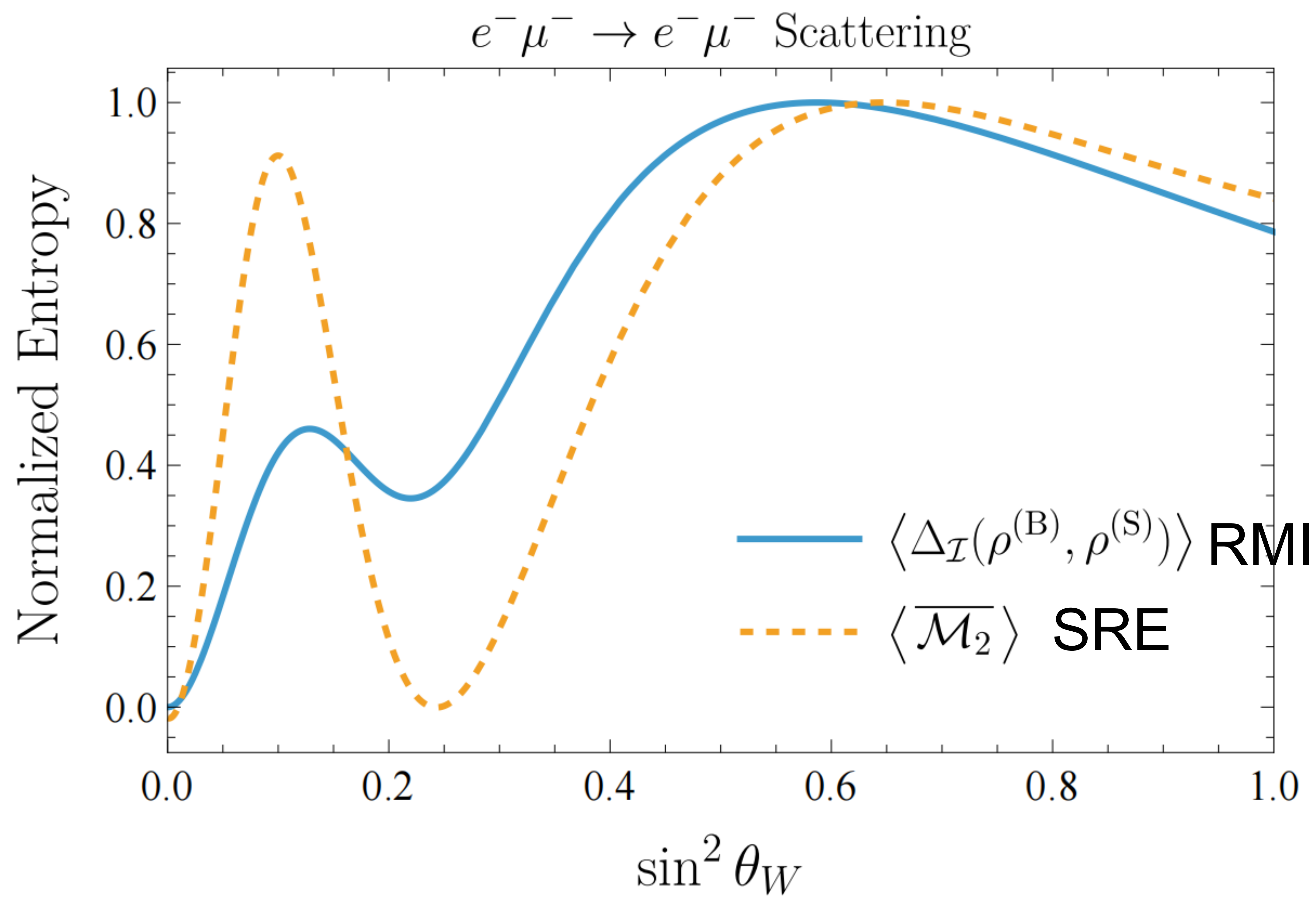
$$\mathscr{P}_n = \bigotimes_i P_i, P_i \in \{I, X, Y, Z\}$$

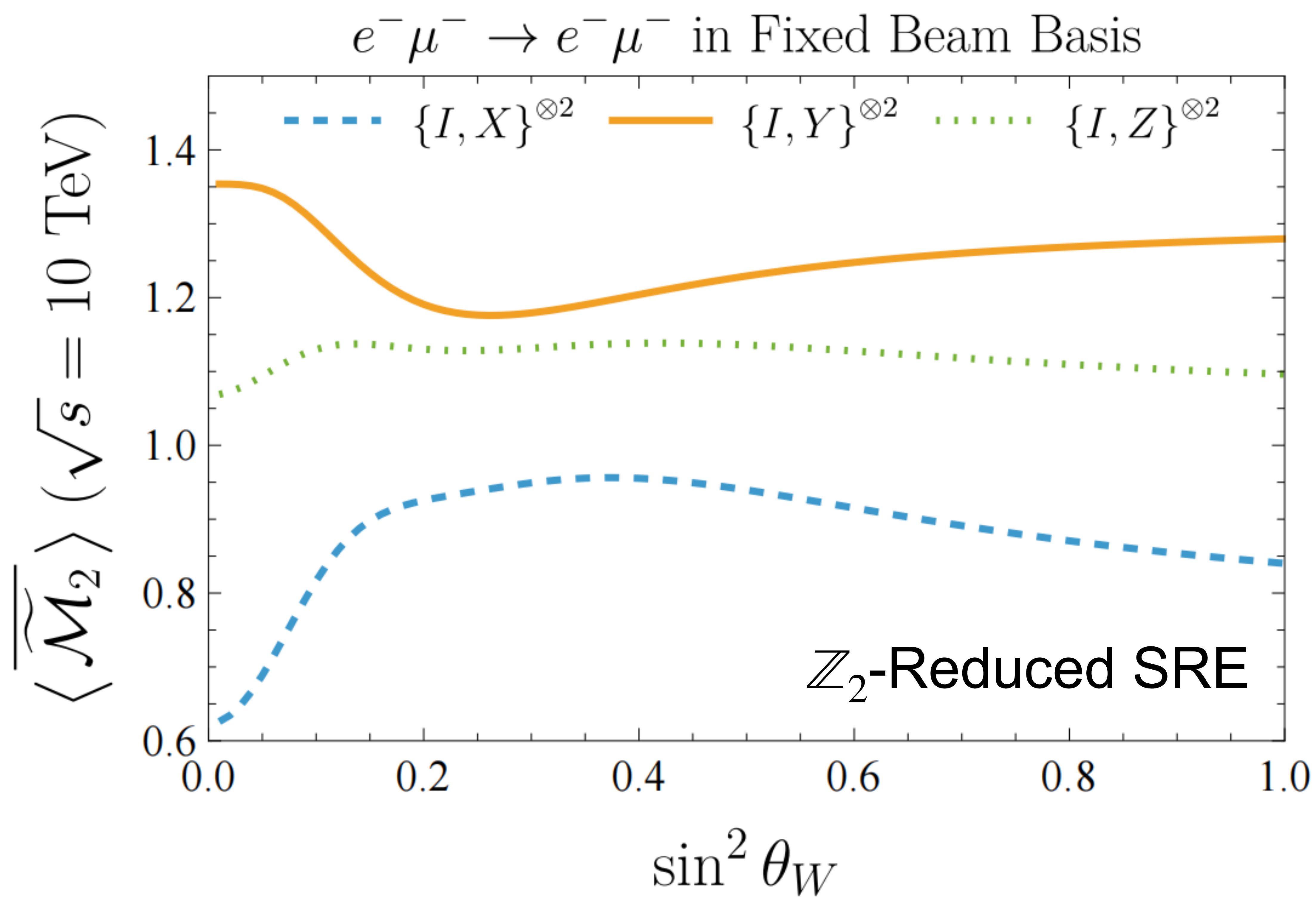
To verify which gate is dominant, consider a **sub-string** $\widetilde{\mathscr{P}}_n$ of $\mathscr{P}_n$

$$\widetilde{\mathscr{P}}_n = \bigotimes_i \widetilde{P}_i, \widetilde{P}_i \in \{I, X\}, \{I, Y\}, \{I, Z\}$$

SRE based on $\widetilde{\mathscr{P}}_n$ is referred to the $\mathbb{Z}_2$-Reduced SRE

Global Agrees Local: EWSB as Quantum Gate

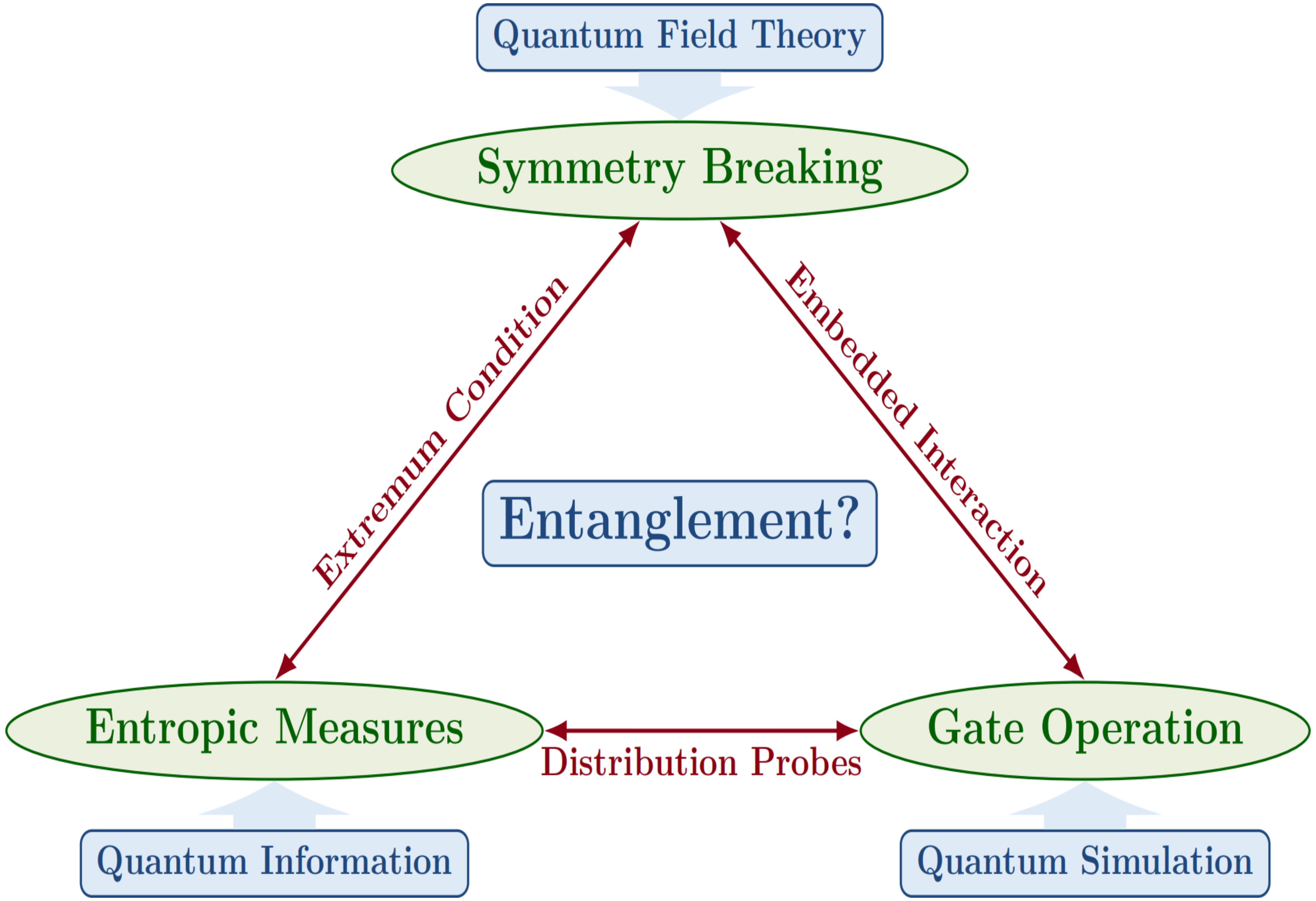




- **Consistency on s_W^2** for RMI (**local** correlation) & SRE (**global** resource)
- Physical origin of this consistency is the ***Y* quantum gate** realized by

Mass Insertion (RMI) ⟷ Measurement Subset (SRE)

Triad Correspondence in QFT & QI & QS



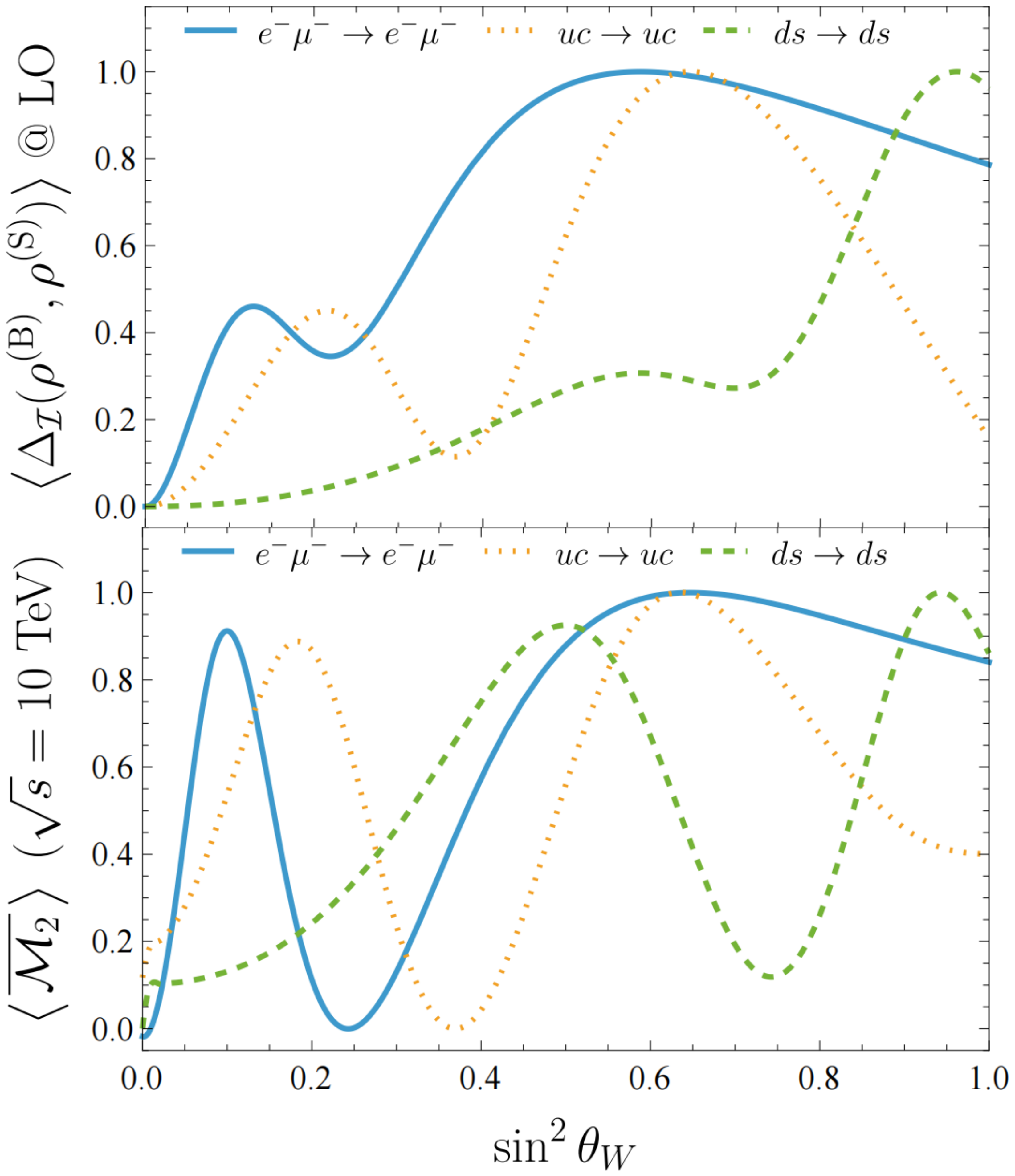
- Parameter controls symmetry breaking, selected by extremum condition of entanglement entropy
- Entanglement entropy is a probe to statistical distribution of a given measurement set realized by gate
- Direct correspondence between gate operation and symmetry breaking

Further discussion: Is extremum point universal to all scattering ?

Universality in Extremal Entanglement ?

NO ! Entropy extremum is sensitive to fermion flavors

- Consider different fermion species carrying electroweak charges $\left(Y^W, T^{W,3}\right)$



- Entropy extremum only diagnose **axial coupling** $\kappa_L^{(Z)} = -\kappa_R^{(Z)}$

	$e^-\mu^- \to e^-\mu^-$	$uc \to uc$	$ds \to ds$
RMI	0.23	0.37	0.72
SRE	0.24	0.37	0.74

- Nevertheless, consistency between RMI and SRE still holds
- If universal, origin of leptons must be different from quarks.

Universal Constraints From Self-Consistency

[Q.-H Cao et al., in preparation]

- Unsatisfied with the extremal point sensitive to manual choice
- **Absolute bound** of entanglement profiles gives common parameter constraints **by definition**, arising opportunity to examine **the self-consistency** of theory
- Typical self-consistent condition is the **unitarity** (perturbativity)

[B. Lee et al., Phys. Rev. D 16, 1519 (1977)]

Weak interactions at very high energies: The role of the Higgs-boson mass

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(Received 20 April 1977)

We give an S-matrix-theoretic demonstration that if the Higgs-boson mass exceeds $M_c = (8\pi\sqrt{2}/3G_F)^{1/2}$, parital-wave unitarity is not respected by the tree diagrams for two-body scattering of gauge bosons, and the weak interactions must become strong at high energies. We exhibit the relation of this bound to the structure of the Higgs-Goldstone Lagrangian, and speculate on the consequences of strongly coupled Higgs-Goldstone systems. Prospects for the observation of massive Higgs scalars are noted.

Tree level unitarity bound on the Higgs boson

$m_h \leq 1.2$ TeV

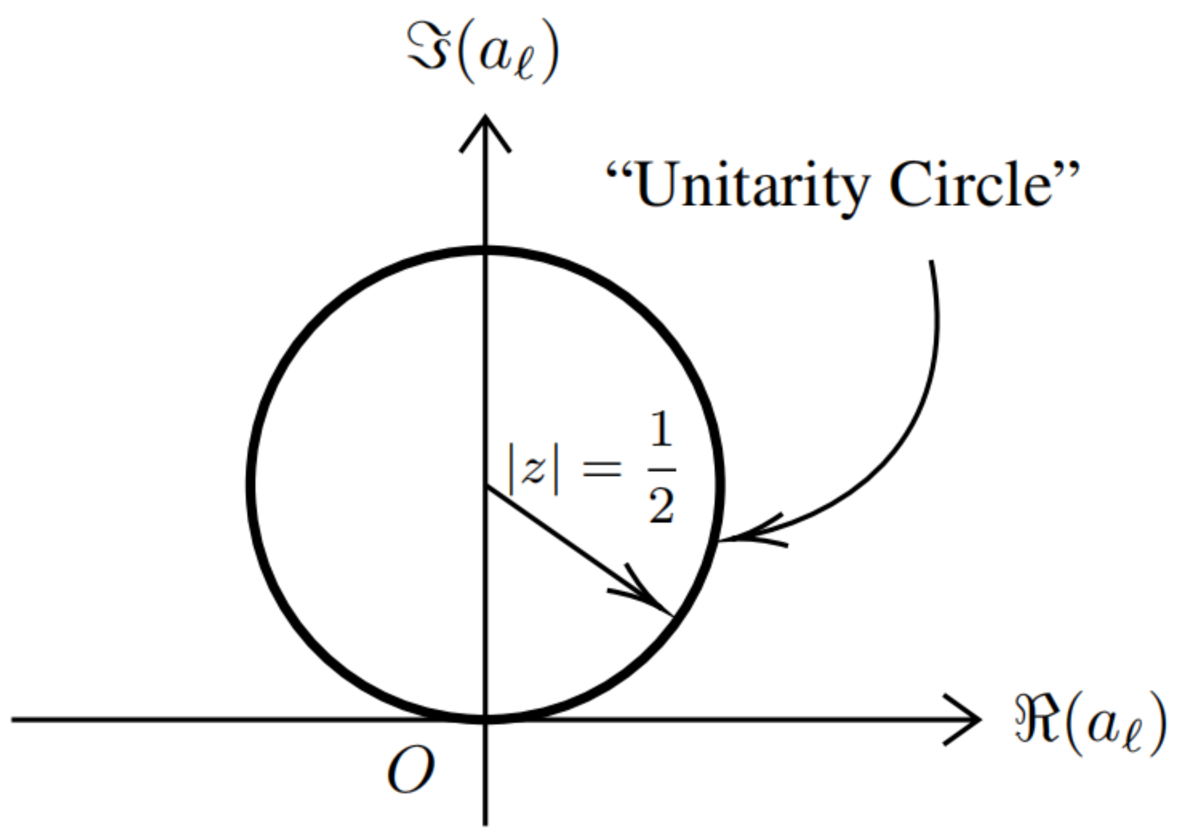
Perturbative Unitarity From Entropic Constraints

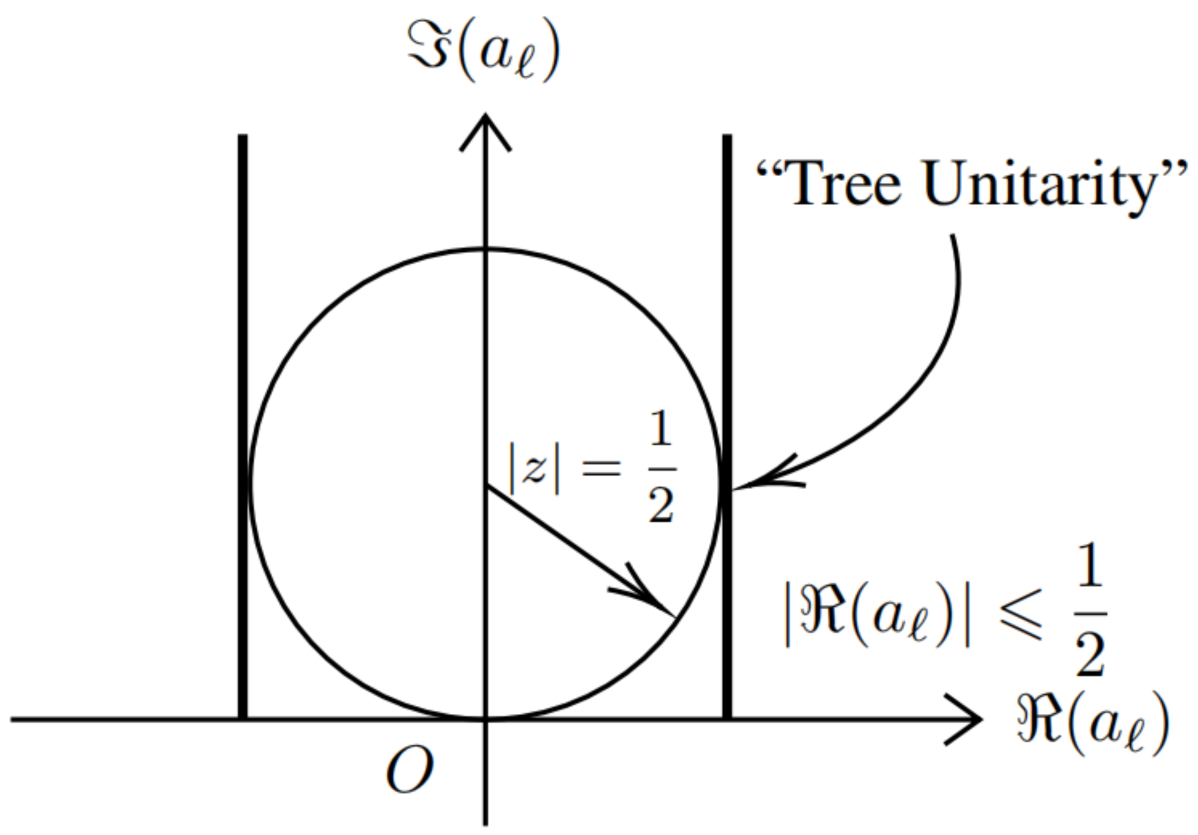
[R. Aoude et al., arXiv:2402.16956]
[C. Pueyo et al., arXiv:2410.23709]

- Purity (Linear entropy) directly related to unitarity

$$\gamma(\rho) := \mathrm{Tr}\left(\rho^2\right) = \sum_i p_i^2 \leqslant \left(\sum_i p_i\right)^2 = 1 \qquad \rho = \sum_i p_i \left|i\right\rangle \left\langle i\right|$$

- Perturbative unitarity of partial-wave amplitude





$$\mathrm{Im}\{a_l\} = |a_l|^2 \xrightarrow{\text{Tree Unitarity}} |\mathrm{Re}\{a_l\}| \leqslant 1/2$$

- **Pseudo-purity Bound on unitarity**

$$\gamma(\tilde{\sigma}), \tilde{\sigma} = T_2 \rho^{(\mathrm{in})} T_2^\dagger$$

T_2 is the **truncated transition matrix** for $2 \to 2$ scattering and of no forward region ($\theta \neq 0$)

Unitarity $\gamma(\tilde{\sigma}) \leqslant 16$

Perturbativity $\gamma(\tilde{\sigma}) \leqslant 1$

Sufficiency on Pseudo-Purity Bound

- Cauchy inequality for Hermitian matrices $T_2^\dagger T_2$ $\qquad \gamma(\tilde{\sigma}) \leqslant \left[\mathrm{Tr}\left(T_2\rho^{(\mathrm{in})}T_2^\dagger\right)\right]^2$

- Sufficiency on two body phase space truncation $\quad S = \mathbf{1} + \mathrm{i}T, T_2 = P_2 T P_2$

$$\mathcal{H} = \bigoplus_n \mathcal{H}_n \quad \text{Orthogonal Projector } P_n = P_n^2 = P_n^\dagger, \sum_n P_n = \mathbf{1}$$

Positive semi-definite $\rho^{(\mathrm{in})} \in \mathscr{H}_2 \longrightarrow \left[\mathrm{Tr}\left(T_2\rho^{(\mathrm{in})}T_2^\dagger\right)\right]^2 \leqslant \left[\mathrm{Tr}\left(T\rho^{(\mathrm{in})}T^\dagger\right)\right]^2$

- **Unitarity $SS^\dagger = S^\dagger S = 1$**

$$\mathrm{Tr}\left(T\rho^{(\mathrm{in})}T^\dagger\right) \leqslant 4$$

$$\downarrow$$

$$\gamma(\tilde{\sigma}) \leqslant 16$$

- **Perturbativity $|T| \lesssim |S|$**

$$\mathrm{Tr}\left(T\rho^{(\mathrm{in})}T^\dagger\right) \lesssim \mathrm{Tr}\left(S\rho^{(\mathrm{in})}S^\dagger\right) \equiv 1$$

$$\downarrow$$

$$\gamma(\tilde{\sigma}) \leqslant 1$$

Saturation on Pseudo-Purity Bound

- Flexible choices on **initial state** lead to optimization

- Hermicity of $T_2^\dagger T_2$ allows diagonalization $\quad 0 \leqslant \alpha_i, p_i \leqslant \alpha_j, p_j \ (i \leqslant j)$

$$(T_2^\dagger T_2) = \mathrm{diag}\{\alpha_1, \ldots, \alpha_d\} \quad \rho^{(\mathrm{in})} = U.\mathrm{diag}\{p_1, \ldots, p_d\}.U^\dagger = UQU^\dagger$$

- Hermicity of $X = U^\dagger T_2^\dagger T_2 U$ allows following inequality

$$\tilde{\gamma}(\tilde{\sigma}) = \mathrm{Tr}\big[(XQ)^2\big] \leqslant \mathrm{Tr}\big(X^2 Q^2\big) \leqslant \sum_{i=1}^{d} \alpha_i^2 p_i^2$$

- Equality can be satisfied by setting $\quad U = \mathbf{1}, \ Q = \mathrm{diag}\{1, 0, \ldots, 0\}$

Partial-wave Discretization For Phase Space

- To relate transition matrix T_2 with partial wave amplitude a^J
- To “trace out” continuous momentum from flavor degree of freedom

We choose the angular momentum eigenstate initially, labeled by (J, M)

$$|\boldsymbol{p}, i; \boldsymbol{q}, j\rangle = \frac{1}{\sqrt{1+\delta_{ij}}} b^\dagger_{\boldsymbol{p},i} b^\dagger_{\boldsymbol{q},j} |0\rangle$$

$$|J, M; i, j\rangle = \int \mathrm{d}\Omega_{\boldsymbol{p}} Y_J^{M*}(\Omega_{\boldsymbol{p}}) |\boldsymbol{p}, i; -\boldsymbol{p}, j\rangle$$

Integrate the phase space, we derive

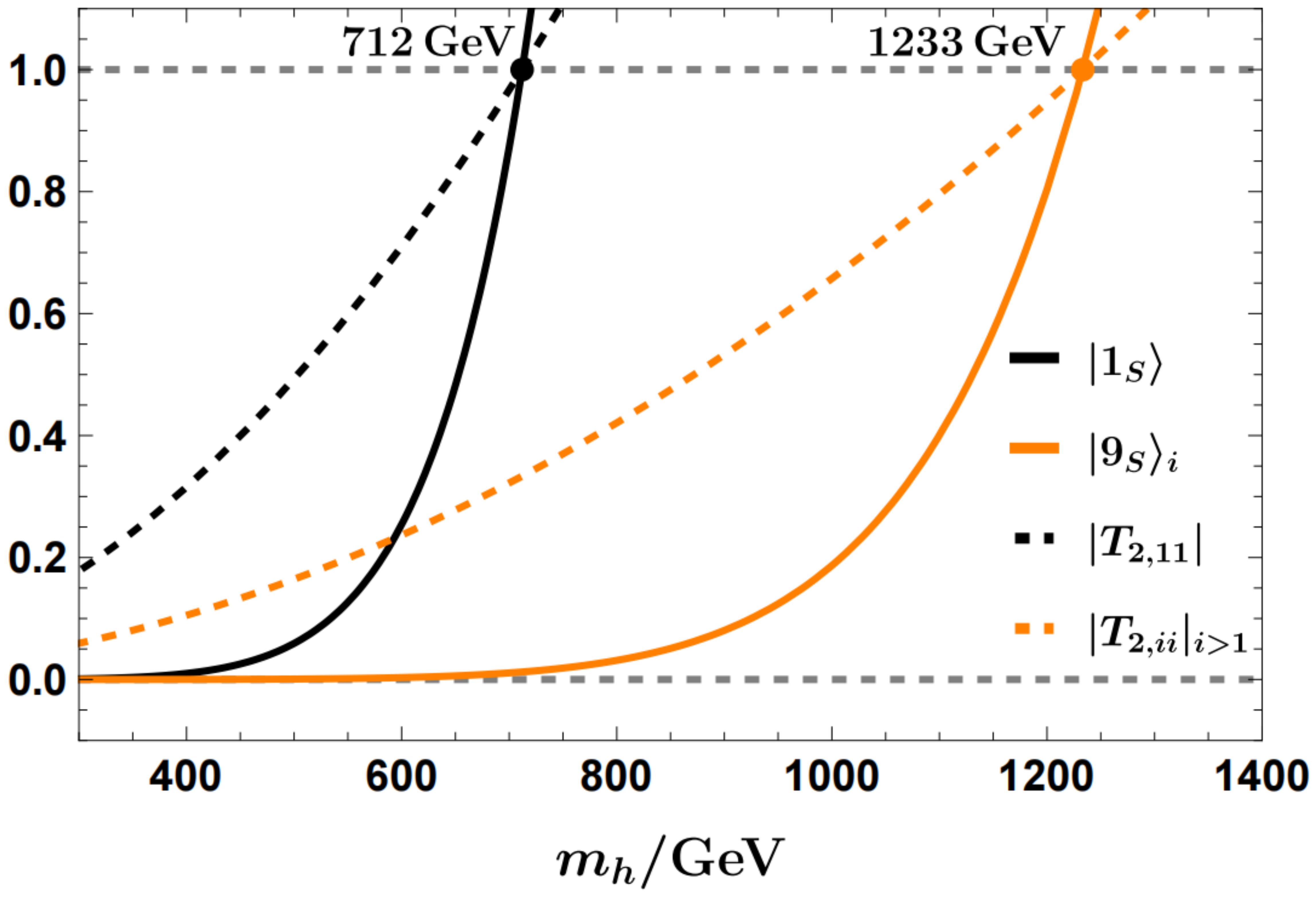
$$T_2 |J, M; i, j\rangle = \sum_{k\leqslant l} \frac{2a^J_{ij;kl}(s)}{\sqrt{(1+\delta_{ij})(1+\delta_{kl})}} |J, M; k, l\rangle$$

Application: Recover Lee-Quigg-Thacker Bound

[Q.-H Cao et al., in preparation]

Higgs boson + 3 Goldstones in asymptotic global $O(4)$ flavor symmetry

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \text{ with } v = \sqrt{\mu^2/\lambda} = 246 \text{ GeV fixed}$$

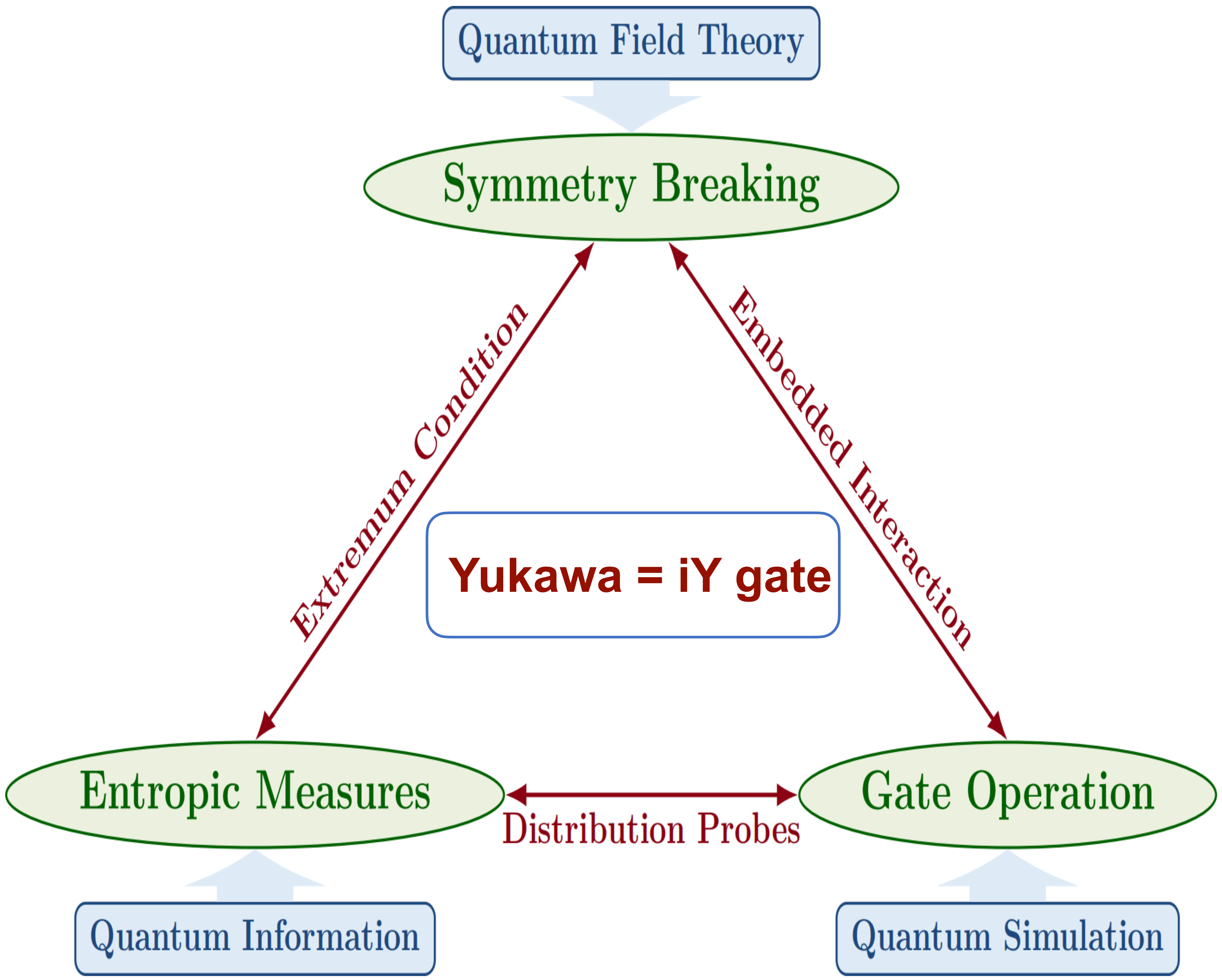


$$\mathbf{4} \otimes \mathbf{4} = \mathbf{9}_S \oplus \mathbf{6}_A \oplus \mathbf{1}_S$$

$$T_2 = -\frac{m_h^2}{8\pi v^2}\left(3\mathbb{1}_{1\times1} \oplus \mathbb{1}_{9\times9}\right)$$

- $\boldsymbol{\gamma \leqslant 1}$ **reproduce LQT Bound for** $\mathbf{Re}\{\boldsymbol{a^J}\} \leqslant \mathbf{1/2}$ **in tree level**
- Can $\gamma \leqslant 1$ reproduce Perturbativity with quantum correction ?

Take Home Message



- Local correlation and global resource coincide in the EW symmetry breaking, originated from the Y quantum gate.

- Entropy extremum is NOT universal.

- Absolute bounds on purity can be served as probe of unitarity violation

Thanks !